45

Applications of Nuclear Physics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ45.1 Answer (c). We compute the change in mass number A: 235 + 1 – 137 – 96 = 3. All the protons that start out in the uranium nucleus end up in the fission product nuclei.

OQ45.2 Answer (d). The best particles to trigger a fission reaction of the uranium nuclei are slow moving neutrons. Fast moving neutrons may not stay in close proximity with a uranium nucleus long enough to have a good probability of being captured by the nucleus so that a reaction can occur. Positively charged particles, such as protons and alpha particles, have difficulty approaching the target nuclei because of Coulomb repulsion.

OQ45.3 Answer (c). The total energy released was

$$E = (17 \times 10^3 \text{ ton})\left(4.2 \times 10^9 \text{ J/1 ton}\right) = 7.1 \times 10^{13} \text{ J}$$

and according to the mass-energy equivalence, the mass converted was

$$m = \frac{E}{c^2} = \frac{7.1 \times 10^{13} \text{ J}}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} = 7.9 \times 10^{-4} \text{ kg} = 0.79 \text{ g} \sim 1 \text{ g}$$

OQ45.4 The ranking is (b) > (c) > (a) > (d). See Table 45.1 for the RBE factors. Dose (a) is 1 rem. Dose (b) is (1 rad × 10) = 10 rem. Doses (c) and (d) are (1 rad × 4 or 5) = 4 to 5 rem, but dose (d) is to the hands only (less mass has absorbed the radiation). If we assume that (a) and (b) as well as (c) were whole-body doses to many kilograms of tissue (more mass has absorbed the radiation), we find the ranking stated.

OQ45.5 Answer (c). The function of the moderator is to slow down the neutrons released by one fission so that they can efficiently cause more fissions.

OQ45.6 The ranking is $Q_1 > Q_2 > Q_3 > 0$. Because all of the reactions involve 108 nucleons, we can look just at the change in binding-energy-per-nucleon as shown on the curve of binding energy. The jump from lithium to carbon is the biggest jump (~ 5.4 → 7.7 MeV), and next the jump from $A = 27$ to $A = 54$ (~ 8.3 → 8.8 MeV), which is near the peak of the curve. The step up for fission from $A = 108$ to $A = 54$ (~ 8.7 → 8.8 MeV) is smallest. All the reactions result in an increase in binding-energy-per-nucleon, so both of the fusion reactions described and the fission reaction put out energy, so Q is positive for all.

Imagine turning the curve of binding energy upside down so that it bends down like a cross-section of a bathtub. On such a curve of total energy per nucleon versus mass number it is easy to identify the fusion of small nuclei, the fission of large nuclei, and even the alpha decay of uranium, as exoenergetic processes. The most stable nucleus is at the drain of the bathtub, with minimum energy.

OQ45.7 Answer (d). The particles lose energy by collisions with nuclei in the bubble chamber to make their speed and their cyclotron radii $r = mv/qB$ decrease.

OQ45.8 Answer (b). The cyclotron radius is given by

$$r = mv/qB = \sqrt{2\left(\tfrac{1}{2}m^2v^2\right)}\Big/qB = \sqrt{2mK}\Big/qB$$

K and B are the same for both particles, but the ratio $\sqrt{m}/q$ is smaller for the electron; therefore, the path of the electron has a smaller radius, meaning the electron is deflected more.

OQ45.9 Answer (b). The nuclei must be energetic enough to overcome the Coulomb repulsion between them so that they can get close enough to fuse, and numerous enough for many collisions to occur in a short period of time so that the reaction produces more energy than it requires to operate.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ45.1 The two factors presenting the most technical difficulties are the requirements of a high plasma density and a high plasma temperature. These two conditions must occur simultaneously.

CQ45.2 For the deuterium nuclei to fuse, they must be close enough to each other for the nuclear forces to overcome the Coulomb repulsion of the protons—this is why the ion density is a factor. The more time that the nuclei in a sample spend in close proximity, the more nuclei will fuse—hence the confinement time is a factor.

CQ45.3 The products of fusion reactors are generally not themselves unstable, while fission reactions result in a chain of reactions which almost all have some unstable products, because they have an excess of neutrons.

CQ45.4 The advantage of a fission reaction is that it can generate much more electrical energy per gram of fuel compared to fossil fuels. Also, fission reactors do not emit greenhouse gases as combustion byproducts like fossil fuels—the only necessary environmental discharge is heat. The cost involved in producing fissile material is comparable to the cost of pumping, transporting, and refining fossil fuel.

The disadvantage is that some of the products of a fission reaction are radioactive—and some of those have long half-lives. The other problem is that there will be a point at which enough fuel is spent that the fuel rods do not supply power economically and need to be replaced. The fuel rods are still radioactive after removal. Both the waste and the "spent" fuel rods present serious health and environmental hazards that can last for tens of thousands of years. Accidents and sabotage involving nuclear reactors can be very serious, as can accidents and sabotage involving fossil fuels.

CQ45.5 Fusion of light nuclei to a heavier nucleus releases energy. Fission of a heavy nucleus to lighter nuclei releases energy. Both processes are steps towards greater stability on the curve of binding energy, Figure 44.5. The energy release per nucleon is typically greater for fusion, and this process is harder to control.

CQ45.6 The excitation energy comes from the binding energy of the extra nucleon.

CQ45.7 Advantages of fusion: high energy yield, no emission of greenhouse gases, fuel very easy to obtain, reactor cannot go supercritical like a fission reactor and low amounts of radioactive waste.

Disadvantages: requires high energy input to sustain reaction, lithium and helium are scarce, and neutrons released by the reaction cause structural damage to reactor housing.

CQ45.8 For each additional dynode, a larger applied voltage is needed, and hence a larger output from a power supply—"infinite" amplification would not be practical. Nor would it be desirable: the goal is to connect the tube output to a simple counter, so a massive pulse amplitude is not needed. If you made the detector sensitive to weaker and weaker signals, you would make it more and more sensitive to background noise.

CQ45.9 The hydrogen nuclei in water molecules have mass similar to that of a neutron, so that they can efficiently rob a fast-moving neutron of kinetic energy as they scatter it. A neutron bouncing off a more massive nucleus would lose less energy, so it would continue to travel through the shield. Once the neutron is slowed down, a hydrogen nucleus can absorb it in the reaction $\mathrm{n} + {}^1_1\mathrm{H} \to {}^2_1\mathrm{H}$.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 45.1 Interactions Involving Neutrons

Section 45.2 Nuclear Fission

***P45.1** The energy consumed by a 100-W lightbulb in a 1.0-h time period is

$$E = P\Delta t = (100\ \mathrm{J/s})(1.0\ \mathrm{h})\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 3.6\times10^{5}\ \mathrm{J}$$

The number of fission events, yielding an average of 208 MeV each, required to produce this quantity of energy is

$$n = \frac{E}{208\ \mathrm{MeV}} = \frac{3.6\times10^{5}\ \mathrm{J}}{208\ \mathrm{MeV}}\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) = \boxed{1.1\times10^{16}}$$

P45.2 The mass of U-235 producing the same amount of energy as 1 000 kg of coal is

$$\begin{aligned} m &= \left(3.30\times10^{10}\ \mathrm{J}\right)\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) \\ &\quad\times\left(\frac{1\ \text{U-235 nucleus}}{200\ \mathrm{MeV}}\right)\left(\frac{235\ \mathrm{g}}{6.02\times10^{23}\ \mathrm{nucleus}}\right) \\ &= \boxed{0.403\ \mathrm{g}} \end{aligned}$$

P45.3 Three different fission reactions are possible:

$$ {}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{144}_{54}\text{Xe} + 2\,{}^{1}_{0}\text{n} \qquad \boxed{{}^{144}_{54}\text{Xe}} $$

$$ {}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{143}_{54}\text{Xe} + 3\,{}^{1}_{0}\text{n} \qquad \boxed{{}^{143}_{54}\text{Xe}} $$

$$ {}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{142}_{54}\text{Xe} + 4\,{}^{1}_{0}\text{n} \qquad \boxed{{}^{142}_{54}\text{Xe}} $$

P45.4 If the electrical power output of 1.00 GW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$ \frac{1.00\text{ GW}}{0.400} = \left(2.50\times10^{9}\text{ J/s}\right)\left(8.64\times10^{4}\text{ s/d}\right) = 2.16\times10^{14}\text{ J/d} $$

The number of fissions per day is

$$ \left(2.16\times10^{14}\text{ J/d}\right)\left(\frac{1\text{ fission}}{200\times10^{6}\text{ eV}}\right)\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 6.75\times10^{24}\text{ d}^{-1} $$

This also is the number of ^{235}U nuclei used, so the mass of ^{235}U used per day is

$$ \left(6.75\times10^{24}\text{ nuclei/d}\right)\left(\frac{235\text{ g/mol}}{6.02\times10^{23}\text{ nuclei/mol}}\right) = 2.63\times10^{3}\text{ g/d} = \boxed{2.63\text{ kg/d}} $$

In contrast, a coal-burning steam plant producing the same electrical power uses more than 6×10^{6} kg/d of coal.

P45.5 First, the thorium is bombarded:

$$ {}^{1}_{0}\text{n} + {}^{232}_{90}\text{Th} \rightarrow {}^{233}_{90}\text{Th} $$

Then, the thorium decays by beta emission:

$$ {}^{233}_{90}\text{Th} \rightarrow {}^{233}_{91}\text{Pa} + {}^{0}_{-1}\text{e} + \bar{\nu} $$

Protactinium-233 has more neutrons than the more stable protactinium-231, so it too decays by beta emission:

$$ {}^{233}_{91}\text{Pa} \rightarrow {}^{233}_{92}\text{U} + {}^{0}_{-1}\text{e} + \bar{\nu} $$

P45.6 (a) The energy released is equal to the Q value, given by

$$ Q = (\Delta m)c^2 = \left[m_n + M_{\text{U-235}} - M_{\text{Ba-141}} - M_{\text{Kr-92}} - 3m_n\right]c^2 $$

with

$$ \Delta m = [1.008\,665\text{ u} + 235.043\,923\text{ u} - 140.914\,4\text{ u} - 91.926\,2\text{ u} - 3(1.008\,665\text{ u})] = 0.185\,993\text{ u} $$

Then,

$$Q = (0.185\ 993\ \text{u})(931.5\ \text{MeV/u}) = \boxed{173\ \text{MeV}}$$

(b) The fraction of rest energy transformed is

$$f = \frac{\Delta m}{m_i} = \frac{0.185\ 993\ \text{u}}{236.05\ \text{u}} = 7.88 \times 10^{-4} = \boxed{0.078\ 8\%}$$

P45.7 The energy released in the reaction ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{88}_{38}\text{Sr} + {}^{136}_{54}\text{Xe} + 12\,{}^{1}_{0}\text{n}$ is

$$\begin{aligned} Q = (\Delta m)c^2 &= \left[m_{{}^{235}_{92}\text{U}} - 11m_{\text{n}} - m_{{}^{88}_{38}\text{Sr}} - m_{{}^{136}_{54}\text{Xe}} \right] c^2 \\ &= [235.043\ 923\ \text{u} - 11(1.008\ 665\ \text{u}) \\ &\qquad -87.905\ 614\ \text{u} - 135.907\ 220\ \text{u}](931.5\ \text{MeV/u}) \\ &= \boxed{126\ \text{MeV}} \end{aligned}$$

P45.8 In N collisions, the energy is reduced from 2.00 MeV to 0.039 eV:

$$\left(2.00 \times 10^6\ \text{eV}\right)\left(\frac{1}{2}\right)^N \leq 0.039\ \text{eV}$$

$$\left(\frac{1}{2}\right)^N \leq \frac{0.039}{2.00 \times 10^6}$$

$$N \ln\left(\frac{1}{2}\right) \leq \ln\left(\frac{0.039}{2.00 \times 10^6}\right)$$

$$N \ln(2) \geq \ln\left(\frac{2.00 \times 10^6}{0.039}\right)$$

which gives

$$N \geq 25.6 \quad \rightarrow \quad N = \boxed{26}$$

P45.9 The mass defect is

$$\begin{aligned} \Delta m &= (m_n + M_{\text{U}}) - (M_{\text{Zr}} + M_{\text{Te}} + 3m_n) \\ \Delta m &= [1.008\ 665\ \text{u} + 235.043\ 923\ \text{u} \\ &\qquad - 97.912\ 7\ \text{u} - 134.916\ 5\ \text{u} - 3(1.008\ 665\ \text{u})] \\ &= 0.197\ 393\ \text{u} \end{aligned}$$

The energy equivalent is

$$\Delta mc^2 = (0.197\ 393\ \text{u})c^2 \left(\frac{931.5\ \text{MeV}/c^2}{\text{u}}\right) = \boxed{184\ \text{MeV}}$$

P45.10 (a) At a concentration of $c = 3\text{ mg/m}^3 = 3 \times 10^{-3}\text{ g/m}^3$, the mass of uranium dissolved in the oceans covering two-thirds of Earth's surface to an average depth of $h_{\text{avg}} = 4$ km is

$$m_{\text{U}} = cV = c\left(\tfrac{2}{3}A\right)\cdot h_{\text{av}} = c\left[\tfrac{2}{3}\left(4\pi R_E^2\right)\right]\cdot h_{\text{av}}$$

or

$$m_{\text{U}} = \left(3\times 10^{-3}\ \frac{\text{g}}{\text{m}^3}\right)\left(\frac{2}{3}\right)4\pi\left(6.38\times 10^6\ \text{m}\right)^2\left(4\times 10^3\ \text{m}\right)$$
$$= \boxed{4\times 10^{15}\ \text{g}}$$

(b) Fissionable ^{235}U makes up 0.700% of the mass of uranium computed above. If we assume all of the ^{235}U is collected and caused to undergo fission, with the release of about 200 MeV per event, the potential energy supply is

$$E = \left(\text{number of }^{235}\text{U atoms}\right)(200\text{ MeV})$$
$$= \frac{0.700}{100}\left(\frac{m_{\text{U}}}{m_{^{235}U_{\text{atom}}}}\right)(200\text{ MeV})$$

and at a consumption rate of $P = 1.5 \times 10^{13}$ J/s, the time interval this could supply the world's energy needs is $\Delta t = E/P$, or

$$\Delta t = \frac{0.700}{100}\left(\frac{m_{\text{U}}}{m_{^{235}U_{\text{atom}}}}\right)\frac{(200\text{ MeV})}{P}$$
$$= \frac{0.700}{100}\left[\frac{4\times 10^{15}\ \text{g}}{(235\text{ u})\left(1.66\times 10^{-27}\ \text{kg/u}\right)}\left(\frac{1\text{ kg}}{10^3\ \text{g}}\right)\right]$$
$$\times\left[\left(\frac{200\text{ MeV}}{1.50\times 10^{13}\ \text{J/s}}\right)\left(\frac{1.60\times 10^{-13}\ \text{J}}{1\text{ MeV}}\right)\left(\frac{1\text{ yr}}{3.16\times 10^7\ \text{s}}\right)\right]$$
$$= \boxed{5\times 10^3\ \text{yr}}$$

(Compare this value to that in part (b) of Problem 17, which is a more realistic estimate of the time interval for the uranium that can be extracted reasonably from the Earth.)

(c) $\boxed{\text{The uranium comes from rocks and minerals dissolved in water and carried into the ocean by rivers.}}$

(d) $\boxed{\text{No.}}$ Uranium cannot be replenished by the radioactive decay of other elements on Earth.

P45.11 One kg of enriched uranium contains 3.40% ${}^{235}_{92}\text{U}$, so the mass of uranium-235 is

$$m_{235} = 0.034\,0(1\,000\text{ g}) = 34.0\text{ g}$$

In terms of number of nuclei, this is equivalent to

$$N_{235} = (34.0\text{ g})\left(\frac{1}{235\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right)$$
$$= 8.71\times10^{22}\text{ nuclei}$$

If all these nuclei fission, the energy released is equal to

$$\left(8.71\times10^{22}\text{ nuclei}\right)\left(200\times10^{6}\text{ eV/nucleus}\right)$$
$$\times\left(1.602\times10^{-19}\text{ J/eV}\right) = 2.79\times10^{12}\text{ J}$$

Now, for the engine,

$$\text{efficiency} = \frac{\text{work output}}{\text{heat input}} \quad \text{or} \quad e = \frac{P\Delta r\cos\theta}{Q_h}$$

So the distance the ship can travel per kilogram of uranium fuel is

$$\Delta r = \frac{eQ_h}{P\cos(0°)} = \frac{0.200\left(2.79\times10^{12}\text{ J}\right)}{1.00\times10^{5}\text{ N}} = \boxed{5.58\times10^{6}\text{ m}}$$

Section 45.3 Nuclear Reactors

***P45.12** (a) With a specific gravity of 4.00, the density of soil is $\rho = 4.00\times10^{3}\text{ kg/m}^3$. Thus, the mass of the top 1.00 m of soil is

$$m = \rho V = \left(4.00\times10^{3}\text{ kg/m}^3\right)\left[(1.00\text{ m})\left(43\,560\text{ ft}^2\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)^2\right]$$
$$= 1.62\times10^{7}\text{ kg}$$

At a rate of 1 part per million, the mass of uranium in this soil is

$$m_{\text{U}} = \frac{m}{10^6} = \frac{1.62\times10^{7}\text{ kg}}{10^6} = \boxed{16.2\text{ kg}}$$

(b) Since 0.720% of naturally occurring uranium is ${}^{235}_{92}\text{U}$, the mass of ${}^{235}_{92}\text{U}$ in the soil of part (a) is

$$m_{{}^{235}_{92}\text{U}} = \left(7.20\times10^{-3}\right)m_{\text{U}} = \left(7.20\times10^{-3}\right)(16.2\text{ kg})$$
$$= 0.117\text{ kg} = \boxed{117\text{ g}}$$

P45.13 In one minute there are $N = \dfrac{60.0\text{ s}}{1.20 \times 10^{-3}\text{ s}} = 5.00 \times 10^4$ fissions.

So the rate increases by a factor of $(1.000\,25)^{50\,000} = \boxed{2.68 \times 10^5}$.

P45.14 (a) For a sphere: $V = \dfrac{4}{3}\pi r^3 \rightarrow r = \left(\dfrac{3V}{4\pi}\right)^{1/3}$, so

$$\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = \frac{3}{r} = \left(\frac{36\pi}{V}\right)^{1/3} = \boxed{4.84V^{-1/3}}$$

(b) For a cube: $V = \ell^3 \rightarrow \ell = V^{1/3}$, so

$$\frac{A}{V} = \frac{6\ell^2}{\ell^3} = \frac{6}{\ell} = \boxed{6V^{-1/3}}$$

(c) For a parallelepiped: $V = 2a^3 \rightarrow a = \left(\dfrac{V}{2}\right)^{1/3}$, so

$$\frac{A}{V} = \frac{(2a^2 + 8a^2)}{2a^3} = \frac{5}{a} = 5\left(\frac{2}{V}\right)^{1/3} = \left(\frac{250}{V}\right)^{1/3} = \boxed{6.30V^{-1/3}}$$

(d) The answers show that the sphere has the smallest surface area for a given volume and the brick has the greatest surface area of the three. Therefore, $\boxed{\text{The sphere has minimum leakage and the parallelepiped has maximum leakage.}}$

P45.15 Recall the radius of a nucleus of mass number A is $r = aA^{1/3}$, where $a = 1.2$ fm. The center to center distance of the nuclei of helium ($A = 4$) and gold ($A = 197$) is the sum of their combined radii:

$$r = (1.2\text{ fm})(4)^{1/3} + (1.2\text{ fm})(197)^{1/3} = 8.9\text{ fm} = 8.9 \times 10^{-15}\text{ m}$$

The electric potential energy is

$$U = qV = \frac{k_e q_1 q_2}{r}$$

$$= \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ (2)(79)(1.60 \times 10^{-19}\ \text{C})\ e}{8.9 \times 10^{-15}\ \text{m}}$$

$$= 2.6 \times 10^7\ \text{eV} = \boxed{26\ \text{MeV}}$$

P45.16 The power after three months is $P = 10.0$ MW $= 1.00 \times 10^7$ J/s. If each decay delivers 1.00 MeV $= 1.60 \times 10^{-13}$ J, then the number of decays/s

$$= \frac{1.00 \times 10^7\ \text{J/s}}{1.60 \times 10^{-13}\ \text{J}} = \boxed{6.25 \times 10^{19}\ \text{Bq}}$$

P45.17 (a) Do not think of the "reserve" as being held in reserve. We are depleting it as fast as we choose. The remaining current balance of irreplaceable ^{235}U is 0.7% of the whole mass of uranium:

$$(0.007\ 00)\left(4.40\times10^{6}\text{ tons}\right)\left(\frac{10^{3}\text{ kg}}{1\text{ ton}}\right)\left(\frac{10^{3}\text{ g}}{1\text{ kg}}\right)=\boxed{3.08\times10^{10}\text{ g}}$$

(b) The number of moles of ^{235}U in the reserve is

$$n=\frac{m}{M}=\frac{3.08\times10^{10}\text{ g}}{235\text{ g/mole}}=\boxed{1.31\times10^{8}\text{ mole}}$$

(c) The number of moles found in part (b) corresponds to

$$N=nN_{\text{A}}=\left(1.31\times10^{8}\text{ mole}\right)\left(\frac{6.02\times10^{23}\text{ atom}}{1\text{ mole}}\right)\left(\frac{1\text{ nucleus}}{1\text{ atom}}\right)$$
$$=\boxed{7.89\times10^{31}\text{ nuclei}}$$

(d) We imagine each nucleus as fissioning, to release

$$\left(7.89\times10^{31}\text{ fissions}\right)\left(\frac{200\text{ MeV}}{1\text{ fission}}\right)\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)=\boxed{2.52\times10^{21}\text{ J}}$$

(e) The definition of power is represented by $P=(\text{energy converted})/\Delta t$, so we have

$$\Delta t=\frac{\text{energy}}{P}=\frac{2.52\times10^{21}\text{ J}}{1.5\times10^{13}\text{ J/s}}=\left(1.68\times10^{8}\text{ s}\right)\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right)$$
$$=\boxed{5.33\text{ yr}}$$

(f) Fission is not sufficient to supply the entire world with energy at a price of \$130 or less per kilogram of uranium.

P45.18 Assuming that the impossibility is *not* that he can have this control over the process (which, as far as we know presently, *is* impossible), let's see what else might be wrong. The reaction can be written

$${}^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\rightarrow{}^{141}_{57}\text{La}+{}^{94}_{35}\text{Br}+n\left({}^{1}_{0}\text{n}\right)$$

where n is the number of neutrons released in the fission reaction. By balancing the equation for electric charge and number of nucleons, we find that $n=1$. If one incoming neutron results in just one outgoing neutron, the possibility of a chain reaction is not there, so this nuclear reactor will not work.

***P45.19** The total energy required for one year is

$$E = (2\ 000\ \text{kWh/month})\left(3.60\times10^{6}\ \text{J/kWh}\right)(12.0\ \text{months})$$
$$= 8.64\times10^{10}\ \text{J}$$

The number of fission events needed will be

$$N = \frac{E}{E_{\text{event}}} = \frac{8.64\times10^{10}\ \text{J}}{(208\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)} = 2.60\times10^{21}$$

and the mass of this number of ^{235}U atoms is

$$m = \left(\frac{N}{N_A}\right)M_{\text{mol}} = \left(\frac{2.60\times10^{21}\ \text{atoms}}{6.02\times10^{23}\ \text{atoms/mol}}\right)(235\ \text{g/mol})$$
$$= \boxed{1.01\ \text{g}}$$

P45.20 (a) Since $K = p^2/2m$, we have

$$p = \sqrt{2mK} = \sqrt{2m\left(\frac{3}{2}k_B T\right)}$$
$$= \sqrt{3(1.675\times10^{-27}\ \text{kg})(1.38\times10^{-23}\ \text{J/K})\ (300\ \text{K})}$$
$$= \boxed{4.56\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(b) The de Broglie wavelength of the particle is

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{4.56\times10^{-24}\ \text{kg}\cdot\text{m/s}} = 1.45\times10^{-10}\ \text{m} = \boxed{0.145\ \text{nm}}$$

(c) This size has the same order of magnitude as an atom's outer electron cloud, and is vastly larger than a nucleus.

Section 45.4 Nuclear Fusion

P45.21 (a) Helium fusion proceeds according to

$${}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} \to \boxed{{}^{8}_{4}\text{Be}} + \gamma$$

(b) The beryllium produced by helium fusion fuses with another alpha particle according to

$${}^{8}_{4}\text{Be} + {}^{4}_{2}\text{He} \to \boxed{{}^{12}_{6}\text{C}} + \gamma$$

(c) The total energy released in this pair of fusion reactions is

$$Q = (\Delta m)c^2 = \left[2m_{^4\text{He}} - m_{^8B}\right]c^2 + \left[m_{^8B} + m_{^4\text{He}} - m_{^{12}\text{C}}\right]c^2$$
$$= \left[3m_{^4\text{He}} - m_{^{12}\text{C}}\right]c^2$$
$$= \left[3(4.002\ 602\ \text{u}) - 12.000\ 000\ \text{u}\right](931.5\ \text{MeV/u})$$
$$= \boxed{7.27\ \text{MeV}}$$

P45.22 From Equation 45.2, the energy released in the reaction ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$ is 17.59 MeV per event. The total energy required for the year is

$$E = (2\ 000\ \text{kWh/month})(12.0\ \text{months})(3.60\times10^6\ \text{J/kWh})$$
$$= 8.64\times10^{10}\ \text{J}$$

so the number of fusion events needed for the year is

$$N = \frac{E}{Q} = \frac{8.64\times10^{10}\ \text{J}}{(17.59\ \text{MeV/event})(1.602\times10^{-13}\ \text{J/MeV})}$$
$$= \boxed{3.07\times10^{22}\ \text{events}}$$

P45.23 The energy released in the reaction ${}^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \gamma$ is

$$Q = (\Delta m)c^2 = \left[m_{{}^1_1\text{H}} + m_{{}^2_1\text{H}} - m_{{}^3_2\text{He}}\right]c^2$$
$$= \left[1.007\ 825\ \text{u} + 2.014\ 102\ \text{u} - 3.016\ 029\ \text{u}\right](931.5\ \text{MeV/u})$$
$$= \boxed{5.49\ \text{MeV}}$$

P45.24 (a) We assume that the nuclei are stationary at closest approach, so that the electrostatic potential energy equals the total energy E. Then, from the isolated system model,

$$K_f + U_f = K_i + U_i \quad \rightarrow \quad U_f = E$$

then,

$$\frac{k_e(Z_1 e)(Z_2 e)}{r_{\min}} = E$$

$$E = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2 Z_1 Z_2}{1.00\times10^{-14}\ \text{m}}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right)$$
$$= (144\ \text{keV})Z_1 Z_2$$

or $\boxed{E = 144 Z_1 Z_2 \text{ where } E \text{ is in keV.}}$

(b) $\boxed{\text{The energy is proportional to each atomic number.}}$

(c) $\boxed{\text{Take } Z_1 = 1 \text{ and } Z_2 = 59 \text{ or vice versa. This choice minimizes the product } Z_1 Z_2.}$ If extra cleverness is allowed, take $Z_1 = 0$ and $Z_2 = 60$: use neutrons as the bombarding particles. A neutron is a nucleon but not an atomic nucleus.

(d) For both the D-D and the D-T reactions, $Z_1 = Z_2 = 1$. Thus, the minimum energy required in both cases is

$$E = \left(2.30 \times 10^{-14}\ \text{J}\right)\left(\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right)$$
$$= \boxed{144\ \text{keV for both, according to this model.}}$$

Section 45.4 in the text gives more accurate values for the critical ignition temperatures, of about 52 keV for D-D fusion and 6 keV for D-T fusion. The nuclei can fuse by tunneling. A triton moves more slowly than a deuteron at a given temperature. Then D-T collisions last longer than D-D collisions and have much greater tunneling probabilities.

P45.25 (a) The Q value for the D-T reaction is 17.59 MeV (from Equation 45.4). Specific energy content in fuel for D-T reaction (from Table 44.2, mass = 2.014 u + 3.016 u = 5.030 u):

$$\frac{(17.59\ \text{MeV})\left(1.60 \times 10^{-13}\ \text{J/MeV}\right)}{(5.030\ \text{u})\left(1.6605 \times 10^{-27}\ \text{kg/u}\right)} = 3.37 \times 10^{14}\ \text{J/kg}$$

The rate of fuel burning for the D-T reaction is then

$$r_{\text{DT}} = \frac{\left(3.00 \times 10^{9}\ \text{J/s}\right)\left(3\,600\ \text{s/hr}\right)}{\left(3.37 \times 10^{14}\ \text{J/kg}\right)\left(10^{-3}\ \text{kg/g}\right)}$$
$$= \boxed{32.1\ \text{g/h burning of D and T}}$$

(b) Using energy values from Equation 45.4, the specific energy content in fuel for D-D reaction is:

$$Q = \frac{1}{2}(3.27 + 4.03) = 3.65\ \text{MeV}$$

From Table 44.2, the D-D mass is = 2(2.014 u) = 4.018 u. The specific energy content in D-D fuel is

$$\frac{(3.65\ \text{MeV})\left(1.60 \times 10^{-13}\ \text{J/MeV}\right)}{(4.028\ \text{u})\left(1.6605 \times 10^{-27}\ \text{kg/u}\right)} = 8.73 \times 10^{13}\ \text{J/kg}$$

and the rate of fuel burning for the D-D reaction is

$$r_{\mathrm{DD}} = \frac{(3.00\times 10^{9}\ \mathrm{J/s})(3\,600\ \mathrm{s/hr})}{(8.73\times 10^{13}\ \mathrm{J/kg})(10^{-3}\ \mathrm{kg/g})} = \boxed{124\ \mathrm{g/h}}$$

P45.26 (a) The radius of a nucleus with mass number A is $r = aA^{1/3}$, where $a = 1.2$ fm. The distance of closest approach is equal to the center to center distance of the two nuclei:

$$r_f = r_{\mathrm{D}} + r_{\mathrm{T}} = (1.20\times 10^{-15}\ \mathrm{m})\left[(2)^{1/3} + (3)^{1/3}\right]$$
$$= 3.24\times 10^{-15}\ \mathrm{m} = \boxed{3.24\ \mathrm{fm}}$$

(b) At this distance, the electric potential energy is

$$U_f = \frac{k_e e^2}{r_f} = \frac{(8.99\times 10^{9}\ \mathrm{N\cdot m^2/C^2})(1.60\times 10^{-19}\ \mathrm{C})^2}{3.24\times 10^{-15}\ \mathrm{m}}$$
$$= 7.10\times 10^{-14}\ \mathrm{J} = \boxed{444\ \mathrm{keV}}$$

(c) Conserving momentum, $m_{\mathrm{D}} v_i = (m_{\mathrm{D}} + m_{\mathrm{T}}) v_f$ or

$$v_f = \left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right) v_i = \boxed{\frac{2}{5} v_i}$$

(d) To find the minimum initial kinetic energy of the deuteron, we use $K_i + U_i = K_f + U_f$, where $U_i = 0$ because the deuteron starts from very far away (infinity), and with the result from part (c),

$$K_i + 0 = \frac{1}{2}(m_{\mathrm{D}} + m_{\mathrm{T}}) v_f^2 + U_f$$
$$K_i = \frac{1}{2}(m_{\mathrm{D}} + m_{\mathrm{T}})\left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right)^2 v_i^2 + U_f$$

With some re-arrangement, we have

$$K_i = \left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right)\left(\frac{1}{2} m_{\mathrm{D}} v_i^2\right) + U_f = \left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right) K_i + U_f$$

or

$$\left(1 - \frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right) K_i = U_f$$

solving for the initial kinetic energy then gives

$$K_i = U_f \left(\frac{m_{\mathrm{D}} + m_{\mathrm{T}}}{m_{\mathrm{T}}}\right) = \frac{5}{3}(444\ \mathrm{keV}) = \boxed{740\ \mathrm{keV}}$$

(e) The nuclei can fuse $\boxed{\text{possibly by tunneling}}$ through the potential energy barrier.

P45.27 (a) $V = \left(317\times10^{6}\ \text{mi}^3\right)\left(\dfrac{1\,609\ \text{m}}{1\ \text{mi}}\right)^3 = 1.32\times10^{18}\ \text{m}^3$

From the periodic table, H has atomic mass 1.007 9 and O has atomic mass 15.999 4, so water has atomic mass 18.015 2.

$$m_{\text{water}} = \rho V = \left(10^3\ \text{kg/m}^3\right)\left(1.32\times10^{18}\ \text{m}^3\right) = 1.32\times10^{21}\ \text{kg}$$

$$\begin{aligned} m_{H_2} &= \left(\frac{M_{H_2}}{M_{H_2O}}\right)m_{H_2O} = \left(\frac{2.016}{18.015}\right)\left(1.32\times10^{21}\ \text{kg}\right) \\ &= 1.48\times10^{20}\ \text{kg} \end{aligned}$$

$$\begin{aligned} m_{\text{Deuterium}} &= (0.030\,0\%)m_{H_2} = \left(0.030\,0\times10^{-2}\right)\left(1.48\times10^{20}\ \text{kg}\right) \\ &= 4.43\times10^{16}\ \text{kg} \end{aligned}$$

The number of deuterium nuclei in this mass is

$$N = \frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}} = \frac{4.43\times10^{16}\ \text{kg}}{(2.014\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)} = 1.33\times10^{43}$$

Since two deuterium nuclei are used per fusion, ${}^2_1\text{H} + {}^2_1\text{H} \to {}^4_2\text{He}$, the number of events is $\dfrac{N}{2} = 6.63\times10^{42}$.

The energy released per event is

$$\begin{aligned} Q &= \left[M_{{}^2\text{H}} + M_{{}^2\text{H}} - M_{{}^4\text{He}}\right]c^2 \\ &= [2(2.014\ 102) - 4.002\ 603]\text{u}(931.5\ \text{MeV/u}) \\ &= 23.8\ \text{MeV} \end{aligned}$$

The total energy available is then

$$\begin{aligned} E &= \left(\frac{N}{2}\right)Q = \left(6.63\times10^{42}\right)(23.8\ \text{MeV})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) \\ &= \boxed{2.53\times10^{31}\ \text{J}} \end{aligned}$$

(b) The time this energy could possibly meet world requirements is

$$\begin{aligned} \Delta t &= \frac{E}{P} = \frac{2.53\times10^{31}\ \text{J}}{100\left(1.50\times10^{13}\ \text{J/s}\right)} = \left(1.69\times10^{16}\ \text{s}\right)\left(\frac{1\ \text{yr}}{3.16\times10^{7}\ \text{s}}\right) \\ &= \boxed{5.34\times10^{8}\ \text{yr}} \end{aligned}$$

P45.28 (a) Including both ions and electrons, the number of particles in the plasma is $N = 2nV$, where n is the ion density and V is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E = \frac{3}{2}Nk_BT = 3nVk_BT$$

$$= 3\left(2.00\times10^{13}\ \text{cm}^{-3}\right)\left[\left(50.0\ \text{m}^3\right)\left(\frac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right]$$

$$\times\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.00\times10^8\ \text{K}\right)$$

$$E = \boxed{1.66\times10^7\ \text{J}}$$

(b) The specific heat of water is $c = 4\,186$ J/kg · °C, and the energy required to raise the temperature of one kilogram of water from 27.0°C to 100°C is given by Equation 20.4:

$$Q = mc\Delta T = (1.00\ \text{kg})(4\,186\ \text{J/kg}\cdot°\text{C})(100°\text{C} - 27.0°\text{C})$$

$$= 3.06\times10^5\ \text{J}$$

From Table 20.2, the heat of vaporization of water is $L_v = 2.26\times10^6\ \text{J/kg}$, so that a total of

$$E_{1\ \text{kg}} = 3.06\times10^5\ \text{J} + 2.26\times10^6\ \text{J} = 2.57\times10^6\ \text{J}$$

is required to boil away each kilogram of water initially at 27.0°C. The mass of water that could be boiled away is therefore

$$m = \frac{E}{E_{1\ \text{kg}}} = \frac{1.66\times10^7\ \text{J}}{2.57\times10^6\ \text{J/kg}} = \boxed{6.45\ \text{kg}}$$

P45.29 (a) Taking $m \approx 2m_p$ for deuterons, we have

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

The root-mean-square speed is

$$v_{rms} = \sqrt{\frac{3k_BT}{2m_p}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.00\times10^8\ \text{K}\right)}{2\left(1.67\times10^{-27}\ \text{kg}\right)}}$$

$$= \boxed{2.23\times10^6\ \text{m/s}}$$

(b) The confinement time in the absence of confinement measures is

$$\Delta t = \frac{x}{v} = \frac{0.100\ \text{m}}{2.23\times10^6\ \text{m/s}} \sim 10^{-7}\ \text{s}$$

P45.30 (a) By adding 1 + 6 = 7 and 1 + 12 = 13, we have ${}^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma$ so nucleus *A* is $\boxed{{}^{13}\mathrm{N}.}$

(b) Now 13 – 0 = 13 and 7 – 1 = 6, so the positron decay is ${}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + {}^{0}_{1}\mathrm{e} + \nu$ and nucleus *B* is $\boxed{{}^{13}\mathrm{C}.}$

(c) Similarly, we have ${}^{1}_{1}\mathrm{H} + {}^{13}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma$ and nucleus *C* is $\boxed{{}^{14}\mathrm{N}.}$

(d) The hydrogen nuclei keep piling on like rugby players after a tackle. We have ${}^{1}_{1}\mathrm{H} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{15}_{8}\mathrm{O} + \gamma$ and nucleus *D* is $\boxed{{}^{15}\mathrm{O}.}$

(e) Now ${}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + {}^{0}_{1}\mathrm{e} + \nu,$ so nucleus *E* is $\boxed{{}^{15}\mathrm{N}.}$

(f) We calculate 15 + 1 – 4 = 12 and 7 + 1 – 2 = 6 to identify ${}^{1}_{1}\mathrm{H} + {}^{15}_{7}\mathrm{N} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}$ and nucleus *F* is $\boxed{{}^{12}\mathrm{C}.}$

(g) $\boxed{\text{The original carbon-12 nucleus is returned.}}$ One carbon nucleus can participate in the fusions of colossal numbers of hydrogen nuclei, four after four. Carbon is a catalyst.

The two positrons immediately annihilate with electrons according to ${}^{0}_{1}\mathrm{e} + {}^{0}_{-1}\mathrm{e} \rightarrow 2\gamma.$ The overall reaction, obtained by adding all eight reactions, can be represented as

$${}^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + 2\,{}^{0}_{-1}\mathrm{e} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{12}_{6}\mathrm{C} + 7\gamma + 2\nu$$

This simplifies to $\boxed{4\left({}^{1}_{1}\mathrm{H}\right) + 2\,{}^{0}_{-1}\mathrm{e} \rightarrow {}^{4}_{2}\mathrm{He} + 2\nu}$. The net reaction is identical to the net reaction in the proton–proton cycle which predominates in the Sun. In energy terms the reaction can be considered as $4\left({}^{1}_{1}\mathrm{H\ atom}\right) \rightarrow {}^{4}_{2}\mathrm{He\ atom} + 26.7\ \mathrm{MeV},$ where the *Q* value of energy output was computed in Chapter 39, Problem 67 and again in Problem 59 in this chapter.

P45.31 (a) Lawson's criterion for the D-T reaction is $n\tau \geq 10^{14}\ \mathrm{s/cm^3}$. For a confinement time of $\tau = 1.00$ s, this requires a minimum ion density of $n = \boxed{10^{14}\ \mathrm{cm^{-3}}}$.

(b) At the ignition temperature of $T = 4.5 \times 10^7$ K and the ion density found above, the plasma pressure is

$$P = 2nk_{\mathrm{B}}T$$
$$= 2\left[\left(10^{14}\ \mathrm{cm^{-3}}\right)\left(\frac{10^6\ \mathrm{cm^3}}{1\ \mathrm{m^3}}\right)\right]\left(1.38 \times 10^{-23}\ \mathrm{J/K}\right)\left(4.5 \times 10^7\ \mathrm{K}\right)$$
$$= \boxed{1.2 \times 10^5\ \mathrm{J/m^3}}$$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \geq 10P = 10\left(1.2\times10^5 \text{ J/m}^3\right) = 1.2\times10^6 \text{ J/m}^3$$

which requires a magnetic field of magnitude

$$B \geq \sqrt{2\mu_0(10P)} = \sqrt{2\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(1.24\times10^6\text{ J/m}^3\right)}$$
$$= \boxed{1.8\text{ T}}$$

This is a very strong field.

Section 45.5 Radiation Damage

P45.32 (a) The number of x-ray images made per year is (assuming a 2-week vacation)

$$n = (8\text{ x-ray/d})(5\text{ d/wk})(50\text{ wk/yr}) = 2.0\times10^3\text{ x-ray/yr}$$

The average dose per photograph is

$$\frac{5.0\text{ rem/yr}}{2.0\times10^3\text{ x-ray/yr}} = 2.5\times10^{-3}\text{ rem/x-ray} = \boxed{2.5\text{ mrem/x-ray}}$$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The ration dose of 5.0 rem/yr received as a result of the job to background is

$$\frac{5.0\text{ rem/yr}}{0.13\text{ rem/yr}} = 38$$

$\boxed{\text{The technician's occupational exposure is high compared to background radiation—it is 38 times 0.13 rem/yr.}}$

P45.33 (a) $I = I_0 e^{-\mu x}$, so $x = -\frac{1}{\mu}\ln\left(\frac{I}{I_0}\right)$, with $\mu = 1.59\text{ cm}^{-1}$.

When the intensity $I = \frac{I_0}{2}$,

$$x = -\frac{1}{1.59\text{ cm}^{-1}}\ln\left(\frac{1}{2}\right) = \boxed{0.436\text{ cm}}$$

(b) When $I = \frac{I_0}{1.00\times10^4}$,

$$x = -\frac{1}{1.59\text{ cm}^{-1}}\ln\left(\frac{1}{1.00\times10^4}\right) = \boxed{5.79\text{ cm}}$$

P45.34 (a) $I = I_0 e^{-\mu x}$, so $x = -\dfrac{1}{\mu}\ln\left(\dfrac{I}{I_0}\right)$.

When intensity $I = \dfrac{I_0}{2}$, $x = -\dfrac{1}{\mu}\ln\left(\dfrac{I}{I_0}\right) = -\dfrac{1}{\mu}\ln\left(\dfrac{1}{2}\right) = \boxed{\dfrac{\ln(2)}{\mu}}$.

(b) When intensity $I = f I_0$, $x = -\dfrac{1}{\mu}\ln\left(\dfrac{I}{I_0}\right) = -\dfrac{1}{\mu}\ln(f) = \boxed{-\dfrac{\ln f}{\mu}}$.

P45.35 The source delivers 100 mrad of 2.00-MeV γ-rays/h at a 1.00-m distance. The RBE for these γ-rays is 1.0 (from Table 45.1).

(a) From Equation 45.6,

$$\text{dose in rem} = \text{dose in rad} \times \text{RBE}$$

$$1.00 \text{ rem} = \text{dose in rad} \times 1.0$$

or, $\text{dose in rad} = 1.00 \text{ rad} = \left(100\times10^{-3} \text{ rad/h}\right)\Delta t$

which gives $\Delta t = 10.0$ h.

Thus a person would have to stand there $\boxed{\text{10.0 hours}}$ to receive 1.00 rem from a 100-mrad/h source.

(b) If the γ-radiation is emitted isotropically, the dosage rate falls off as $\dfrac{1}{r^2}$.

Thus a dosage 10.0 mrad/h would be received at a distance $r = \sqrt{10.0}$ m $= \boxed{3.16 \text{ m}}$.

***P45.36** For each gray (GY) or radiation, 1 J of energy is delivered to each kilogram of absorbing material. Thus, the total energy delivered in this whole body dose to a 75.0-kg person is

$$E = (0.250 \text{ Gy})\left(1 \ \frac{\text{J/kg}}{\text{Gy}}\right)(75.0 \text{ kg}) = \boxed{18.8 \text{ J}}$$

P45.37 By definition, one rad increases the energy of one kilogram of the absorbing material by 1.00×10^{-2} J. The energy starts as energy carried by electromagnetic radiation, and turns entirely into internal energy. The 1 000 rad or 10.0 gray = 10.0 Gy will then put 10.0 J/kg into the body, to raise its temperature by the same amount as 10.0 J/kg of energy input by heat from a higher-temperature energy source. In $Q = mc\Delta T$ we have Q/m = 10.0 J/kg and

$$\Delta T = \frac{Q}{m}\frac{1}{c} = (10.0 \text{ J/kg})\left(\frac{1}{4\ 186 \text{ J/kg}\cdot{}^\circ\text{C}}\right) = \boxed{2.39\times10^{-3}\ {}^\circ\text{C}}$$

P45.38 Assume all the energy from the x-ray machine is absorbed by the water and that no energy leaves the cup of water by heat or thermal radiation. The energy input to the cup and the temperature of the water are related by

$$T_{ER} = mc\Delta T$$

Because the power input P is equal to $T_{ER}/\Delta T$, we have

$$P\Delta t = mc\Delta T \quad \rightarrow \quad \Delta t = \frac{mc\Delta T}{P}$$

where we have solved for the time interval required to raise the temperature of the water. We note that the temperature of the water will increase until it is 100°C, after which the latent heat of vaporization of $L_v = 2.26 \times 10^6$ J/kg would have to be added to boil the water. For the purposes of this problem, we limit ourselves to increasing the temperature of the water to 100°C. Substituting numerical values gives

$$\Delta t = \frac{m(4\ 186\ \text{J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C})}{(10.0\ \text{rad/s})(1 \times 10^{-2}\ \text{J/kg})m} = 2.09 \times 10^6\ \text{s} = 24.2\ \text{d}$$

Therefore, it would take over 24 days just to increase the water's temperature to 100°C, and much longer to boil it, and this technique will not work for a 20-minute coffee break!

P45.39 The number of nuclei in the original sample is

$$N_0 = \frac{\text{mass present}}{\text{mass of nucleus}} = \frac{5.00\ \text{kg}}{(89.907\ 7\ \text{u})(1.66 \times 10^{-27}\ \text{kg/u})}$$
$$= 3.35 \times 10^{25}\ \text{nuclei}$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{29.1\ \text{yr}} = 2.38 \times 10^{-2}\ \text{yr}^{-1} = 4.53 \times 10^{-8}\ \text{min}^{-1}$$

The original activity is

$$R_0 = \lambda N_0 = (4.53 \times 10^{-8}\ \text{min}^{-1})(3.35 \times 10^{25}\ \text{nuclei})$$
$$= 1.52 \times 10^{18}\ \text{decays/min}$$

The law of decay then gives us

$$\frac{R}{R_0} = \frac{10.0\ \text{decays/min}}{1.52 \times 10^{18}\ \text{decays/min}} = 6.59\ \times 10^{-18} = e^{-\lambda t}$$

and the time interval is

$$t = \frac{-\ln(R/R_0)}{\lambda} = \frac{-\ln\left(6.59\times10^{-18}\right)}{2.38\times10^{-2}\ \text{yr}^{-1}} = \boxed{1.66\times10^{3}\ \text{yr}}$$

P45.40 If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\ \text{MeV})}{2}\left[\left(\frac{1.00\times10^{-8}\ \text{g}}{98.9\ \text{g/mol}}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{1\ \text{mol}}\right)\right]$$

$$= \left(4.26\times10^{12}\ \text{MeV}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right) = 0.682\ \text{J}$$

Thus, the dose received is $\text{Dose} = \dfrac{0.682\ \text{J}}{60.0\ \text{kg}}\left(\dfrac{1\ \text{rad}}{10^{-2}\ \text{J/kg}}\right) = \boxed{1.14\ \text{rad}}$

P45.41 The decay constant is $\lambda = \ln 2/T_{1/2} = \ln 2/17.0\ \text{d}$. The number of nuclei remaining after 30.0 days is

$$N = N_0 e^{-\lambda T} = N_0 \exp\left[\left(\frac{-\ln 2}{17.0\ \text{d}}\right)30.0\ \text{d}\right] = 0.294N_0$$

The number decayed is $N_0 - N = N_0(1 - 0.294) = 0.706N_0$.

Then the energy release is

$$2.12\ \text{J} = \left(0.706N_0\right)\left(21.0\times10^{3}\ \text{eV}\right)\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$

$$N_0 = \frac{2.12\ \text{J}}{2.37\times10^{-15}\ \text{J}} = 8.94\times10^{14}$$

(a) The initial activity is

$$R_0 = \lambda N_0 = \frac{\ln 2}{17.0\ \text{d}}\left(8.94\times10^{14}\right)\left(\frac{1\ \text{d}}{86\,400\ \text{s}}\right) = \boxed{4.22\times10^{8}\ \text{Bq}}$$

(b) We find the total mass contained in the seeds from

$$\text{original sample mass} = m = N_0 m_{\text{one atom}}$$

$$= 8.94\times10^{14}\left[(103\ \text{u})\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right)\right]$$

Then,

$$m = \boxed{1.53\times10^{-10}\ \text{kg}} = 1.53\times10^{-7}\ \text{g} = 153\ \text{ng}$$

P45.42 The nuclei initially absorbed are (mass from Table 44.2)

$$N_0 = \left(1.00\times10^{-9}\ \text{g}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei/mol}}{89.9\ \text{g/mol}}\right) = 6.70\times10^{12}$$

The number of decays in time t is

$$\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$$

At the end of 1 year,

$$\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left\{1-\exp\left[\left(\frac{-\ln 2}{29.1\ \text{yr}}\right)1.00\ \text{yr}\right]\right\}$$
$$= 1.58\times10^{11}$$

The energy deposited is

$$E = \left(1.58\times10^{11}\right)\left(1.10\ \text{MeV}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right) = 0.027\,7\ \text{J}$$

Thus, the dose received is

$$\text{Dose} = \left(\frac{0.027\,7\ \text{J}}{70.0\ \text{kg}}\right) = \boxed{3.96\times10^{-4}\ \text{J/kg}} = 0.039\,6\ \text{rad}$$

Section 45.6 Uses of Radiation

P45.43 (a) With $I(x) = \frac{1}{2}I_0$, $I(x) = I_0 e^{-\mu x}$ becomes

$$\frac{1}{2}I_0 = I_0 e^{-0.72x/\text{mm}}$$
$$2 = e^{+0.72x/\text{mm}} \to \ln 2 = 0.72x/\text{mm} \to x = \frac{(\ln 2)\ \text{mm}}{0.72}$$
$$= \boxed{0.963\ \text{mm}}$$

(b) The intensity reaching the detector through $x_1 = 0.800$ mm of steel is $I_1 = I_0 e^{-\mu x_1}$. That transmitted by thickness $x_2 = 0.700$ mm is $I_2 = I_0 e^{-\mu x_2}$. The fractional change is

$$\frac{I_2 - I_1}{I_1} = \frac{I_0 e^{-\mu x_2} - I_0 e^{-\mu x_1}}{I_0 e^{-\mu x_1}} = e^{\mu(x_1 - x_2)} - 1 = e^{(0.720/\text{mm})(0.100\ \text{mm})} - 1$$
$$= e^{0.0720} - 1 = +0.074\ 7 = 7.47\%$$

As the thickness decreases, $\boxed{\text{the intensity increases by 7.47\%.}}$

P45.44 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \quad \text{so} \quad dN = (R - \lambda N)dt.$$

The variables are separable.

$$\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt: \qquad -\frac{1}{\lambda}\ln\left(\frac{R - \lambda N}{R}\right) = t$$

so $\quad \ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t \quad$ and $\quad \left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t}.$

Therefore $\quad 1 - \frac{\lambda}{R}N = e^{-\lambda t} \quad \rightarrow \quad N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right).$

(b) The maximum number of radioactive nuclei would be $\boxed{\dfrac{R}{\lambda}}$.

P45.45 (a) The number of photons is $\dfrac{10^4 \text{ MeV}}{1.04 \text{ MeV}} = 9.62 \times 10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\frac{3}{4}N_0 = 1.92 \times 10^4$ and $N_0 = 2.56 \times 10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is 2.56×10^6 $\boxed{\sim 10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{65}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is $N_{63} = 0.691\,7\,N_{\text{Cu}}$ and $N_{65} = 0.308\,3\,N_{\text{Cu}}$. Therefore,

$$\frac{N_{63}}{N_{65}} = \frac{0.691\,7}{0.308\,3}$$

or $\quad N_{63} = \left(\frac{0.6917}{0.3083}\right)N_{65} = \left(\frac{0.6917}{0.3083}\right)\left(2.56 \times 10^6\right) = 5.75 \times 10^6$

The total mass of copper present is then

$$m_{\text{Cu}} = (62.93 \text{ u})N_{63} + (64.93 \text{ u})N_{65}$$

$$m_{\text{Cu}} = \left[(62.93 \text{ u})\left(5.75 \times 10^6\right) + (64.93 \text{ u})\left(2.56 \times 10^6\right)\right] \times \left(1.66 \times 10^{-24} \text{ g/u}\right)$$

$$= 8.77 \times 10^{-16} \text{ g} \; \boxed{\sim 10^{-15} \text{ g}}$$

Additional Problems

P45.46 (a) The energy released by the ${}^{1}_{1}\mathrm{H} + {}^{11}_{5}\mathrm{B} \to 3\left({}^{4}_{2}\mathrm{He}\right)$ reaction is

$$Q = \left[M_{{}^{1}_{1}\mathrm{H}} + M_{{}^{11}_{5}\mathrm{B}} - 3M_{{}^{4}_{2}\mathrm{He}}\right]c^2$$

$$Q = \left[1.007\,825\text{ u} + 11.009\,305\text{ u} - 3(4.002\,603\text{ u})\right] \times (931.5\text{ MeV/u})$$

$$= \boxed{8.68\text{ MeV}}$$

(b) $\boxed{\text{The particles must have enough kinetic energy to overcome their mutual electrostatic repulsion so that they can get close enough to fuse.}}$

P45.47 From momentum conservation, we have

$$0 = m_{\mathrm{Li}}\vec{\mathbf{v}}_{\mathrm{Li}} + m_{\alpha}\vec{\mathbf{v}}_{\alpha} \text{ or } m_{\mathrm{Li}}v_{\mathrm{Li}} = m_{\alpha}v_{\alpha}$$

Thus,

$$K_{\mathrm{Li}} = \frac{1}{2}m_{\mathrm{Li}}v_{\mathrm{Li}}^2 = \frac{1}{2}\frac{(m_{\mathrm{Li}}v_{\mathrm{Li}})^2}{m_{\mathrm{Li}}} = \frac{(m_{\alpha}v_{\alpha})^2}{2m_{\mathrm{Li}}} = \left(\frac{m_{\alpha}^2}{2m_{\mathrm{Li}}}\right)v_{\alpha}^2$$

$$K_{\mathrm{Li}} = \left[\frac{(4.002\,6\text{ u})^2}{2(7.016\,0\text{ u})}\right](9.25\times10^{6}\text{ m/s})^2$$

$$= (1.14\text{ u})(1.66\times10^{-27}\text{ kg/u})(9.25\times10^{6}\text{ m/s})^2$$

$$K_{\mathrm{Li}} = 1.62\times10^{-13}\text{ J} = \boxed{1.01\text{ MeV}}$$

P45.48 (a) We have $I = \frac{1}{2}\rho v(\omega s_{\max})^2$, and from Equation 17.10, $I = \frac{(\Delta P_{\max})^2}{2\rho v}$.

Substituting the second expression for I into the first and solving for $s_{\max}$ gives

$$s_{\max} = \frac{1}{\omega}\left(\frac{2I}{\rho v}\right)^{1/2} = \frac{1}{\omega}\left[\frac{2}{\rho v}\frac{(\Delta P_{\max})^2}{2\rho v}\right]^{1/2} = \frac{\Delta P_{\max}}{\omega\rho v}$$

Solving for $\Delta P_{\max}$ and assuming $s_{\max} \sim 2.5$ m,

$$\Delta P_{\max} = \omega\rho v s_{\max} = (1\text{ s}^{-1})(1.20\text{ kg/m}^3)(343\text{ m/s})(2.5\text{ m})$$

$$\sim \boxed{10^{3}\text{ Pa}}$$

(b) The change in volume is given by

$$\Delta V = 4\pi r^2 \Delta r = 4\pi\left(14.0\times 10^3 \text{ m}\right)^2 (2.5 \text{ m})$$
$$= 1.23\times 10^8 \text{ m}^3 \sim \boxed{6\times 10^9 \text{ m}^3}$$

(c) The energy carried by the blast wave is

$$W = \left(\Delta P_{\max}\right)\left(\Delta V\right) = \left(10^3 \text{ Pa}\right)\left(6\times 10^9 \text{ m}^3\right) = \boxed{6\times 10^{12} \text{ J}}$$

(d) Since the blast wave carries only 10% of the bomb's energy, $6\times 10^{16} \text{ J} = \dfrac{1}{10}(\text{yield})$, and the bomb yield is then

$$\text{yield} = 6\times 10^{13} \text{ J} \boxed{\sim 10^{14} \text{ J}}$$

(e) The yield in terms of tons of TNT is

$$\frac{6\times 10^{13} \text{ J}}{4.2\times 10^9 \text{ J/ton TNT}} = 1.42\times 10^4 \text{ ton TNT} \boxed{\sim 10^4 \text{ ton TNT}}$$

***P45.49** The Japanese call it the *original child bomb*.

(a) Suppose each ^{235}U fission releases 208 MeV of energy. Then, the number of nuclei that must have undergone fission is

$$N = \frac{\text{total release}}{\text{energy per nuclei}} = \frac{5\times 10^{13} \text{ J}}{(208 \text{ MeV})\left(1.60\times 10^{-13} \text{ J/MeV}\right)}.$$
$$= \boxed{1.5\times 10^{24} \text{ nuclei}}$$

(b) $$\text{mass} = \left(\frac{1.5\times 10^{24} \text{ nuclei}}{6.02\times 10^{23} \text{ nuclei/mol}}\right)(235 \text{ g/mol}) \approx \boxed{0.6 \text{ kg}}$$

P45.50 (a) Subtracting the background counts, the decay counts are $N_1 = 372 - 5(15) = 297$ in the first 5.00 min interval and $N_2 = 337 - 5(15) = 262$ in the second. The midpoints of the time intervals are separated by $T = 5.00$ min. We use $R = R_0 e^{-\lambda t}$, taking $t = T$ and identifying $R_0 = N_1/T = 297/5$ min and $R = N_2/T = 262/5$ min. We have then

$$\frac{N_2}{T} = \left(\frac{N_1}{T}\right)e^{-\lambda T} \quad \text{or} \quad \frac{262}{5 \text{ min}} = \left(\frac{297}{5 \text{ min}}\right)e^{-\left(\ln 2/T_{1/2}\right)(5.00 \text{ min})}$$

which gives

$$e^{-\left(\ln 2/T_{1/2}\right)T} = \frac{N_2}{N_1} \quad \text{or} \quad e^{-\left(\ln 2/T_{1/2}\right)(5.00 \text{ min})} = \frac{262}{297}$$

Solving,

$$-\frac{\ln 2}{T_{1/2}}T = \ln\left(\frac{N_2}{N_1}\right) \qquad \text{or} \qquad -\frac{\ln 2}{T_{1/2}}T = \ln\left(\frac{262}{297}\right)$$

The half-life is then

$$T_{1/2} = \frac{-\ln 2}{\ln(N_2/N_1)}T = \frac{-\ln 2}{\ln(262/297)}(5.00\ \text{min}) = \boxed{27.6\ \text{min}}$$

NOTE: If it seems questionable to set instantaneous decay rates equal to average decay rates, to let $R_0 = N_1/T$ and $R = N_2/T$, see the Alternate Solution to (a) below. The results are the same.

(b) The average count rate is about

$$\frac{1}{2}\left(\frac{262}{5\ \text{min}} + \frac{297}{5\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \sim 1\ \text{s}^{-1}$$

but the counts are randomly spaced in time, meaning some counts near the beginning and end of each 5.00-min interval should or should not have been counted. Let's assume that the count incidence could vary by as much as 5 seconds, so we shall assume a count uncertainty of ±5. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right) = -\frac{\ln 2}{T_{1/2}}(5.00\ \text{min}), \text{ giving } \left(T_{1/2}\right)_{\text{min}} = 21.5\ \text{min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right) = -\frac{\ln 2}{T_{1/2}}(5.00\ \text{min}), \text{ yielding } \left(T_{1/2}\right)_{\text{max}} = 38.7\ \text{min}$$

Thus, the half-life is about

$$T_{1/2} = \left(\frac{38.5+21.7}{2}\right) \pm \left(\frac{38.5-21.7}{2}\right)\ \text{min}$$
$$= (30 \pm 8)\ \text{min} = \boxed{30\ \text{min} \pm 27\%}$$

<u>Alternate Solution to (a)</u> The amount of the radioactive sample at time t is $N = N_0 e^{-\lambda t}$, where we do not know N_0. The number of decay counts between $t = 0$ and $t = T$ are

$$N_1 = N_0\left(1 - e^{-\lambda T}\right) = 297$$

and the number of decay counts between $t = 0$ and $t = 2T$ are

$$N_1 + N_2 = N_0\left(1 - e^{-\lambda 2T}\right) = 297 + 262 = 559$$

To eliminate N_0, we consider the ratio of the counts:

$$r = \frac{N_1 + N_2}{N_1} = \frac{N_0\left(1 - e^{-\lambda 2T}\right)}{N_0\left(1 - e^{-\lambda T}\right)} = \frac{559}{297}$$

$$r = \frac{\left(1 - e^{-\lambda 2T}\right)}{\left(1 - e^{-\lambda T}\right)} = \frac{\left(1 - e^{-\lambda T}\right)\left(1 + e^{-\lambda T}\right)}{\left(1 - e^{-\lambda T}\right)} = 1 + e^{-\lambda T}$$

solving,

$$e^{-\lambda T} = r - 1 = \frac{N_1 + N_2}{N_1} - 1 = \frac{N_2}{N_1} \quad \rightarrow \quad e^{-\left(\ln 2/T_{1/2}\right)T} = \frac{N_2}{N_1}$$

which leads to the same result as above, $T_{1/2} = \dfrac{-\ln 2}{\ln\left(N_2/N_1\right)}T$.

P45.51 (a) The energy amplification is

$$\frac{E}{E_0} = \frac{\frac{1}{2}C\Delta V^2}{0.500\ \text{MeV}} = \frac{\frac{1}{2}\left(5.00 \times 10^{-12}\ \text{F}\right)\left(1.00 \times 10^3\ \text{V}\right)^2}{\left(0.500\ \text{MeV}\right)\left(1.60 \times 10^{-13}\ \text{J/MeV}\right)}$$
$$= \boxed{3.12 \times 10^7}$$

(b) The number of electrons is

$$N = \frac{Q}{e} = \frac{C\Delta V}{e} = \frac{\left(5.00 \times 10^{-12}\ \text{F}\right)\left(1.00 \times 10^3\ \text{V}\right)}{1.60 \times 10^{-19}\ \text{C}}$$
$$= \boxed{3.12 \times 10^{10}\ \text{electrons}}$$

P45.52 (a) To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad v_2 = \left(\frac{m_1}{m_2}\right)v_1$$

The kinetic energies after the break-up are then

$$K_1 = \frac{1}{2}m_1 v_1^2 \quad \text{and} \quad K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_2\left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right)K_1$$

The fraction of the total kinetic energy carried off by m_1 is

$$\frac{K_1}{K_{\text{tot}}} = \frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + \left(m_1/m_2\right)K_1} = \frac{m_2}{m_1 + m_2}$$

and the fraction carried off by m_2 is

$$\frac{K_2}{K_{\text{tot}}} = 1 - \frac{K_1}{K_{\text{tot}}} = 1 - \frac{m_2}{m_1 + m_2} = \frac{m_1}{m_1 + m_2}$$

(b) The disintegration energy is

$$Q = (236.045\,562\text{ u} - 86.920\,711\text{ u} - 148.934\,370\text{ u}) \times (931.5\text{ MeV/u})$$

$$= 177.4\text{ MeV} = \boxed{177\text{ MeV}}$$

(c) Immediately after fission, this Q-value is the total kinetic energy of the fission products. From part (a),

$$\frac{K_1}{K_{\text{tot}}} = \frac{m_2}{m_1 + m_2} = \frac{K_{\text{Br}}}{Q}$$

Then,

$$K_{\text{Br}} = Q\frac{m_{\text{La}}}{m_{\text{Br}} + m_{\text{La}}} = (177.4\text{ MeV})\left(\frac{149\text{ u}}{87\text{ u} + 149\text{ u}}\right) = \boxed{112.0\text{ MeV}}$$

and $K_{\text{La}} = Q - K_{\text{Br}} = 177.4\text{ MeV} - 112.0\text{ MeV} = \boxed{65.4\text{ MeV}}$

(d) The speed of the fragments is given by

$$v_{\text{Br}} = \sqrt{\frac{2K_{\text{Br}}}{m_{\text{Br}}}} = \sqrt{\frac{2(112\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(87\text{ u})(1.66\times10^{-27}\text{ kg/u})}}$$

$$= 1.58\times10^7\text{ m/s} = \boxed{15.8\text{ Mm/s}}$$

and

$$v_{\text{La}} = \sqrt{\frac{2K_{\text{La}}}{m_{\text{La}}}} = \sqrt{\frac{2(65.4\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(149\text{ u})(1.66\times10^{-27}\text{ kg/u})}}$$

$$= 9.20\times10^6\text{ m/s} = \boxed{9.30\text{ Mm/s}}$$

***P45.53** (a) For each of the following six steps, the subscripts a - f of Q refer to the corresponding step in Problem 45.30.

For ${}^{12}\text{C} + {}^{1}\text{H} \to {}^{13}\text{N} + Q$,

$$Q_a = (12.000\ 000 + 1.007\ 825 - 13.005\ 739)(931.5\text{ MeV})$$

$$= \boxed{1.94\text{ MeV}}$$

For the second step, add seven electrons to both sides to get:

$${}^{13}\text{N atom} \to {}^{13}\text{C atom} + e^+ + e^- + Q$$

Then,

$$Q_b = [13.005\ 739 - 13.003\ 355 - 2(0.000\ 549)](931.5\text{ MeV})$$
$$= \boxed{1.20\text{ MeV}}$$
$$Q_c = [13.003\ 355 + 1.007\ 825 - 14.003\ 074](931.5\text{ MeV})$$
$$= \boxed{7.55\text{ MeV}}$$
$$Q_d = [14.003\ 074 + 1.007\ 825 - 15.003\ 065](931.5\text{ MeV})$$
$$= \boxed{7.30\text{ MeV}}$$
$$Q_e = [15.003\ 065 - 15.000\ 109 - 2(0.000\ 549)](931.5\text{ MeV})$$
$$= \boxed{1.73\text{ MeV}}$$
$$Q_f = [15.000\ 109 + 1.007\ 825 - 12 - 4.002\ 603](931.5\text{ MeV})$$
$$= \boxed{4.97\text{ MeV}}$$

(b) The energy released in the annihilations is

$$Q_3 = Q_7 = 2(0.000\ 549)(931.5\text{ MeV})$$
$$= \boxed{1.02\text{ MeV}}$$

(c) The sum is $\boxed{26.7\text{ MeV}}$, the same as for the proton-proton cycle.

(d) Not all of the energy released appears as internal energy in the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

P45.54 The original activity per area is

$$\frac{5.00\times 10^6\text{ Ci}}{10^4\text{ km}^2}\left(\frac{1\text{ km}}{10^3\text{ m}}\right)^2 = 5.00\times 10^{-4}\text{ Ci/m}^2$$

The half-life is 29.1 yr. The decay law, $N = N_0 e^{-\lambda t}$, becomes the law of decrease of activity, $R = R_0 e^{-\lambda t}$. If the material is not transported, it describes the time evolution of activity per area, $R/A = R_0/A\ e^{-\lambda t}$. Solving for the time t gives

$$e^{\lambda t} = \frac{R_0 / A}{R / A} \quad \rightarrow \quad t = \frac{1}{\lambda}\ln\left(\frac{R_0 / A}{R / A}\right)$$

Substituting numerical values,

$$t = \left(\frac{29.1\text{ yr}}{\ln 2}\right)\ln\left(\frac{R_0 / A}{R / A}\right) = \frac{29.1\text{ yr}}{\ln 2}\ln\left(\frac{5.00\times 10^{-4}\text{ Ci/m}^2}{2.00\times 10^{-6}\text{ Ci/m}^2}\right)$$
$$= \boxed{232\text{ yr}}$$

P45.55 The number of nuclei in 3.80 kg of ${}^{238}_{94}\text{Pu}$ is

$$N_0 = \left(\frac{3\ 800\ \text{g}}{238.049\ 560\ \text{g/mol}}\right)\left(6.022\times 10^{23}\ \text{nuclei/mol}\right)$$
$$= 9.61\times 10^{24}\ \text{nuclei}$$

The half-life of ${}^{238}_{94}\text{Pu}$ is 87.7 years, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(87.7\ \text{yr})\left(3.155\times 10^{7}\ \text{s/yr}\right)} = 2.51\times 10^{-10}\ \text{s}^{-1}$$

The initial activity is

$$R_0 = \lambda N_0 = \left(2.51\times 10^{-10}\ \text{s}^{-1}\right)\left(9.61\times 10^{24}\ \text{nuclei}\right) = 2.41\times 10^{15}\ \text{Bq}$$

The energy released in each ${}^{238}_{94}\text{Pu} \to {}^{234}_{92}\text{U} + {}^{4}_{2}\text{He}$ reaction is

$$Q = \left[M_{{}^{238}_{94}\text{Pu}} - M_{{}^{234}_{92}\text{U}} - M_{{}^{4}_{2}\text{He}}\right]c^2:$$

$$Q = [238.049\ 560\ \text{u} - 234.040\ 952\ \text{u} - 4.002\ 603\ \text{u}] \times (931.5\ \text{MeV/u})$$
$$= 5.59\ \text{MeV}$$

Thus, assuming a conversion efficiency of 3.20%, the initial power output of the battery is

$$P = (0.032\ 0)R_0Q$$
$$= (0.032\ 0)\left(2.41\times 10^{15}\ \text{decays/s}\right)(5.59\ \text{MeV/decay}) \times \left(1.602\times 10^{-13}\ \text{J/MeV}\right)$$
$$= \boxed{69.0\ \text{W}}$$

P45.56 The number of hydrogen-3 nuclei is

$$N = \left(50.0\ \text{m}^3\right)\left(2.00\times 10^{14}\ \frac{\text{particles}}{\text{cm}^3}\right)(100\ \text{cm/m})^3$$
$$= 1.00\times 10^{22}\ \text{particles}$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \left(\frac{0.693}{12.3\ \text{yr}}\right)\left(\frac{1\ \text{yr}}{3.16\times 10^{7}\ \text{s}}\right) = 1.78\times 10^{-9}\ \text{s}^{-1}$$

The activity is then

$$R = \lambda N = \left(1.78\times 10^{-9}\ \text{s}^{-1}\right)\left(1.00\times 10^{22}\ \text{nuclei}\right) = 1.78\times 10^{13}\ \text{Bq}$$

In curies this is

$$R = \left(1.78\times 10^{13}\ \text{Bq}\right)\left(\frac{1\ \text{Ci}}{3.70\times 10^{10}\ \text{Bq}}\right) = 482\ \text{Ci}$$

$\boxed{\text{482 Ci, which is less than the fission inventory by on the order of a hundred million times.}}$

P45.57 The complete fissioning of 1.00 gram of ^{235}U releases

$$Q = \left(\frac{1.00\ \text{g}}{235\ \text{grams/mol}}\right)\left(\frac{6.02\times 10^{23}\ \text{atoms}}{\text{mol}}\right) \times\left(\frac{200\ \text{MeV}}{\text{fission}}\right)\left(\frac{1.60\times 10^{-13}\ \text{J}}{\text{MeV}}\right)$$

$$= 8.20\times 10^{10}\ \text{J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values), then

$$Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$$

$$Q = m\left[\left(4\ 186\ \text{J/kg °C}\right)\left(80.0\ \text{°C}\right) + 2.26\times 10^{6}\ \text{J/kg} + \left(2\ 010\ \text{J/kg °C}\right)\left(300\ \text{°C}\right)\right]$$

Therefore, $m = \dfrac{8.20\times 10^{10}\ \text{J}}{3.20\times 10^{6}\ \text{J/kg}} = \boxed{2.56\times 10^{4}\ \text{kg}}$

P45.58 When mass m of ^{235}U undergoes complete fission, releasing energy E per fission event, the total energy released is

$$Q = \left(\frac{m}{M_{\text{U-235}}}\right)N_{\text{A}}E$$

where N_A is Avogadro's number. If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then

$$Q = m_w\left[c_w\left(100\text{°C} - T_c\right) + L_v + c_s\left(T_h - 100\text{°C}\right)\right]$$

where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w = \frac{Q}{\left[c_w\left(100\text{°C} - T_c\right) + L_v + c_s\left(T_h - 100\text{°C}\right)\right]}$$

$$= \boxed{\frac{mN_{\text{A}}E}{M_{\text{U-235}}\left[c_w\left(100 - T_c\right) + L_v + c_s\left(T_h - 100\right)\right]}}$$

P45.59 (a) $Q_{\rm I} = \left[M_{\rm A} + M_{\rm B} - M_{\rm C} - M_{\rm E}\right]c^2$, and

$$Q_{\rm II} = \left[M_{\rm C} + M_{\rm D} - M_{\rm F} - M_{\rm G}\right]c^2$$

$$Q_{\rm net} = Q_{\rm I} + Q_{\rm II} = \left[M_{\rm A} + M_{\rm B} - M_{\rm C} - M_{\rm E} + M_{\rm C} + M_{\rm D} - M_{\rm F} - M_{\rm G}\right]c^2$$

$$Q_{\rm net} = Q_{\rm I} + Q_{\rm II} = \left[M_{\rm A} + M_{\rm B} + M_{\rm D} - M_{\rm E} - M_{\rm F} - M_{\rm G}\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives

$${}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + {}^{0}_{-1}\text{e} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + {}^{0}_{-1}\text{e} \to {}^{4}_{2}\text{He} + 2\nu$$

$$4\,{}^{1}_{1}\text{H} + 2\,{}^{0}_{-1}\text{e} \to {}^{4}_{2}\text{He} + 2\nu$$

Adding two electrons to each side gives

$$4\,{}^{1}_{1}\text{H atom} \to {}^{4}_{2}\text{He atom} + 2\nu$$

Thus,

$$\begin{aligned} Q_{\rm net} &= \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2 \\ &= \left[4(1.007\ 825\ \text{u}) - 4.002\ 603\ \text{u}\right](931.5\ \text{MeV/u}) \\ &= \boxed{26.7\ \text{MeV}} \end{aligned}$$

P45.60 (a) From the definition of the volume of a cube and the definition of mass density, we have $V = \ell^3 = \dfrac{m}{\rho}$, so

$$\ell = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{70.0\ \text{kg}}{19.1\times10^3\ \text{kg/m}^3}\right)^{1/3} = 0.154\ \text{m} = \boxed{15.4\ \text{cm}}$$

(b) We add 92 electrons to both sides of the given nuclear reaction. Then it becomes

$${}^{238}_{92}\text{U atom} \to 8\,{}^{4}_{2}\text{He atom} + {}^{206}_{82}\text{Pb atom}$$

The Q value of this reaction is

$$\begin{aligned} Q &= \left[M_{{}^{238}_{92}\text{U}} - 8M_{{}^4_2\text{He}} - M_{{}^{206}_{82}\text{Pb}}\right]c^2 \\ &= \left[238.050\,783 - 8(4.002\,603) - 205.974\,449\right](931.5\ \text{MeV/u}) \\ Q &= \boxed{51.7\ \text{MeV}} \end{aligned}$$

(c) $\boxed{\text{The number of decays per second is the decay rate } R\text{, and the energy released in each decay is } Q\text{. Then the energy released per unit time interval is } P = QR.}$

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00\times10^{4}\ \text{g}}{238\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{nuclei/mol}\right)$$
$$= 1.77\times10^{26}\ \text{nuclei}$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47\times10^{9}\ \text{yr}} = 1.55\times10^{-10}\ \frac{1}{\text{yr}}$$

and the rate of decays is then

$$R = \lambda N = \left(1.55\times10^{-10}\ \frac{1}{\text{yr}}\right)\left(1.77\times10^{26}\ \text{nuclei}\right)$$
$$= 2.75\times10^{16}\ \text{decays/yr}$$

so, $P = QR = (51.7\ \text{MeV})\left(2.75\times10^{16}\ \text{yr}^{-1}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right)$
$$= \boxed{2.27\times10^{5}\ \text{J/yr}}$$

(e) We know that

dose in rem = dose in rad × RBE

or

5.00 rem/yr = (dose in rad/yr)(1.10)

giving

(dose in rad/yr) = 4.55 rad/yr

The allowed whole-body dose is then

$$(70.0\ \text{kg})(4.55\ \text{rad/yr})\left(\frac{10^{-2}\ \text{J/kg}}{1\ \text{rad}}\right) = \boxed{3.18\ \text{J/yr}}$$

P45.61 (a) The mass of the pellet is

$$m = \rho V = \rho\frac{4\pi}{3}r^{3} = \left(0.200\ \text{g/cm}^{3}\right)\left[\frac{4\pi}{3}\left(\frac{1.50\times10^{-2}\ \text{cm}}{2}\right)^{3}\right]$$
$$= 3.53\times10^{-7}\ \text{g}$$

The pellet consists of equal numbers of ^{2}H and ^{3}H atoms, so the average molar mass is 2.50 and the total number of atoms is

$$N = \left(\frac{3.53\times10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)$$
$$= 8.51\times10^{16}\ \text{atoms}$$

When the pellet is vaporized, the plasma will consist of $2N$ particles (N nuclei and N electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E = (2N)\left(\frac{3}{2}k_{\text{B}}T\right)$ as

$$T = \frac{E}{3Nk_{\text{B}}} = \frac{2.00\times10^{3}\ \text{J}}{3\left(8.51\times10^{16}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{5.68\times10^{8}\ \text{K}}$$

(b) Each fusion event uses 2 nuclei, so N/2 events will occur. From Equation 45.4, the energy released by one fusion event is 17.59 MeV, so the total energy released will be

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{8.51\times10^{16}}{2}\right)(17.59\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)$$
$$= 1.20\times10^{5}\ \text{J} = \boxed{120\ \text{kJ}}$$

P45.62 (a) From the given equation, the ratio of the two intensities is

$$\frac{I_2}{I_1} = \frac{I_0e^{-\mu_2 x}}{I_0e^{-\mu_1 x}} = \boxed{e^{-(\mu_2-\mu_1)x}}$$

(b) Substituting numerical values into the equation in part (a) gives

$$\frac{I_{50}}{I_{100}} = \exp\left[-\left(5.40\ \text{cm}^{-1} - 41.0\ \text{cm}^{-1}\right)(0.100\ \text{cm})\right] = e^{3.56} = \boxed{35.2}$$

(c) Here, x = 10.0 mm = 1.00 cm, and

$$\frac{I_{50}}{I_{100}} = \exp\left[-\left(5.40\ \text{cm}^{-1} - 41.0\ \text{cm}^{-1}\right)(1.00\ \text{cm})\right] = e^{35.6}$$
$$= \boxed{2.89\times10^{15}}$$

Thus, a 1.00-cm-thick aluminum plate has essentially removed the long-wavelength x-rays from the beam.

P45.63 The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n\vec{\mathbf{v}}_n + m_\alpha\vec{\mathbf{v}}_\alpha = 0 \quad \text{or} \quad (1.008\,7\ \text{u})v_n = (4.002\,6\ \text{u})\,v_\alpha$$

At the same time, their kinetic energies must add to 17.6 MeV:

$$E = \frac{1}{2}m_n v_n^2 + \frac{1}{2}m_\alpha v_\alpha^2 = \frac{1}{2}(1.008\ 7\ \text{u})v_n^2 + \frac{1}{2}(4.002\ 6\ \text{u})v_\alpha^2$$
$$= 17.6\ \text{MeV}$$

Substitute $v_\alpha = 0.252\ 0v_n$ to obtain

$$E = (0.504\ 35\ \text{u})v_n^2 + (0.127\ 10\ \text{u})v_n^2$$
$$= 17.6\ \text{MeV}\left(\frac{1\ \text{u}}{931.494\ \text{MeV}/c^2}\right)$$

Solving for v_n then gives

$$v_n = \sqrt{\frac{0.018\ 9c^2}{0.631\ 45}} = 0.173c = 5.19 \times 10^7\ \text{m/s}$$

Since this speed is not too much greater than $0.1c$, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.008\ 7\ \text{u})(0.173c)^2\left(\frac{931.494\ \text{MeV}/c^2}{\text{u}}\right)$$
$$= \boxed{14.1\ \text{MeV}}$$

For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of energy for this reaction requires that

$$E_\text{n} + E_\alpha = \left(m_\text{n}c^2 + K_\text{n}\right) + \left(m_\alpha c^2 + K_\alpha\right) = m_\text{n}c^2 + m_\alpha c^2 + K \qquad \textbf{[1]}$$

where $K = 17.6$ MeV is the total kinetic energy, and conservation of momentum for this reaction requires that

$$\vec{\mathbf{p}}_\text{n} + \vec{\mathbf{p}}_\alpha = 0 \quad \rightarrow \quad p_\text{n} = p_\alpha \qquad \textbf{[2]}$$

From the relation between total energy, mass, and momentum of a particle, we have

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \quad \rightarrow \quad p^2c^2 = E^2 - \left(mc^2\right)^2 \qquad \textbf{[3]}$$

From equations [2] and [3], we may write

$$p_\text{n}^2c^2 = p_\alpha^2c^2$$
$$E_\text{n}^2 - \left(m_\text{n}c^2\right)^2 = E_\alpha^2 - \left(m_\alpha c^2\right)^2$$
$$E_\text{n}^2 - E_\alpha^2 = \left(m_\text{n}c^2\right)^2 - \left(m_\alpha c^2\right)^2$$
$$\left(E_\text{n} - E_\alpha\right)\left(E_\text{n} + E_\alpha\right) = \left(m_\text{n}c^2\right)^2 - \left(m_\alpha c^2\right)^2$$

Substituting the above expression into equation [1] gives

$$(E_n - E_\alpha)(m_n c^2 + m_\alpha c^2 + K) = (m_n c^2)^2 - (m_\alpha c^2)^2$$

$$E_n - E_\alpha = \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)}$$

$$E_\alpha = E_n - \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)}$$

Substituting this result back into equation [1] gives

$$E_n + \left[E_n - \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)} \right] = m_n c^2 + m_\alpha c^2 + K$$

$$2E_n = (m_n c^2 + m_\alpha c^2 + K) + \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)}$$

$$E_n = \frac{(m_n c^2 + m_\alpha c^2 + K)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + K)}$$

To find the kinetic energy of the neutron, we note that $E_n = m_n c^2 + K_n$:

$$E_n = \frac{(m_n c^2 + m_\alpha c^2 + K)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + K)} = m_n c^2 + K_n$$

$$K_n = \frac{(m_n c^2 + m_\alpha c^2 + K)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + K)} - m_n c^2$$

For $K = 17.6$ MeV,

$$m_n c^2 = (1.008\,7\text{ u})c^2(931.494\text{ MeV}/c^2 \cdot \text{u}) = 939.60\text{ MeV}$$

and $$m_\alpha c^2 = (4.002\,6\text{ u})c^2(931.494\text{ MeV}/c^2 \cdot \text{u}) = 3\,728.4\text{ MeV}$$

we find that $K_n = \boxed{14.0\text{ MeV}}$.

P45.64 (a) The number of Pu nuclei in 1.00 kg is

$$\frac{6.02 \times 10^{23}\text{ nuclei/mol}}{239.05\text{ g/mol}}(1\,000\text{ g}) = 2.52 \times 10^{24}\text{ nuclei}$$

The total energy is

$$(25.2 \times 10^{23}\text{ nuclei})\left(\frac{1\text{ fission}}{\text{nucleus}}\right)\left(\frac{200\text{ MeV}}{\text{fission}}\right) = 5.04 \times 10^{26}\text{ MeV}$$

$$E = \left(5.04 \times 10^{26}\ \text{MeV}\right)\left(4.44 \times 10^{-20}\ \text{kWh/MeV}\right)$$
$$= \boxed{2.24 \times 10^{7}\ \text{kWh}}$$

or 22 million kWh.

(b) $E = \Delta m c^2 = \left(3.016\,049\ \text{u} + 2.014\,102\ \text{u} - 4.002\,603\ \text{u} - 1.008\,665\ \text{u}\right) \times \left(931.5\ \text{MeV/u}\right)$

$$E = \boxed{17.6\ \text{MeV for each D-T fusion}}$$

(c) $E_n = (\text{total number of D nuclei})(17.6\ \text{MeV})\left(4.44 \times 10^{-20}\ \text{kWh/MeV}\right)$

$$E_n = \left(\frac{6.02 \times 10^{23}}{\text{mol}}\right)\left(\frac{1\,000\ \text{g}}{2.014\ \text{g/mol}}\right)(17.6\ \text{MeV}) \times \left(4.44 \times 10^{-20}\ \text{kWh/MeV}\right)$$
$$= \boxed{2.34 \times 10^{8}\ \text{kWh}}$$

(d) $E_n = \left(\text{the number of C atoms in 1.00 kg}\right) \times \left(\dfrac{4.20\ \text{eV}}{\text{kg}}\right)$

$$E_n = \left(\frac{6.02 \times 10^{26}}{12\ \text{g}}\right)\left(4.20 \times 10^{-6}\ \text{MeV}\right)\left(4.44 \times 10^{-20}\ \text{kWh/MeV}\right)$$
$$= \boxed{9.36\ \text{kWh}}$$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels. Burning coal in the open puts carbon dioxide into the atmosphere, worsening global warming. Plutonium is a very dangerous material to have sitting around.

P45.65 (a) We have 1.00 kg – (1.00 kg)(0.007 20) – (1.00 kg)(0.000 0500) = 0.993 kg of ^{238}U, comprising

$$N = \left(0.993\ \text{kg}\right)\left(\frac{6.02 \times 10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.238\ \text{kg}}\right)$$
$$= 2.51 \times 10^{24}\ \text{nuclei}$$

with activity

$$R = \lambda N = \frac{\ln 2}{4.47 \times 10^{9}\ \text{yr}}\left(2.51 \times 10^{24}\ \text{nuclei}\right) \times \left(\frac{1\ \text{yr}}{3.16 \times 10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70 \times 10^{10}\ \text{s}^{-1}}\right)$$

$$= \boxed{3.3 \times 10^{-4}\ \text{Ci}} = 330\ \mu\text{Ci}$$

We have (1.00 kg)(0.007 20) = 0.007 2 kg of ^{235}U, comprising

$$N = \left(0.007\ 2\ \text{kg}\right)\left(\frac{6.02 \times 10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.235\ \text{kg}}\right)$$

$$= 1.84 \times 10^{22}\ \text{nuclei}$$

with activity

$$R = \lambda N = \frac{\ln 2}{7.04 \times 10^{8}\ \text{yr}}\left(1.84 \times 10^{22}\ \text{nuclei}\right) \times \left(\frac{1\ \text{yr}}{3.16 \times 10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70 \times 10^{10}\ \text{s}^{-1}}\right)$$

$$= 1.6 \times 10^{-5}\ \text{Ci} = \boxed{16\ \mu\text{Ci}}$$

We have (1.00 kg)(0.000 0500) = 5.00 × 10^{-5} kg of ^{234}U, comprising

$$N = \left(5.00 \times 10^{-5}\ \text{kg}\right)\left(\frac{6.02 \times 10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.234\ \text{kg}}\right)$$

$$= 1.29 \times 10^{20}\ \text{nuclei}$$

with activity

$$R = \lambda N = \frac{\ln 2}{2.44 \times 10^{5}\ \text{yr}}\left(1.29 \times 10^{20}\ \text{nuclei}\right) \times \left(\frac{1\ \text{yr}}{3.16 \times 10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70 \times 10^{10}\ \text{s}^{-1}}\right)$$

$$= \boxed{3.1 \times 10^{-4}\ \text{Ci}} = 310\ \mu\text{Ci}$$

(b) The total activity is (330 + 16 + 310) μCi = 656 μCi, so the fractional contributions are, respectively, 330/656 = $\boxed{50\%}$, 16/656 = $\boxed{2.4\%}$, and 310/656 = $\boxed{47\%}$

(c) It is dangerous, notably if the material is inhaled as a powder. With precautions to minimize human contact, however, microcurie sources are routinely used in laboratories.

P45.66 (a) The number of molecules in 1.00 liter of water (mass = 1 000 g) is

$$N = \left(\frac{1.00\times 10^{3}\ \text{g}}{18.0\ \text{g/mol}}\right)\left(6.02\times 10^{23}\ \text{molecules/mol}\right)$$
$$= 3.34\times 10^{25}\ \text{molecules}$$

The number of deuterium nuclei contained in these molecules is

$$N' = \left(3.34\times 10^{25}\ \text{molecules}\right)\left(\frac{1\ \text{deuteron}}{3\ 300\ \text{molecules}}\right)$$
$$= 1.01\times 10^{22}\ \text{deuterons}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is $\frac{N'}{2} = 5.07\times 10^{21}$ reactions, and the energy released is

$$E_{\text{fusion}} = \left(5.07\times 10^{21}\ \text{reactions}\right)\left(3.27\ \text{MeV/reaction}\right)$$
$$= 1.66\times 10^{22}\ \text{MeV}$$

$$E_{\text{fusion}} = \left(1.66\times 10^{22}\ \text{MeV}\right)\left(1.60\times 10^{-13}\ \text{J/MeV}\right) = \boxed{2.65\times 10^{9}\ \text{J}}$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_{\text{fusion}}}{E_{\text{gasoline}}} = \frac{2.65\times 10^{9}\ \text{J}}{3.40\times 10^{7}\ \text{J}} = \boxed{78.0\ \text{times larger}}$$

P45.67 (a) At 6×10^{8} K, the average kinetic energy of a carbon atom is

$$\frac{3}{2}k_{\text{B}}T = (1.5)\left(8.62\times 10^{-5}\ \text{eV/K}\right)\left(6\times 10^{8}\ \text{K}\right) = \boxed{8\times 10^{4}\ \text{eV}}$$

Note that 6×10^{8} K is about $6^{2} = 36$ times larger than 1.5×10^{7} K, the core temperature of the Sun. This factor corresponds to the higher potential-energy barrier to carbon fusion compared to hydrogen fusion. It could be misleading to compare it to the temperature ~ 10^{8} K required for fusion in a low-density plasma in a fusion reactor.

(b) The energy released is

$$Q = \left[2M_{\text{C}^{12}} - M_{\text{Ne}^{20}} - M_{\text{He}^{4}}\right]c^{2}$$
$$Q = \left[2(12.000\ 000\ \text{u}) - 19.992\ 440\ \text{u} - 4.002\ 603\ \text{u}\right]$$
$$\times\left(931.5\ \text{MeV/u}\right)$$
$$= \boxed{4.62\ \text{MeV}}$$

In the second reaction,

$$Q=\left[2M_{\mathrm{C}^{12}}-M_{\mathrm{Mg}^{24}}\right]c^2$$

$$Q=[2(12.000\ 000\ \mathrm{u})-23.985\ 042\ \mathrm{u}](931.5\ \mathrm{MeV/u})$$

$$=\boxed{13.9\ \mathrm{MeV}}$$

(c) The energy released is the energy of reaction of the number of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E=\left(2.00\times10^{3}\ \mathrm{g}\right)\left(\frac{6.02\times10^{23}\ \mathrm{atoms/mol}}{12.0\ \mathrm{g/mol}}\right)\times\left(\frac{4.62\ \mathrm{MeV/fusion\ event}}{2\ \mathrm{nuclei/fusion\ event}}\right)\left(\frac{1\ \mathrm{kWh}}{2.25\times10^{19}\ \mathrm{MeV}}\right)$$

$$\Delta E=\frac{\left(1.00\times10^{26}\right)(4.62)}{2\left(2.25\times10^{19}\right)}\mathrm{kWh}=\boxed{1.03\times10^{7}\ \mathrm{kWh}}$$

P45.68 From Table 44.2 of isotopic masses, the half-life of ^{32}P is 14.26 d. Thus, the decay constant is

$$\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{14.26\ \mathrm{d}}=0.048\,6\ \mathrm{d}^{-1}=5.63\times10^{-7}\ \mathrm{s}^{-1}$$

and the initial number of nuclei is

$$N_0=\frac{R_0}{\lambda}=\frac{5.22\times10^{6}\ \mathrm{decay/s}}{5.63\times10^{-7}\ \mathrm{s}^{-1}}=9.28\times10^{12}\ \mathrm{nuclei}$$

At t = 10.0 days, the number remaining is

$$N=N_0e^{-\lambda t}=\left(9.28\times10^{12}\ \mathrm{nuclei}\right)\exp\left[-\left(0.048\,6\ \mathrm{d}^{-1}\right)(10.0\ \mathrm{d})\right]$$

$$=5.71\times10^{12}\ \mathrm{nuclei}$$

so the number of decays has been $N_0 - N = 3.57 \times 10^{12}$ and the energy released is

$$E=\left(3.57\times10^{12}\right)(700\ \mathrm{keV})\left(1.60\times10^{-16}\ \mathrm{J/keV}\right)=0.400\ \mathrm{J}$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\mathrm{Dose}=\left(\frac{0.400\ \mathrm{J}}{0.100\ \mathrm{kg}}\right)\left(\frac{1\ \mathrm{rad}}{10^{-2}\ \mathrm{J/kg}}\right)=\boxed{400\ \mathrm{rad}}$$

P45.69 (a) The thermal power transferred to the water is P_w = 0.970 (waste heat):

$$P_w=0.970(3\,065\ \mathrm{MW}-1\,000\ \mathrm{MW})=2.00\times10^{9}\ \mathrm{J/s}$$

r_w is the mass of water heated per hour:

$$r_w = \frac{P_w}{c(\Delta T)} = \frac{(2.00\times10^9\ \text{J/s})(3600\ \text{s/h})}{(4186\ \text{J/kg}\cdot{}^\circ\text{C})(3.50\ {}^\circ\text{C})} = \boxed{4.92\times10^8\ \text{kg/h}}$$

Then, the volume used per hour is

$$\frac{4.91\times10^8\ \text{kg/h}}{1.00\times10^3\ \text{kg/m}^3} = \boxed{4.92\times10^5\ \text{m}^3\text{/h}}$$

(b) The ^{235}U fuel is consumed at a rate

$$r_f = \left(\frac{3\,065\times10^6\ \text{J/s}}{7.80\times10^{10}\ \text{J/g}}\right)\left(\frac{1\ \text{kg}}{1\,000\ \text{g}}\right)\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = \boxed{0.141\ \text{kg/h}}$$

P45.70 We add two electrons to both sides of the given reaction.

Then, $4\ {}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + 2\nu,$

where $Q = (\Delta m)c^2 = \left[4(1.007\ 825\ \text{u}) - 4.002\,603\ \text{u}\right](931.5\ \text{MeV/u})$
$= 26.7\ \text{MeV}$

or $Q = (26.7\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV}) = 4.28\times10^{-12}\ \text{J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.85\times10^{26}\ \text{J/s}}{(4.28\times10^{-12}\ \text{J})/(4\ \text{protons})}$$
$$= \boxed{3.60\times10^{38}\ \text{protons/s}}$$

Challenge Problems

P45.71 The initial specific activity of ^{59}Fe in the steel is

$$(R/m)_0 = \frac{20.0\ \mu\text{Ci}}{0.200\ \text{kg}} = \left(\frac{100\ \mu\text{Ci}}{\text{kg}}\right)\left(\frac{3.70\times10^4\ \text{Bq}}{1\ \mu\text{Ci}}\right)$$
$$= 3.70\times10^6\ \text{Bq/kg}$$

The decay constant of ^{59}Fe is $\lambda = \dfrac{\ln 2}{45.1\ \text{d}}\left(\dfrac{1\ \text{d}}{24\ \text{h}}\right)$.

After 1 000 h, the activity is

$$\frac{R}{m} = \left(\frac{R}{m}\right)_0 e^{-\lambda t}$$

$$= \left(3.70 \times 10^6 \text{ Bq/kg}\right) \exp\left[-\left(\frac{\ln 2}{45.1 \text{ d}}\right)\left(\frac{1 \text{ d}}{24 \text{ h}}\right)(1\,000 \text{ h})\right]$$

$$= 1.95 \times 10^6 \text{ Bq/kg}$$

The activity of the oil is

$$R_{\text{oil}} = \left(\frac{800}{60.0} \text{ Bq/liter}\right)(6.50 \text{ liters}) = 86.7 \text{ Bq}$$

Therefore,

$$m_{\text{in oil}} = \frac{R_{\text{oil}}}{(R/m)} = \frac{86.7 \text{ Bq}}{1.95 \times 10^6 \text{ Bq/kg}} = 4.44 \times 10^{-5} \text{ kg}$$

So that the wear rate is $\dfrac{4.45 \times 10^{-5} \text{ kg}}{1\,000 \text{ h}} = \boxed{4.44 \times 10^{-8} \text{ kg/h}}$.

P45.72 (a) The number of fissions occurring in the zeroth, first, second, …, nth generation is

$$N_0,\ N_0K,\ N_0K^2,\ \ldots,\ N_0K^n$$

The total number of fissions that have occurred up to and including the nth generation is

$$N = N_0 + N_0K + N_0K^2 + \cdots + N_0K^n = N_0\left(1 + K + K^2 + \cdots + K^n\right)$$

Note that the factoring of the difference of two squares, $a^2 - 1 = (a + 1)(a - 1)$, can be generalized to a difference of two quantities to any power,

$$a^3 - 1 = \left(a^2 + a + 1\right)(a - 1)$$

$$a^{n+1} - 1 = \left(a^n + a^{n-1} + \cdots + a^2 + a + 1\right)(a - 1)$$

Thus, $K^n + K^{n-1} + \cdots + K^2 + K + 1 = \dfrac{K^{n+1} - 1}{K - 1}$

and $\boxed{N = N_0 \dfrac{K^{n+1} - 1}{K - 1}}$

(b) The number of U-235 nuclei is

$$N = (5.50 \text{ kg})\left(\frac{1 \text{ atom}}{235 \text{ u}}\right)\left(\frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}}\right) = 1.41 \times 10^{25} \text{ nuclei}$$

We solve the equation from part (a) for n, the number of generations:

$$\frac{N}{N_0}(K-1) = K^{n+1} - 1$$

$$\frac{N}{N_0}(K-1) + 1 = K^n(K)$$

$$n\ln K = \ln\left(\frac{N(K-1)/N_0 + 1}{K}\right) = \ln\left(\frac{N(K-1)}{N_0} + 1\right) - \ln K$$

$$n = \frac{\ln\left(N(K-1)/N_0 + 1\right)}{\ln K} - 1 = \frac{\ln\left(1.41\times10^{25}(0.1)/10^{20} + 1\right)}{\ln 1.1} - 1$$
$$= 99.2$$

Therefore time must be allotted for 100 generations:

$$\Delta t_b = 100\left(10\times10^{-9}\text{ s}\right) = 1.00\times10^{-6}\text{ s} = \boxed{1.00\ \mu\text{s}}$$

(c) The speed of sound in uranium is

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{150\times10^9\text{ N/m}^2}{18.7\times10^3\text{ kg/m}^3}} = 2.83\times10^3\text{ m/s} = \boxed{2.83\text{ km/s}}$$

(d) From the definitions of volume and mass density, $V = \frac{4}{3}\pi r^3 = \frac{m}{\rho}$, and

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3(5.5\text{ kg})}{4\pi\left(18.7\times10^3\text{ kg/m}^3\right)}\right)^{1/3} = 4.13\times10^{-2}\text{ m}$$

then, the time interval is given by

$$\Delta t_d = \frac{r}{v} = \frac{4.13\times10^{-2}\text{ m}}{2.83\times10^3\text{ m/s}} = 1.46\times10^{-5}\text{ s} = \boxed{14.6\ \mu\text{s}}$$

(e) 14.6 μs is greater than 1 μs, so the entire bomb can fission. The destructive energy released is

$$\left(1.41\times10^{25}\text{ nuclei}\right)\left(\frac{200\times10^6\text{ eV}}{\text{fissioning nucleus}}\right)\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)$$
$$= 4.51\times10^{14}\text{ J}\left(\frac{1\text{ ton TNT}}{4.20\times10^9\text{ J}}\right)$$
$$= 1.08\times10^5\text{ ton TNT}$$
$$= \boxed{108\text{ kilotons of TNT}}$$

What if? If the bomb did not have an "initiator" to inject 10^{20} neutrons at the moment when the critical mass is assembled, the number of generations would be

$$n = \frac{\ln\left(1.41\times 10^{25}(0.1)/1+1\right)}{\ln 1.1} - 1 = 582.4$$

requiring $583\left(10\times 10^{-9}\text{ s}\right) = 5.83\ \mu\text{s}$

This time is not very short compared with 14.6 μs, so this bomb would likely release much less energy.

P45.73 (a) $E_I = 10.0$ eVis the energy required to liberate an electron from a dynode. Let n_i be the number of electrons incident upon a dynode, each having gained energy $e\Delta V$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e\dfrac{\Delta V}{E_I}$.

At the <u>first</u> dynode, $n_i = 1$ and

$$N_1 = \frac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^1\text{ electrons}}$$

(b) For the <u>second</u> dynode, $n_i = N_1 = 10^1$, so

$$N_2 = \frac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$$

At the <u>third</u> dynode, $n_i = N_2 = 10^2$ and

$$N_3 = \frac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$$

Observing the developing pattern, we see that the number of electrons incident on the nth dynode is $n_n = N_{n-1} = 10^{n-1}$, so for the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$$E = n_i e(\Delta V) = 10^6 e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P45.2 0.403 g

P45.4 2.63 kg/d

P45.6 (a) 173 MeV; (b) 0.078 8%

P45.8 26

P45.10 (a) 4×10^{15} g; (b) 5×10^{3} yr; (c) The uranium comes from rocks and minerals dissolved in water and carried into the ocean by rivers; (d) No.

P45.12 (a) 16.2 kg; (b) 117 g

P45.14 (a) $4.84V^{-1/3}$; (b) $6V^{-1/3}$; (c) $6.30V^{-1/3}$; (d) The sphere has minimum leakage and the parallelepiped has minimum leakage.

P45.16 6.25×10^{19} Bq

P45.18 By balancing the equation for electric charge and number of nucleons, we find that $n = 1$. If one incoming neutron results in just one outgoing neutron, the possibility of a chain reaction is not there, so this nuclear reactor will not work.

P45.20 (a) $4.56\times10^{-24}\ \text{kg}\cdot\text{m/s}$; (b) 0.145 nm; (c) This size has the same order of magnitude as an atom's outer electron cloud, and is vastly larger than a nucleus.

P45.22 3.07×10^{22} events

P45.24 (a) $E = 144Z_1Z_2$ where E is in keV; (b) The energy is proportional to each atomic number; (c) Take $Z_1 = 1$ and $Z_2 = 59$ or vice versa. This choice minimizes the product $Z_1 Z_2$; (d) 144 keV for both, according to this model

P45.26 (a) 3.24 fm; (b) 444 keV; (c) $\frac{2}{5}v_i$; (d) 740 keV; (e) possibly by tunneling

P45.28 (a) 1.66×10^{7} J; (b) 6.45 kg

P45.30 (a) ${}^{13}_{7}\text{N}$; (b) ${}^{13}_{6}\text{C}$; (c) ${}^{14}_{7}\text{N}$; (d) ${}^{15}_{8}\text{O}$; (e) ${}^{15}_{7}\text{N}$; (f) ${}^{12}_{6}\text{C}$; (g) The original carbon-12 nucleus is returned so the overall reaction is $4\left({}^{1}_{1}\text{H}\right) \rightarrow {}^{4}_{2}\text{He}$.

P45.32 (a) 2.5 mrem/x-ray; (b) The technician's occupational exposure is high compared to background radiation; it is 38 times 0.13 rem/yr.

P45.34 (a) $\dfrac{\ln(2)}{\mu}$; (b) $-\dfrac{\ln f}{\mu}$

P45.36 18.8 J

P45.38 It would take over 24 days to raise the temperature of the water to 100°C and even longer to boil it, so this technique will not work for a 20-minute coffee break!

P45.40 1.14 rad

P45.42 3.96×10^{-4} J/kg

P45.44 (a) See P45.44(a) for full explanation; (b) $\dfrac{R}{\lambda}$

P45.46 (a) 8.68 MeV; (b) The particles must have enough kinetic energy to overcome their mutual electrostatic repulsion so that they can get close enough to fuse.

P45.48 (a) 10^3 Pa; (b) 6×10^9 m^3; (c) 6×10^{12} J; (d) ~10^{14} J; (e) ~ 10^4 ton TNT

P45.50 (a) 27.6 min; (b) 30 min ± 27%

P45.52 (a) See P45.52(a) for full explanation; (b) 177 MeV; (c) K_{Br} = 112.0 MeV, K_{La} = 65.4 MeV; (d) v_{Br} = 15.8 Mm/s, v_{La} = 9.30 Mm/s

P45.54 232 yr

P45.56 482 Ci, less than the fission inventory by on the order of a hundred million times.

P45.58 $$\frac{mN_{\text{A}}E}{M_{\text{U-235}}\left[c_w\left(100 - T_c\right) + L_v + c_s\left(T_h - 100\right)\right]}$$

P45.60 (a) 15.4 cm; (b) 51.7 MeV; (c) The number of decays per second is the decay rate *R*, and the energy released in each decay is *Q*. Then the energy released per unit time interval is $P = QR$; (d) 2.27×10^5 J/yr; (e) 3.18 J/yr

P45.62 (a) See P45.62(a) for full explanation; (b) 35.2; (c) 2.89×10^{15}

P45.64 (a) 2.24×10^7 kWh; (b) 17.6 MeV for each D-T fusion; (c) 2.34×10^8 kWh; (d) 9.36 kWh; (e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels. Burning coal in the open puts carbon dioxide into the atmosphere, worsening global warming. Plutonium is a very dangerous material to have sitting around.

P45.66 (a) 2.65×10^9 J; (b) 78.0 times larger

P45.68 400 rad

P45.70 3.60×10^{38} protons/s

P45.72 (a) See P45.72(a) for full explanation; (b) 1.00 μs; (c) 2.83 km/s; (d) 14.6 μs; (e) 108 kilotons of TNT