

# Spectral Learning for Structured Partially Observable Environments

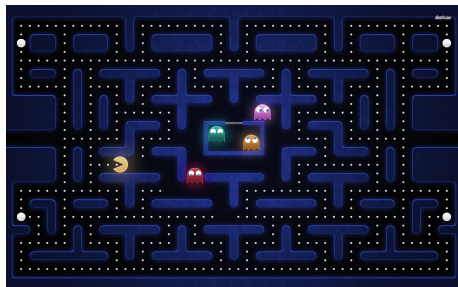
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## Structured Environments

- ① Goal: Predict observations
- ② Plan: Exploit patterns
- ③ Example: Pacman



# PSR: The Timing Case

- Model: Predictive State Representation (PSR)

Timing: one observation symbol:  $\sigma$

Notation:  $\sigma$ : one time unit,  $\sigma^k$ : k time units

- PSR defined by:  $\langle \alpha_0, \{A_\sigma\}, \alpha_\infty \rangle$

$\alpha_0$ : Initial weighting on states  $1 \times n$

$A_\sigma$ : Transition matrix  $n \times n$

$\alpha_\infty$ : Normalizer  $n \times 1$

- PSRs compute probabilities of observations

$$f(\sigma^k) = \alpha_0 \cdot A_\sigma^k \cdot \alpha_\infty$$

Ex: HMMs

# Spectral Learning of PSRs

- Step 1: Represent Data as a Hankel Matrix
- Step 2: Spectral: Singular Value Decomposition
- Step 3: Pick Model Size
- Step 4: Learn PSR:  $\langle \alpha_0, \{A_\sigma\}, \alpha_\infty \rangle$

# The Base System

- Idea: Learn  $\{A_\sigma, A_{\sigma^2}, A_{\sigma^4}, A_{\sigma^8}, \dots A_{\sigma^N}\}$  as extra transition operators

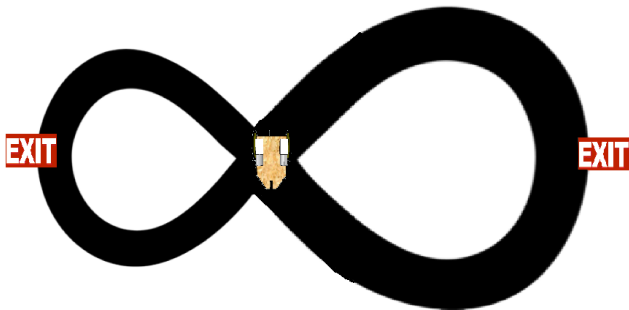
Note: operators learned separately

- Timing queries:  $f(\sigma^{11}) = \alpha_0 \cdot A_{\sigma^8} \cdot A_{\sigma^2} \cdot A_{\sigma^1} \cdot \alpha_\infty$

- Motivation: express structure directly

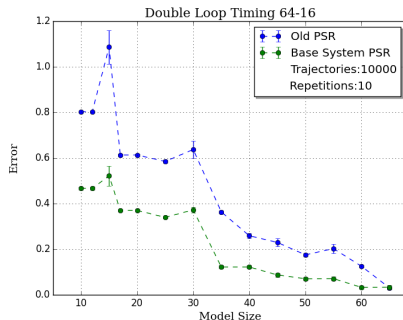
# Timing with the Base

Agent drives around loops until leaving through an exit state.

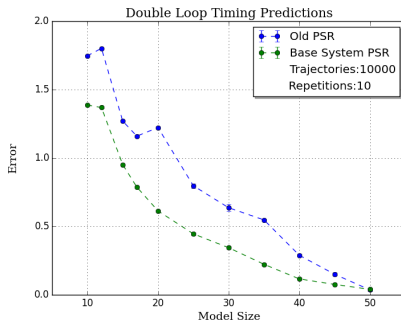


# Base System Performance for Loops

## 64-16 Loop Lengths



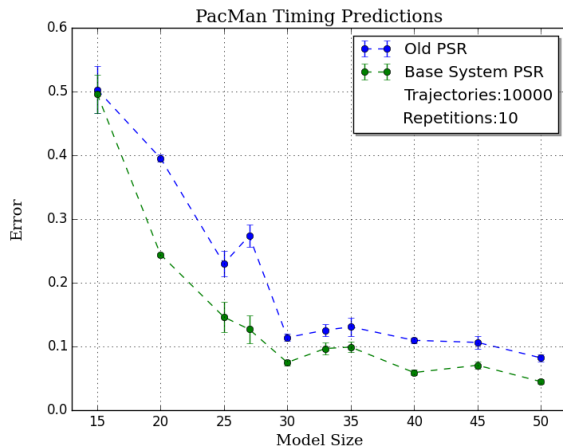
## 47-27 Loop Lengths



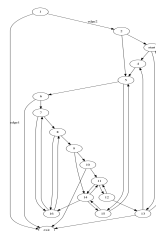
Base System dominates for smaller models

$$\|f - \hat{f}\| = \sqrt{\sum_{x \in \text{observations}} (f(x) - \hat{f}(x))^2}$$

# Pacman Labyrinth



(a) Pacman



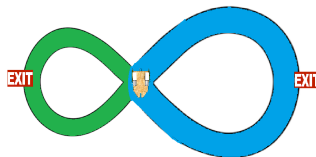
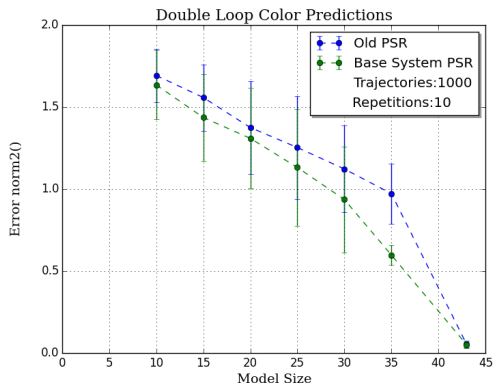
(b) Graph

$$\|f - \hat{f}\| = \sqrt{\sum_{x \in \text{observations}} (f(x) - \hat{f}(x))^2}$$



# Wall Color Predictions

We paint the first loop green and the second loop blue



$$\|f - \hat{f}\| = \sqrt{\sum_{x \in \text{observations}} (f(x) - \hat{f}(x))^2}$$

# Picking the Base System

- Observations:  $\{ "a^{30}":10, "a^{60}":5, "b^{18}":15 \}$

Desired Base System:  $A_{a^{30}}, A_{b^{18}}, A_a, A_b$

- Substring properties: **long, frequent, diverse**

Low entropy view of structure

- Solution: iterative greedy heuristic

# Computing with the Base System

- How should we execute queries?

Goal: **minimize number of matrices**

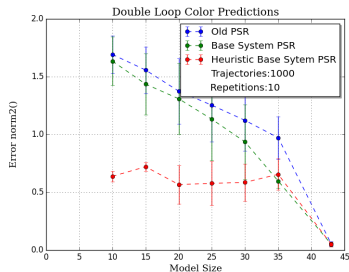
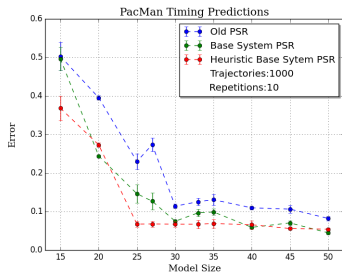
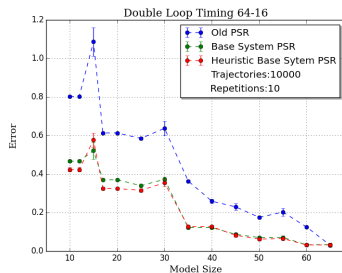
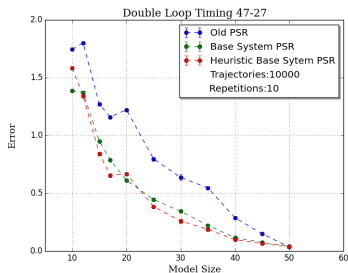
Query string: "abcacb", Base System =  $\{A_{ab}, A_{bca}, A_{cb}, A_a, A_b\}$

Desired partition: "a—bca—cb"

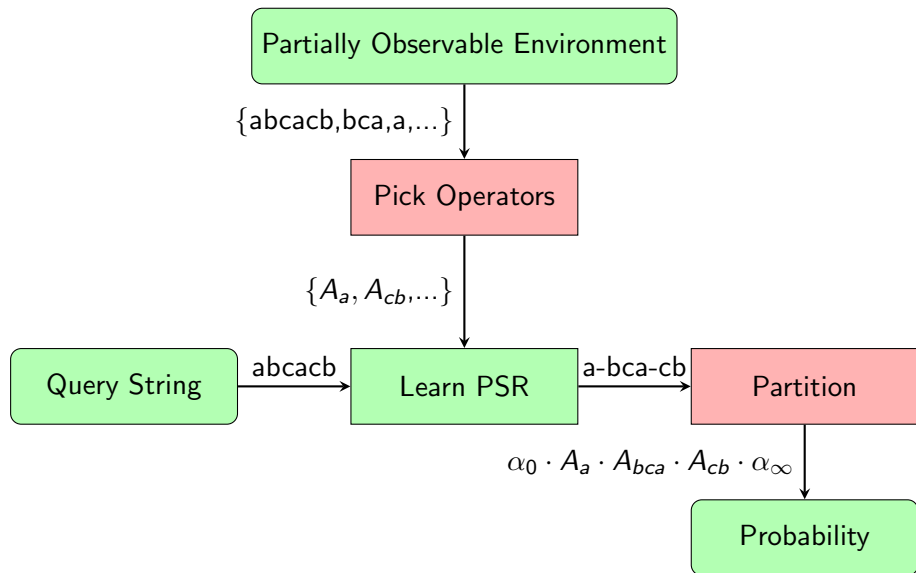
Computation:  $f(abcacb) = \alpha_0 \cdot A_a \cdot A_{bca} \cdot A_{cb} \cdot \alpha_\infty$

- Solution: dynamic programming

# Performance of Heuristics



# The Big Picture



# Questions?