Problem 3

a) inner loop executes $i<n$ times.

k	i
0	2
1	4
2	16
3	256
$\vdots$	$\vdots$
K	2^{2^K}

$\rightarrow 2^{2^K} < n$

$2^K < \log n$

$K < \log(\log n))$

$$\sum_{i=2}^{\log(\log(n))} \theta(1) = \boxed{\theta(\log(\log(n)))}$$

b) Innermost loop runs $k<i^3$ times, so we can say it is $\theta(i^3)$.
Check how many times the if-statement gets triggered.

K	n
1	$\sqrt{n}$
2	$2\sqrt{n}$
$\vdots$	$\vdots$
K	$K\sqrt{n}$

$$\rightarrow O\left(\sum_{K=0}^{i^3} \theta(1)\right)$$

$$= \sum_i \sum_{k=0}^{\sqrt{n}} \theta(i^3)$$

$$= \theta(\sqrt{n}\, n^3)$$

$$\boxed{= \theta(n^{7/2})}$$

Note: We can ignore the first loop b/c the # of times we don't enter the if-statement are constant

c) $$\sum_{i=1}^{n} \sum_{K=1}^{n} (\theta(1)) + \theta(n)\theta(\log(n)))$$

$\hookrightarrow$ complexity of innermost loop

$\hookrightarrow$ max # of times we can enter innermost loop $\rightarrow$ worst case is entering every time but we can only do that once and never again, or we can enter the loop once for each value of i n times

$$= \theta(n^2) + \theta(n \log n)$$

$$\boxed{= \theta(n^2)}$$

d) Inside the loop is $\Theta(i)$ → calculate i

k	size(i)
1	10
2	15
3	22
⋮	⋮
k	$10\left(\frac{3}{2}\right)^k$

→ so for $i < n$

$$10\left(\frac{3}{2}\right)^k < n$$

$$\left(\frac{3}{2}\right)^k < \frac{n}{10}$$

$$k < \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$$

this is the worst case for entering if-statement

so $$\sum_{i=0}^{\log_{\frac{3}{2}}\left(\frac{n}{10}\right)} \Theta(i) = 10 \sum_{j=0}^{k} \left(\frac{3}{2}\right)^j = \Theta\left(\frac{3}{2}\right)^k$$

↳ plug in $k = \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$ from earlier

$$= \Theta\left(\frac{3}{2}\right)^{\log_{\frac{3}{2}}\left(\frac{n}{10}\right)} = \boxed{\Theta(n)}$$