Coding Project 1: Detecting objects through frequency signatures

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Abstract

The objective of this project is to estimate the location of a large underground vibration source through seismograph readings. To achieve this, frequency analysis of the three-dimensional seismograph readings is conducted. Specifically, The Fast Fourier transform, spectral averaging, and spectral filtering techniques were used. Averaging of the spectral decompositions of the seismograph realizations reveals the frequency the signals are centered about. A gaussian filter, centered about the primary frequency, is then used to estimate the location of the vibration source for each seismograph reading.

1 Introduction

In the 1930s, scientists and engineers began to experiment with detecting the presence of objects using radio waves. This technology, known as radar detection, was created primarily for military purposes used to detect incoming aircraft and ships. Today, radar detection is used in a wide range of industries. From meteorology to air traffic control, radar detection has become more complex and refined, providing more detailed information on an objects location and movement.

Radar detection has made numerous significant contributions to society. In the military, it has been critical in detecting and tracking enemy aircraft and ships, helping to ensure the safety and security of nations. In aviation, it has made air travel safer and more efficient providing critical information to air traffic controllers. In medicine, radar has revolutionized imaging and treatment, where it can be used to produce images of internal body structures through non-invasive means. Overall, radar detection has had a profound impact on many aspects of modern life, helping to keep people safe, improving efficiency, and deepening our understanding of the world around us.

This paper reports the theoretical background, the numerical methods, and the results of the methods used in the analysis of a hypothetical situation. Specifically, the situation where the location of a Kraken, who's movement under a the Kraken hockey team ice rink causes vibrations only measurable by a seismograph, over time wants to be known by the team.

2 Theoretical Background

This section briefly covers the origins and derivations of the numerical methods used in the project.

2.1 The Fourier Series

The Fourier series is a mathematical tool used to represent a periodic function as an infinite sum of sine and cosine functions. The derivation of the Fourier series begins with the observation that any periodic function can be expressed as a sum of its harmonics, which are sine and cosine functions with frequencies that are integer multiples of the fundamental frequency of the periodic function.

That is, for a periodic function $f(x)$,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos{(\frac{n\pi x}{L})} + b_n \sin{(\frac{n\pi x}{L})}\ L(-L, L)$$

Applying Euler's formula, the Fourier Series becomes:

$$f(x) = \sum_{n=infty}^{\infty} C_n e^{\frac{in\pi x}{L}}$$

where

$$C_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{\frac{-in\pi x}{L}} dx$$

This is also known as the exponential form of the Forier Series.

2.2 The Fourier Transform

Perhaps the biggest limitation to the Fourier Series is that it is bounded from $(-L, L)$ and expresses periodic functions. The Fourier Transform is a generalization of the Fourier Series to non-periodic signals from $[-\infty, \infty]$.

The Fourier Transform of a continuous time signal, f(t), is defined as $\hat{f}(k)$ where k is the frequency.

$$\hat{f}(k) = F(f(t)) \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-ikt} dt$$

The Inverse Fourier Transform is then:

$$f(t) = F^{-1}(\hat{f}(k)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(k) e^{ikt} dk$$

The Fourier Transform is a powerful tool for analyzing and processing signals in various applications such as signal processing, image processing, and communication systems. It provides a frequency-domain representation of a signal, which is useful for identifying dominant frequencies and understanding the spectral content of a signal.

2.3 The Fast Fourier Transform

The Fast Fourier Transform (FFT) is a discretized version of the Four Transform. The FFT is the means by which computers calculate the discrete fourier transform of a given a sequence. In MATLAB, the fft(X) function reurns the discrete fourier transform of X using the FFT algorithm. This method can be used for signals of two dimensions. For signals of higher dimensions, however, the fftn(X) function should be used instead.

2.4 Spectral Averaging

Spectral averaging refers to the averaging of signals transformed onto the frequency domain. Naturally occurring noise, such as gaussian white noise, is random. As such, taking the average of the signal in the frequency domain "filters" those frequencies out as they become smaller. Artifical noise, however, is persistent and tends to be above a certain threshold. Consequently, averaging of signals in the frequency domain results in noisy frequencies diminsihing and aritifical frequencies persisting. A similar approach is taken in physics, where multiple experiments are done to reduce random error.

2.5 Spectral Filtering

Spectral Filtering refers to the technique of multiplying the spectral decomposition of a signal with a filter function to isolate desired signals. In this project, a Gaussian filter, centered about the previously found peak signal frequencies. Applying the filter and transforming the signal back in to the spacial domain reveals a good estimate of the position of the signal source. Spectral Filtering is also commonly used to conduct time/frequency analysis of non-stationary signals (Gabor Transform).

3 Numerical Methods

The MATLAB code used in the project is provided below

```
% Clean workspace
clear all; close all; clc
load('Kraken.mat') %load data matrix
L = 10; % spatial domain
n = 64; % Fourier modes
```

```
x2 = linspace(-L,L,n+1); x = x2(1:n); y =x; z = x; %create 3D axis arrays with 64 points
k = (2*pi/(2*L))*[0:(n/2 - 1) -n/2:-1]; %create frequency array and rescale them to be 2pi periodic
ks = fftshift(k); %shift values to order them correctly
x1 = linspace(-L,L,64);
%Create 3D grids for both spatial domain and frequency domain
X,Y,Z  = meshgrid(x,y,z);Kx,Ky,Kz  = meshgrid(ks,ks,ks);
threshold = 0.7; %lower bound for frequencies that are recognized
```

The first section of code above loads in the readings from the seismograph ($64^3 \times 49$ matrix) and creates spatial domain and frequency domain in which the data will be plotted, transformed, and analyzed.

Afterwards, each reading from the seismograph is reshaped into a 3D tensor ($64 \times 64 \times 64$ Matrix) and the discrete fourier transform of the reshapes tensor is obtained using the fftn() function mentioned in section 2.3. The results of the transformation are then added to the transformsSum matrix. After all 49 transformed realizations have been added, the total sum is divided by 49 to obtain the average (Spectral Averaging).

```
   % sum all realizations in frequency space
transformsSum = zeros(n,n,n);
for j = 1:49
Un(:, :, :)  = reshape(Kraken(:, j), n, n, n);
transformsSum= transformsSum + fftn(Un);
end

% after for loop ends, save the sum as variable A1
A1 = transformsSum;
% Average the sum over the 49 realizations (i.e., A1/49) and save as A2
A2 = A1/49;
```

Now that the frequencies are averaged, the frequencies with the highest peaks/values should be the frequency that the signals are centered about. To find the max values, the MATLAB function max() is used. This returns the max value of the whole matrix as well as the linear index of the value. To obtain the three-dimensional index, the function ind2sub() is used. Given an matrix size and a linear index, ind2sub() returns the multi-dimensional index corresponding to the linear index. Finally, as ks is the frequency domain of each dimension, the maximum frequencies in each direction correspond to the frequency of ks at the previously obtained three-dimensional index.

```
% use max function to find max value and linear index in all of A2
A2_analysis = fftshift(A1)/49;
[maxVal,maxIndex] = max(A2_analysis(:));
% use ind2sub to obtain 3D array index of maxvalue
[i,j,k]=ind2sub(size(A2), maxIndex);
% save max x frequency
A3 = ks(i);
% save max y frequency
A4 = ks(j);
% save
A5 = ks(k);
```

The next step is to create the three-dimensional gaussian filter (i.e. filter in the space (Kx, Ky, Kz)). The filter should be centered around the previously found maximum frequencies to keep the desired information.

```
%create an appropriate Gaussian filter and save it as A6
tau = 0.025;
A6 = filter;
```

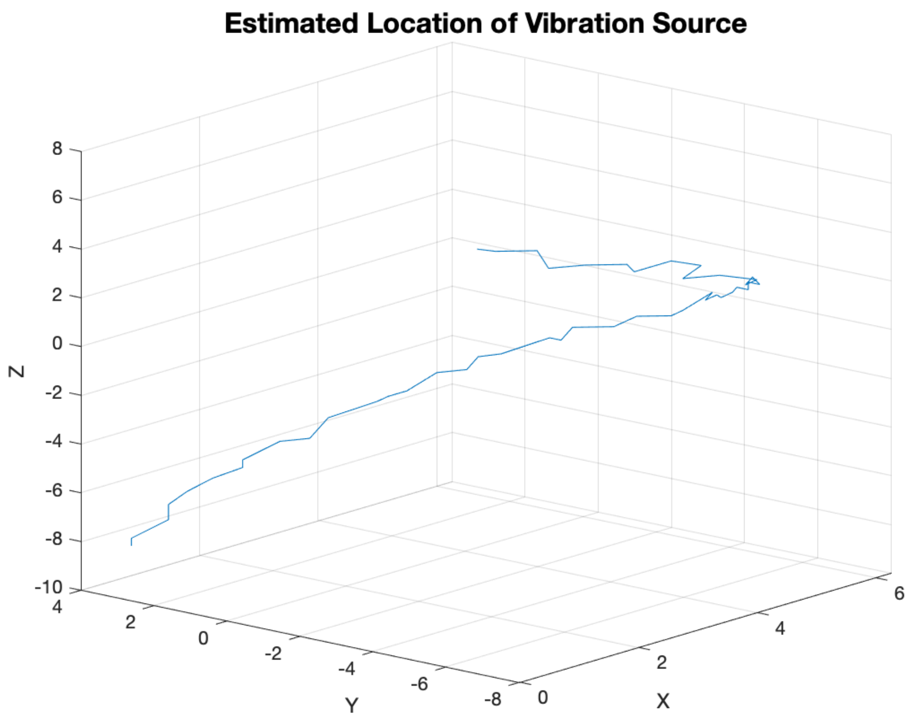
Finally, apply the gaussian the filter to each realization in the frequency domain. Afterwards, apply the inverse transform to obtain the spatial plot of the filtered frequencies. The x,y,and z coordinates with maximum value, most likely the location of the source, is then saved using the same method as before.

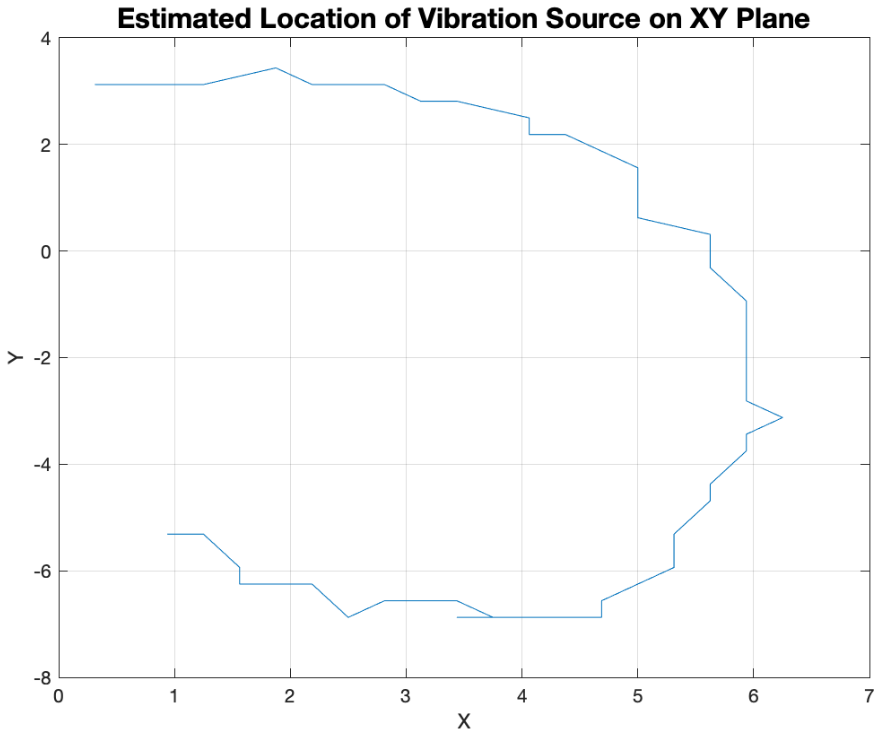
```
% Using the peak frequencies for the filtered signal, estimate the x, y, and z coordinates of the Kraken over time and save as A7, A8, A9
A7 = [];
A8 = [];
A9 = [];
for c =1:49
%obtain realization
Un(:,:,:)  = reshape(Kraken(:,c),n,n,n);
%apply gaussain filter to fourier transform of realization
Ufreqfilt = A6.*fftshift(fftn(Un));
% obtain inverse fourier transform of filtered realization transform
Unfilt = ifftn(Ufreqfilt);
% obtain max value element (this should be estimate of location)maxValfilt,maxIndexfilt  = max(abs(Unfilt(:)));i,j,k  = ind2sub(size(Unfilt),maxIndexfilt);
% obtain and save x location estimate
A7(c) = x(i);
```

```
% obtain and save y location estimate
A8(c) = y(j);
% obtain and save z location estimate
A9(c) = z(k);
end
```

4 Results

The estimated locations of the Kraken are shown below:





The estimated beginning position of the Kraken is at $(0.3125, 3.1250, -8.1250)$. As can be seen, the path the Kraken takes is similar to that of a circle and moves in a clockwise direction in the xy plane while elevating in the z plane (getting closer to the surface of the ice rink).

5 Conclusion

The Kraken hockey team can use the estimated positions of the Kraken and it's vibrations to their advantage. By aligning the position in which they execute a hard check with the position of the Kraken at that time (e.g. at position $(0.3125, 3.1250)$ at the beginning of the game), they can improve their defensive efficiency, increasing their likelihood of victory.

One limitation of using the Fourier Transform for spectral analysis is that although it provides a comprehensive view of the frequency content of the signal, it does a poor job at providing a representation of how the frequencies change with time. Other instance of the Fourier Transform like the Gabor Transform may be better at determining the signal changes over time.

There are many things I learned from this project. Of the many, understanding what a tensor is and being able to manipulate it (applying the FFT) was definitely new to me. In addition, I also learned the differnece between the fft() method and the fftn() method. I really enjoyed this project, seeing a real life application of something we learned in class and getting hands on experience was very satisfying.

Acknowledgment

Stackoverflow and offical MATLAB pages were referenced during this project.