

TP3 - Ejercicio 4

Hallar $Z(s)$ a partir de su reactancia $X(j\omega) = \frac{-\omega^3 + \omega}{\omega^4 - \omega^2 + 1}$

$$X(j\omega) \Big|_{\omega = \frac{s}{j}} = j \left(\frac{\frac{s^3}{j} + \frac{s}{j}}{s^4 + s^2 + 1} \right) = \frac{s^3 + s}{s^4 + s^2 + 1} = X(s)$$

$$X(s) = \frac{s^3 + s}{s^4 + s^2 + 1} = \frac{s(s^2 + 1)}{s^4 + s^2 + 1} = \frac{n_1 \cdot m_2 - m_1 \cdot n_2}{m_2^2 - n_2^2}$$

$$\text{Ca} \\ (s^2 + 1)^2 = s^4 + 2s^2 + 1$$

Del numerador:

$$s^3 + s = n_1 m_2 - m_1 n_2 \Rightarrow \underline{n_2 = s} \wedge \underline{m_1 = 0} \\ \underline{n_1 = s} \wedge \underline{m_2 = s^2 + 1}$$

Verificamos el denominador: $(s^2 + 1)^2 - s^2 = s^4 + 2s^2 + 1 - s^2 = s^4 + s^2 + 1$

Ahora armamos $F(s) = \frac{m_1 + n_1}{m_2 + n_2} = \frac{s}{s^2 + s + 1}$

Corroboramos:

$$F(s) \Big|_{s=j\omega} = \frac{j\omega}{(1 - \omega^2 + j\omega)} \cdot \frac{(1 - \omega^2 - j\omega)}{(1 - \omega^2 - j\omega)} = \frac{j\omega - j\omega^3 + \omega^2}{(1 - \omega^2)^2 + \omega^2}$$

$$F(\omega) = R(\omega) + jX(\omega) = \frac{\omega^2}{\omega^4 - \omega^2 + 1} + j \frac{-\omega^3 + \omega}{\omega^4 - \omega^2 + 1}$$