CMC-12: Lista 7

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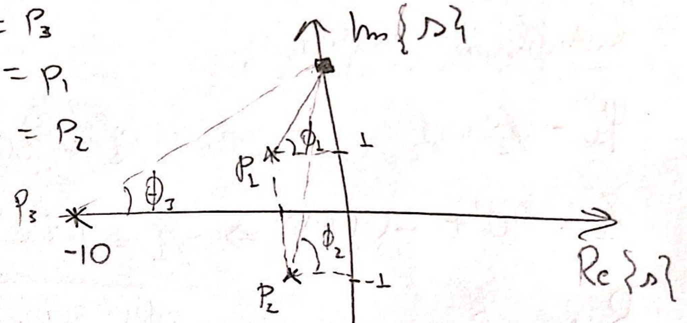
Q1 $G(s) = \dfrac{10}{(s^2+2s+2)(s+10)}$

Polos: $\begin{cases} -10 = p_3 \\ -1+j = p_1 \\ -1-j = p_2 \end{cases}$



- Condição Angular:

$-\phi_1 - \phi_2 - \phi_3 = -180° \rightarrow \phi_2 \approx 90°$

$\phi_1 = 90° - \angle(p_1 - p_3) = 90° - \angle(9+j)$

$= 90° - \text{arctg}(1/9)$

- $G_f(s) = \dfrac{10K}{s^3+12s^2+22s+20+10K}$

$G_f(j\omega) = \dfrac{10K}{-j\omega^3 - 12\omega^2 + 22j\omega + 20 + 10K}$

$\begin{cases} -\omega^3 + 22\omega = 0 \rightarrow \omega = \sqrt{22} \text{ rad/s} \\ -12\omega^2 + 20 + 10K = 0 \rightarrow K = \dfrac{12\omega^2 - 20}{10} = 24,4 \end{cases}$

Q2) $G(s) = \frac{1}{10}\left(\frac{s+10}{s^2+2s+2}\right)$

Zeros: -10

Polos: $(-1+j)$; $(-1-j)$

- Condição Angular:

$\psi_1 - \phi_1 - \phi_2 = -180° \rightarrow \phi_2 \approx 90°$ e $\psi_1 = \angle(p_1 - z_1)$

$\phi_1 = 90° + \angle(9+j) \Rightarrow \boxed{\phi_1 = 90° + \operatorname{arctg}(9+j)}$

- Regra 5:

$\frac{d}{ds}\left[\frac{10(s^2+2s+2)}{(s+10)}\right]_{s=s^*} = 0 \rightarrow \frac{\overbrace{(s+10)(2s+2)}^{2s^2+22s+20} - (s^2+2s+2)}{(s^*+10)^2} = 0$

$= s^{*2} + 20s^* + 18 = 0 \rightarrow \boxed{s^* = -10 + \sqrt{82}}$

$K^* = -\frac{D(s^*)}{N(s^*)} = -\frac{10(s^{*2}+2s^*+2)}{(s^*+10)} = \frac{10(s^{*2}+2s^*+2)}{\sqrt{82}}$

$= \frac{10}{\sqrt{82}} \cdot \left(100+82+20\sqrt{82} - 20 - 2\sqrt{82} + 2\right)$

$= \frac{10}{\sqrt{82}}(164 + 18\sqrt{82}) = 10\left(\frac{164}{82}\sqrt{82} + 18\right)$

$\boxed{K^* = 20\sqrt{82} + 180}$

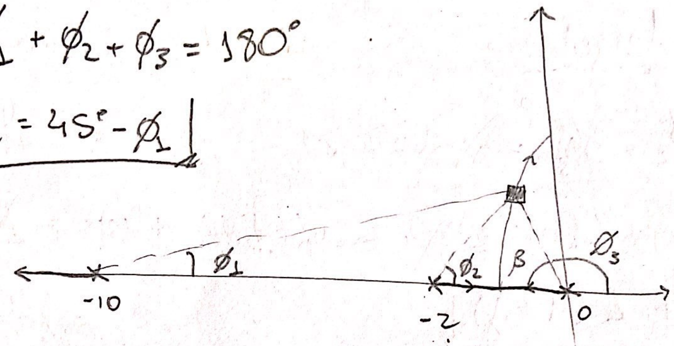
Q3 $G(s) = \dfrac{5}{s(s+2)(s+10)}$

• Condição de magnitude:

$$K = \left|\frac{1}{G(s)}\right| = \frac{|s||s+2||s+10|}{5} = \frac{l_1 \cdot l_2 \, l_3}{5}$$

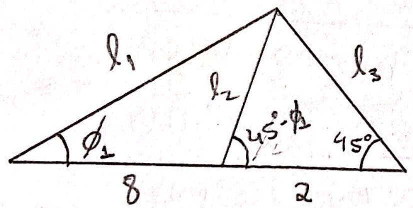
$$\phi_1 + \phi_2 + \phi_3 = 180°$$

$$\boxed{\phi_2 = 45° - \phi_1}$$



$$M_p = 0{,}0432 = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right) \rightarrow \xi = \frac{-\ln(M_p)}{\sqrt{\pi^2 + (\ln(M_p))^2}}$$

$$\xi = 0{,}7071 \Rightarrow \boxed{\beta = \arccos(\xi) = 45°}$$



Lei dos Senos:

• $\dfrac{l_1}{\text{sen}\,45°} = \dfrac{10}{\text{sen}(135-\phi_1)} = \dfrac{l_3}{\text{sen}\,\phi_1}$

$$\boxed{l_1 = \frac{10 \cdot \text{sen}\,45°}{\text{sen}(135-\phi_1)}}$$

• $\dfrac{l_2}{\text{sen}\,45°} = \dfrac{2}{\cos\phi_1} = \dfrac{l_3}{\text{sen}(45°-\phi_1)}$

$$\boxed{l_2 = \frac{2\,\text{sen}\,45°}{\cos\phi_1}} \quad \text{e} \quad \boxed{l_3 = \frac{2\,\text{sen}(45°-\phi_1)}{\cos\phi_1} = \frac{10\,\text{sen}\,\phi_1}{\text{sen}(135°-\phi_1)}}$$

De l_3, temos: $10 \operatorname{sen}\phi_1 \cos\phi_1 = 2 \operatorname{sen}(45° - \phi_1) \cdot \operatorname{sen}(90 + (45° - \phi_1))$

$\Rightarrow 5 \operatorname{sen} 2\phi_1 = \operatorname{sen}(90° - 2\phi_1) = \cos 2\phi_1$

$\operatorname{tg} 2\phi_1 = 1/5 \rightarrow \boxed{\phi_1 = \frac{1}{2} \operatorname{arctg}(1/5)}$

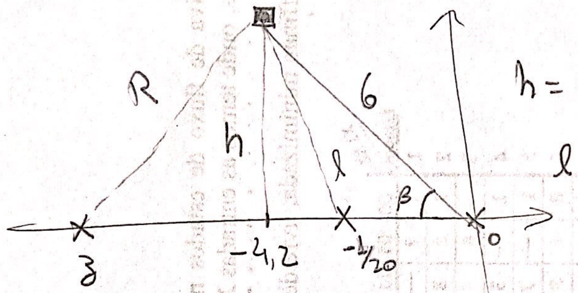
$$K = \frac{l_1 l_2 l_3}{5} = \frac{200 \operatorname{sen}^2 45° \cdot \operatorname{sen}\phi_1}{5 \cdot \cos^2(45° - \phi_1) \cos\phi_1} = \frac{100 \operatorname{tg}\phi_1}{5 \cdot \cos^2(45 - \phi_1)}$$

$$K = \frac{100 \cdot 0{,}099}{5 \cdot 0{,}5981} = 3{,}311$$

Q4 $G(s) = \dfrac{1}{m \cdot s + b} \rightarrow \begin{cases} \omega_n = 6 \text{ rad/s} \\ \xi = 0{,}7 \end{cases}$

$\Rightarrow \begin{cases} \beta = \arccos(0{,}7) = \boxed{45{,}6°} \\ \sigma = \xi \omega_n = \boxed{4{,}2} \end{cases}$ $\qquad C(s) = K \cdot \underbrace{\dfrac{(s - z)}{s}}_{C'(s)}$

$G'(s) = C'(s) \cdot G(s) = \dfrac{1}{m} \cdot \dfrac{(s - z)}{s \cdot (s + b/m)} \rightarrow \begin{cases} \underline{Z}: z \\ \underline{P}: 0;\ -b/m \end{cases}$



$h = 6 \cdot \text{sen}\, \beta = 4{,}2849$

$l^2 = h^2 + (4{,}15)^2$

$l = 5{,}9851$

$R^2 = (z - p_1) \cdot (z - p_2) \Rightarrow z \cdot (z + 1/20) = h^2 + (z + 4{,}2)^2$

$\Rightarrow \boxed{z = -4{,}3114} \rightarrow \begin{cases} K_p = K \\ K_i = -Kz \end{cases}$

- Condição de magnitude:

$K = \left| \dfrac{1}{G'(s)} \right| = m \cdot \dfrac{|s| \cdot |s + b/m|}{|s - z|} = \dfrac{m \cdot 6 \cdot l}{R} = 8350{,}67$

$\begin{cases} \boxed{K_p = 8350{,}67} \\ \boxed{K_i = 36002{,}87} \end{cases}$

Q5 $G(s) = \dfrac{10(s+a)}{s(s+1)(s+2)} = (s+a)\cdot G'(s)$

$s(s+1)(s+2) \to G'(s)$

$$G_f(s) = \frac{G(s)}{1+G(s)} = \frac{(s+a)\,G'(s)}{1+(s+a)\,G'(s)}$$

$$D_f(s) = 1+(s+a).G'(s) = 0 \Rightarrow 1+s\cdot G'(s)+aG'(s) = 0$$

$$-a = \frac{1+s.G'(s)}{G'(s)} \to \frac{G'(s)}{1+s\cdot G'(s)} = -\frac{1}{a}$$

$$Q(s) = \frac{G'(s)}{1+s\cdot G'(s)} = -\frac{1}{a}$$

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