Introduction to Bayesian statistics with R
4. Priors

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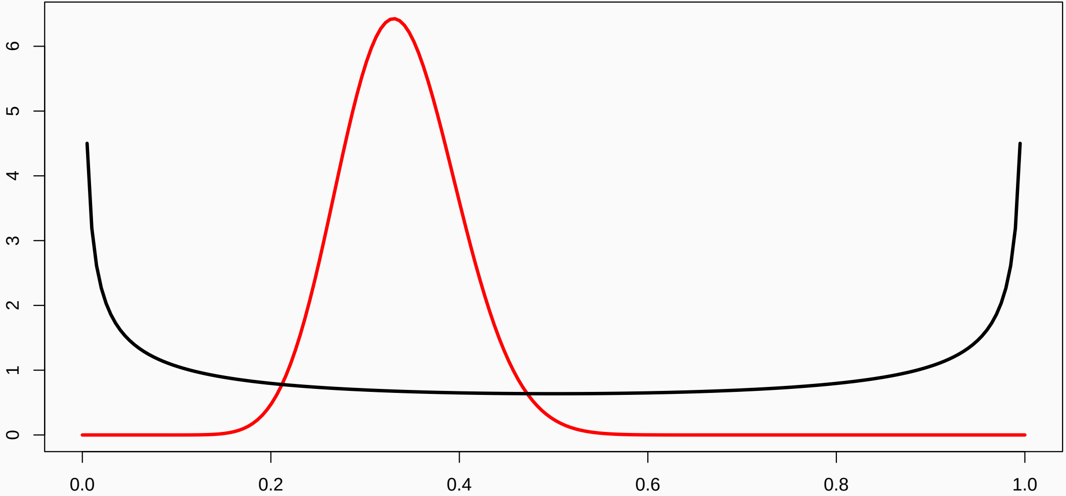
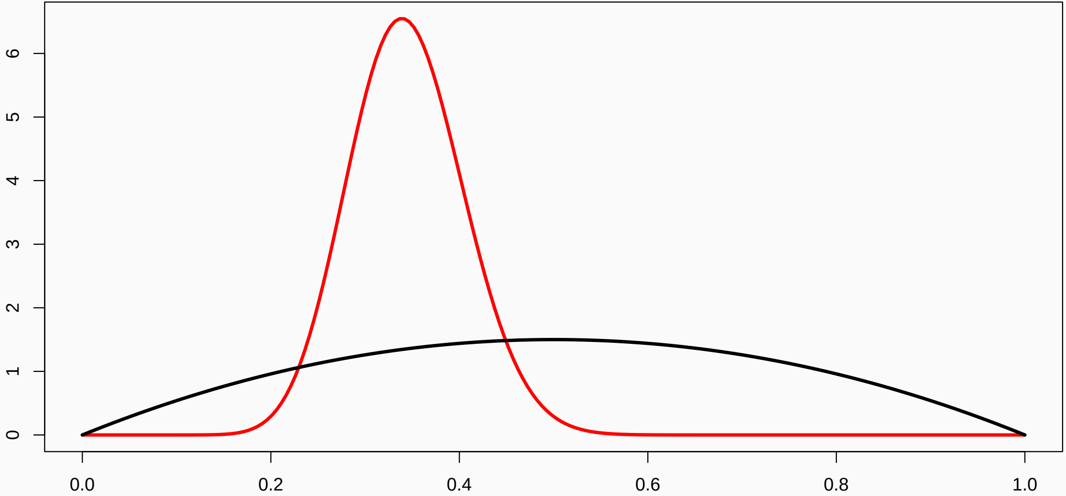
Influence of the prior: Back to our guiding example

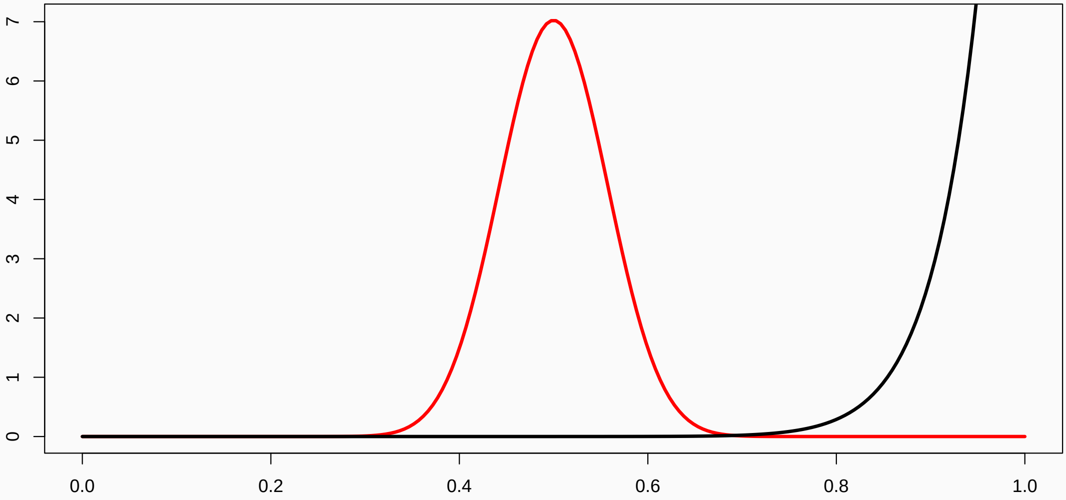
Prior $Beta(0.5, 0.5)$ and posterior survival $Beta(19.5, 38.5)$



Prior $Beta(2,2)$ and posterior survival $Beta(21,40)$



Prior $Beta(20,1)$ and posterior survival $Beta(39,49)$



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- With sufficiently large and informative datasets the prior typically has little effect on the results.
- This can be assessed with a sensitivity analysis.

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- In the absence of any prior information on one or more model parameters we wish to ensure that this lack of knowledge is properly reflected in the prior.

How to incorporate prior information?

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- We assume a vague prior:

$$\phi_{prior} \sim \text{Beta}(1, 1) = \text{Uniform}(0, 1)$$

European dippers in Eastern France (1981-1987)

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- No increase of precision in posterior inference.

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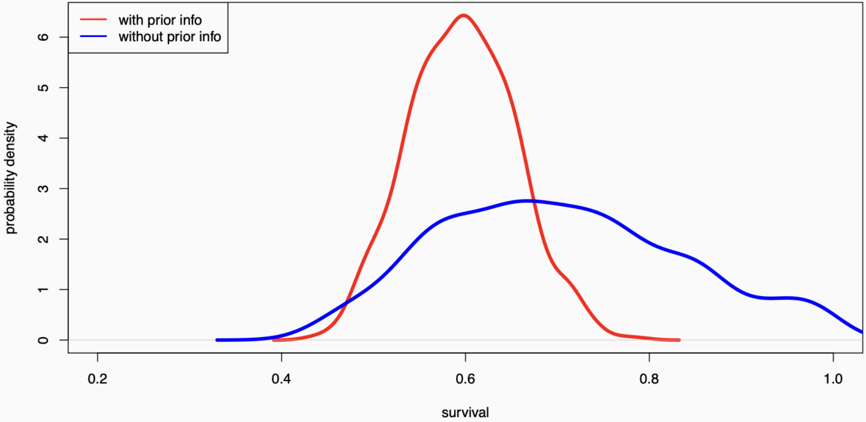
- Width of credible interval is 0.47 (vague prior) vs. 0.30 (informative prior).
- Huge increase of precision in posterior inference (40% gain)!

Compare vague vs. informative prior



Prior elicitation via moment matching

Remember the Beta distribution

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- If $X \sim Beta(a, b)$, then the first and second moments of X are:

$$\mu = \mathrm{E}(X) = \frac{a}{a+b}$$

$$\sigma^2 = \mathrm{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

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- Parameters μ and σ^2 are seen as the moments of a $Beta(a, b)$ distribution.
- Now we look for values of a and b that match the observed moments of the Beta distribution (μ and σ^2).
- We need another set of equations:

$$a = \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu}\right)\mu^2$$

$$b = a\left(\frac{1}{\mu} - 1\right)$$

- For our model, that means:

```
(a <- ( (1 - 0.57)/(0.073*0.073) - (1/0.57) )*0.57^2)
#> [1] 25.64636
(b <- a * ( (1/0.57) - 1))
#> [1] 19.34726
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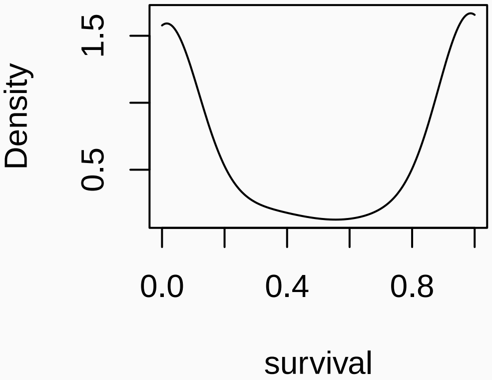
- Now use $\phi_{prior} \sim \text{Beta}(a = 25.6, b = 19.3)$ instead of $\phi_{prior} \sim \text{Normal}(0.57, 0.073^2)$

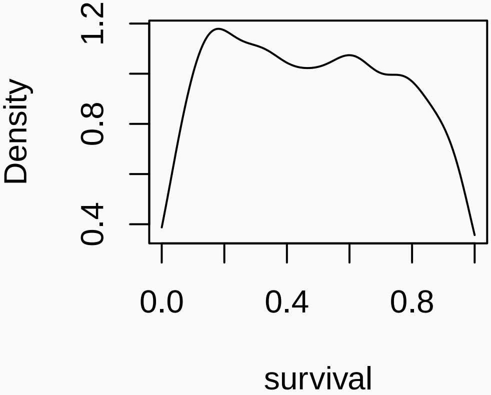
Beware of so-called non-informative priors

Logistic regression

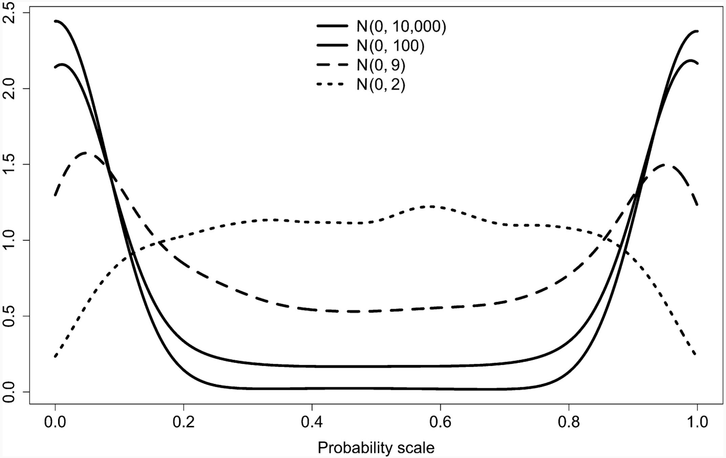
Unreasonable $\text{logit}(\phi) = \beta \sim N(0, 10^2)$



Reasonable $\text{logit}(\phi) = \beta \sim N(0, 1.5^2)$



Further illustration of the issue



Banner KM, Irvine KM, Rodhouse TJ (2020). The use of Bayesian priors in Ecology: The good, the bad and the not great. *Methods Ecol Evol* 11: 882–889.

Prior predictive checks: Use simulations to check your priors.

```r
naive_prior_logit <- rnorm(1000,0,10) # on logit scale
naive_prior <- plogis(naive_prior_logit) # on natural scale bw 0 and 1
hist(naive_prior)
```

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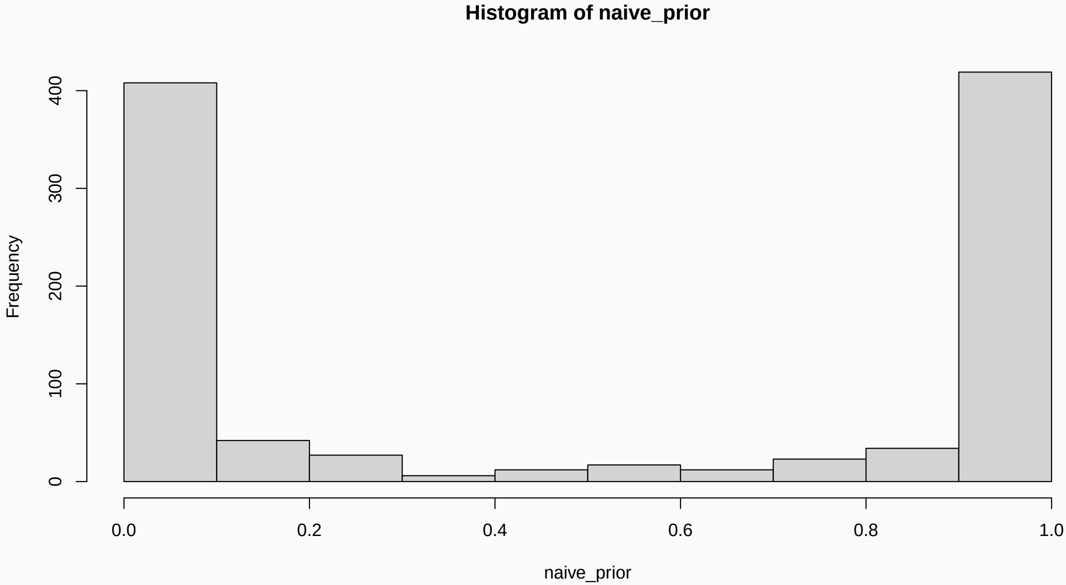

Prior predictive checks: Use simulations to check your priors.

```r
prior_logit <- rnorm(1000,0,1.5) # on logit scale
prior <- plogis(prior_logit) # on natural scale bw 0 and 1
hist(prior)
```

Prior predictive checks: Use simulations to check your priors.

Further reading

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- Banner KM, Irvine KM, Rodhouse TJ (2020). The use of Bayesian priors in Ecology: The good, the bad and the not great. *Methods Ecol Evol* 11: 882–889.
- Lemoine NP (2019). Moving beyond noninformative priors: why and how to choose weakly informative priors in Bayesian analyses. *Oikos* 128: 912–928.
- McCarthy MA, Masters P (2005). Profiting from prior information in Bayesian analyses of ecological data. *Journal of Applied Ecology* 42: 1012–1019.
- Mikkola P et al. (2024). Prior Knowledge Elicitation: The Past, Present, and Future. *Bayesian Analysis* 19: 1129-1161.
- Northrup JM, Gerber BD (2018). A comment on priors for Bayesian occupancy models. *PLoS ONE* 13(2): e0192819