$$f' - 2x f = 2x \quad , \quad f(0) = 1 \qquad (\mathrm{I})$$

$$\left[\begin{array}{l} y' + P(x)\,y = Q(x) \\ \\ \text{say that } \; y = u \cdot v \; , \text{ for functions } u, v. \\ \\ y' = u v' + v u' \\ \\ u v' + v u' + P u v = Q \end{array}\right] \qquad (\alpha)$$

Applying α: $P(x) = -2x$, $Q(x) = 2x$

$$u v' + v u' - 2x u v = 2x$$

$$u v' + \left(u' - 2xu \right) v = 2x \qquad (\mathrm{II})$$

$u' - 2xu = 0$	$\ln(u) = x^2$
$u' = 2xu$	$u = \exp(x^2) + C_u$
$\dfrac{u'}{u} = 2x$	
	$C_u = 0$ otherwise $u' - 2xu \neq 0$
$\displaystyle\int \frac{u'}{u} = \int 2x$	

Substituting u back in (II):

$\exp(x^2)v' = 2x$ for $v = -\exp(-x^2)$

$$v' = [-\exp(-x^2)]\cdot[-2x] = 2x\exp(-x^2)$$

$$v'\cdot\exp(x^2) = 2x \quad \checkmark$$

$$v = -\exp(-x^2) + C_v$$

Finally: $y = u\cdot v = \exp(x^2)\cdot[-\exp(-x^2) + C_v] =$

$$= -1 + C_v\exp(x^2) = C_v\exp(x^2) - 1$$

Applying the boundary condition:

$$y(0) = C_v\cdot\exp(0) - 1 = 1 \Rightarrow$$

$$\Rightarrow C_v - 1 = 1 \Rightarrow C_v = 2$$

We conclude that:

$$f(x) = 2\exp(x^2) - 1$$

(which clearly satisfy the given equation and boundary cond.)