# Respondent driven-sampling

#### Procedure to sample from hidden or hard-to-reach populations

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- Introduction
- 2 Mathematical formulation
- 3 Examples and usages
- Applications with real data

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- 3 Examples and usages
- 4 Applications with real data

# Hidden and hard-to-reach populations

- ▶ No sampling frame exists: size and boundaries of the population are unknown.
- Privacy concerns: stigmatized or illegal behavior.
- ► Fear of exposition or prosecution complicates the enumeration and learning about these populations.
- ► High logistic cost when the occurrence frequency is low.
- Examples: Heavy drug users, sex workers, homeless people, and men who have sex with men.

# Existing sampling methods

### ► Snowball [Goodman, 1961]

From starting individuals, each subject provides a list of names of known individuals from the target population. The researcher invites this person to participate, who can agree or deny it.

### ► Key informant [Deaux and Callaghan, 1985]

Expert respondents are selected to answer about others' behavior. For instance, social workers, drug abuse counselors, official, etc.

## ► Targeted [Watters and Biernacki, 1989]

Field researchers build an ethnographic mapping of a target population, and recruit a number of individuals at sites identified by this map.

# Problems with snowball sampling

- ▶ Inferences about the individuals depend on the initial sample.
- Bias towards individuals who are more cooperative and agree to participate.
- ▶ Bias because of masking, that is, protecting friends by not referring them.
- Individuals with more links may be oversampled.

# Respondent-driven sampling

- The researchers select a handful of individuals from a target population who serve as seeds.
- 2 Each participant receives a fixed number of recruitment coupons and invite members of their own social network to participate in exchange of a reward.
- The sampling is without replacement.
- 4 If the individual accepts to participate, they answer a questionnaire and inform the network degree. One important point is that the recruiter doesn't say the name of the other members, reducing the mask effect.

# System of incentives

Two different sources of theoretical incentive (dual incentive system):

- ▶ Individual-sanction based control: reward for participating in the research.
- ▶ **Group-mediated social control:** reward for recruiting peers. When social approval is important, it's more efficient and cheaper. Symbolic incentive is also important.

Introduction

- 2 Mathematical formulation
- Examples and usages
- 4 Applications with real data

#### Formal model

The RDS can be seen mathematically in two different approaches.

### ► Stochastic process [Heckathorn, 1997]

Each recruiter's social characteristics affect the characteristics of the recruits. There are a limited number of states that subjects can assume and the recruits are function of the recruiter characteristics.

## ► Graphical structure [Crawford, 2016]

A hidden population is an undirected graph, and we observe it partially in the *recruitment graph*, as also the coupon matrix and recruitment times. The unobserved graph is treated as *missing data* what can be interpreted as as Exponential Random Graph Model.

### Markov chain model

- ▶ In a survey, questions about ethnicity, location, and gender create states in which each participant will be. By statistical tests, one can verify association between the recruiter and recruited responses.
- Heckathorn concluded that the recruitment was a memoryless process, and concluded RDS was a first-oder Markov process. At any instant, the current location of the point indicates the most recent recruit's characteristic.
- ► The markov chain must be ergodic.

# Example of RDS in Markov chain

# Consequences of Markov chain theory

#### Theorem,

An equilibrium mix of recruits will be attained when the number of waves goes to infinity, and it is independent from which recruitment began.

#### Theorem

The subject pool generated approaches the equilibrium in a geometric rate.

### Theorem (Remark)

If there is a substancial in-group selection bias, this convergence will take longer.

# Assessing bias in RDS

### Network model

- Let G = (V, E) be an undirected graph representing the hidden population. The Recruitment Graph is  $G_R = (V_R, E_R)$ , where  $V_S$  represents the recruited individuals, and  $E_R$  the recruitment edge. The Recruitment-induced Subgraph is the induced subgraph by  $V_R$ .
- ▶ The Coupon Matrix C has elements  $C_{ii} = 1$  if the subject i has at least one coupon just before the jth recruitment event.
- We observe  $Y = (G_R, d, t, C)$ .
- The time to recruitment along a susceptible edge has Exponential distribution, independent of the identity, neighbor, and all the other waiting times.

15/22

# Example of observed data

# Consequences

## Theorem (Waiting time for a recruitment)

Let u be a recruiter and  $v \in S_u$  a susceptible neighbor. The waiting time to u recruit v conditioned on the recruitment event has distribution Exponential with rate  $\lambda |S_u|$ . The probability of  $v \in S_u$  to be the next recruited is uniform.

### Theorem (Waiting time for some recruitment to occur)

The waiting time to the next recruitment is distributed as Exponential with rate  $\lambda \sum_{u \in R} |S_u|$ .

## Likelihood of the recruitment time series

Introduction

- 2 Mathematical formulation
- Second Examples and usages
- 4 Applications with real data

Introduction

- 2 Mathematical formulation
- Examples and usages
- 4 Applications with real data

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