Problem Description

Consider Cayley Trees $T(k, P)$ as defined in Problem 4 of Barabasi's book, Chapter 3 on Random Networks, for $k >= 3$:

1. Find a formula for the diameter $d_{\max}(k, P)$ of $T(k, P)$ in terms of k and P.
2. Find a formula for the number of vertices $N(k, P)$ of $T(k, P)$ in terms of k and P.
3. For a fixed k which is different for each student (see below), plot a graph showing the values of $d_{\max}(k, P)$ and $\log(N(k, p))$, for P ranging from 1 to 10. Do you see a tendency as P grows?

The attached table contains the values of k to be used by each student. Find your first two initials in the table and use the corresponding value of k.

student initials	k
AI	16
FG	3
GO	4
LO	5
LG	6
IB	7
MA	8
MF	9
MM	10
MI	11
PH	12
RB	13
RC	14
VJ	15

Table 1: value of k for each student.

1 Formula for the diameter

maximum diameter $d_{\max}$: the maximum shortest path in the network

For a Cayley tree, $d_{\max}$ is the distance between two leaf vertices which are not in the same branch of the main vertex:

$$d_{\max}(k, P) = 2 \cdot P = P + P \tag{1}$$

the first P is the distance from one leaf vertex to the root of the tree. The second is the distance to the target vertex (the other leaf vertex).

2 Formula for the number of vertices

1. The level 0 has 1 vertex: the root of the tree.
2. The level l ($l \geq 1$) has $N_L(l, k)$ vertices:

$$N_L(l, k) = k \cdot (k-1)^{l-1} \tag{2}$$

3. The total number of vertices is the sum of the number of vertices of each level up to the level P:

$$N(k, P) = \sum_{i=0}^{P} N_L(l, k) = 1 + k \cdot \frac{(k-1)^P - 1}{k-2} \tag{3}$$

3 Plot of the number of nodes and the diameter

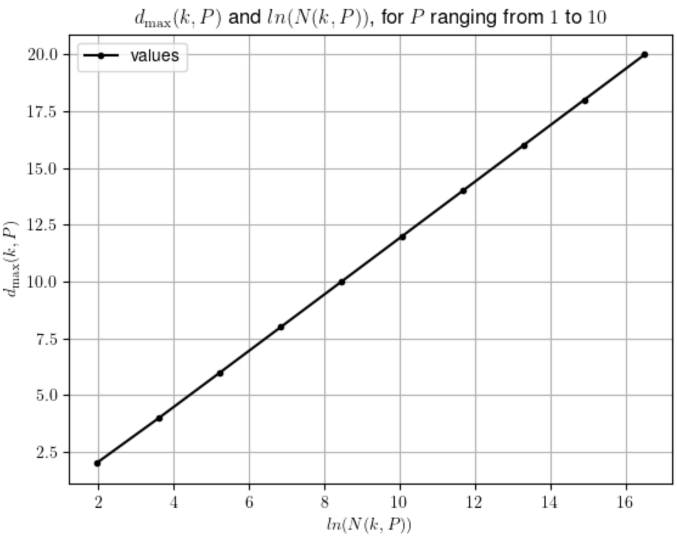

Figure 1: $d_{\max}(k, P)$ as a function of $ln(N(k, P))$

Notice that there is a clear exponential tendency in the Figure 1. One can describe the relation with the function:

$$d_{\max}(N) = \alpha \cdot \ln(N) + \beta \tag{4}$$

For the data used to produce the Figure 1 ($k = 6$, P from 1 to 10), using curve fit, one finds the values:

1. $\alpha = 1.2392486252985329 \pm 0.0015234289193245037$
2. $\beta = -0.4615606209981668 \pm 0.015760537821112773$