Teoria dos números e corpos finitos

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1. (4.6) For each of the following equations, find an integer x that satisfies the equation.

a. $5x \equiv 4 \pmod{3}$

$$x = 2$$
$$5 \times 2 = 10$$
$$10 - 4 = 6 = 3 \times 2$$

b. $7x \equiv 6 \pmod{5}$

$$x = 3$$
$$7 \times 3 = 21$$
$$21 - 6 = 15 = 5 \times 3$$

c. $9x \equiv 8 \pmod{7}$

$$x = 4$$
$$9 \times 4 = 36$$
$$36 - 8 = 28 = 7 \times 4$$

2. (4.7) In this text, we assume that the modulus is a positive integer. But the definition of the expression $a \mod n$ also makes perfect sense if n is negative. Determine the following:

Usando $a \mod n = a - \lfloor a/n \rfloor \times n$.

a. $5 \mod 3$

$$2$$

b. $5 \quad mod \quad -3$

$$5 - \lfloor 5/-3 \rfloor \times -3$$
$$5 - (-2 \times -3)$$
$$5 - 6$$
$$-1$$

c. $-5 \quad mod \quad 3$

$$-5 - \lfloor -5/3 \rfloor \times 3$$
$$-5 - (-2 \times 3)$$
$$-5 + 6$$
$$1$$

d. $-5 \quad mod \quad -3$

$$-5 - \lfloor -5/-3 \rfloor \times -3$$
$$-5 - (1 \times -3)$$
$$-5 + 3$$
$$-2$$

3. (4.8) A modulus of 0 does not fit the definition but is defined by convention as follows: $a \quad mod \quad 0 = a$. With this definition in mind, what does the following expression mean: $a \equiv b \pmod{0}$?

Significa que a e b são iguais.

4. (4.1) For the group S_n of all permutations of n distinct symbols:

a. what is the number of elements in S_n?

$n!$

b. show that S_n is not abelian for $n > 2$.

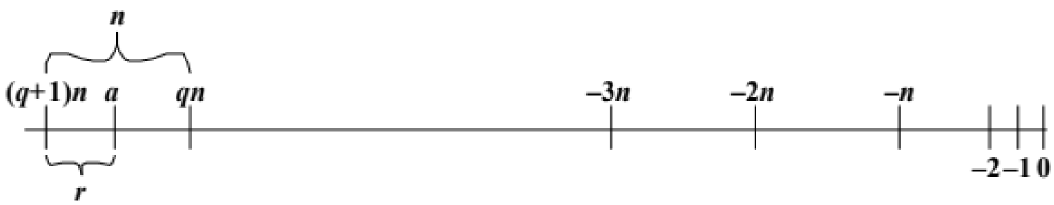
Um contra exemplo com o S_3 seria:

$$\{3,2,1\} \cdot \{1,3,2\} = \{2,3,1\}$$
$$\{1,3,2\} \cdot \{3,2,1\} = \{3,1,2\}$$

5. (4.4) Reformulate Equation (4.1), removing the restriction that a is a nonnegative integer. That is, let a be any integer.

A equação continua a mesma.

6. (4.5) Draw a figure similar to Figure 4.1 for $a < 0$.



7. (4.13) Find the multiplicative inverse of each nonzero element in Z_5.

Para todo $a \in Z_5$, precisamos encontrar um $b \in Z_5$ onde $ab \equiv 1 \pmod{5}$

$1 \to 1$
$2 \to 3$
$3 \to 2$
$4 \to 4$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

w	$-w$	w^{-1}
0	0	—
1	4	1
2	3	3
3	2	2
4	1	4

8. (4.20) Develop a set of tables similar to Table 4.3 for GF(5).

9. (4.10) What is the smallest positive integer that has exactly k divisors, for $1 \leq k \leq 6$?

$k = 1 \rightarrow 1 \rightarrow \{1\}$
$k = 2 \rightarrow 2 \rightarrow \{1, 2\}$
$k = 3 \rightarrow 4 \rightarrow \{1, 2, 4\}$
$k = 4 \rightarrow 6 \rightarrow \{1, 2, 3, 6\}$
$k = 5 \rightarrow 16 \rightarrow \{1, 2, 4, 8, 16\}$
$k = 6 \rightarrow 12 \rightarrow \{1, 2, 3, 4, 6, 12\}$

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.