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**Bayesian analysis of respondent-driven surveys  
with outcome uncertainty**

Rio de Janeiro, Brazil

16 de junho de 2021

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of Mathematics Applied (FGV) as a partial  
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work.

Getulio Vargas Foundation – FGV

School of Applied Mathematics

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# 1 Introduction

This work proposes to study the survey method Respondent-Driven Sampling (RDS), a chain-referral method with the objective of sampling from hard-to-reach populations when necessary to estimate the prevalence of some binary condition from this population. The modeling also accounts for sensibility and sensitivity since the imperfection of the detection tests.

Hidden or hard-to-reach populations have two main features: no sampling frame exists, given that their size and boundaries are unknown, and there are privacy concerns because the subjects are stigmatized or have illegal behavior (HECKATHORN, 1997). Fear of exposition or prosecution complicates the enumeration of the populations and the learning about them. Moreover, if the occurrence frequency of the condition is low, there are high logistic costs involved. Some examples are heavy drug users, sex workers, homeless people, and men who have sex with men.

Researches have been done with the development of some methods to reach these populations, such as, for example, snowball sampling (GOODMAN, 1961), key important sampling (DEAUX; CALLAGHAN, 1985), and targeted sampling (WATTERS; BIER-NACKI, 1989). HECKATHORN introduced the Respondent-Driven Sampling (RDS) to fill some gaps from other methods he depicted in his work. In his proposed approach, the researchers select a handful of individuals from the target population and give them coupons to recruit their peers. The individuals receive a reward for being recruited and for recruiting, which creates a dual incentive system. After 1997, several papers studied this topic more deeply.

Following the sampling from the target population, a questionnaire or a disease test is conducted. This work considers binary outcomes. For instance, asking about smoking status or testing for HIV infections. However, the diagnoses are subject to measure error, and regard their accuracy is a vital step (REITSMA et al., 2005). In particular, we propose the joint use of sensitivity (the ability to detect the condition) and specificity (the ability to identify the absence of it).

Nevertheless, because of our lack of knowledge about nature itself, it is necessary to model the uncertainty of this process, and Bayesian Statistics is the indicated area of study. In the Bayesian view, the parameters are random variables, and the beliefs about them are updated given new data. The idea is to propagate uncertainty about the outcome through the network of contacts, which has its probability distribution.

The objective of this work is to analyze the network structure as a stochastic object, along with the sensibility and sensitivity. We also intend to apply this framework efficiently,

comparing Monte Carlo algorithms and Laplace approximations.

## 1.1 Respondent-driven sampling

RDS is commonly used to survey hidden or hard-to-reach populations when no sampling frame exists (HECKATHORN, 1997). In this approach, the researchers select some individuals, called *seeds* from the target population, and give them a fixed amount of *recruitment coupons* to recruit their peers. Each recipient of the coupons reclaims it in the study site, is interviewed, and receives more coupons to continue the recruitment. This process occurs until it reaches some criteria. The sampling is without replacement, so the participants cannot be recruited more than once. Moreover, the respondents inform their *network degree*.

The subjects receive a reward for being interviewed and for each recruitment which establishes a dual system incentive. The *primary incentive* is the *individual-sanction-based control*, so there is a reward for participating. The second one is the *group-mediated social control* that influences the participants seeking to induce others to comply. When social approval is important, recruitment can be even more efficient and cheaper. Moreover, the material incentive can be converted into symbolic by the individuals.

Descrever o modelo de Heckathorn

Descrever o modelo de Crawford

Discutir detalhes adicionais de outros artigos como aplicações

## 1.2 Prevalence estimation with imperfect tests

Consider a population of interest and a known condition, such as, for example, a disease or a binary behavior. It is important to understand the proportion of individuals in this population exposed at time  $t$ , called *prevalence*. In hidden networks, the population size is unknown prior to the study, what makes the proceeding to be hard. Suppose a diagnostic test is done to measure the presence or the absence of this condition in the individuals.

Mathematically, let  $\theta \in (0, 1)$  be the prevalence of a condition and  $Y_i$  be an indicator function of the condition in the  $i^{th}$  individual. Assuming for simplicity that all tests are performed at time  $t$ , and the sample is  $\{y_1, \dots, y_n\}$ , the RDS estimator was proposed based largely on Markov chain theory and social network theory (HECKATHORN, 1997; HECKATHORN, 2002).

$$a = 1 \tag{1.1}$$

Citar (VOLZ; HECKATHORN, 2008) para o estimador RDS II.

## 1.3 Bayesian statistics

Breve discussão sobre o teorema de Bayes e estatística bayesiana

## 2 Justification

There are two justifications for the importance of this work. First, hidden populations are often omitted from national representative surveys since they do not have fixed addresses or fear prosecution. However, the individuals can have a greater risk of drug abuse or having sexually transmitted infections. This combination creates an environment of aid absence from the government to these people. The second reason is mathematical. This topic has lots of gaps in Statistics that deserve attention. The correct sampling probabilities for the recruited members under RDS are hard to obtain since not all links and nodes are observed, constituting missing data ([CRAWFORD, 2016](#)). In this fertile area, regression approaches to prevalence estimation taking the network structure can be built ([BASTOS et al., 2012](#)) and are still in development.

### 3 Objectives

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## 4 Methodology

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## 5 Preliminary results

- a) Descrição do problema em termos matemáticos e revisão bibliográfica: material sobre RDS (formalização matemática em forma de cadeia ou processo de ramificação), regressão logística em que a resposta tem incerteza e aplicações em usuários de drogas, infecções transmissíveis, entre outros.
- b) Incerteza sobre especificidade e sensibilidade do teste e como propagar a classificação errada na rede. Comparação de prioris e, por isso, estudo de métodos Bayesianos. Justificar utilização desses métodos com argumento da incerteza.
- c) Estudo do MCMC e Aproximação de Laplace, comparação dos algoritmos em alguns artigos e, quem sabe, codificação em Python e R.
- d) Implementação de inferência eficiente em INLA, com possibilidades abertas em Python (talvez Julia?)

## 6 Schedule

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## Final considerations

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