CMC-12: Lista 10

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1) $\omega_{CG} \rightarrow \angle G(j\omega_{CG}) = -180°$ (nº real negativo)

GM $\rightarrow GM \cdot |G(j\omega_{CG})| = 1 \rightarrow GM = \dfrac{1}{|G(j\omega_{CG})|}$

$\omega_{CP} \rightarrow |G(j\omega_{CP})| = 1$

PM $\rightarrow 180° + \angle G(j\omega_{CP})$

2) Do diagrama de blocos:

$$X = \frac{1}{s}\left[\frac{K_v(K_p\cdot(R-X) - sX)}{ms+b}\right]$$

$$X(ms^2 + bs) = K_p K_v R - K_v X(s + K_p)$$

$$G_f(s) = \frac{X(s)}{R(s)} = \frac{K_p K_v}{ms^2 + (b+K_v)s + K_p K_v}$$

$$G_f(s) = \frac{K_p K_v / m}{s^2 + \left(\frac{b+K_v}{m}\right)s + K_p K_v / m}$$

$$\begin{cases} \omega_n = \dfrac{\omega_b}{\sqrt{1 - 2\xi^2 + \sqrt{4\xi^4 - 4\xi^2 + 2}}} \\ PM = 60^\circ = \operatorname{arctg}\left(\dfrac{2\xi}{\sqrt{-2\xi^2 + \sqrt{4\xi^4 + 1}}}\right) \end{cases}$$

Daí: $\xi = 0{,}6124$ e $\omega_n = 5{,}5519$ rad/s

$$\begin{cases} (b + K_v)/m = 2\xi\omega_n \\ K_p \cdot K_v / m = \omega_n^2 \end{cases} \Rightarrow \begin{aligned} K_p &= \frac{m\omega_n^2}{K_v} \\ K_v &= 2\xi\omega_n m - b \end{aligned}$$

3) Do gráfico:

$\angle G(j\omega) = -180° \rightarrow |G(j\omega)|_{dB} = -15\, dB$

Logo, $GM_{dB} = 15\, dB$

$|G(j\omega)|_{dB} = 0 \rightarrow \angle G(j\omega) = -100°$

$PM = \angle G(j\omega_{cp}) + 180°$

$PM = 80°$

4) $G(s) = \frac{1}{s^2}$ e $C(s) = \frac{6(1{,}5s+1)}{(0{,}1s+1)}$

$$G_a(s) = C(s) \cdot e^{-\tau s} \cdot G(s)$$

$$G_a(s) = \frac{6 \cdot (1{,}5s+1)}{(0{,}1s+1)} \cdot \left(\frac{-s + 2/\tau}{s + 2/\tau}\right) \cdot \frac{1}{s^2}$$

ω_{cp} não é alterado pelo atraso:

$\underline{\omega_{cp}} \Rightarrow |G_f(j\omega)| = 1 \rightarrow \left| \frac{6 \cdot (1{,}5j\omega+1)}{0{,}1j\omega+1} \cdot \frac{1}{-\omega^2} \right| = 1$

$$\sqrt{81\omega^2+36} = \omega^2 \sqrt{0{,}01\omega^2+1}$$

$$81\omega^2+36 = 0{,}01\omega^6 + \omega^4 \rightarrow \frac{\omega^6}{100} + \omega^4 - 81\omega^2 - 36 = 0$$

$$\Rightarrow \underline{\omega_{cp} = 7{,}3 \text{ rad/s}}$$

$$PM_1 = 180° + \angle G(j\omega_{cp}) - \tau_1 \omega_{cp}$$

$$PM_2 = 180° + \angle G(j\omega_{cp}) - \tau_2 \omega_{cp}$$

$$|PM_2 - PM_1| = \omega_{cp}(\tau_1 - \tau_2) = 70 \cdot 10^{-3} \cdot 7{,}3$$

$$\Delta PM = 0{,}511 \text{ rad} = 29{,}3°$$

5) Expressão geral do diagrama de blocos

$$Y(s) = G(s) \cdot \left[D(s) + C(s) \cdot \left(R(s) \cdot F(s) - F_m(s) \cdot (Y(s) + N(s)) \right) \right]$$

$\underline{\underline{G_R}}$: $D(s) = 0$ e $N(s) = 0$

$$Y = G(C \cdot R \cdot F - C F_m \cdot Y) \rightarrow Y(1 + CGF_m) = CGRF$$

$$G_R(s) = \frac{Y(s)}{R(s)} = \frac{C(s) \cdot G(s) \cdot F(s)}{1 + C(s) G(s) F_m(s)}$$

$\underline{\underline{G_N}}$: $R(s) = 0$ e $D(s) = 0$

$$Y = -G \cdot C \cdot F_m (Y + N) \rightarrow Y(1 + C \cdot G F_m) = -CGF_m N$$

$$G_N(s) = \frac{Y(s)}{N(s)} = -\frac{C(s) \cdot G(s) F_m(s)}{1 + C(s) G(s) F_m(s)}$$

$\underline{\underline{G_D}}$: $R(s) = 0$ e $N(s) = 0$

$$Y = G(D - C \cdot F_m \cdot Y) \rightarrow Y(1 + CGF_m) = GD$$

$$G_D(s) = \frac{Y(s)}{D(s)} = \frac{G(s)}{1 + C(s) G(s) F_m(s)}$$