Lista 5 - CMC-12

Q1) Sabemos que:

$$\begin{cases} M_p = \exp\left(-\frac{\pi \varepsilon}{\sqrt{1-\varepsilon^2}}\right) \rightarrow \ln(M_p) = \frac{-\pi\varepsilon}{\sqrt{1-\varepsilon^2}} \\ t_r\Big|_0^{100\%} = \frac{\pi - \arccos(\varepsilon)}{\omega_n\sqrt{1-\varepsilon^2}} \end{cases}$$

$$(1-\varepsilon^2)\ln^2(M_p) = \pi^2\varepsilon^2$$

$$\boxed{\varepsilon = \left(1 + \frac{\pi^2}{\ln^2(M_p)}\right)^{-1/2}}$$

$$\Rightarrow \ln(M_p) = -\frac{\pi\varepsilon}{\sqrt{1-\varepsilon^2}} = -\frac{\pi\sigma}{\omega_d} \rightarrow \boxed{\sigma = \frac{-\omega_d \ln(M_p)}{\pi}}$$

$$\Rightarrow \omega_d\, t_r\Big|_0^{100\%} = \pi - \arccos(\varepsilon) \rightarrow \boxed{\omega_d = \frac{\pi - \arccos(\varepsilon)}{t_r\Big|_0^{100\%}}}$$

Deixando mais organizado:

$$\rightarrow \varepsilon = \left(1 + \left(\frac{\pi}{\ln(M_p)}\right)^2\right)^{-1/2}$$

$$\rightarrow \sigma = \frac{\ln(M_p)}{\pi} \cdot \left[\frac{\arccos(\varepsilon) - \pi}{t_r\Big|_0^{100\%}}\right]$$

$$\rightarrow \omega_d = \frac{\pi - \arccos(\varepsilon)}{t_r\Big|_0^{100\%}}$$

$$P_1 = -\sigma + \omega_d j$$

$$P_2 = -\sigma - \omega_d j$$

Q2) Resposta ao degrau unitário:

$$y(t) = 1 - \frac{\omega_n}{\omega_d} \cdot e^{-\sigma t} \cdot sen(\omega_d \cdot t + \beta)$$

$$1 - \frac{\omega_n}{\omega_d} e^{-\sigma t} \leq y(t) \leq 1 + \frac{\omega_n}{\omega_d} e^{-\sigma t}$$

$$exp(-\sigma t) = 0{,}02 \rightarrow t_s\big|_{2\%} = \frac{-\ln(0{,}02)}{\xi \omega_n}$$

Q3) Do diagrama, temos:

$$I = (R - I) \cdot \underbrace{\left(\frac{K_p \cdot s + K_i}{s}\right) \cdot \left(\frac{1}{L \cdot s + R}\right)}_{x}$$

$$\frac{I(s)}{R(s)} = G(s) = \frac{x}{1 + x}$$

$$G(s) = \frac{K_p \cdot s + K_i}{L s^2 + (R + K_p) s + K_i}$$

Polos: raízes de $s^2 + \frac{(R + K_p)}{L} s + \frac{K_i}{L}$

$$P_1 + P_2 = -2 \varepsilon \omega_n = -\frac{(R + K_p)}{L}$$

$$P_1 \cdot P_2 = \omega_n^2 = \frac{K_i}{L}$$

Logo:
$$\begin{cases} K_i = \omega_n^2 L \\ K_p = 2 \varepsilon \omega_n L - R \end{cases}$$

Q4) Do diagrama de blocos:

$$X = (R \cdot F - X) \cdot \underbrace{\frac{(K_p + K_d s)}{(ms^2 + bs)}}_{L}$$

$$\frac{X}{R} = G(s) = \frac{F \cdot L}{1 + L}$$

$$G(s) = \frac{F(s) \cdot (K_p + K_d s)}{ms^2 + (K_d + b)s + K_p}$$

Comparando com $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

temos: $2\xi\omega_n = \dfrac{K_d + b}{m}$ e $\omega_n^2 = \dfrac{K_p}{m}$

$\boxed{\omega_n = \sqrt{\dfrac{K_p}{m}}}$ e $\boxed{\xi = \dfrac{K_d + b}{2\sqrt{K_p \cdot m}}}$. Com os valores do enunciado $\rightarrow$ $K_p = m\omega_n^2$ e $K_d = 2\xi\omega_n \cdot m - b$

$\boxed{C(s) = K_p + K_d \cdot s}$ e $\dfrac{F(s)}{m} \cdot (K_p + K_d \cdot s) = \omega_n^2$

$$\boxed{F(s) = \frac{K_p}{(K_p + K_d s)}}$$