5 Investment Planning for Electricity Generation

[1]The demand for electricity fluctuates heavily with time. During peak-hours much more is needed than in quiet times, and in winter the demand is higher than in summer. Electricity producers usually represent the demand over a prespecified period of time (a year, say) in terms of the so-called *load-curve*, see Figure 1.

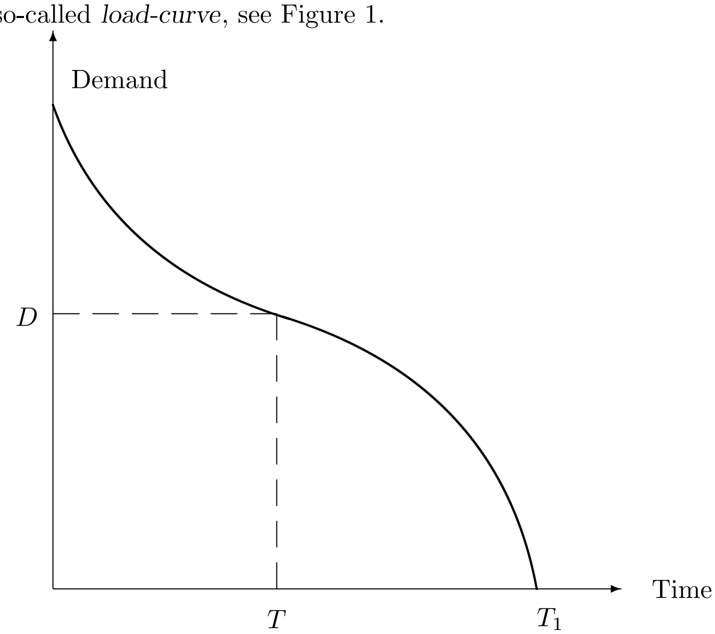

Figure 1: Load-curve, $D = L(T)$, $0 \leq T \leq T_1$.

Here T_1 indicates the number of time units (a day, say) in the considered period. The load-curve is constructed by representing the days in the order of decreasing demands. As a result, the load-curve is a nonincreasing function of T on $[0, T_1]$. Note that the load-curve does not reveal on which days the corresponding demand occurs, only the frequency is depicted. That is, $D = L(T)$ means, that in the period of T_1 time units the demand is at least D for in total T time units.
The load-curve of a year is a practical means of representing the variability of the demand over time, in particular for analyzing the technology to be used to generate electricity, as is the subject of this case. On an aggregated level, one may think of production technologies based on coal, natural gas, sun, wind, nuclear power and so on. Each technology has its own advantages and disadvantages. From the point of view of costs, equipment with relatively low operating cost will be suitable, even if the investment costs are large, if it is exploited during a large part of the year, where it may be the other way around for peak-load demand.

In the article of Louveaux and Smeers (on which this case is based) a multistage stochastic programming model is formulated for the investment planning for electricity generation. In this case we restrict ourselves to a two-stage model with recourse. The first stage will deal with decisions on investments in the technologies, whereas the second stage deals with the distribution of the technologies over the various parts of the load-curve. Since there is a long lead-time for the investments, the load-curve is not known exactly in advance, and will be modeled in terms of random variables, see Figure 2.

[1]For further information about this case, see *Optimal investments for electricity generation: a stochastic model and a test problem*, Louveaux, F.V. and Y. Smeers. This article is published as Chapter 24 in *Numerical techniques for stochastic optimization*, Ermoliev, Yu and R.J-B. Wets, Springer-Verlag, Berlin, 1988, p445-453

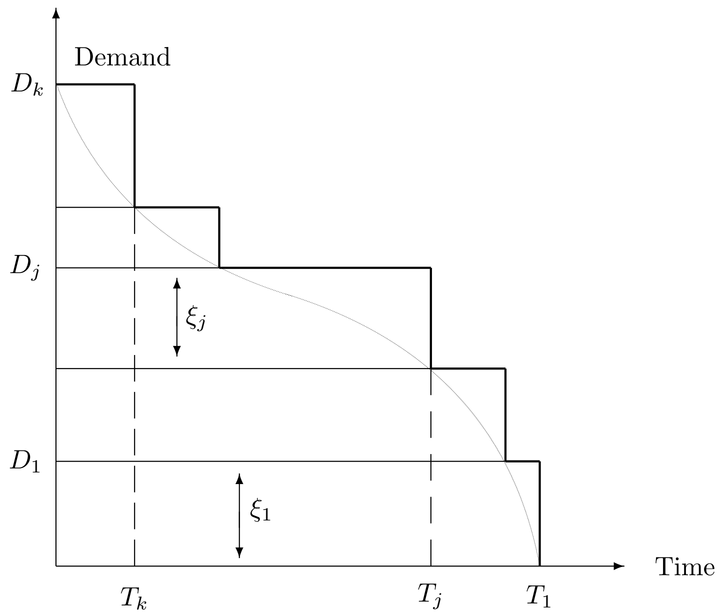

Figure 2: Approximate load curve, with k modes.
The random demand in mode j is $\xi_j := D_j - D_{j-1}$, $j = 1, \ldots, k$ with $D_0 := 0$.

The approximate load-curve is a piecewise constant curve, indicating that during T_j time units the demand is equal to D_j, $j = 1, \ldots, k$. In our model we will assume that the values of $T_1, \ldots, T_k$ are known whereas $D_1, \ldots, D_k$ are random variables, specified as $D_h = \sum_{j=1}^{h} \xi_j$, $h = 1, \ldots, k$. That is, ξ_j is the demand related with the j-th block in the approximate load-curve, to be called the j-th "mode". For simplicity, we will assume that there does not exist any capacity in any technology in the beginning of the investment period.

Define the following variables.

n	number of technologies
k	number of modes
x_i	capacity for technology i, $i = 1, \ldots, n$
y_{ij}	capacity of technology i effectively used in mode j, $i = 1, \ldots, n$, $j = 1, \ldots, k$
ξ_j	(stochastic) demand for mode j, $j = 1, \ldots, k$
$\xi_j^{\max}$	(deterministic) maximum possible value for ξ_j, $j = 1, \ldots, k$
T_j	duration of mode j, $j = 1, \ldots, k$
c_i	investment and maintenance cost for technology i per unit capacity, $i = 1, \ldots, n$ (on a yearly equivalent basis)
q_i	production cost for technology i per unit capacity per unit of time, $i = 1, \ldots, n$
$c_{\max}$	maximum allowed investment cost

The first-stage constraint will look like this:

- $\sum_{i=1}^{n} x_i \;\geq \sum_{j=1}^{k} \xi_j^{\max}$
  the total available capacity has to be enough to cover maximum demand

- $\sum_{i=1}^{n} c_i x_i \ \leq \ c_{\max}$

  budget constraint

The second-stage constraints look like this:

- $\sum_{j=1}^{k} y_{ij} \ \leq \ x_i, \qquad i \ = \ 1, \ldots, n$

  the modes cannot use more capacity than is available for any technology

- $\sum_{i=1}^{n} y_{ij} \ \geq \ \xi_j, \qquad j \ = \ 1, \ldots, k$

  demand has to be met in each mode

Our objective is to minimize expected total cost, which is represented by the following formula:

$$\min_{x \geq 0} \left\{ \sum_{i=1}^{n} c_i x_i + E_\xi \min_{y \geq 0} \left\{ \sum_{i=1}^{n} \sum_{j=1}^{k} q_i T_j y_{ij} \ : \ \begin{array}{c}\text{second-stage} \\ \text{constraints}\end{array} \right\} \ : \ \begin{array}{c}\text{first-stage} \\ \text{constraints}\end{array} \right\}$$

We will assume that four technologies are available, which can all be operated in three different modes. We assume that $T_1 \ = \ 10$, $T_2 \ = \ 0.6 \cdot T_1$ and $T_3 \ = \ 0.1 \cdot T_1$. The upper bound on the total investment costs is equal to 120, while the investment costs per unit for the separate technologies are 10, 7, 16 and 6, respectively. Production costs are 4.0, 4.5, 3.2 and 5.5, respectively, per unit.

(a) We will analyze the recourse model described above.

Assume that the distributions of the *independent* random variables ξ_i are:

Prob	ξ_1	ξ_2	ξ_3	
.3	3	2	1	
.4	5	3	2	(altogether 27 realizations)
.3	7	4	3	

Compute (i) the Expected Value Solution, and (ii) the Expected Result (of the expected value solution). Based on the results, discuss the usefulness of solving the recourse model. Remark: To compute (i), the maximum demand in the first-stage constraint can be adjusted.

Solve the recourse model and compare the outcome with (i) and (ii) above. In particular, discuss the so-called Value of the Stochastic Solution.

Compute the Wait-and-See solution and compare the outcome with previous results. In particular, discuss the so-called Expected Value of Perfect Information.

(b) Assume now, in addition, that the operational availability of each of the technologies is random: if a capacity of x_i is installed, the actual amount available is $\alpha_i x_i$, with α_i a random parameter, $i = 1, .., n$.

$$\begin{aligned} &\alpha_1 \sim \mathcal{U}(0.6, 0.9) \\ &\alpha_2 \sim \mathcal{U}(0.7, 0.8) \\ &\alpha_3 \sim \mathcal{U}(0.5, 0.8) \\ &\alpha_4 \sim \mathcal{U}(0.9, 1) \end{aligned}$$

Moreover, assume that it is possible to 'import' electricity to cover any observed power shortage. This virtual fifth technology has zero investment cost, but high production cost 10.

Formulate the corresponding recourse model. (Hint: the possibility to import electricity has implications for both stages of the model.)

Analyze this model, and compare the results with (a). In particular, discuss the expected result (in model (b)) of the solution obtained under (a).

(c) Assume, in addition, that $T_i = \tau_i,\ i = 2, 3$, where the distribution of the independent random variables τ_i is

Prob	τ_2	τ_3	
.6	5	.5	
.4	7.5	1.75	(altogether 4 realizations)

Formulate this model and analyze it. Compare the results with the other models.