The Capital Asset Pricing Model

INTRODUCTION TO PORTFOLIO RISK MANAGEMENT IN PYTHON

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The founding father of asset pricing models

CAPM

The Capital Asset Pricing Model is the fundamental building block for many other asset pricing models and factor models in finance.

Excess returns

To calculate excess returns, simply subtract the risk free rate of return from your total return:

$$\text{Excess Return} = \text{Return} - \text{Risk Free Return}$$

Example:

Investing in Brazil:

10% Portfolio Return - 15% Risk Free Rate = -5% Excess Return

Investing in the US:

10% Portfolio Return - 3% Risk Free Rate = 7% Excess Return

The Capital Asset Pricing Model

$$E(R_P) - RF = \beta_P(E(R_M) - RF)$$

- $E(R_P) - RF$: The excess expected return of a stock or portfolio P
- $E(R_M) - RF$: The excess expected return of the broad market portfolio B
- RF: The regional risk free-rate
- β_P: Portfolio beta, or exposure, to the broad market portfolio B

Calculating Beta using co-variance

To calculate historical beta using co-variance:

$$\beta_P = \frac{Cov(R_P, R_B)}{Var(R_B)}$$

- β_P: Portfolio beta
- $Cov(R_P, R_B)$: The co-variance between the portfolio (P) and the benchmark market index (B)
- $Var(R_B)$: The variance of the benchmark market index

Calculating Beta using co-variance in Python

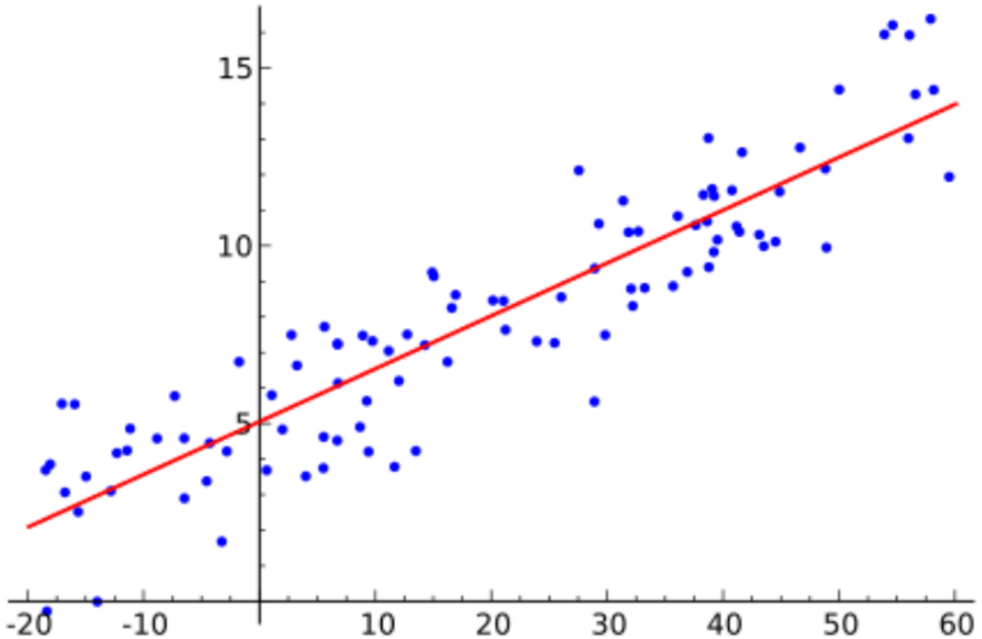
Assuming you already have excess portfolio and market returns in the object `Data` :

```
covariance_matrix = Data[["Port_Excess","Mkt_Excess"]].cov(
covariance_coefficient = covariance_matrix.iloc[0, 1]
benchmark_variance = Data["Mkt_Excess"].var()
portfolio_beta = covariance_coefficient / benchmark_varianc
portfolio_beta
```

```
0.93
```

Linear regressions

Example of a linear regression:



Regression formula in matrix notation:

$$\mathbf{y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix},$$

$$X = \begin{pmatrix} \mathbf{x}_1^{\mathsf{T}} \\ \mathbf{x}_2^{\mathsf{T}} \\ \vdots \\ \mathbf{x}_n^{\mathsf{T}} \end{pmatrix} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1p} \\ 1 & x_{21} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{np} \end{pmatrix},$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix}, \quad \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}.$$

Calculating Beta using linear regression

Assuming you already have excess portfolio and market returns in the object `Data` :

```
import statsmodels.formula.api as smf
model = smf.ols(formula='Port_Excess ~ Mkt_Excess', data=Data)
fit = model.fit()
beta = fit.params["Mkt_Excess"]
beta
```

```
0.93
```

R-Squared vs Adjusted R-Squared

To extract the adjusted r-squared and r-squared values:

```
import statsmodels.formula.api as smf
model = smf.ols(formula='Port_Excess ~ Mkt_Excess', data=Data)
fit = model.fit()
r_squared = fit.rsquared
r_squared
```

```
0.70
```

```
adjusted_r_squared = fit.rsquared_adj
```

```
0.65
```

Let's practice!

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Alpha and multi-factor models

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The Fama-French 3 factor Model

$$R_P =$$

$$RF + \beta_M(R_M - RF) + b_{SMB} \cdot SMB + b_{HML} \cdot HML + \alpha$$

- SMB: The small minus big factor
- b_{SMB}: Exposure to the SMB factor
- HML: The high minus low factor
- b_{HML}: Exposure to the HML factor
- α: Performance which is unexplained by any other factors
- β_M: Beta to the broad market portfolio B

The Fama-French 3 factor model

Date	Portfolio	Market_Excess	SMB	HML	RF
2013-01-03	-0.005066	-0.0014	0.0014	0.0004	0.0
2013-01-04	0.004024	0.0055	0.0019	0.0043	0.0
2013-01-07	0.004421	-0.0031	-0.0009	-0.0037	0.0
2013-01-08	-0.004659	-0.0027	0.0004	-0.0007	0.0
2013-01-09	0.004636	0.0034	0.0024	-0.0041	0.0

The Fama-French 3 factor model in Python

Assuming you already have excess portfolio and market returns in the object `Data` :

```
import statsmodels.formula.api as smf
model = smf.ols(formula='Port_Excess ~ Mkt_Excess + SMB + HML',
                data=Data)
fit = model.fit()
adjusted_r_squared = fit.rsquared_adj
adjusted_r_squared
```

```
0.90
```

P-values and statistical significance

To extract the `HML` p-value, assuming you have a fitted regression model object in your workspace as `fit` :

```
fit.pvalues["HML"]
```

```
0.0063
```

To test if it is statistically significant, simply examine whether or not it is less than a given threshold, normally 0.05:

```
fit.pvalues["HML"] < 0.05
```

```
True
```

Extracting coefficients

To extract the `HML` coefficient, assuming you have a fitted regression model object in your workspace as `fit`:

```
fit.params["HML"]
```

```
0.502
```

```
fit.params["SMB"]
```

```
-0.243
```

Alpha and the efficient market hypothesis

Assuming you already have a fitted regression analysis in the object `fit`:

```
portfolio_alpha = fit.params["Intercept"]
portfolio_alpha_annualized = ((1 + portfolio_alpha) ** 252) - 1
portfolio_alpha_annualized
```

```
0.045
```

Let's practice!

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Expanding the 3-factor model

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Fama French 1993

The original paper that started it all:

ELSEVIER

Journal of Financial Economics

Volume 33, Issue 1, February 1993, Pages 3-56

Common risk factors in the returns on stocks and bonds ☆

Eugene F. Fama, Kenneth R. French

Cliff Assness on Momentum

A paper published later by Cliff Asness from AQR:

PERSPECTIVE

Fama on Momentum

February 5, 2016 - Cliff Asness

Topics - Factor/Style Investing, Momentum

The Fama-French 5 factor model

In 2015, Fama and French extended their previous 3-factor model, adding two additional factors:

- **RMW:** Profitability
- **CMA:** Investment

The RMW factor represents the returns of companies with high operating profitability versus those with low operating profitability.

The CMA factor represents the returns of companies with aggressive investments versus those who are more conservative.

The Fama-French 5 factor model

	Portfolio	Market_Excess	SMB	HML	RMW	CMA	RF
Date							
2013-01-03	-0.005066	-0.0014	0.0014	0.0004	0.0020	0.0023	0.0
2013-01-04	0.004024	0.0055	0.0019	0.0043	-0.0037	0.0027	0.0
2013-01-07	0.004421	-0.0031	-0.0009	-0.0037	-0.0013	-0.0012	0.0
2013-01-08	-0.004659	-0.0027	0.0004	-0.0007	-0.0012	0.0009	0.0
2013-01-09	0.004636	0.0034	0.0024	-0.0041	-0.0007	-0.0015	0.0

The Fama-French 5 factor model in Python

Assuming you already have excess portfolio and market returns in the object `Data` :

```
import statsmodels.formula.api as smf
model = smf.ols(formula='Port_Excess ~ Mkt_Excess + SMB + HML + RMW + CMA',
                data=Data)
fit = model.fit()
adjusted_r_squared = fit.rsquared_adj
adjusted_r_squared
```

```
0.92
```

Let's practice!