Introduction to Bayesian statistics with R
1. Motivations

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This workshop

Objectives

- Try and demystify Bayesian statistics, and what we call MCMC.
- Make the difference between Bayesian and Frequentist analyses.
- Understand the Methods section of ecological papers doing Bayesian stuff.
- Run Bayesian analyses, safely hopefully.

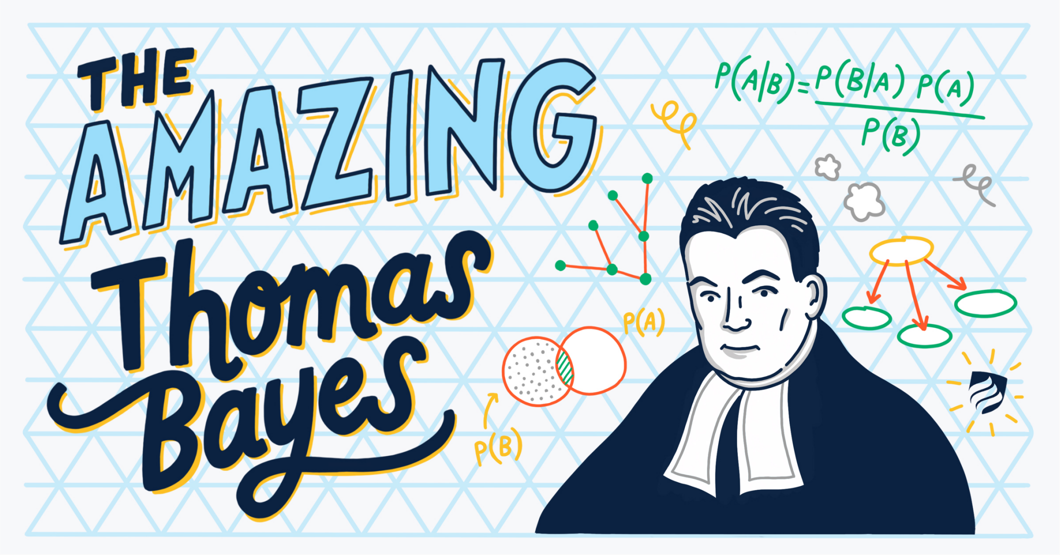
What's on our plate?

- Section 1 - Motivation and Bayes theorem.
- Section 2 - Markov chain Monte Carlo algorithms (MCMC).
- Section 3 - Introduction to `NIMBLE` and `brms`.
- Section 4 - Priors.
- Section 5 - Case studies and GLMMs.

Material

- Material from previous workshops, a book in progress and a paper on ten quick tips to get you started with Bayesian statistics.
- All material prepared with `R`.
- `R Markdown` used to write reproducible material.
- Slides, data and R codes on GitHub at https://github.com/oliviergimenez/bayes-workshop

What is Bayesian inference?


THE AMAZING
Thomas Bayes
P(A|B)=P(B|A) P(A) / P(B)
P(A)
P(B)

A reminder on conditional probabilities

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HOW TO CURE VAMPIRES?
imgflip.com

Screening for vampirism

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- The chance of a negative test given you are mortal is $\Pr(-|\text{mortal}) = 0.95$ (**specificity**).

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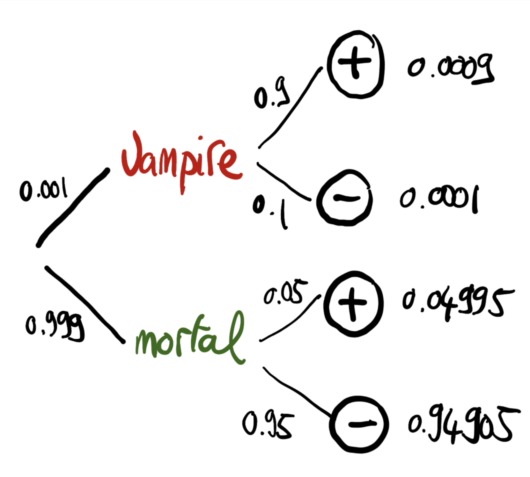
- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$
- Assume that vampires are rare, and represent only 0.1% of the population. This means that $\Pr(\text{vampire}) = 0.001$.

What is the answer? Bayes' theorem to the rescue!



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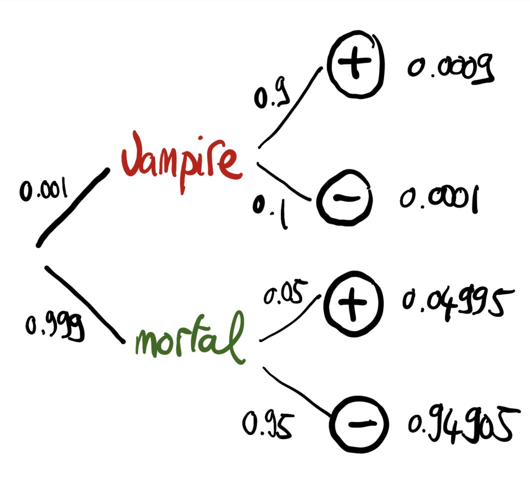

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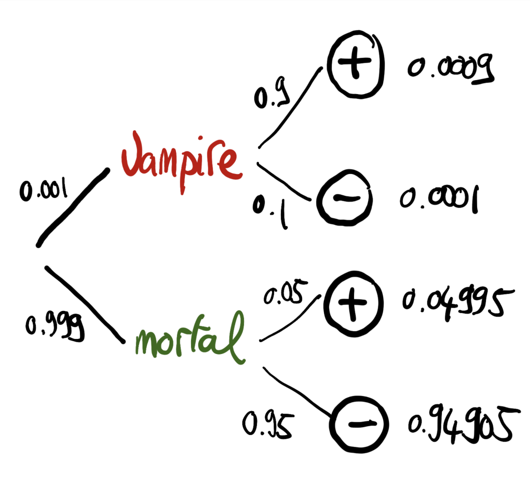

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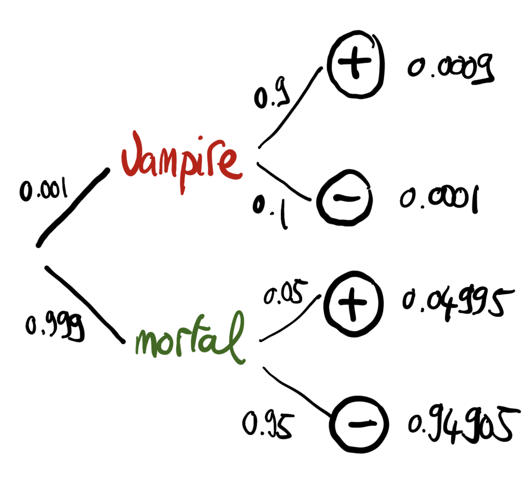

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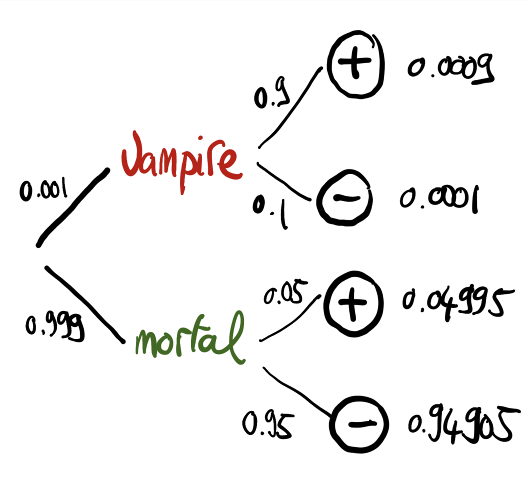

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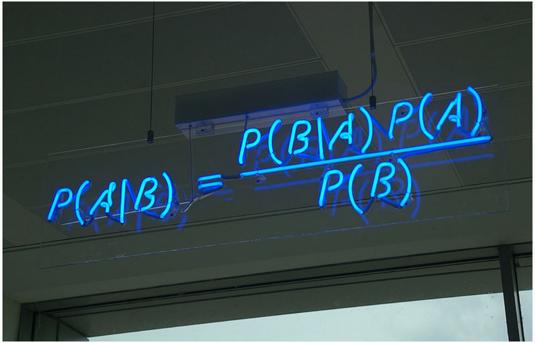
$$\Pr(\text{vampire}|+) = \frac{\Pr(+|\text{vampire})\ \Pr(\text{vampire})}{\Pr(+)}$$

Bayes' theorem

- A theorem about conditional probabilities.
- $\Pr(B \mid A) = \dfrac{\Pr(A \mid B)\ \Pr(B)}{\Pr(A)}$



Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

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- The "hypothesis" is typically something unobserved or unknown. It's what you want to learn about using the data.
- For regression models, the "hypothesis" is a parameter (intercept, slopes or error terms).
- Bayes theorem tells you the probability of the hypothesis given the data.

What is doing science after all?

How plausible is some hypothesis given the data?

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \ \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

Why is Bayesian statistics not the default?

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- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.
- Recent advances in computational power coupled with the development of new methodology have led to a great increase in the application of Bayesian methods within the last two decades.

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- The frequentist approach (**maximum likelihood estimation** – MLE) assumes that the parameters are fixed, but have unknown values to be estimated.
- Classical estimates generally provide a point estimate of the parameter of interest.
- The Bayesian approach assumes that the parameters are not fixed but have some fixed unknown distribution - a distribution for the parameter.

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- The approach is based upon the idea that the experimenter begins with some prior beliefs about the system.
- And then updates these beliefs on the basis of observed data.
- This updating procedure is based upon the Bayes' Theorem:

$$\Pr(A \mid B) = \frac{\Pr(B \mid A)\ \Pr(A)}{\Pr(B)}$$

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- Schematically if $A = \theta$ and $B = \text{data}$, then
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$$\Pr(A \mid B) = \frac{\Pr(B \mid A)\ \Pr(A)}{\Pr(B)}$$

- Translates into:

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\ \Pr(\theta)}{\Pr(\text{data})}$$

Bayes' theorem

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- Prior distribution: Represents what you know before seeing the data. The source of much discussion about the Bayesian approach.
- $\Pr(\text{data}) = \int \Pr(\text{data} \mid \theta)\ \Pr(\theta) d\theta$: Possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods.

Summary

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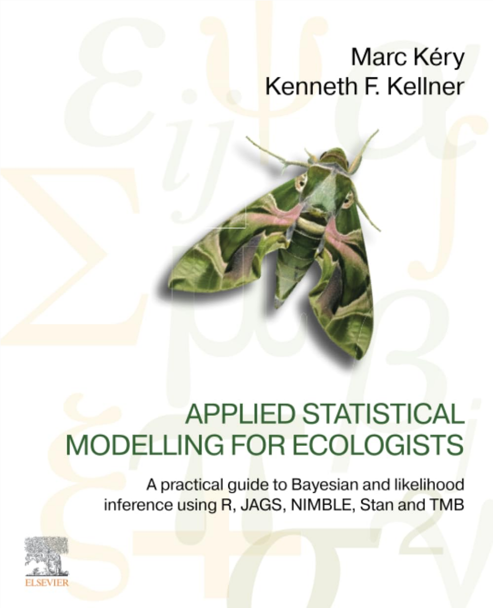
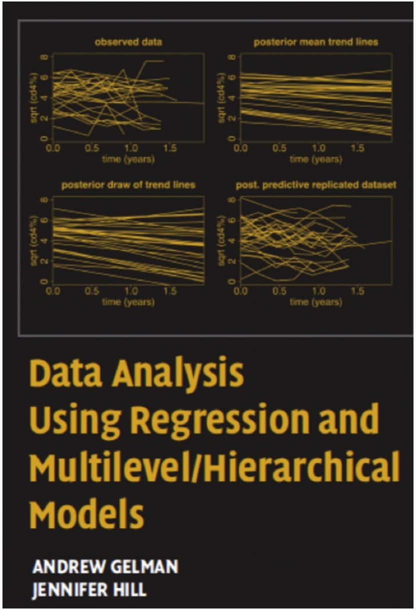
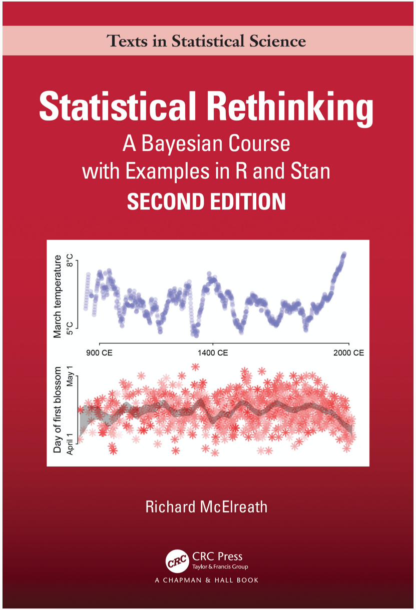
- With the Bayes' theorem, you update your beliefs (prior) with new data (likelihood) to get posterior beliefs (posterior): posterior $\propto$ likelihood $\times$ prior.
- When applying the Bayes' theorem, you get possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods.

Further reading


Marc Kéry
Kenneth F. Kellner
APPLIED STATISTICAL MODELLING FOR ECOLOGISTS
A practical guide to Bayesian and likelihood inference using R, JAGS, NIMBLE, Stan and TMB
ELSEVIER


observed data
posterior mean trend lines
posterior draw of trend lines
post. predictive replicated dataset
sqrt (cd4%)
time (years)
Data Analysis Using Regression and Multilevel/Hierarchical Models
ANDREW GELMAN
JENNIFER HILL


Texts in Statistical Science
Statistical Rethinking
A Bayesian Course
with Examples in R and Stan
SECOND EDITION
March temperature
8°C
5°C
900 CE
1400 CE
2000 CE
Day of first blossom
May 1
April 1
Richard McElreath
CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK