Introduction to Bayesian statistics with R
1. Motivations

Olivier Gimenez

last updated: 2025-03-09

This workshop

Objectives

- Try and demystify Bayesian statistics, and what we call MCMC.
- Make the difference between Bayesian and Frequentist analyses.
- Understand the Methods section of ecological papers doing Bayesian stuff.
- Run Bayesian analyses, safely hopefully.

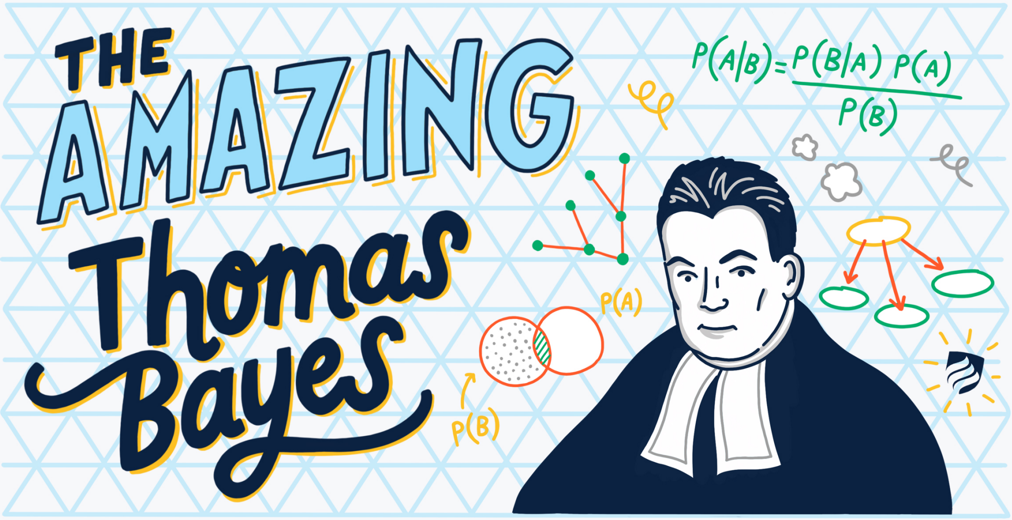
What's on our plate?

- Section 1 - Motivation and Bayes theorem.
- Section 2 - Markov chain Monte Carlo algorithms (MCMC).
- Section 3 - Introduction to `NIMBLE` and `brms`.
- Section 4 - Priors.
- Section 5 - Case studies and GLMMs.

Material

- Material from previous workshops, a book in progress and a paper on ten quick tips to get you started with Bayesian statistics.
- All material prepared with `R`.
- `R Markdown` used to write reproducible material.
- Slides, data and R codes on GitHub for easier manipulation
- Dedicated website https://oliviergimenez.github.io/bayesian-stats-with-R/.

What is Bayesian inference?


THE AMAZING
Thomas Bayes
P(A|B)=P(B|A) P(A) / P(B)
P(A)
P(B)

A reminder on conditional probabilities

- $\Pr(A \mid B)$: Probability of A given B

A reminder on conditional probabilities

- $\Pr(A \mid B)$: Probability of A given B
- The ordering matters: $\Pr(A \mid B)$ is not the same as $\Pr(B \mid A)$.

A reminder on conditional probabilities

- $\Pr(A \mid B)$: Probability of A given B
- The ordering matters: $\Pr(A \mid B)$ is not the same as $\Pr(B \mid A)$.
- $\Pr(A \mid B) = \dfrac{\Pr(A \text{ and } B)}{\Pr(B)}$


HOW TO CURE VAMPIRES?
imgflip.com

Screening for vampirism

- The chance of the test being positive given you are a vampire is $\Pr(+|\text{vampire}) = 0.90$ (**sensitivity**).

Screening for vampirism

- The chance of the test being positive given you are a vampire is $\Pr(+|\text{vampire}) = 0.90$ (**sensitivity**).
- The chance of a negative test given you are mortal is $\Pr(-|\text{mortal}) = 0.95$ (**specificity**).

What is the question?

- From the perspective of the test: Given a person is a vampire, what is the probability that the test is positive? $\Pr(+|\text{vampire}) = 0.90$.

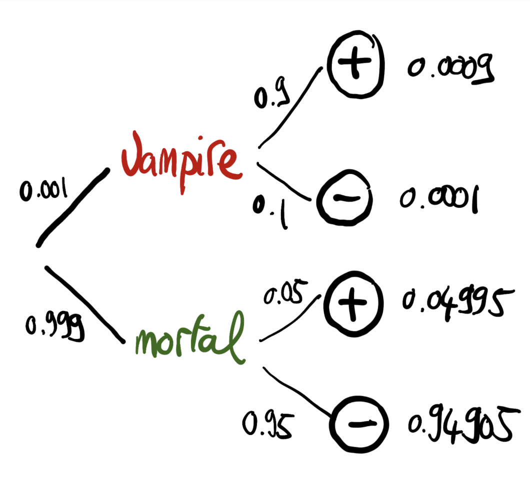
What is the question?

- From the perspective of the test: Given a person is a vampire, what is the probability that the test is positive? $\Pr(+|\text{vampire}) = 0.90$.
- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$

What is the question?

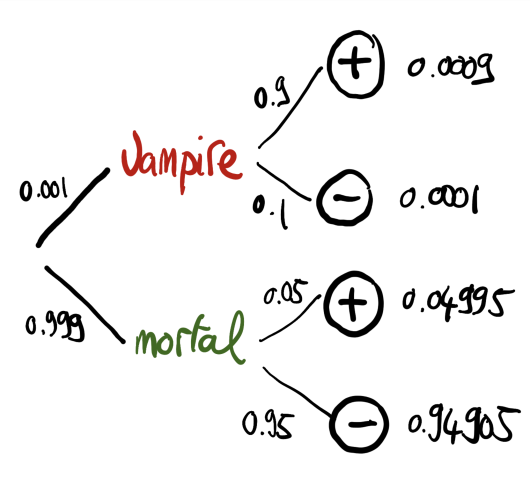
- From the perspective of the test: Given a person is a vampire, what is the probability that the test is positive? $\Pr(+|\text{vampire}) = 0.90$.
- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$
- Assume that vampires are rare, and represent only 0.1% of the population. This means that $\Pr(\text{vampire}) = 0.001$.

What is the answer? Bayes' theorem to the rescue!



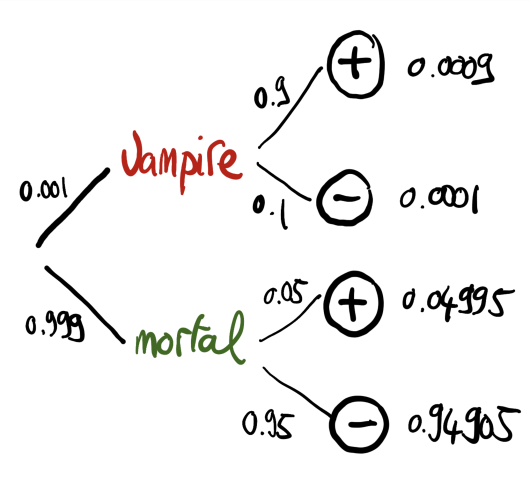
- $\Pr(\text{vampire}|+) = \dfrac{\Pr(\text{vampire and } +)}{\Pr(+)}$

What is the answer? Bayes' theorem to the rescue!



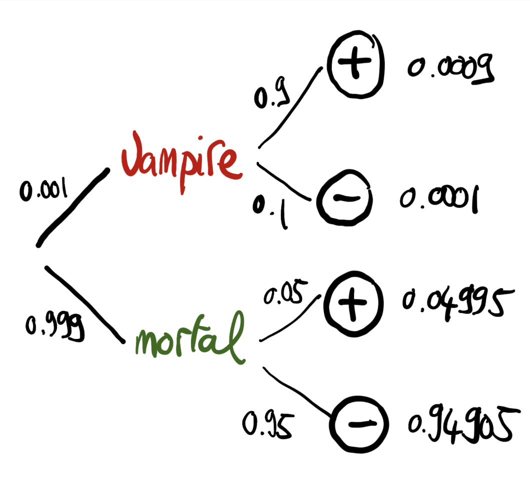
- $\Pr(\text{vampire}|+) = \dfrac{\Pr(\text{vampire and } +)}{\Pr(+)}$
- $\Pr(\text{vampire and } +) =$
  $\Pr(\text{vampire}) \ \Pr(+|\text{vampire}) = 0.0009$

What is the answer? Bayes' theorem to the rescue!



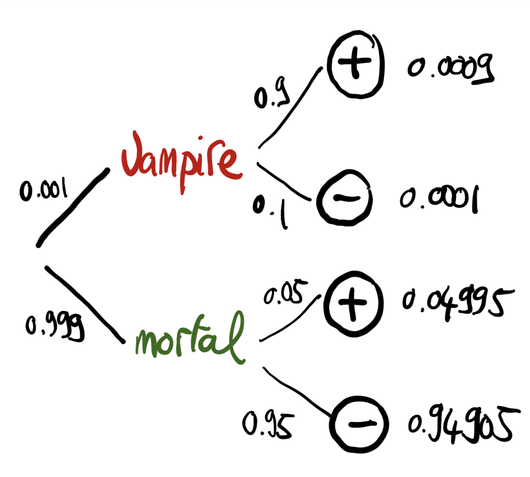
- $\Pr(\text{vampire}|+) = \dfrac{\Pr(\text{vampire and } +)}{\Pr(+)}$
- $\Pr(\text{vampire and } +) =$
  $\Pr(\text{vampire}) \ \Pr(+|\text{vampire}) = 0.0009$
- $\Pr(+) = 0.0009 + 0.04995 = 0.05085$

What is the answer? Bayes' theorem to the rescue!



- $\Pr(\text{vampire}|+) = \dfrac{\Pr(\text{vampire and } +)}{\Pr(+)}$
- $\Pr(\text{vampire and } +) = \Pr(\text{vampire}) \; \Pr(+|\text{vampire}) = 0.0009$
- $\Pr(+) = 0.0009 + 0.04995 = 0.05085$
- $\Pr(\text{vampire}|+) = 0.0009/0.05085 = 0.02$

What is the answer? Bayes' theorem to the rescue!

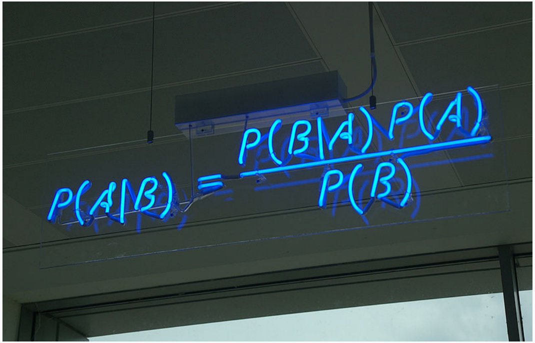


- $\Pr(\text{vampire}|+) = \dfrac{\Pr(\text{vampire and } +)}{\Pr(+)}$
- $\Pr(\text{vampire and } +) = \Pr(\text{vampire})\ \Pr(+|\text{vampire}) = 0.0009$
- $\Pr(+) = 0.0009 + 0.04995 = 0.05085$
- $\Pr(\text{vampire}|+) = 0.0009/0.05085 = 0.02$

$$\Pr(\text{vampire}|+) = \frac{\Pr(+|\text{vampire})\ \Pr(\text{vampire})}{\Pr(+)}$$

Bayes' theorem

- A theorem about conditional probabilities.
- $\Pr(B \mid A) = \dfrac{\Pr(A \mid B)\ \Pr(B)}{\Pr(A)}$



Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \; \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

- The "hypothesis" is typically something unobserved or unknown. It's what you want to learn about using the data.

Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \ \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

- The "hypothesis" is typically something unobserved or unknown. It's what you want to learn about using the data.
- For regression models, the "hypothesis" is a parameter (intercept, slopes or error terms).

Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis})\ \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

- The "hypothesis" is typically something unobserved or unknown. It's what you want to learn about using the data.
- For regression models, the "hypothesis" is a parameter (intercept, slopes or error terms).
- Bayes theorem tells you the probability of the hypothesis given the data.

What is doing science after all?

How plausible is some hypothesis given the data?

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \; \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

Why is Bayesian statistics not the default?

- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.

Why is Bayesian statistics not the default?

- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.
- Recent advances in computational power coupled with the development of new methodology have led to a great increase in the application of Bayesian methods within the last two decades.

Frequentist versus Bayesian

- Typical stats problems involve estimating parameter θ with available data.

Frequentist versus Bayesian

- Typical stats problems involve estimating parameter θ with available data.
- The frequentist approach (**maximum likelihood estimation** – MLE) assumes that the parameters are fixed, but have unknown values to be estimated.

Frequentist versus Bayesian

- Typical stats problems involve estimating parameter θ with available data.
- The frequentist approach (**maximum likelihood estimation** – MLE) assumes that the parameters are fixed, but have unknown values to be estimated.
- Classical estimates generally provide a point estimate of the parameter of interest.

Frequentist versus Bayesian

- Typical stats problems involve estimating parameter θ with available data.
- The frequentist approach (**maximum likelihood estimation** – MLE) assumes that the parameters are fixed, but have unknown values to be estimated.
- Classical estimates generally provide a point estimate of the parameter of interest.
- The Bayesian approach assumes that the parameters are not fixed but have some fixed unknown distribution - a distribution for the parameter.

What is the Bayesian approach?

- The approach is based upon the idea that the experimenter begins with some prior beliefs about the system.

What is the Bayesian approach?

- The approach is based upon the idea that the experimenter begins with some prior beliefs about the system.
- And then updates these beliefs on the basis of observed data.

What is the Bayesian approach?

- The approach is based upon the idea that the experimenter begins with some prior beliefs about the system.
- And then updates these beliefs on the basis of observed data.
- This updating procedure is based upon the Bayes' Theorem:

$$\Pr(A \mid B) = \frac{\Pr(B \mid A)\ \Pr(A)}{\Pr(B)}$$

What is the Bayesian approach?

- Schematically if $A = \theta$ and $B = \text{data}$, then

What is the Bayesian approach?

- Schematically if $A = \theta$ and $B =$ data, then
- The Bayes' theorem

$$\Pr(A \mid B) = \frac{\Pr(B \mid A)\ \Pr(A)}{\Pr(B)}$$

What is the Bayesian approach?

- Schematically if $A = \theta$ and $B =$ data, then
- The Bayes' theorem

$$\Pr(A \mid B) = \frac{\Pr(B \mid A)\ \Pr(A)}{\Pr(B)}$$

- Translates into:

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\ \Pr(\theta)}{\Pr(\text{data})}$$

Bayes' theorem

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta) \Pr(\theta)}{\Pr(\text{data})}$$

Bayes' theorem

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\,\Pr(\theta)}{\Pr(\text{data})}$$

- Posterior distribution: Represents what you know after having seen the data. The basis for inference, a distribution, possibly multivariate if more than one parameter (θ).

Bayes' theorem

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\,\Pr(\theta)}{\Pr(\text{data})}$$

- Posterior distribution: Represents what you know after having seen the data. The basis for inference, a distribution, possibly multivariate if more than one parameter (θ).
- Likelihood: We know that quantity, same as in the MLE approach.

Bayes' theorem

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\,\Pr(\theta)}{\Pr(\text{data})}$$

- Posterior distribution: Represents what you know after having seen the data. The basis for inference, a distribution, possibly multivariate if more than one parameter (θ).
- Likelihood: We know that quantity, same as in the MLE approach.
- Prior distribution: Represents what you know before seeing the data. The source of much discussion about the Bayesian approach.

Bayes' theorem

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\ \Pr(\theta)}{\Pr(\text{data})}$$

- Posterior distribution: Represents what you know after having seen the data. The basis for inference, a distribution, possibly multivariate if more than one parameter (θ).
- Likelihood: We know that quantity, same as in the MLE approach.
- Prior distribution: Represents what you know before seeing the data. The source of much discussion about the Bayesian approach.
- $\Pr(\text{data}) = \int \Pr(\text{data} \mid \theta)\ \Pr(\theta) d\theta$: Possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods.

Summary

Summary

- With the Bayes' theorem, you update your beliefs (prior) with new data (likelihood) to get posterior beliefs (posterior): posterior $\propto$ likelihood $\times$ prior.
- When applying the Bayes' theorem, you get possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods.