Lista 1 - CMC-12

Questão 2.

$$m\ddot{x}(t) + b\cdot\dot{x}(t) + K\cdot x(t) = f$$

$$\begin{cases} m = 1\,kg \\ b = 1,4\ N\cdot s/m \\ K = 1\ N/m \\ f = 1\ N \end{cases}$$

PVI: $\begin{cases} x(0) = 0 \\ \dot{x}(0) = 0 \end{cases}$

$\Rightarrow \ddot{x} + 1,4\dot{x} + x = 1$ $\xrightarrow{\text{homogênea}}$ $\lambda^2 + 1,4\lambda + 1 = 0$

Particular: $x_p(t) = f/K = 1$

$\Delta = 1,4^2 - 4\cdot 1\cdot 1 = -2,04$

$\lambda = \underbrace{-1,4}_{\sigma} \pm i\,\underbrace{1,43}_{\omega}$

$x_h(t) = c_1\cdot e^{-\sigma t}\cos(\omega t) + c_2\cdot e^{-\sigma t}\,\text{sen}(\omega t)$

Dessa forma:

$$x(t) = c_1\cdot e^{-1,4t}\cos(1,43t) + c_2\cdot e^{-1,4t}\,\text{sen}(1,43t) + 1$$

PVI: $x(0) = c_1 + 1 = 0 \rightarrow c_1 = -1$

$\dot{x}(0) = c_1\cdot(-1,4) + c_2\cdot 1,43 = 0 \rightarrow c_2 = -0,98$

Com isso,

$$x(t) = -e^{-1,4t}\cdot\cos(1,43t) - 0,98\cdot e^{-1,4t}\cdot\text{sen}(1,43t) + 1$$

Questão 3.

Da EDO: $\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{f}{m}$, pode-se fazer a transformação $x_1 = x$ e $x_2 = \dot{x}$, sendo $x(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x(t) & \dot{x}(t) \end{bmatrix}^T$

Dessa forma:

$$\dot{x}_2 + \frac{b}{m}x_2 + \frac{k}{m}x_1 = \frac{f}{m}$$

$$\dot{x}_2 = \frac{f}{m} - \frac{b}{m}x_2 - \frac{k}{m}x_1 \quad \text{e} \quad \dot{x}_1 = x_2$$

Portanto,

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{f}{m} - \frac{b}{m}x_2 - \frac{k}{m}x_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f$$

$$\frac{d}{dt}x(t) = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f \qquad A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \text{ e } B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

Questão 4

EDO $\rightarrow$ $ml\ddot{\theta} + b\dot{\theta} + mg\,\text{sen}\,\theta = 0$

$$\ddot{\theta} + \frac{b}{ml}\dot{\theta} + \frac{g}{l}\,\text{sen}\,\theta = 0$$

Analogamente à questão 3:

$$\frac{d}{dt}\theta(t) = d\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \theta_2 \\ -\frac{b}{ml}\theta_2 - \frac{g}{l}\,\text{sen}\,\theta_1 \end{bmatrix}$$