1- $T: R^2 \Rightarrow R^2$, $T(x,y) = (2x+5y, 5x-y)$

$\alpha = \{(1,0), (0,1)\}$ $[T]_{\alpha}^{\alpha} = \begin{pmatrix} 2 & 5 \\ 5 & -1 \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\left[\begin{pmatrix} 2 & 5 \\ 5 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right] = 0$

$\begin{pmatrix} 2-\lambda & 5 \\ 5 & -1-\lambda \end{pmatrix} = 0$

$(2-\lambda) - (-1-\lambda) - 25 = 0$

$\lambda^2 - \lambda - 27 = 0$

$\lambda_1 = 5,7$

$\lambda_2 \simeq -4,7$

- Autovetores:

$\underline{\lambda_1 = 5,7}$

$\begin{pmatrix} 2-5,7 & 5 \\ 5 & -1-5,7 \end{pmatrix} = \begin{pmatrix} -3,7 & 5 \\ 5 & -6,7 \end{pmatrix}$

$\begin{cases} -3,7x + 5y = 0 \\ 5x - 6,7y = 0 \end{cases}$

$\begin{pmatrix} 3,7 & 5 \\ 5 & 6,7 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$-3,7x = -5y \cdot (-1)$

$x = \frac{5}{3,7} = 1,34y$

$V_{\lambda=5,7}: \{(x,y) / \in \mathbb{R}^2; x = 1,3y, y\}$

$V_{\lambda=5,7}: \{(x,y) ; y \in \mathbb{R}\}$

$V_{\lambda=5,7}: [(1,3, 1)]$

$\lambda = 4,7$

$$\begin{pmatrix} 2+4,7 & 5 \\ 5 & -1+4,7 \end{pmatrix} = \begin{pmatrix} 6,7 & 5 \\ 5 & 3,7 \end{pmatrix}$$

$$\begin{pmatrix} 6,7 & 5 \\ 5 & 3,7 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{cases} 6,7x + 5y = 0 \\ 5x + 3,7y = 0 \end{cases}$$

$V\lambda = 4,7 : \{(x,y) \in \mathbb{R}^2 ; y = -1,3x , x\}$

$V\lambda = 4,7 : \{(x,y) ; x \in \mathbb{R}\}$

$V\lambda = 4,7 : [(1, -1,3)]$

$5 \cdot y = -6,7x$

$y = -\frac{6,7x}{5} = 1,3x$

Logo, os vetores $(1,3 , 1)$ e $(1, -1,3)$ foram gerados, então existe uma base de autovetores, pois o espaço referente tem dimensão igual a 2. Portanto é diagonalizável.

2 -

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & c \\ 0 & c & b \end{pmatrix}$$

1) $\det(A - \lambda I) = 0$

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & c \\ 0 & c & b \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} a-\lambda & 0 & 0 \\ 0 & b-\lambda & c \\ 0 & c & b-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \left| \begin{array}{ccc|cc} a-\lambda & 0 & 0 & a-\lambda & 0 \\ 0 & b-\lambda & c & 0 & b-\lambda \\ 0 & c & b-\lambda & 0 & c \end{array} \right.$$

$$\rightarrow (a-\lambda).(b-\lambda).(b-\lambda) - \left(c^2 + (a-\lambda)\right) = 0$$

$$(a-\lambda)\cdot(b-\lambda)^2-\left(c^2\cdot(a-\lambda)\right)=0$$

$\lambda = a$

$\lambda = b+c$

$\lambda = b-c$

- $\lambda = a$:

$$(\cancel{a-a})\cdot(b-a)^2-\left(c^2\cdot(\cancel{a-a})\right)=0$$

$$(b-a)^2-c^2$$

- $\lambda = b+c$:

$$(a-b-c)\cdot(\cancel{b-b}-c)^2-\left(c^2\cdot(a-b-c)\right)=0$$

$$(a-b-c)\cdot c^2-\left(c^2\cdot(a-b-c)\right)=0$$

- $\lambda = b-c$:

$$(a-b+c)\cdot(\cancel{b-b}+c)^2-\left(c^2\cdot(a-b+c)\right)=0$$

$$(a-b+c)\cdot c^2-\left(c^2\cdot(a-b+c)\right)=0$$

ii)

$\lambda = b + c$

$$\begin{pmatrix} a-b-c & 0 & 0 \\ 0 & -c & c \\ 0 & c & -c \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} a-b-c & 0 & 0 \\ 0 & c & -c \\ 0 & -c & c \end{pmatrix} = \begin{pmatrix} a-b-c & 0 & 0 \\ 0 & c & -c \\ 0 & 0 & 0 \end{pmatrix}$$

Redução

Cancelamento de

$R_3 \leftrightarrow R_3 + 1 \cdot R_2$

3- $T(x,y) = (x+y, x-y)$

$B = \{(1,2), (0,-1)\}$

i) $T(1,2) = (3,1) = 3.(1,2) + 7(0,-1)$

$T(0,-1) = (-1,1) = 1.(1,2) + 3(0,-1)$

$[T]_{\beta}^{\beta} = \begin{pmatrix} 3 & -1 \\ 7 & 3 \end{pmatrix}$ → Autovalores

ii) $v(4,2)$

$T(v) = (6,2)$

$T(v)_{\beta} = 6.(1,2) + 2.(0,-1) = (6,10)$

iii) $[T]_{\beta}^{\alpha} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ $\quad \alpha = \{\vec{v_1}, \vec{v_2}\}$

$\vec{v_1} = 1.(1,2) + 3(0,-1) = (1,-1)$

$\vec{v_2} = 2.(1,2) + 1(0,-1) = (2,3)$

$\alpha = \{(1,-1), (2,3)\}$

4-

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\det(A - \lambda I) = 0 \rightarrow \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} 1-\lambda & 3 & 1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{pmatrix}.$$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 3 & 1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} \begin{matrix} 1-\lambda & 3 \\ 0 & 2-\lambda \\ 0 & 0 \end{matrix}$$

$$= (1-\lambda).(2-\lambda).(3-\lambda)$$

$$\rightarrow (1-\lambda).(2-\lambda).(3-\lambda) = 0$$

$$\text{Autovalores} \begin{cases} 1-\lambda = 0 \rightarrow \lambda_1 = 1 \\ 2-\lambda = 0 \rightarrow \lambda_2 = 2 \\ 3-\lambda = 0 \rightarrow \lambda_3 = 3 \end{cases}$$

$$\boxed{\begin{matrix} A.X = \lambda.A \\ (A-\lambda).X = 0 \end{matrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}}$$

$\lambda_1 = 1$

$$\begin{pmatrix} 0 & 3 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\begin{cases} 0 + 3y + z = 0 \\ 0 + y + 0 = 0 \\ 0 + 0 + 2z = 0 \end{cases}$$

$$V_{\lambda=1} : \{(x, y, z) / \in \mathbb{R} / y = 0 \text{ e } z = 0\}$$

$$V_{\lambda=1} : \{(x, 0, 0) / x \in \mathbb{R}^3\}$$

$$\boxed{V_{\lambda=1} : [(1,0,0)]}$$

$$\begin{cases} 3y + z = 0 \\ y = 0 \\ z = 0 \end{cases}$$

$\lambda = 2$

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$-x + 3y + z = 0 \rightarrow \boxed{x = 3y}$

$z = 0$

$V_{\lambda=2} : \{(x, y, z) / \in \mathbb{R} / x = 3y \text{ e } z = 0\}$

$V_{\lambda=2} : \{(3y, y, 0) / y \in \mathbb{R}^3\}$

$\boxed{V_{\lambda=2} : [(3, 1, 0)]}$

$\lambda = 3$

$$\begin{pmatrix} -2 & 3 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$-2x + 3y + z = 0$

$y = 0$

$z = 0$

$V_{\lambda=3} : \{(x, y, z) / \in \mathbb{R} / y = 0 \text{ e } z = 0\}$

$V_{\lambda=3} : \{(x, 0, 0) / x \in \mathbb{R}^3\}$

$\boxed{V_{\lambda=3} : [(1, 0, 0)]}$

. Logo os vetores gerados são $(1,0,0)$ e $(3,1,0)$. Então não há base de autovetores para o espaço.