1 -

$$W = \left\{ \begin{pmatrix} a & 0 \\ b & 2a \end{pmatrix} ; a, b \in \mathbb{R} \right\} \subset M_{2\times2}$$

$$A_1 = \begin{pmatrix} a_1 & 0 \\ b_1 & 2a_1 \end{pmatrix} \qquad A_2 = \begin{pmatrix} a_2 & 0 \\ b_2 & 2a_2 \end{pmatrix}$$

1. $\begin{pmatrix} a_1 & 0 \\ b_1 & 2a_1 \end{pmatrix} + \begin{pmatrix} a_2 & 0 \\ b_2 & 2a_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 & 0 \\ b_1 + b_2 & 2(a_1+a_2) \end{pmatrix} \in W$

2. $\alpha \cdot \begin{pmatrix} a_1 & 0 \\ b_1 & 2a_1 \end{pmatrix} = \begin{pmatrix} \alpha a_1 & 0 \\ \alpha b_1 & \alpha 2 a_1 \end{pmatrix} \in W$

2 - $v = (-2, -1, 3)$

$v_1 = (1,1,2)$

$v_2 = (0,1,3)$

$v_3 = (0,1,2)$

$(-2,-1,3) = a\cdot(1,1,2) + b(0,1,3) + c(0,1,2)$

$(-2,-1,3) = (a,a,2a) + (0,b,3b) + (0,c,2c)$

$(-2,-1,3) = (a, a+b+c, 2a+3b+2c)$

$$\begin{cases} a = -2 \\ a+b+c = -1 \\ 2a+3b+2c = 3 \end{cases} \Rightarrow \begin{cases} -2+b+c = -1 \\ -4+3b+2c = 3 \end{cases}$$

$$\begin{cases} b+c = 1 \\ 3b+2c = 7 \end{cases} \Rightarrow \begin{cases} -2b - \cancel{2c} = -2 \\ 3b + \cancel{2c} = 7 \end{cases} \Rightarrow \boxed{b = 5}$$

$5 + c = 1$

$\boxed{c = -4}$

Logo,

$(-2,-1,3) = (-2).(1,1,2) + 5(0,1,3) + (-4).(0,1,2)$

$(-2,-1,3) = (-2,-2,-4) + (0,5,15) + (0,-4,-8)$

$(-2,-1,3) = (-2,-1,3)$

3- $\beta = \{(-2,1), (1,1)\} \in \mathbb{R}^2$

$a(-2,1) + b(1,1) = (0,0)$

$(-2a, a) + (b, b) = (0,0)$

$(-2a+b, a+b) = (0,0)$

$$\begin{cases} -2a + b = 0 \\ a + b = 0 \end{cases} \Rightarrow \begin{cases} -\cancel{2a} + b = 0 \\ \cancel{2a} + 2b = 0 \end{cases} \to 3b = 0 \quad \boxed{b = 0}$$

$a + 0 = 0$

$\boxed{a = 0}$

$\boxed{a = b = 0 \text{ e } \beta \text{ é l.i.}}$

Seja $(x,y) \in \mathbb{R}^2$

$$(x,y) = a(-2,1) + b(1,1)$$

$$(x,y) = (-2a, a) + (b, b)$$

$$(x,y) = (-2a + b, a + b)$$

$$\begin{cases} -2a + b = x \\ a + b = y \end{cases} \Rightarrow \begin{cases} -\cancel{2a} + b = x \\ \cancel{2a} + 2b = 2y \end{cases} \Rightarrow 3b = x + 2y$$

$$\boxed{b = \frac{x+2y}{3}}$$

$$a = y - b$$

$$a = y - \left(\frac{x+2y}{3}\right)$$

$$a = \frac{3y - x - 2y}{3} \Rightarrow \boxed{a = \frac{y-x}{3}}$$

Logo $\boxed{(x,y) = \frac{y-x}{3}(-2,1) + \frac{x+2y}{3}(1,1)}$

$$(1,4) = \frac{4-1}{3} \cdot (-2,1) + \frac{1+8}{3}(1,1)$$

$$(1,4) = 1 \cdot (-2,1) + 3 \cdot (1,1)$$

$$(1,4) = (-2,1) + (3,3)$$

$$(1,4) = (1,4)$$

4 - $B_1 = \{(1,1,1), (-1,1,0), (2,0,-2)\} \in \mathbb{R}^3$

$$a(1,1,1) + b(-1,1,0) + c(2,0,-2) = (0,0,0)$$
$$(a,a,a) + (-b,b,0) + (2c,0,-2c) = (0,0,0)$$
$$(a-b+2c, a+b, a-2c) = (0,0,0)$$

$$\begin{cases} a - b + 2c = 0 \longrightarrow a - (-a) + 2\left(\frac{a}{2}\right) = 0 \\ a + b = 0 \Rightarrow b = -a \\ a - 2c = 0 \Rightarrow c = \frac{a}{2} \end{cases}$$

$$2a + \frac{2a}{2} = 0$$

$$\frac{6a}{2} = 0 \rightarrow \boxed{a=0} \text{ e } \boxed{b=c=0}$$

$\boxed{B_1 \text{ é L.I.}}$

$$(x,y,z) = a(1,1,1) + b(-1,1,0) + c(2,0,-2)$$
$$(x,y,z) = (a,a,a) + (-b,b,0) + (2c,0,-2c)$$

$$\begin{cases} a - b + 2c = x \longrightarrow a - (y-a) + 2\cdot\left(\frac{-z+a}{2}\right) = x \\ a + b = y \rightarrow b = y - a \\ a - 2c = z \rightarrow c = \frac{-z+a}{2} \end{cases}$$

$$a - y + a - z + a = x$$
$$3a - y - z = x$$
$$3a = x + y + z$$
$$\boxed{a = \frac{x+y+z}{3}}$$

$$b = y - \left(\frac{x+y+z}{3}\right)$$
$$b = \frac{3y-x-y-z}{3}$$
$$\boxed{b = \frac{2y-x-z}{3}}$$

$$c = \frac{-z + \frac{x+y+z}{3}}{2} \rightarrow c = \frac{\frac{-3z+x+y}{3}}{2} \rightarrow \boxed{c = \frac{-2z+x+y}{6}}$$

$$\boxed{(x,y,z) = \frac{x+y+z}{3}\cdot(1,1,1) + \frac{2y-x-z}{3}\cdot(-1,1,0) + \left(\frac{-2z+x+y}{6}\right)\cdot(2,0,-2)}$$

$V=(7,1,1)$ , $x=7$, $y=1$, $z=1$

$$(7,1,1) = \frac{7+1+1}{3}\cdot(1,1,1) + \frac{2-7-1}{3}\cdot(-1,1,0) + \left(\frac{-2+7+1}{6}\right)\cdot(2,0,-2)$$

$$(7,1,1) = 3\cdot(1,1,1) + (-3)\cdot(-1,1,0) + 1\cdot(2,0,-2)$$

$$\boxed{\left[(7,1,1)\right]_{B_1} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}}$$