In-situ Backpropagation in Photonic Neural Networks

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Abstract: Recently, integrated optics has gained interest as a hardware platform for implementing machine learning algorithms. Here, we introduce a method that enables highly efficient, in situ training of a photonic artificial neural network. We use the adjoint variable method to derive the photonic analogue of the backpropagation algorithm, which is the standard method for computing gradients for conventional neural networks. We further show how these gradients can be obtained exactly through intensity measurements inside the device. Beyond the training of photonic machine learning implementations, our method may also be of broad interest to experimental sensitivity analysis of photonic systems and the optimization of reconfigurable optics platforms.
OCIS codes: (130.3120) Integrated optics devices; (200.4260) Neural networks;

Artificial neural networks (ANNs) are now used for an impressively large number of applications. This has brought them in the focus of research not only of computer science, but also of engineering, and hardware specifically suited to perform neural network operations is actively developed. Photonic implementations of such a hardware are particularly interesting since, due to the non-interacting nature of photons, linear operations - like the repeated matrix multiplications found in every neural network algorithm - can be performed in parallel, and at a low energy cost. However, a key requirement for the utility of any ANN platform is the ability to train the network using algorithms such as error backpropagation. Here, we propose a procedure in which this can be done for a chip-integrated photonic ANN through the use of only *in situ* intensity measurements. Our procedure works by *physically* implementing the adjoint variable method, which has previously been implemented computationally in the optimization and inverse design of photonic structures [1]. The method scales in constant time with respect to the number of control parameters, and is thus highly efficient. Although we focus our discussion on one particular recently proposed hardware implementation [2], our conclusions are derived starting from Maxwell's equations, and the ideas could therefore extend to other photonic neural network platforms, as well as to other applications.

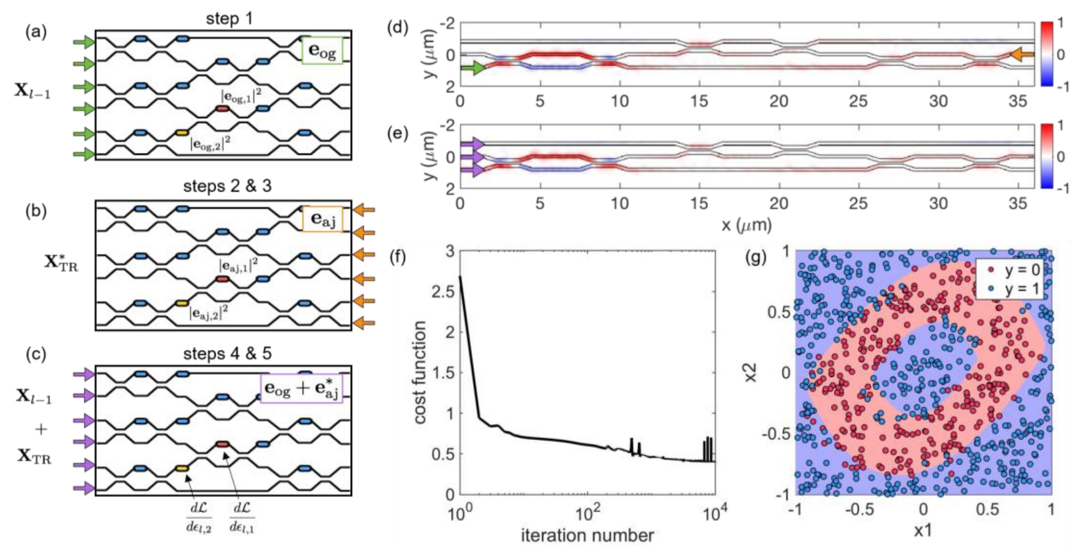

Fig. 1. (a)-(c): Schematic illustration of the proposed method for experimental measurement of gradient information within an optical interference unit; (a): forward pass; (b): backpropagation of the error signal; (c): interference measurement to recover the gradient of the objective function with respect to all degrees of freedom simultaneously. (d)-(e): Using a finite-difference frequency-domain simulation of a 3x3 OIU, the gradient $\partial\mathcal{L}/\partial\epsilon$ normalized by its maximum value as obtained (d): directly by the adjoint method; (e): by the method introduced in this work. (f): The cost function for the training of a numerically simulated photonic neural network. (g): The training examples, blue and red dots correspond to y = 0 and y = 1 labels on a given x1 and x2 input. The background shows the prediction of the network over the whole domain.

We first summarize briefly forward propagation and backpropagation in a generic ANN. The forward pass starts with a vector $\boldsymbol{X}_0$ and repeatedly applies a multiplication by a matrix W_l and an element-wise activation function f_l at each network layer l. The final output $\boldsymbol{X}_L$ is then used to compute a cost function $\mathcal{L}$, which typically defines how closely

$\boldsymbol{X}_L$ matches a target output associated to the input $\boldsymbol{X}_0$. In the backpropagation step, the derivative of $\mathcal{L}$ with respect to the controllable degrees of freedom in each layer can be computed using an error signal, which starts with the derivative of $\mathcal{L}$ with respect to the final output of the forward pass, and then, at layer l, is multiplied element-wise by the derivative of f_l, and then by the transposed matrix W_l^T. In Ref. [2], the use of an Optical Interference Unit (OIU) consisting of a mesh of reconfigurable Mach-Zehnder interferometers (Fig. 1(a)) to implement the matrices W_l was proposed and experimentally demonstrated. However, the training of the phase-shifter settings for this system was performed using a model implemented on a standard computer, which does not take into account experimental errors, and furthermore loses all the potential advantages in time and energy of the photonic implementation. Alternatively, training using a brute-force in-situ computation of the gradient was also proposed, but this strategy is highly inefficient for a large number of tunable parameters.

In our work [3], we show how the gradient computation can be performed efficiently in the hybrid opto-electronic network of Ref. [2]. The only additional component that is required is a means to measure the light intensity in the vicinity of each of the tunable phase shifters. This can be done for example by recording the unavoidable light scattering outside of the waveguides, or by integrated detectors as proposed in [4]. Starting from Maxwell's equations at steady state, and using the adjoint variable method [1], we can derive the gradient of the cost function with respect to the permittivity ϵ_l of a given phase shifter in layer l:

$$\frac{d\mathcal{L}}{d\epsilon_l} = k_0^2 \mathcal{R}\left\{\sum_{\mathbf{r}\in\mathbf{r}_\phi} \mathbf{e}_{\mathrm{aj}}(\mathbf{r})\mathbf{e}_{\mathrm{og}}(\mathbf{r})\right\}, \quad (1)$$

where $\mathcal{R}$ denotes the real part, the summation is over positions $\mathbf{r}_\phi$ inside the phase shifter, $\mathbf{e}_{og}$ is the electric field distribution given a source $\mathbf{X}_{l-1}$ on the left, and $\mathbf{e}_{aj}$ is the electric field given the backpropagation error source $\boldsymbol{\delta}_l$ on the right. The crucial insight then is that this sensitivity is proportional to the intensity pattern produced when interfering the electric field of the forward signal with a time-reversed copy of the error signal. Thus, the sensitivity of the cost function with respect to *every* phase shifter in layer l can be computed in parallel using the following procedure (Fig. 1(a)-(c)): (1) Send in the original field amplitudes $\mathbf{X}_{l-1}$ and measure the intensities at each phase shifter. (2) Send $\boldsymbol{\delta}_l$ into the output ports on the right and measure the intensities at each phase shifter. (3) Record the output on the left, and compute $\mathbf{X}_{\mathrm{TR}}$ as the complex conjugate of that output. (4) Interfere the original and the time-reversed adjoint fields in the device, measuring again the resulting intensities at each phase shifter. (5) Subtract the constant intensity terms from steps 1 and 2 and multiply by k_0^2 to recover the gradient as in eq. (1).

This procedure for the gradient computation is exact if we assume a lossless, reciprocal, feed-forward propagation inside the OIU, which is to a good approximation true [2, 4]. However, we have also checked numerically that it also works to a good precision in a structure with non-negligible, mode-dependent losses, as shown in Fig. 1(d)-(e). We used a finite-difference frequency-domain method to simulate the gradient computation for a cost function that maximizes the transmission from the bottom port on the left to the middle port on the right for a 3x3 OIU. Despite the fact that about 40% of the light is lost due to back-scattering and radiation losses, the exact computation (d) and the computation using our intensity measurement procedure (e) match to a very good approximation. Finally, we also present a numerical simulation of a photonic ANN optimized using the presented method. Specifically, we generate a set of one thousand training examples represented by input and target $\mathbf{X}_0 \rightarrow \mathbf{T}$ pairs, assigning one of two classes to a two-variable input x_1 and x_2. The class labels are generated from an underlying distribution that resembles an oblong disk, with some additional random noise (Fig. 1(g)). Using a six-layer simulated photonic ANN and gradient descent minimization with the gradients computed as outlined above, the cost function was successfully minimized (Fig. 1(f)) and the resulting ANN prediction matches the training and the test sets with 91% precision (Fig. 1(g)).

In conclusion, we present an efficient approach to compute the sensitivity of an optical interference unit with respect to an arbitrary objective function and with respect to all control parameters in parallel. This paradigm is particularly attractive as a means for in situ training of photonic neural networks as implemented in Ref. [2], but can also have further-reaching applications, for example for the setup and tuning of reconfigurable quantum optical circuits [5].

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