ESTA020-17 LISTA 2

1) a) $\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) + x_2(t)(1 - 3x_1^2(t) - 2x_2^2(t) \end{cases}$

$$\dot{x}_1(t) = 0 \Rightarrow \boxed{x_2(t) = 0}$$

$$\dot{x}_2(t) = 0 \Rightarrow -x_1(t) + x_2(t)(1 - 3x_1^2(t) - 2x_2^2(t)) = 0 \Rightarrow$$

$$\Rightarrow \boxed{x_1(t) = 0}$$

$$f(x) = \begin{bmatrix} x_2(t) \\ -x_1(t) + x_2(t)(1 - 3x_1^2(t) - 2x_2^2(t) \end{bmatrix} =$$

$$= \begin{bmatrix} x_2(t) \\ -x_1(t) + x_2(t) - 3x_1^2(t)x_2(t) - 2x_2^3(t) \end{bmatrix}$$

$$\frac{\partial f(x)}{\partial x} = \begin{bmatrix} 0 & 1 \\ -1 - 6x_1(t)x_2(t) & 1 - 3x_1^2(t) - 6x_2^2(t) \end{bmatrix}$$

$$x_e = (0,0) \quad A = \left.\frac{\partial f(x)}{\partial x}\right|_{x = x_e} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$

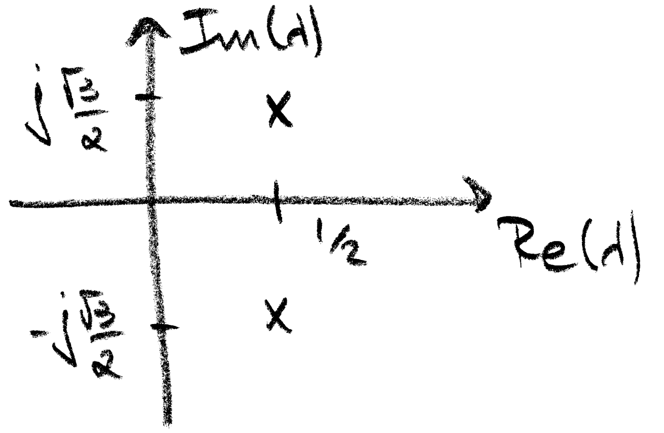
$$\det(\lambda I - A) = 0 \Rightarrow$$

$$\Rightarrow \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}\right) = 0 \Rightarrow$$

$$\Rightarrow \det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}\right) = 0 \Rightarrow$$

$$\Rightarrow \det\left(\begin{bmatrix} \lambda & -1 \\ 1 & \lambda - 1 \end{bmatrix}\right) = 0 \Rightarrow \lambda^2 - \lambda + 1 = 0 \Rightarrow$$

$$\Rightarrow \lambda = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \Rightarrow \boxed{\lambda = \frac{1}{2} \pm j\frac{\sqrt{3}}{2}}$$



Pelo primeiro método de Lyapunov, $X_e = (x_1, x_2) = (0,0)$ é um ponto de equilíbrio instável do sistema não linear.

1) b) $\begin{cases} \dot{x}_1(t) = -x_1(t) + x_2(t) \\ \dot{x}_2(t) = (x_1(t) + x_2(t)) \operatorname{sen} x_1(t) - 3x_2(t) \end{cases}$

$$f(x) = \begin{bmatrix} -x_1(t) + x_2(t) \\ (x_1(t) + x_2(t)) \operatorname{sen} x_1(t) - 3x_2(t) \end{bmatrix}$$

$$= \begin{bmatrix} -x_1(t) + x_2(t) \\ x_1(t) \operatorname{sen} x_1(t) + x_2(t) \operatorname{sen} x_1(t) - 3x_2(t) \end{bmatrix}$$

$$\frac{\partial f(x)}{\partial x} = \begin{bmatrix} -1 & 1 \\ 1 . \operatorname{sen} x_1(t) + x_1(t) \cos x_1(t) + x_2(t) \cos x_1(t) & \operatorname{sen} x_1(t) - 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ \operatorname{sen} x_1(t) + (x_1(t) + x_2(t)) \cos x_1(t) & \operatorname{sen} x_1(t) - 3 \end{bmatrix}$$

$$x_e = (0,0) \quad A = \left. \frac{\partial f(x)}{\partial x} \right|_{x = x_e} = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}$$

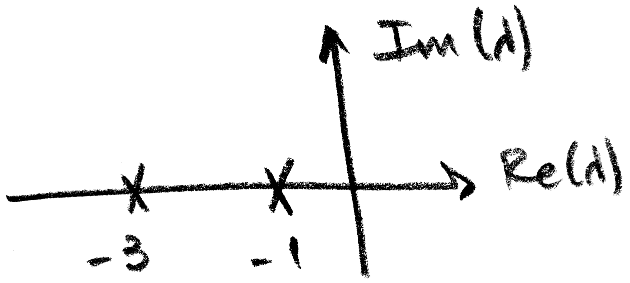
$$\det(\lambda I - A) = 0 \Rightarrow$$

$$\Rightarrow \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}\right) = 0 \Rightarrow$$

$$\Rightarrow \det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}\right) = 0 \Rightarrow$$

$$\Rightarrow \det\left(\begin{bmatrix} \lambda+1 & -1 \\ 0 & \lambda+3 \end{bmatrix}\right) = 0 \Rightarrow$$

$$\Rightarrow (\lambda+1)(\lambda+3) = 0 \Rightarrow \begin{cases} \boxed{\lambda = -1} \\ \boxed{\lambda = -3} \end{cases}$$



Pelo primeiro método de Lyapunov, $x_e = (x_1, x_2) = (0,0)$ é um ponto de equilíbrio assintoticamente estável em seu entorno no sistema não linear.

2) $\ddot{q}(t) = -\dfrac{1}{LC} q(t) - \dfrac{R}{L} \dot{q}(t)$

$$E(t) = \frac{1}{2C} q^2(t) + \frac{L}{2} \dot{q}^2(t) > 0 \quad \forall q(t), \dot{q}(t) \in \mathbb{R}^2 - \{0\}$$

$$\dot{E}(t) = \frac{1}{\cancel{2}C} \cdot \cancel{2} q(t) \dot{q}(t) + \frac{L}{\cancel{2}} \cdot \cancel{2} \dot{q}(t) \cdot \ddot{q}(t) =$$

$$= \frac{1}{2} q(t) \dot{q}(t) + L \dot{q}(t) \underbrace{\left(-\frac{1}{LC} q(t) - \frac{R}{L} \dot{q}(t) \right)}_{\ddot{q}(t)} =$$

$$= \cancel{\frac{1}{C} q(t) \dot{q}(t)} - \cancel{\frac{1}{C} q(t) \dot{q}(t)} - R \cancel{\dot{q}^2(t)}^{\,i^2(t)} \Rightarrow$$

$$\Rightarrow \dot{E}(t) = -R i^2(t) < 0 \quad \forall q(t), \dot{q}(t) \in \mathbb{R}^2 - \{0\}$$

Pelo segundo método de Lyapunov, a origem $x_e = (q(t), \dot{q}(t)) = 0$ é um ponto de equilíbrio assintoticamente estável.

3) A energia cinética é dada por

$$T = \frac{1}{2} m \dot{w}^2(t)$$

A energia potencial elástica da mola é dada por

$$V = \int_0^{w(t)} [k(1 + a^2 w^2(t)) w(t)] \, dw \Rightarrow$$

$$\Rightarrow V = \int_0^{w(t)} [k w(t) + k a^2 w^3(t)] \, dw \Rightarrow$$

$$\Rightarrow V = k \int_0^{w(t)} w(t) \, dw + k a^2 \int_0^{w(t)} w^3(t) \, dw \Rightarrow$$

$$\Rightarrow V = \frac{1}{2} k w^2(t) \Big|_0^{w(t)} + \frac{1}{4} k a^2 w^4(t) \Big|_0^{w(t)} \Rightarrow$$

$$\Rightarrow V = \frac{1}{2} k w^2(t) + \frac{1}{4} k a^2 w^4(t)$$

$$L = T - V = \frac{1}{2} m \dot{w}^2(t) - \frac{1}{2} k w^2(t) - \frac{1}{4} k a^2 w^4(t)$$

$$\frac{\partial L}{\partial w(t)} = -\frac{1}{\cancel{2}}\cdot\cancel{2}\,k\,w(t) - \frac{1}{\cancel{4}}\cdot\cancel{4}\,k a^2 w^3(t) \Rightarrow$$

$$\Rightarrow \frac{\partial L}{\partial w(t)} = -k\,w(t) - k a^2 w^3(t)$$

$$\frac{\partial L}{\partial \dot{w}(t)} = \frac{1}{\cancel{2}}\cdot\cancel{2}\,m\,\dot{w}(t) \Rightarrow \frac{\partial L}{\partial \dot{w}(t)} = m\,\dot{w}(t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{w}(t)}\right) = m\,\ddot{w}(t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{w}(t)}\right) - \frac{\partial L}{\partial w(t)} = H(t) \Rightarrow$$

$$\Rightarrow m\,\ddot{w}(t) + k\,w(t) + k a^2 w^3(t) = -b\,\dot{w}(t) + u(t) \Rightarrow$$

$$\Rightarrow m\,\ddot{w}(t) + b\,\dot{w}(t) + k\,w(t) + k a^2 w^3(t) = u(t) \Rightarrow$$

$$\Rightarrow \boxed{\ddot{w}(t) = -\frac{k}{m}w(t) - \frac{k a^2}{m}w^3(t) - \frac{b}{m}\dot{w}(t) + \frac{1}{m}u(t)}$$