Opdracht 1

1. Values outside the image interpreted as 0. The result is:
   $(f * g) = \{\ 0\ 0\ 0\ 1\ \underline{3}\ 4\ 4\ 4\ 3\ \}$
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   $(f * g) =\{\ 0\ 0\ 0\ 1\ \underline{3}\ 4\ 4\ 4\ 3\ \}$
3. Values outside the image interpreted as 0. $(f * g) =\{\ 0\ 0\ 0\ 1\ \underline{0}\ 0\ 0\ 0\ \}$
4. the proof of commutativity of convolution:
   $(f * g)(x) = \sum f(y)g(x - y)$

   $Y = x - y$
   $y = x - Y$
   $y = \infty \rightarrow Y = -\infty$
   $y = -\infty \rightarrow Y = \infty$

   $(f * g)(x) = \sum f(x - Y)g(Y)$
   $(f * g)(x) = \sum g(Y)f(x - Y) = (g * f)(x)$ QED.
5. the proof of associativity of convolution:
   $((f * g) * h)(u) = \int (f * g)(x)h(u - x)dx$
   $((f * g) * h)(u) = \int [\int f(y)g(x - y)dx]h(u - x)dx$
   $((f * g) * h)(u) = \int\int f(y)g(x - y)h(u - x)dydx.$

   $((f * g) * h)(u) = \int\int f(y)g(x - y)h(u - x)dxdy$
   $((f * g) * h)(u) = \int f(y)[\int g(x - y)h(u - x)dx]dy$

   $\int g(x - y)h(u - x)dx = \int g((x + y) - y)h(u - (x + y))dx$
   $\int g(x - y)h(u - x)dx = \int g(x)h((u - y) - x)dx$
   $\int g(x - y)h(u - x)dx = (g * h) * (u - y)$

   $((f * g) * h)(u) = \int f(y)(g * h)(u - y)dy$
6. The identity kenel: $\begin{Bmatrix} 0 & 0 & 0 \\ 0 & \underline{1} & 0 \\ 0 & 0 & 0 \end{Bmatrix}$
7. Multiply the image intesity by 3: $3* \begin{Bmatrix} 0 & 0 & 0 \\ 0 & \underline{1} & 0 \\ 0 & 0 & 0 \end{Bmatrix}$
8. Translating the vector $[-3, 1]^T$ in a kernel: $\begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{0} \end{Bmatrix}$
9. This operation depends on the parameter ϕ, so the operation is not always the same. Convolution only is possible for local linear operations, and this is not one of those local linear operations. Rotation is not possible with a kernel.

10. The average kernel: $1/9 * \begin{Bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{Bmatrix}$

11. To compute the median we need the values in the neighbourhood to be sorted, this is not usual. So getting the median through a kernel is not possible.

12. Comparison between values in the neighbourhood is not possible when we are working with kernels. It is not possible through convolution.

13. Motion blur: $\begin{Bmatrix} 0.2 & 0.2 & \underline{0.2} & 0.2 & 0.2 \end{Bmatrix}$

14. kernel of optical blur: σ is 3. $g(\vec{x}) = \frac{1}{2\pi 9} e^{-\frac{||\vec{x}||^2}{18}}$

15. unsharp masking: $\begin{Bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{Bmatrix}$

16. Kernel $= \frac{1}{2}\{-1\underline{0}1\}$

17. Kernel $= \frac{1}{4}\{10\underline{-2}01\}$

18. Zooming is not possible, because zooming and interpolation depends on the point. It is different for every point in the image, and thus not possible by convolution.

19. the use of conditional statements is not in the scope of convolution kernels. This will not be possible.

20. We can differentiate the Gaussian infinite times because is has an exponent (e). Because of eulers number, the gaussian had smooth derivatives of arbitrary order.

21. It is a linear operator, and invariant, so it can be a convolution. It should be, because it is way easier this way.

22. $\partial(f * G_\sigma) = D * (f * G_\sigma) = f * (D * G_\sigma) = f * (\partial G_\sigma)$, the derivative of an image at scale σ can be computed by convolution with the derivative of a Gaussian kernel of that scale, all because of the associative and commutative properties.

23. Derivatives with e are easy, just use the chain rule. $G_\sigma(x) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$
$\frac{\partial G_\sigma(x)}{\partial x} = -\frac{4\sigma^2 x}{4\sigma^4} G_\sigma(x) = -\frac{x}{\sigma^2} G_\sigma(x)$

24. Use chainrule again on $\frac{\partial G_\sigma(x)}{\partial x}$.
$\frac{\partial^2 G_\sigma(x)}{\partial x^2} = \partial_x(-x/\sigma^2) G_\sigma + (-x/\sigma^2) * \partial_x G_\sigma = (\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}) G_\sigma$