3-
d)

$$A = \begin{bmatrix} 3 & -1 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 3 & -1 & -3 \\ 0 & 2 & -3 \\ 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right) = 0$$

$$\det \begin{pmatrix} 3-\lambda & -1 & -3 \\ 0 & 2-\lambda & -3 \\ 0 & 0 & -1-\lambda \end{pmatrix} \begin{matrix} 3-\lambda & -1 \\ 0 & 2-\lambda \\ 0 & 0 \end{matrix} = 0$$

$$(3-\lambda) \cdot (2-\lambda) \cdot (-1-\lambda) = 0$$

$$(3-\lambda) \cdot (2 - 2\lambda + \lambda + \lambda^2) = 0 \qquad \lambda' = 3$$

$$\lambda'' = -1$$

$$(3-\lambda) \cdot (\lambda^2 - \lambda - 2) = 0 \qquad \lambda''' = 2$$

Para $\lambda = 3$

$$\begin{pmatrix} 3-3 & -1 & -3 \\ 0 & 2-3 & -3 \\ 0 & 0 & -1-3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -y - 3z = 0 \\ -y - 3z = 0 \quad \to \underline{y = 0} \\ -4z = 0 \quad \to \underline{z = 0} \end{cases}$$

$$V\lambda = 3 = \{ (x, y, z) \in \mathbb{R} / y = 0 \text{ e } z = 0 \}.$$

$$V\lambda = 3 = \{ (x, 0, 0) \ y / x \in \mathbb{R}^3 \}$$

$$V\lambda = 3 = [(1, 0, 0)]$$

Para $\lambda = 2$:

$$\begin{pmatrix} 3-2 & -1 & -3 \\ 0 & 2-2 & -3 \\ 0 & 0 & -1-2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & -1 & -3 \\ 0 & 0 & -3 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x - y = 3z \\ -3z = 0 \\ -3z = 0 \end{cases} \quad x = y \quad \longrightarrow z = 0$$

$V\lambda = 2 = \{(x, y, z) \in R \ / x = y \text{ e } z = 0\}$

$V\lambda = 2 = \{(y, y, 0) \ / y \in R^3\}$

$V\lambda = 2 = [(1, 1, 0)]$

Para $\lambda = -1$:

$$\begin{pmatrix} 3-(-1) & -1 & -3 \\ 0 & 2-(-1) & -3 \\ 0 & 0 & -1-(-1) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 & -3 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 4x - y - 3z = 0 \longrightarrow x = y \\ 3y - 3z = 0 \longrightarrow y = z \end{cases}$$

$V\lambda = -1 = \{(x, y, z) \in R \ / x = y \text{ e } y = z\}$

$V\lambda = -1 = \{(y, y, y) \ / y \in R^3\}$

$V\lambda = -1 = [(1, 1, 1)]$

5-

$\lambda_1 = 5$ e $v_1(1,1)$ $\qquad$ $v(4,1)$

$\lambda_2 = -1$ e $v_2(2,-1)$

$T(1,1) = 5(1,1) = (5,5)$

$T(2,-1) = -1(2,-1) = (-2,1)$

$(x,y) = \alpha_1(1,1) + \alpha_2(2,-1)$

$(x,y) = (\alpha_1,\alpha_1) + (2\alpha_2,-\alpha_2)$

$(x,y) = (\alpha_1 + 2\alpha_2, \alpha_1 - \alpha_2)$

$$\begin{cases} \alpha_1 + 2\alpha = x & \rightarrow \alpha_2 = \dfrac{x-y}{3} \\ \alpha_1 - \alpha_2 = y & \rightarrow \alpha_1 = \dfrac{2y+x}{3} \end{cases}$$

$T(x,y) = \alpha_1(1,1) + \alpha_2(2,-1)$

$T(x,y) = \dfrac{2y+x}{3} \cdot T(1,1) + \dfrac{x-y}{3} \cdot T(2,-1)$

$T(x,y) = \dfrac{2y+x}{3} \cdot (5,5) + \dfrac{x-y}{3} \cdot (-2,1)$

$T(x,y) = \left(\dfrac{10y+5x}{3}, \dfrac{10y+5x}{3}\right) + \left(\dfrac{-2x+2y}{3}, \dfrac{x-y}{3}\right)$

$T(x,y) = \left(\dfrac{10y+5x}{3} + \left(\dfrac{-2x+2y}{3}\right), \dfrac{10y+5x}{3} + \dfrac{x-y}{3}\right)$

$T(x,y) = \left(\dfrac{12y+3x}{3}, \dfrac{9y+6x}{3}\right)$

$T(x,y) = (4y+x, 3y+2x)$

$\boxed{T(x,y) = (x+4y, 2x+3y)}$ $\rightarrow$ $(4,1) = (4+4.1, 2.4+3.1)$

$\boxed{(4,1) = (8,11)}$

6-

a) I $\lambda_1 = 1$ $\quad v_1 = (y - y)$

II $\lambda_2 = 3$ $\quad v_2 = (0, y)$

I $T(y, -y) = 1(y, -y) \to yT(1,-1) \to y(1,-1) \to T(1,-1) = (1,-1)$

II $T(0, y) = 3(0, y) = (0, 3y) \to y\,T(0,1) = y(0,3) \to T(0,1) = (0,3)$

$\to (x, y) = \alpha_1 (1,-1) + \alpha_2 (0,1)$

$\begin{cases} \alpha_1 = x \to x = \alpha_1 \\ -\alpha_1 + \alpha_2 = y \to \alpha_2 = y + x \end{cases}$

$T(x,y) = T[x(1,-1) + (x,y).(0,1)]$

$T(x,y) = T[x(1,-1) + T[(x+y)(0,1)]$

$T(x,y) = xT(1,-1) + (x+y)\,T(0,1)$

$T(x,y) = x(1,-1) + (x+y)(0,3)$

$T(x,y) = (x, -x) + (0, 3y + 3y)$

$\boxed{T(x,y) = (x, 2x + 3y)} \to$ Operedor Linear T

b)

I $\lambda_1 = 3$ $\quad v_1 = x(1,2)$ | II $\lambda_2 = -2$ $\quad v_2 = x(-1,0)$

III $T(-1,0) = -2(-1,0) = (2,0)$

$(x,y) = \alpha_1 (1,2) + \alpha_2 (-1,0)$

$(x,y) = (\alpha_1, 2\alpha_1) + (-\alpha_2, 0)$

$\begin{cases} \alpha_1 - \alpha_2 = x \\ 2x_1 = y \end{cases}$

8-

• Transpormações T nos vetores $\underline{v}$ e $\underline{u}$:

$T(u) = 2.(2,1) = (4,2)$

$T(v) = 3.(1,2) = (3,6)$

• Criação de um vetor genérico (a,b) e fazer uma combinação linear dos outros dois vetores:

$(a,b) = \alpha(2,1) + \beta(1,2)$

$(a,b) = (2\alpha, 1\alpha) + (1\beta, 2\beta)$

$(a,b) = (2\alpha + \beta, \alpha + 2\beta)$

• Pela comparação, é possível afirmar:

$$\begin{cases} a = 2\alpha + \beta \ (i) \\ b = 1\alpha + 2\beta \ (ii) \end{cases}$$

• Multiplicando a equação (ii) por 2, subtraindo as duas para o α sumir, tornando possível encontrar β:

$$\begin{cases} a = 2\alpha + \beta \ (i) \\ b = \alpha + 2\beta \ (ii) \end{cases} \rightarrow \begin{cases} a = 2\alpha + \beta \\ 2b = 2\alpha + 4\beta \end{cases} \rightarrow a - 2b = -3\beta \quad \boxed{\beta = \frac{a-2b}{-3}}$$

• Da equação (i), teremos:

$2\alpha + \beta = a$

$2\alpha = a - \beta$

$$\alpha = \frac{a - \frac{a-2b}{-3}}{2} \rightarrow \alpha = \frac{\frac{4a-2b}{3}}{2}$$

$$\boxed{\alpha = \frac{4a-2b}{6}}$$

• Tendo os valores de α e β:

$$(a,b) = \frac{4a-2b}{6}(2,1) + \frac{a-2b}{-3}(1,2)$$

• Aplicando a transformação linear nos termos da equação:

$$T(a,b) = T\left[\frac{4a-2b}{6}(2,1)\right] + T\left[\frac{a-2b}{-3}(1,2)\right]$$

$$T(a,b) = \frac{4a-2b}{6}(4,2) + \frac{a-2b}{-3}(3,6)$$

$$T(a,b) = \left(\frac{16a-8b}{6}, \frac{8a-4b}{6}\right) + \left(\frac{3a-6b}{-3}, \frac{6a-12b}{-3}\right)$$

$$T(a,b) = \left(\frac{16a-8b}{6} + \frac{3a-6b}{-3}, \frac{8a-4b}{6} + \frac{6a-12b}{-3}\right)$$

$$\boxed{T(a,b) = \left(\frac{-5a+2b}{-3}, \frac{2a-10b}{-3}\right)} \longrightarrow \text{Lei da formação.}$$

a) $T(0,3)$ aplicado na lei de formação:

$$T(0,3) = \left(\frac{-5.0+2.3}{-3}, \frac{2.0-10.3}{-3}\right)$$

$$\boxed{T(0,3) = (-2,10)}$$

b) É a própria Lei da formação já encontrada:

$$\boxed{T(x,y) = \left(-\frac{5a+2b}{3}, \frac{2a-10b}{-3}\right)}$$

c) Aplicando os vetores de base na ordem apresentada:

$T(2,1) = (4,2)$

$T(2,1) = (3,6)$ $\qquad [T] = \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix}$