Fokker-Plank equation

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Abstract

This text aims to summarise the content studied during the period in the Department of Mathematics at Imperial College London with professor Dante Kalise. The object of study is the Fokker-Planck equation and Optimal Control.

Contents

Notation

This list is part of the notation we use throughout the text:

- $\langle X \rangle = \mathbb{E}[X]$ is the expected value of the random variable X.

1 Stochastic Differential Equations

The most used Langevin equation is of the form

$$\frac{dx}{dt} = a(x,t) + b(x,t)\xi(t),$$

where $\xi(t)$ is a random term. We suppose that for $t \neq t'$, $\xi(t)$ and $\xi(t')$ are independent. It is also assumed that $\langle \xi(t) \rangle = 0$. This is not restrictive since $a(x,t)$ can incorporate the mean of $\xi(t)$. Finally, we require that $\xi(t)$ has infinity variance. The problem with this choice is the discontinuities of dx/dt. Let

$$u(t) = \int_0^t \xi(s)\,ds$$

and suppose that u is a continuous function. This implies that u is a Markov Process. Notice that

$$\langle u(t+\Delta t) - u_0 | [u_0, t] \rangle = \langle \int_t^{t+\Delta t} \xi(s)\,ds \rangle = 0$$

and

$$\langle [u(t+\Delta t) - u_0]^2 | [u_0, t] \rangle =$$

TOFINISH

1.1 Stochastic Integration

Suppose G is an arbitrary function and $W(t)$ is a Wiener process. The stochastic integral

$$I := \int_{t_0}^t G(s) dW(s)$$

is a kind of Riemann-Stieltjes integral. More specifically, consider the sum

$$S_n = \sum_{i=1}^n G(\tau_i)[W(t_i) - W(ti-1)],$$

with $t_0 \leq t_1 \leq \cdots \leq t_{n-1} \leq t$ and $\tau_i \in [t_{i-1}, t_i]$, The convergence of S_n, as this, depends on the choice of the values of τ_i. By this reason, we specify that $\tau_i = t_{i-1}$ leading to the *Itô Stochastic Integral*.

Remark 1.1.1. *The limit taken for S_n is the mean square limit, that is, we want that*

$$\lim_{n\to\infty} \langle (S_n - I)^2 \rangle = 0.$$

An alternative definition comes from *Stratonovich*.

References

[1]