

Computational Project - Project Planning

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1 checklist what to do

- Título
 - denominação do problema
 - metodologia de solução
- Resumo
 - objetivos
 - informações sobre o problema
 - metodologia de solução
 - como será feita a avaliação dos resultados
- Introdução
 - descrição formal do problema (formulação matemática)
 - revisão bibliográfica do problema (e/ou problemas relacionados)
 - metodologias previamente utilizadas
- Metodologia
 - justificativa
 - descrição das técnicas de otimização
 - descrever as técnicas de otimização contextualizando-as ao problema de otimização combinatória proposto
- Avaliação dos Resultados
 - proposta de experimentos
 - descrição das instâncias
 - como será feita a avaliação dos resultados
- Referências Bibliográficas

2 Problem Statement

2.1 Input

1. a directed acyclic graph $G = \langle V, E \rangle$;
2. a multi-dimensional weight function $w : V \rightarrow \mathbb{Z}_{>}^{n_w}$, where $n_w \in \mathbb{N}$;
We will usually write $w_v = w(v)$
3. a maximum capacity $W \in \mathbb{Z}_{>}^{n_w}$;

2.1.1 Partial Order

Definition 1 (Partial Order on Directed Acyclic Graph). Given a directed acyclic graph $G = \langle V, E \rangle$, we define the set:

$$\prec = \{\langle v, v' \rangle : \text{there is a path from the first to the second}\} \quad (1)$$

and so \prec is a partial order over the set V .

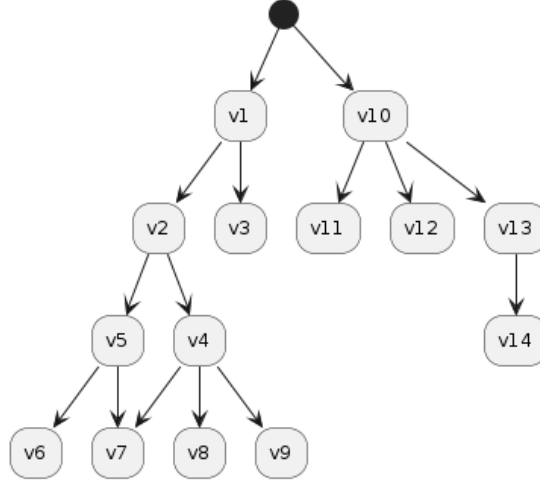


Figure 1: Example of a directed acyclic graph. The black dot indicates the root vertices. For this case, the induced partial order satisfy: $v5 \prec v2$, $v7 \prec v1$, $v14 \prec v10$.

2.2 Output

A subset $S \subseteq V$ of the vertices which satisfy:

$$\sum_{v \in S} w_v \leq W \quad (2)$$

$$\forall v (v \in S \rightarrow \forall v' (v' \prec v \rightarrow v' \in S)) \quad (3)$$

Equation (2) is the maximum weight constraint, the total weight of all vertices in the solution set S must not be greater than the weight limit W .

Equation (3)¹ says that, if a v is included in the solution, then all the v' lower than it (in the sense of the partial order \prec) must also be included.

2.3 Objective

Find S that maximizes $|S|$. In other words: find the solution with the maximum number of vertices.

3 Integer Linear Programming Model

3.1 Decision Variables

$$x_v = \begin{cases} 1 & , v \in S \\ 0 & , v \notin S \end{cases} \quad (4)$$

3.2 Mathematical Model

$$\max_{S \subseteq V} \sum_{v \in S} x_v \quad (5)$$

$$s.t. \sum_{v \in S} x_v w_v \leq W \quad (6)$$

$$x_v \leq x_{v'} \quad \forall v' \prec v \quad (7)$$

$$x \in \{0, 1\}^{n_w} \quad (8)$$

Equation (5) is the objective function: maximize the number of vertices in the solution. Equation (6) is the knapsack capacity constraint. Equation (7) constraints that if a vertex v is in the solution, then all vertices v' for which there is a path from v must also be in the solution.

References

- [1] Cezar A Mortari. *Introdução à lógica*. Unesp, 2001.

¹It is a First-order logic expression [1].