Lista 3 - CMC-12

1) Do esquema, temos a equação:

$$V(t) = L\cancel{\dot{i}}^{\,0} + Ri + V_b \text{, com } L \approx 0 \text{, para:}$$

$$\boxed{Ri + V_b = V(t)}$$ Além disso,

$$\begin{cases} V_b = K_t \cdot \omega \\ \tau_m = K_t \cdot i \\ \tau_m - b\omega = J \cdot \dot{\omega} \end{cases} \Rightarrow \begin{cases} Ri + K_t \cdot \omega = V(t) \\ J \cdot \dot{\omega} + b\omega = K_t \cdot i \end{cases}$$

$$i = \frac{J\dot{\omega} + b\omega}{K_t} = \frac{V(t) - K_t \cdot \omega}{R}$$

$$\dot{\omega} + \left(\frac{Rb + K_t^2}{R \cdot J}\right)\omega = \frac{K_t}{J \cdot R} \cdot V(t)$$

Fazendo $V(t) = K_{ff} \cdot \omega_r + K_p \cdot (\omega_r - \omega)$,

$$\dot{\omega} + \left(\frac{R \cdot b + K_t \cdot K_p + K_t^2}{R \cdot J}\right)\omega = \frac{K_t \cdot \omega_r}{J \cdot R}(K_{ff} + K_p)$$

Queremos que $\underline{\omega_r = \omega_\infty}$.

continua →

Da EDO, temos que:

$$\omega(t) = \left[\frac{K_t \cdot \omega_r (K_p + K_{ff})}{Rb + K_t \cdot K_p + K_t^2}\right] \cdot (1 - e^{-t/\tau})$$

Com $\tau = \lambda^{-1} = \dfrac{R \cdot J}{R \cdot b + K_t \cdot K_p + K_t^2}$

Fazendo $\omega_r = \omega_\infty$, temos:

$$\frac{K_t \cancel{\omega_r} (K_p + K_{ff})}{R \cdot b + K_t \cdot K_p + K_t^2} = \cancel{\omega_r}$$

$$\cancel{K_t \cdot K_p} + K_t \cdot K_{ff} = R \cdot b + \cancel{K_t K_p} + K_t^2$$

$$\boxed{K_{ff} = \frac{Rb + K_t^2}{K_t}}$$

Isolando K_p em τ, temos:

$$\frac{RJ}{\tau} = Rb + K_t \cdot K_p + K_t^2$$

$$\boxed{K_p = \frac{RJ}{\tau K_t} - \frac{R \cdot b}{K_t} - K_t}$$

2) $$\begin{cases} \omega_r(t) = K_p\left(\theta_r - \theta(t)\right) \\ V(t) = K_v\left(\omega_r(t) - \omega(t)\right) \end{cases}$$

Logo,

$$V(t) = K_v\left[K_p\left(\theta_r - \theta(t)\right) - \omega(t)\right]$$

$$\dot{\omega} + \left(\frac{R\cdot b + K_t^2}{R\cdot J}\right)\cdot\omega = \frac{K_t}{R\cdot J}\cdot V(t)$$

$$\dot{\omega} + \left(\frac{R\cdot b + K_t\cdot K_v + K_t^2}{R\cdot J}\right)\cdot\omega = \frac{K_t\cdot K_v K_p}{R\cdot J}\left(\theta_r - \theta(t)\right)$$

Fazendo que $\omega = \dot{\theta}$ e $\dot{\omega} = \ddot{\theta}$, temos:

$$\ddot{\theta} + \left(\frac{R\cdot b + K_t\cdot K_v + K_t^2}{R\cdot J}\right)\dot{\theta} + \frac{K_t K_v K_p}{RJ}\theta = \frac{K_t K_v K_p}{RJ}\theta_r$$

$$\omega_n^2 = \sqrt{\frac{K_t K_v K_p}{RJ}} \quad e \quad 2\xi\omega_n = \left(\frac{R\cdot b + K_t\cdot K_v + K_t^2}{RJ}\right)$$

Logo, $$\boxed{K_v = \frac{2\xi\omega_n RJ - Rb - K_t^2}{K_t}}$$ e

$$K_p = \frac{R\cdot J\cdot\omega_n^2}{2\xi\omega_n RJ - Rb - K_t^2}$$

3) $\begin{cases} \psi_r(t) = K_p \cdot (h_r - h(t)) \\ \omega(t) = K_\psi (\psi_r(t) - \psi(t)) \end{cases}$

e

$\begin{cases} \dot{h}(t) = v \cdot \psi(t) & \text{(usando aproximação)} \\ \dot{\psi}(t) = \omega(t) \end{cases}$

$\Rightarrow$ Seja $x \triangleq \begin{bmatrix} h \\ \psi \end{bmatrix}$, então:

$$\frac{d}{dt}\begin{bmatrix} h \\ \psi \end{bmatrix} = \begin{bmatrix} v\psi \\ K_\psi [K_p (h_r - h) - \psi] \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} h \\ \psi \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & v \\ -K_\psi K_p & -K_\psi \end{bmatrix}}_{A} \begin{bmatrix} h \\ \psi \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ K_\psi K_p \end{bmatrix}}_{B} h_r$$

$$y = h = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} h \\ \psi \end{bmatrix} + \underbrace{0}_{D} \cdot h_r$$