AMMM Course Project

Adria Lisa Bou[1] Lucas Estevao Bazilio[2]

Algorithmic Methods for Mathematical Models

Bachelor Degree in Informatics Engineering (Computing)

12/12/2023

Contents

Variables used in the ILP formulation

- $f_i \in T \cup \{0\}$: Finish baking time of an order i.

$$y_i = 0 \implies f_i = 0$$

- $x_{ij} \in \{0, 1\}$: Schedule variable.

$$x_{ij} = 1 \iff \text{order } i \in N \text{ is being baked at time slot } j \in T.$$

- $z_{ij} \in \{0, 1\}$: Indicates a change of value between $x_{i(j-1)}$ and x_{ij}.

$$z_{i1} = x_{i1} \quad z_{ij} = \begin{cases} 1 & \text{if } x_{i(j-1)} \neq x_{ij} \\ 0 & \text{if } x_{i(j-1)} = x_{ij} \end{cases} \qquad \forall i \in N$$

To ensure the orders are baked continuously, $\sum_{j \in T} z_{ij} \leq 2, \forall i \in N$

Some remarkable constrains

- Our formulation for the objective function and most constraints is the natural one.
- Four constraints to define z were inspired by a XOR gate:

$$\forall (i,j) \in N \times T \setminus \{1\} \quad \begin{cases} z_{ij} \geq x_{ij} - x_{i(j-1)} \\ z_{ij} \geq x_{i(j-1)} - x_{ij} \\ z_{ij} \leq x_{ij} + x_{i(j-1)} \\ z_{ij} \leq 2 - x_{ij} - x_{i(j-1)} \end{cases}$$

- Defining the finishing time f_i was the hardest part.

Some remarkable constrains

$$1 + f_i \geq jz_{ij} \tag{1}$$

The tightest bound will correspond to the 2nd change of values in x_{ij}.

$$f_i \geq (2y_i - \sum_{j \in T} z_{ij})t \tag{2}$$

When an order finishes at the last time $j = t$, we will have $\sum_{j \in T} z_{ij} = 1$.

$$1 + f_i \leq jz_{ij} + (2 - \sum_{k=1}^{j} z_{ik})(t+1) + (1 - z_{ij})(t+1) \tag{3}$$

The tightest bound will correspond to the 2nd change of values in x_{ij}.

$$q(i,S) = \begin{cases} p_i & \text{if } S \cup \{(i,f)\} \text{ is feasible for some } f \in T \\ -\infty & \text{otherwise.} \end{cases} \qquad \forall i \in N$$

- A first-improving local search was performed

$$RCL(i) = \{j \in N \mid j \text{ is a feasible addition to } S \wedge p_j \geq p_{\text{cota}}\}$$

$$p_{\text{cota}} = p_n + \alpha \cdot (p_i - p_n)$$

Tuning of the α parameter

- 100 instances of size $n = 5000$ were generated
- Values of $m = 20$ and $m = 60$ of the GRASP algorithm were chosen

Tuning of the α parameter: Results for $m = 20$

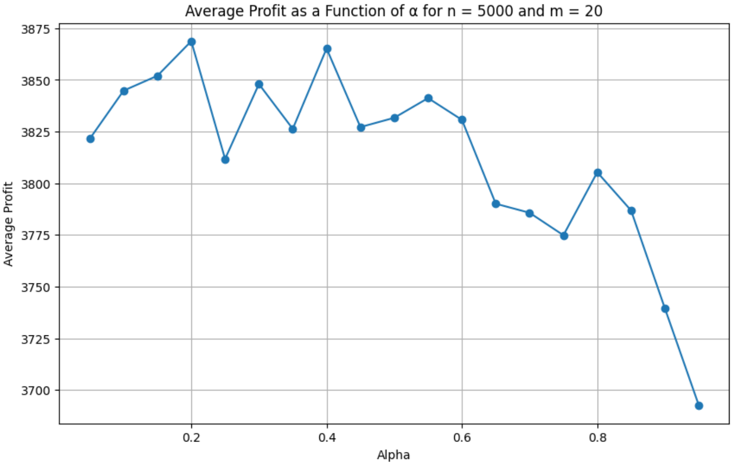

Figure 1: Average profit as a function of α for $n = 5000$ and $m = 20$

Tuning of the α parameter: Results for $m = 60$

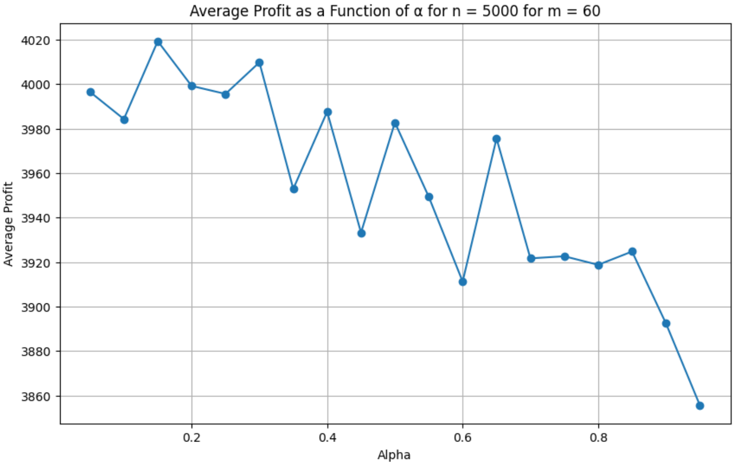

Figure 2: Average profit as a function of α for $n = 5000$ and $m = 60$

Performance Comparison

The comparative analysis has been made in terms of

- Solution quality
- Computation time

Solution Quality

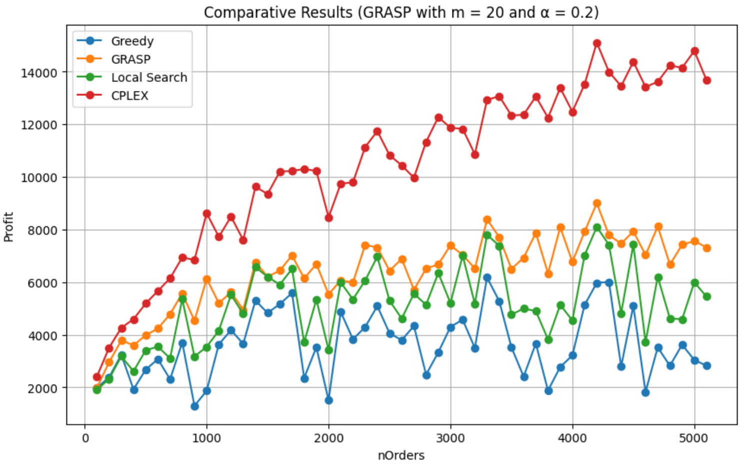

Figure 3: Comparative Results regarding the quality of the solution among CPLEX and heuristic algorithms. (GRASP with $m = 20$ and $\alpha = 0.2$)

Solution Quality

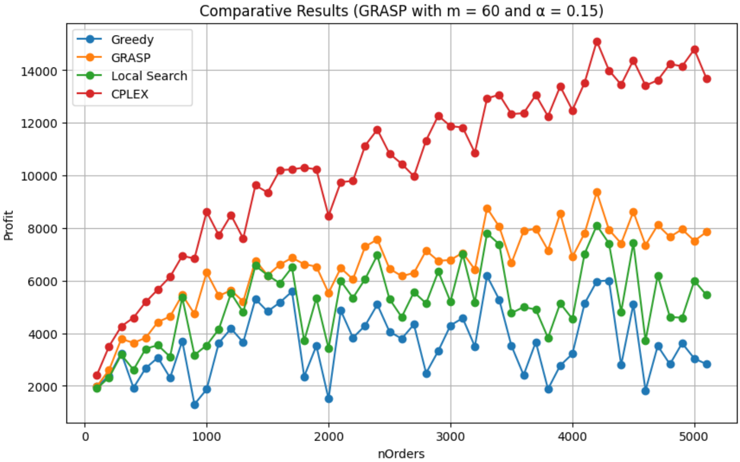

Figure 4: Comparative Results regarding the quality of the solution among CPLEX and heuristic algorithms. (GRASP with $m = 60$ and $\alpha = 0.15$)

Solution Quality

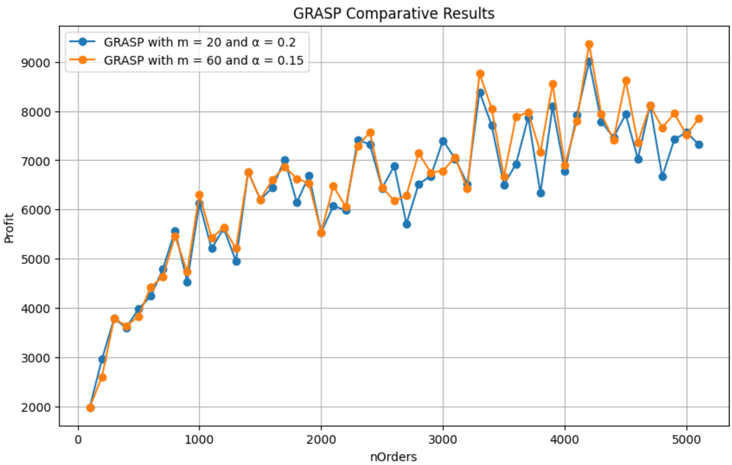

Figure 5: Comparative Results regarding the quality of the solution among GRASP algorithm as a function of *m* and tuned α

Computation Time

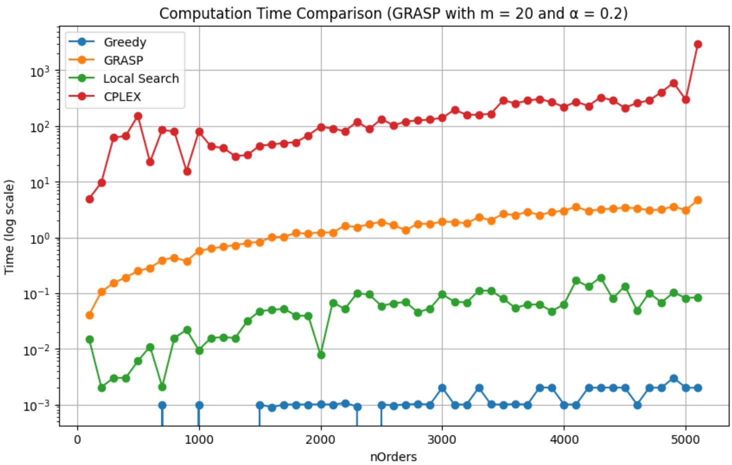

Figure 6: Comparative Results regarding the computation time of the solution among CPLEX and heuristic algorithms. (GRASP with $m = 20$ and $\alpha = 0.2$)

Computation Time

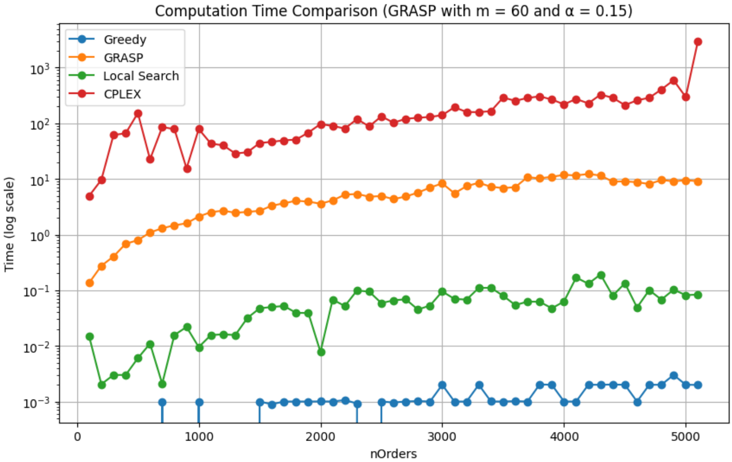

Figure 7: Comparative Results regarding the computation time of the solution among CPLEX and heuristic algorithms. (GRASP with $m = 60$ and $\alpha = 0.15$)

Computation Time

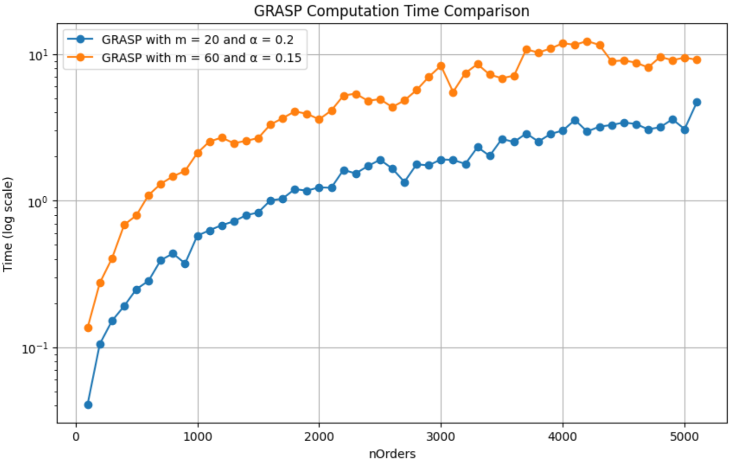

Figure 8: Comparative Results regarding the computation time among GRASP algorithm as a function of *m* and tuned α

Final Discussion

- It is observed a discernible trade-off between solution quality and computation time.
- We should determine the level of optimization required for the problem versus the time constraints within which the solution is needed.

Thank You