Universidad Tecnológica Nacional

Facultad Regional Buenos Aires

Departamento De Electrónica

Teoría de los circuitos II

Año: 2021 **Curso: R4052**

Profesor: Llamedo, Mariano **JTP: Fuoco Cesar**

Trabajo Práctico Nro 5

GRUPO N°: 3

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OBSERVACIONES:

Presentado: 30/10/2021
Aprobado:
Firmado:

HOJA N°

FECHA

TP.5

2 a)

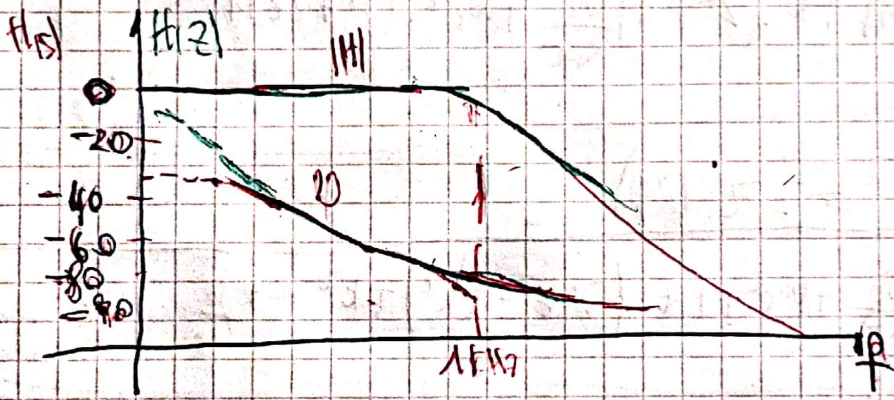
$$H(s) = \frac{(2\pi \cdot 1\cdot 10^3)^2}{s^2 + s\sqrt{2}\,1\cdot 10^3 2\pi + (1\cdot 10^3 2\pi)^2} = \frac{w_c^2}{s^2 + s\sqrt{2}\,w_c + w_c^2}$$

$$H(z) = H(s)\Big|_{s = k\frac{z-1}{z+1}} = \frac{w_c^2}{k^2 \cdot \frac{(z-1)^2}{(z+1)^2} + k \cdot \frac{(z-1)}{(z+1)} \cdot \sqrt{2}\,w_c + w_c^2}$$

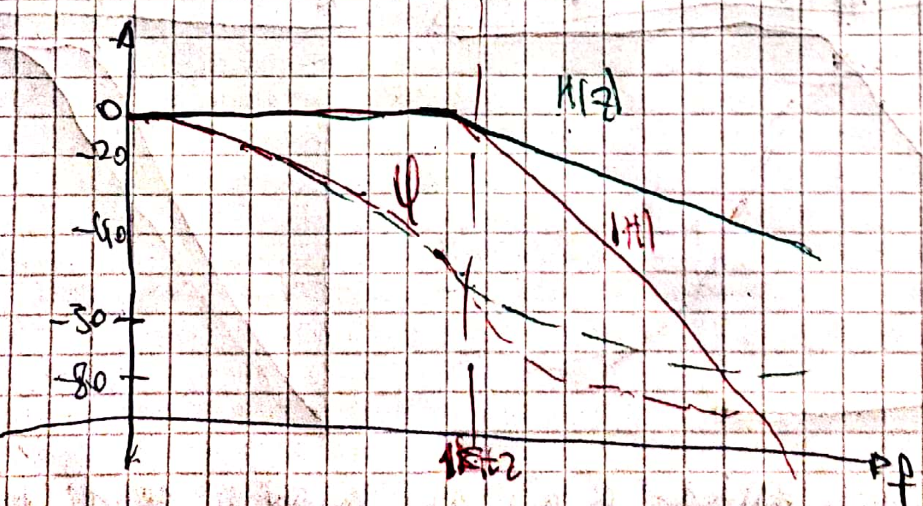
$$H(z) = \frac{(z+1)^2 \cdot w_c^2}{k^2 \cdot (z-1)^2 + k \cdot (z-1) \cdot (z+1) \cdot \sqrt{2}\,w_c + w_c^2 \cdot (z+1)^2}$$

$$H(z) = \frac{w_c^2 \cdot (z^2 + 2z + 1)}{k^2 \cdot (z^2 - 2z + 1) + k \cdot (z^2 - 1)\sqrt{2}\,w_c + w_c^2 \cdot (z^2 + 2z + 1)}$$

$$\boxed{H(z) = \frac{w_c^2 \,(z^2 + 2z + 1)}{z^2 \cdot (k^2 + k\sqrt{2}w_c + w_c^2) + z \cdot (2w_c^2 - 2k^2) + k^2 - k\sqrt{2}\,w_c + w_c^2}}$$



b)



NOTA

(c)

$$H_{(s)} = \frac{w_c^2}{s^2 + s\cdot w_c \cdot \sqrt{2} + (w_c^2)}$$

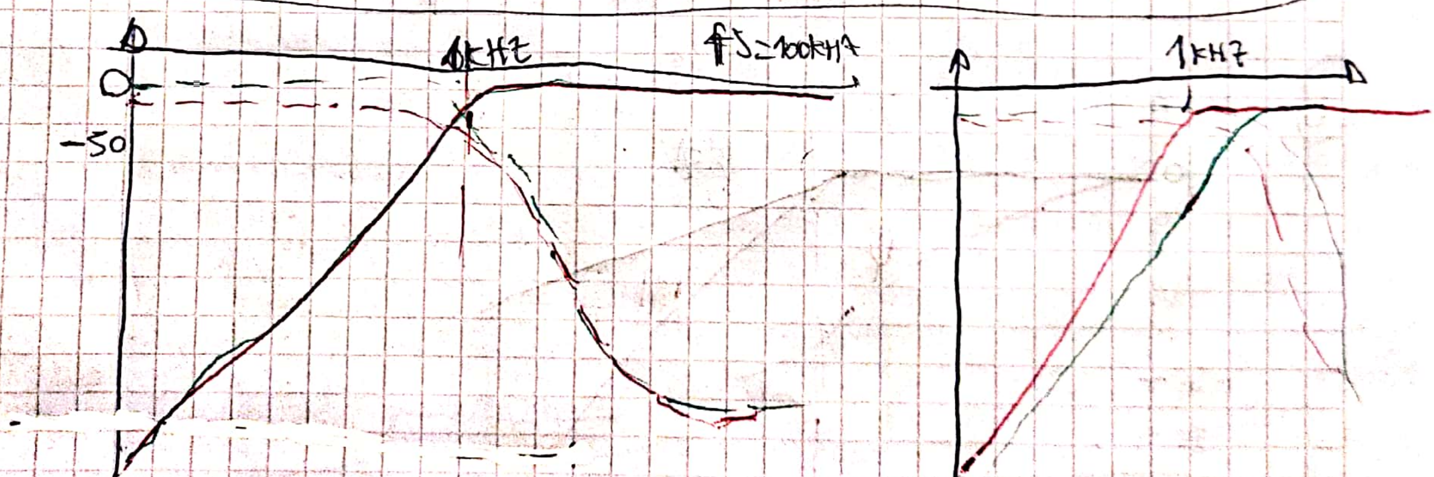
$$H_{(s)He} = \frac{w_c^2}{\left(\frac{1}{s}\right)^2 + \frac{w_c \sqrt{2}}{s} + w_c^2} = w_c^2 \frac{s^2}{1 + s\, w_c \sqrt{2} + w_c^2 \cdot s^2}$$

$$H_{(s)HP} = \frac{s^2}{s^2 + \frac{s\sqrt{2}}{w_c} + w_c^2}$$

$$H_{(z)} = \frac{k^2 \cdot \left(\frac{z-1}{z+1}\right)^2}{k^2 \cdot \left(\frac{z-1}{z+1}\right)^2 + k \cdot \frac{(z-1)}{z+1} \frac{\sqrt{2}}{w_c} + w_c^2} = \frac{k^2 \cdot \cancel{\frac{(z-1)^2}{(z+1)^2}}}{\frac{k^2 \cdot (z-1)^2 + k \cdot (z+1)\cdot(z-1)\frac{\sqrt{2}}{w_c} + w_c^2 \cdot (z+1)^2}{\cancel{(z+1)^2}}}$$

$$H_{(z)} = \frac{k^2 \cdot (z^2 - 2z + 1)}{k^2 \cdot (z^2 - 2z + 1) + k \cdot (z^2 - 1)\frac{\sqrt{2}}{w_c} + w_c^2 \cdot (z^2 + 2z + 1)}$$

$$\boxed{H_{(z)} = \frac{k^2 \cdot (z^2 - 2z + 1)}{z^2 \cdot \left(k^2 + k\frac{\sqrt{2}}{w_c} + w_c^2\right) + z \cdot (2w_c^2 - 2k^2) + k^2 - k\frac{\sqrt{2}}{w_c} + w_c^2}}$$



(d) En el último caso debido a que por requisito se necesitaría al menos 12kHz para trabajar sin problemas

3) $H(z)$, ung. y respuesta en modulo y fase

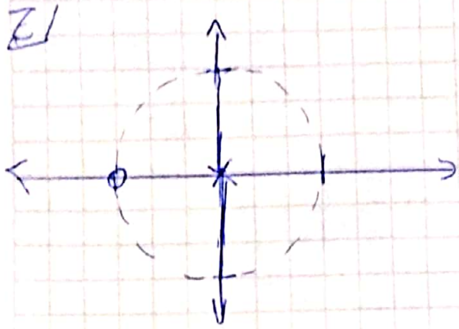
a) Media movil

$$h_1(k) = (1,1) \rightarrow y(k) = \overbrace{\delta[k]}^{x[k]} + \overbrace{\delta[k-1]}^{x[k-1]}$$

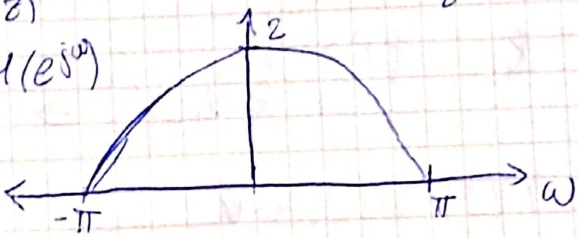
$$Y(z) = X(z) + z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-1} = \frac{z+1}{z}$$

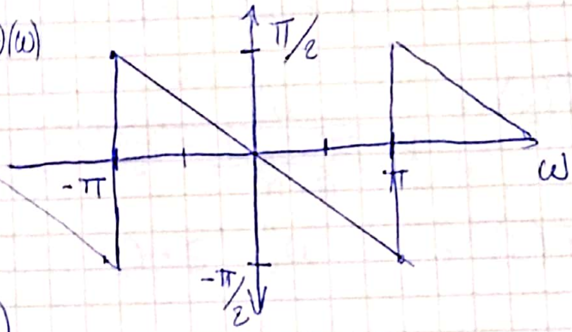
Z/



$H(e^{j\omega})$



$\Theta(\omega)$



$$|H(e^{j\omega})| = \frac{\prod |e^{j\omega} - z_i|}{\prod |e^{j\omega} - p_i|}$$

$$\Theta(e^{j\omega}) = \sum \Theta(e^{j\omega} - z_i) - \sum \Theta(e^{j\omega} - p_i)$$

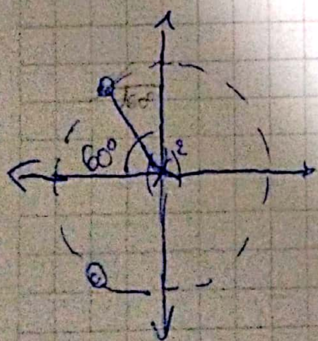
$$h_2[k] = (1,1,1)$$

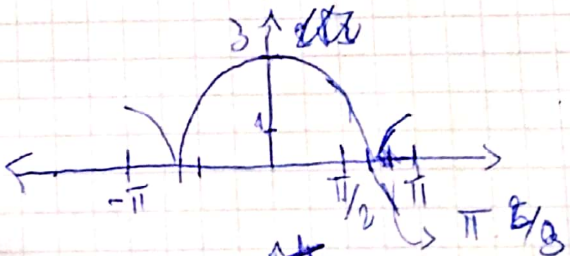
$$\hookrightarrow Y(z) = X(z) + z^{-1}X(z) + X(z)z^{-2}$$

$$H(z) = \frac{z^2 + z + 1}{z^2}$$

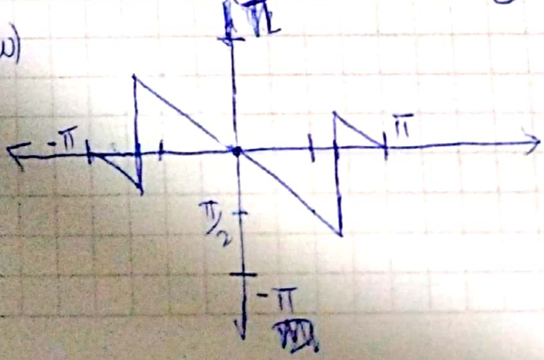
Ceros: $-0{,}5 \pm j\frac{\sqrt{3}}{2}$

Polos: 2 en 0





$\Theta(\omega)$



90°
~~180°~~ en $\frac{\pi}{2}$

360° en π

[illegible] en el todo

a) para que sean medias aritméticas debemos dividir por el número de muestras

b) $\frac{f_s}{2}$ —— 180° $\qquad 120° = \frac{50Hz \cdot 180°}{f_s/2}$

50Hz —— 120°

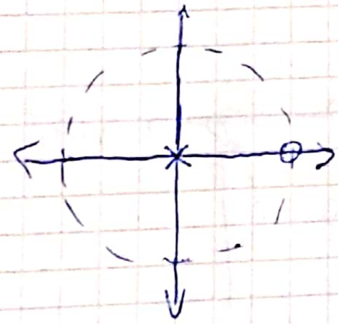
$f_s = 150 Hz$

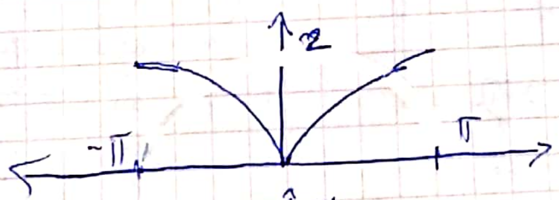
Filtro diferenciador

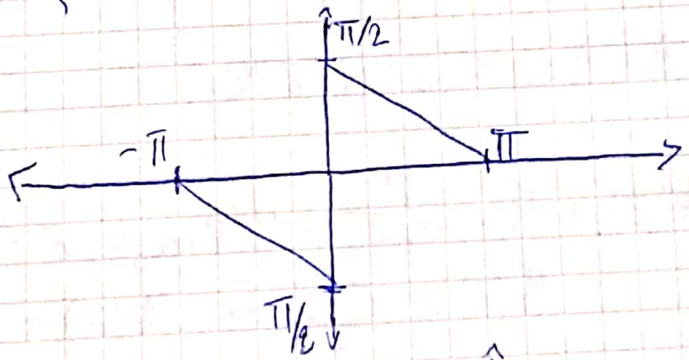
$h(k) = (1, -1) \rightarrow y(k) = \delta[k] - \delta(k-1) \quad x(k) = \delta(k)$

$Y(z) = X(z)(1 - z^{-1})$

$H(z) = 1 - z^{-1} = \frac{z-1}{z}$

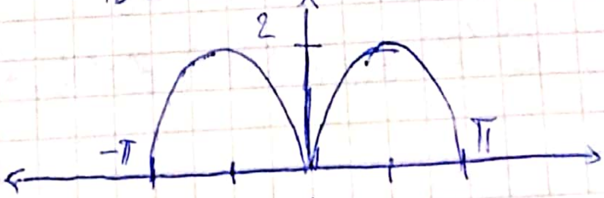


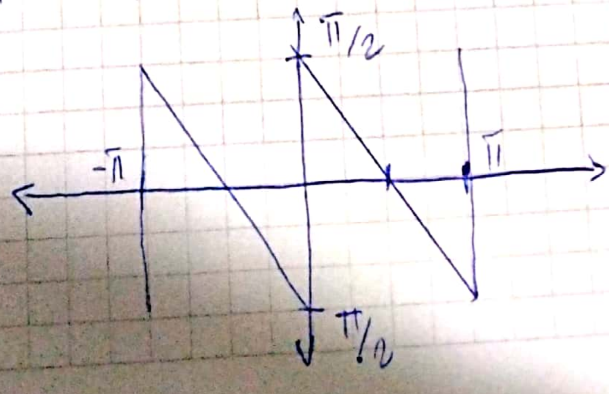


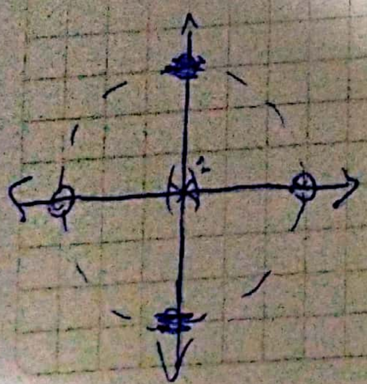


$h(k) = (1, 0, -1)$

$H(z) = 1 - z^{-2} = \frac{z^2 - 1}{z^2}$







a) Ambos introducen una demora igual al orden del filtro

b) $H(\Omega) = 1 - e^{-j\Omega} = e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} - e^{-j\Omega/2}\right)$

$= e^{j\left(\frac{\pi}{2}-\frac{\Omega}{2}\right)} 2 \operatorname{sen}(\Omega/2)$

$0{,}95 \leq \frac{2 \operatorname{sen}(\Omega/2)}{\Omega} \leq 1{,}05$

$\Omega = 0 \rightarrow \frac{|H(\Omega)|}{\Omega} = 1$

$\Omega = \pi \rightarrow \frac{|H(\Omega)|}{\Omega} =$

$\frac{|H(\Omega)|}{\Omega} = 0{,}95 \rightarrow \Omega = 1{,}1038$

HOJA Nº

FECHA

(1) (a) $\underbrace{-X_{(z)}\cdot z^{-N}\cdot c_1 + X_{(z)} + W_{(z)}\cdot a_0\cdot z^{-1}\cdot a_1 + W_{(z)}\cdot a_0\cdot a_2\cdot z^{-2}}_{b_{W(z)}} = W_{(z)}$

$$Y_{(z)} = W_{(z)}\cdot a_0\left(b_0 + z^{-1}\cdot b_1 + z^{-2}\cdot b_2\right)$$

$$W_{(z)} = \frac{Y_{(z)}}{a_0\cdot\left(b_0 + z^{-1}\cdot b_1 + z^{-2}\cdot b_2\right)}$$

$$X_{(z)}\cdot\left(1 - z^{-N}c_1\right) + \frac{Y_{(z)}\cdot a_1\cdot z^{-1}}{\left(b_0 + z^{-1}\cdot b_1 + z^{-2}\cdot b_2\right)} + \frac{Y_{(z)}\cdot a_2\cdot z^{-2}}{b_0 + z^{-1}\cdot b_1 + z^{-2}\cdot b_2} = \frac{Y_{(z)}}{a_0\cdot\left(b_0 + z^{-1}\cdot b_1 + z^{-2}\cdot b_2\right)}$$

$$X_{(z)}\cdot\left(1 - z^{-N}\cdot c_1\right) = Y_{(z)}\cdot\left(\frac{1}{a_0\cdot\left(b_0 + z^{-1}b_1 + z^{-2}b_2\right)} - \frac{a_1\cdot z^{-1}}{b_0 + z^{-1}b_1 + z^{-2}b_2} - \frac{a_2\cdot z^{-2}}{b_0 + z^{-1}\ldots}\right)$$

$$X_{(z)}\cdot\left(1 - z^{-N}\cdot c_1\right) = Y_{(z)}\left(\frac{1 - a_0\cdot a_1\cdot z^{-1} - a_0\cdot a_2\cdot z^{-2}}{a_0\cdot\left(b_0 + z^{-1}\cdot b_1 + z^{-2}\cdot b_2\right)}\right)$$

$$H(z) = \left(1 - c_1\cdot z^{-N}\right)\cdot\left(\frac{z^{-2}\cdot a_0 b_2 + z^{-1}\cdot a_0 b_1 + a_0\cdot b_0}{-z^{-2}\cdot a_0 a_2 - z^{-1}\cdot a_0 a_1 + 1}\right)$$

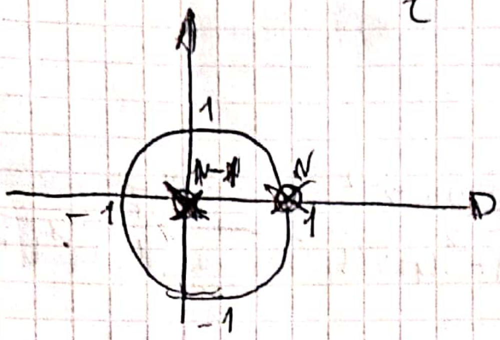
$$\boxed{H(z) = \left(1 - c_1\cdot z^{-N}\right)\cdot\left(\frac{z^{-2}\cdot b_2 + z^{-1}\cdot b_1 + b_0}{-z^{-2}\cdot a_2 - z^{-1}\cdot a_1 + \frac{1}{a_0}}\right)}$$

$$H(z) = \left(\frac{z^N - c_1}{z^N}\right)\cdot\left(\frac{z^2\cdot b_0 + z\cdot b_1 + b_2}{\frac{z^2}{a_0} - z\cdot a_1 - a_2}\right)$$

NOTA

(b) $\boxed{H(z) = (1-z^{-N})} \cdot \left(\frac{1/N}{1-z^{-1}}\right) = \boxed{\frac{1}{N}\cdot\left(\frac{1-z^{-N}}{1-z^{-1}}\right)}$ ✓

$$H(z) = \frac{1}{N}\cdot\frac{\frac{z^N-1}{z^N}}{\frac{z-1}{z}} = \frac{1}{N}\cdot\frac{(z^N-1)}{z^{N-1}(z-1)}$$



según su definición es un filtro IIR ya que $|a_i| \neq 0$, pero se comporta como FIR ya que es siempre estable

$$H_6(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}$$

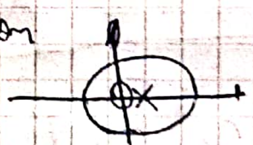
se puede lograr usando 3 de estas redes en cascada

(c) Para diferenciador de 1er orden: $\boxed{b_0 = a_2 = 0}$ $\boxed{a_0 \to \infty}$
$\boxed{b_1 = 1}$ $\boxed{b_2 = -1}$ $\boxed{a_1 = -1}$

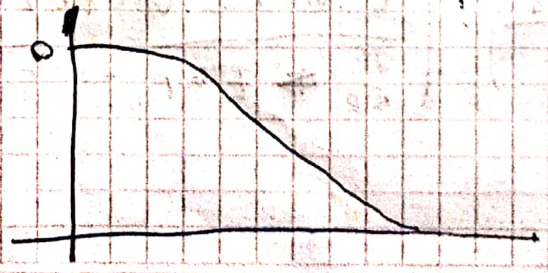
de 2º orden: $\boxed{b_0 = a_0 = 1}$ $\boxed{b_1 = a_1 = a_2 = 0}$ $\boxed{b_2 = -1}$

(d) Para $\alpha = 0,9 \Rightarrow \boxed{H(z) = \frac{0,9z^2}{z^2 - 0,1z}}$

$\boxed{H(z) = \frac{0,9z}{z - 0,1}}$ Es un integrador



$$\frac{0,9\, e^{j\Omega}}{e^{j\Omega} - 0,1}$$



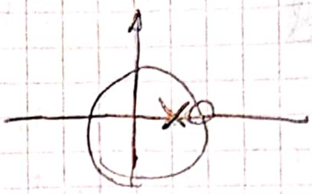
$$\frac{0,9\, e^{j\Omega}}{0,1\, e^{j\Omega/2}\left(\frac{e^{j\Omega}}{0,1} + e^{-j\Omega/2}\right)}$$

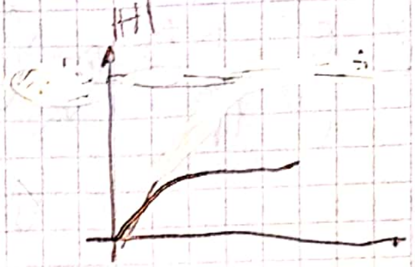
HOJA Nº

FECHA

(e) $H_{(z)} = \dfrac{z^2 - z}{z^2 - \alpha \cdot z} = \dfrac{z-1}{z-\alpha}$

Para $\alpha = 0{,}9$ $\quad \dfrac{z-1}{z-0{,}9}$





$$H_{(e^{j\Omega})} = \frac{e^{j\Omega}-1}{e^{j\Omega}-0{,}9} = \frac{\cos\Omega + j\,\text{sen}\,\Omega - 1}{\cos\Omega + j\,\text{sen}\,\Omega - 0{,}9} \qquad |He^{j\Omega}| = \frac{\sqrt{(\cos\Omega-1)^2 + \text{sen}^2\Omega}}{\sqrt{(\cos\Omega-0{,}9)^2 + \text{sen}^2\Omega}}$$

$$|He^{j\Omega}| = \frac{\sqrt{2(1-\cos\Omega)}}{\sqrt{1{,}81 - 1{,}8\cos\Omega}} \qquad |H_{(\Omega=\pi)}| = 20 \cdot \log\left(\frac{\sqrt{2\cdot(1-\cos\pi)}}{\sqrt{1{,}81 - 1{,}8\cdot\cos\pi}}\right)$$

$$|He^{j\Omega}| = \frac{\sqrt{2\cdot(1-\cos(\Omega))}}{\sqrt{(1+\alpha^2) - 2\alpha\cdot\cos\Omega}}$$

$-1{,}74$

$$3 = 20\log\left(\frac{\sqrt{2\cdot(1-\cos\pi)}}{\sqrt{(1+\alpha^2) - 2\alpha\cdot\cos\pi}}\right) - 20\log\left(\frac{\sqrt{2\cdot(1-\cos(0{,}1\pi))}}{\sqrt{(1+\alpha^2) - 2\alpha\cdot\cos 0{,}1\pi}}\right)$$

$1{,}2596$

$$\frac{3}{20} = \log\left(\frac{\dfrac{2}{\sqrt{(1+\alpha^2)+2\alpha}}}{\dfrac{0{,}31}{\sqrt{(1+\alpha^2) - \alpha 1{,}9}}}\right)$$

$$10^{0{,}15} = \frac{2}{0{,}31} \cdot \frac{\sqrt{\alpha^2 - 1{,}9\alpha + 1}}{\sqrt{\alpha^2 + 2\alpha + 1}} \Rightarrow 0{,}21894 = \frac{\sqrt{\alpha^2 - 1{,}9\alpha + 1}}{\sqrt{\alpha^2 + 2\alpha + 1}}$$

$$0{,}04793 \cdot (\alpha^2 + 2\alpha + 1) = \alpha^2 - 1{,}9\alpha + 1$$

NOTA

$$\alpha^2\, 0{,}95206 - \alpha\, 1{,}99386 + 0{,}95206 \qquad \boxed{\alpha = 0{,}73}$$

f) $H(z)=\left(1-C_1 z^{-N}\right)\dfrac{b_0+b_1z^{-1}+b_2z^{-2}}{\frac{1}{a_0}-a_1z^{-1}-a_2z^{-2}}$; $a_0=1$, $a_1=-R$, $a_2=\emptyset$, $C=\emptyset$; $b_0=R$, $b_1=1$, $b_2=\emptyset$; $R=\dfrac{-D}{D+2}$, $D\in[-0{,}5\,;\,+0{,}5]$

$$\hookrightarrow H(z)=\frac{R+z^{-1}}{1+Rz^{-1}}\Rightarrow H(e^{-j\Omega})=\frac{R-1+1+e^{-j\Omega}}{1+e^{-j\Omega}+(R-1)e^{-j\Omega}}=\frac{\cancel{e^{-j\Omega/2}}}{\cancel{e^{-j\Omega/2}}}\frac{e^{j\Omega/2}+e^{-j\Omega/2}+(R-1)e^{j\Omega/2}}{e^{j\Omega/2}+e^{-j\Omega/2}+(R-1)e^{-j\Omega/2}}=$$

$$H(e^{+j\Omega})=\frac{2\cos(\Omega/2)+(R-1)\cos(\Omega/2)+j(R-1)\operatorname{sen}(\Omega/2)}{2\cos(\Omega/2)+(R-1)\cos(\Omega/2)-j(R-1)\operatorname{sen}(\Omega/2)}=$$

$$=\frac{(R+1)\cos(\Omega/2)+j(R-1)\operatorname{sen}(\Omega/2)}{(R+1)\cos(\Omega/2)-j(R-1)\operatorname{sen}(\Omega/2)}$$

$$\hookrightarrow |H(\Omega)|^2=\frac{(R+1)^2\cos^2(\Omega/2)+(R-1)^2\operatorname{sen}^2(\Omega/2)}{(R+1)^2\cos^2(\Omega/2)+(R-1)^2\operatorname{sen}^2(\Omega/2)}=1\Rightarrow \boxed{|H(\Omega)|=1}$$

$$\varphi(\Omega)=\operatorname{arctg}\left[\frac{R-1}{R+1}\operatorname{tg}(\Omega/2)\right]+\operatorname{arctg}\left[+\frac{R-1}{R+1}\operatorname{tg}(\Omega/2)\right]=\boxed{2\operatorname{arctg}\left[\frac{R-1}{R+1}\operatorname{tg}(\Omega/2)\right]}$$

$$-\frac{d\varphi(\Omega)}{d\Omega}=\tau(\Omega)=-\cancel{2}\frac{1}{1+\frac{(R-1)^2}{(R+1)^2}\operatorname{tg}^2(\Omega/2)}\left(\frac{R-1}{R+1}\right)\frac{1}{\cancel{2}\cos^2(\Omega/2)}=$$

$$=\frac{1-R}{1+R}\frac{1}{\cos^2(\Omega/2)+\left(\frac{1-R}{1+R}\right)^2\operatorname{sen}^2(\Omega/2)}\ ;$$

$$\boxed{R=\frac{-D}{D+2}\Rightarrow\frac{1-R}{1+R}=\frac{1+\frac{D}{D+2}}{1-\frac{D}{D+2}}=\frac{2D+2}{2}=D+1}$$

$$\Rightarrow \boxed{\tau(\Omega)=\frac{1}{\frac{1}{D+1}\cos^2(\Omega/2)+(D+1)\operatorname{sen}^2(\Omega/2)}}$$

$$\hookrightarrow \Omega=0\rightarrow \boxed{\tau(\Omega=0)=D+1}$$

MARGEN 5% RESPECTO DE $\tau(\Omega=0)=D+1$

$$0{,}95<\frac{\tau(\Omega)}{\tau(0)}<1{,}05$$

$$0{,}95<\frac{1}{D+1}\frac{1}{\frac{1}{D+1}\cos^2(\Omega/2)+(D+1)\operatorname{sen}^2(\Omega/2)}<1{,}05$$

$$0{,}95<\frac{1}{\cos^2(\Omega/2)+(D+1)^2\operatorname{sen}^2(\Omega/2)}<1{,}05$$

$$\frac{20}{19}>\cos^2(\Omega/2)+(D^2+2D)\operatorname{sen}^2(\Omega/2)+\operatorname{sen}^2(\Omega/2)<\frac{20}{21}$$

$$\frac{20}{19}>1+(D^2+2D)\operatorname{sen}^2(\Omega/2)>\frac{20}{21}$$

$$\rightarrow\frac{1}{19}>(D^2+2D)\operatorname{sen}^2(\Omega/2)>-\frac{1}{21}$$

$\hookrightarrow \operatorname{sen}^2(\Omega/2)\geq 0$

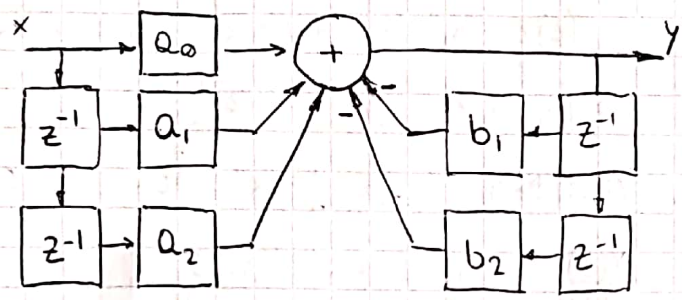
$\hookrightarrow (D^2+2D)\begin{cases}>0 & \text{si } D>0\\ <0 & \text{si } D<0\\ =0 & \text{si } D=0\end{cases}$

$$\hookrightarrow \underline{D>0}\Rightarrow \Omega<2\arcsin\left[\sqrt{\frac{1}{19(D^2+2D)}}\right]$$

$$\hookrightarrow \underline{D<0}\Rightarrow \Omega>2\arcsin\left[\sqrt{\frac{1}{-21(D^2+2D)}}\right]$$

7)

a)



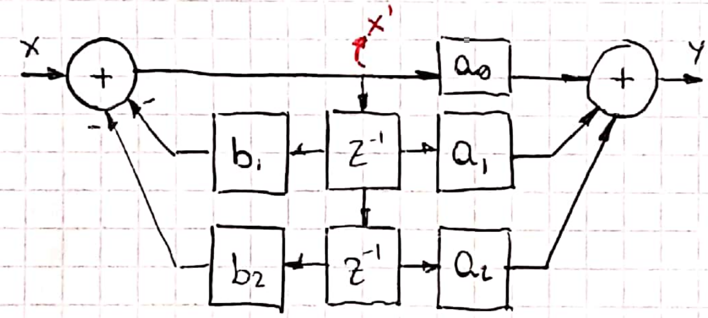
$$Y(k) = a_0 X(k) + a_1 X(k-1) + a_2 . X(k-2) - b_1 . Y(k-1) - b_2 Y(k-2)$$

$$Y(z) = a_0 X(z) + a_1 X(z) z^{-1} + a_2 X(z) z^{-2} - b_1 Y(z) z^{-1} - b_2 Y(z) . z^{-2}$$

$$Y(z) \left[1 + b_1 z^{-1} + b_2 z^{-2} \right] = X(z) \left[a_0 + a_1 z^{-1} + a_2 z^{-2} \right]$$

$$\frac{Y(z)}{X(z)} = H(z) = \boxed{\frac{a_0 + a_1 z^{-1} + a_2 . z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}} \rightarrow \text{FILTRO IIR}$$

b)



$$x'(k) = x(k) - b_1 . x'(k-1) - b_2 . x'(k-2)$$

$$x'(z) = x(z) - b_1 x'(z) z^{-1} - b_2 x'(z) z^{-2}$$

$$x'(z) \left[1 + b_1 z^{-1} + b_2 z^{-2} \right] = x(z)$$

$$x'(z) = \frac{x(z)}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

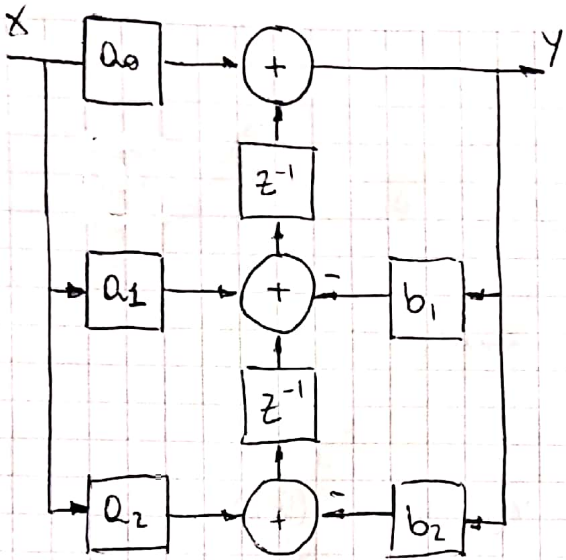
$$Y(k) = a_0 . x'(k) + a_1 x'(k-1) + a_2 x'(k-2)$$

$$Y(z) = a_0 x'(z) + a_1 x'(z) z^{-1} + a_2 x'(z) z^{-2}$$

$$Y(z) = x'(z) \left[a_0 + a_1 z^{-1} + a_2 z^{-2} \right]$$

$$Y(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} \, x(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \boxed{\frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}}$$

c) 

$$\Big[\big[a_2 \cdot X(z) - b_2 \cdot Y(z)\big] z^{-1} + a_1 X(z) - b_1 Y(z)\Big] z^{-1} + X(z) a_0 = Y(z)$$

$$\big[a_2 X(z) z^{-1} - b_2 Y(z) z^{-1} + a_1 X(z) - b_1 Y(z)\big] z^{-1} + X(z) a_0 = Y(z)$$

$$a_2 X(z) z^{-2} - b_2 Y(z) z^{-2} + a_1 X(z) z^{-1} - b_1 Y(z) z^{-1} + a_0 \cdot X(z) = Y(z)$$

$$X(z)\big[a_0 + a_1 z^{-1} + a_2 z^{-2}\big] - Y(z)\big[b_1 z^{-1} + b_2 z^{-2}\big] = Y(z)$$

$$Y(z)\big[1 + b_1 z^{-1} + b_2 z^{-2}\big] = X(z)\big[a_0 + a_1 z^{-1} + a_2 z^{-2}\big]$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

↳ Los 3 esquemas representan la misma $H(z)$, que es un filtro IIR bicuadrático