Introduction to Bayesian statistics with R

4. Priors

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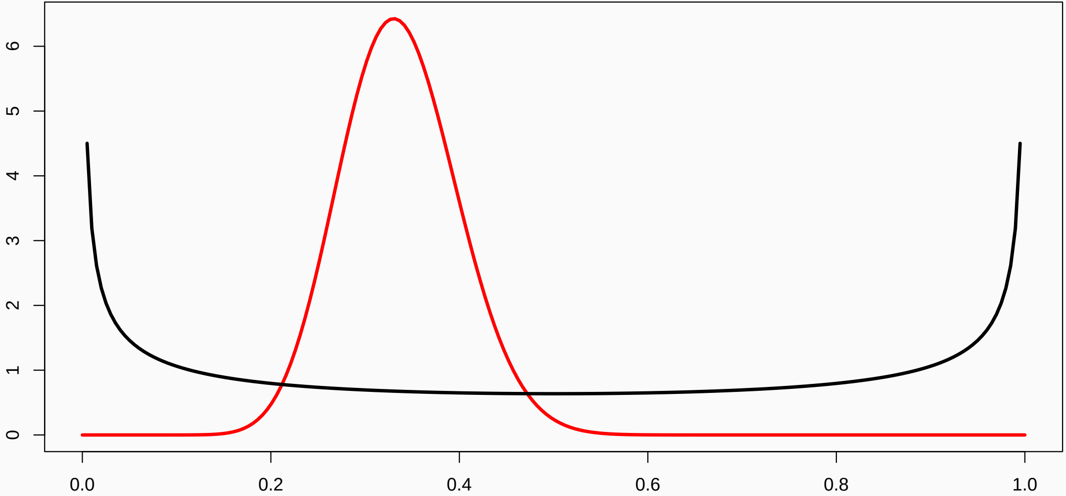
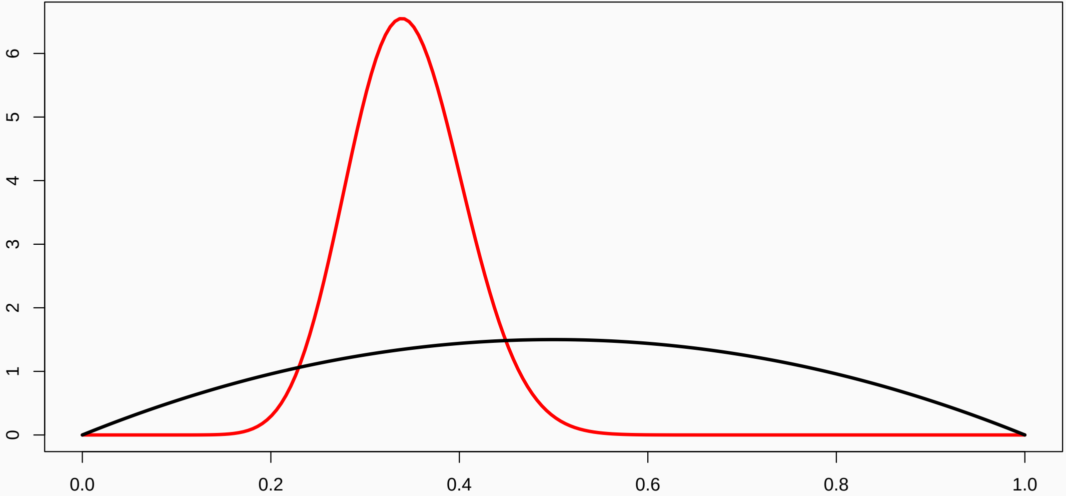
A detour to explore priors

Influence of the prior

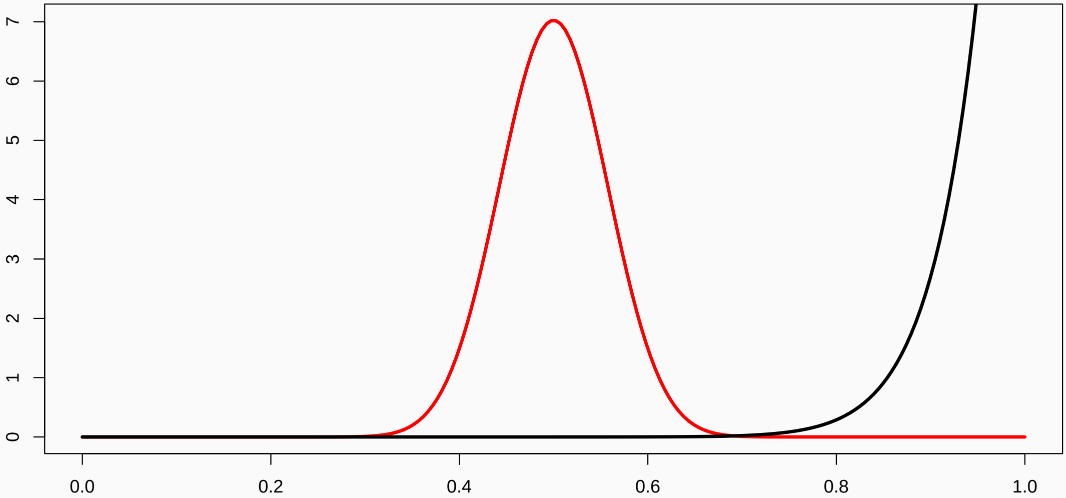
Prior $Beta(0.5, 0.5)$ and posterior survival $Beta(19.5, 38.5)$



Prior $Beta(2, 2)$ and posterior survival $Beta(21, 40)$



Prior $Beta(20, 1)$ and posterior survival $Beta(39, 49)$



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- Always perform a sensitivity analysis.

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How to incorporate prior information?

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- We assume a vague prior:

$$\phi_{prior} \sim \text{Beta}(1, 1) = \text{Uniform}(0, 1)$$

Notation

- $y_{i,t} = 1$ if individual i detected at occasion t and 0 otherwise
- $z_{i,t} = 1$ if individual i alive between occasions t and $t+1$ and 0 otherwise

$$
\begin{aligned}
y_{i,t} \mid z_{i,t} &\sim \text{Bernoulli}(p\, z_{i,t}) && \text{[likelihood (observation eq.)]} \\
z_{i,t+1} \mid z_{i,t} &\sim \text{Bernoulli}(\phi\, z_{i,t}) && \text{[likelihood (state eq.)]} \\
\phi &\sim \text{Beta}(1,1) && \text{[prior for } \phi\text{]} \\
p &\sim \text{Beta}(1,1) && \text{[prior for p]}
\end{aligned}
$$

European dippers in Eastern France (1981-1987)

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- Width of credible interval is 0.47 (vague prior) vs. 0.30 (informative prior).
- Huge increase of precision in posterior inference (40% gain)!

Compare vague vs. informative prior

Prior elicitation via moment matching

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- If $X \sim \mathit{Beta}(\alpha, \beta)$, then the first and second moments of X are:

$$\mu = \mathsf{E}(X) = \frac{\alpha}{\alpha + \beta}$$

$$\sigma^2 = \mathsf{Var}(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

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- Now we look for values of α and β that match the observed moments of the Beta distribution (μ and σ^2).
- We need another set of equations:

$$\alpha = \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu}\right)\mu^2$$

$$\beta = \alpha\left(\frac{1}{\mu} - 1\right)$$

- For our model, that means:

```r
(alpha <- ( (1 - 0.57)/(0.073*0.073) - (1/0.57) )*0.57^2)
#> [1] 25.64636
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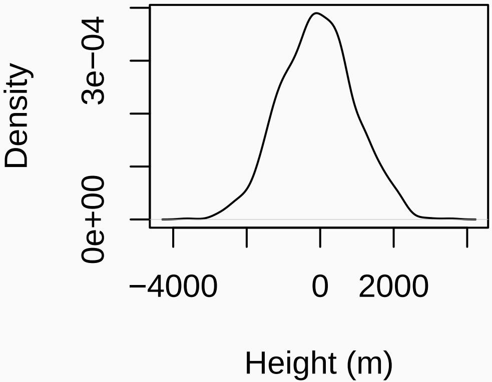
- Now use $\phi_{prior} \sim \text{Beta}(\alpha = 25.6, \beta = 19.3)$ instead of $\phi_{prior} \sim \text{Normal}(0.57, 0.073^2)$

Your turn: Practical 3

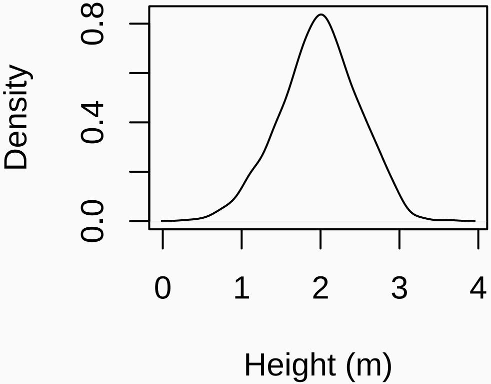
Prior predictive checks

Linear regression

Unreasonable prior $\beta \sim N(0, 1000^2)$

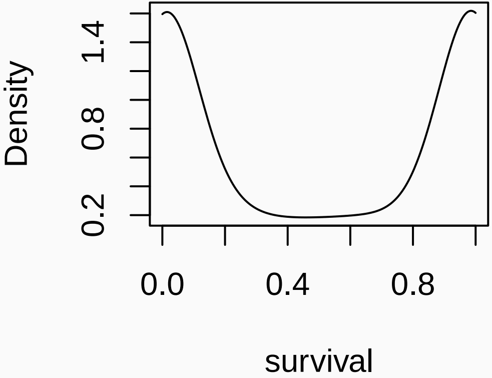


Reasonable prior $\beta \sim N(2, 0.5^2)$

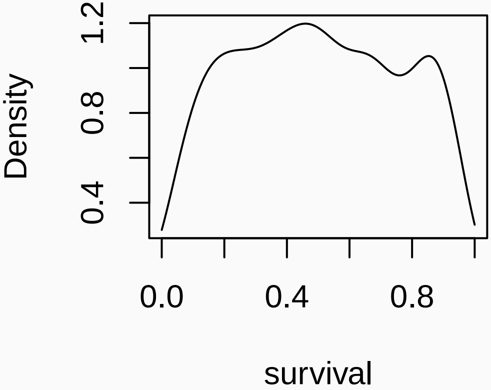


Logistic regression

Unreasonable $\text{logit}(\phi) = \beta \sim N(0, 10^2)$



Reasonable $\text{logit}(\phi) = \beta \sim N(0, 1.5^2)$



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