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Professor, peço perdão pela letra horrorosa kkkk, tive que terminar algumas questões e passar o rascunho a limpo correndo, qualquer necessidade eu mando uma outra foto para ti posteriormente, abraço!

9 -

$$\frac{u^4-(1^4)}{u^3-(1^3)} =$$

$$\lim_{u \to 1} \frac{u^4-1}{u^3-1} \to \frac{(u^2-1^2)\cdot(u^2+1)}{(u-1)\cdot(u^2+u+1)} \to \frac{\cancel{(u-1)}\cdot(u+1)\cdot(u^2+1)}{\cancel{(u-1)}\cdot(u^2+u+1)} \to$$

$$\boxed{\lim_{u \to 1} \frac{u^4-1}{u^3-1} = \frac{4}{3}}$$

$$\frac{u^3+u^2+u+1}{u^2+u+1} \to \frac{1^3+1^2+1+1}{1^2+1+1} \to \boxed{\frac{4}{3}}$$

$$\lim_{v \to 2} \frac{v^3-8}{v^4-16} \to \frac{v^3-2^3}{v^4-2^4} \to \frac{(v-2)\cdot(v^2+2v+4)}{(v^2-4)\cdot(v^2+4)} \to$$

$$\frac{(v-2)\cdot(v^2+2v+4)}{(v^2-2^2)\cdot(v^2+2^2)} \to$$

$$\boxed{\lim_{v \to 2} \frac{v^3-8}{v^4-16} = \frac{3}{8}}$$

$$\frac{\cancel{(v-2)}\cdot(v^2+2v+4)}{\cancel{(v-2)}\cdot(v+2)\cdot(v^2+4)} \to \frac{v^2+2v+4}{(v+2)\cdot(v^2+4)} \to$$

$$\frac{v^2+2v+4}{v^3+4v+2v^2+8} \to \frac{2^2+4+4}{8+8+8+8} \to \frac{12}{32} \to \boxed{\frac{3}{8}}$$

$$\lim_{x \to 9} \frac{\sqrt{x}-3}{\underbrace{x}_{=\sqrt{x}^2}-9} \to \frac{\sqrt{x}-3}{\sqrt{x}^2-3^2} \to \frac{\cancel{\sqrt{x}-3}^{\,1}}{\cancel{(\sqrt{x}-3)}\cdot(\sqrt{x}+3)} \to \frac{1}{\sqrt{9}+3} \to \boxed{\frac{1}{6}}$$

$$\boxed{\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \frac{1}{6}}$$

$$\lim_{x \to 4} \frac{4x - x^2}{2 - \sqrt{x}} \to \frac{x(4 - \overset{= \sqrt{x}^2}{x})}{2 - \sqrt{x}} \to \frac{x(2^2 - \sqrt{x}^2)}{2 - \sqrt{x}} \to \frac{x \cdot \cancel{(2 - \sqrt{x})} \cdot (2 + \sqrt{x})}{\cancel{2 - \sqrt{x}}}$$

$$4 \cdot (2+2) \to \boxed{16}$$

$$\boxed{\lim_{x \to 4} \frac{4x - x^2}{2 - \sqrt{x}} = 16}$$

$$\lim_{x \to 1} \frac{x+1}{\sqrt{x+3} - 2} \to \underbrace{\frac{x+1}{\sqrt{x+3} - 2} \cdot \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2}}_{\text{produto notável}} \to \frac{(x+1) \cdot (\sqrt{x+3} + 2)}{\cancel{\sqrt{x+3}^2} - 2^2} \to \frac{\cancel{(x+1)} \cdot (\sqrt{x+3} + 2)}{\cancel{x - 1}}$$

$$\sqrt{4} + 2 \to \boxed{4}$$

$$\boxed{\lim_{x \to 1} \frac{x+1}{\sqrt{x+3} - 2} = 4}$$

$$\lim_{x \to -1} \frac{\sqrt{x^2+8} - 3}{x+1} \to \frac{\sqrt{x^2+8} - 3}{x+1} \cdot \frac{\sqrt{x^2+8} + 3}{\sqrt{x^2+8} + 3} \to \frac{(\sqrt{x^2+8} - 3) \cdot (\sqrt{x^2+8} + 3)}{(\sqrt{x^2+8} + 3) \cdot (x+1)}$$

$$\frac{\cancel{\sqrt{x^2+8}^2} - 3^2}{(\sqrt{x^2+8} + 3) \cdot (x+1)} \to \frac{x^2 - 1^2}{(\sqrt{x^2+8} + 3) \cdot (x+1)} \to \frac{(x-1) \cdot \cancel{(x+1)}}{(\sqrt{x^2+8} + 3) \cdot \cancel{(x+1)}} \to \frac{-2}{\sqrt{9} + 3} \to \frac{-2}{6} \to$$

$$\boxed{\frac{-1}{3}}$$

$$\boxed{\lim_{x \to -1} \frac{\sqrt{x^2+8} - 3}{x+1} = \frac{-1}{3}}$$

$$\lim_{x \to 2} \frac{\sqrt{x^2+12}-4}{x-2} \rightarrow \frac{\sqrt{x^2+12}-4}{x-2} \cdot \frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4} \rightarrow \frac{(\sqrt{x^2+12}-4)\cdot(\sqrt{x^2+12}+4)}{(x-2)\cdot(\sqrt{x^2+12}+4)} \rightarrow$$

$$\frac{\sqrt{x^2+12}^{\,2}-4^2}{(x-2)(\sqrt{x^2+12}+4)} \rightarrow \frac{x^2-4^{\rightarrow (2)^2}}{(x-2)\cdot(\sqrt{x^2+12}+4)} \rightarrow \frac{(x+2)\cdot\cancel{(x-2)}}{\cancel{(x-2)}\cdot(\sqrt{x^2+12}+4)} \rightarrow \frac{x+2}{\sqrt{x^2+12}+4} \rightarrow$$

$$\frac{4}{\sqrt{16}+4} \rightarrow \frac{4}{8} \rightarrow \boxed{\frac{1}{2}}$$

$$\boxed{\lim_{x \to 2} \frac{\sqrt{x^2+12}-4}{x-2} = \frac{1}{2}}$$

$$\lim_{x \to -2} \frac{x+2}{\sqrt{x^2+5}-3} \rightarrow \frac{x+2}{\sqrt{x^2+5}-3} \cdot \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3} \rightarrow \frac{(x+2)\cdot(\sqrt{x^2+5}+3)}{\sqrt{x^2+5}^{\,2}-3^2} \rightarrow$$

$$\frac{(x+2)\cdot(\sqrt{x^2+5}+3)}{x^2-4_{\rightarrow (2)^2}} \rightarrow \frac{\cancel{(x+2)}\cdot(\sqrt{x^2+5}+3)}{\cancel{(x+2)}\cdot(x-2)} \rightarrow \frac{\sqrt{9}+3}{-4} \rightarrow \frac{6}{-4} \rightarrow \boxed{\frac{3}{-2}}$$

$$\boxed{\lim_{x \to 2} \frac{x+2}{\sqrt{x^2+5}+3} = -\frac{3}{2}}$$

$$\lim_{x \to -3} \frac{2-\sqrt{x^2-5}}{x+3} \rightarrow \frac{(2-\sqrt{x^2-5})}{(x+3)} \cdot \frac{2+\sqrt{x^2-5}}{2+\sqrt{x^2-5}} \rightarrow \frac{(2-\sqrt{x^2-5})\cdot(2+\sqrt{x^2-5})}{(x+3)\cdot(2+\sqrt{x^2-5})} \rightarrow$$

$$\frac{2^2-(\sqrt{x^2-5})^2}{(x+3)\cdot(2+\sqrt{x^2-5})} \rightarrow \frac{4-x^2+5}{(x+3)\cdot(2+\sqrt{x^2-5})} \rightarrow \frac{9^{\rightarrow 3^2}-x^2}{(x+3)\cdot(2+\sqrt{x^2-5})} \rightarrow \frac{\cancel{(3+x)}\cdot(3-x)}{\cancel{(x+3)}\cdot(2\sqrt{x^2-5})} \rightarrow$$

$$\frac{3+3}{2+\sqrt{4}} \rightarrow \frac{6}{4} \rightarrow \boxed{\frac{3}{2}}$$

$$\boxed{\lim_{x \to -3} \frac{2-\sqrt{x^2-5}}{x+3} = \frac{3}{2}}$$

10 - $\lim\limits_{x \to 0} \dfrac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x} \to \dfrac{\lim\limits_{x \to 0} x^2 - 4x + 4 = 4}{\lim\limits_{x \to 0} x^3 + 5x^2 - 14x} \to$

a. $\dfrac{4}{\lim\limits_{x \to 0} x^3 + 5x^2 - 14x} \to \dfrac{4}{0}$, logo é inexistente, pois o denominador da expressão é igual a 0.

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b. $\lim\limits_{x \to 2} \dfrac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x} \to \dfrac{4}{\lim\limits_{x \to 2} x^3 + 5x^2 - 14x} \to \dfrac{4}{28 - 28} \to \dfrac{4}{0}$, logo também é inexistente.

$\lim\limits_{x \to 0} \dfrac{x^2 + x}{x^5 + 2x^4 + x^3} \to \dfrac{\cancel{x}(x+1)}{\cancel{x}(x^4 + 2x^3 + x^2)} \to \dfrac{x+1}{x(x^3 + 2x^2 + x)} \to \dfrac{x+1}{x.x(x^2 + 2x + 1)} \to$

a. $\dfrac{\cancel{x+1}}{x.x.(x+1)^{\cancel{2}}} \to \dfrac{1}{x^2 . (x+1)} \to \dfrac{1}{x^3 + x^2} \to \dfrac{1}{0}$, logo é inexistente, pois o denominador é igual a 0

b. $\lim\limits_{x \to -1} \dfrac{x^2 + x}{x^5 + 2x^4 + x^3} \to \dfrac{1}{x^3 + x^2} \to \dfrac{1}{-1+1} \to \dfrac{1}{0}$, logo também é inexistente.

$\lim\limits_{x \to 1} \dfrac{1 - \sqrt{x}}{1 - x} \to \dfrac{1 - \sqrt{x}}{1^2 - \sqrt{x}^2} \to \dfrac{\cancel{1 - \sqrt{x}}}{(\cancel{1 - \sqrt{x}}).(1 + \sqrt{x})} \to \dfrac{1}{1 + \sqrt{x}} \to \dfrac{1}{2}$

$$\boxed{\lim\limits_{x \to 1} \dfrac{1 - \sqrt{x}}{1 - x} = \dfrac{1}{2}}$$

$$\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4} \to \frac{\cancel{x^2 - a^2}}{\cancel{(x^2 - a^2)} \cdot (x^2 + a^2)} \to \frac{1}{x^2 + a^2} \to \frac{1}{a^2 + a^2} \to \boxed{\frac{1}{2a^2}}$$

$$\boxed{\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4} = \frac{1}{2a^2}}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \to \frac{(\cancel{x^2} + 2xh + h^2) - \cancel{x^2}}{h} \to \frac{\cancel{h}(2x + h)}{\cancel{h}} \to \boxed{2x}$$

$$\boxed{\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = 2x}$$

$$\lim_{x \to 0} \frac{(x+h)^2 - x^2}{h} \to \frac{(0+h)^2 - 0^2}{h} \to \frac{h^{\cancel{2}}}{\cancel{h}} \to \boxed{h} \qquad \boxed{\lim_{x \to 0} \frac{(x+h)^2 - x^2}{h} = h}$$

$$\lim_{x \to \pi} \operatorname{sen}\left(\frac{x}{2} + \operatorname{sen} x\right) \to \frac{\pi}{2} + \operatorname{sen} \pi \xrightarrow{0} \operatorname{sen}\left(\frac{\pi}{2}\right) \to \boxed{1}$$

$$\boxed{\lim_{x \to \pi} \operatorname{sen}\left(\frac{x}{2} + \operatorname{sen} x\right) \to 1}$$

$$\lim_{x \to \pi} \cos^2 (x - \operatorname{tg} x)$$

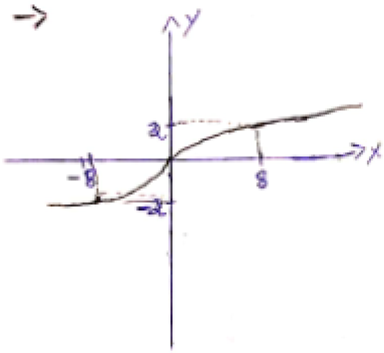
11 -

a) $F(x) = x^{\frac{1}{3}} \rightarrow$

$F(8) \rightarrow 2$

$F(0) \rightarrow 0$

$F(-8) \rightarrow -2$



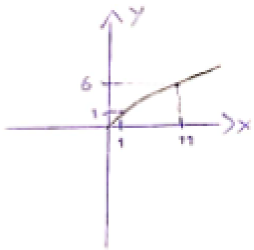
Verificando para $x \rightarrow 8$

1. $F(8) = 2$
2. $\lim_{x \rightarrow 8} x^{\frac{1}{3}} = 8^{\frac{1}{3}}$
3. $F(8) = \lim_{x \rightarrow 8} x^{3}$, logo é contínua em qualquer $x > 0$ e $x < 0$.

b) $g(x) = x^{\frac{3}{4}}$

$g(1) = 1$

$g(11) = 11^{\frac{3}{4}} \rightarrow \sqrt[4]{11^3} \rightarrow 6$



Verificando para $g \rightarrow 1$

1. $g(1) = 1$
2. $\lim_{x \rightarrow 1} x^{\frac{3}{4}} = 1^{\frac{3}{4}}$
3. $g(1) = \lim_{x \rightarrow 1} x^{\frac{3}{4}}$, também é contínua, porém apenas para todo $x \geq 1$.

12- $$\lim_{x \to \infty} \sqrt{\frac{8x^2-3}{2x^2+x}} \to \frac{8x^2-3 \cdot \frac{1}{x^2}}{2x^2+x \cdot \frac{1}{x^2}} \to \frac{8 - \cancel{\frac{3}{x^2}} \to 0}{2 + \cancel{\frac{1}{x}} \to 0} \to \frac{8}{2} \to 4 \to \sqrt{4}$$

$$\to 2$$

$$\boxed{\lim_{x \to \infty} \sqrt{\frac{8x^2-3}{2x^2+x}} = 2}$$

$$\lim_{x \to -\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{\frac{1}{3}} \to \left(\frac{\cancel{x^2}\left(1 + \cancel{\frac{1}{x}}^{\,0} - \cancel{\frac{1}{x^2}}^{\,0}\right)}{\cancel{x^2}\left(8 - \cancel{\frac{3}{x^2}}^{\,0}\right)}\right)^{\frac{1}{3}} \to \left(\frac{1}{8}\right)^{\frac{1}{3}}$$

$$\downarrow$$

$$\sqrt[3]{\frac{1}{8}}$$

$$\downarrow$$

$$\frac{\sqrt[3]{1}}{\sqrt[3]{8}} \to \boxed{\frac{1}{2}}$$

$$\boxed{\lim_{x \to -\infty} \left(\frac{x^2+x-1}{8x^2-3}\right)^{\frac{1}{3}} \to \frac{1}{2}}$$

$$\lim_{x \to +\infty} \frac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}} \to \frac{\frac{1}{x}+\frac{1}{x^4}}{\frac{1}{x^2}-\frac{1}{x^3}} \to \frac{\frac{x^3+1}{x^4}}{\frac{x-1}{x^3}} \to \frac{x^3+1}{\cancel{x^4}} \cdot \frac{\cancel{x^3}}{x-1} \to$$

$$\frac{x^3+1}{x^2-x} \to \frac{\cancel{x^2}\left(x+\frac{1}{x^2}\right)}{\cancel{x^2}\left(1-\frac{1}{x}\right)} \to \frac{x + \cancel{\frac{1}{x^2}} \to 0}{1 - \cancel{\frac{1}{x}} \to 0} \to x+1 \to +\infty$$

$$\lim_{x \to +\infty} \frac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}} = +\infty$$