Probability

$\boxed{Q1}$ 15 students, 8 questions p(no student answers more than 1 question)

= 1 − p(student is asked more than 1 q)

$\frac{1}{15}$ chance of being asked a question

~~$1 = \left(\frac{1}{15}\right)5$~~

~~$\frac{8}{15}$ = avg # of questions a student is asked~~

~~$\frac{5}{15} = \left(5\left(\frac{1}{15}\right)\right) \cdot 8$~~

~~$5 = 1$~~

~~$\left(\frac{1}{15}\right)^8 = \frac{1}{15^2}$ = prob of being asked 2 questions~~

$\boxed{Q2}$ # of strings meeting criteria

~~$1 - \left(\frac{1}{15^2}\right)$~~

~~$= \frac{224}{225}$~~

$$\frac{15}{15} \cdot \frac{14}{15} \cdot \frac{13}{15} \cdot \frac{12}{15} \cdot \frac{11}{15} \cdot \frac{10}{15} \cdot \frac{9}{15} \cdot \frac{8}{15} = \boxed{\frac{259459200}{2562890625}}$$

1 − chance of 1 student (who has been asked a question) getting asked each of the next 7 questions

$\boxed{22}$ 00000 − 99999

total ints: $10^5 = 100000$ ~~= 59049~~

~~− (ints that don't start with 2 odd digits)~~

~~− (ints with non-unique digits~~

~~− odd ints~~

~~or total ints w/ all unique digits = $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 5040$~~

~~− ints that don't start w/ 2 odd digits = 10^5 −~~

~~$5^2 = 25$~~

ints matching criteria

$5 \cdot 4 \cdot 7 \cdot 6 \cdot 5 = 4200$

5 odd #'s for position 1

4 odds for position 2

7 & 6 options left for middle after choosing first 2 and last digit

5 evens available for the last digit

so $\frac{4200}{100000}$ ints match → that's our probability of generating one on e

$= \frac{21}{500}$

so the probability of getting 5/8 =

$= \binom{8}{5}\left(\frac{21}{500}\right)^5\left(\frac{479}{500}\right)^3 = \boxed{56\left(\frac{21}{500}\right)^5\left(\frac{479}{500}\right)^3}$

$\boxed{Q3}$ A = 2 or more dice show 4 or above   B = all 3 dice show the same value

P(A):

4 or above in one dice $= \frac{1}{2}$

$P(A) = \underset{2}{p}(\text{all } \underset{2}{4}\text{'s}) + \underset{4}{p}(\text{exactly 2 in 4's}) = \frac{1}{216} + \frac{15}{216} = \frac{16}{216} = \frac{2}{27}$

possible values that could be the same

$P(B): \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216} \cdot 6 = \frac{1}{36}$

? $P(A \cap B): \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 3 = \frac{3}{216}$

now we can only have 4, 5, or 6 be the same value - so that's 3 out of 216 possible events

$\frac{3}{216} \stackrel{?}{=} \frac{1}{36} \cdot \frac{2}{27}$

$\frac{3}{216} \stackrel{?}{=} \frac{2}{972}$

$\frac{3}{216} \neq \frac{2}{972}$

since $P(A \cap B) \neq P(A) \cdot P(B)$, the events are $\boxed{\text{not independent}}$

$\boxed{Q4}$ $\frac{\text{number of flushes}}{\text{number of hands}}$ = % chance of getting a flush

$\#$ of hands = $\binom{52}{5} = 2598960$

$\#$ of flushes = $\binom{4}{1} \cdot \binom{13}{5} = 4(1287) = 5148$

$\frac{5148}{2598960} \approx \frac{1}{505}$ hands is a flush

the player should expect to play about $\boxed{505}$ hands of poker to get a flush

(exact expectation is $\frac{2598960}{5148}$ hands of poker to get a flush)

$\boxed{Q5}$ $p(\text{win} \mid \text{plays}) = 0.7$ Want $p(\text{plays} \mid \text{won } 4)$

$p(\text{win} \mid \text{does not play}) = 0.5$

~~$\binom{5}{4}(0.7)^4(0.5) = 0.6$ = 60% chance he played in those 5 games~~

$$p(\text{plays} \mid \text{won } 4) = \frac{p(\text{won } 4 \mid \text{plays})\, p(\text{plays})}{p(\text{won } 4)}$$

$$= \frac{p(\text{won } 4 \mid \text{plays})\, p(\text{plays})}{p(\text{won } 4 \mid \text{plays})\, p(\text{plays}) + p(\text{won } 4 \mid \text{does not play})\, p(\text{does not play})}$$

$$= \frac{\binom{5}{4}(0.7)^4(0.3)(0.75)}{\binom{5}{4}(0.7)^4(0.3)(0.75) + \binom{5}{4}(0.5)^4(0.5)(0.25)}$$

$$= \frac{5(0.7)^4(0.3)(0.75)}{5(0.7)^4(0.3)(0.75) + 5(0.5)^5(0.25)}$$

$$= \frac{0.270}{0.270 + 0.039} = \boxed{87.3\%}$$