Respondent driven-sampling

Procedure to sample from hidden or hard-to-reach populations

Lucas Moschen

School of Applied Mathematics
Fundação Getulio Vargas

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Table of Contents

1. Introduction

2. Mathematical formulation

3. Examples and usages

4. Real-world application

Table of Contents

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2. Mathematical formulation
3. Examples and usages
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Hidden and hard-to-reach populations

- No sampling frame exists: unknown size and boundaries;
- Privacy concerns: stigmatized or illegal behavior;
- Fear of exposition or prosecution complicates the enumeration and learning about these populations;
- Examples: Heavy drug users, sex workers, homeless people, and men who have sex with men.

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Existing sampling methods

- **Snowball** [Goodman, 1961]
  From starting individuals, each subject provides a list of names of known individuals from the target population. The researcher invites this person to participate, who can agree or deny it.

- **Key informant** [Deaux and Callaghan, 1985]
  Expert respondents are selected to answer about others' behavior. For instance, social workers, drug abuse counselors, official, etc.

- **Targeted** [Watters and Biernacki, 1989]
  Field researchers build an ethnographic mapping of a target population, and recruit a number of individuals at sites identified by this map.

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Problems with snowball sampling

- Inferences about the individuals depend on the initial sample;
- Bias towards individuals who are more cooperative;
- Bias by masking, that is, protecting friends by not referring them;
- Individuals with more links may be oversampled.

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Respondent-driven sampling

- Proposed by [Heckathorn, 1997] as an approach to estimate proportions in a hard-to-reach population;
- Theory based on Markov chains;
- [Crawford, 2016] models as an interaction network and defines a probability distribution over the observed subgraph;
- The sampling is without replacement.

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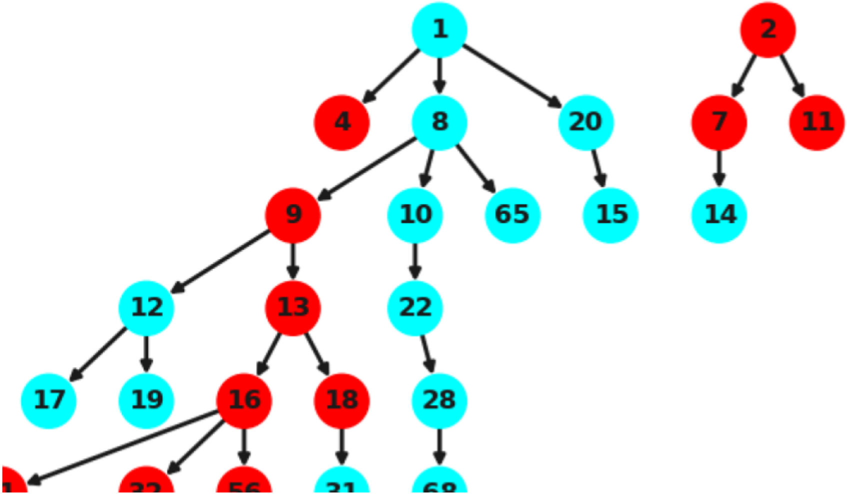

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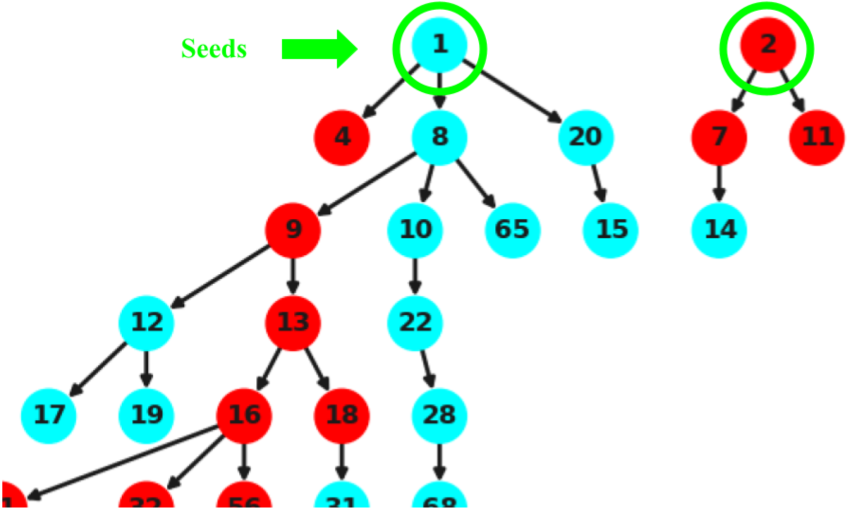

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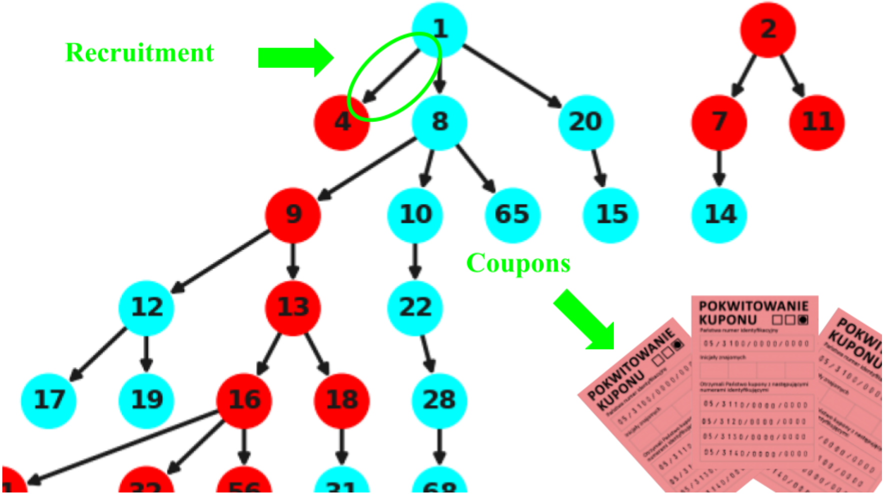

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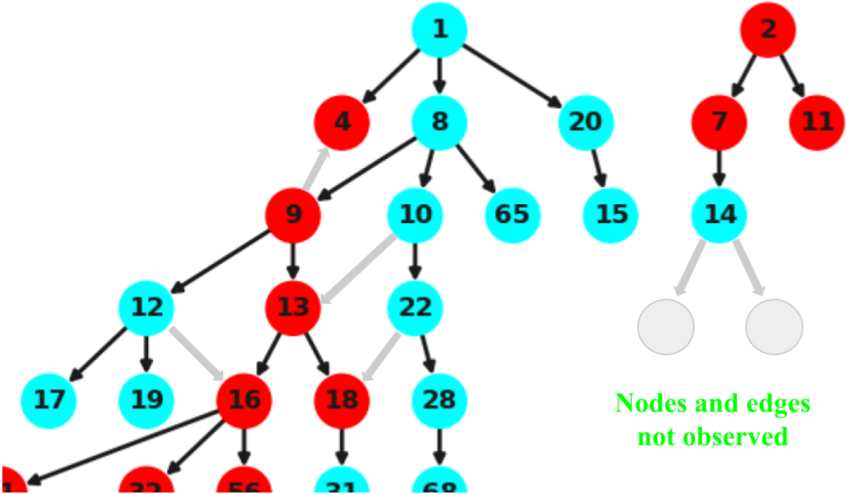

Dual system of incentives

Two different sources of theoretical incentive (dual incentive system):

- **Individual-sanction based control:** reward for participating in the research.

- **Group-mediated social control:** reward for recruiting peers. When social approval is important, it's more efficient and cheaper. Symbolic incentive is also important.

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Formal model

The RDS can be built mathematically with different approaches:

- **Markov process** [Heckathorn, 1997]
  Each recruiter's social characteristics affect the characteristics of the recruits. There are a limited number of states that subjects can assume and the recruits are function of the recruiter characteristics.

- **Graphical structure** [Crawford, 2016]
  A hidden population is an undirected graph, and we observe it partially in the *recruitment graph*, as also the coupon matrix and recruitment times. The unobserved graph is treated as *missing data* and can be interpreted as an Exponential Random Graph Model.

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Markov chain model

- In a survey, questions create states describing the participant;
- Heckathorn concluded that the recruitment was a memoryless process and first-order Markov process.
- The Markov chain indicates the most recent recruit's characteristic;
- The Markov chain must be ergodic.

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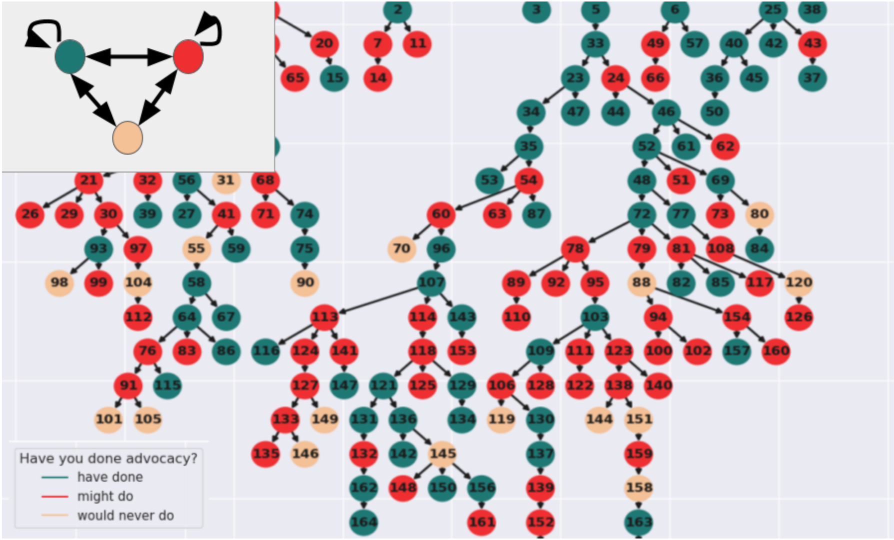

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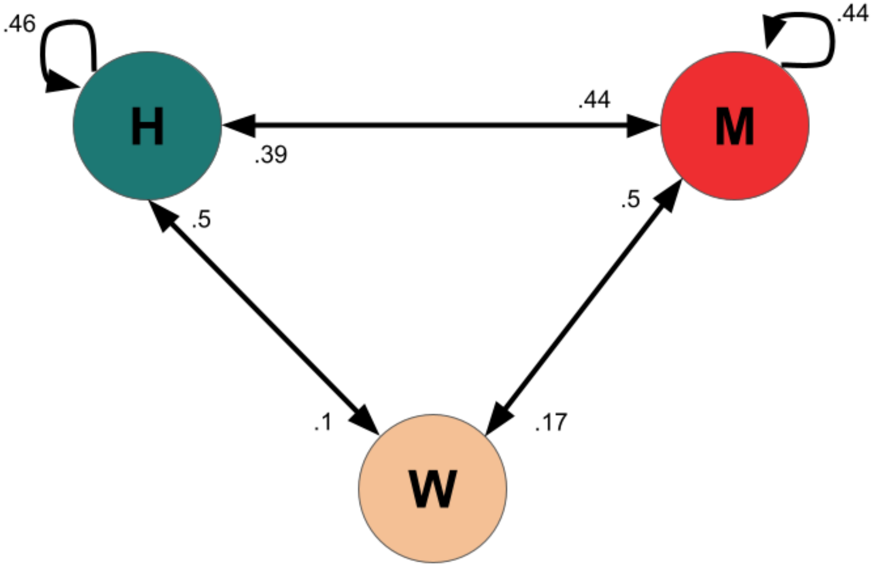
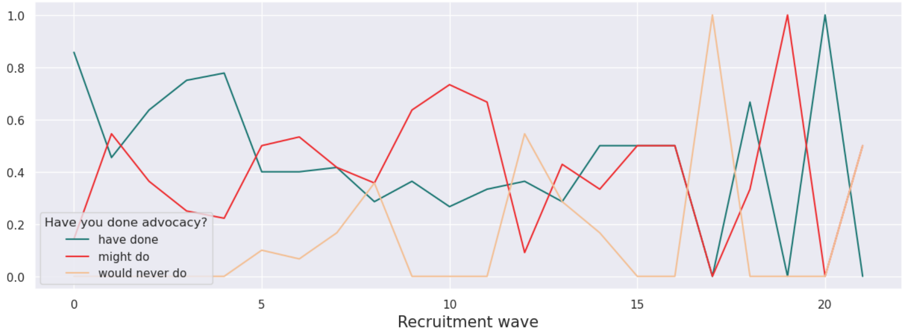


Consequences of Markov chain theory

Theorem

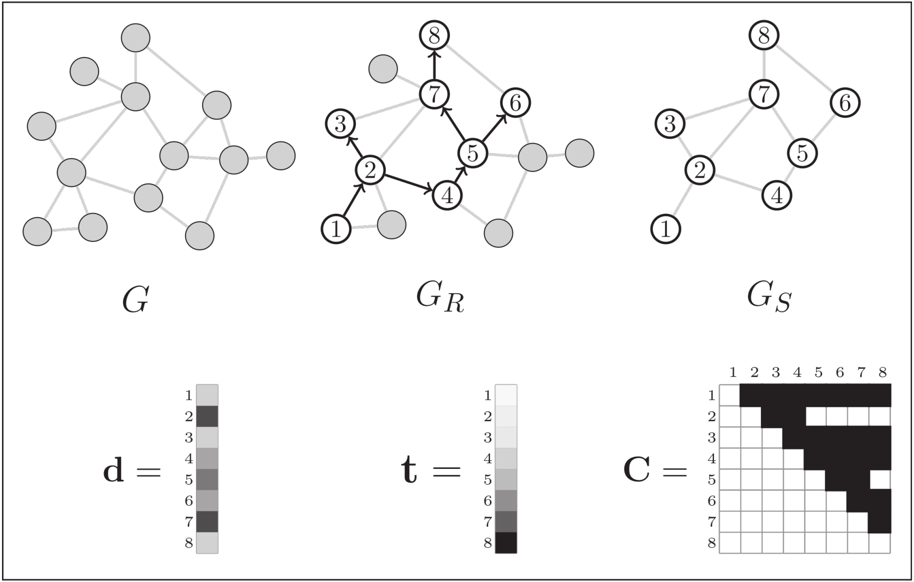
An equilibrium mix of recruits will be attained when the number of waves goes to infinity, and it is independent from which recruitment began. The pooling approaches the equilibrium in a geometric rate.

Convergence analysis



Assessing bias in RDS

Network model



$$\mathbf{C} = \begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & □ & ■ & ■ & ■ & ■ & ■ & ■ & ■ \\ 2 & □ & □ & ■ & ■ & □ & □ & □ & □ \\ 3 & □ & □ & □ & ■ & ■ & ■ & ■ & ■ \\ 4 & □ & □ & □ & □ & ■ & ■ & ■ & ■ \\ 5 & □ & □ & □ & □ & □ & ■ & ■ & □ \\ 6 & □ & □ & □ & □ & □ & □ & ■ & ■ \\ 7 & □ & □ & □ & □ & □ & □ & □ & ■ \\ 8 & □ & □ & □ & □ & □ & □ & □ & □ \end{array}$$

Network model

- Let $G = (V, E)$ be an undirected graph representing the hidden population. The *Recruitment Graph* is $G_R = (V_R, E_R)$, where V_R represents the recruited individuals, and E_R the recruitment edge. The *Recruitment-induced Subgraph* is the induced subgraph by V_R.

- The *Coupon Matrix* C has elements $C_{ij} = 1$ if the subject i has at least one coupon just before the jth recruitment event.

- We observe $Y = (G_R, d, t, C)$.

- The time to recruitment along a *susceptible edge* has Exponential distribution, independent of the identity, neighbor, and all the other waiting times.

Consequences

Theorem (Waiting time for a recruitment)

Let u be a recruiter and $v \in S_u$ a susceptible neighbor. The waiting time to u recruit v conditioned on the recruitment event has distribution Exponential with rate $\lambda|S_u|$. The probability of $v \in S_u$ to be the next recruited is uniform.

Theorem (Waiting time for some recruitment to occur)

The waiting time to the next recruitment is distributed as Exponential with rate $\lambda \sum_{u \in R} |S_u|$.

Likelihood of the recruitment time series

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References I

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