Prediction of ozone level in Boston

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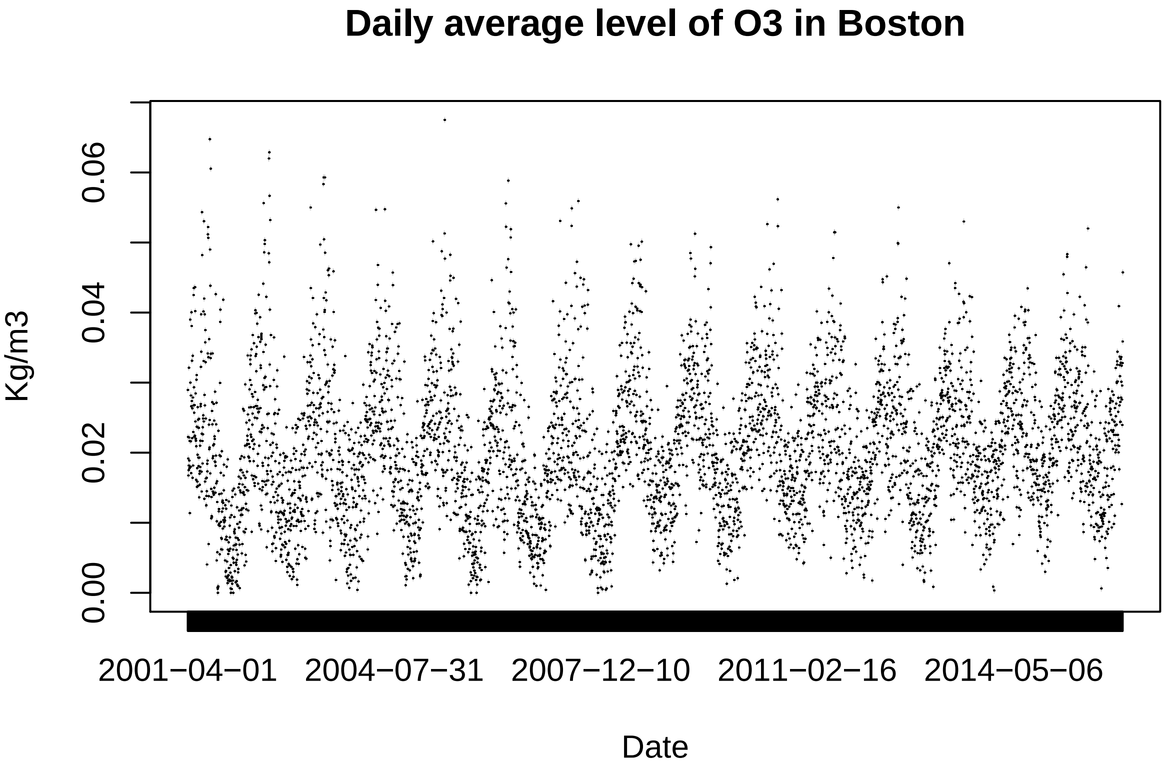
TO DO

1. Week model: the same analysis
2. Escrever melhor o texto em algumas transições.
3. Alguma combinação de modelo?
4. Residual analysis with adjusted error, not predictive one.

Load and visualize



Data treatment

We noticed that some days do not exist in the dataset, for example, the day August 31, 2001 does not have information in the dataset.

```
##       X   City         State Site.Num Date.Local  O3.Mean
## 148 148 Boston Massachusetts       42 2001-08-28 0.024583
## 149 149 Boston Massachusetts       42 2001-08-29 0.015000
## 150 150 Boston Massachusetts       42 2001-08-30 0.022333
## 151 151 Boston Massachusetts       42 2001-09-01 0.021958
```

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```
## 152 152 Boston Massachusetts       42 2001-09-02 0.018750
## 153 153 Boston Massachusetts       42 2001-09-03 0.028708
```

Also, there is duplicated days, as June 9, 2002:

```
##       X   City         State Site.Num Date.Local  O3.Mean
## 412 412 Boston Massachusetts       42 2002-06-08 0.022917
## 413 413 Boston Massachusetts       42 2002-06-09 0.036190
## 414 414 Boston Massachusetts       42 2002-06-09 0.037000
## 415 415 Boston Massachusetts       42 2002-06-10 0.023389
```

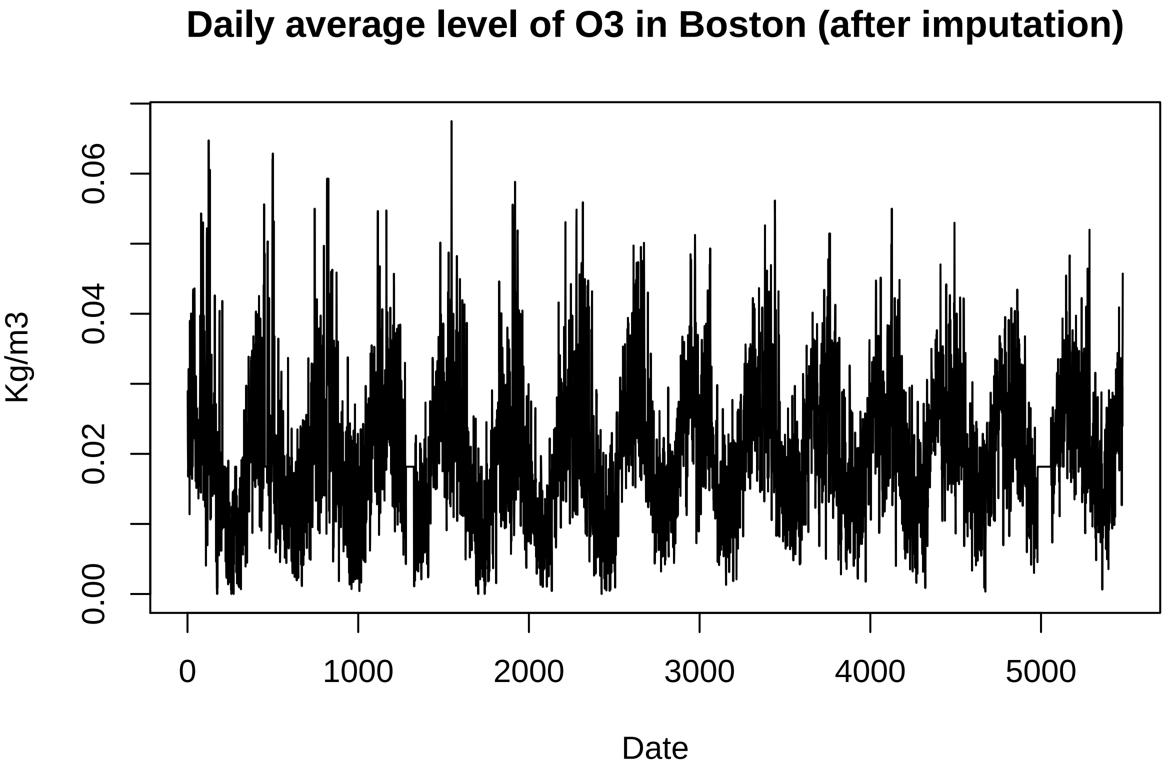
The duplicated one is easier to deal, but the missing values are harder. First we calculate the mean value between the duplicated.

The rate of missing values is almost 5% of the dataset.

```
## [1] 0.04453367
```

So as to solve that problem, we make a knn imputation using the month ($k = 30$)

```
o3.clean <- knn.impute(as.matrix(o3.ts), k = 30)
o3.clean <- as.ts(o3.clean)
plot(o3.clean, main = 'Daily average level of O3 in Boston (after imputation)',
                xlab = 'Date', ylab = 'Kg/m3')
```



We also separate our data between training and test because of modelling best practices.

```
o3_train = o3.clean[1:(length(o3)[1] - 365),]
o3_test = o3.clean[(length(o3)[1] - 365 + 1):length(o3)[1],]
```

Models: case 1

Now we develop some models using the train data.

The metric to compare is the Mean Absolute Error (MAE) in the predictions:

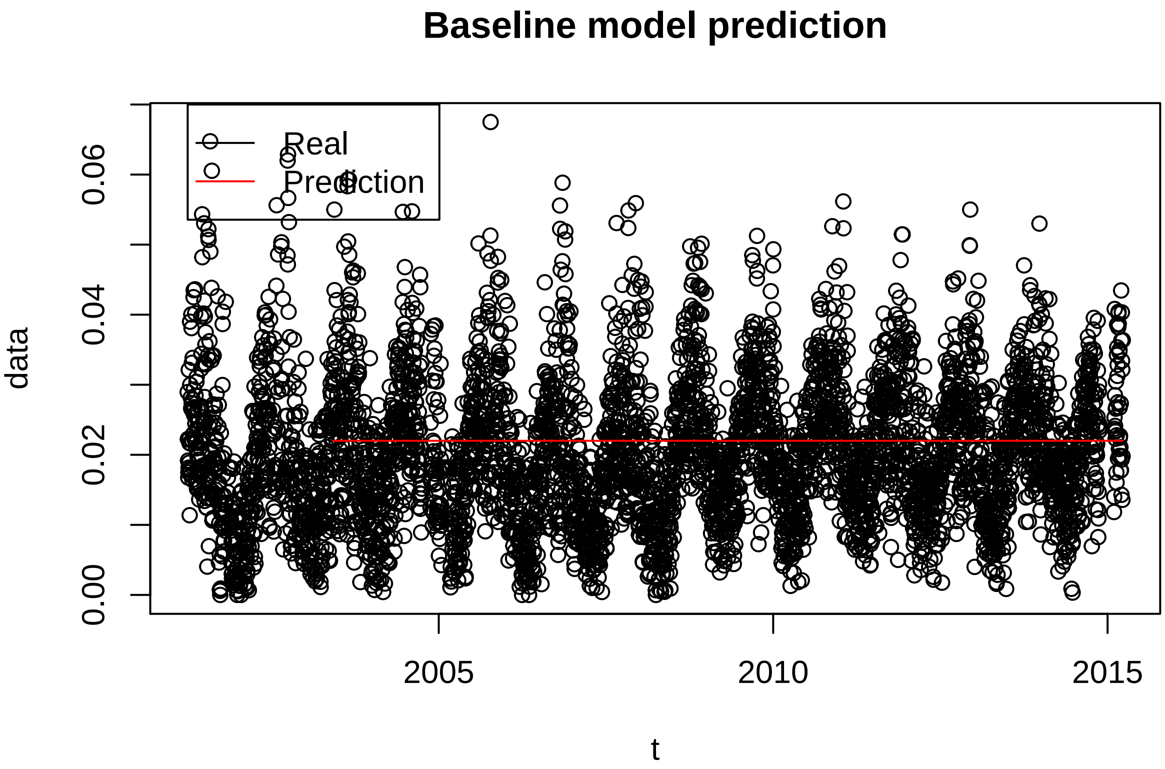
```
mae <- function(ytrue, ypred)
{
    return(mean(abs(ytrue - ypred)))
}
```

We will use `rollapply` in order to calculate the error, considering the last two years to predict one week forward.

Baseline Model

We will do the naive forecast to the baseline model.

```
## [1] 0.008015428
```



Decompose

First of all we make a seasonality test using Kruskal-Wallis. Actually it tests whether samples originate from the same distribution. We can organize it to be samples for each corresponding day. We compare two different frequencies: monthly and yearly. The second one showed the smallest p-value, in particular less than 0.05. For that reason, we will use 365 as the seasonality.

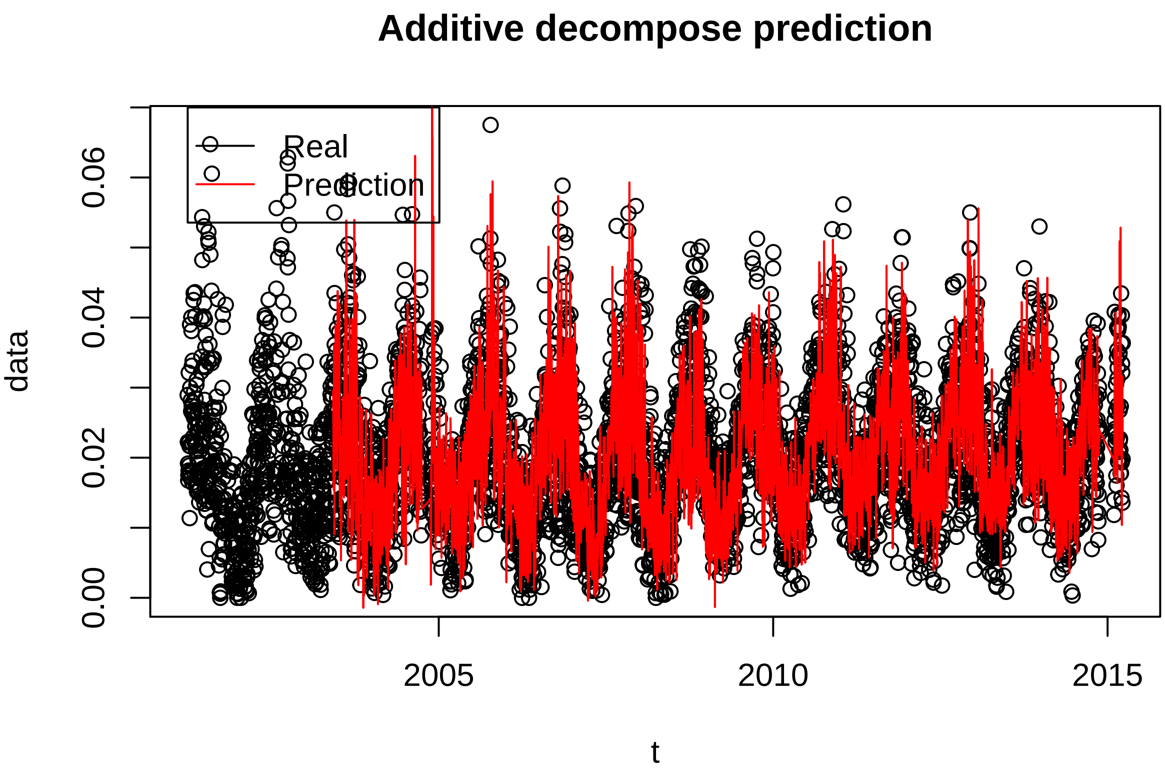
```
##
##  Kruskal-Wallis rank sum test
##
## data:  o3_train and g
## Kruskal-Wallis chi-squared = 32.983, df = 30, p-value = 0.3233
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  o3_train and g
## Kruskal-Wallis chi-squared = 2122.9, df = 364, p-value < 2.2e-16
```

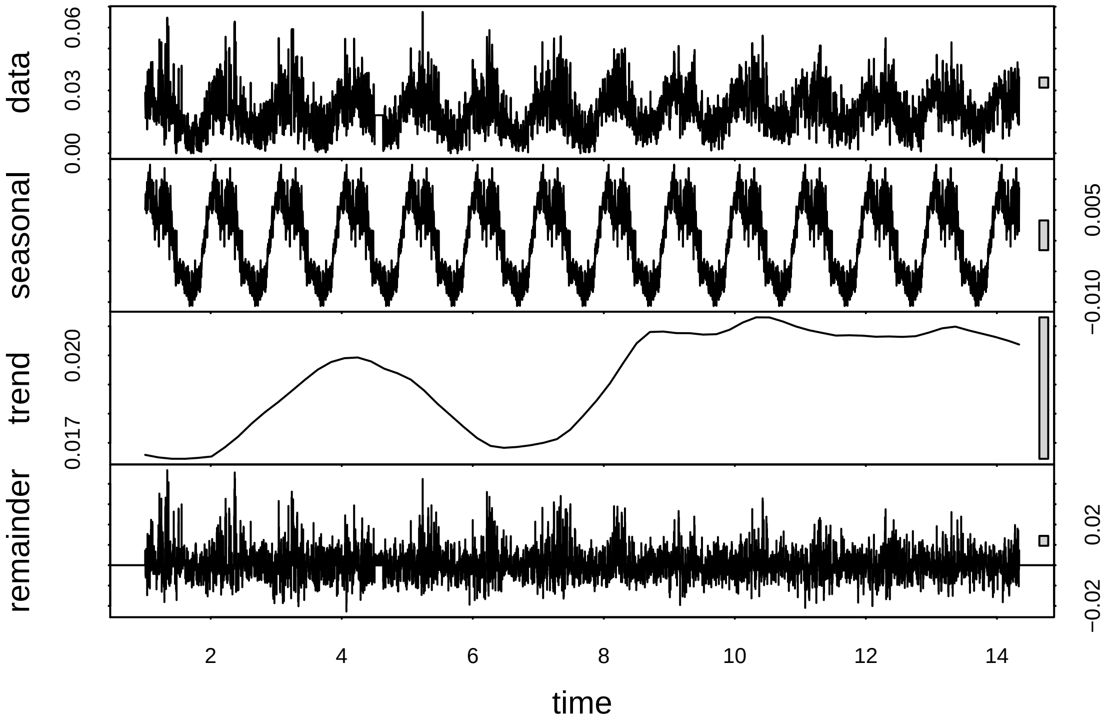
Additive model

First we analyse the MAE.

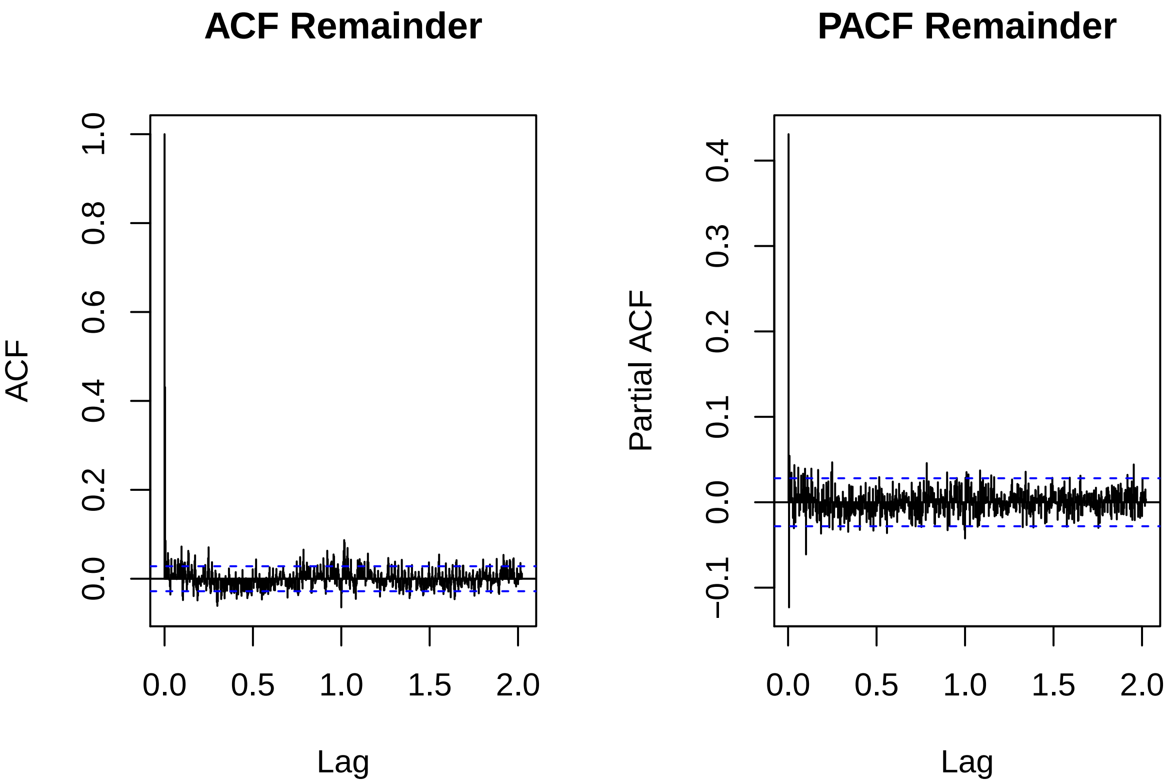
```
## [1] 0.007963981
```



We also can fit the model using `t.window` and analyse the reminder of the method.



The ACF and the PACF of the reminder:

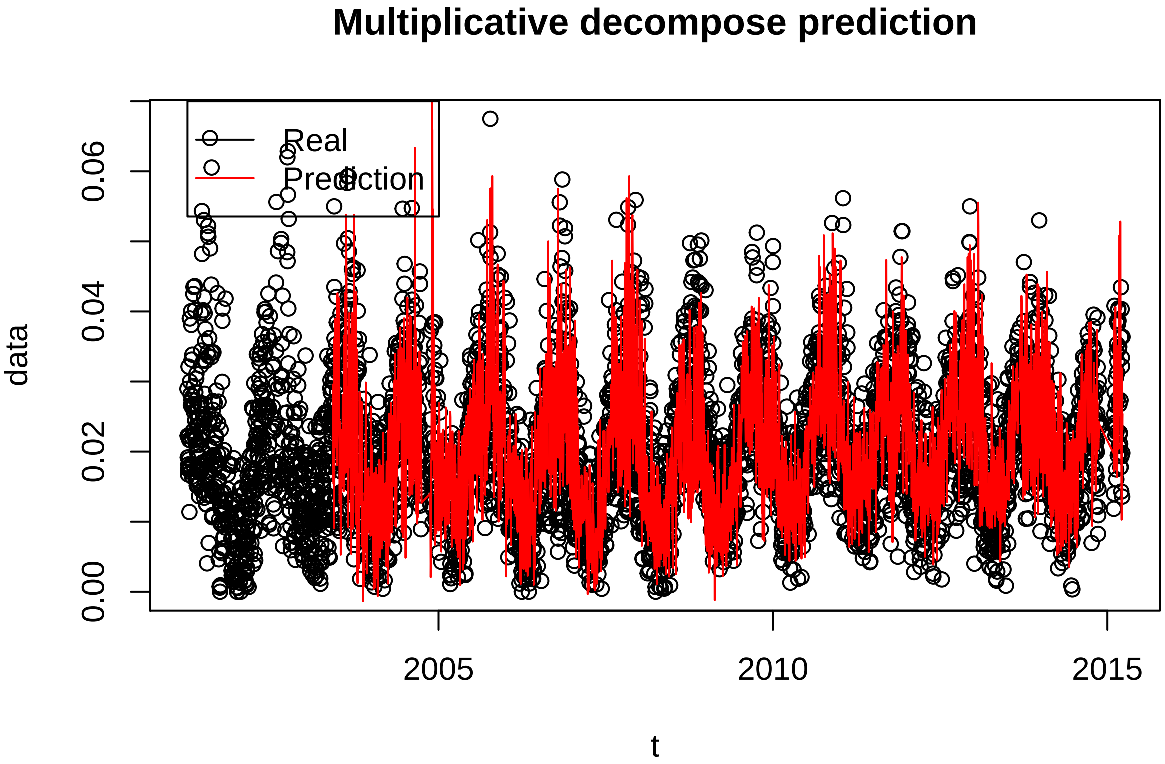


We see that there are a big spike when lag = 365. It seems not so good for a reminder. We could fit an ARMA model in this reminder yet.

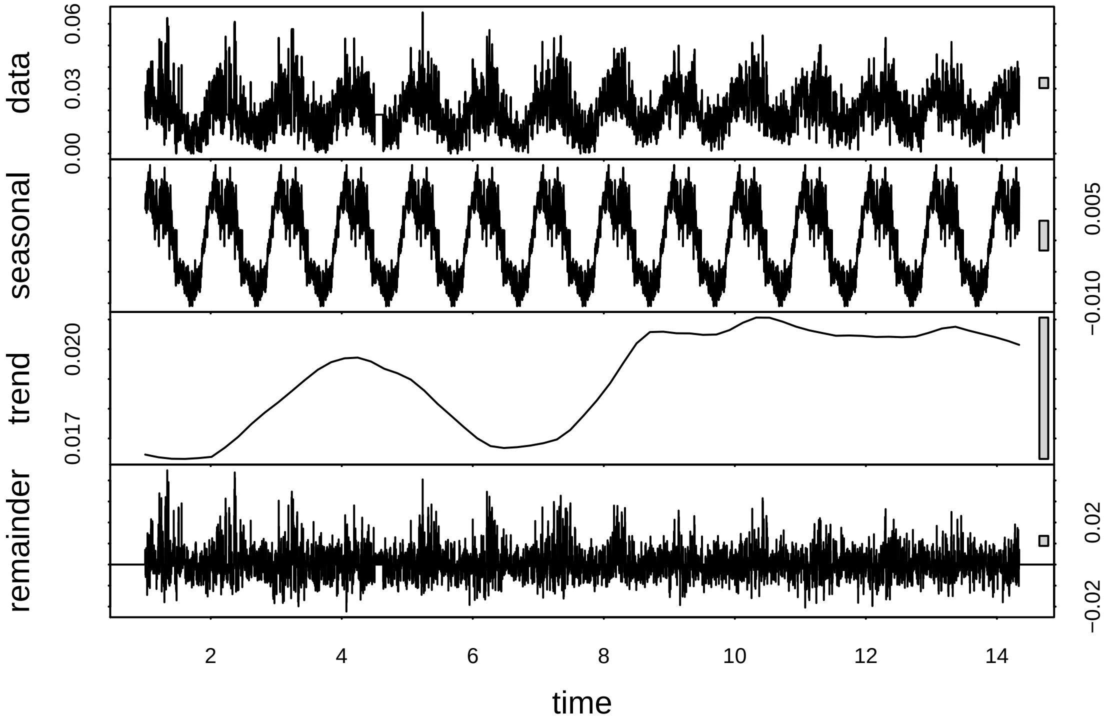
Multiplicative model

First we analyse the MAE.

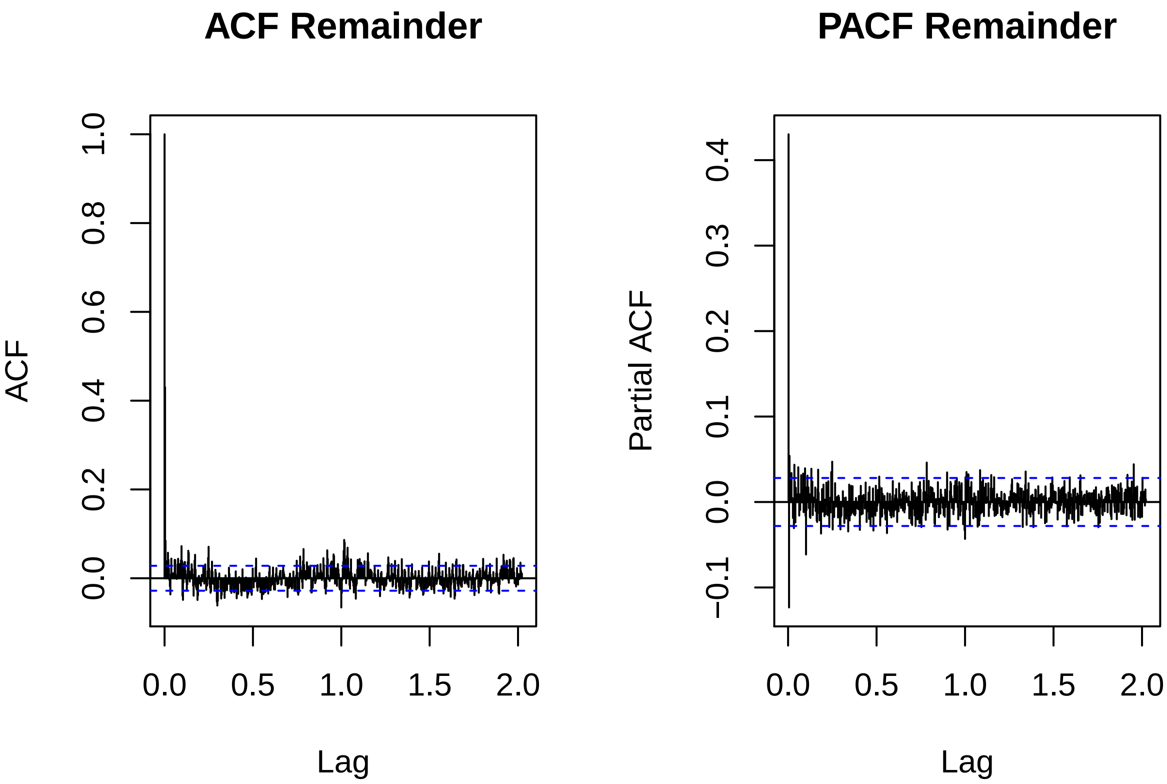
```
## [1] 0.00795386
```



We also can fit the model using `t.window` and analyse the reminder of the method.



The ACF and the PACF of the reminder:

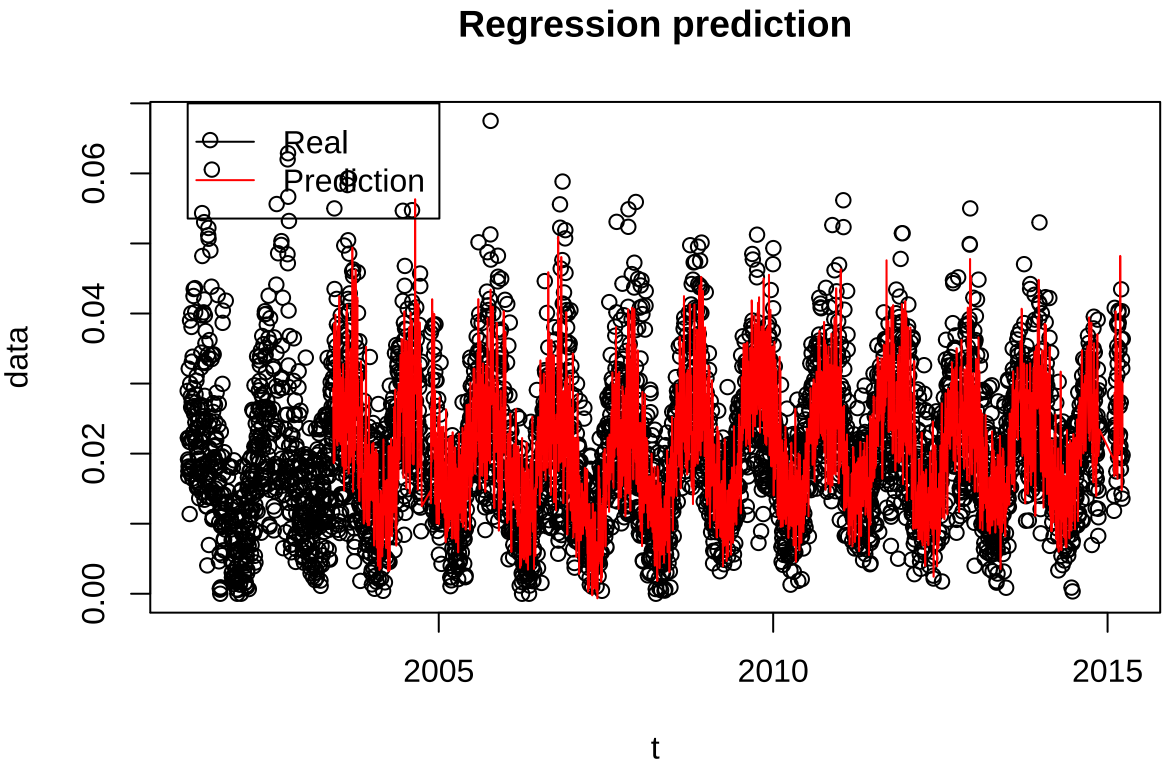


We see that there are a big spike when lag = 365. It seems not so good for a reminder. We could fit an ARMA model in this reminder yet. The same problem as before.

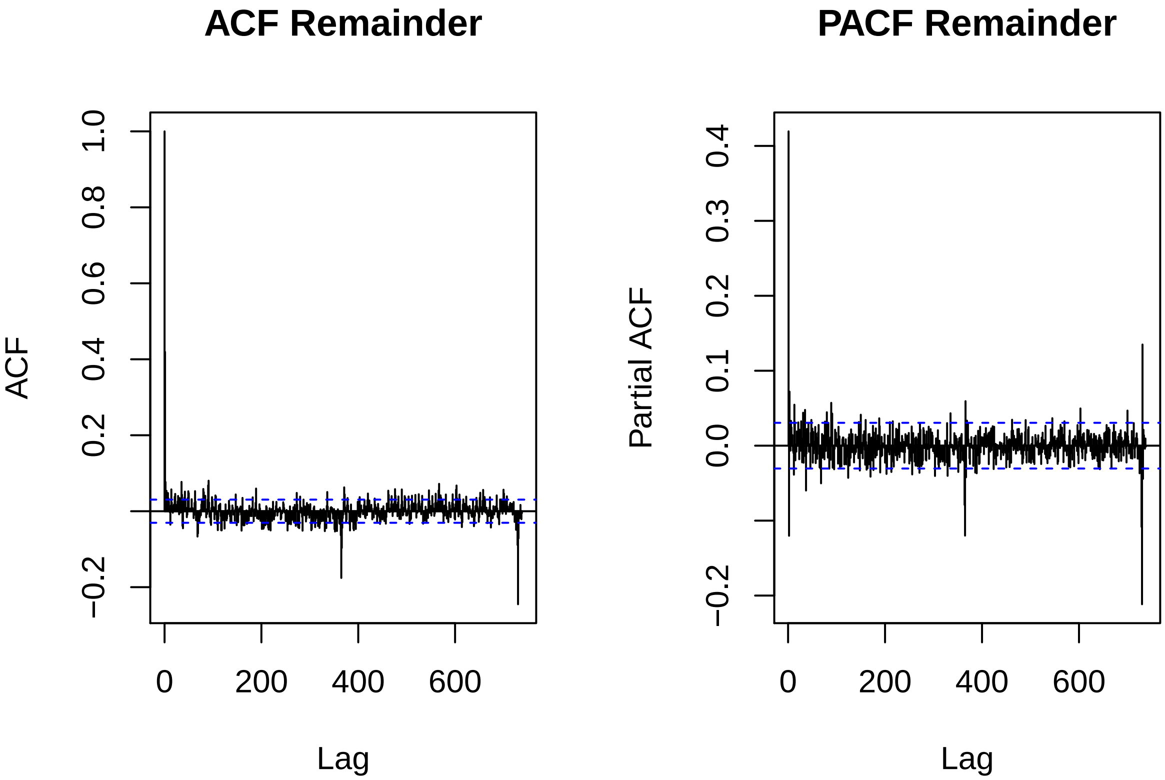
Regression

We tested for seasonalities, and we settled with 365. So we will fit a regression model with the seasonality dummies. We see the MAE:

```
## [1] 0.007518997
```



Now we analyse the residuals. The analysed residuals will be those from each model in the rolling window. It's valid because every model has (the assumption of) white noise with the same variance. Let's see the ACF and PACF:



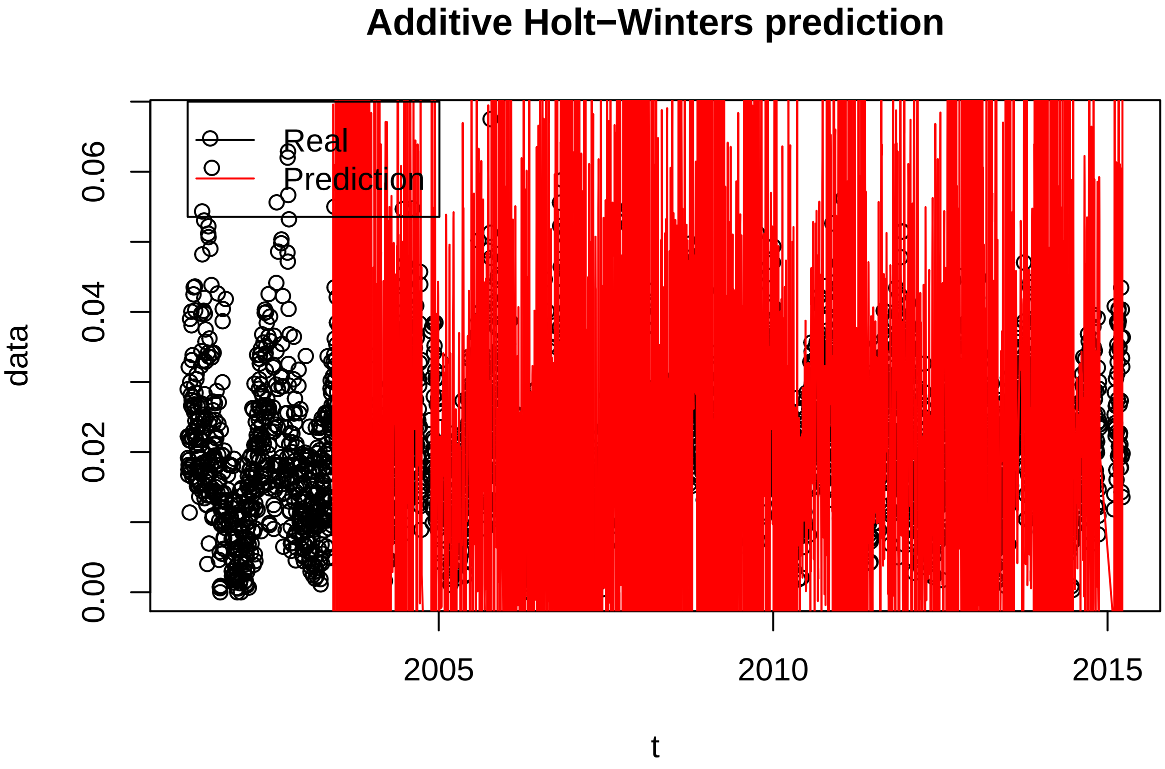
As before, wee see spikes in lag $= 365, 730$. We expect a WN to not have this.

Holt-Winters

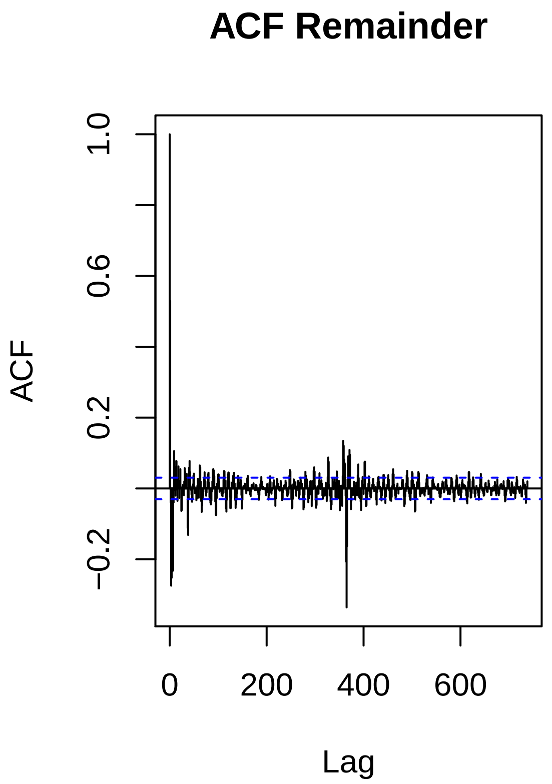
Now we will try Holt-Winters models. In fact, because of apparently seasonality, we will consider complete Holt-Winters models, both additive and multiplicative.

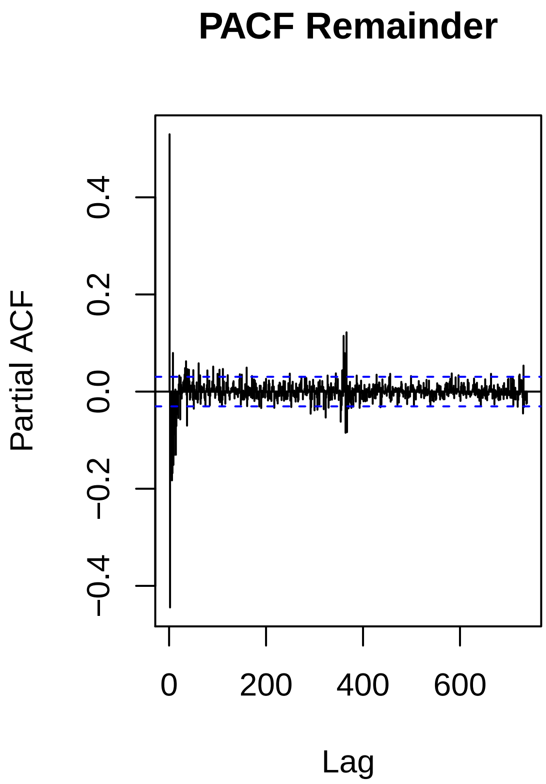
Additive

```
## [1] 0.03493751
```



MAE is not so good, nor is the resulting graph. HW tends to “explode”. We have seem better results. Let’s analyse the residuals:

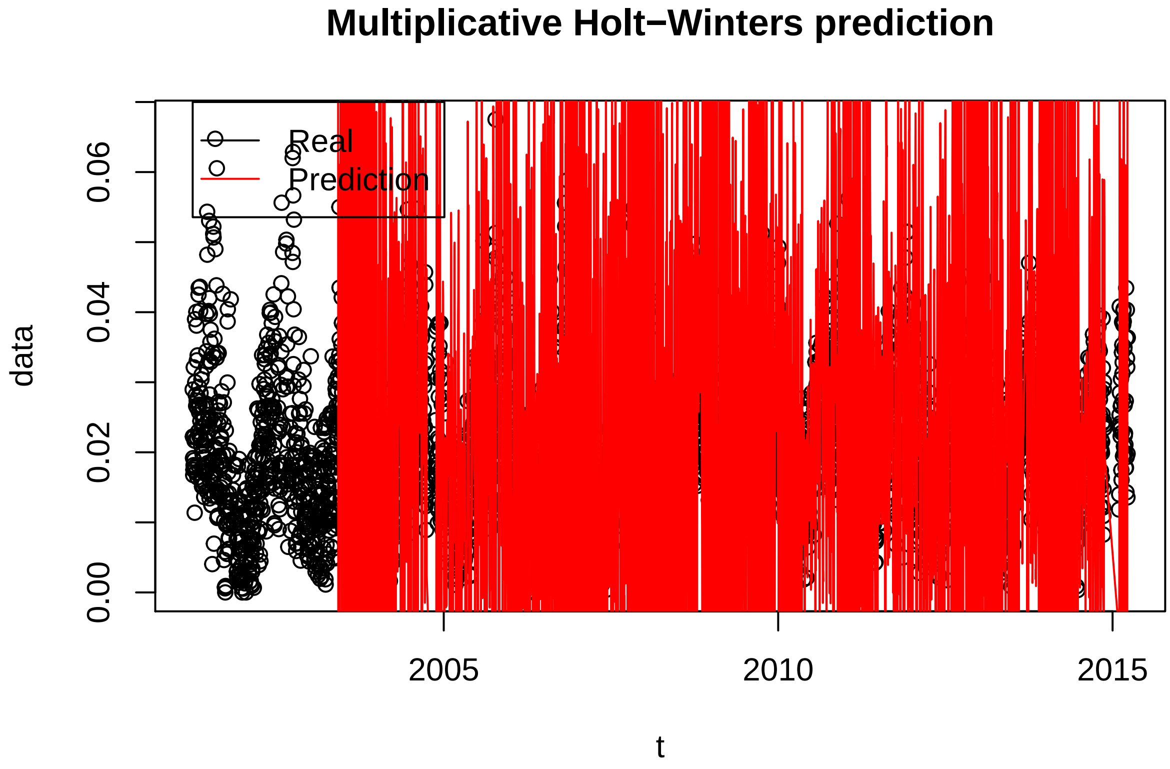




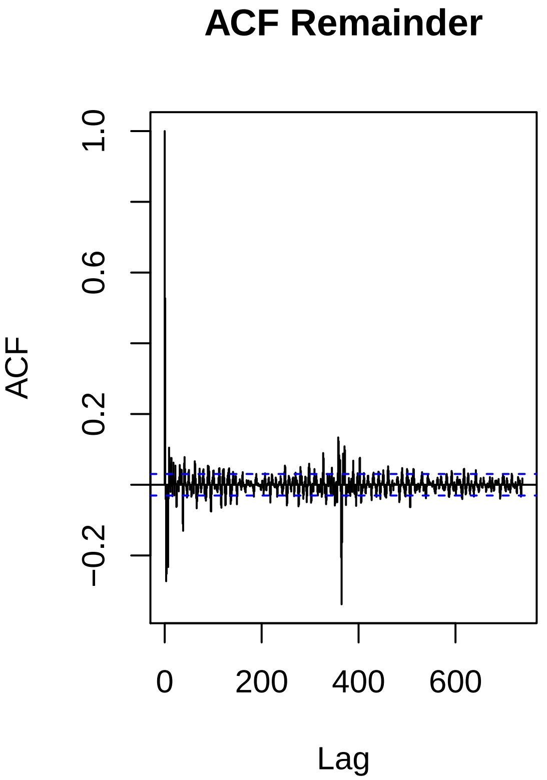
We see the same problems as before: high correlated lag = 365, evidence of this not beeing a WN.

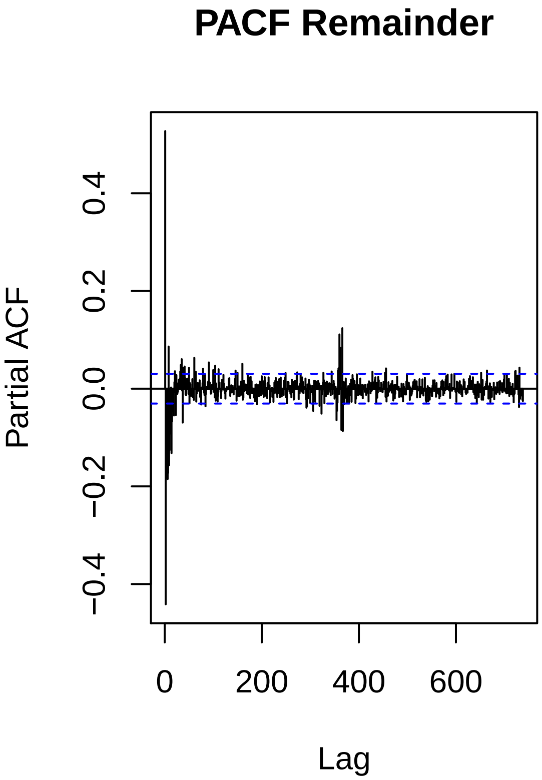
Multiplicative

```
## [1] 0.0348543
```



We see also a not so good MAE and graph. Let's analyse the residuals:

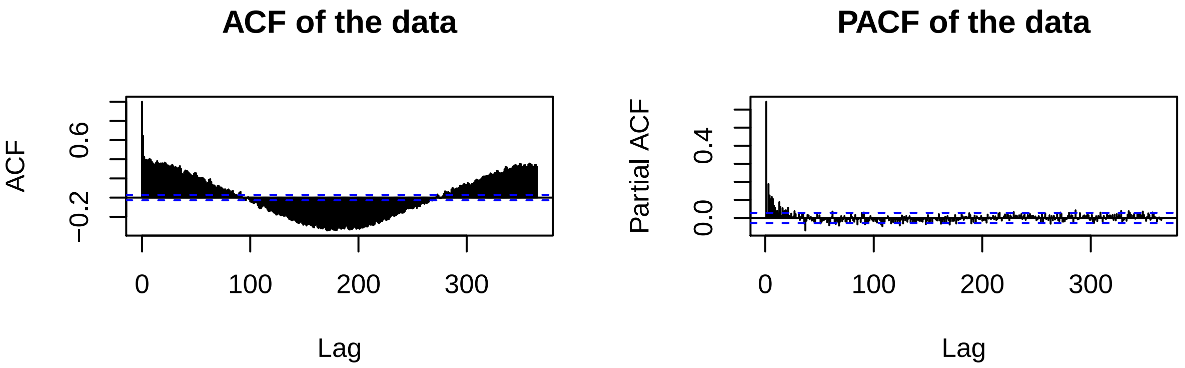


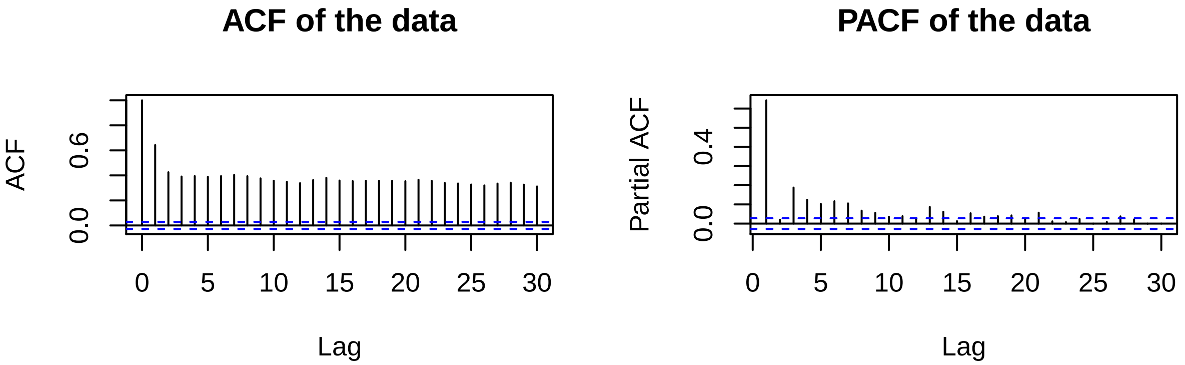


Same problems as before. Holt-Winters doesn't appear to work well in this case.

ARMA

We can see the ACF and PACF:





Based on these graphs, we see both graphs has a exponentially decay, the first after the $p-q=1$ or $p-q=2$. In order to identify the model, we will compare the adjusted ARMA models with different p and q. First we simply fit it to look at the Akaike Information Criteria (AIC) and the significance of the parameters estimated.

The AIC measures the goodness of fit and the simplicity of the model into a single statistic. Generally we aim to reduce the AIC.

$$AIC = 2k - 2\ln(\hat{L}),$$

where $k = p + q + 2$ and $\hat{L}$ is the maximum value of the likelihood for the model.

```
##
## Call:
## arma(x = o3_train, order = c(2, 1))
##
## Model:
## ARMA(2,1)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0256895 -0.0050812 -0.0003623  0.0043066  0.0398679
##
## Coefficient(s):
##             Estimate  Std. Error  t value Pr(>|t|)
## ar1        1.4018803   0.0150148   93.367  < 2e-16 ***
## ar2       -0.4073756   0.0146509  -27.805  < 2e-16 ***
## ma1       -0.9294688   0.0059259 -156.849  < 2e-16 ***
## intercept  0.0001116   0.0000292    3.822 0.000132 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Fit:
## sigma^2 estimated as 5.396e-05,  Conditional Sum-of-Squares = 0.26,  AIC = -34030.29
```

```
##
## Call:
## arma(x = o3_train, order = c(3, 1))
##
## Model:
## ARMA(3,1)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0280340 -0.0049450 -0.0003378  0.0043396  0.0391002
##
## Coefficient(s):
##             Estimate  Std. Error  t value Pr(>|t|)
## ar1        1.436e+00   1.616e-02   88.837  < 2e-16 ***
## ar2       -5.972e-01   2.357e-02  -25.337  < 2e-16 ***
## ar3        1.537e-01   1.485e-02   10.346  < 2e-16 ***
## ma1       -9.060e-01   8.615e-03 -105.163  < 2e-16 ***
## intercept  1.531e-04   3.928e-05    3.897 9.75e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Fit:
## sigma^2 estimated as 5.282e-05,  Conditional Sum-of-Squares = 0.26,  AIC = -34132.4
```

```
##
## Call:
## arma(x = o3_train, order = c(3, 2))
##
## Model:
## ARMA(3,2)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.027737 -0.004941 -0.000356  0.004364  0.039341
##
## Coefficient(s):
##             Estimate  Std. Error  t value Pr(>|t|)
## ar1        1.2360037   0.0891579   13.863  < 2e-16 ***
## ar2       -0.3130929   0.1287091   -2.433 0.014992 *
## ar3        0.0686044   0.0422242    1.625 0.104213
## ma1       -0.7035439   0.0887237   -7.930 2.22e-15 ***
## ma2       -0.1896764   0.0832069   -2.280 0.022633 *
## intercept  0.0001719   0.0000448    3.837 0.000124 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Fit:
## sigma^2 estimated as 5.279e-05,  Conditional Sum-of-Squares = 0.26,  AIC = -34133.17
```

```
##
```

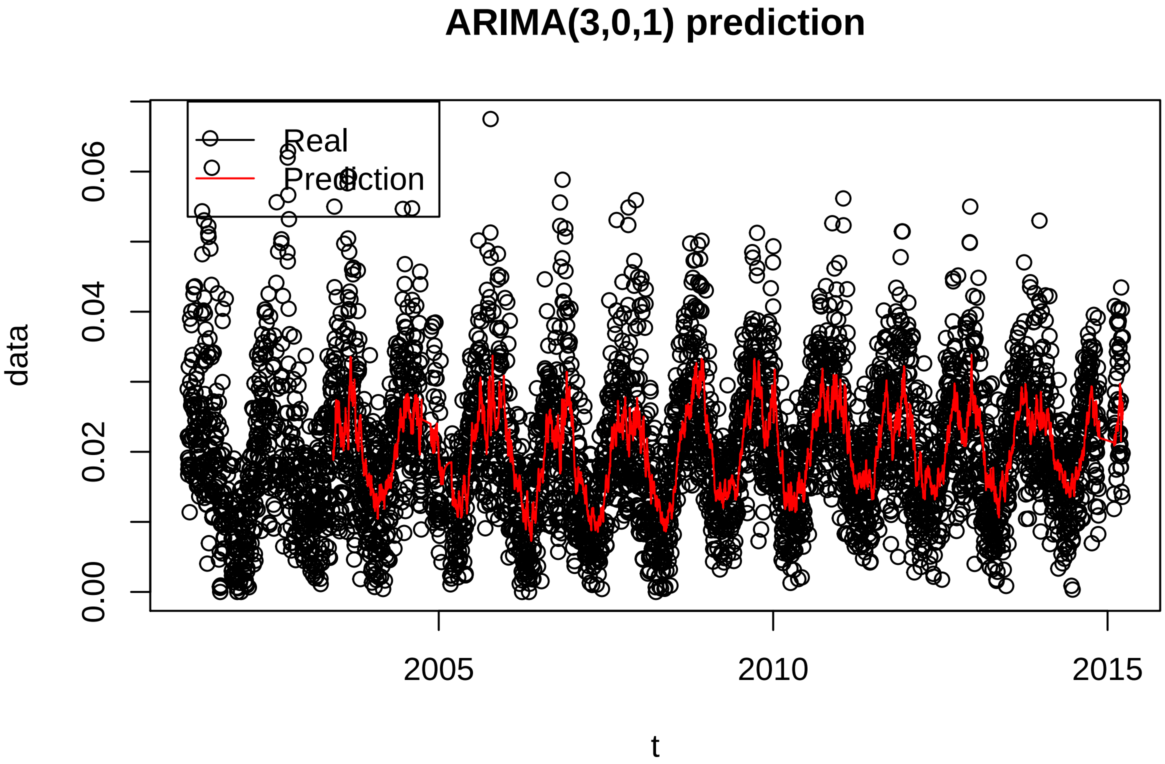
```
## Call:
## arma(x = o3_train, order = c(4, 3))
##
## Model:
## ARMA(4,3)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0275745 -0.0049606 -0.0003479  0.0043427  0.0391448
##
## Coefficient(s):
##             Estimate  Std. Error  t value Pr(>|t|)
## ar1        0.9060239   0.3974686    2.279  0.02264 *
## ar2       -0.0265556   0.4953457   -0.054  0.95725
## ar3        0.1412857   0.1831460    0.771  0.44045
## ar4       -0.0326660   0.0554639   -0.589  0.55589
## ma1       -0.3723047   0.3971375   -0.937  0.34852
## ma2       -0.2997470   0.2906608   -1.031  0.30242
## ma3       -0.1781035   0.1224956   -1.454  0.14596
## intercept  0.0002418   0.0000921    2.625  0.00866 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Fit:
## sigma^2 estimated as 5.281e-05,  Conditional Sum-of-Squares = 0.26,  AIC = -34127.1
```

```
##
## Call:
## arma(x = o3_train, order = c(4, 2))
##
## Model:
## ARMA(4,2)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0278979 -0.0049234 -0.0003225  0.0043674  0.0393150
##
## Coefficient(s):
##             Estimate  Std. Error  t value Pr(>|t|)
## ar1        1.059e+00   4.642e-01    2.281   0.0226 *
## ar2       -6.008e-02   6.680e-01   -0.090   0.9283
## ar3       -4.541e-02   2.895e-01   -0.157   0.8754
## ar4        3.665e-02   8.276e-02    0.443   0.6579
## ma1       -5.259e-01   4.625e-01   -1.137   0.2555
## ma2       -3.481e-01   4.171e-01   -0.834   0.4040
## intercept  2.042e-04   9.095e-05    2.246   0.0247 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Fit:
## sigma^2 estimated as 5.281e-05,  Conditional Sum-of-Squares = 0.26,  AIC = -34128.87
```

First we see that in the last two model, there is no statistical significance in the major part of the parameters, so we'll no consider it. The first, second and third models have significant parameters and similar AIC. In especial the 2° and 3° has the smallest AIC. So we will compare them both.

We see the MAE of the ARIMA(3, 0, 1):

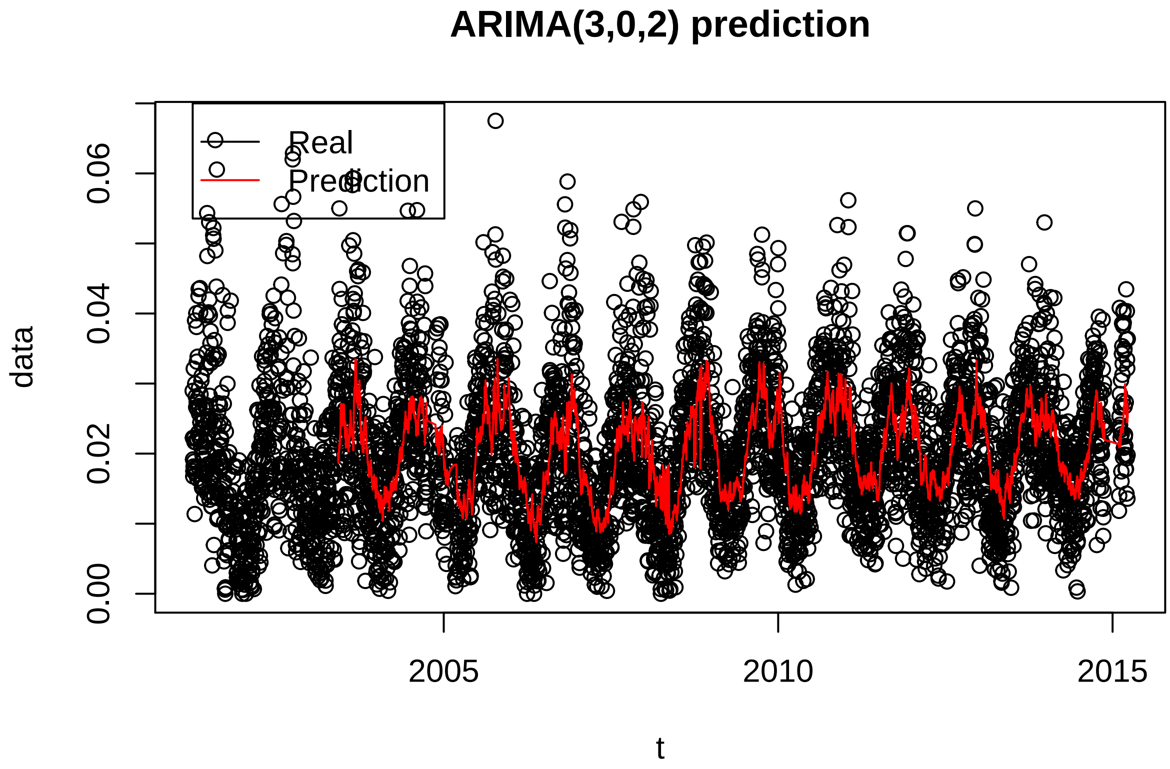
```
## [1] 0.006463936
```



And the MAE of the ARIMA(3, 0, 2):

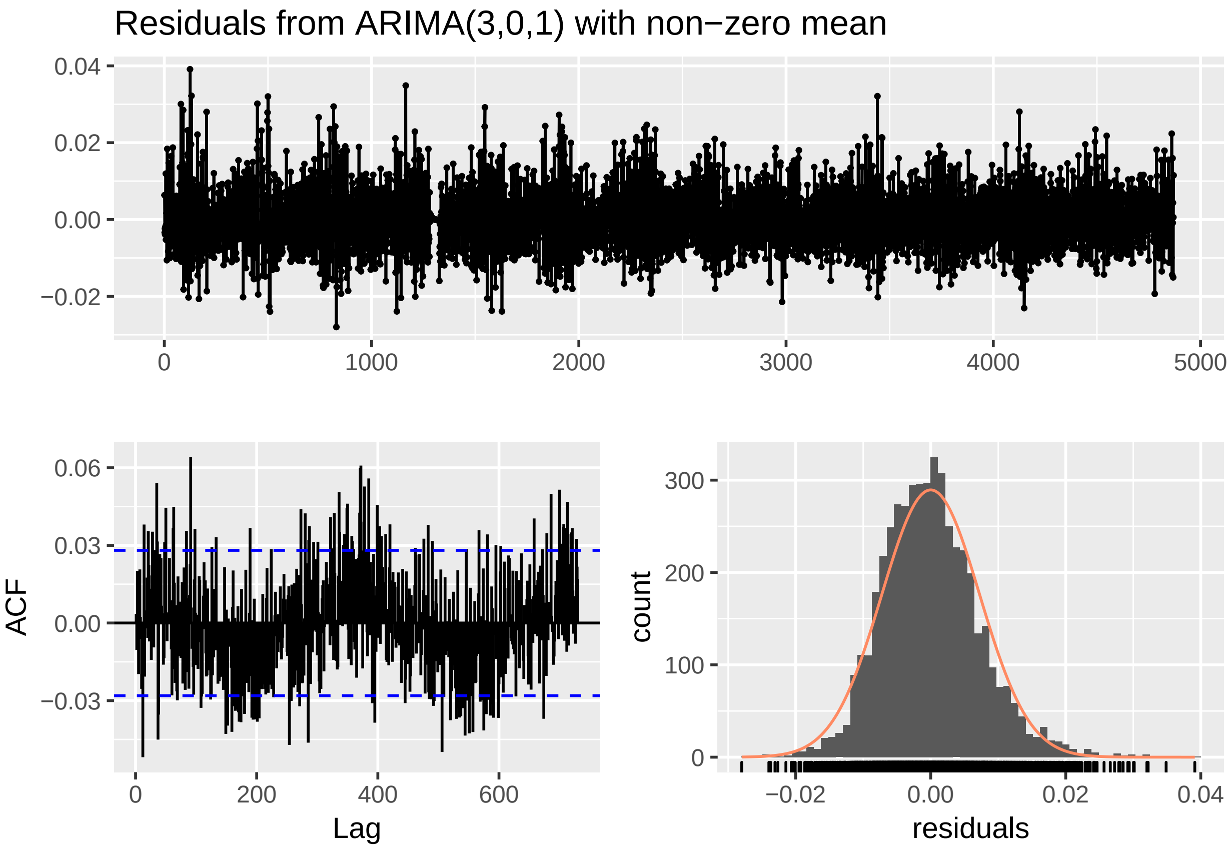
```
## [1] "MAE ARIMA(3,0,2)"
```

```
## [1] 0.006483882
```



The first model seems a little better. So, let's check the residuals to observe it's problems.

```
model <- arima(o3_train, order = c(3,0,1))
checkresiduals(model, lag = 2*freq, lag.max = 2*freq)
```



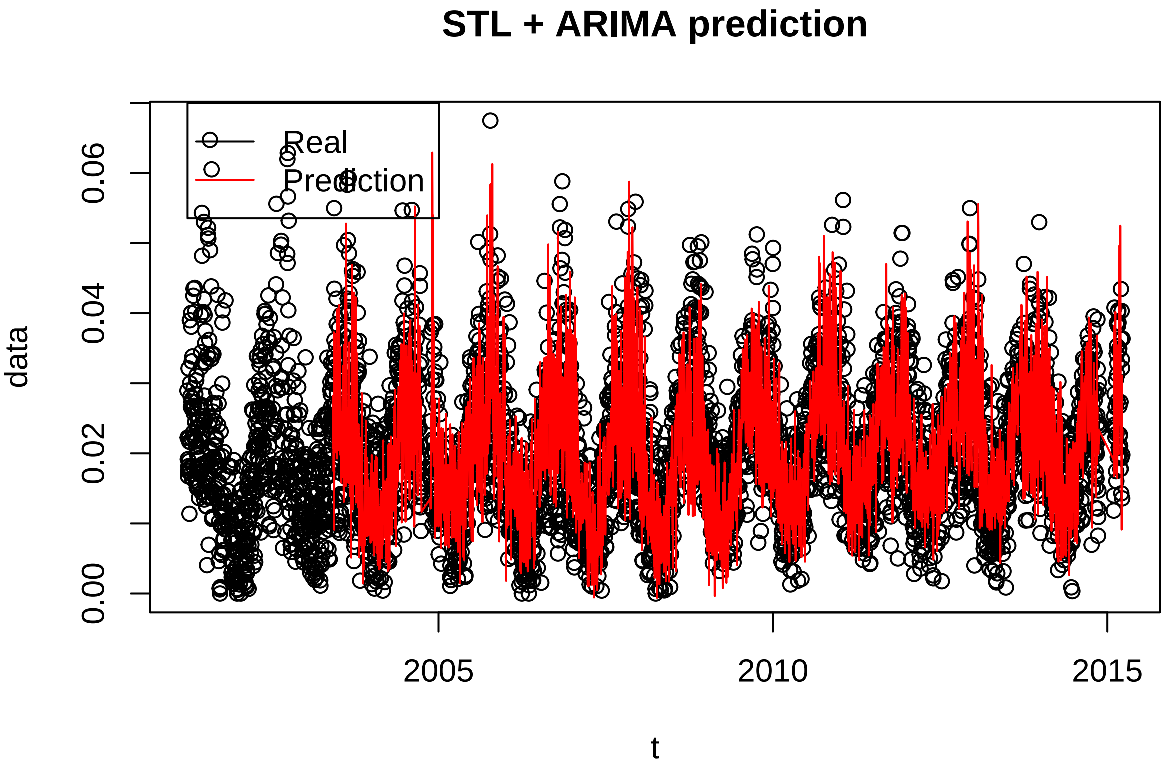
```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(3,0,1) with non-zero mean
## Q* = 1627, df = 725, p-value < 2.2e-16
##
## Model df: 5.   Total lags used: 730
```

It's interesting to note the ACF has a peak around the 365, so the ARMA model did not seem to capture the seasonality. It would be better to fit an Seasonal ARIMA further. The histogram is pretty similar to normal distribution. The test made analyses if the mean is 0. It may be because the p-value is really small. That's the reason to adapt ARMA with STL, that is, fit an ARMA model in the residuals.

Adapting ARMA

The ARMA seems to fit well as we can see so far. However, it's not capturing other caracteristics on the data, as seasonality. For that reason, we will combine the stl and arma model and extract the best of each one. We will decompose the series in trend and seasonality and in the reminder, we fit an arima model with `auto.arima()`.

```
## [1] 0.007854377
```

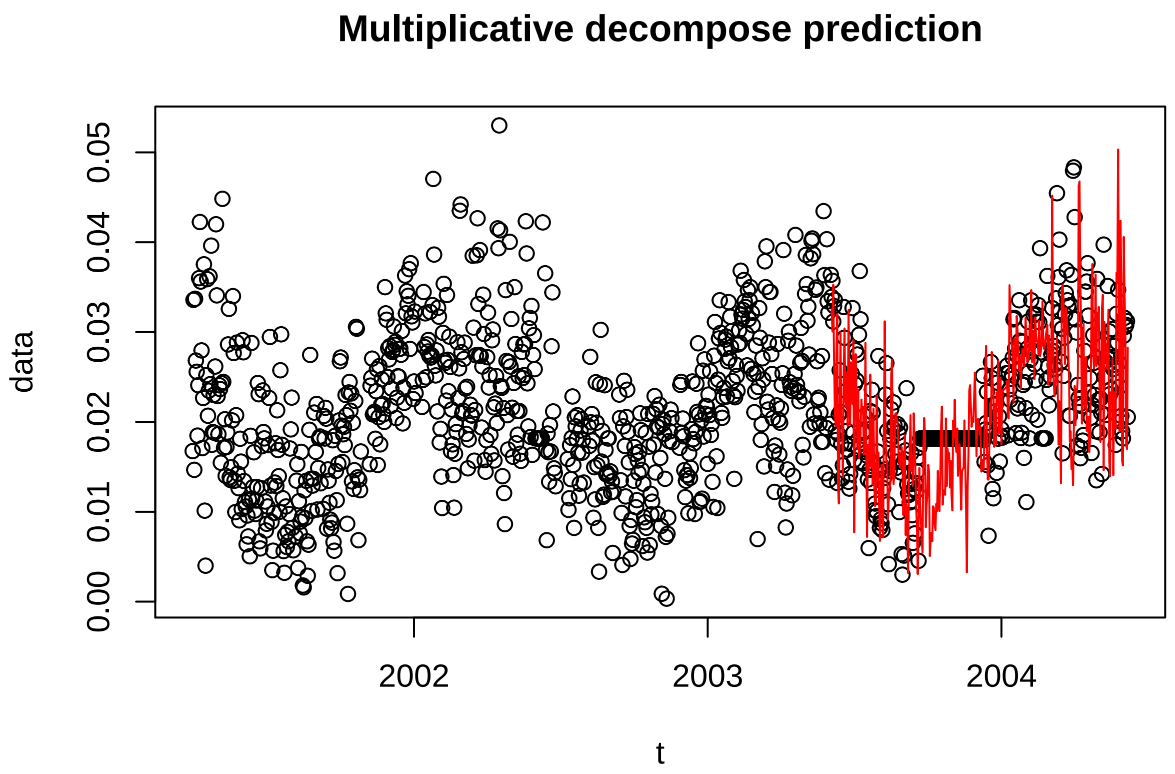


However the MAE isn't improved by this model. For that reason, this model was disregarded.

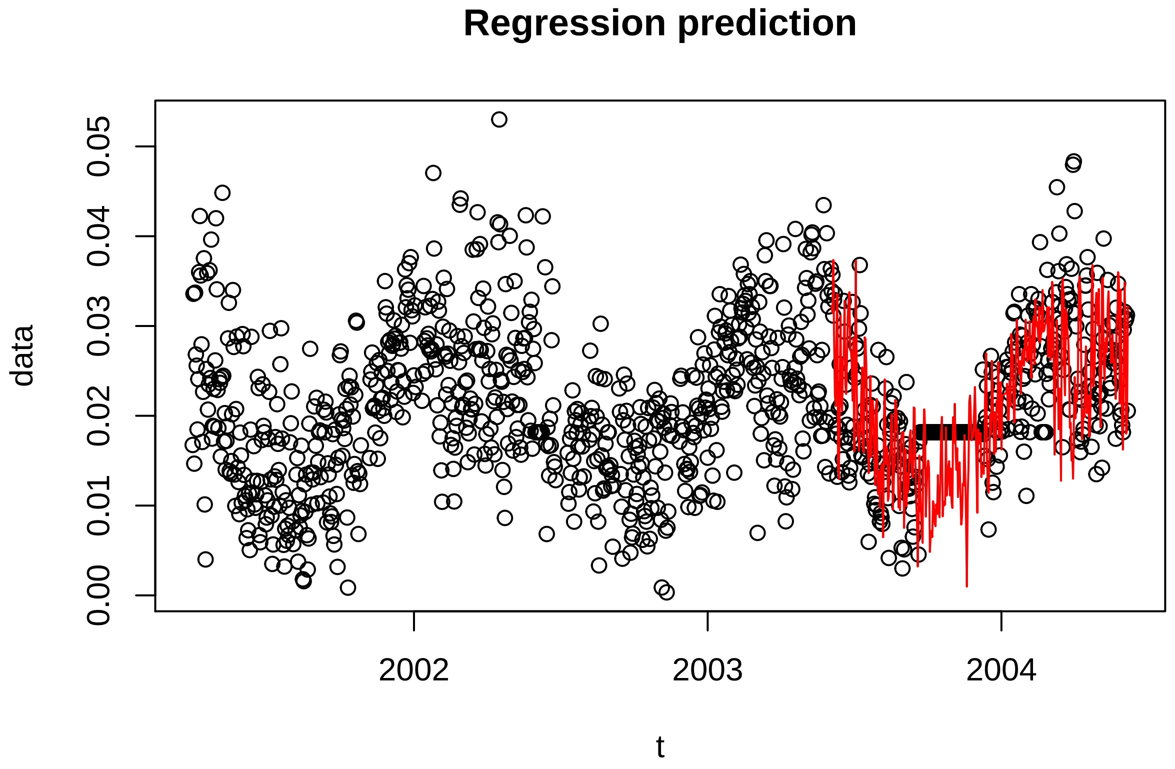
Comparing models in the test data.

We chose some of the best models to compare with the testing data.

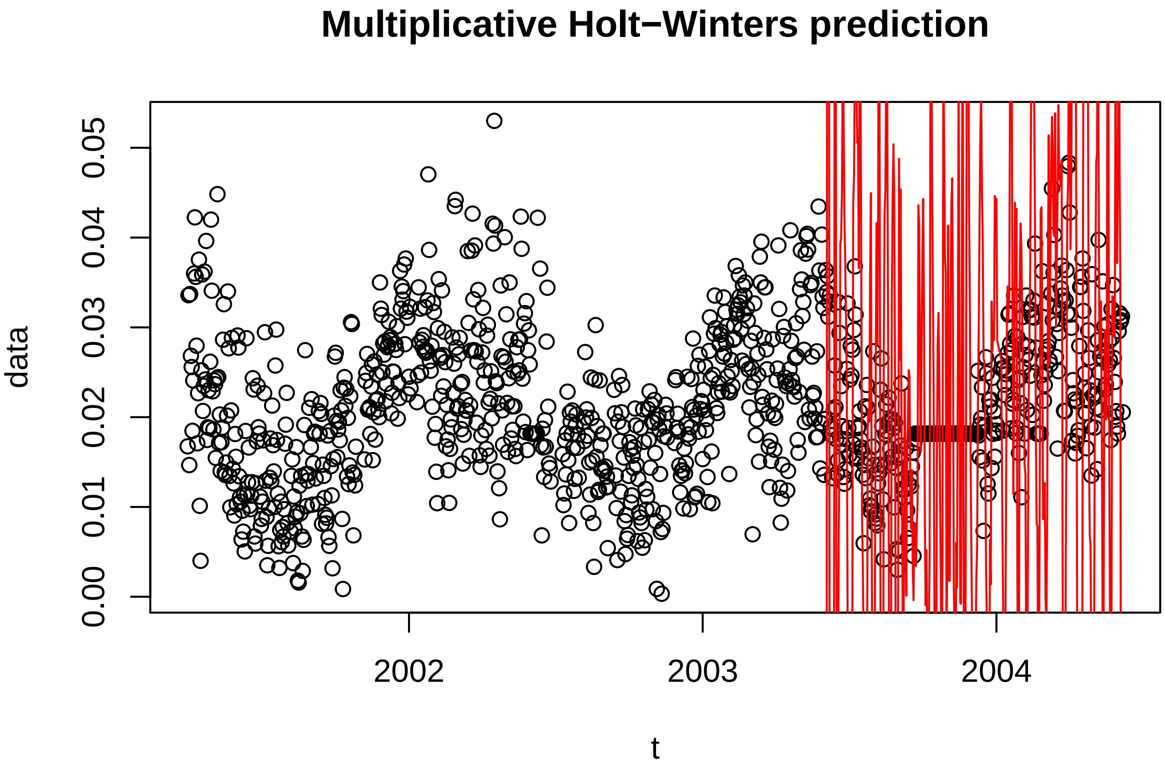
```
## [1] 0.006599825
```


Multiplicative decompose prediction

```
## [1] 0.006393855
```

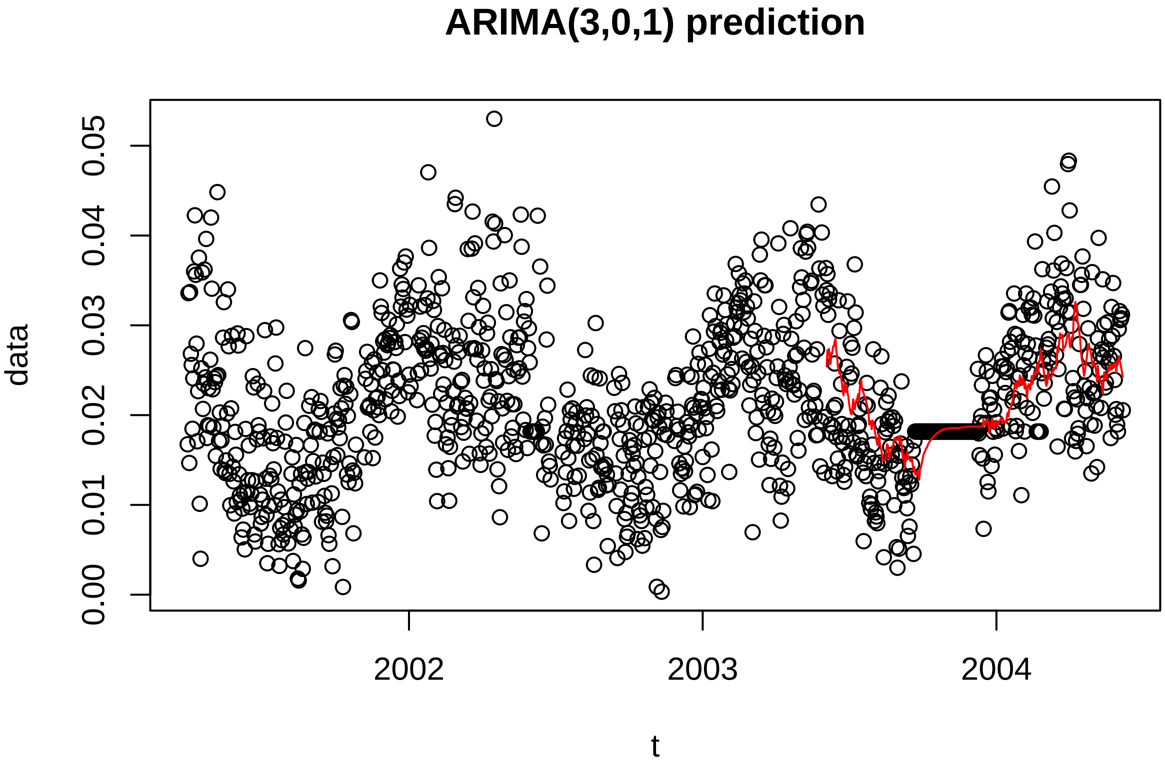

Regression prediction

```
## [1] 0.0256431
```

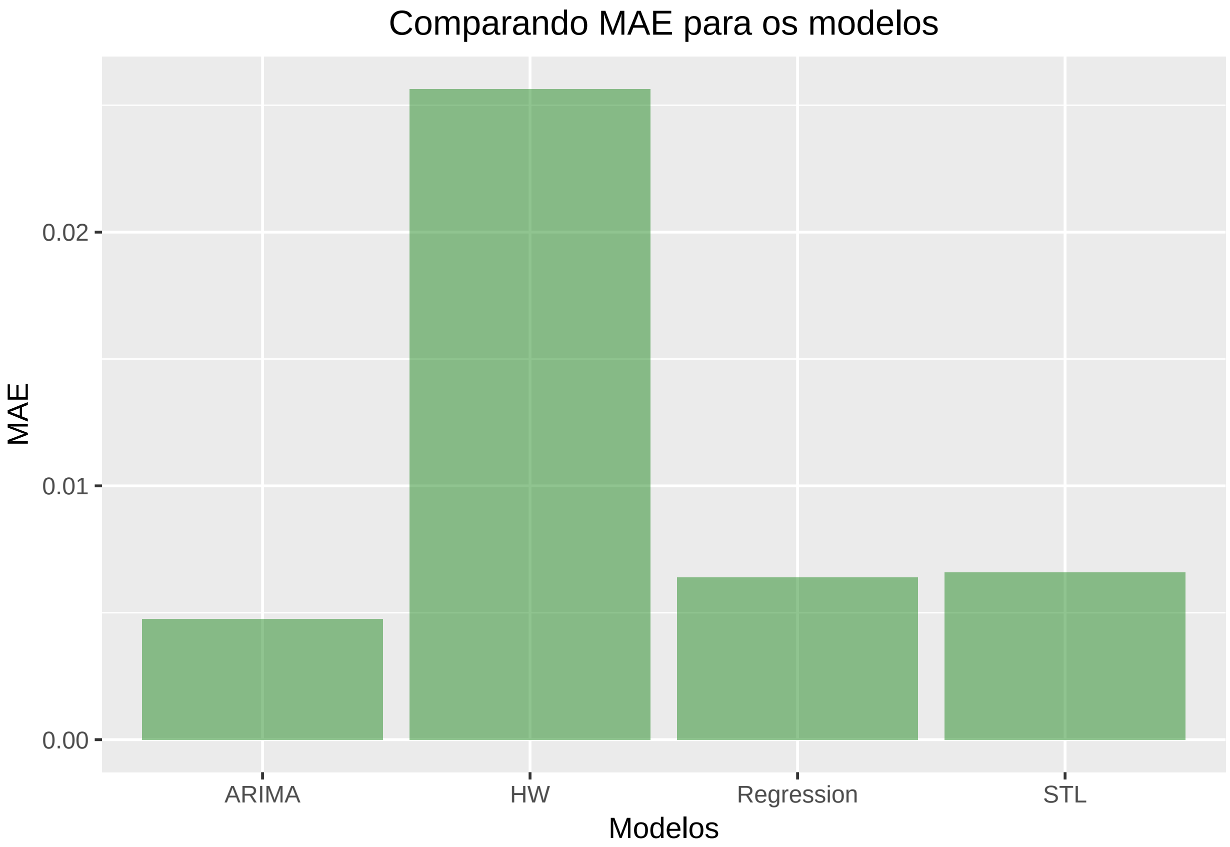


```
## [1] "MAE ARIMA(3,0,1)"

## [1] 0.004755896
```



Finally, we have this bar graphic to compare the values:

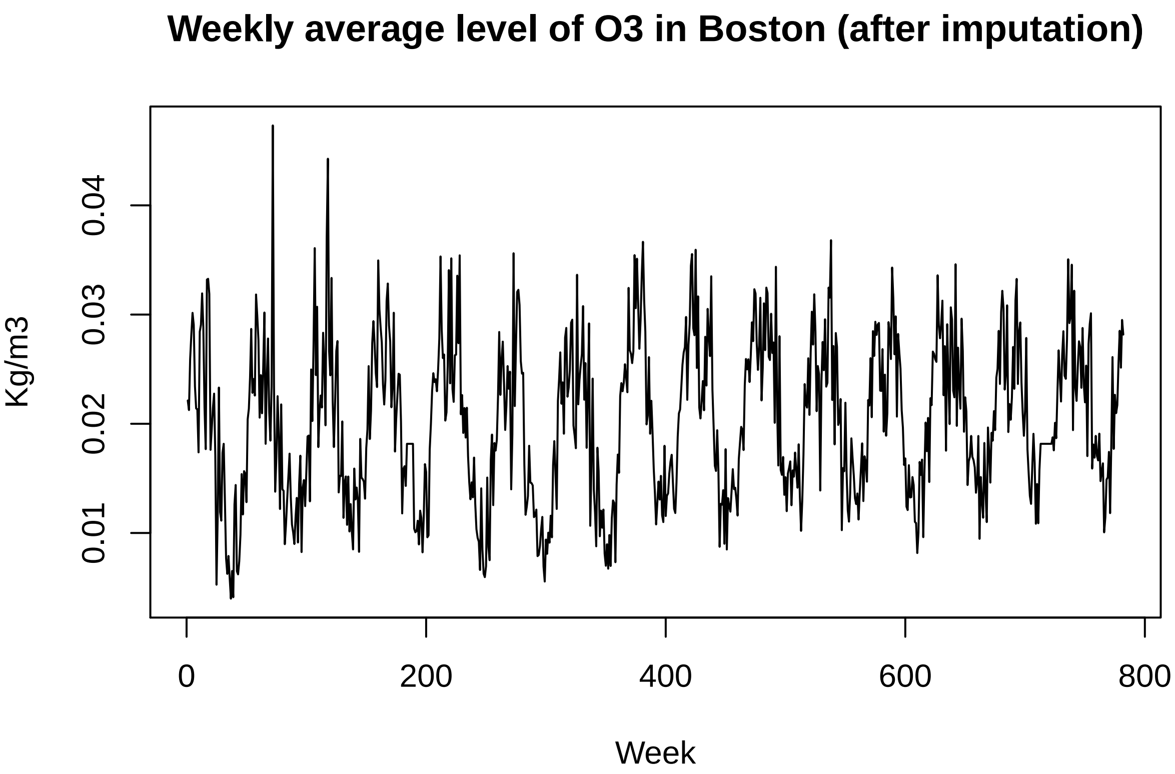


Models: case 2

In case two, we have to aggregate the diary days in a week, starting from the sunday, as requested. So we calculate the mean value in the week to be its representant. The models may be very similar to the previous. We may see less outliers. We also will separate train and test data. The first day in the data is April 1, 2001, a Sunday. SO we do not worry about that.

```
o3_week <- c(1:floor(length(o3.clean)/7))
for(i in seq(1,length(o3.clean)-7, 7)){
  o3_week[ceiling(i/7)] = mean(o3.clean[i:(i+6)])
}

plot(ts(o3_week), main = 'Weekly average level of O3 in Boston (after imputation)',
     xlab = 'Week', ylab = 'Kg/m3')
```



```
o3_train_week = o3_week[1:(length(o3_week)[1] - 52)]
o3_test_week = o3_week[-c(1:(length(o3_week)[1] - 52))]
```