

Mandatory assignment Week 4

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Mandatory exercise - Markov decision process

Part a

State space

$$S : \{(l_1, l_2) | 0 \leq l_1 \leq 20, 0 \leq l_2 \leq 20\}$$

Where l_1 is the number of available cars at location 1 and l_2 is the number of available cars at location 2.

Action space

$$A : \{m_1, m_2 | 0 \leq m_1 \leq l_1, 0 \leq m_2 \leq l_2\}$$

Where m_1 is the amount of moved cars from location 1 to location 2, and m_2 is the amount of moved cars from location 2 to location 1.

Part b

Transition function

$$P(s' | (l_1, l_2), a) = \begin{cases} s' = (l_1 - m_1, l_2 + m_1) & \text{if } a = m_1 \\ s' = (l_1 + m_2, l_2 - m_2) & \text{if } a = m_2 \end{cases}$$

This gives the of transitioning from state $s = (l_1, l_2)$ to state s' using action a which is either m_1 or m_2 .

Reward function

It cost the owner 2\$ to move a car. And the owner gets 10\$ per rented car. The reward function can then be defined like this:

$$R((l_1, l_2), a, (l'_1, l'_2)) = ((l_1 - l'_1) * 10) + ((l_2 - l'_2) * 10) - (m_1 + m_2) * 2$$

The reward function consist of the amount of revenue from rented cars at location 1, plus the amount of revenue from rented cars at location 2, minus the cost of moved cars from location 1 to location 2 and the cost of moved cars from location 2 to location 1.

Part c

At location 2, Jack loses two cars, since four cars are rented out and two are returned. Which means that he at some point will run out of cars at location 2. At location 1, Jack loses zero cars, since three cars are rented out and three are returned. The optimal policy for maximizing profit would be to always make sure that there are three cars available at location 1, and transfer all the other 17 cars to location 2.

If Jack has less than three cars at location 1, then he will have to move cars from location 2 to location 1 with a maximum cost of 6\$ (the scenario where location 1 is out of cars). But at day one he will always have three cars at location 1, so it is unlikely that he will move cars from location 2 to location 1.

It is also unlikely that Jack will keep up with the total rental demand, since he eventually will run out of cars at location 2.