Lista 2 - CMC-12

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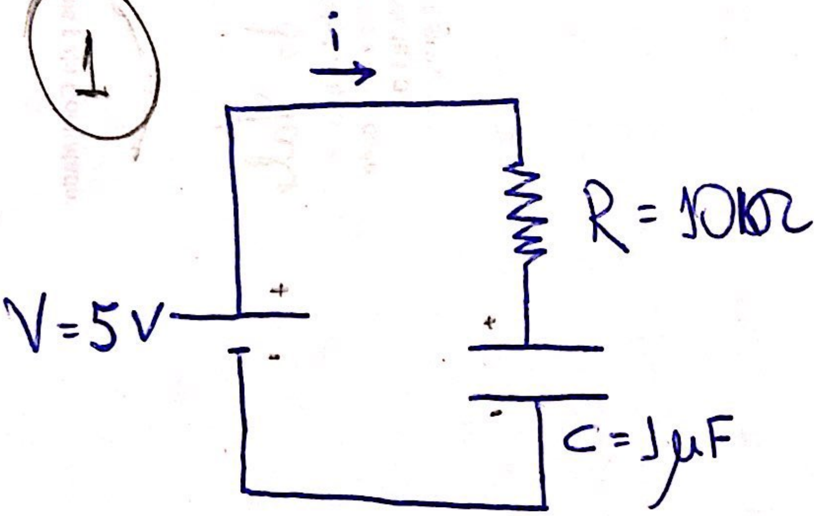

a) EDO: $V - Ri - \frac{Q}{C} = 0$

$i = \dot{Q} \Rightarrow \boxed{\dot{Q} + \frac{Q}{RC} = \frac{V}{R}}$

Resolvendo a EDO:

$$\boxed{Q(t) = C \cdot V \left(1 - e^{-t/RC}\right)}$$

Define-se $\tau = RC$ (constante de tempo)

$$\boxed{\tau = 0{,}01\ s}$$

b) $Q(0{,}02) = 5 \cdot 10^{-6} \cdot \left(1 - e^{-2}\right) = \boxed{4{,}323\ \mu C}$

② Do esquema, temos:

$$\begin{cases} V = Ri + \cancel{L\dot{i}}^{\,0} + V_b \\ \tau_r = \tau_m - b\omega = J\cdot\dot{\omega} \end{cases} \quad e \quad \begin{cases} \tau_m = K_t \cdot i \\ \omega = K_\omega \cdot V_b \end{cases}$$

$$P_{ele} = P_{mec} \rightarrow \underline{K_t = K_\omega^{-1}}$$

Para $L \approx 0$:

$$i = \frac{J\dot{\omega} + b\omega}{K_t} = \frac{V - K_t\,\omega}{R}$$

$$\dot{\omega} + \left(\frac{R\cdot b + K_t^2}{R\cdot J}\right)\omega = \frac{K_t\,V}{J\cdot R}$$

$$\omega(t) = \frac{K_t\cdot V}{(K_t^2 + R\cdot b)}\left(1 - e^{-t/\tau}\right)$$

com $\tau = \dfrac{R\cdot J}{K_t^2 + Rb}$

Para $t \rightarrow \infty$, $\omega = \dfrac{K_t\cdot V}{K_t^2 + R\cdot b} \approx 1374{,}34^3$ rad/s

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$$\begin{cases} K(x_2 - x_1) + b(\dot{x}_2 - \dot{x}_1) = m_1 \ddot{x}_1 \\ f - b(\dot{x}_2 - \dot{x}_1) - K(x_2 - x_1) = m_2 \ddot{x}_2 \end{cases}$$

$$\ddot{x}_1 = \frac{-Kx_1 - b\dot{x}_1 + Kx_2 + b\dot{x}_2}{m_1} \quad , \quad \ddot{x}_2 = \frac{f + Kx_1 + b\dot{x}_1 - Kx_2 - b\dot{x}_2}{m_2}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dfrac{-Kx_1 - b\dot{x}_1 + Kx_2 + b\dot{x}_2}{m_1} \\ \dot{x}_2 \\ \dfrac{f + Kx_1 + b\dot{x}_1 - Kx_2 - b\dot{x}_2}{m_2} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & -b/m_1 & K/m_1 & b/m_1 \\ 0 & 0 & 0 & 1 \\ K/m_2 & b/m_2 & -K/m_2 & -b/m_2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix}}_{B} f$$

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \underbrace{0_{2\times1}}_{D} f$$