Learning Multi-Step Predictive State Representations

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PSRs, WFA, and OOMs

Goals:

1. Learn a representation of hidden states
2. Model state to state transitions
3. Predict observation sequences

   f: strings -¿ [0, 1]

Motivation

1. HMMs are a specific case
2. Globally optima guaranteed
3. Learn smaller representations

PSR: The single observation case

- PSR defined by: $\langle \alpha_0, \{A_\sigma\}, \alpha_\infty \rangle$

  where

  α_0 is an initial weighting on states $1xn$

  A_σ is a transition matrix nxn

  α_∞ is a normalizer $nx1$

- PSRs compute probabilities of observations

  $f(\sigma^k) = \alpha_0 \cdot A_\sigma^k \cdot \alpha_\infty$

Spectral Learning of PSRs

Step 1: Represent Data as a Hankel Matrix

Step 2: Singular Value Decomposition

Step 3: Pick Model Size

Step 4: Learn PSR: $\langle \alpha_0, \{A_\sigma\}, \alpha_\infty \rangle$

The Base System

- Idea: Learn $\{A_\sigma, A_{\sigma^2}, A_{\sigma^4}, A_{\sigma^8}, \ldots A_{\sigma^N}\}$ as extra transition operators
  Note: operators learned separately

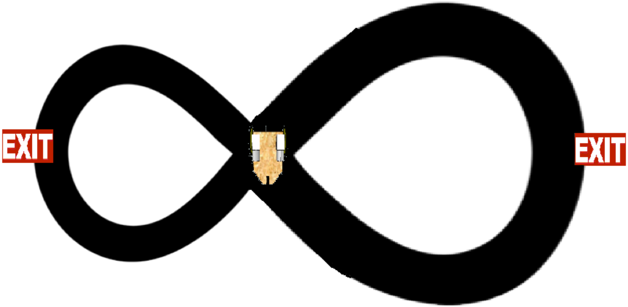
- $f(\sigma^{11}) = \alpha_0 \cdot A_{\sigma^8} \cdot A_{\sigma^2} \cdot A_{\sigma^1} \cdot \alpha_\infty$

Why might this help?

- Computations become more direct
- Capture structure directly
- Reduce error build up

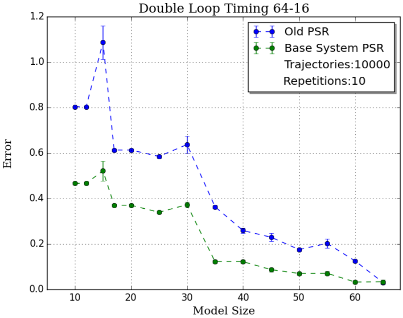
Timing with the Base

Agent drives around loops until leaving through an exit state.

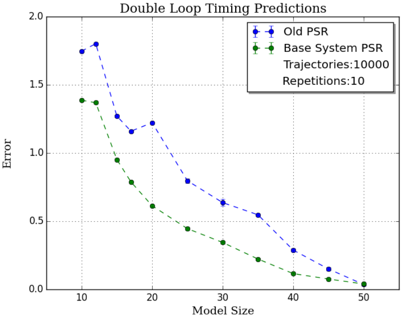


Base System Performance for Loops

64-16 Loop Lengths



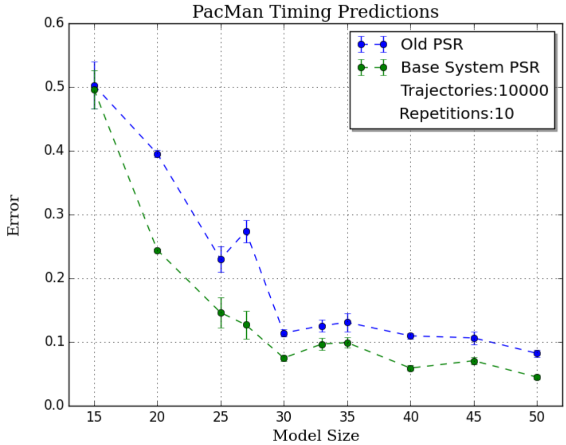
47-27 Loop Lengths



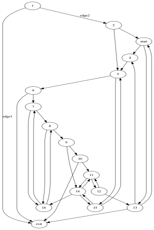
Base System dominates for smaller models

$$||f - \hat{f}|| = \sqrt{\sum_{x \in observations} (f(x) - f(\hat{x}))^2}$$

Pacman Labyrinth



(a) Pacman

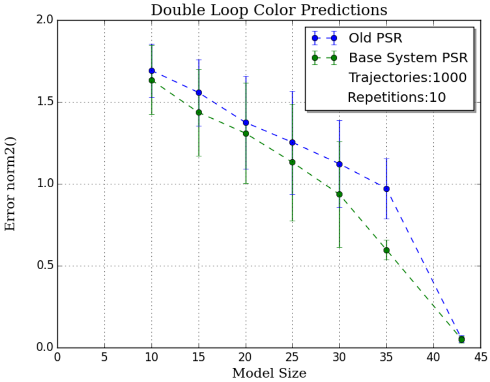


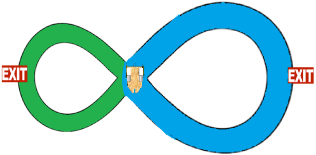
(b) Graph

$$||f - \hat{f}|| = \sqrt{\sum_{x \in observations} (f(x) - f(\hat{x}))^2}$$

Wall Color Predictions

We paint the first loop green and the second loop blue





$$||f - \hat{f}|| = \sqrt{\sum_{x \in observations} (f(x) - \hat{f(x)})^2}$$

Picking the Base System

- Observations: $\{"a^{30}":10, "a^{60}":5, "b^{18}":15\}$
  Desired Base System: $A_{a^{30}}, A_{b^{18}}, A_a, A_b$
- Substring properties: **long**, **frequent**, **diverse**
  Low entropy view of structure
- Solution: iterative greedy heuristic

Computing with the Base System

- How should we execute queries?

  Goal: **minimize number of matrices**

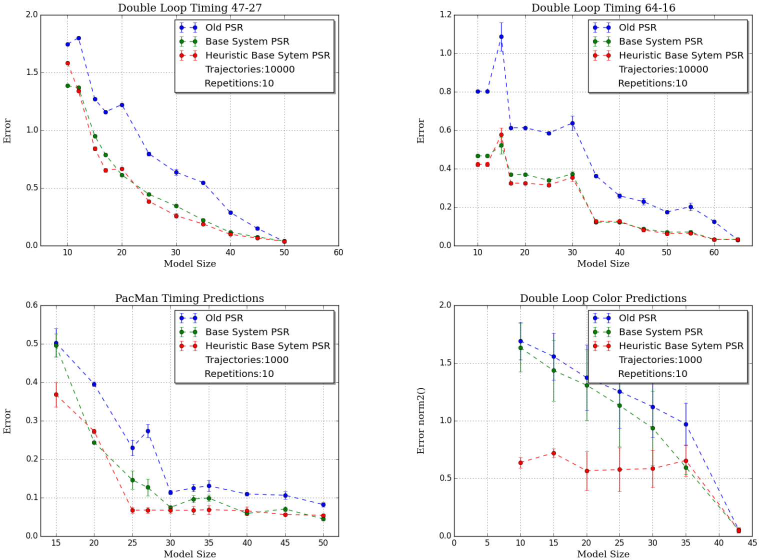
  Query string: "abcacb", Base System $= \{A_{ab}, A_{bca}, A_{cb}, A_a, A_b\}$

  Desired partition: "a—bca—cb"

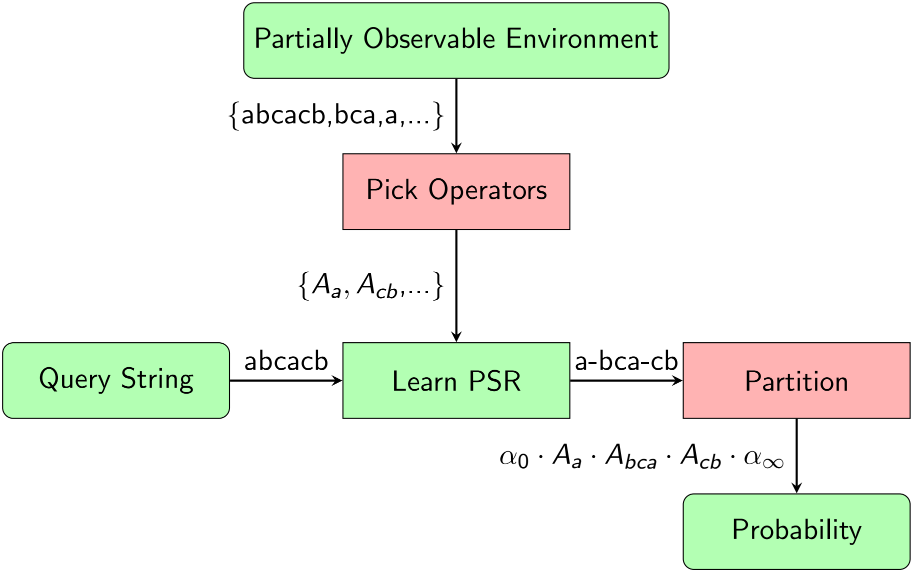
  Computation: $f(abcacb) = \alpha_0 \cdot A_a \cdot A_{bca} \cdot A_{cb} \cdot \alpha_\infty$

- Solution: dynamic programming

Performance of Heuristics



The Big Picture



Questions?