Spectral learning for structured partially observable environments

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Overview

PSRs

Predictive state representations (PSRs) are for computing a probability distribution over observations in a dynamical system [Littman et al.]

Also known as Weighted Automata (motivation dependent)

PSRs compute a function on finite strings of observations sequences f(abaab...).

Defined by three parameters: $< \alpha, A_x, \beta >$

α : Weighting on states E.g $\alpha = [0.5, 0.5]$

A_x : Transition operator for symbol x

$$A_x = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

β : Normalizer on states E.g

$$M = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$f(abaaba) = \alpha * A_a * A_b * A_a * A_a * A_b * \beta$

HMMs

- HMMs are an example of PSRs
- $A_x = O_x * T_x$
- O_x being an observation matrix
- T_x being a transition matrix
- So why bother the general framework of PSRs?

Why PSRs?

- For global optimum can learn a PSR but not an HMM
- Computational Equivalence of PSRs
  $< \alpha, A_x, \beta >$ v.s $< \alpha * M^-1, M * A_x * M^-1, M * \beta >$
  $f(x) = (\alpha * M\_1) * (M * A_x * M\_1) * (M * \beta)$
  $= \alpha * (M\_1 * M) * A_x * (M\_1 * M) * \beta)$
  $= \alpha * A_x * \beta)$

Hankel Matrices

Block 1

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Block 3

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Learning a PSR

Heading

1. Statement
2. Explanation
3. Example

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Learning a PSR Cont.

Heading

1. Statement
2. Explanation
3. Example

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The Base System

Number representations:
$39 = 1 * 2^5 + 0 * 2^4 + 0 * 2^3 + 1 * 2^2 + 1 * 2^1 + 1 * 2^0$
Timing Queries $f(a^39) = \alpha * A_a^32 * A_a^4 * A_a^2 * A_a^1$
Motivation: express longer transitions directly to avoid error compounding

The Base System Cont.

When are compounding errors a threat –¿ Truncating states

Analogy to rounding Round(51.63*34.12) v.s Round(51.63) * Round(34.12)

Using π as the projection operator onto less states (Borja's notation)

$f_Base(x) =$

$f_Naive(x) =$

Why would it help?

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table : Table caption

Timing in Labyrinths

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table : Table caption

Multiple Observations

Treatments	**Response 1**	**Response 2**
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table : Table caption

Picking the Base System

- For timing the GCD is of interest
- In general, one wants long and frequent sub-strings
- Want to make sure Base System is diverse
- Iterative greedy heuristic

Computing with the Base System

- Goal of Heuristic: minimize matrices in query
- Solution: Dynamic programming + trips
- Example:

Questions? Comments?

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End