Maastricht University

EBC2086 Time Series Modelling

Final Case: An Analysis of Belgian Economic Data

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1 Data and Motivation

In this paper, we will apply the learned theory from the course Time Series Modelling from the Maastricht University to selected data series of Belgium. We acquired the data from the FRED Economic Research department of the Federal Bank of St. Louis [1]. The series contains quarterly information about the real gross domestic product (rgdp), the export rate (export), industry production (prod_ind) and three more series which were eventually not included in the analysis. Most variables were available in seasonally adjusted or not seasonally adjusted values. We compiled all these series into a single data set and then excluded the values of the last two quarters (2020Q1, 2020Q2) as these contained the shock of the economic impact of the Corona-Crisis. In the instructions of our data selection process, we were permitted exclude them as including the outliers would have complicated our analysis. However, we kept a spare set and might repeat parts of the following analysis in the future to compare the result deviations. Our motivation behind selecting Belgium was mainly curiosity since one of the group members lived in Belgium and it would be interesting to compare the analysis with knowledge about the Dutch economy.

Eventually, we have a data set with nine different series ranging from 1995Q1 to 2019Q4 which sum up to 100 observations.

[1] https://fred.stlouisfed.org/series/CLVMNACSCAB1GQBE

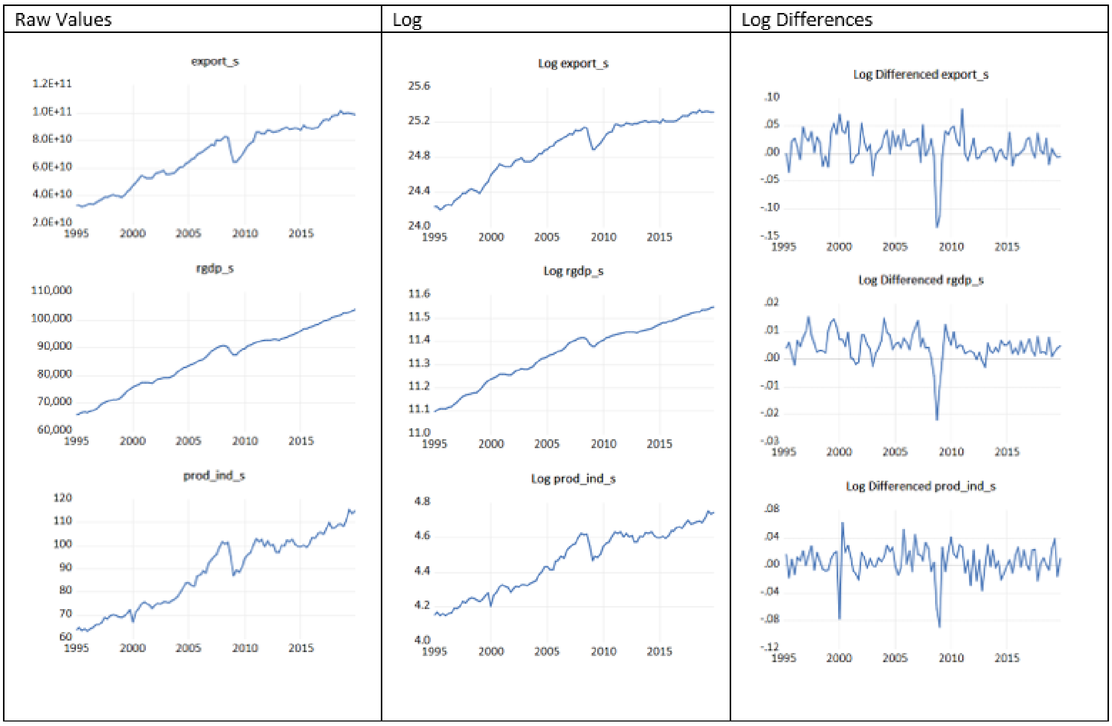


Figure 1: Visual comparison of three series in different levels

We will use the seasonally adjusted real gross domestic product (measured in millions) as our main dependant variable and the other two series displayed in Figure 1 as our explanatory variables. As the three series contain greatly differing scales of raw levels we find it useful to compute our analysis with log levels. This will also make coefficients easier to interpret. Further, regarding the plotted series, a steady positive trend can be seen in all three series with a strong outlier around the financial crisis in 2008.

2 Univariate Analysis

Commencing our analysis, we first investigate our dependant variable - the *rgdp_s*. As previously mentioned a trend is obvious. To determine whether this trend is stochastic or deterministic we compute the augmented Dickey-Fuller (ADF) test. The null hypothesis of not having a unit root is not rejected and therefore we have a difference stationary series (Figure 2). After recomputing the ADF test with the series in first difference the null hypothesis is rejected meaning that taking the series in second difference will not be necessary such as it is not necessary to incorporate further deterministic components such as a trend dynamic.

Lag Length: 1 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.343196	0.4068
Test critical values:	1% level	-4.054393	
	5% level	-3.456319	
	10% level	-3.153989	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(RGDP_S))
Method: Least Squares
Date: 10/20/20 Time: 14:27
Sample (adjusted): 1995Q3 2019Q4
Included observations: 98 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(RGDP_S(-1))	-0.043123	0.018404	-2.343196	0.0212
D(LOG(RGDP_S(-1)))	0.557729	0.084363	6.611097	0.0000
C	0.482862	0.204822	2.357475	0.0205
@TREND("1995Q1")	0.000173	8.24E-05	2.100533	0.0384

R-squared	0.362562	Mean dependent var	0.004556
Adjusted R-squared	0.342218	S.D. dependent var	0.005062
S.E. of regression	0.004106	Akaike info criterion	-8.112981
Sum squared resid	0.001584	Schwarz criterion	-8.007472
Log likelihood	401.5361	Hannan-Quinn criter.	-8.070305
F-statistic	17.82174	Durbin-Watson stat	1.924186
Prob(F-statistic)	0.000000		

Figure 2: Output for ADF test on *rgdp_s*

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 2 lags

F-statistic	2.373292	Prob. F(2,94)	0.0987
Obs*R-squared	4.710696	Prob. Chi-Square(2)	0.0949

Heteroskedasticity Test: Breusch-Pagan-Godfrey
Null hypothesis: Homoskedasticity

F-statistic	4.144069	Prob. F(1,96)	0.0445
Obs*R-squared	4.055345	Prob. Chi-Square(1)	0.0440
Scaled explained SS	13.33332	Prob. Chi-Square(1)	0.0003

Figure 3: Test outputs LM test (t) and heteroskedasticity (b) *rgdp_s*

Next, we made sure our model was not auto-correlated. A first LM test rejected the null hypothesis of no auto-correlation but after adding on lag *dlog(rgdp_s(-1))* to the model the test showed that auto-correlation had been removed. When testing for heteroskedasticity we rejected the null hypothesis of homoskedasticity (Figure 3). We then generated an equation which included lags for the quarters Q2 to Q4 assuming that the intercept output includes values for Q1. The equation looked as follows:

$$dlog(rgdp\_s) = c + dlog(rgdp\_s(-1)) + @quarter2 + @quarter3 + @quarter4 \quad (1)$$

The output of this estimation already indicated that the coefficients for Q2 to Q4 were insignificant but we proceeded in computing the Wald test for joint significance of the latter variables. With the null hypothesis being: $@quarter2 = @quarter3 = @quarter4 = 0$.

We fail to reject the null hypothesis (Figure 4) meaning that each of the output coefficients could be regarded as equal to zero thus we exclude explanatory components for seasonality from our model. This could have been expected since we are already working with seasonally adjusted data.

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	0.470407	(3, 93)	0.7036
Chi-square	1.411221	3	0.7029

Null Hypothesis: C(3) = C(4) = C(5) = 0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(3)	0.001012	0.001192
C(4)	-0.000104	0.001118
C(5)	-0.000147	0.001431

Restrictions are linear in coefficients.

Figure 4: Output for Wald test on joint significance

Thereafter we had to evaluate whether we have an AR, MA or ARMA model. When regarding the correlogram (Figure 5) it can be seen that in the autocorrelation column the first two values and in the partial autocorrelation column the first value exceeds the critical value. If this is an AR model then the partial autocorrelation should be significant while the autocorrelation decreases. For MA models it is vice versa. In an ARMA model, both columns should decrease. Hence an AR(1) model could fit as the first bar is surpassing the critical value and autocorrelation is decreasing to 0 but then rises again. An MA(2) model could also be considered as the first two bars of autocorrelation are significant and partial autocorrelation stays around 0 with a few outliers.

Date: 09/29/20 Time: 01:14
Sample (adjusted): 1995Q2 2019Q4
Included observations: 99 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.563	0.563	32.323	0.000
		2	0.250	-0.097	38.777	0.000
		3	-0.034	-0.197	38.896	0.000
		4	-0.138	-0.020	40.903	0.000
		5	-0.150	-0.015	43.305	0.000
		6	-0.146	-0.075	45.591	0.000
		7	-0.110	-0.020	46.899	0.000
		8	-0.106	-0.065	48.140	0.000
		9	-0.080	-0.022	48.849	0.000
		10	0.098	0.221	49.925	0.000
		11	0.082	-0.134	50.694	0.000
		12	0.107	0.039	51.999	0.000
		13	0.045	-0.002	52.238	0.000
		14	0.020	0.003	52.283	0.000
		15	0.021	0.043	52.334	0.000
		16	-0.027	-0.062	52.421	0.000
		17	0.001	0.046	52.422	0.000
		18	-0.031	-0.022	52.539	0.000
		19	-0.068	-0.042	53.115	0.000
		20	0.040	0.142	53.312	0.000
		21	0.026	-0.062	53.401	0.000
		22	0.052	-0.009	53.755	0.000
		23	0.022	0.034	53.816	0.000
		24	-0.042	-0.105	54.054	0.000
		25	-0.038	0.037	54.254	0.001
		26	-0.070	-0.016	54.930	0.001
		27	0.080	0.157	55.824	0.001
		28	0.169	0.110	59.835	0.000
		29	0.232	0.104	67.524	0.000
		30	0.215	-0.014	74.229	0.000
		31	0.096	-0.013	75.581	0.000
		32	-0.081	-0.188	76.558	0.000
		33	-0.100	0.151	78.065	0.000
		34	-0.119	-0.043	80.260	0.000

Figure 5: Correlogram for *dlog(rgdp_s)*

Dependent Variable: DLOG(RGDP_S)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 09/29/20 Time: 01:29
Sample: 1995Q2 2019Q4
Included observations: 99
Convergence achieved after 14 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004551	0.001032	4.408800	0.0000
AR(1)	0.557314	0.093494	5.960980	0.0000
SIGMASQ	1.72E-05	1.87E-06	9.179755	0.0000

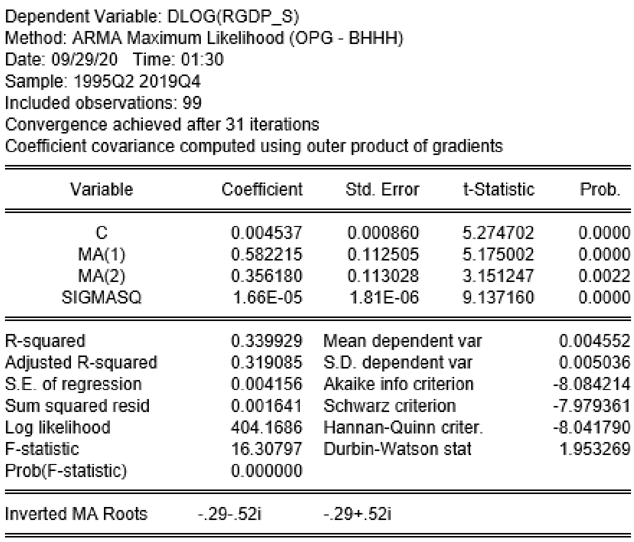
R-squared	0.316792	Mean dependent var	0.004552
Adjusted R-squared	0.302558	S.D. dependent var	0.005036
S.E. of regression	0.004206	Akaike info criterion	-8.071005
Sum squared resid	0.001698	Schwarz criterion	-7.992365
Log likelihood	402.5148	Hannan-Quinn criter.	-8.039187
F-statistic	22.25675	Durbin-Watson stat	1.879841
Prob(F-statistic)	0.000000		
Inverted AR Roots	.56		

Date: 09/29/20 Time: 01:32
Sample (adjusted): 1995Q2 2019Q4
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.060	0.060	0.3669	
		2	0.048	0.045	0.6082	0.435
		3	-0.157	-0.163	3.1608	0.206
		4	-0.115	-0.101	4.5508	0.208
		5	-0.056	-0.029	4.8885	0.299
		6	-0.068	-0.080	5.3806	0.371
		7	-0.005	-0.029	5.3830	0.496
		8	-0.049	-0.069	5.6502	0.581
		9	-0.146	-0.182	8.0161	0.432
		10	0.185	0.195	11.882	0.220
		11	-0.009	-0.048	11.891	0.292
		12	0.101	0.014	13.053	0.290
		13	-0.016	0.004	13.082	0.363
		14	-0.017	-0.019	13.115	0.439
		15	0.046	0.067	13.372	0.497
		16	-0.070	-0.050	13.954	0.529
		17	0.050	0.030	14.253	0.580
		18	-0.005	0.018	14.256	0.649
		19	-0.137	-0.111	16.604	0.550
		20	0.110	0.115	18.131	0.514
		21	-0.025	0.007	18.209	0.574
		22	0.062	-0.024	18.704	0.604
		23	0.033	0.078	18.845	0.655
		24	-0.067	-0.081	19.448	0.675
		25	0.018	0.000	19.492	0.725
		26	-0.169	-0.103	23.413	0.553
		27	0.074	0.041	24.180	0.566
		28	0.069	0.075	24.845	0.583
		29	0.132	0.152	27.318	0.501
		30	0.145	0.077	30.372	0.396
		31	0.074	0.135	31.175	0.407
		32	-0.152	-0.197	34.628	0.299
		33	-0.028	0.065	34.746	0.338

Figure 6: AR(1) model with corresponding correlogram

After running the AR(1) model it seems that the AR(1) coefficient is highly significant and all autocorrelation seems to be removed (Figure 6). We can thus conclude that the AR(1) model does succeed to explain the dynamics of the *rgdp_s* variable. Adding a second (AR(2)) component resulted in an insignificant coefficient.

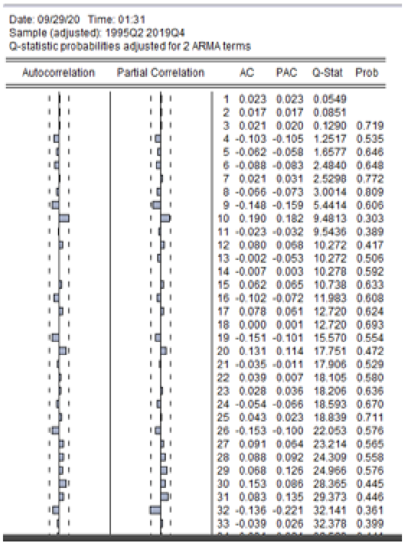


Dependent Variable: DLOG(RGDP_S)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 09/29/20 Time: 01:30
Sample: 1995Q2 2019Q4
Included observations: 99
Convergence achieved after 31 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004537	0.000860	5.274702	0.0000
MA(1)	0.582215	0.112505	5.175002	0.0000
MA(2)	0.356180	0.113028	3.151247	0.0022
SIGMASQ	1.66E-05	1.81E-06	9.137160	0.0000

R-squared	0.339929	Mean dependent var	0.004552
Adjusted R-squared	0.319085	S.D. dependent var	0.005036
S.E. of regression	0.004156	Akaike info criterion	-8.084214
Sum squared resid	0.001641	Schwarz criterion	-7.979361
Log likelihood	404.1686	Hannan-Quinn criter.	-8.041790
F-statistic	16.30797	Durbin-Watson stat	1.953269
Prob(F-statistic)	0.000000		

Inverted MA Roots	-.29-.52i	-.29+.52i



Date: 09/29/20 Time: 01:31
Sample (adjusted): 1995Q2 2019Q4
Q-statistic probabilities adjusted for 2 ARMA terms

	AC	PAC	Q-Stat	Prob
1	0.023	0.023	0.0549	
2	0.017	0.017	0.0851	
3	0.021	0.020	0.1290	0.719
4	-0.103	-0.105	1.2517	0.535
5	-0.062	-0.058	1.6577	0.646
6	-0.088	-0.083	2.4840	0.648
7	0.021	0.031	2.5298	0.772
8	-0.066	-0.073	3.0014	0.809
9	-0.148	-0.159	5.4414	0.606
10	0.190	0.182	9.4813	0.303
11	-0.023	-0.032	9.5436	0.389
12	0.080	0.068	10.272	0.417
13	-0.002	-0.053	10.272	0.506
14	-0.007	0.003	10.278	0.592
15	0.062	0.065	10.738	0.633
16	-0.102	-0.072	11.983	0.608
17	0.078	0.061	12.720	0.624
18	0.000	0.001	12.720	0.693
19	-0.151	-0.101	15.570	0.554
20	0.131	0.114	17.751	0.472
21	-0.035	-0.011	17.906	0.529
22	0.039	0.007	18.105	0.580
23	0.028	0.036	18.206	0.636
24	-0.054	-0.066	18.593	0.670
25	0.043	0.023	18.839	0.711
26	-0.153	-0.100	22.053	0.576
27	0.091	0.064	23.214	0.565
28	0.088	0.092	24.309	0.558
29	0.068	0.126	24.966	0.576
30	0.153	0.086	28.365	0.445
31	0.083	0.135	29.373	0.446
32	-0.136	-0.221	32.141	0.361
33	-0.039	0.026	32.378	0.399

Figure 7: MA(2) model with corresponding correlogram

The MA(2) model achieved similar successful results. Both MA coefficients were significant and the correlogram also indicated that autocorrelation had been removed (Figure 7). Since the AR(1) model is more parsimonious than the MA(2) model, we decide to continue with the former. The AR(1) component is based in the immediately lag of the dependant variable and thus we continue with the misspecification tests. Autocorrelation and heteroskedasticity had already been tested, the null hypothesis for normality was rejected (Figure 8) and the Ramsey test null hypothesis was also not rejected (Figure 9) indicating that the fitted values are not linear. Hence, our final model equation for the univariate model is:

$$dlog(rgdp\_s) = c + dlog(rgdp\_s(-1)) \tag{2}$$

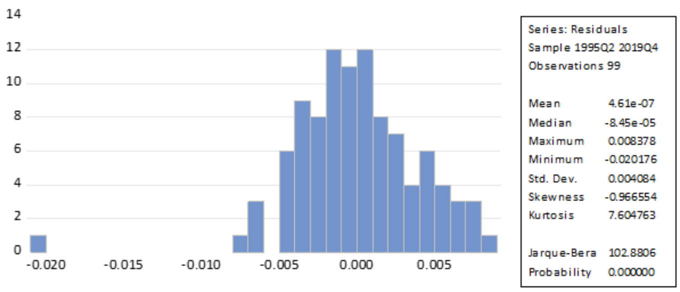


Figure 8: Output of normality test

Ramsey RESET Test
Equation: UNTITLED
Omitted Variables: Squares of fitted values
Specification: DLOG(RGDP_S) C AR(1)

	Value	df	Probability
t-statistic	0.566317	95	0.5725
F-statistic	0.320714	(1, 95)	0.5725
Likelihood ratio	0.296086	1	0.5863

Figure 9: Output of Ramsey test

Finally, we recompute our model for the time frame 1995Q1 to 2014Q4. Hence we excluded the values for the last 20 observations. Then we forecasted those last 20 observations based on the newly computed model. The outputs for the dynamic and static model can be seen in Figure 10. The dynamic model scored a root mean squared error (RMSE) of 803.50 and the static model a RMSE of 313.95. Note that although these values give a forecast accuracy measurement of the model the results of the static and dynamic forecast should not directly be compared with each other. We will further evaluate these forecasts in part 6 where we compare the results from different type of models between each other.

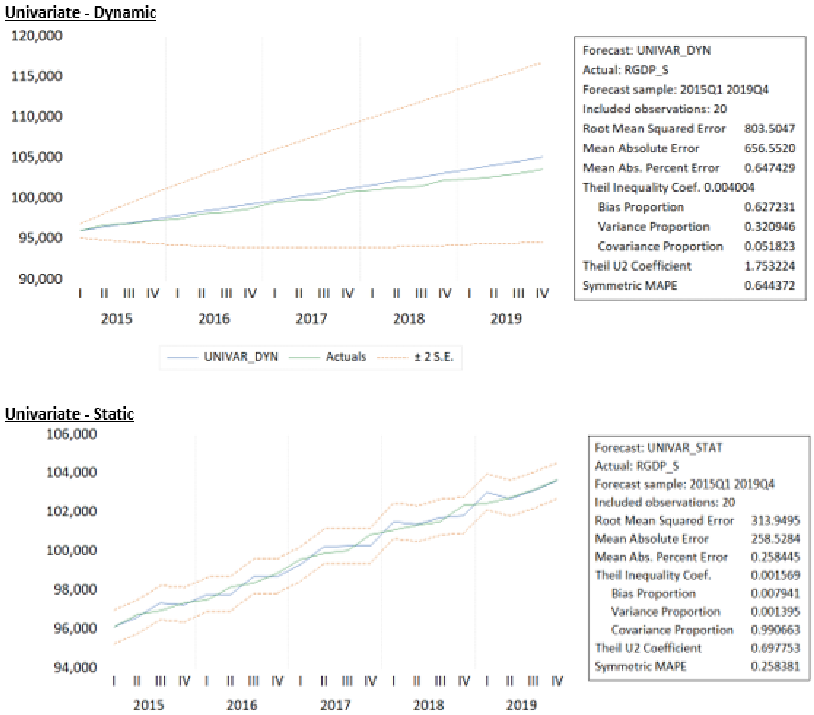


Figure 10: Forecasts of next 20 observations for the univariate model based on 1995Q1 to 2014Q4

3 Multivariate Models part I

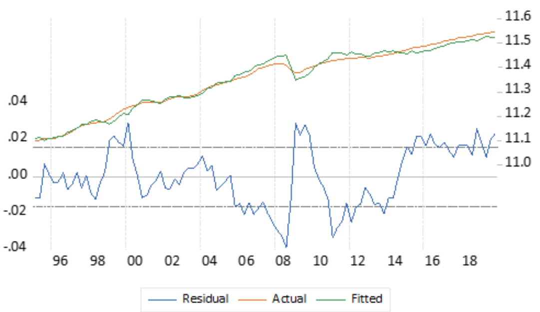
The first step in setting up the model will be to determine whether the explanatory variables also have a unit root and whether they should be taken level, first or second difference. To investigate this, we compute the intermediate ADF test. The results can be seen in Figure 11 and indicate that all three series have a unit root since the null hypothesis of unit root failed to be rejected. Computing the test again while having the three series in first differences gives us significant p-values and again second differences are not necessary.

To investigate the three variables for cointegration we computed a model with *log(rgdp_s)* as the dependent variable and the other two as independent variables. The residuals which were a plotted in Figure 12 were than saved as a new series called *resid01*. We manually computed the Engle-Granger test for cointegration by running the ADF unit root test on *resid01*. Considering the critical value for the 5%-level for this test (-2.89), the null hypothesis was not rejected (Also Figure 12) meaning that the residuals have a unit root, consequently, we conclude that or dependent variable Y_t does not cointegrate with the X_t explanatory series and they will tend to deviate away from each other in the long-run. We can thus remain with using all series in first differences. It can be investigated here if the test should be run again while having all three series in first differences.

Intermediate ADF test results UNTITLED

Series	Prob.	Lag	Max Lag	Obs
LOG(EXPORT_S)	0.5533	1	12	98
LOG(PROD_IND_S)	0.3120	0	12	99
LOG(RGDP_S)	0.4068	1	12	98

Figure 11: Intermediate ADF test



Null Hypothesis: RESID01 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.772278	0.0660
Test critical values:	1% level	-3.497727	
	5% level	-2.890926	
	10% level	-2.582514	

Figure 12: Plotted residuals for multivariate equation and corresponding unit root test on residual values

4 Multivariate Models part II

We have now investigated our model for cointegration and will set up the best conditional (multivariate) model to explain the real gross domestic product for Belgium. Therefore we need to determine if further dynamic components need to be added. We already know that seasonal components for *rgdp_s* were not needed and since the other two series are also already seasonally adjusted it can be assumed that they too do not need seasonal dummies. Nonetheless we still need to look into autocorrelation. To determine the amount of lags needed per series to remove autocorrelation, we use the Autoregressive Distributed Lag Model method (ARDL) based on the Akaike information criteria.

The output (Figure 13) suggests three lags for *dlog(rgdp_s)* and none for the other two series. However, the second and third lags are insignificant. They will therefore be excluded. The coefficient for (prod_ind_s) is - as before - still insignificant which we find quite bizarre regarding the plotted values in Figure 1.

Dependent Variable: DLOG(RGDP_S)
Method: ARDL
Date: 10/03/20 Time: 14:49
Sample (adjusted): 1996Q1 2019Q4
Included observations: 96 after adjustments
Maximum dependent lags: 8 (Automatic selection)
Model selection method: Akaike info criterion (AIC)
Dynamic regressors (8 lags, automatic): DLOG(EXPORT_S) DLOG(PROD_IND_S)
Fixed regressors: C
Number of models evalulated: 648
Selected Model: ARDL(3, 0, 0)
Note: final equation sample is larger than selection sample

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
DLOG(RGDP_S(-1))	0.288094	0.096310	2.991324	0.0036
DLOG(RGDP_S(-2))	0.028981	0.097672	0.296714	0.7674
DLOG(RGDP_S(-3))	-0.192243	0.083434	-2.304120	0.0235
DLOG(EXPORT_S)	0.090103	0.014512	6.208996	0.0000
DLOG(PROD_IND_S)	0.013871	0.017750	0.781459	0.4366
C	0.002894	0.000530	5.456553	0.0000

R-squared	0.575697	Mean dependent var	0.004567
Adjusted R-squared	0.552125	S.D. dependent var	0.005107
S.E. of regression	0.003418	Akaike info criterion	-8.459338
Sum squared resid	0.001051	Schwarz criterion	-8.299066
Log likelihood	412.0482	Hannan-Quinn criter.	-8.394553
F-statistic	24.42256	Durbin-Watson stat	1.901290
Prob(F-statistic)	0.000000		

*Note: p-values and any subsequent tests do not account for model selection.

Figure 13: ARDL model estimation

Our multivariate model thus looks as follows:

$$dlog(rgdp\_s) = c + dlog(rgdp\_s(-1)) + dlog(export\_s) + dlog(prod\_ind\_s) \qquad (3)$$

Further misspecification tests resulted in no autocorrelation, no normality and heteroskedasticity. The outputs of which have not been included in this paper since this would be somewhat repetitive. The same methods as in part 2 were used.

We use our model to forecast values based on the same procedure as the univariate model. First, the model was re-estimated removing the last 20 periods and using the Hubert-White method to be heteroskedasticity-consistent. Then the dynamic and static forecast was computed for the period 2015Q1 to 2019Q4 (Figure 14). The dynamic model achieved a RMSE of 260.07 and the static model a RMSE of 281.28.

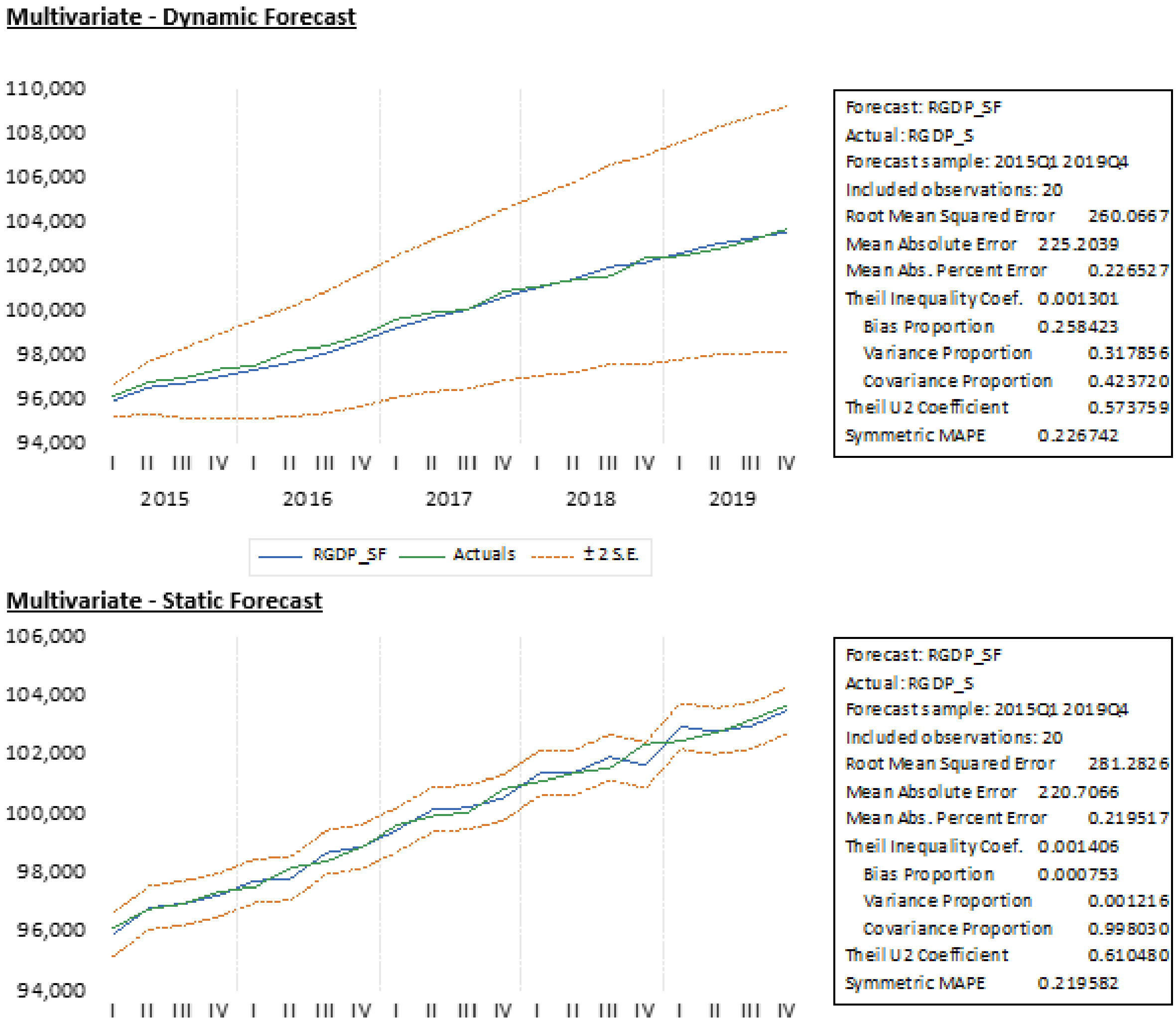


Figure 14: Forecasts of the conditional model for 2015Q1 to 2019Q4

5 Multivariate Models part III

In this section, we will estimate another multivariate model by using the Vector Autoregression Model (VAR). Because we did not have cointegration we used the first differences for all variables (and did not make use of an Error Correction Model). In Figure 15 the output for the VAR(1) model can be seen.

Standard errors in ( ) & t-statistics in [ ]

	DLOG(RGDP_S)	DLOG(EXPORT_S)	DLOG(PROD_IND_S)
DLOG(RGDP_S(-1))	0.628082	3.282341	2.013149
	(0.11770)	(0.72457)	(0.56878)
	[5.33624]	[4.53005]	[3.53944]
DLOG(EXPORT_S(-1))	-0.018936	-0.041412	0.090735
	(0.02040)	(0.12556)	(0.09856)
	[-0.92839]	[-0.32982]	[0.92058]
DLOG(PROD_IND_S(-1))	0.006362	0.018226	-0.265326
	(0.02157)	(0.13280)	(0.10424)
	[0.29493]	[0.13725]	[-2.54527]
C	0.001871	-0.003520	-0.002725
	(0.00059)	(0.00366)	(0.00287)
	[3.14868]	[-0.96225]	[-0.94878]
R-squared	0.323037	0.279251	0.226513
Adj. R-squared	0.301432	0.256249	0.201827
Sum sq. resids	0.001683	0.063768	0.039294
S.E. equation	0.004231	0.026046	0.020446
F-statistic	14.95181	12.13999	9.175869
Log likelihood	398.5883	220.4802	244.2050
Akaike AIC	-8.052822	-4.417963	-4.902143
Schwarz SC	-7.947313	-4.312454	-4.796635
Mean dependent	0.004556	0.011061	0.005869
S.D. dependent	0.005062	0.030201	0.022885

Figure 15: Output for VAR(1) estimation

We are however not yet certain whether we need a VAR(1) or VAR(2) etc. model. The lag length criteria (Figure 16) gives a suggestion which used lag length gives the most optimal information criteria for each model. The stars in Figure 16 indicate that all information criteria point towards one lag. In consequence, we decide to stick with the VAR(1) model.

Lag	LogL	LR	FPE	AIC	SC	HQ
0	797.6796	NA	5.22e-12	-17.46549	-17.38271	-17.43209
1	825.1827	52.58824*	3.47e-12*	-17.87215*	-17.54104*	-17.73857*
2	829.1415	7.308711	3.88e-12	-17.76135	-17.18192	-17.52759
3	835.4228	11.18197	4.13e-12	-17.70160	-16.87384	-17.36765
4	837.6259	3.776717	4.81e-12	-17.55222	-16.47613	-17.11808
5	840.0945	4.069150	5.58e-12	-17.40867	-16.08426	-16.87435
6	843.7357	5.761910	6.33e-12	-17.29089	-15.71816	-16.65639
7	850.1982	9.800365	6.76e-12	-17.23513	-15.41406	-16.50044
8	858.1286	11.50338	7.02e-12	-17.21162	-15.14223	-16.37675

Figure 16: Lag length criteria for VAR(1)

Next, we investigate the model for autocorrelation. Figure 17 displays a matrix of correlation plots between the respective variables with the grey horizontal line indicating the critical value for the presence of autocorrelation. The critical values are only exceeded for a few outlier periods, in all other cases autocorrelation is not an issue.

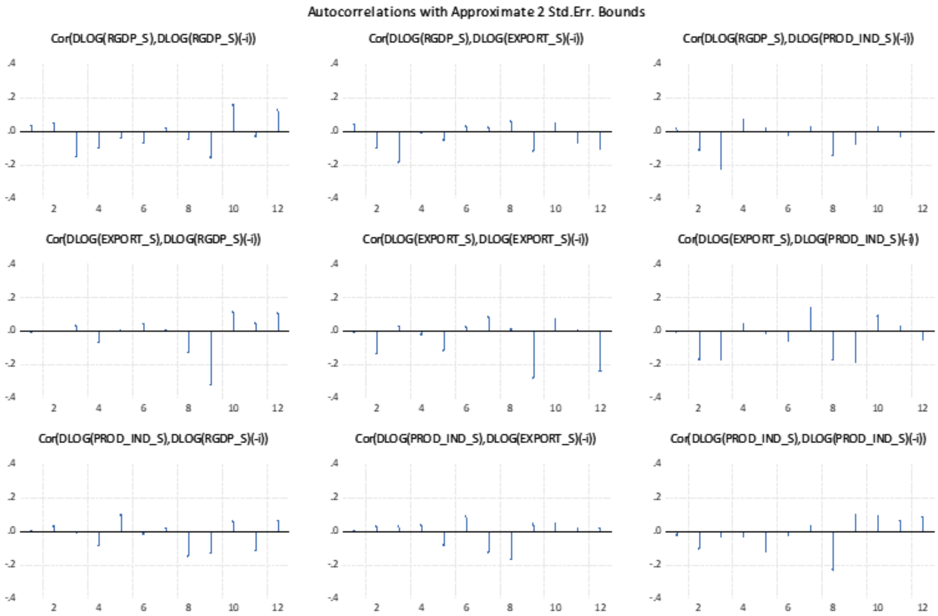


Figure 17: VAR(1) autocorrelation plot

The following step is to compute the Granger-Causality test. The results are displayed in Figure 18. The null hypothesis for the Granger-Causality test is that there is no causality between the listed dependent and explanatory variables. Here the test shows that only *rgdp_s* G-causes the other two variables and the other two variable G-cause none. We find this fact interesting because *rgdp_s* is supposed to be our dependent variable in the original equations.
To visualise the results of the Granger-Causality test we plot the impulse responses (Figure 19). The plot displays the effect of a shocked explanatory variable on the listed dependent variable. If we shock *rgdp_s* it has a positive impact on *export_s* and *prod_ind_s* in the period after. This impact lasts in both cases until period 7 (see bottom left two graphs). However, if we shock*rgdp_s* it has little influence on both other variables. This again is bizarre but complies with the last result.

Dependent variable: DLOG(RGDP_S)

Excluded	Chi-sq	df	Prob.
DLOG(EXPORT S)	0.861911	1	0.3532
DLOG(PROD_IND_S)	0.086987	1	0.7680
All	0.862786	2	0.6496

Dependent variable: DLOG(EXPORT_S)

Excluded	Chi-sq	df	Prob.
DLOG(RGDP_S)	20.52138	1	0.0000
DLOG(PROD IND S)	0.018838	1	0.8908
All	21.16474	2	0.0000

Dependent variable: DLOG(PROD IND S)

Excluded	Chi-sq	df	Prob.
DLOG(RGDP S)	12.52761	1	0.0004
DLOG(EXPORT_S)	0.847470	1	0.3573
All	27.47118	2	0.0000

Figure 18: Plotted impulse responses of VAR(1) model

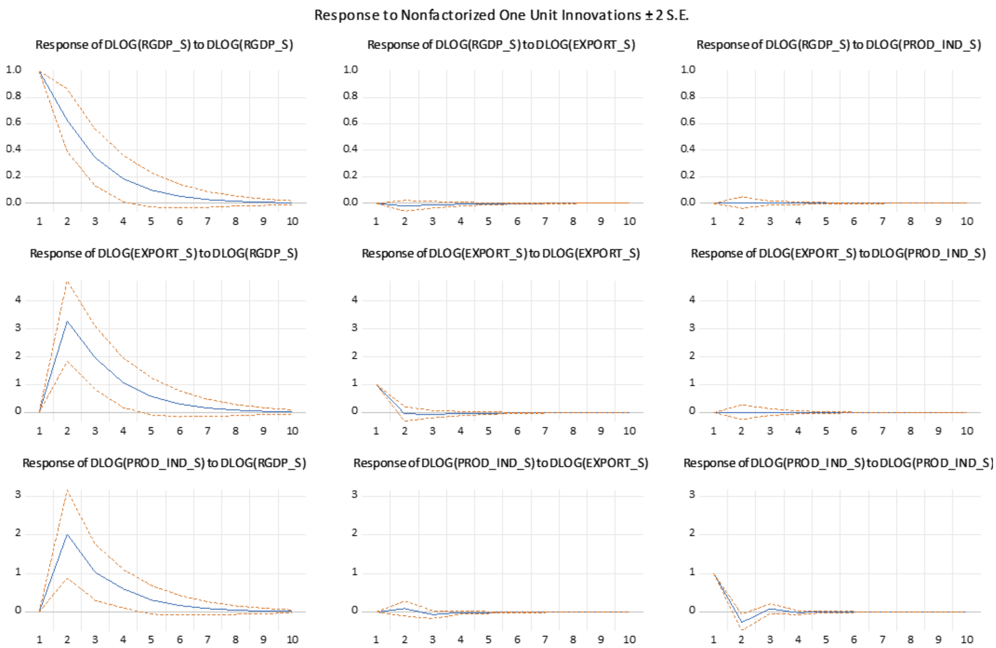


Figure 19: Plotted impulse responses of VAR(1) model

Concluding the VAR(1) model we do some forecasts again. First, we recompute the model while removing the last 20 periods (up to 2014Q4). Then we execute dynamic and static forecasts for the removed 20 periods (up to 2019Q4). The outputs for the static forecasts including the plots are displayed in Figure 21. The forecast with *rgdp_s* as the dependent variable had a RMSE of 335.36 and the dynamic forecast a RMSE of 942.49.

Forecast Evaluation
Date: 10/10/20 Time: 18:30
Sample: 2015Q1 2019Q4
Included observations: 20

Variable	Inc. obs.	RMSE	MAE	MAPE	Theil
EXPORT_S	20	8.65E+09	7.75E+09	7.355260	0.043789
PROD_IND_S	20	1.763517	1.332292	1.243164	0.008257
RGDP_S	20	942.4908	799.3195	0.780924	0.004693

RMSE: Root Mean Square Error
MAE: Mean Absolute Error
MAPE: Mean Absolute Percentage Error
Theil: Theil inequality coefficient

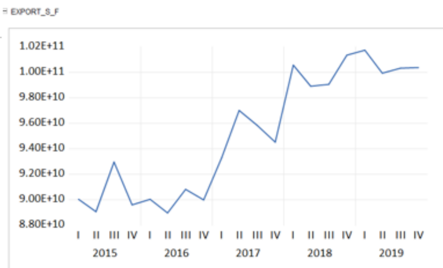
Figure 20: Results of dynamic forecast of VAR(1) model for 2015Q1 to 2019Q4

Evaluation Table

Forecast Evaluation
Date: 10/10/20 Time: 18:24
Sample: 2015Q1 2019Q4
Included observations: 20

Variable	Inc. obs.	RMSE	MAE	MAPE	Theil
EXPORT_S	20	1.89E+09	1.64E+09	1.727316	0.009954
PROD_IND_S	20	1.897954	1.473460	1.386649	0.008882
RGDP_S	20	335.3638	268.4919	0.267906	0.001676

RMSE: Root Mean Square Error
MAE: Mean Absolute Error
MAPE: Mean Absolute Percentage Error
Theil: Theil inequality coefficient



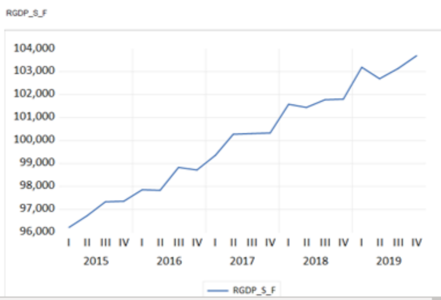


Figure 21: Results of static forecast of VAR(1) model for 2015Q1 to 2019Q4

6 Forecast Comparison

To conclude this paper we will evaluate which of the three models estimated up to now are significantly the most efficient ones. We assume that the conditional model might still need some adjustments in the forecast (we did not achieve to include the forecast for the X_t series when forecasting our explanatory variable); therefore we will mainly focus on the forecast comparison of the univariate and the VAR(1) model. A summary of the RMSEs from the forecasts can be found in the table below.

$model/forecast$	$dynamic$	$static$
1: univariate	803.50	313.95
2: conditional	260.07	281.28
3: VAR(1)	942.49	335.36

Since we focus on the forecasts of models 1 and 3 we will compute the Diebold-Mariano test to assess if the forecast with the lower error is also significantly better than the other one - more precisely we test whether the two RMSEs are significantly different from each other. For this we create a new series which is the difference in of the forecasted values of the two series: $dm_1 3 = rgdp\_s\_forecast_1 - rgdp\_s\_forecast_3$. This is done for the forecast values of the dynamic and the static forecasts. Then the new series is regressed only on an intercept. If the intercept is significant (p-value of $c < 0.05$) then the difference in the forecast errors is also significant.

Diebold-Mariano Dynamic
Dependent Variable: DM13
Method: Least Squares
Date: 10/12/20 Time: 18:29
Sample: 1995Q1 2019Q4
Included observations: 100

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	30.87868	6.338850	4.871338	0.0000

R-squared	0.000000	Mean dependent var	30.87868
Adjusted R-squared	0.000000	S.D. dependent var	63.38850
S.E. of regression	63.38850	Akaike info criterion	11.14639
Sum squared resid	397792.1	Schwarz criterion	11.17244
Log likelihood	-556.3196	Hannan-Quinn criter.	11.15694
Durbin-Watson stat	0.015782		

Diebold-Mariano Static
Dependent Variable: DM13
Method: Least Squares
Date: 10/12/20 Time: 18:11
Sample: 1995Q1 2019Q4
Included observations: 100

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.112587	2.144550	0.985096	0.3270

R-squared	0.000000	Mean dependent var	2.112587
Adjusted R-squared	0.000000	S.D. dependent var	21.44550
S.E. of regression	21.44550	Akaike info criterion	8.978856
Sum squared resid	45531.04	Schwarz criterion	9.004908
Log likelihood	-447.9428	Hannan-Quinn criter.	8.989400
Durbin-Watson stat	2.711217		

Figure 22: Results of the Diebold-Mariano test between model 1 and 3 for static and dynamic forecasts

Regarding Figure 22 we note that the p-value for the coefficient of the Diebold-Mariano test on the dynamic forecasts is highly significant. This means that the null is rejected and the accuracy in these forecasts are different. According to the Diebold-Mariano test one forecast is significantly better than the other one and in this case it is the dynamic forecast of the VAR(1) model.

Regarding the intercept of the static forecast comparison, the difference in the forecasts results is not significant. According to the Diebold-Mariano test one forecast is not significantly better than the other one.

As a final step in the analysis we create a combination model of model 1 and 3 based on the simple average combination with the inverse of the variance we call it:

$$\begin{aligned} comb13dyn &= 0.5 * univar\_dyn + 0.5 * rgdp\_s\_v\_d \\ comb13stat &= 0.5 * univar\_stat + 0.5 * rgdp\_s\_v\_s \end{aligned} \tag{4}$$

We then compute the Diebold-Mariano test between model 1 and 3 (static and dynamic) and the comb13 models respectively. This is a total of four tests. This time we used the forecast evaluation function which our software (EViews) offers. We did not include all the output results for each test here but two examples can be found in Figure 23 and 24. Here the p-value for the Diebold-Mariano test indicates a rejection of the null hypothesis that the RMSEs are the same and comparing the RMSEs at the bottOm of the output we can conclude that the dynamic forecast for the univariate model was significantly better than the combination model. A summary of the results can be seen in the matrix below:

	comb13dyn	comb13stat
1: univar_dyn	univar_dyn better	-
2: univar_stat	-	none better
3: rgdp_s_v_d	comb13dyn better	-
4: rgdp_s_v_s	-	none better

We conclude that the difference between the *comb13stat* model and its corresponding models was so small that it was not significant. The *comb13dyn* was only better compared to the *rgdp_s_v_d* model and worse compared to the *univar_dyn* model indicating that the *rgdp_s_v_d* model contributed to a significantly higher RMSE in the combination model.

Forecast Evaluation
Date: 10/24/20 Time: 18:58
Sample: 2015Q1 2019Q4
Included observations: 20
Evaluation sample: 2015Q1 2019Q4
Number of forecasts: 2

Combination tests
Null hypothesis: Forecast i includes all information contained in others

Forecast	F-stat	F-prob
COMB13DYN	49.57089	0.0000
UNIVAR_DYN	46.86620	0.0000

Diebold-Mariano test (HLN adjusted)
Null hypothesis: Both forecasts have the same accuracy

Accuracy	Statistic	<> prob	> prob	< prob
Abs Error	5.807178	0.0000	1.0000	0.0000
Sq Error	4.924136	0.0001	1.0000	0.0000

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
COMB13DYN	878.3135	701.7173	0.692529	0.688865	0.004374	1.433923
UNIVAR_DYN	813.2998	638.9252	0.630455	0.627338	0.004052	1.326842

Figure 23: Forecast evaluation between the dynamic combination and the dynamic univariate model

Forecast Evaluation
Date: 10/24/20 Time: 19:45
Sample: 2015Q1 2019Q4
Included observations: 20
Evaluation sample: 2015Q1 2019Q4
Number of forecasts: 2

Combination tests
Null hypothesis: Forecast i includes all information contained in others

Forecast	F-stat	F-prob
COMB13STAT	28.48337	0.0000
RGDP_S_V_S	27.72014	0.0001

Diebold-Mariano test (HLN adjusted)
Null hypothesis: Both forecasts have the same accuracy

Accuracy	Statistic	<> prob	> prob	< prob
Abs Error	0.204812	0.8399	0.5801	0.4199
Sq Error	-0.129804	0.8981	0.4490	0.5510

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
COMB13STAT	878.3135	701.7173	0.685247	0.688865	0.004374	1.837266
RGDP_S_V_S	878.9978	700.5921	0.684230	0.687858	0.004378	1.838724

Figure 24: Forecast evaluation between the static combination and the static VAR(1) model

7 Appendix: Declaration of Originality

By filling out and handing in this statement, we hereby declare that the submitted paper

Course code: EBC 2086

Course name: Time Series Modelling

was produced independently by us, without external help.

Wherever we paraphrase or cite literally, a reference to the original source (journal, book, report, internet, etc.) is provided.

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