Lista 4 - CMC-12

Q1) $F(s) = \dfrac{3s+5}{s^3+4s^2+5s+2} = \dfrac{3(s+2)-1}{(s+1)^2(s+2)}$

$= \dfrac{3}{(s+1)^2} - \dfrac{1}{(s+1)^2(s+2)} = \dfrac{3}{(s+1)^2} + A$

$A = \dfrac{-1}{(s+1)^2(s+2)} = \dfrac{s}{(s+1)^2} - \dfrac{1}{(s+2)}$

$F(s) = \dfrac{3}{(s+1)^2} + \dfrac{\overbrace{(s+1)-1}^{s}}{(s+1)^2} - \dfrac{1}{(s+2)}$

$= \dfrac{2}{(s+1)^2} + \dfrac{1}{(s+1)} - \dfrac{1}{(s+2)}$

$\mathcal{L}^{-1}(F(s)) = f(t) = 2 \cdot t \cdot e^{-t} + e^{-t} - e^{-2t}$

$f(t) = (1+2t) \cdot e^{-t} - e^{-2t}$

Q2) $\ddot{x} + 6\dot{x} + 18x = 18 \cdot 1(t)$, $x(0) = 1$ e $\dot{x}(0) = 3$

$$\ddot{X} = s^2 X(s) - s\,x(0) - \dot{x}(0) = s^2 X(s) - s - 3$$

$$\dot{x} = s\,X(s) - x(0) = s\,X(s) - 1$$

Logo: $s^2 X(s) - s - 3 + 6sX(s) - 6 + 18X(s) = 18 \cdot \frac{1}{s}$

$$X(s)\left[s^2 + 6s + 18\right] = s + 9 + \frac{18}{s} = \frac{s^2 + 9s + 18}{s}$$

$$X(s) = \frac{s^2 + 9s + 18}{s(s^2 + 6s + 18)} = \frac{(s+3)(s+6)}{s\left[(s+3)^2 + 3^2\right]}$$

$$X(s) = \frac{(s+3)}{(s+3)^2 + 3^2} + \frac{6}{s} \cdot \frac{(s+3)}{(s+3)^2 + 3^2}$$

$$\mathcal{L}^{-1}\{X(s)\} = x(t) = e^{-3t}\cos(3t) + 6 \cdot \int_0^t e^{-3\tau}\cos(3\tau)\,d\tau$$

$$x(t) = e^{-3t}\cos(3t) + \left[e^{-3\tau}\operatorname{sen}(3\tau) - e^{-3t}\cos(3\tau)\right]_0^t$$

$$x(t) = 1 + e^{-3t}\operatorname{sen}(3t)$$

Q3) EDO para o motor elétrico:

$$\begin{cases} L\dot{i} + Ri + V_b = V(t), \text{ com } V_b = K_t \cdot \omega \\ J\dot{\omega} + b\omega = K_t \cdot i \rightarrow i = \dfrac{J\dot{\omega} + b\omega}{K_t} \end{cases}$$

$$L \cdot \left(\frac{J \cdot \ddot{\omega} + b\dot{\omega}}{K_t} \right) + R\left(\frac{J\dot{\omega} + b\omega}{K_t} \right) + K_t \omega = V(t)$$

$\omega = \dot{\theta} \rightarrow$ Logo, temos a EDO final:

$$\dddot{\theta} + \left(\frac{b}{J} + \frac{R}{L} \right) \ddot{\theta} + \left(\frac{Rb + K_t^2}{J \cdot L} \right) \dot{\theta} = \frac{K_t}{J \cdot L} V(t)$$

$$\Rightarrow s^3 \Theta(s) + \left(\frac{b}{J} + \frac{R}{L} \right) s^2 \Theta(s) + \left[\frac{Rb + K_t^2}{JL} \right] s \cdot \Theta(s) = \frac{K_t}{J \cdot L} V(s)$$

$$\Theta(s) \left[s^3 + \left(\frac{b}{J} + \frac{R}{L} \right) s^2 + \left(\frac{Rb + Kt^2}{JL} \right) s \right] = \frac{K_t}{JL} V(s)$$

$$G(s) = \frac{\Theta(s)}{V(s)} = \frac{K_t}{JLs^3 + (bL + RJ)s^2 + (Rb + K_t^2)s}$$

Q4) $m\ddot{x} + (b + K_v)\dot{x} + K_p K_v x = K_p K_v \underbrace{x_r(t)}_{t}$

$\Rightarrow m s^2 X(s) + (b + K_v) \cdot s X(s) + K_p K_v X(s) = K_p K_v \cdot \frac{1}{s^2}$

$$X(s) = \frac{K_p K_v}{s^2 (m s^2 + (b + K_v) s + K_p K_v)}$$

$$G(s) = \frac{X(s)}{X_r(s)} = \frac{K_p K_v}{m s^2 + (b + K_v) s + K_p K_v}$$

↓

função de transferência