

1-

$$i) A = \{(1,1), (2,1), (2,2), (3,2), (3,3)\}$$

$$B = \{(0,0), (0,1)\}$$

$$B(1,1) = \{(1,1), (1,2)\} \subseteq A \text{ Falso}$$

$$B(2,1) = \{(2,1), (2,2)\} \subseteq A \text{ Verdadeiro}$$

$$B(2,2) = \{(2,2), (2,3)\} \subseteq A \text{ Falso}$$

$$B(3,2) = \{(3,2), (3,3)\} \subseteq A \text{ Verdadeiro}$$

$$B(3,3) = \{(3,3), (3,4)\} \subseteq A \text{ Falso}$$

Logo, temos que:

$$\text{Dilata\c{c}\~ao} = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3), (3,4)\} //$$

Imagem dilata\c{c}\~ao:

	0	1	2	3	4
0					
1		1	1		
2		1	1	1	
3			1	1	1
4					

$$\text{Eros\c{a}\~o} = \{(2,1), (3,2)\} //$$

Imagem da eros\c{a}\~o:

	0	1	2	3	4
0					
1					
2		1			
3			1		
4					

Gradiente: Dilatação - Erosão

$$\text{Gradiente} = \{(1,1), (1,2), (2,2), (2,3), (3,3), (3,4)\} //$$

	0	1	2	3	4
0					
1		1	1		
2			1	1	
3				1	1
4					

→ Gradiente

Bordas externas: Imagem dilatação - Imagem original

$$\text{Bordas externas} = \{(1,2), (2,3), (3,4)\} //$$

	0	1	2	3	4
0					
1					
2			1		
3				1	
4					1

→ Bordas externas

Fechamento: Erosão da Dilatação

$$B = \{(0,0), (0,1)\}$$

$$\text{Imagem dilatação} = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3), (3,4)\}$$

$$B(1,1) = \{(1,1), (1,2)\} \subseteq A \text{ Verdadeiro}$$

$$B(1,2) = \{(1,2), (1,3)\} \subseteq A \text{ Falso}$$

$$B(2,1) = \{(2,1), (2,2)\} \subseteq A \text{ Verdadeiro}$$

$$B(2,2) = \{(2,2), (2,3)\} \subseteq A \text{ Verdadeiro}$$

$$B(2,3) = \{(2,3), (2,4)\} \subseteq A \text{ Falso}$$

$$B(3,2) = \{(3,2), (3,3)\} \subseteq A \text{ Verdadeiro}$$

$$B(3,3) = \{(3,3), (3,4)\} \subseteq A \text{ Verdadeiro}$$

$$B(3,4) = \{(3,4), (3,5)\} \subseteq A \text{ Falso}$$

• Portanto, temos que:

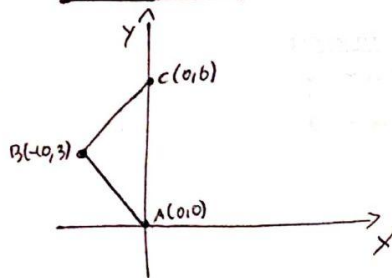
$$\text{Fechamento} = \{(1,1), (2,1), (2,2), (3,2), (3,3)\} //$$

	0	1	2	3	4
0					
1		1			
2		1	1		
3			1	1	
4					

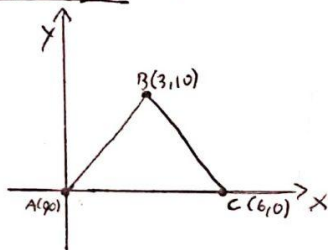
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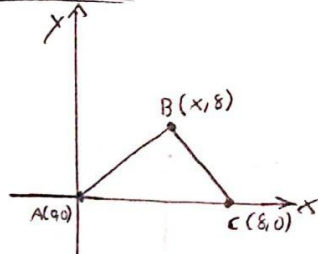
• Primeiramente:



• Segundamente:



• Terceiramente



• Devemos então definir a escala

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{aligned} P' &= M \cdot P \\ 6s_x &= 8 \\ 10s_y &= 8 \end{aligned}$$

$$\begin{bmatrix} 8 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 10 \\ 1 \end{bmatrix}$$

$$\begin{aligned} s_x &= \frac{8}{6} = \frac{4}{3} \\ s_y &= \frac{8}{10} = \frac{4}{5} \end{aligned}$$

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$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\underline{I}$                        $\underline{R}$                        $\underline{S}$                       Cisalhamento

$T_x = 0$                        $-90$                        $s_x = \frac{4}{3}$                        $\text{Cis } A = \frac{1}{2}$   
 $T_y = -2$                                             $s_y = \frac{4}{5}$                        $\text{Cis } B = 0$

• Observando o ponto B do terceiro passo, devemos determinar X;

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

• Logo:

$$x' = \frac{4}{3} \cdot 3 = 4$$

$$y' = \frac{4}{5} \cdot 10 = 8$$

• Por fim, determino-se o cisalhamento:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 8 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}$$

• Logo:

$$8 = 4 + 8a$$

$$8 = 8 + 4b$$

$$4 = 8a$$

$$0 = 4b$$

$$a = \frac{1}{2}$$

$$b = 0 //$$

$$\underline{a = \frac{1}{2} //}$$

4-

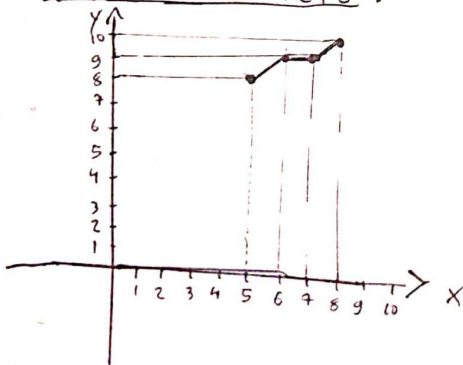
$$\left. \begin{array}{l} dx = \Delta x = 3 \\ dy = \Delta y = 5 \\ ds = 2dx - dy = 1 \\ incE = 2 \cdot dx = 6 \\ incNE = 2(dx - dy) = -4 \end{array} \right\} \text{ Para o 2º octante}$$

$x = 5$   
 $y = 8$

	ds	x	y
1	1	5	8
2	-3	6	9
3	3	7	9
4	-1	8	10

→ Iterações do algoritmo do ponto médio.

Desenho da reta:



5-

$$\sin(\theta) = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\cos(\theta) = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$d_{AD} = \sqrt{(0-1)^2 + (3-1)^2} = \sqrt{5}$$

1- Traz a figura para a origem

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

2- Realiza a rotação em  $R(\theta)$

$$\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3- Realiza o escalonamento na VPN

$$\begin{bmatrix} \frac{1}{2\sqrt{5}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4- Realiza a translação para a VPN

$$\begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 100 \\ 0 & 0 & 1 \end{bmatrix}$$



$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2\sqrt{5}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 100 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & 39 \\ -\frac{1}{10} & \frac{2}{5} & 29 \\ 0 & 0 & 1 \end{bmatrix}$$

