Lista 6 - CMC-12

Q1) $C(s) = K$ e $G(s) = \dfrac{5}{(s+2)(s+3)(s+4)}$

$$G_f(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} \Rightarrow G_f(s) = \frac{5K}{(s+2)(s+3)(s+4)+5K}$$

$$D_{mf}(s) = s^3 + 9s^2 + 26s + (24+5K)$$

Routh-Hurwitz:

s^3: 1 $\quad 26$

s^2: 9 $\quad (24+5K)$

s^1: $\left(\dfrac{210-5K}{9}\right)$

s^0: $(24+5K)$

* Estabilidade:

- $\dfrac{210-5K}{9} > 0 \Rightarrow K < 42$
- $24 + 5K > 0 \Rightarrow K > -\dfrac{24}{5}$

Resp.: $-24/5 < K < 42$

Q2) Efeito da perturbação:

$$V_o(s) = \frac{G(s)\,D(s)}{1 + C(s)G(s)} \Rightarrow e_\infty = \lim_{s \to 0} s \cdot V_o(s)$$

$$V_o(s) = \frac{\dfrac{1}{ms+b} \cdot \dfrac{1}{s^2}}{1 + \dfrac{K_p \cdot s + K_i}{s \cdot (ms+b)}} = \frac{1/m}{s\left(s^2 + \dfrac{(b+K_p)}{m}s + \dfrac{K_i}{m}\right)}$$

Dessa forma, temos:

$$e_\infty = -\lim_{s\to 0} s \cdot V_D(s) = -\lim_{s\to 0} \frac{1/m}{s^2 + \frac{(b+K_p)}{m}s + \frac{K_i}{m}}$$

$$\boxed{e_\infty = -\frac{1}{K_i}}$$

Q3) $\begin{cases} \dot{y} = v \\ m\dot{v} = f - bv - mg \\ f = K_f \dfrac{u^2}{(y_{máx} - y)^2} \end{cases} \rightarrow$ Fazendo $x_1 = y$

$x_2 = \dot{x}_1 = v$

Nova EDO: $\begin{cases} x_2 = \dot{x}_1 \\ \dot{x}_2 = \dfrac{K_f}{m} \cdot \dfrac{u^2}{(y_{máx} - x_1)^2} - \dfrac{bx_2}{m} - g \end{cases}$

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \underbrace{\frac{K_f u^2}{m} \frac{1}{(y_{máx} - x_1)^2} - \frac{b x_2}{m} - g}_{f_2(x_1, x_2, u)} \end{bmatrix}$$

$$\frac{dx_2}{dt} = \frac{\partial f_2}{\partial x_1}\bigg|_{x_{10}, x_{20}, u_0} \partial x_1 + \frac{\partial f_2}{\partial x_2}\bigg|_{x_{10}, x_{20}, u_0} \partial x_2 + \frac{\partial f_2}{\partial u}\bigg|_{x_{10}, x_{20}, u_0} \partial u$$

$$= -\frac{b}{m}\partial v + \frac{2K_f}{m} \cdot \frac{\sqrt{\frac{mg}{K_f}} \cdot (y_{máx} - y_0)}{(y_{máx} - y_0)^2} \partial u + \frac{2\cancel{K_f}}{\cancel{m}} \cdot \frac{\frac{\cancel{m}g}{\cancel{K_f}}(y_{máx} - y_0)^2}{(y_{max} - y_0)^3} \partial y$$

$$= + 2g \cdot \frac{\partial y}{(y_{máx} - y_0)} - \frac{b}{m} \partial v + 2\sqrt{\frac{K_f g}{m}} \frac{\partial u}{(y_{máx} - y_0)}$$

$$\frac{d}{dt}\begin{bmatrix} \partial y \\ \partial v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2g/(y_{máx} - y_0) & -\frac{b}{m} \end{bmatrix}}_{A} \begin{bmatrix} \partial y \\ \partial v \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 2\sqrt{\frac{K_f \cdot g}{m}} \cdot \frac{1}{(y_{máx} - y_0)} \end{bmatrix}}_{B} \partial u$$

Q4) $G_f(s) = \dfrac{1}{(s+10)(s+20)(s^2+2s+4)}$

Polos: $P_1 = -10$; $P_2 = -20$

$P_3 = -2 + 2i\sqrt{3}$; $P_4 = -2 - 2i\sqrt{3}$

Como P_1 e P_2 satisfazem a relação $a \geq 5\xi\omega_n$:

$$G_f(s) \approx \frac{1}{200} \cdot \frac{1}{s^2+2s+4} \Rightarrow \omega_n^2 = 4 \rightarrow \underline{\omega_n = 2}$$

$$2\xi\omega_n = 2 \rightarrow \underline{\xi = 1/2}$$

- $e_\infty = 1 - G_f(0) = 1 - \frac{1}{800} = \underline{\frac{799}{800}}$

- $t_r\big|_{10\%}^{90\%} \approx \dfrac{2,16\,\xi + 0,60}{\omega_n} = \underline{0,84}$

- $t_s\big|_{2\%} \approx \dfrac{3,9}{\xi\omega_n} = \underline{3,9}$

Q5) Requisitos: $t_r\Big|_0^{100\%} = 10s$ e $M_p = 0{,}046$

$$\Rightarrow M_p = 0{,}046 = exp\left(\frac{-\pi\varepsilon}{\sqrt{1-\varepsilon^2}}\right)$$

$$-3{,}079 = \frac{-\pi\varepsilon}{\sqrt{1-\varepsilon^2}} \rightarrow 0{,}98 = \frac{\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$\varepsilon^2 = 0{,}962 - 0{,}962\varepsilon^2 \rightarrow \varepsilon^2 = 0{,}49 \rightarrow \boxed{\varepsilon = 0{,}7}$$

$$\Rightarrow t_r\Big|_0^{100\%} = \frac{\pi - \arccos(\varepsilon)}{\omega_n\sqrt{1-\varepsilon^2}} \rightarrow \boxed{\omega_n = 0{,}329 \text{ rad/s}}$$

Do circuito: $\boxed{\dot{Q} + \frac{Q}{RC} = \frac{V}{R}}$

$$\boxed{G(s) = \frac{1/R}{s + \frac{1}{RC}}}$$

Com o pré-filtro, temos:

$$G_f(s) = \frac{K_i \cdot K}{s^2 + (K_p \cdot K + a)s + K_i \cdot K} \cdot \begin{cases} K = 1/R \\ a = \frac{1}{RC} \end{cases}$$

Logo, $\frac{K_i}{R} = \omega_n^2$ e $\frac{K_p}{R} + \frac{1}{RC} = 2\varepsilon\omega_n$

$\boxed{K_i = 1082{,}4}$ $\boxed{K_p = -995{,}394}$