

Respostas - Lista 8

Funções de Uma Variável

Técnicas de Integração

Exercício 2

$$\text{a) } \int \sin(6x) \cos(x) dx = -\frac{1}{10} \cos(5x) - \frac{1}{14} \cos(7x)$$

$$\text{b) } \int \sin(x) \cos(6x) dx = \frac{1}{10} \cos(5x) - \frac{1}{14} \cos(7x)$$

$$\text{c) } \int \sin(3x) \cos(2x) dx = -\frac{\cos(x)}{2} - \frac{1}{10} \cos(5x)$$

$$\text{d) } \int \sin(mx) \cos(nx) dx = \\ -\frac{\cos((m-n)x)}{2(m-n)} - \frac{\cos((m+n)x)}{2(m+n)}$$

Exercício 3)

$$\text{b) } \int \sin(3x) \sin(7x) dx = \frac{1}{8} \sin(4x) - \frac{1}{20} \sin(10x)$$

$$\text{c) } \int \sin(nx) \sin(mx) dx = \frac{\sin((m-n)x)}{2(m-n)} - \frac{\sin((m+n)x)}{2(m+n)}$$

Exercício 4)

$$\text{a) } \int \cos(x) \cos(5x) dx = \frac{1}{8} \sin(4x) + \frac{1}{12} \sin(6x)$$

$$\text{b) } \int \cos(x) \cos(7x) dx = \frac{1}{12} \sin(6x) + \frac{1}{16} \sin(8x)$$

$$\text{c) } \int \cos(nx) \cos(mx) dx = \frac{\sin((m-n)x)}{2(m-n)} + \frac{\sin((m+n)x)}{2(m+n)}$$

Exercício 6

$$\text{a) } \int \frac{1}{(x+1)(x-1)} dx = \frac{1}{2} \ln(1-x) - \frac{1}{2} \ln(1+x)$$

$$\text{b) } \int \frac{x-1}{x(x-2)} dx = \frac{1}{2} \ln(-2+x) + \frac{\ln(x)}{2}$$

$$\text{c) } \int \frac{x-1}{(x^2-4)} dx = \frac{3}{4} \ln(-2-x) + \frac{1}{4} \ln(-2+x)$$

$$\text{d) } \int \frac{x-3}{x^2+3x+2} dx = -4 \ln(9(1+x)) + 5 \ln(-9(2+x))$$

Exercício 7

$$\text{c) } \int \frac{3x+1}{5+x^2} dx = \frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{\sqrt{5}} + \frac{3}{2} \ln(5+x^2)$$

Exercício 8

$$\text{a) } \int \sqrt{1+x^2} dx = \frac{1}{2} (x\sqrt{1+x^2} + \operatorname{arcsenh}(x))$$

$$\text{b) } \int \sqrt{1-4x^2} dx = \frac{1}{2} x\sqrt{1-4x^2} + \frac{1}{4} \operatorname{arcsen}(2x)$$

$$\text{c) } \int \sqrt{1-\cos(x)} dx = -2\sqrt{1-\cos(x)} \cot\left(\frac{x}{2}\right)$$

$$\text{d) } \int \sqrt{3+4x^2} dx = \frac{1}{2} x\sqrt{3+4x^2} + \frac{3}{4} \operatorname{arcsenh}\left(\frac{2x}{\sqrt{3}}\right)$$

$$\text{e) } \int \sqrt{x^2+2x+2} dx =$$

$$\frac{1}{2} \left((1+x)\sqrt{2+2x+x^2} + \operatorname{arcsenh}(1+x) \right)$$

Exercício 11

- a) $\int \frac{1}{x^2-9} dx = \frac{1}{6} \ln(3-x) - \frac{1}{6} \ln(3+x)$
- b) $\int \frac{x}{x^2-5x+6} dx = 3 \ln(-5(-3+x)) - 2 \ln(5(-2+x))$
- c) $\int \frac{x+3}{(x-1)^2} dx = -\frac{4}{-1+x} + \ln(-1+x)$
- d) $\int \frac{x^3+x+1}{x^2-2x+1} dx = \frac{1}{2} \left(-5 - \frac{6}{-1+x} + 4x + x^2 + 8 \ln(-1+x) \right)$
- e) $\int \frac{x^2+2}{x^2-4} dx = x + \frac{3}{2} \ln(-3(-2+x)) - \frac{3}{2} \ln(3(2+x))$
- f) $\int \frac{x+1}{x(x-2)(x+3)} dx = \frac{3}{10} \ln(-13(-2+x)) - \frac{\ln(x)}{6} - \frac{2}{15} \ln(13(3+x))$

Exercício 12

- a) $\int (\sin(x))^3 dx = -\frac{3 \cos(x)}{4} + \frac{1}{12} \cos(3x)$
- b) $\int (\cos(4x))^2 dx = \frac{x}{2} + \frac{1}{16} \sin(8x)$
- c) $\int \sin(x)(\cos(5x))^4 dx = -\frac{3 \cos(x)}{8} + \frac{1}{36} \cos(9x) - \frac{1}{44} \cos(11x) + \frac{1}{304} \cos(19x) - \frac{1}{336} \cos(21x)$