1- $\int_6^7 \frac{x^3}{\sqrt{x^2-25}}\,dx$

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• Cálculo da integral indefinida:

$$\int \frac{x^3}{\sqrt{x^2-25}}\,dx$$

• Usando a substituição no diferencial $dx = \frac{1}{t'}\cdot dt$ onde $t = x^2-25$ e $t' = 2x$:

$$\int \frac{x^{\cancel{3}\,2}}{\sqrt{x^2-25}}\cdot\frac{1}{2\cancel{x}}\,dt$$

$$= \int \frac{x^2}{2\sqrt{x^2-25}}\,dt$$

• Substituindo x^2-25 por t:

$$= \int \frac{x^2}{2\sqrt{t}}\,dt$$

• Adicionando dois termos que se anulam:

$$= \int \frac{x^2-25+25}{2\sqrt{t}}\,dt$$

• Substituindo x^2-25 por t:

$$= \int \frac{t+25}{2\sqrt{t}}\,dt$$

$$= \frac{1}{2}\int \frac{t+25}{\sqrt{t}}\,dt$$

$$= \frac{1}{2}\int \frac{t+25}{t^{\frac{1}{2}}}\,dt$$

$$= \frac{1}{2}\int \frac{t}{t^{\frac{1}{2}}} + \frac{25}{t^{\frac{1}{2}}}\,dt$$

$$= \frac{1}{2}\int t^{1-\frac{1}{2}} + \frac{25}{t^{\frac{1}{2}}}\,dt$$

$$= \frac{1}{2}\int t^{\frac{1}{2}} + \frac{25}{t^{\frac{1}{2}}}\,dt$$

$$= \frac{1}{2}\left(\int t^{\frac{1}{2}}\,dt + \int \frac{25}{t^{\frac{1}{2}}}\,dt\right)$$

$$= \frac{1}{2}\left(\frac{2t\sqrt{t}}{3} + 50\sqrt{t}\right) + C$$

$$\boxed{= \frac{1}{2}\left(\frac{2(x^2-25)\cdot\sqrt{x^2-25}}{3} + 50\sqrt{x^2-25}\right) + C}$$

$$= \frac{(x^2-25)\sqrt{x^2-25}}{3} + 25\sqrt{x^2-25} + C$$

• Integral definida:

$$\left(\frac{(x^2-25)\sqrt{x^2-25}}{3} + 25\sqrt{x^2-25}\right)\Bigg|_6^7$$

$$\int t^{\frac{1}{2}}\,dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2t^{\frac{3}{2}}}{3} = \frac{2\sqrt{t^3}}{3}$$

$$\boxed{= \frac{2t\sqrt{t}}{3}}$$

$$25\int \frac{\cancel{1}}{t^{\frac{1}{2}}} \longrightarrow 1$$

$$= 25\cdot 2\sqrt{t}$$

$$\boxed{= 50\sqrt{t}}$$

$$= \frac{(7^2-25)\sqrt{7^2-25}}{3} + 25\sqrt{7^2-25} - \left(\frac{(6^2-25)\sqrt{6^2-25}}{3} + 25\sqrt{6^2-25}\right)$$

$$= \frac{\cancel{24}\sqrt{7^2-25}}{\cancel{3}} + 25\sqrt{7^2-25} - \left(\frac{11\sqrt{11}}{3} + 25\sqrt{11}\right)$$

$$= 8\sqrt{7^2-25} + 25\sqrt{7^2-25} - \frac{86\sqrt{11}}{3}$$

$$= 33\sqrt{7^2-25} - \frac{86\sqrt{11}}{3}$$

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$$= 33\sqrt{24} - \frac{86\sqrt{11}}{3}$$

$$\boxed{= 66\sqrt{6} - \frac{86\sqrt{11}}{3}}$$

a- $\int_0^{+\infty} x^3 . e^{x^4} dx$

$\boxed{\text{• Para avaliar a integral imprópria, reescreve-se usando o limite de um integral definido:}}$

$$\lim_{a \to +\infty} \left(\int_0^a x^3 . e^{x^4} dx \right)$$

• Calculando a integral indefinida:

$$\int x^3 e^{x^4} dx$$

• Usando a substituição $t = x^4$:

$$= \int \frac{e^t}{4} dt$$

$$= \frac{1}{4} . \int e^t dt$$

$$= \frac{1}{4} e^t + c$$

$$= \frac{1}{4} e^{x^4} + c$$

$$\boxed{= \frac{e^{x^4}}{4} + c}$$

• Calculando a integral definida:

$$\left. \frac{e^{x^4}}{4} \right|_0^a$$

$$\frac{e^{a^4}}{4} - \frac{e^{0^4}}{4} = \frac{e^{a^4} - 1}{4}$$

• Calculando o limite:

$$\lim_{a \to +\infty} (e^{a^4} - 1) = +\infty$$

$$\lim_{a \to +\infty} (4) = 4$$

$$\therefore \text{Logo} \lim_{a \to +\infty} \left(\frac{e^{a^4} - 1}{4} \right) = +\infty$$

$\boxed{\text{• Portanto, já que o limite é igual a } +\infty \text{, a integral imprópria é Divergente.}}$

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3- b) $\int t^3 \cos^5(2t^4+1)\,dt$

• Usando a substituição com $dt = \frac{1}{u}\cdot du$, onde $u = 2t^4+1$ e $u = 2.4t^3$

$$\int \cancel{t^3}\cdot \cos^5(2t^4+1)\cdot \frac{1}{2.4\cancel{t^3}}\,du$$

$$= \int \cos^5(2t^4+1)\cdot \frac{1}{8}\,du$$

$$= \int \frac{\cos^5(2t^4+1)}{8}\,du$$

• Substituindo $2t^4+1$ por u:

$$= \int \frac{\cos^5(u)}{8}\,du$$

$$= \frac{1}{8}\int \cos^5(u)\,du$$

$$= \frac{1}{8}\int \cos^4(u)\cdot\cos(u)\,du$$

Usando a substituição com $du = \frac{1}{v}\cdot dv$, onde $v = \operatorname{sen}(u)$ e $v = \cos(u)$:

$$= \frac{1}{8}\int \cos^4(u)\cdot\cancel{\cos(u)}\cdot\frac{1}{\cancel{\cos(u)}}\,dv$$

$$= \frac{1}{8}\int \cos^4(u)\,dv$$

$$= \frac{1}{8}\int (\cos^2(u))^2\,dv$$

• Sabendo que $\operatorname{sen}(t)^2 = 1-\cos(t)^2$:

$$= \frac{1}{8}\int (1-\operatorname{sen}^2(u))^2\,dv$$

• Substituindo $\operatorname{sen}(u)$ por v:

$$= \frac{1}{8}\int (1-v^2)^2\,dv$$

$$= \frac{1}{8}\int 1-2v^2+v^4\,dv$$

$$= \frac{1}{8}\left(\int 1\,dv - \int 2v^2\,dv + \int v^4\,dv\right)$$

$$= \frac{1}{8}\left(v - \frac{2v^3}{3} + \frac{v^5}{5}\right) + c$$

• Substituindo $v = \operatorname{sen}(u)$

$$= \frac{1}{8}\left(\operatorname{sen}(u) - \frac{2\operatorname{sen}(u)^3}{3} + \frac{\operatorname{sen}(u)^5}{5}\right) + c$$

• Substituindo $u = 2t^4+1$:

$$= \frac{1}{8}\left(\operatorname{sen}(2t^4+1) - \frac{2\operatorname{sen}(2t^4+1)^3}{3} + \frac{\operatorname{sen}(2t^4+1)^5}{5}\right) + c$$

$$= \boxed{\frac{\operatorname{sen}(2t^4+1)}{8} - \frac{\operatorname{sen}(2t^4+1)^3}{12} + \frac{\operatorname{sen}(2t^4+1)^5}{40} + c}$$

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