CMC-12: Lista 11

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1) $G_a(s) = \dfrac{K}{s(s+1)(s+2)}$ e $\begin{cases} e_{\infty,rampa} \leq 1 \\ GM \geq 6dB \end{cases}$

• $e_{\infty,rampa} = \dfrac{1}{K_{v_0}} \rightarrow K_{v_0} = \lim_{s \to 0} s \cdot G_a(s)$

$K_{v_0} = \lim_{s \to 0} \dfrac{K}{(s+1)(s+2)} = \dfrac{K}{2} \rightarrow e_{\infty,rampa} = \dfrac{2}{K} \leq 1$

$\underline{K \geq 2}$

• $G_a(jw) = \dfrac{K}{(jw)^3 + 3(jw)^2 + 2(jw)} = \dfrac{K}{-3w^2 + j(2w - w^3)}$

$\angle G_a(jw_{c0}) = -180^\circ \rightarrow 2w - w^3 = 0 \rightarrow w_{c0} = \sqrt{2}$ rad/s

$|G_a(jw_{c0})| = \left| \dfrac{K}{-3 \cdot 2} \right| = \dfrac{K}{6} \rightarrow \underline{GM = 6/K}$

$GM_{dB} \geq 6dB \Rightarrow GM \geq 10^{6/20} \rightarrow \dfrac{6}{K} \geq 10^{6/20}$

$K \leq \dfrac{6}{10^{6/20}} \rightarrow K \leq 3{,}0071$. Logo:

$$\underline{2 \leq K \leq 3{,}0071}$$

2) Sem o Lag: $G_a(s) = \dfrac{54}{(s+2)(s+3)}$ e

$$G_f(s) = \frac{54}{s^2+5s+60} \Rightarrow e_\infty = 1 - G_f(0) = 1 - \frac{9}{10} = \underline{\underline{\frac{1}{10}}}$$

$$|G_a(j\omega_{cp})| = 1 \rightarrow \left|\frac{54}{6-\omega^2+5j\omega}\right| = 1 \rightarrow 54^2 = (6-\omega^2)^2 + 25\omega^2$$

$$54^2 = 36 - 12\omega^2 + \omega^4 + 25\omega^2 \rightarrow \omega^4 + 13\omega^2 - 2880 = 0$$

$\omega_{cp} = 6,8962$ rad/s

Com o Lag: $C(s) = \alpha \dfrac{(Ts+1)}{(\alpha Ts+1)}$. $z = -\dfrac{1}{T} = -\dfrac{\omega_{cp}}{10}$

$T = 1,4501$ $\quad e_\infty' = \dfrac{e_\infty}{2} = \dfrac{1}{20} = 1 - G_f(0)$

$$0,05 = 1 - \frac{54 \cdot C(0)}{6 + 54C(0)} = 1 - \frac{54\alpha}{6+54\alpha} = \frac{6}{6+54\alpha}$$

$$6 + 54\alpha = 120 \rightarrow \alpha = \frac{114}{54} = 2,111$$

3) $G(s) = \dfrac{1}{Ls+R}$, $C(s) = \dfrac{K(s-z)}{s(s-p)}$ e $\begin{cases} e_{\infty,rampa} \leq 0{,}05 \\ \omega_b \geq 15\,rad/s \\ PM \geq 50° \end{cases}$

$$G_a(s) = \frac{C(s)}{Ls+R} \quad e \quad G_f(s) = \frac{C(s)}{Ls+R+C(s)}$$

- $e_{\infty,rampa} = \dfrac{1}{K_{v_0}} \rightarrow K_{v_0} = \lim\limits_{s\to 0} s \cdot G_a(s) = \lim\limits_{s\to 0} \dfrac{s \cdot C(s)}{Ls+R}$

$$K_{v_0} = \frac{Kz}{R \cdot p} \rightarrow \frac{R\,p}{Kz} \leq 0{,}05 \rightarrow \boxed{Kz \geq 20p}$$

- $|G_f(j\omega_b)| = \dfrac{\sqrt{2}}{2} G_f(0) \rightarrow G_f(s) = \dfrac{K(s-z)}{Ls^3 + (R-pL)s^2 + (K-pR)s - Kz}$

$$\Rightarrow \left| \frac{K \cdot (j\omega - z)}{\omega^2 pL - \omega^2 R - Kz + j\omega(K - pR - L\omega^2)} \right| = \frac{\sqrt{2}}{2} \cdot 1$$

$$2 \cdot K^2 \cdot (z^2 + \omega^2) = (\omega^2(pL - R) - Kz)^2 + \omega^2(K - pR - L\omega^2)^2$$

$\omega = 15\,rad/s \rightarrow$ outro lim inferior p/ K.

- $PM \geq 50° \Rightarrow 180° + \angle G_a(j\omega_{cp}) \geq 50°$

4) Função de transferência final:

$$G_f(s) = \frac{(K_p + K_d \cdot s)/m}{s^2 + \frac{(b + K_d)}{m} s + K_p/m} \Rightarrow \begin{cases} \omega_n = 2\pi \\ \xi = 0,7 \end{cases}$$

$$\Rightarrow K_p/m = \omega_n^2 \text{ e } \frac{b + K_d}{m} = 2\xi\omega_n$$

$$G_a(s) = \frac{(K_p + K_d \cdot s)/m}{s(s + b/m)} \Rightarrow |G_a(j\omega_{cp})| = 1$$

$$\left| \frac{K_p + K_d \cdot j\omega}{j\omega(j\omega \cdot m + b)} \right| = 1 \rightarrow K_p^2 + K_d^2\omega^2 = $$

$$= \omega^2(b^2 + m^2\omega^2)$$

$$m^2\omega^4 + \omega^2(b^2 - K_d^2) - K_p^2 = 0$$

$$\omega^4 + \omega^2\left(\frac{b^2 - K_d^2}{m^2}\right) - \frac{K_p^2}{m^2} = 0 \Rightarrow \omega_{cp} = 9,6549 \text{ rad/s}$$

$$\text{Atraso} = \left(\frac{T_{am}}{2} + \tau\right) = \left(\frac{0,1}{2} + 0,05\right) = 0,1 \text{ s}$$

$$\Delta PM = \text{Atraso} \cdot \omega_{cp} \Rightarrow \Delta PM = 0,96549 \text{ rads}$$

$$\Delta PM(°) = 55,3185°$$

5) $$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$F_m(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$F(z) = \frac{\omega_n^2}{\frac{2^2}{T^2}\left(\frac{z-1}{z+1}\right)^2 + 2\xi\omega_n \cdot \frac{2}{T}\left(\frac{z-1}{z+1}\right) + \omega_n^2}$$

$$F(z) = \frac{\omega_n^2 \cdot T^2 \cdot (z+1)^2}{4(z-1)^2 + 4\xi\omega_n \cdot (z-1)(z+1)T + \omega_n^2 \cdot T^2 (z+1)^2}$$

$$F(z) = \frac{\omega_n^2 \cdot T^2 \cdot (z^2 + 2z + 1)}{z^2(\omega_n^2T^2 + 4\xi\omega_n T + 4) + z(2T^2\omega_n^2 - 8) + (\omega_n^2T^2 - 4\xi\omega_n T + 4)}$$

$$\Rightarrow D(z) \cdot U(z) = N(z) \cdot E(z)$$

$$D(z) = \overbrace{(\omega_n^2T^2 + 4\xi\omega_n T + 4)}^{D} + (2T^2\omega_n^2 - 8)z^{-1} + (\omega_n^2T^2 - 4\xi\omega_n T + 4)z^{-2}$$

$$N(z) = \omega_n^2T^2 \cdot (1 + 2z^{-1} + z^{-2})$$

$$u[k] = \frac{\left[\omega_n^2T^2 \cdot (e[k] + 2e[k-1] + e[k-2]) + (2T^2\omega_n^2 - 8)\,u[k-1] + (\omega_n^2T^2 - 4\xi\omega_n T + 4)\,u[k-2]\right]}{D}$$