CMC-12: Lista 8

Lucas do Vale Bezerra

Q1 Do diagrama de blocos:

$$Y = \frac{\left[(R-Y)\cdot(K_p + K_d\, s) + D(s)\right]}{s(ms+b)} \Rightarrow Y\left(1 + \frac{K_p + K_d s}{s(ms+b)}\right) = \frac{D}{s(ms+b)}$$

$$\frac{Y(s)}{D(s)} = \frac{1}{m}\cdot\frac{1}{s^2 + \left(\frac{K_d + b}{m}\right)s + K_p/m} = G_d(s)$$

- $d(t) = A_d \cdot sen(\omega t + \phi_d)$
- $y(t) = A_y \cdot sen(\omega t + \phi_y)$

$$\Rightarrow \begin{cases} |G(j\omega)| = A_y/A_d \\ \angle G(j\omega) = \phi_y - \phi_d \end{cases}$$

$$G(j\omega) = \frac{1}{m}\cdot\frac{1}{\left(K_p/m - \omega^2\right) + \left(\frac{K_d + b}{m}\right)\omega j}$$

- $$|G(j\omega)| = \frac{1}{m}\cdot\frac{1}{\sqrt{\left(\frac{K_p}{m} - \omega^2\right)^2 + \omega^2\left(\frac{K_d + b}{m}\right)^2}} = \frac{A_y}{A_d}$$

- $$\angle G(j\omega) = arctg\left(\frac{\omega(K_d + b)}{m\omega^2 - K_p}\right) = \phi_y - \phi_d$$

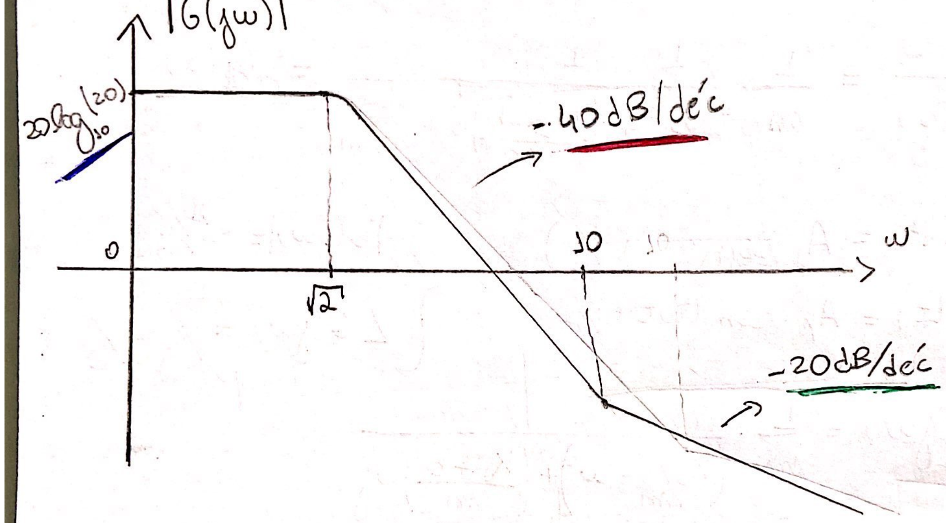
$$A_y = \frac{A_d}{\sqrt{(m\omega^2 - K_p)^2 + \omega^2(K_d + b)^2}} \quad e \quad \phi_y = \phi_d + arctg\left(\frac{\omega(K_d+b)}{m\omega^2 - K_p}\right)$$

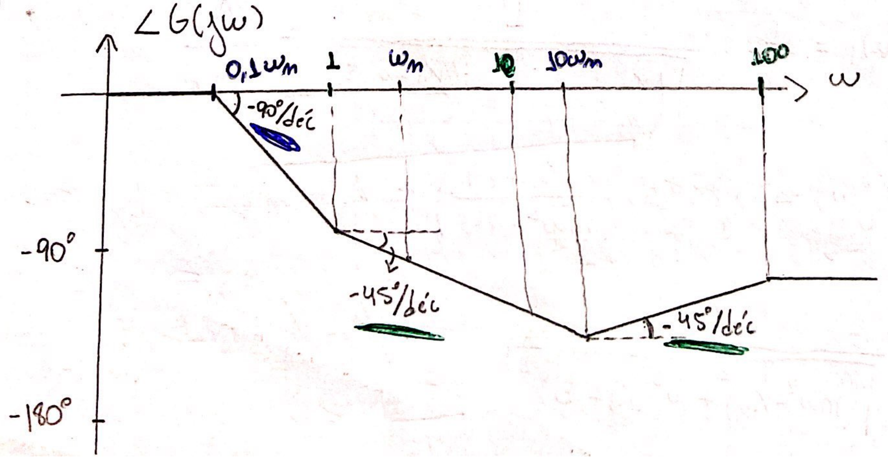
Q2 $G(s) = \dfrac{2(s+10)}{(s^2+2s+2)} = \dfrac{2\,(s+10)}{(s-(-1+j))\,(s-(-1-j))}$

Zero: -10

Polos Comp. Conj.: $(-1+j)$; $(-1-j)$ $\Rightarrow \begin{cases} \omega_n = \sqrt{2} \\ \xi = \sqrt{2}/2 \end{cases}$

$|G(j\omega)|_{dB} = 20\log_{10}(2\cdot 10) + \text{Polos} - \text{Zeros}$





Q4 Do diagrama de blocos:

$$Y = (R - Y - N)\cdot\frac{K}{ms+b} \quad \begin{cases} \boxed{G_R(s) = \dfrac{K}{ms+b+K}} \\ \boxed{G_N(s) = -\dfrac{K}{ms+b+K}} \end{cases}$$

- Erro em regime:

$$e_{\infty} = \lim_{s\to 0} s\cdot R(s)\cdot(1 - G_R(s)) \leq 0{,}1$$

$$\lim_{s\to 0}\left(\frac{ms+b}{ms+b+K}\right) = \frac{b}{b+K} \leq 0{,}1$$

$$50 \leq 5 + 0{,}1K \rightarrow \boxed{K \geq 450}$$

- Atenuamento:

$$|G_N(j\omega)| = \left|\frac{K}{b+K+m\omega\cdot j}\right| = \frac{K}{\sqrt{(b+K)^2+(m\omega)^2}}$$

$$20\log_{10}|G_N(j\omega)| = |G_N(j\omega)|_{dB} = \underbrace{20\log_{10}K}_{\text{Ganho DC}} - \underbrace{20\log_{10}\sqrt{(b+K)^2+(m\omega)^2}}_{\text{Atenuamento}}$$

$$-20\log_{10}\left(\frac{K}{\sqrt{(b+K)^2+(m\omega)^2}}\right) \leq -20$$

$$\Rightarrow 10K \leq \sqrt{(50+K)^2+(10000)^2}$$

$$100K^2 \leq 2500 + 100K + K^2 + 10^8$$

$$99K^2 - 100K - (2500 + 10^8) \leq 0$$

$$\boxed{K \leq \frac{9050}{9}}$$

Q5) $\begin{cases} \omega_b = 6 rad/s = \omega_n \cdot \sqrt{1 - 2\xi^2 + \sqrt{4\xi^4 - 4\xi^2 + 2}} \\ M_r = 0,3546 dB = \left| \omega_n \cdot \sqrt{1 - 2\xi^2} \right|_{dB} \end{cases}$

$$1,0417 = \omega_n \sqrt{1 - 2\xi^2}$$

$$\frac{\omega_b}{M_r} = 5,76 = \sqrt{1 + \frac{\sqrt{4\xi^2 - 4\xi^2 + 2}}{1 - 2\xi^2}}$$

$$32,1774 = \sqrt{\frac{4\xi^4 - 4\xi^2 + 2}{4\xi^4 - 4\xi^2 + 1}} \Rightarrow$$

$$\Rightarrow 4\xi^4 - 4\xi^2 + 1 = 9,67 \cdot 10^{-4}$$

$$\Rightarrow \begin{cases} \xi = 0,6 \\ \omega_n = 5,225 \end{cases}$$

Do diagrama, temos:

$$G(s) = \frac{K_p \cdot K_\varphi \cdot v}{s^2 + K_\varphi \cdot s + K_p \cdot K_\varphi \cdot v}$$

$$\begin{cases} K_\varphi = 2\xi\omega_n = 2 \cdot 0,6 \cdot 5,225 = \boxed{6,27} \\ K_p \cdot K_\varphi \cdot v = \omega_n^2 \Rightarrow \boxed{K_p = 4,3542} \end{cases}$$