

DESIGN OF AN RESONANT WIRELESS POWER TRANSMISSION SYSTEM USING PRINTED INDUCTORS

L. M. D. Pereira¹, U. C. Resende¹, S. T. M. Gonçalves¹, M. M. Afonso¹, C. Vollaire² and P. M. Pereira¹

¹CEFET-MG, Amazonas Av.7675, Belo Horizonte, Minas Gerais, Brazil,
lucasmdpereira@gmail.com, resendeursula@des.cefetmg.br, sandro@div.cefetmg.br,
marciomatias@des.cefetmg.br, asal@des.cefetmg.br

²Laboratoire AMPERE - UMR 5005, EC-INSa Lyon, France, christian.vollaire@ec-lyon.fr

This paper presents the conception and the analysis of a simple, compact and low cost resonant system for wireless power transmission. The system design is performed considering a geometry of copper coils printed on FR4 substrates in order to obtain good efficiency values in the energy transfer process. The coils inductance and resistance are determined using a proposed approach on Maxwell's method. Measured results and that obtained from a commercial software are presented to show the proposed approach accuracy and system functionality and efficiency.

Introduction

In 1899, Nikola Tesla arrives in Colorado Springs with the goal of establishing a laboratory for his new big ideas. Using a huge transformer, known today as Tesla Coil, he was able to light 200 lamps, located 42 kilometres away [1]. Currently, several devices exchange information and commands using wireless transmission, but all these devices still require a physical connection to be fed with electricity, either through cables or through batteries. The advance use of so-called "electro portables" creates a new necessity of power transmission, allowing these devices to be truly portable. Currently, one of the most investigated ways to transfer energy wirelessly with the purpose of feeding devices at a considerable distance from the source is the strong resonant coupling. The feasibility of this technology was demonstrated in 2007 by [2] when a wireless power transmission system, constituted of two self-resonant coils, was able to transfer 60W with 40% of efficiency and distance of 2m using a 9.9 MHz resonance frequency. Actually the main challenges of this technology are to increase the efficiency and transmission distance and to design compact systems. Coils printed over different kind of dielectric substrate has been one the most investigated kind of compact wireless power transmission system to supply small loads [3]. The main advantages of this kind of system are the low cost, the simplicity of construction and the ease of to design different coils geometries.

In this work the technology developed in [1] is used to design a wireless power transmission system constituted of planar and square copper coils printed on dielectric substrates of FR-4. The design was carried out in order to obtain a compact and efficient system and low operation frequencies. Additionally an approach of the Maxwell method [4] is presented in order to accurately determine the coils inductance and resistance.

Coils Parameters

The design of a wireless system for efficient energy transfer requires that some important parameters are carefully determined. In addition to the system efficiency and the amount of power transferred, for the system approximation by a circuit model, it is necessary to obtain the coil parameters R , L and C . Through these parameters, we can get the quality factor, Q , which is dimensionless quantity and is directly associated with the coil ability to generate electromagnetic field. Theoretically, there is no limit to the value of this factor, but it is very difficult to get values greater than 100, especially at low frequencies, Q values above 10 are already considered satisfactory. The Q factor can be determined using different equations depending on the geometry of the analysed circuit under analysis. The coil investigated in this work can be represented by a RLC series circuit, however as the natural capacitance of the coils have a small value it is neglected. Then Q factor is calculated using (1) considering a RL series approach, as illustrated in Fig. 1.

$$Q = \frac{\omega \cdot L}{R}. \quad (1)$$

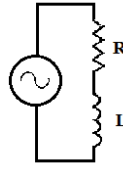


Fig 1. Circuit approach and respective Q factor for a RL model.

Inductance

The calculation of the coil's total inductance, L , is divided into two parts. The first is the calculation of the mutual inductance between each turns of the coil, which is performed using the Maxwell's method [4]. This method produces accurate mutual inductance values for two turns, being necessary simply performing a sum of all turns combined two by two to obtain the coil total mutual inductance. For wired coils, equation (2) and (3) can be used to find two turns mutual inductance, where a and b are the values of radius of each turns, μ_0 is the vacuum magnetic permeability and $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively. In the case of a conventional wired coil, $a = b$. However for planar coils there are different values of radius for each turn, as can be seen in Fig. 2. In addition to this fact, a square coil, obviously, do not have a constant radius. In order to overcome the differences between wired and planar coils and use (2) and (3) to determinate the planar turn mutual inductance some approach can be adopted. It is possible establish equivalents values for a and b , as indicated by (4), where d_o is the length of one side of the square that represent the turn for which the mutual inductance is calculated, as illustrated in Fig. 2.

$$M_{ij} = \mu_0 \cdot \sqrt{a \cdot b} \left\{ \left(\frac{2}{k} - k \right) \cdot K(k) - \frac{2}{k} \cdot E(k) \right\}, \quad (2)$$

$$k = \left(\frac{4ab}{(a+b)^2} \right)^{1/2}, \quad (3)$$

$$(a, b)_{eq} = d_o / \pi. \quad (4)$$

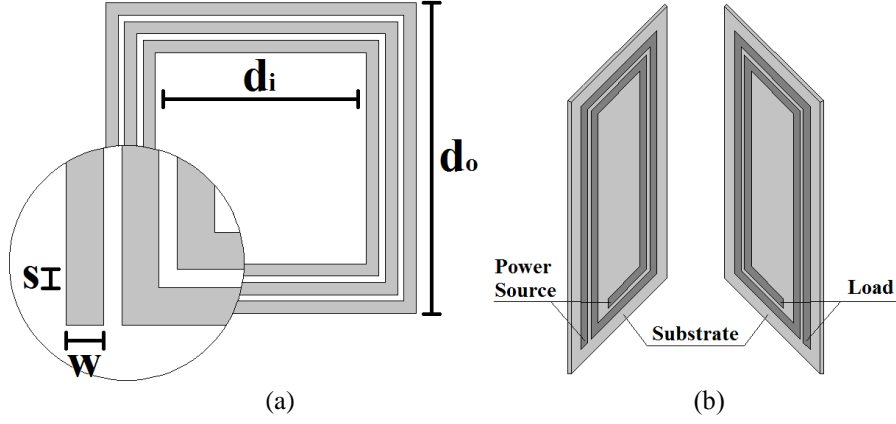


Fig 2. (a) Coils geometry. (b) Wireless System.

The second part of the calculus of the coil's total inductance consists in determining the self-inductance of each turn of the coil. The self-inductance of wire turns may be approximated by (5) [5]. Therefore it is necessary to determine an equivalent radius for planar square printer coil in order to use (5). Once the cross section of the planar square printer coils is rectangular, equalling the areas of the circle and the rectangle it is possible to obtain an equivalent radius, r_{eq} , according to (6), where w is the track width and t is the coil thickness, as illustrated in Fig. 2.

$$L_{self}(a, r) = \mu_0 \cdot a \left(\ln \left(\frac{8 \cdot a}{r} \right) - 2 \right), \quad (5)$$

$$r_{eq} = \sqrt{w \cdot t / \pi}, \quad (6)$$

The total inductance, L , of the coil is given by the sum of self-inductance of all turns plus the mutual inductance of all turns taken in account two by two.

Resistance

To calculate the resistance, three phenomena are observed: Resistance characteristic (R_{cc}), non-uniformity current effect and proximity effect. The resistance characteristic uses the concept of resistivity of the material and can be calculated by (7), where ρ is the material resistivity (in Ωm), l_c the total length of the coil, w and t track width and thickness, respectively. In this work l_c is calculated using (8) together geometric parameters presented in Fig. 2.

$$R_{cc} = \frac{l_c}{\sigma \cdot w \cdot t}, \quad (7)$$

$$l_c = 4 \cdot (d_o - w) + \sum_{i=1}^n 4 \cdot d_o - n \cdot (w - 2 \cdot s), \quad (8)$$

where n is the number of turns of the coil.

The effects of non-uniformities are related to non-uniform current distribution in the conductor surface, this effect influences the total resistance of the coil and tends to be most significant at high frequencies (ω). The resistance characteristic plus the non-uniformity effects is called skin resistance, R_{skin} , given by [6]:

$$R_{skin}(\omega) = \frac{\rho \cdot l_c}{w \cdot t_{skin}(\omega)} = \frac{\rho \cdot l_c}{w \cdot \delta \cdot \left(1 - e^{-\frac{t}{\delta}} \right)}, \quad (9)$$

where

$$t_{skin}(\omega) = \delta \cdot \left(1 - e^{-\frac{t}{\delta}}\right), \quad (10)$$

$$\delta = \sqrt{\frac{2}{\omega \cdot \mu_0 \cdot \sigma}}. \quad (11)$$

The next step is finding the resistance due to the proximity effect. The increase in the resistance due to the proximity effect occurs owing to Lenz's law where eddy currents are created in adjacent turns due to the magnetic flux of his neighbour track, the current generated has the opposite direction of the original current causing an additional resistance to current flow introduced into coil. The Fig. 3 demonstrate the iteration of these fields and resulting currents.

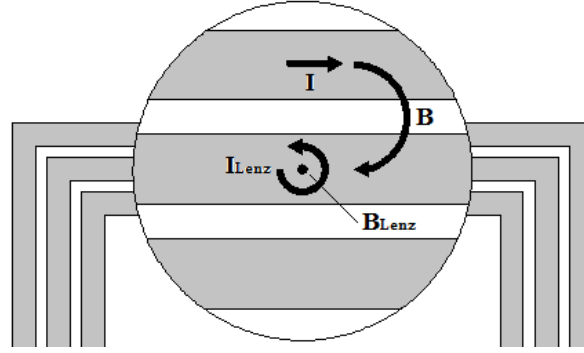


Fig 3. Eddy currents effect.

According to [7] the resistance due to the proximity effect can be calculated by the magnetic flux B that penetrates to the adjacent line. Equation (12) provides resistance proximity calculated from adjacent magnetic fields.

$$R_{prox} = \frac{l_c \cdot t \cdot \sigma \cdot \omega^2 \cdot \mu_0^2}{6 \cdot \pi^2 \cdot (w+s)^2} \left\{ \left(\frac{w}{2}\right)^3 - \left[\frac{w}{2} - \delta \left(1 - e^{-\frac{w}{\delta}}\right)\right]^3 \right\}. \quad (12)$$

The total resistance of the planar square printed coils is given by the sum of R_{skin} with R_{prox} .

$$R_{coil} = R_{skin} + R_{prox} = \frac{\rho \cdot l_c}{w \cdot \delta \cdot \left(1 - e^{-\frac{t}{\delta}}\right)} + \frac{l_c \cdot t \cdot \sigma \cdot \omega^2 \cdot \mu_0^2}{6 \cdot \pi^2 \cdot (w+s)^2} \cdot \left\{ \left(\frac{w}{2}\right)^3 - \left[\frac{w}{2} - \delta \cdot \left(1 - e^{-\frac{w}{\delta}}\right)\right]^3 \right\} \quad (13)$$

Simulation results

To validate the proposed approach (PA), the Advanced Assign System software (ADS) was used. Three configurations were tested, changing the dimensions shown in Fig. 2. The frequency range was set between 400 kHz and 1.4 MHz and divided between 11 points (one point for every 100 kHz). The numerical method used by ADS is based on the momentum method and produces accurate results. The Table I show the dimension used for each (PT01, PT02 and PT03) coil investigated. A comparison between the simulation results from ADS and PA are presented in Figs. 4, 5 and 6. For obtain accuracy values in ADS the mesh frequency used was 2.8 MHz (two times the maximum frequency simulated), the density mesh was set to 200 cells pour wavelength (ten times more the minimum recommendation) an edge mesh is generated and no mesh reduction was made.

TABLE I
DIMENSION OF SIMULATED COILS

Parameters	PT01	PT02	PT03
d_0 (mm)	30	30	130
d_i (mm)	8	6.6	12
n (turns)	37	13	10
w (mm)	0.150	0.750	5
s (mm)	0.150	0.150	1

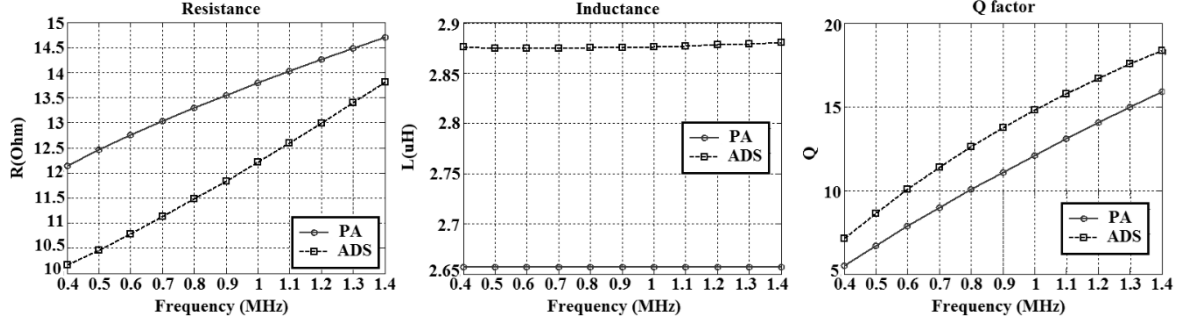


Fig 4. Numerical results from ADS and PA for the prototype PT01

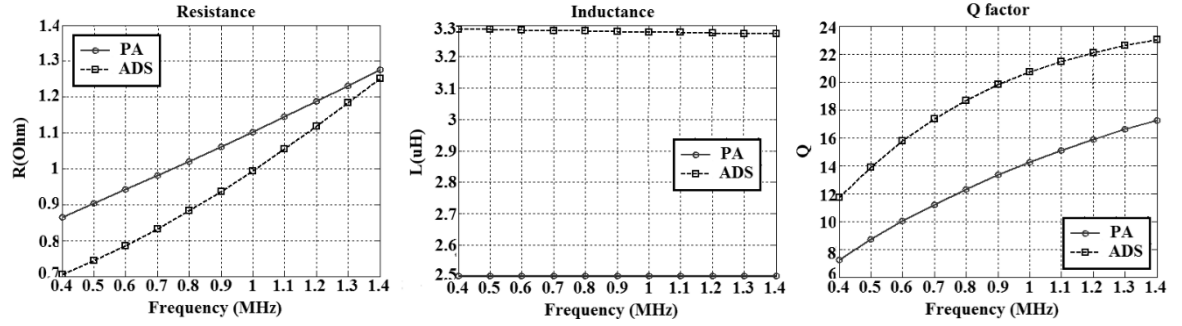


Fig 5. Numerical results from ADS and PA for the prototype PT02

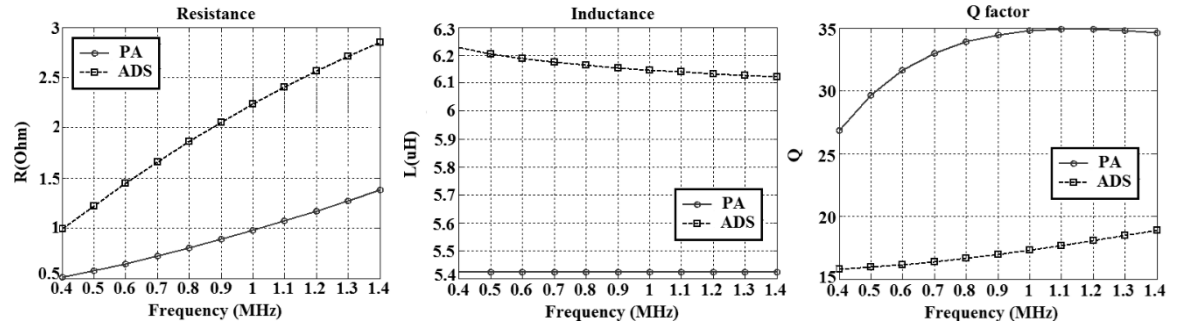


Fig 6. Numerical results from ADS and PA for the prototype PT03

As can be observed the values of resistance and inductance obtained by ADS and PA for three prototypes are closed and present the same behavior as the frequency increases. These facts demonstrate the PA adequacy to obtain, in as fast and simple way, R and L parameters for a planar square printed coil. The prototypes PT01, PT02 and PT03 have significant amounts of Q factor, which increase as the frequency increases.

Geometry optimization

Upon successful demonstration of adequacy of the PA, the conditions to optimize the dimensions of the coil in order to find higher values of Q was reached. However, some technical limitations for the construction of coils must be observed and included as the initial parameters in the optimization process. Because of some limitations in the coil manufacturing process it is not possible to construct coils tracks or gaps less than 1mm, and the larger dimension of the planar coil can not be greater than 150 mm. Subject to these conditions, a routine in MATLAB was developed, varying dimensions w , s and d_i to find high Q -values. This routine takes into account 1345 different geometrical configurations, compares all Q values obtained and return the values of w , s and d_i which lead to the higher value of Q . Table II show the dimensions found and used to build the prototype 04 (PT04). Coils with a high Q -value is a very important characteristic to guarantee a good magnetic couple between the them and therefore the high rate of energy transfer.

TABLE II
DIMENSION OF PT04

Parameters	PT04
d_0 (mm)	120
d_i (mm)	10
n (turns)	9
w (mm)	3
s (mm)	3

After coil manufacturing, the values of R , L and Q were simulated using ADS and PA and measured for comparison. Frequency range for the analysis was from 400 kHz to 1.4 MHz and the results obtained are presented in Fig. 7. Table III present the mean absolute error for numerical results calculated in relation to measured ones, according (14), where n is the number of points considered.

$$Error = \frac{\sum_{i=1}^n \frac{|Measured(i) - Expected(i)|}{Measured(i)}}{n}. \quad (14)$$

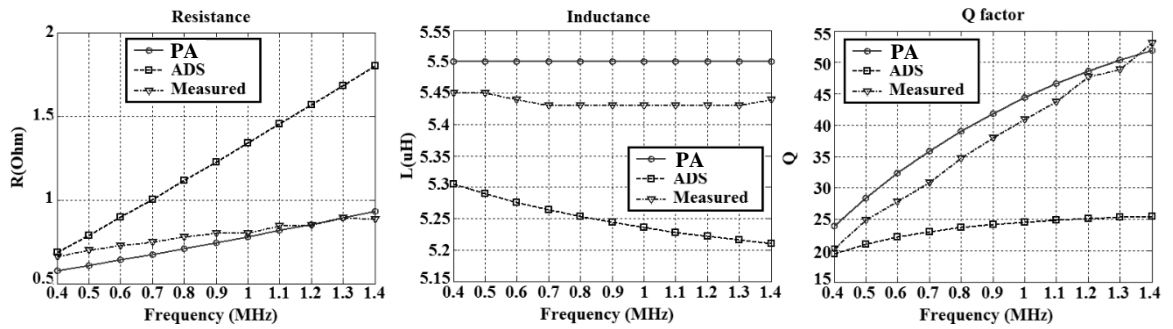


Fig 7. Numerical results from ADS and PA and mensuared results for the prototype PT04

TABLE III
RELATIVE MEAN ABSOLUTE ERROR (%)

Parameters	Simulated	Calculated
L	3.47	1.20
R	53.28	5.57
Q	33.02	9.54

As can be observed the PA results produces enhanced results than ADS one. Is important to observe the behavior of the curves, the PA curves follows the measured curve much more closely. By the results it is possible to see that the coil resistance and Q factor increases as the frequency

increases and the inductance is practically constant. It is important to note that for the whole frequency range investigated, the Q factor values obtained for the PT04 prototype are larger than the values found for the prototypes PT01, PT02 and PT03. These results validate the optimization process used.

Resonance frequency

The PT04 presented natural resonance in the MHz range, as can be verified in Fig. 8 (a). An external capacitor is solder in parallel at the coils feed for reduce this resonance frequency to kHz range. Low frequencies are better because is possible to obtain high values of transferred power and power source operating in a kHz frequencies have strong values of efficiency, being possible to elevate overall efficiency system. It was used a 10 nF capacitor to reduce the resonance frequency to 724 kHz, as can be observed in Fig. 8(b), so the measuring range was maintained between 400 kHz to 1.4 MHz.

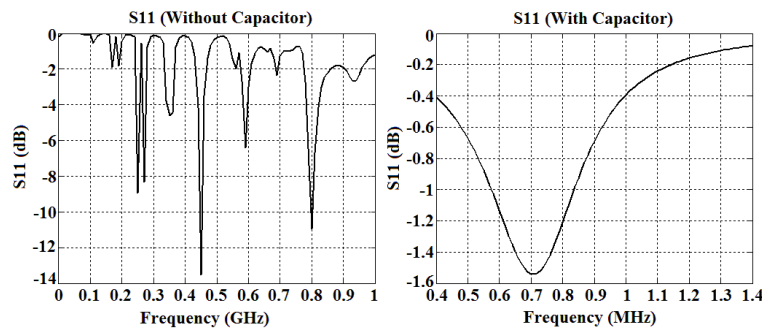


Fig 8. Return loss (S11) simulated for PT04 without capacitor (left) and with a 10 nF capacitor (right).

Efficiency

A wireless power transmission system, constituted of two PT04 coils each connected in parallel with a 10 nF capacitor, was investigated. The transmitting unit was fed by signal generate adjusted to a frequency of 724 kHz. The receiver coil was connected to a 100 Ω load and the distance between the coils was varied from 24 to 120 mm. The power input was calculated from the relation between voltage and current and the output power was calculated using the relation between voltage and load resistance. The Table. 3 summarizes the efficiencies found.

TABLE IV
COMPARISON BETWEEN MEASURE, CALCULATED AND SIMULATED VALUES

Distance between coils (mm)	24	36	48	60	72	84	96	108	120
Efficiency (%)	57.89	47.11	34.32	21	14.24	8.28	5.22	2.96	1.89

The values of efficient found in Table III evidence the electrical viability of the system. One of important features of printed coils is you small thickness, being possible to adapt in diverse situations.

Conclusion

This article presents an empirical method to determine, in a fast and simple way, the parameters necessary to the design planar square printed coils. The initial objective was submit equations for encounter the R and L parameters in square planar printed coils, but the equations obtained can be easily extend to any coil geometry. Once determined R and L coil parameters is possible to find a

geometry that produces a high Q factor and therefore a strong magnetic coupling between the coils. The results obtained demonstrated the accuracy and the reliability of the proposed approach that was used to design of a wireless power transmission system that reaches expressive values of efficiency.

Acknowledgment

This work was partially supported by FAPEMIG, CAPES, CNPq and CEFET-MG.

References

- [1] V.J. Ivan, N. Tesla, the man time forgot, IEEE Potentials, Vol.9, pp.53-54, 1990.
- [2] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljacic, Wireless Power Transfer via Strongly Coupled Magnetic Resonances, Science, Vol.317, 2007
- [3] J. J. Casanova, Z. N. Low, J. Lin, and R. Tseng, Transmitting coil achieving uniform magnetic field distribution for planar wireless power transfer system, in Proc. IEEE Radio Wireless Symp., pp. 530-533, 2009.
- [4] J. C. Maxwell, A Treatise on Electricity and Magnetism, Oxford: Clarendon, 1892.
- [5] C.M. Zierhofer, E.S. Hochmair, Geometric Approach for Coupling Enhancement of Magnetically Coupled Coils, IEEE Transactions on Biomedical Engineering, pp.708-714, 1996.
- [6] U. Jow, M. Ghovanloo, Design and Optimization of Printed Spiral Coils for Efficient Transcutaneous Inductive Power Transmission, IEEE Transactions on Biomedical Circuits and Systems, Vol. 1, pp.192-202, 2007.
- [7] R. Chan, J. Guo, Analysis and Modeling of Skin and Proximity Effects for Millimeter-Wave Inductors Design in Nonoscale Si CMOS, Proceedinfs of the 9th European Microwave Integrated Circuits Conference, pp. 13-16, 2014.