

Teoria dos números e corpos finitos

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1. (4.6) For each of the following equations, find an integer x that satisfies the equation.

a. $5x \equiv 4 \pmod{3}$

$$x = 2$$

$$5 \times 2 = 10$$

$$10 - 4 = 6 = 3 \times 2$$

b. $7x \equiv 6 \pmod{5}$

$$x = 3$$

$$7 \times 3 = 21$$

$$21 - 6 = 15 = 5 \times 3$$

c. $9x \equiv 8 \pmod{7}$

$$x = 4$$

$$9 \times 4 = 36$$

$$36 - 8 = 28 = 7 \times 4$$

2. (4.7) In this text, we assume that the modulus is a positive integer. But the definition of the expression $a \pmod{n}$ also makes perfect sense if n is negative. Determine the following:

$$\text{Usando } a \pmod{n} = a - \lfloor a/n \rfloor \times n.$$

a. $5 \pmod{3}$

b. $5 \bmod -3$

$$\begin{aligned} 5 - \lfloor 5 / -3 \rfloor \times -3 \\ 5 - (-2 \times -3) \\ 5 - 6 \\ -1 \end{aligned}$$

c. $-5 \bmod 3$

$$\begin{aligned} -5 - \lfloor -5 / 3 \rfloor \times 3 \\ -5 - (-2 \times 3) \\ -5 + 6 \\ 1 \end{aligned}$$

d. $-5 \bmod -3$

$$\begin{aligned} -5 - \lfloor -5 / -3 \rfloor \times -3 \\ -5 - (1 \times -3) \\ -5 + 3 \\ -2 \end{aligned}$$

3. (4.8) A modulus of 0 does not fit the definition but is defined by convention as follows: $a \bmod 0 = a$. With this definition in mind, what does the following expression mean: $a \equiv b \pmod{0}$?

Significa que a e b são iguais.