# Introduction to Bayesian statistics with R
# 4. Priors

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# To-do list

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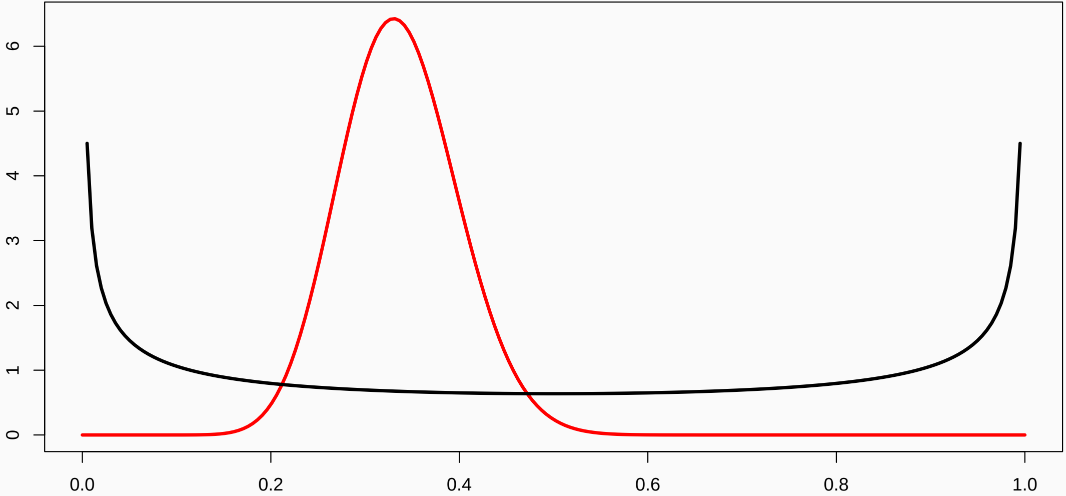
- add graph with gamma distribution
- check McCarthy paper w/ example ANOVA
- check paper big bad ugly prior
- check tips paper
- add further reading (McCarthy JAE, paper bad and ugly and paper on problem with priors in occupancy)
- check to-do list on github page of stats course https://github.com/oliviergimenez/bayesian-stats-with-R?tab=readme-ov-file#to-do-list
- prior elicitation (voir papiers JOSS suggérés par Osvaldo pour papier tips ; moment matching, voir aussi https://umr-astre.pages.mia.inra.fr/training/crashbayes/articles/crashbayes.html#prior-elicitation).

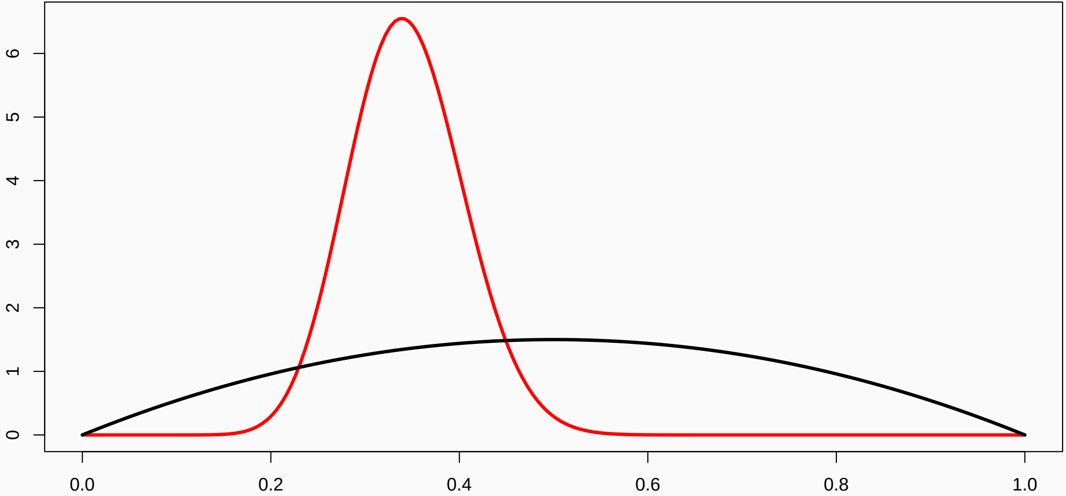
# A detour to explore priors

# Influence of the prior

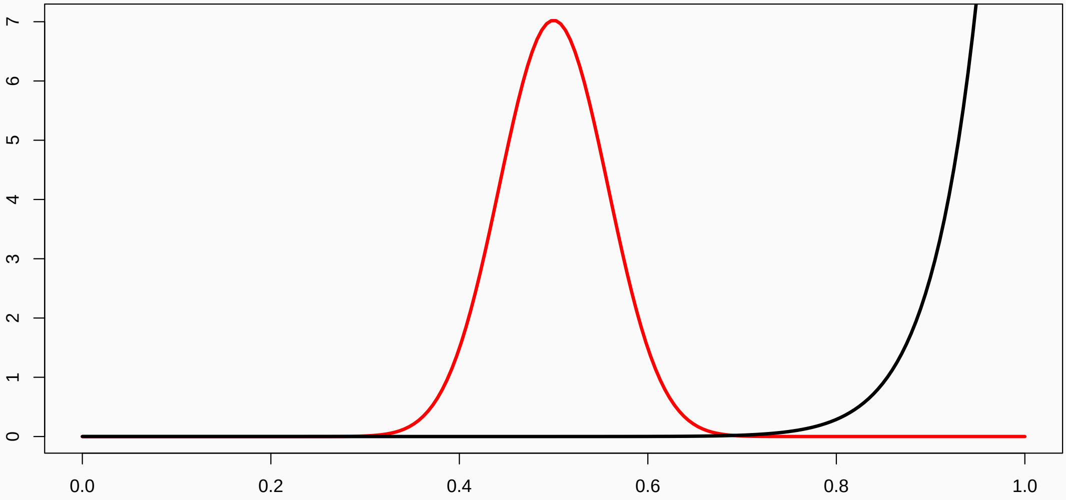
# Prior $Beta(0.5, 0.5)$ and posterior survival $Beta(19.5, 38.5)$



# Prior $Beta(2, 2)$ and posterior survival $Beta(21, 40)$



# Prior $Beta(20, 1)$ and posterior survival $Beta(39, 49)$



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- With sufficiently large and informative datasets the prior typically has little effect on the results.
- Always perform a sensitivity analysis.

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# How to incorporate prior information?

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- We assume a vague prior:

$$\phi_{prior} \sim \text{Beta}(1, 1) = \text{Uniform}(0, 1)$$

# Notation

- $y_{i,t} = 1$ if individual $i$ detected at occasion $t$ and 0 otherwise
- $z_{i,t} = 1$ if individual $i$ alive between occasions $t$ and $t+1$ and 0 otherwise

$$
\begin{aligned}
y_{i,t} \mid z_{i,t} &\sim \text{Bernoulli}(p \, z_{i,t}) & \text{[likelihood (observation eq.)]} \\
z_{i,t+1} \mid z_{i,t} &\sim \text{Bernoulli}(\phi \, z_{i,t}) & \text{[likelihood (state eq.)]} \\
\phi &\sim \text{Beta}(1,1) & \text{[prior for } \phi \text{]} \\
p &\sim \text{Beta}(1,1) & \text{[prior for p]}
\end{aligned}
$$

# European dippers in Eastern France (1981-1987)

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- Assuming an informative prior $\phi_{prior} \sim \text{Normal}(0.57, 0.073^2)$.
- Mean posterior $\phi_{posterior} = 0.56$ with credible interval $[0.52, 0.60]$.
- No increase of precision in posterior inference.

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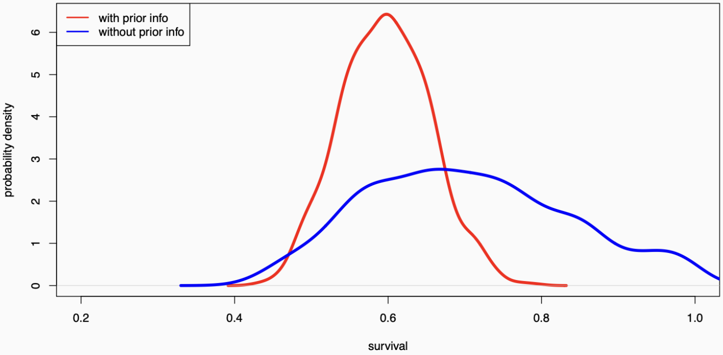
- Now if you had only the three first years of data, what would have happened?
- Width of credible interval is 0.47 (vague prior) vs. 0.30 (informative prior).
- Huge increase of precision in posterior inference (40% gain)!

# Compare vague vs. informative prior



# Prior elicitation via moment matching

## Remember the Beta distribution

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- If $X \sim Beta(\alpha, \beta)$, then the first and second moments of $X$ are:

$$\mu = \mathrm{E}(X) = \frac{\alpha}{\alpha + \beta}$$

$$\sigma^2 = \mathrm{Var}(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

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- Parameters $\mu$ and $\sigma^2$ are seen as the moments of a *Beta*$(\alpha, \beta)$ distribution.
- Now we look for values of $\alpha$ and $\beta$ that match the observed moments of the Beta distribution ($\mu$ and $\sigma^2$).
- We need another set of equations:

$$\alpha = \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu}\right)\mu^2$$

$$\beta = \alpha\left(\frac{1}{\mu} - 1\right)$$

- For our model, that means:

```
(alpha <- ( (1 - 0.57)/(0.073*0.073) - (1/0.57) )*0.57^2)
#> [1] 25.64636
(beta <- alpha * ( (1/0.57) - 1))
#> [1] 19.34726
```

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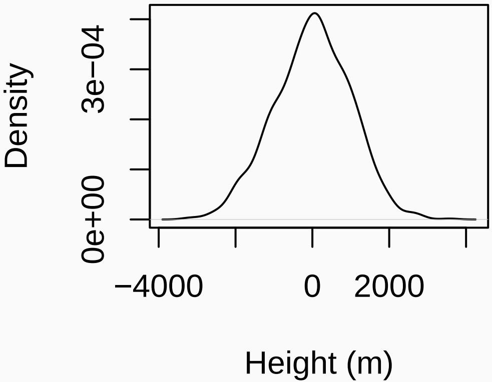
- Now use $\phi_{prior} \sim \text{Beta}(\alpha = 25.6, \beta = 19.3)$ instead of $\phi_{prior} \sim \text{Normal}(0.57, 0.073^2)$

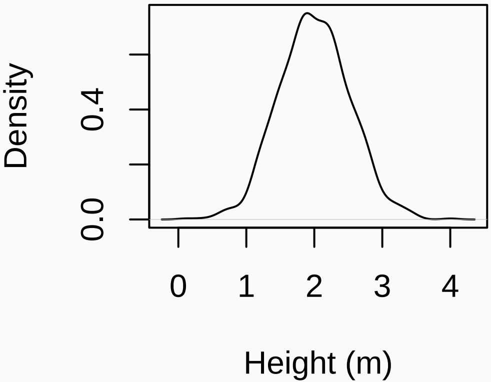
# Prior predictive checks

## Linear regression

Unreasonable prior $\beta \sim N(0, 1000^2)$

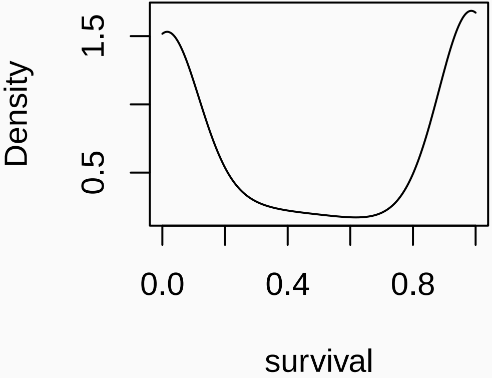



Reasonable prior $\beta \sim N(2, 0.5^2)$



# Logistic regression

Unreasonable $\text{logit}(\phi) = \beta \sim N(0, 10^2)$




Reasonable $\text{logit}(\phi) = \beta \sim N(0, 1.5^2)$