1-

a) $T: R^2 \to R^3$, onde $T(x,y) = (x+y, x-y, y)$

$u = (x_1, y_1)$

$v = (x_2, y_2)$

<u>1° Propriedade a ser verificada:</u>

$$T(u+v) = T(x_1+x_2, y_1+y_2) = (x_1+x_2+y_1+y_2 \,;\, x_1+x_2-y_1-y_2 \,;\, x_1+y_2)$$
$$= (x_1+y_1 \,;\, x_1-y_1, y_1) + (x_2+y_2, x_2-y_2, y_2)$$
$$= T(u) + T(v)$$

<u>2° Propriedade a ser verificada:</u>

$$T(\alpha u) = T(\alpha x_1, \alpha y_1)$$
$$= (\alpha x_1, \alpha y_1, \alpha x_1 - 2y_1, 2y_1)$$
$$= \alpha . (x_1+y_1, x_1-y_1, y_1) = \alpha . T(u)$$

1-

b) $U = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{Aplicando a matrizada}} \begin{bmatrix} 2 & 6 \\ -3 & 6 \end{bmatrix}$

$V = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix} \longrightarrow \begin{bmatrix} 6 & 3 \\ -4 & 11 \end{bmatrix}$

$U+V = \begin{bmatrix} 4 & 3 \\ 7 & 9 \end{bmatrix} \longrightarrow \begin{bmatrix} 8 & 9 \\ -7 & 17 \end{bmatrix}$

Sendo $\alpha = 3$, $\alpha . U = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 18 \\ -9 & 18 \end{bmatrix}$

• Pegando A matriz U e multiplicando por $\alpha$, temos;

$$\boxed{\underbrace{3}_{\alpha} \underbrace{\begin{bmatrix} 2 & 6 \\ -3 & 6 \end{bmatrix}}_{\text{Matriz } U} = \begin{bmatrix} 6 & 18 \\ -9 & 18 \end{bmatrix}}$$

O resultado coincidiu.

2 - $T: R^3 \to R^2$, tal que $T(1,1,1) = (1,2)$, $T(1,1,0) = (2,3)$ e $T(1,0,0) = (3,4)$.

$(x,y,z) = a(1,1,1) + b(1,1,0) + c(1,0,0)$

Sistema:

$$\begin{cases} a+b+c = x \\ a+b = y \\ a = z \end{cases}$$

Solução do Sistema: $\boxed{\begin{array}{l} \cdot\, a = z \\ \cdot\, b = y-z \\ \cdot\, c = x-y \end{array}}$

Logo: $(x,y,z) = z(1,1,1) + (y-z)(1,1,0) + (x-y)(1,0,0)$

Aplicando T e usando a linearidade:

$$T(x,y,z) = T\big(z(1,1,1) + (y-z)(1,1,0) + (x-y)(1,0,0)\big)$$
$$= zT(1,1,1) + (y-z)T(1,1,0) + (x-y)T(1,0,0)$$

Substituindo os valores:

$$T(x,y,z) = z(1,2) + (y-z)(2,3) + (x-y)(3,4)$$

$$T(x,y,z) = (z + 2y - 2z + 3x - 3y,\ 2z + 3y - 3z + 4x - 4y)$$

$$\boxed{T(x,y,z) = (3x - y - z,\ 4x - y - z)}$$

3- $T: R^3 \to R^3$

$T(x,y,z) = (6x+y-z, z-y, 2x-z)$

<u>Logo temos:</u>

$N(T) = \{(x,y,z) \in R^3 / T(x,y,z) = (0,0,0)\}$

$(6x+y-z, z-y, 2x-z) = (0,0,0)$

<u>Sistema:</u>

$$\begin{cases} 6x+y-z=0 \\ z-y=0 \\ 2x-z=0 \end{cases} \qquad \boxed{y=z} \qquad \boxed{x=\frac{-z}{2}}$$

<u>Logo:</u>

$N(T)\{(x,y,z) \in R^3 / y=z \text{ e } x = \frac{-z}{2}\}$

$N(T)\{(\frac{-z}{2}, z, z) / z \in R^3\}$, $\dim(T) = 1$

$\boxed{N(T)[(-\frac{1}{2}, 1, 1)]}$

$Im(T) = \{(6x+y-z, z-y, 2x-z) / x,y,z \in R^3\}$

$Im(T) = \{(6,0,2), (1,-1,0), (-1,1,-1)\}$

$T: R^3 \to R^3$

$T(x,y,z) = (6x+y-z, z-y, 2x-z)$

$ImT = \{(6x+y-z, z-y, 2x-z) / x,y,z \in R\}$

$ImT = [(6,0,2), (1,-1,0), \cancel{(-1,1,-1)}]$

$\beta\, Im(T) = \{(6,0,2), (1,-1,0)\}$

$\boxed{\dim R^3 = 3}$ //

4- $T : R^2 -> R^3$

$$[T]^{\alpha}_{B} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & -2 \end{bmatrix} \quad \alpha = \{(1,0),(0,1)\}$$
$$B = \{(1,0,0),(0,1,0),(0,0,1)\}$$

$$\boxed{\begin{array}{l} (x,y) = a.(1,0) + b(0,1) \\ (x,y) = (a,0) + (0,b) \end{array} \quad \boxed{\begin{array}{l} a = x \\ b = y \end{array}}}$$

$$[T]^{\alpha}_{B}[v]_{\alpha} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x+y \\ 3x-2y \end{pmatrix}$$

$$T(x,y) = x.(1,0,0) + 2x+y.(0,1,0) + 3x-2y(0,0,1)$$
$$T(x,y) = (x,0,0) + (0,2x+y,0) + (0,0,3x-2y)$$
$$\boxed{T(x,y) = (x, 2x+y, 3x-2y)}$$

Obs: Nesta questão no primeiro quadrado que desenhei não é o dígito 2, mas sim o caractere “a”.