TP 5

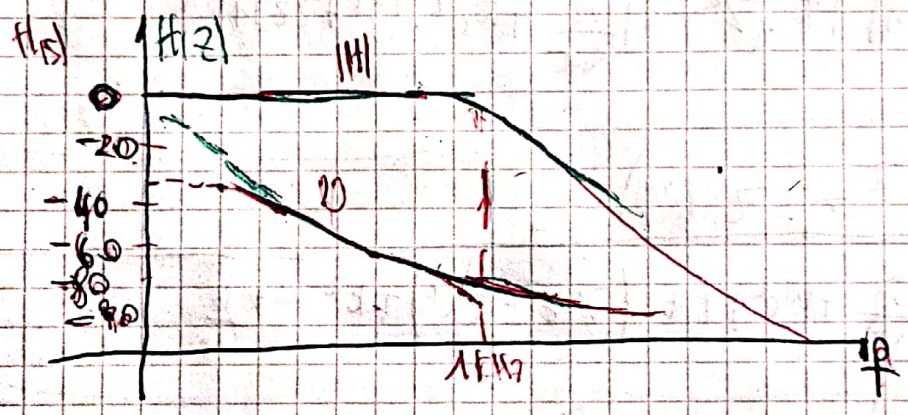
② ⓐ $H(s) = \frac{(2\pi \cdot 1 \cdot 10^3)^2}{s^2 + 5\sqrt{2}\cdot 1\cdot 10^3 \pi s + (1\cdot 10^3 2\pi)^2} = \frac{w_c^2}{s^2 + 5\sqrt{2} w_c s + w_c^2}$

$H(z) = H(s)\Big|_{s = k\frac{z-1}{z+1}} = \frac{w_c^2}{k^2\frac{(z-1)^2}{(z+1)^2} + k\frac{(z-1)}{(z+1)}\sqrt{2} w_c + w_c^2}$

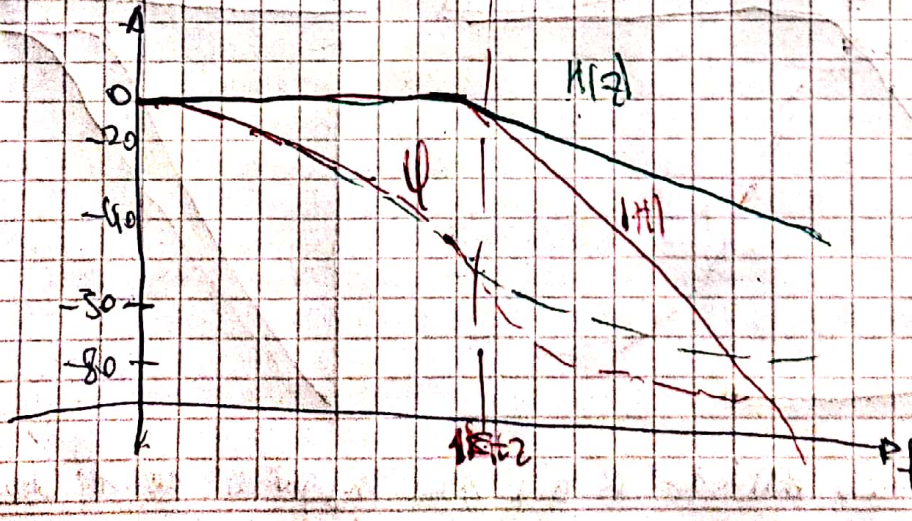
$H(z) = \frac{(z+1)^2 \cdot w_c^2}{k^2 (z-1)^2 + k(z-1)(z+1)\sqrt{2} w_c + w_c^2 (z+1)^2}$

$H(z) = \frac{w_c^2 (z^2 + 2z + 1)}{k^2 (z^2 - 2z + 1) + k(z^2 - 1)\sqrt{2} w_c + w_c^2 (z^2 + 2z + 1)}$

$$H(z) = \frac{w_c^2 (z^2 + 2z + 1)}{z^2 (k^2 + k\sqrt{2} w_c + w_c^2) + z(2w_c^2 - 2k^2) + k^2 - k\sqrt{2} w_c + w_c^2}$$



ⓑ



(c)

$$H_{(s)} = \frac{w_c^2}{s^2 + s \cdot w_c \cdot \sqrt{2} + w_c^2}$$

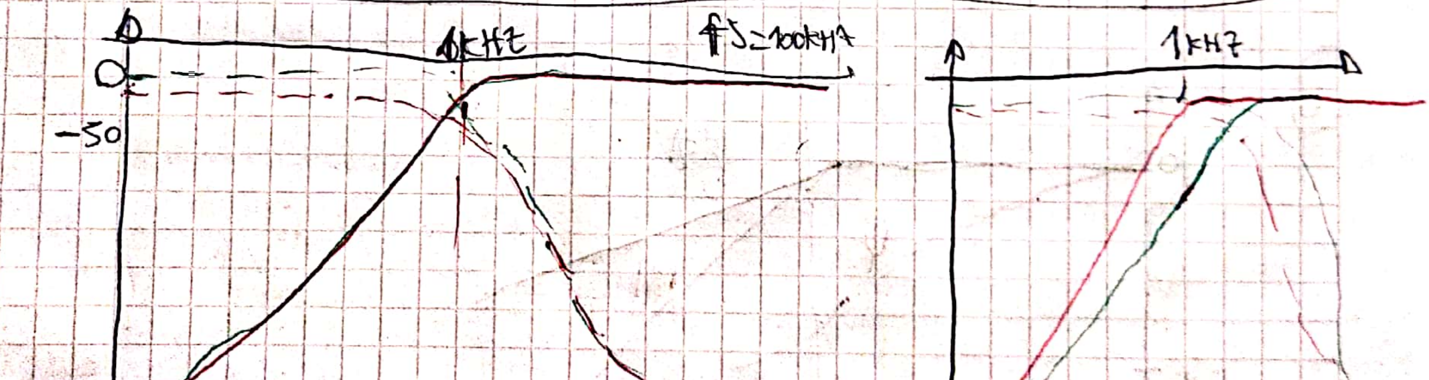
$$H_{(s)HP} = \frac{w_c^2}{\left(\frac{1}{s}\right)^2 + \frac{w_c \sqrt{2}}{s} + w_c^2} = \frac{w_c^2 \, s^2}{1 + s w_c \sqrt{2} + w_c^2 s^2}$$

$$H_{(s)HP} = \frac{s^2}{s^2 + \frac{s\sqrt{2}}{w_c} + w_c^2}$$

$$H_{(z)} = \frac{k^2 \cdot \left(\frac{z-1}{z+1}\right)^2}{k^2 \cdot \left(\frac{z-1}{z+1}\right)^2 + k \cdot \frac{(z-1)\sqrt{2}}{(z+1)\, w_c} + w_c^2} = \frac{k^2 \cdot \frac{(z-1)^2}{(z+1)^2}}{\frac{k^2 \cdot (z-1)^2 + k \cdot (z+1)(z-1)\frac{\sqrt{2}}{w_c} + w_c^2 \cdot (z+1)^2}{(z+1)^2}}$$

$$H_{(z)} = \frac{k^2 \cdot (z^2 - 2z + 1)}{k^2 \cdot (z^2 - 2z + 1) + k \cdot (z^2 - 1)\frac{\sqrt{2}}{w_c} + w_c^2 \cdot (z^2 + 2z + 1)}$$

$$\boxed{H_{(z)} = \frac{k^2 \cdot (z^2 - 2z + 1)}{z^2 \cdot \left(k^2 + k\frac{\sqrt{2}}{w_c} + w_c^2\right) + z \cdot (2w_c^2 - 2k^2) + k^2 - k\frac{\sqrt{2}}{w_c} + w_c^2}}$$



(d) En el último caso debido a que por requisito se necesitaría al menos 12kHz para trabajar sin problemas