# 46

# Particle Physics and Cosmology

## CHAPTER OUTLINE



* An asterisk indicates a question or problem new to this edition.

## ANSWERS TO OBJECTIVE QUESTIONS

**OQ46.1** Answers (a), (b), (c), and (d). Protons feel all these forces; but within a nucleus the strong interaction predominates, followed by the electromagnetic interaction, then the weak interaction. The gravitational interaction is very small.

**OQ46.2** Answer (e). Kinetic energy is transformed into internal energy: $Q = -\Delta K$. In the first experiment, momentum conservation requires the final speed be zero:

$$p_1 = mv - mv = 2mv_f \rightarrow \quad v_f = 0$$



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The kinetic energy converted into internal energy is $mv^2$:

$$\Delta K_1 = K_f - K_i = 0 - \left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2\right) = -mv^2 \quad \rightarrow \quad Q_1 = mv^2$$

In the second experiment, momentum conservation requires the final speed be half the initial speed:

$$p_2 = mv + m(0) = 2mv_f \qquad \rightarrow \qquad v_f = \frac{v}{2}$$

The kinetic energy converted into internal energy is $\dfrac{mv^2}{4}$:

$$\Delta K_2 = K_f - K_i = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 = -\frac{mv^2}{4} \qquad \rightarrow \qquad Q_2 = \frac{mv^2}{4}$$

**OQ46.3** Answer (b). There are $(2s+1) = \left(2\tfrac{3}{2}+1\right) = 4$ states: the $z$ component of its spin angular momentum can be 3/2, 1/2, –1/2, or –3/2, in units of $\hbar$.

**OQ46.4** Answer (b). According the Table 46.1, the photon mediates the electromagnetic force, the graviton the gravitational force, and the $W^+$ and Z bosons the weak force.

**OQ46.5** Answer (c). According to Table 46.2, the muon has much more rest energy (105.7 MeV/$c^2$) than the electron (0.511 MeV/$c^2$) and the neutrinos together (< 0.3 MeV/$c^2$). The missing rest energy goes into kinetic energy: $m_\mu c^2 = K_{\text{total}} + m_e c^2 + m_{\bar{\nu}_e} c^2 + m_{\nu_\mu} c^2$.

**OQ46.6** Answer (a). The vast gulfs not just between stars but between galaxies and especially between clusters, empty of ordinary matter, are important to bring down the average density of the Universe. We can estimate the average density defined for the Solar System as the mass of the Sun divided by the volume of a sphere of radius $2 \times 10^{16}$ m:

$$\frac{2\times 10^{30}\ \text{kg}}{\frac{4}{3}\pi(2\times 10^{16}\ \text{m})^3} = 6\times 10^{-20}\ \text{kg/m}^3 = 6\times 10^{-23}\ \text{g/cm}^3$$

This is ten million times larger than the critical density $3H^2/8\pi G$ $= 6 \times 10^{-30}$ g/cm$^3$.

**OQ46.7** Answer (b). Momentum would not be conserved. The electron and positron together have very little momentum. A 1.02-MeV photon has a definite amount of momentum. Production of a single gamma ray could not satisfy the law of conservation of momentum, which must hold true in this—and every—interaction.

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**OQ46.8** The sequence is c, b, d, e, a, f, g. Refer to Figure 46.16 in the textbook. The temperature corresponding to b is on the order of $10^{13}$ K. That for hydrogen fusion d is on the order of $10^7$ K. A fully ionized plasma can be at $10^4$ K. Neutral atoms can exist at on the order of 3 000 K, molecules at 1 000 K, and solids at on the order of 500 K.

## ANSWERS TO CONCEPTUAL QUESTIONS

**CQ46.1** The electroweak theory of Glashow, Salam, and Weinberg predicted the $W^+$, $W^-$, and Z particles. Their discovery in 1983 confirmed the electroweak theory.

**CQ46.2** Hadrons are massive particles with internal structure. There are two classes of hadrons: mesons (bosons) and baryons (fermions). Hadrons are composed of quarks, so they interact via the strong force. Leptons are light particles with no structure. All leptons are fermions. It is believed that leptons are fundamental particles (otherwise, there would be leptonic bosons); leptons are not composed of quarks, so they do not interact via the strong force.

**CQ46.3** Before that time, the Universe was too hot for the electrons to remain bound to any nucleus. The thermal motion of both nuclei and electrons was too rapid for the Coulomb force to dominate. The Universe was so filled high energy photons that any nucleus that managed to captured an electron would immediately lose it because of Compton scattering or the photoelectric effect.

**CQ46.4** Baryons are heavy hadrons; they are fermions with spin $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$; they are composed of three quarks. (Antibaryons are composed of three antiquarks.) Mesons are light hadrons; they are bosons with spin 0, 1, 2, …; they are composed of a quark and an antiquark.

**CQ46.5** The decay is slow, relatively speaking. The decays by the weak interaction typically take $10^{-10}$ s or longer to occur. This is slow in particle physics. The decay does not conserve strangeness: the $\Xi^0$ has strangeness of $-2$, the $\Lambda^0$ has strangeness –1, and the $\pi^0$ has strangeness 0. (Refer to Table 46.2.)

**CQ46.6** The word "color" has been adopted in *analogy* to the properties of the three primary colors (and their complements) in additive color mixing. Each flavor of quark can have colors, designated as red, green, and blue. Antiquarks are colored antired, antigreen, and antiblue. We call baryons and mesons *colorless*. A baryon consists of three quarks, each having a different color: the analogy is three

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primary colors combine to form no color: colorless white. A meson consists of a quark of one color and antiquark with the corresponding anticolor: the analogy is a primary color and its complementary color combine to form no color: colorless white.

**CQ46.7** No. Antibaryons have baryon number –1, mesons have baryon number 0, and baryons have baryon number +1. The reaction cannot occur because it would not conserve baryon number, unless so much energy is available that a baryon-antibaryon pair is produced.

**CQ46.8** The Standard Model consists of quantum chromodynamics (to describe the strong interaction) and the electroweak theory (to describe the electromagnetic and weak interactions). The Standard Model is our most comprehensive description of nature. It fails to unify the two theories it includes, and fails to include the gravitational force. It pictures matter as made of six quarks and six leptons, interacting by exchanging gluons, photons, and W and Z bosons. In 2011 and 2012, experiments at CERN produced evidence for the Higgs boson, a cornerstone of the Standard Model.

**CQ46.9** (a) Baryons consist of three quarks.
(b) Antibaryons consist of three antiquarks.
(c) and (d) Mesons and antimesons consist of a quark and an antiquark.

Since quarks have spin quantum number $\frac{1}{2}$ and can be spin-up or spin-down, it follows that the baryons and antibaryons must have a half-integer spin ($\frac{1}{2}, \frac{3}{2}, \ldots$), while the mesons and antimesons must have integer spin (0, 1, 2, …).

**CQ46.10** We do know that the laws of conservation of momentum and energy are a consequence of Newton's laws of motion; however, conservation of baryon number, lepton number, and strangeness cannot be traced to Newton's laws. Even though we do not know what electric charge *is*, we do know it is conserved, so too we do not know what baryon number, lepton number, or strangeness *are*, but we do know they are conserved—or in the case of strangeness, sometimes conserved—from observations of how elementary particles interact and decay. You can think of these conservation laws as regularities which we happen to notice, as a person who does not know the rules of chess might observe that one player's two bishops are always on squares of opposite colors. (From the observation of the behavior of baryon number, lepton number, and strangeness in particle interactions, *gauge theories*, which are not discussed in the textbook, have been developed to describe that behavior.)

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**CQ46.11** The interactions and their field particles are listed in Table 46.1.

Strong Force—Mediated by gluons.
Electromagnetic Force—Mediated by photons.
Weak Force—Mediated by $W^+$, $W^-$, and $Z^0$ bosons.
Gravitational Force—Mediated by gravitons (not yet observed).

**CQ46.12** Hubble determined experimentally that all galaxies outside the Local Group are moving away from us, with speed directly proportional to the distance of the galaxy from us, by observing that their light spectra were red shifted in direct relation to their distance from the Local Group.

**CQ46.13** The baryon number of a proton or neutron is one. Since baryon number is conserved, the baryon number of the kaon must be zero. See Table 46.2.

## SOLUTIONS TO END-OF-CHAPTER PROBLEMS

## Section 46.1 The Fundamental Forces in Nature

## Section 46.2 Positrons and Other Antiparticles

**P46.1** (a) The rest energy of a total of 6.20 g of material is converted into energy of electromagnetic radiation:

$$E = mc^2 = \left(6.20\times10^{-3}\ \text{kg}\right)\left(2.998\times10^{8}\ \text{m/s}\right)^2 = \boxed{5.57\times10^{14}\ \text{J}}$$

(b)
$$5.57\times10^{14}\ \text{J} = 5.57\times10^{14}\ \text{J}\left(\frac{\$0.11}{\text{kWh}}\right)\left(\frac{\text{k}}{1\ 000}\right)\left(\frac{\text{W}}{\text{J/s}}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right)$$
$$= \boxed{\$1.70\times10^{7}}$$

**P46.2** (a) The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must have the same magnitude of momentum, and $p = E/c$, so each photon must carry away one-half the energy.

Thus $E_{\min} = \dfrac{2E_0}{2} = E_0 = 938.3\ \text{MeV} = hf_{\min}$.

Thus, $$f_{\min} = \frac{(938.3\ \text{MeV})\left(1.602\times10^{-13}\ \text{J/MeV}\right)}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{2.27\times10^{23}\ \text{Hz}}.$$

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(b) $\lambda = \dfrac{c}{f_{\min}} = \dfrac{2.998 \times 10^8 \text{ m/s}}{2.27 \times 10^{23} \text{ Hz}} = \boxed{1.32 \times 10^{-15} \text{ m}}$

**P46.3** (a) Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy $E$ of the photon must be

$$E = 2E_0 = 2(938.3 \text{ MeV}) = 1\ 876.6 \text{ MeV}\left(\frac{1.602 \times 10^{-13} \text{ J}}{1 \text{ MeV}}\right)$$
$$= 3.01 \times 10^{-10} \text{ J}$$

Thus, $E = hf = 3.01 \times 10^{-10}$ J, so

$$f = \frac{3.01 \times 10^{-10} \text{ J}}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{4.53 \times 10^{23} \text{ Hz}}$$

(b) $\lambda = \dfrac{c}{f} = \dfrac{2.998 \times 10^8 \text{ m/s}}{4.53 \times 10^{23} \text{ Hz}} = \boxed{6.61 \times 10^{-16} \text{ m}}$

**P46.4** The half-life of $^{14}$O is 70.6 s, so the decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{70.6 \text{ s}}$.

The number of $^{14}$O nuclei remaining after five minutes is

$$N = N_0 e^{-\lambda t} = (10^{10})\exp\left[-\frac{\ln 2}{70.6 \text{ s}}(300 \text{ s})\right] = 5.26 \times 10^8$$

The number of these in one cubic centimeter of blood is

$$N' = N\left(\frac{1.00 \text{ cm}^3}{\text{total volume of blood}}\right) = (5.26 \times 10^8)\left(\frac{1.00 \text{ cm}^3}{2\ 000 \text{ cm}^3}\right)$$
$$= 2.63 \times 10^5$$

and their activity is

$$R = \lambda N' = \frac{\ln 2}{70.6 \text{ s}}(2.63 \times 10^5) = 2.58 \times 10^3 \text{ Bq} \quad \boxed{\sim 10^3 \text{ Bq}}$$

**P46.5** The total energy of each particle is the sum of its rest energy and its kinetic energy. Conservation of system energy requires that the total energy before this pair production event equal the total energy after. In $\gamma \to \text{p}^+ + \text{p}^-$, conservation of energy requires that

$$E_\gamma \to E_{\text{p}^+} + E_{\text{p}^-}$$
$$E_\gamma \to (m_\text{p}c^2 + K_{\text{p}^+}) + (m_\text{p}c^2 + K_{\text{p}^-})$$

or
$$E_\gamma = (E_{Rp} + K_p) + (E_{R\bar{p}} + K_{\bar{p}})$$

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The energy of the photon is given as

$$E_\gamma = 2.09\ \text{GeV} = 2.09\times 10^3\ \text{MeV}$$

From Table 46.2 or from the problem statement, we see that the rest energy of both the proton and the antiproton is

$$E_{Rp} = E_{R\bar{p}} = m_p c^2 = 938.3\ \text{MeV}$$

If the kinetic energy of the proton is observed to be 95.0 MeV, the kinetic energy of the antiproton is

$$\begin{aligned} K_{\bar{p}} &= E_\gamma - E_{R\bar{p}} - E_{Rp} - K_p \\ &= 2.09\times 10^3\ \text{MeV} - 2(938.3\ \text{MeV}) - 95.0\ \text{MeV} = \boxed{118\ \text{MeV}} \end{aligned}$$

## Section 46.3 Mesons and the Beginning of Particle Physics

**P46.6** The creation of a virtual $Z^0$ boson is an energy fluctuation $\Delta E = m_{Z^0}c^2 = 91\times 10^9\ \text{eV}$. By the uncertainty principle, it can last no longer than $\Delta t = \dfrac{\hbar}{2\Delta E}$ and move no farther than

$$\begin{aligned} c(\Delta t) &= \frac{hc}{4\pi\,\Delta E} \\ &= \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(3.00\times 10^8\ \text{m/s})}{4\pi(91\times 10^9\ \text{eV})}\left(\frac{1\ \text{eV}}{1.60\times 10^{-19}\ \text{J}}\right) \\ &= 1.06\times 10^{-18}\ \text{m} = \boxed{\sim 10^{-18}\ \text{m}} \end{aligned}$$

**P46.7** (a) The particle's rest energy is $mc^2$. The time interval during which a virtual particle of this mass could exist is at most $\Delta t$ in $\Delta E\Delta t = \dfrac{\hbar}{2} = mc^2\Delta t$; or $\Delta t = \dfrac{\hbar}{2mc^2}$; so, the distance it could move (traveling at the speed of light) is at most

$$\begin{aligned} d \approx c\Delta t &= \frac{\hbar c}{2mc^2} = \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(2.998\times 10^8\ \text{m/s})}{4\pi mc^2(1.602\times 10^{-19}\ \text{J/eV})} \\ &= \frac{1.240\times 10^{-6}\ \text{eV}\cdot\text{m}}{4\pi mc^2}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = \frac{1\,240\ \text{eV}\cdot\text{nm}}{4\pi mc^2} \\ &= \frac{98.7\ \text{eV}\cdot\text{nm}}{mc^2} \end{aligned}$$

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or $d \approx \frac{98.7}{mc^2}$, where $d$ is in nanometers and $mc^2$ is in electron volts.

According to Yukawa's line of reasoning, this distance is the range of a force that could be associated with the exchange of virtual particles of this mass.

(b) $\boxed{\text{The range is inversely proportional to the mass of the field particle.}}$

(c) The value of $mc^2$ for the proton in electron volts is $938.3 \times 10^6$. The range of the force is then

$$d \approx \frac{98.7}{mc^2} = \frac{98.7}{938.3 \times 10^6} = \left(1.05 \times 10^{-7}\ \text{nm}\right)\left(\frac{10^{-9}}{1\ \text{nm}}\right)$$
$$= 1.05 \times 10^{-16}\ \text{m} \boxed{\sim 10^{-16}\ \text{m}}$$

## Section 46.4 Classification of Particles

## Section 46.5 Conservation Laws

***P46.8** $\boxed{\text{Baryon number conservation allows the first and forbids the second}}$.

**P46.9** The energy and momentum of a photon are related by $p_\gamma = E_\gamma / c$. By momentum conservation, because the neutral pion is at rest, the magnitudes of the momenta of the two photons are equal; thus, their energies are equal.

(a) From Table 46.2, $m_{\pi^0} = 135\ \text{MeV}/c^2$. Therefore,

$$E_\gamma = \frac{m_{\pi^0} c^2}{2} = \frac{135.0\ \text{MeV}}{2} = \boxed{67.5\ \text{MeV}} \text{ for each photon}$$

(b) $p = \frac{E_\gamma}{c} = \boxed{67.5\ \text{MeV}/c}$

(c) $f = \frac{E_\gamma}{h} = \frac{67.5\ \text{MeV}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}\left(\frac{1.602 \times 10^{-13}\ \text{J}}{\text{MeV}}\right) = \boxed{1.63 \times 10^{22}\ \text{Hz}}$

**P46.10** The time interval for a particle traveling with the speed of light to travel a distance of $3 \times 10^{-15}$ m is

$$\Delta t = \frac{d}{v} = \frac{3 \times 10^{-15}\ \text{m}}{3.00 \times 10^8\ \text{m/s}} = \boxed{\sim 10^{-23}\ \text{s}}$$

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**P46.11** (a) $p + \overline{p} \to \mu^+ + e^-$  $L_\mu: 0 + 0 \to -1 + 0$ and $L_e: 0 + 0 \to 0 + 1$

$\boxed{\text{muon lepton number and electron lepton number}}$

(b) $\pi^- + p \to p + \pi^+$  $\boxed{\text{charge}}: -1 + 1 \to +1 + 1$

(c) $p + p \to p + p + n$  $\boxed{\text{baryon number}}: 1 + 1 \to 1 + 1 + 1$

(d) $\gamma + p \to n + \pi^0$  $\boxed{\text{charge}}: 0 + 1 \to 0 + 0$

(f) $\nu_e + p \to n + e^+$  $L_e: 1 + 0 \to 0 - 1$

$\boxed{\text{electron lepton number}}$

**P46.12** (a) Baryon number and charge are conserved, with respective values of

baryon: 0 + 1 = 0 + 1

charge: 1 + 1 = 1 + 1 in both reactions (1) and (2).

(b) The strangeness values for the reactions are

(1) $S$: 0 + 0 = 1 − 1

(2) $S$: 0 + 0 = 0 − 1

$\boxed{\text{Strangeness is } \textit{not} \text{ conserved in the second reaction.}}$

**P46.13** Check that electron, muon, and tau lepton number are conserved.

(a) $\pi^- \to \mu^- + \boxed{\overline{\nu}_\mu}$  $L_\mu$: $0 \to 1 - 1$

(b) $K^+ \to \mu^+ + \boxed{\nu_\mu}$  $L_\mu$: $0 \to -1 + 1$

(c) $\boxed{\overline{\nu}_e} + p^+ \to n + e^+$  $L_e$: $-1 + 0 \to 0 - 1$

(d) $\boxed{\nu_e} + n \to p^+ + e^-$  $L_e$: $1 + 0 \to 0 + 1$

(e) $\boxed{\nu_\mu} + n \to p^+ + \mu^-$  $L_\mu$: $1 + 0 \to 0 + 1$

(f) $\mu^- \to e^- + \boxed{\overline{\nu}_e} + \boxed{\nu_\mu}$  $L_\mu$: $1 \to 0 + 0 + 1$ and $L_e$: $0 \to 1 - 1 + 0$

**P46.14** The relevant conservation laws are $\Delta L_e = 0$, $\Delta L_\mu = 0$, and $\Delta L_\tau = 0$.

(a) $\pi^+ \to \pi^0 + e^+ + ?$  $L_e$: $0 \to 0 - 1 + L_e$ implies $L_e = 1$, so the particle is $\boxed{\nu_e}$.

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(b) $? + \mathrm{p} \to \mu^- + \mathrm{p} + \pi^+$ $\quad L_\mu$: $L_\mu + 0 \to +1 + 0 + 0$ implies $L_\mu = 1$, so the particle is $\boxed{\nu_\mu}$.

(c) $\Lambda^0 \to \mathrm{p} + \mu^- + ?$ $\quad L_\mu$: $0 \to 0 + 1 + L_\mu$ implies $L_\mu = -1$, so the particle is $\boxed{\bar{\nu}_\mu}$.

(d) $\tau^+ \to \mu^+ + ? + ?$ $\quad L_\mu$: $0 \to -1 + L_\mu$ implies $L_\mu = 1$, so one particle is $\boxed{\nu_\mu}$.

Also, $L_\tau$: $-1 \to 0 + L_\tau$ implies $L_\tau = -1$, so the other particle is $\boxed{\bar{\nu}_\tau}$.

**P46.15** (a) $\mathrm{p}^+ \to \pi^+ + \pi^0$ $\quad$ check baryon number: $1 \to 0 + 0$

$\boxed{\text{It cannot occur because it violates baryon number conservation.}}$

(b) $\mathrm{p}^+ + \mathrm{p}^+ \to \mathrm{p}^+ + \mathrm{p}^+ + \pi^0$ $\quad \boxed{\text{It can occur.}}$

(c) $\mathrm{p}^+ + \mathrm{p}^+ \to \mathrm{p}^+ + \pi^+$ check baryon number: $1 + 1 \to 1 + 0$

$\boxed{\text{It cannot occur because it violates baryon number conservation.}}$

(d) $\pi^+ \to \mu^+ + \nu_\mu$ $\quad \boxed{\text{It can occur.}}$

(e) $\mathrm{n}^0 \to \mathrm{p}^+ + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$ $\quad \boxed{\text{It can occur.}}$

(f) $\pi^+ \to \mu^+ + \mathrm{n}$ $\quad$ check baryon number: $0 \to 0 + 1$

check muon lepton number: $0 \to -1 + 0$

check masses: $m_{\pi^+} < m_{\mu^+} + m_\mathrm{n}$

$\boxed{\begin{array}{l}\text{It cannot occur because it violates baryon number conservation,}\\ \text{muon lepton number conservation, and energy conservation.}\end{array}}$

**P46.16** The reaction is $\mu^+ + \mathrm{e}^- \to \nu + \nu$.

muon-lepton number before reaction: (–1) + (0) = –1

electron-lepton number before reaction: (0) + (1) = 1

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the antineutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons: $\boxed{\bar{\nu}_\mu}$ and $\boxed{\nu_\mathrm{e}}$

Thus, $\quad \mu^+ + \mathrm{e}^- \to \bar{\nu}_\mu + \nu_\mathrm{e}$.

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**P46.17** Momentum conservation for the decay requires the pions to have equal speeds.

The total energy of each is $\dfrac{497.7 \text{ MeV}}{2} = 248.8 \text{ MeV}$, so

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \text{ gives}$$

$$(248.8 \text{ MeV})^2 = (pc)^2 + (139.6 \text{ MeV})^2$$

Solving, $pc = 206 \text{ MeV} = \gamma\, mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$:

$$\frac{pc}{mc^2} = \frac{206 \text{ MeV}}{139.6 \text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$$\frac{v}{c} = 1.48\sqrt{1-\left(\frac{v}{c}\right)^2}$$

and $$\left(\frac{v}{c}\right)^2 = 2.18\left[1-\left(\frac{v}{c}\right)^2\right] = 2.18 - 2.18\left(\frac{v}{c}\right)^2$$

$$3.18\left(\frac{v}{c}\right)^2 = 2.18$$

so $$\frac{v}{c} = \sqrt{\frac{2.18}{3.18}} = 0.828 \quad \text{and} \quad \boxed{v = 0.828c}.$$

**P46.18** (a) In the suggested reaction $\mathrm{p} \to \mathrm{e}^+ + \gamma$.

From Table 46.2, we would have for baryon numbers $+1 \to 0 + 0$; thus $\Delta B \neq 0$, so baryon number conservation would be violated.

(b) From conservation of momentum for the decay: $p_e = p_\gamma$

Then, for the positron,

$$E_e^2 = \left(p_e c\right)^2 + \left(m_e c^2\right)^2$$

becomes

$$E_e^2 = \left(p_\gamma c\right)^2 + \left(m_e c^2\right)^2 = E_\gamma^2 + \left(m_e c^2\right)^2$$

From conservation of energy for the system: $m_\mathrm{p} c^2 = E_e + E_\gamma$

or $E_e = m_\mathrm{p} c^2 - E_\gamma$,

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so $E_e^2 = \left(m_\text{p}c^2\right)^2 - 2\left(m_\text{p}c^2\right)E_\gamma + E_\gamma^2.$

Equating this to the result from above gives

$$\cancel{E_\gamma^2} + \left(m_e c^2\right)^2 = \left(m_\text{p}c^2\right)^2 - 2\left(m_e c^2\right)E_\gamma + \cancel{E_\gamma^2}$$

$$E_\gamma = \frac{\left(m_\text{p}c^2\right)^2 - \left(m_e c^2\right)^2}{2m_\text{p}c^2}$$

$$= \frac{(938.3\text{ MeV})^2 - (0.511\text{ MeV})^2}{2(938.3\text{ MeV})} = 469\text{ MeV}$$

Also, $E_e = m_\text{p}c^2 - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = 469\text{ MeV}$,

Thus, $\boxed{E_e = E_\gamma = 469\text{ MeV}}$.

Also, $p_\gamma = \dfrac{E_\gamma}{c} = \dfrac{469\text{ MeV}}{c}$, so $\boxed{p_e = p_\gamma = 469\text{ MeV}/c}$.

(c) The total energy of the positron is $E_e = 469$ MeV,

but $E_e = \gamma m_e c^2 = \dfrac{m_e c^2}{\sqrt{1-(v/c)^2}}$,

so $\sqrt{1-\left(\dfrac{v}{c}\right)^2} = \dfrac{m_e c^2}{E_e} = \dfrac{0.511\text{ MeV}}{469\text{ MeV}} = 1.09\times 10^{-3}$,

which yields $\boxed{v = 0.000\ 999\ 4c}$.

**P46.19** (a) To conserve charge, the decay reaction is $\Lambda^0 \to \text{p} + \pi^-$.

We look up in the table the rest energy of each particle:

$m_\Lambda c^2 = 1\ 115.6$ MeV  $m_\text{p}c^2 = 938.3$ MeV

$m_\pi c^2 = 139.6$ MeV

The $Q$ value of the reaction, representing the energy output, is the difference between starting rest energy and final rest energy, and is the kinetic energy of the products:

$$Q = 1\ 115.6\text{ MeV} - 938.3\text{ MeV} - 139.6\text{ MeV} = \boxed{37.7\text{ MeV}}$$

(b) The original kinetic energy is zero in the process considered here, so the whole $Q$ becomes the kinetic energy of the products

$$K_\text{p} + K_\pi = \boxed{37.7\text{ MeV}}$$

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(c) The lambda particle is at rest. Its momentum is zero. System momentum is conserved in the decay, so the total vector momentum of the proton and the pion must be $\boxed{\text{zero.}}$

(d) The proton and the pion move in precisely opposite directions with precisely equal momentum magnitudes. Because their masses are different, their kinetic energies are not the same.

The mass of the $\pi$-meson is much less than that of the proton, so it carries much more kinetic energy. We can find the energy of each. Let $p$ represent the magnitude of the momentum of each. Then the total energy of each particle is given by $E^2 = (pc)^2 + (mc^2)^2$ and its kinetic energy is $K = E - mc^2$. For the total kinetic energy of the two particles we have

$$\sqrt{m_p^2c^4 + p^2c^2} - m_pc^2 + \sqrt{m_\pi^2c^4 + p^2c^2} - m_\pi c^2$$
$$= Q = m_\Lambda c^2 - m_pc^2 - m_\pi c^2$$

Proceeding to solve for $pc$, we find

$$m_p^2c^4 + p^2c^2 = m_\Lambda^2c^4 - 2m_\Lambda c^2\sqrt{m_\pi^2c^4 + p^2c^2} + m_\pi^2c^4 + p^2c^2$$

$$\sqrt{m_\pi^2c^4 + p^2c^2} = \frac{m_\Lambda^2c^4 - m_p^2c^4 + m_\pi^2c^4}{2m_\Lambda c^2}$$
$$= \frac{1\ 115.6^2 - 938.3^2 + 139.6^2}{2(1\ 115.6)}\text{ MeV} = 171.9\text{ MeV}$$

$$pc = \sqrt{171.9^2 - 139.6^2}\text{ MeV} = 100.4\text{ MeV}$$

Then the kinetic energies are

$$K_p = \sqrt{938.3^2 + 100.4^2} - 938.3 = 5.35\text{ MeV}$$

and $K_\pi = \sqrt{139.6^2 + 100.4^2} - 139.6 = 32.3\text{ MeV}$

| |
|---|
| No. The mass of the $\pi^-$ meson is much less than that of the proton, so it carries much more kinetic energy. The correct analysis using relativistic energy conservation shows that the kinetic energy of the proton is 5.35 MeV, while that of the $\pi^-$ meson is 32.3 Mev. |

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## Section 46.6 Strange Particles and Strangeness

**P46.20** $\boxed{\begin{array}{l}\text{The } \rho^0 \to \pi^+ + \pi^- \text{ decay must occur via the strong interaction.} \\ \text{The } K_S^0 \to \pi^+ + \pi^- \text{ decay must occur via the weak interaction.}\end{array}}$

**P46.21** (a) $\pi^- + p \to 2\eta$

Baryon number: $0 + 1 \to 0$

$\boxed{\text{It is not allowed because baryon number is not conserved.}}$

(b) $K^- + n \to \Lambda^0 + \pi^-$

Baryon number: $0 + 1 \to 1 + 0$

Charge: $-1 + 0 \to 0 - 1$

Strangeness: $-1 + 0 \to -1 + 0$

Lepton number: $0 \to 0$

The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $K^- \to \pi^- + \pi^0$

Strangeness: $-1 \to 0 + 0$

Baryon number: $0 \to 0$

Lepton number: $0 \to 0$

Charge: $-1 \to -1 + 0$

Strangeness conservation is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \to \Xi^- + \pi^0$

Baryon number: $1 \to 1 + 0$

Lepton number: $0 \to 0$

Charge: $-1 \to -1 + 0$

Strangeness: $-3 \to -2 + 0$

Strangeness conservation is violated by one unit, but everything else is conserved. The reaction may occur by the $\boxed{\text{weak interaction}}$, but not by the strong or electromagnetic interaction.

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(e) $\eta \to 2\gamma$

Baryon number: $0 \to 0$

Lepton number: $0 \to 0$

Charge: $0 \to 0$

Strangeness: $0 \to 0$

No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the $\eta$ is consistent with the $\boxed{\text{electromagnetic interaction}}$.

**P46.22** (a) $\mu^- \to e^- + \gamma$ $\quad L_e: \ 0 \to 1 + 0$

$L_\mu: \ 1 \to 0$

$\boxed{\text{electron and muon lepton numbers}}$

(b) $n \to p + e^- + \nu_e$ $\quad L_e: \ 0 \to 0 + 1 + 1$

$\boxed{\text{electron lepton number}}$

(c) $\Lambda^0 \to p + \pi^0$ $\quad$ Strangeness: $-1 \to 0 + 0$

Charge: $0 \to +1 + 0$

$\boxed{\text{charge and strangeness}}$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0 + 0$

$\boxed{\text{baryon number}}$

(e) $\Xi^0 \to n + \pi^0$ $\quad$ Strangeness: $-2 \to 0 + 0$

$\boxed{\text{strangeness}}$

**P46.23** (a) $K^+ + p \to ? + p$

The strong interaction conserves everything.

Baryon number: $\quad 0 + 1 \to B + 1$ so $\quad B = 0$

Charge: $\quad +1 + 1 \to Q + 1$ so $\quad Q = +1$

Lepton numbers: $\quad 0 + 0 \to L + 0$ so $\quad L_e = L_\mu = L_\tau = 0$

Strangeness: $\quad +1 + 0 \to S + 0$ so $\quad S = 1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

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(b) $\Omega^- \to ? + \pi^-$

Baryon number: $+1 \to B + 0$ so $B = 1$

Charge: $-1 \to Q - 1$ so $Q = 0$

Lepton numbers: $0 \to L + 0$ so $L_e = L_\mu = L_\tau = 0$

Strangeness: $-3 \to S + 0$ so $\Delta S = 1$: $S = -2$

(There is no particle with $S = -4$.)

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \to ? + \mu^+ + \nu_\mu$

Baryon number: $0 \to B + 0 + 0$ so $B = 0$

Charge: $+1 \to Q + 1 + 0$ so $Q = 0$

Lepton numbers: $L_e$: $0 \to L_e + 0 + 0$ so $L_e = 0$

$L_\mu$: $0 \to L_\mu - 1 + 1$ so $L_\mu = 0$

$L_\tau$: $0 \to L_\tau + 0 + 0$ so $L_\tau = 0$

Strangeness: $1 \to S + 0 + 0$ so $\Delta S = \pm 1$: $S = 0$

(There is no meson with $S = 2$.)

The particle must be a neutral meson with strangeness $= 0 \Rightarrow \boxed{\pi^0}$.

**P46.24** (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

Baryon number: $+1 \to +1 + 0 + 0$ Charge: $-1 \to 0 - 1 + 0$

$L_e$: $0 \to 0 + 0 + 0$ $L_\mu$: $0 \to 0 + 1 + 1$

$L_\tau$: $0 \to 0 + 0 + 0$ Strangeness: $-2 \to -1 + 0 + 0$

Conserved quantities are $\boxed{B,\ \text{charge},\ L_e,\ \text{and}\ L_\tau}$.

(b) $K_S^0 \to 2\pi^0$

Baryon number: $0 \to 0$ Charge: $0 \to 0$

$L_e$: $0 \to 0$ $L_\mu$: $0 \to 0$

$L_\tau$: $0 \to 0$ Strangeness: $+1 \to 0$

Conserved quantities are $\boxed{B,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$.

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(c) $K^- + \mathrm{p} \to \Sigma^0 + \mathrm{n}$

Baryon number: $0+1 \to 1+1$ Charge: $-1+1 \to 0+0$

$L_e$: $0+0 \to 0+0$ $L_\mu$: $0+0 \to 0+0$

$L_\tau$: $0+0 \to 0+0$ Strangeness: $-1+0 \to -1+0$

Conserved quantities are $\boxed{S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$.

(d) $\Sigma^0 + \Lambda^0 + \gamma$

Baryon number: $+1 \to 1+0$ Charge: $0 \to 0$

$L_e$: $0 \to 0+0$ $L_\mu$: $0 \to 0+0$

$L_\tau$: $0 \to 0+0$ Strangeness: $-1 \to -1+0$

Conserved quantities are $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$.

(e) $\mathrm{e}^+ + \mathrm{e}^- \to \mu^+ + \mu^-$

Baryon number: $0+0 \to 0+0$ Charge: $+1-1 \to +1-1$

$L_e$: $-1+1 \to 0+0$ $L_\mu$: $0+0 \to +1-1$

$L_\tau$: $0+0 \to 0+0$ Strangeness: $0+0 \to 0+0$

Conserved quantities are $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$.

(f) $\bar{\mathrm{p}} + \mathrm{n} \to \bar{\Lambda}^0 + \Sigma^-$

Baryon number: $-1+1 \to -1+1$ Charge: $-1+0 \to 0-1$

$L_e$: $0+0 \to 0+0$ $L_\mu$: $0+0 \to 0+0$

$L_\tau$: $0+0 \to 0+0$ Strangeness: $0+0 \to +1-1$

Conserved quantities are $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$.

**P46.25** (a) $\Lambda^0 \to \mathrm{p} + \pi^-$ Strangeness: $-1 \to 0+0$, so $\Delta S = +1$

$\boxed{\text{Strangeness is not conserved.}}$

(b) $\pi^- + \mathrm{p} \to \Lambda^0 + \mathrm{K}^0$ Strangeness: $0+0 \to -1+1$, so $\Delta S = 0$

$\boxed{\text{Strangeness is conserved.}}$

(c) $\bar{\mathrm{p}} + \mathrm{p} \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0+0 \to +1-1$, so $\Delta S = 0$

$\boxed{\text{Strangeness is conserved.}}$

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(d) $\pi^- + \text{p} \to \pi^- + \Sigma^+$ Strangeness: $0+0 \to 0-1$, so $\Delta S = -1$

$\boxed{\text{Strangeness is not conserved.}}$

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1+0$, so $\Delta S = +1$

$\boxed{\text{Strangeness is not conserved.}}$

(f) $\Xi^0 \to \text{p} + \pi^-$ Strangeness: $-2 \to 0+0$, so $\Delta S = +2$

$\boxed{\text{Strangeness is not conserved.}}$

**P46.26** As a particle travels in a circle, it experiences a centripetal force, and the centripetal force can be related to the momentum of the particle.

$$\sum F = ma: \quad qvB\sin 90° = \frac{mv^2}{r} \quad \to \quad mv = p = qBr$$

(a) Using $p = qBr$ gives momentum in units of kg · m/s. To convert kg · m/s into units of MeV/$c$, we multiply and divide by $c$:

$$\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right) = \left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\left(\frac{c}{c}\right) = \left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\left(2.998\times10^8 \text{ m/s}\right)\left(\frac{1}{c}\right)$$

$$= \left(2.998\times10^8 \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\right)\left(\frac{1}{c}\right)$$

$$= 2.998\times10^8 \text{ J}\left(\frac{1}{c}\right)\left(\frac{1 \text{ MeV}}{1.602\times10^{-13} \text{ J}}\right)$$

$$= 1.871\times10^{21} \text{ MeV}/c$$

$$p_{\Sigma^+} = eBr_{\Sigma^+}$$

$$= \left(1.602\times10^{-19} \text{ C}\right)(1.15 \text{ T})(1.99 \text{ m})\frac{1.871\times10^{21} \text{ MeV}/c}{\text{kg}\cdot\text{m/s}}$$

$$= \boxed{686 \text{ MeV}/c}$$

$$p_{\pi^+} = eBr_{\pi^+}$$

$$= \left(1.602\times10^{-19} \text{ C}\right)(1.15 \text{ T})(0.580 \text{ m})\left(\frac{1.871\times10^{21} \text{ MeV}/c}{\text{kg}\cdot\text{m/s}}\right)$$

$$= \boxed{200 \text{ MeV}/c}$$

(b) The total momentum equals the momentum of the $\Sigma^+$ particle. The momentum of the pion makes an angle of 64.5° with respect to the original momentum of the $\Sigma^+$ particle. If we take the direction of the momentum of the $\Sigma^+$ particle as an axis of reference, and let $\phi$ be the angle made by the neutron's path with the path of the $\Sigma^+$

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at the moment of its decay, by conservation of momentum, we have these components of momentum:

parallel to the original momentum:

$$p_{\Sigma^+} = p_n \cos\phi + p_{\pi^+} \cos 64.5°$$

thus,

$$p_n \cos\phi = p_{\Sigma^+} - p_{\pi^+} \cos 64.5°$$

$$p_n \cos\phi = 686 \text{ MeV}/c - (200 \text{ MeV}/c)\cos 64.5° \qquad \textbf{[1]}$$

perpendicular to the original momentum:

$$0 = p_n \sin\phi - (200 \text{ MeV}/c)\sin 64.5°$$

$$p_n \sin\phi = (200 \text{ MeV}/c)\sin 64.5° \qquad \textbf{[2]}$$

From [1] and [2]:

$$p_n = \sqrt{(p_n \cos\phi)^2 + (p_n \sin\phi)^2} = \boxed{626 \text{ MeV}/c}$$

(c) $$E_{\pi^+} = \sqrt{(p_{\pi^+}c)^2 + (m_{\pi^+}c^2)^2} = \sqrt{(200 \text{ MeV})^2 + (139.6 \text{ MeV})^2}$$

$$= \boxed{244 \text{ MeV}}$$

$$E_n = \sqrt{(p_n c)^2 + (m_n c^2)^2} = \sqrt{(626 \text{ MeV})^2 + (939.6 \text{ MeV})^2}$$

$$= 1\,129 \text{ MeV} = \boxed{1.13 \text{ GeV}}$$

(d) $$E_{\Sigma^+} = E_{\pi^+} + E_n = 244 \text{ MeV} + 1129 \text{ MeV} = 1\,373 \text{ MeV} = \boxed{1.37 \text{ GeV}}$$

(e) $$m_{\Sigma^+}c^2 = \sqrt{E_{\Sigma^+}^2 - (p_{\Sigma^+}c)^2} = \sqrt{(1\,373 \text{ MeV})^2 - (686 \text{ MeV})^2} = 1\,189 \text{ MeV}$$

$$\therefore m_{\Sigma^+} = 1\,189 \text{ MeV}/c^2 = \boxed{1.19 \text{ GeV}/c^2}$$

(f) From Table 46.2, the mass of the $\Sigma^+$ particle is $1\,189.4 \text{ MeV}/c^2$. The percentage difference is

$$\frac{\Delta m}{m} = \frac{1.19 \times 10^3 \text{ MeV}/c^2 - 1\,189.4 \text{ MeV}/c^2}{1\,189.4 \text{ MeV}/c^2} \times 100\% = 0.0504\%$$

$\boxed{\text{The result in part (e) is within 0.05\% of the value in Table 46.2.}}$

**P46.27** The time-dilated lifetime is

$$T = \gamma T_0 = \frac{0.900 \times 10^{-10} \text{ s}}{\sqrt{1 - v^2/c^2}} = \frac{0.900 \times 10^{-10} \text{ s}}{\sqrt{1 - (0.960)^2}} = 3.214 \times 10^{-10} \text{ s}$$

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During this time interval, we see the kaon travel at $0.960c$. It travels for a distance of

$$\begin{aligned}\text{distance} &= vT = 0.960\left(2.998\times 10^{8}\ \text{m/s}\right)\left(3.214\times 10^{-10}\ \text{s}\right)\\ &= 9.25\times 10^{-2}\ \text{m} = \boxed{9.25\ \text{cm}}\end{aligned}$$

## Section 46.7 Finding Patterns in the Particles

## Section 46.8 Quarks

## Section 46.9 Multicolored Quarks

## Section 46.10 The Standard Model

**P46.28** (a)

| | $K^0$ | d | $\bar{s}$ | total |
|---|---|---|---|---|
| strangeness | 1 | 0 | 1 | 1 |
| baryon number | 0 | 1/3 | –1/3 | 0 |
| charge | 0 | $-e/3$ | $e/3$ | 0 |

(b)

| | $\Lambda^0$ | u | d | s | total |
|---|---|---|---|---|---|
| strangeness | –1 | 0 | 0 | –1 | –1 |
| baryon number | 1 | 1/3 | 1/3 | 1/3 | 1 |
| charge | 0 | $2e/3$ | $-e/3$ | $-e/3$ | 0 |

**P46.29** In the first reaction,

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

the quarks in the particles are

$$\bar{u}d + uud \rightarrow \bar{s}d + uds$$

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There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction,

$$\pi^- + \mathrm{p} \to \mathrm{K}^0 + \mathrm{n}$$

the quarks in the particles are

$$\bar{\mathrm{u}}\mathrm{d} + \mathrm{uud} \to \bar{\mathrm{s}}\mathrm{d} + \mathrm{udd}$$

In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

**P46.30** Compare the given quark states to the entries in Tables 46.4 and 46.5:

(a) $\mathrm{uus} = \boxed{\Sigma^+}$

(b) $\bar{\mathrm{u}}\mathrm{d} = \boxed{\pi^-}$

(c) $\bar{\mathrm{s}}\mathrm{d} = \boxed{\mathrm{K}^0}$

(d) $\mathrm{dss} = \boxed{\Xi^-}$

**P46.31** (a)

| | proton | u | u | d | total |
|---|---|---|---|---|---|
| strangeness | 0 | 0 | 0 | 0 | 0 |
| baryon number | 1 | 1/3 | 1/3 | 1/3 | 1 |
| charge | $e$ | $2e/3$ | $2e/3$ | $-e/3$ | $e$ |

(b)

| | neutron | u | d | d | total |
|---|---|---|---|---|---|
| strangeness | 0 | 0 | 0 | 0 | 0 |
| baryon number | 1 | 1/3 | 1/3 | 1/3 | 1 |
| charge | 0 | $2e/3$ | $-e/3$ | $-e/3$ | 0 |

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**P46.32** (a) $\pi^+ + \text{p} \to \text{K}^+ + \Sigma^+$: $\bar{\text{d}}\text{u} + \text{uud} \to \bar{\text{s}}\text{u} + \text{uus}$

up quarks: $1+2 \to 1+2$, or $3 \to 3$

down quarks: $-1+1 \to 0+0$, or $0 \to 0$

strange quarks: $0+0 \to -1+1$, or $0 \to 0$

$\boxed{\text{The reaction has a net of 3 u, 0 d, and 0 s before and after.}}$

(b) $\text{K}^- + \text{p} \to \text{K}^+ + \text{K}^0 + \Omega^-$: $\bar{\text{u}}\text{s} + \text{uud} \to \bar{\text{s}}\text{u} + \bar{\text{s}}\text{d} + \text{sss}$

up quarks: $-1+2 \to 1+0+0$, or $1 \to 1$

down quarks: $0+1 \to 0+1+0$, or $1 \to 1$

strange quarks: $1+0 \to -1-1+3$, or $1 \to 1$

$\boxed{\text{The reaction has a net of 1 u, 1 d, and 1 s before and after.}}$

(c) $\text{p} + \text{p} \to \text{K}^0 + \text{p} + \pi^+ + ?$: $\text{uud} + \text{uud} \to \bar{\text{s}}\text{d} + \text{uud} + \bar{\text{d}}\text{u} + ?$

The quark combination ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks: $2+2 = 0+2+1+?$ (? has 1 u quark)

down quarks: $1+1 = 1+1-1+?$ (? has 1 d quark)

strange quarks: $0+0 = -1+0+0+?$ (? has 1 s quark)

$\boxed{\text{The reaction must net of 4 u, 2 d, and 0 s before and after.}}$

(d) quark composition $= \text{uds} = \boxed{\Lambda^0 \text{ or } \Sigma^0}$

**P46.33** (a) $\bar{\text{u}}\bar{\text{u}}\bar{\text{d}}$: $\quad \text{charge} = \left(-\frac{2}{3}e\right) + \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{-e}$

(b) $\bar{\text{u}}\bar{\text{d}}\bar{\text{d}}$: $\quad \text{charge} = \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{0}$

(c) $\boxed{\text{antiproton}}$; $\boxed{\text{antineutron}}$

***P46.34** The number of protons in one liter (1 000 g) of water is

$$N_p = (1\ 000\ \text{g})\left(\frac{6.02\times 10^{23}\ \text{molecules}}{18.0\ \text{g}}\right)\left(\frac{10\ \text{protons}}{\text{molecule}}\right)$$
$$= 3.34\times 10^{26}\ \text{protons}$$

and there are

$$N_n = (1\ 000\ \text{g})\left(\frac{6.02\times 10^{23}\ \text{molecules}}{18.0\ \text{g}}\right)\left(\frac{8\ \text{neutrons}}{\text{molecule}}\right)$$
$$= 2.68\times 10^{26}\ \text{neutrons}$$

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So there are, for electric neutrality, $\boxed{3.34\times10^{26}\text{ electrons}}$.

The protonquark content is $\text{p} = \text{uud}$, and the neutron quark content is n = udd, so the number of up quarks is

$$2\left(3.34\times10^{26}\right)+2.68\times10^{26}=\boxed{9.36\times10^{26}\text{ up quarks}}$$

and the number of down quarks is

$$2\left(2.68\times10^{26}\right)+3.34\times10^{26}=\boxed{8.70\times10^{26}\text{ down quarks}}$$

**P46.35** $\Sigma^0+\text{p}\to\Sigma^++\gamma+X$

$\text{uds}+\text{uud}\to\text{uus}+0+?$

The left side has a net 3 u, 2 d, and 1 s. The right-hand side has 2 u and 1 s, leaving 2 d and 1 u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

**P46.36** Quark composition of proton = uud and of neutron = udd. Thus, if we neglect binding energies, we may write

$$m_\text{p}=2m_\text{u}+m_\text{d} \quad \textbf{[1]}$$

and $$m_\text{n}=m_\text{u}+2m_\text{d}. \quad \textbf{[2]}$$

Subtract [2] from 2 × [1]:

$$\begin{aligned}2m_\text{p}&=4m_\text{u}+2m_\text{d}\\ \underline{-m_\text{n}}&\underline{=-\left(m_\text{u}+2m_\text{d}\right)}\\ 2m_\text{p}-m_\text{n}&=3m_\text{u}\end{aligned}$$

We find

$$m_\text{u}=\frac{1}{3}\left(2m_\text{p}-m_\text{n}\right)=\frac{1}{3}\left[2\left(938\ \text{MeV}/c^2\right)-939.6\ \text{meV}/c^2\right]$$
$$=\boxed{312\ \text{MeV}/c^2}$$

and from either [1] or [2], $m_\text{d}=\boxed{314\ \text{MeV}/c^2}$.

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## Section 46.10 The Cosmic Connection

**P46.37** From Equation 39.10,

$$f_{\text{observer}} = f_{\text{source}}\sqrt{\frac{1+v_a/c}{1-v_a/c}}$$

where the velocity of approach, $v$, is the negative of the velocity of mutual recession: $v_a = -v$.

Thus, $$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-v/c}{1+v/c}} \quad \text{and} \quad \boxed{\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}}}$$

**P46.38** (a) We let $r$ in Hubble's law represent any distance.

$$v = Hr = \left(22\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot\text{ly}}\right)(1.85\ \text{m})\left(\frac{1\ \text{ly}}{c\cdot 1\ \text{yr}}\right)$$
$$\times\left(\frac{c}{3.00\times10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)$$
$$= \boxed{4.30\times10^{-18}\ \text{m/s}}$$

This is unobservably small.

(b) $$v = Hr = \left(22\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot\text{ly}}\right)(3.84\times10^{8}\ \text{m})\left(\frac{1\ \text{ly}}{c\cdot 1\ \text{yr}}\right)$$
$$\left(\frac{c}{3.00\times10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)$$
$$= 8.92\times10^{-10}\ \text{m/s} = \boxed{0.892\ \text{nm/s}}$$

Again too small to measure.

**P46.39** (a) From Wien's law,

$$\lambda_{\max}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$$

Thus,

$$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2.73\ \text{K}} = 1.06\times10^{-3}\ \text{m}$$
$$= \boxed{1.06\ \text{mm}}$$

(b) This is a $\boxed{\text{microwave.}}$

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**P46.40** (a) The volume of the sphere bounded by the Earth's orbit is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(1.496\times 10^{11}\text{ m}\right)^3 = 1.40\times 10^{34}\text{ m}^3$$

$$m = \rho V = \left(6\times 10^{-28}\text{ kg/m}^3\right)\left(1.40\times 10^{34}\text{ m}^3\right) = \boxed{8.41\times 10^6\text{ kg}}$$

(b) By Gauss's law, the dark matter would create a gravitational field acting on the Earth to accelerate it toward the Sun. It would shorten the duration of the year in the same way that $8.41 \times 10^6$ kg of extra material in the Sun would. This has the fractional effect of $\frac{8.41\times 10^6\text{ kg}}{1.99\times 10^{30}\text{ kg}} = 4.23\times 10^{-24}$ of the mass of the Sun.

$\boxed{\text{No. It is only the fraction } 4.23 \times 10^{-24} \text{ of the mass of the Sun.}}$

**P46.41** (a) The energy is enough to produce a proton-antiproton pair: $k_B T \approx 2m_p c^2$, so

$$T \approx \frac{2m_p c^2}{k_B} = \frac{2(938.3\text{ MeV})}{\left(1.38\times 10^{-23}\text{ J/K}\right)}\left(\frac{1.60\times 10^{-13}\text{ J}}{1\text{ MeV}}\right)\boxed{\sim 10^{13}\text{ K}}$$

(b) The energy is enough to produce an electron-positron pair: $k_B T \approx 2m_e c^2$, so

$$T \approx \frac{2m_e c^2}{k_B} = \frac{2(0.511\text{ MeV})}{\left(1.38\times 10^{-23}\text{ J/K}\right)}\left(\frac{1.60\times 10^{-13}\text{ J}}{1\text{ MeV}}\right)\boxed{\sim 10^{10}\text{ K}}$$

**P46.42** (a) The Hubble constant is defined in $v = HR$. The gap $R$ between any two far-separated objects opens at constant speed according to $R = v\Delta t$. Then the time interval $\Delta t$ since the Big Bang is found from

$$v = H\,v\Delta t \rightarrow \quad \Delta t = \frac{1}{H}$$

(b) $$\frac{1}{H} = \frac{1}{22\times 10^{-3}\text{ m/s}\cdot\text{ly}}\left[\frac{(1\text{ yr})\cdot\left(3\times 10^8\text{ m/s}\right)}{1\text{ ly}}\right] = \boxed{1.36\times 10^{10}\text{ yr}}$$

$= 13.6$ billion years

***P46.43** The radiation wavelength of $\lambda' = 500\text{ nm}$ that is observed by observers on Earth is not the true wavelength, $\lambda$, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}}$$

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Solving for the true wavelength then gives

$$\lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\text{ nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\text{ nm}$$

The temperature of the star is given by Wien's law,

$$\lambda_{\text{max}}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$$

or $$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{\lambda_{\text{max}}} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{375\times10^{-9}} = \boxed{7.73\times10^{3}\text{ K}}.$$

**P46.44** We assume that the fireball of the Big Bang is a black body. Then,

$$I = e\sigma T^4 = (1)\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(2.73\text{ K})^4$$
$$= \boxed{3.15\times10^{-6}\text{ W/m}^2}$$

**P46.45** (a) We use primed symbols to represent observed Doppler-shifted values and unprimed symbols to represent values as they would be measured by an observer stationary relative to the source. Doppler-shift equations from Chapter 17 do not apply to electromagnetic waves, because the speed of source or observer relative to some medium cannot be defined for these waves. Instead, we use Equation 39.10, expressing it as

$$f' = \frac{c}{\lambda'} = \sqrt{\frac{1+v/c}{1-v/c}}\,f = \sqrt{\frac{1+v/c}{1-v/c}}\left(\frac{c}{\lambda}\right)$$

where $v$ is the velocity of mutual approach. Then we have

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1-v/c}{1+v/c}}$$

Squaring both sides, and solving,

$$\left(\frac{\lambda'}{\lambda}\right)^2 = \frac{1-v/c}{1+v/c}$$

$$\left(\frac{\lambda'}{\lambda}\right)^2 + \left(\frac{\lambda'}{\lambda}\right)^2\frac{v}{c} = 1-\frac{v}{c}$$

$$\left(\frac{\lambda'}{\lambda}\right)^2 - 1 = -\frac{v}{c}\left[\left(\frac{\lambda'}{\lambda}\right)^2 + 1\right]$$

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Solving for $v/c$ then gives

$$\frac{v}{c} = -\frac{(\lambda'/\lambda)^2 - 1}{(\lambda'/\lambda)^2 + 1} = -\frac{(510\text{ nm}/434\text{ nm})^2 - 1}{(510\text{ nm}/434\text{ nm})^2 + 1} = \frac{(1.18)^2 - 1}{(1.18)^2 + 1}$$

$$= -\frac{1.381 - 1}{1.381 + 1} = -0.160$$

The negative sign indicates that the quasar is moving away from us, or us from it. The speed of recession that the problem asks for is then

$$v = \boxed{0.160c} \text{ (or 16.0\% of the speed of light)}$$

(b) Hubble's law asserts that the universe is expanding at a constant rate so that the speeds of galaxies are proportional to their distance $R$ from Earth, as described by $v = HR$.

So, $$R = \frac{v}{H} = \frac{0.160(3.00\times10^8\text{ m/s})}{2.2\times10^{-2}\text{ m/s}\cdot\text{ly}} = \boxed{2.18\times10^9\text{ ly}}.$$

**P46.46** (a) Applying the result from Problem 37, $\lambda_n' = \lambda_n\sqrt{\frac{1+v/c}{1-v/c}}$, to the definition $Z = \frac{\lambda_n' - \lambda_n}{\lambda_n}$, we have

$$Z = \frac{\lambda_n' - \lambda_n}{\lambda_n} \quad \rightarrow \quad (Z+1)\lambda_n = \lambda_n' = \lambda_n\sqrt{\frac{1+v/c}{1-v/c}}$$

$$\frac{1+v/c}{1-v/c} = (Z+1)^2$$

$$1 + \frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2$$

$$\left(\frac{v}{c}\right)(Z^2 + 2Z + 2) = Z^2 + 2Z$$

$$v = \boxed{c\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$$

(b) $$R = \frac{v}{H} = \boxed{\frac{c}{H}\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$$

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**P46.47** First, we calculate $v = HR$, using $H = 22\times10^{-3}$ m/s · ly, and then we use the result of Problem 37, $\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}$, and $c = 2.998 \times 10^8$ m/s, to calculate the wavelength emitted by the galaxy.

(a) $v = \left(22\times10^{-3}\ \text{m/s}\cdot\text{ly}\right)\left(2.00\times10^{6}\ \text{ly}\right) = 4.4\times10^{4}\ \text{m/s}$,

$$\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}} = \lambda\sqrt{\frac{1+\left(4.4\times10^{4}\ \text{m/s}\right)/\left(2.998\times10^{8}\ \text{m/s}\right)}{1-\left(4.4\times10^{4}\ \text{m/s}\right)/\left(2.998\times10^{8}\ \text{m/s}\right)}}$$

$$= (590\ \text{nm})\sqrt{\frac{1+0.000\ 146\ 8}{1-0.000\ 146\ 8}} = \boxed{590.09\ \text{nm}}$$

Similarly,

(b) $v = \left(22\times10^{-3}\ \text{m/s}\cdot\text{ly}\right)\left(2.00\times10^{8}\ \text{ly}\right) = 4.4\times10^{6}\ \text{m/s}$,

$$\lambda' = (590\ \text{nm})\sqrt{\frac{1+0.014\ 68}{1-0.014\ 68}} = \boxed{599\ \text{nm}}$$

(c) $v = \left(22\times10^{-3}\ \text{m/s}\cdot\text{ly}\right)\left(2.00\times10^{9}\ \text{ly}\right) = 4.4\times10^{7}\ \text{m/s}$,

$$\lambda' = (590\ \text{nm})\sqrt{\frac{1+0.146\ 8}{1-0.146\ 8}} = \boxed{684\ \text{nm}}$$

**P46.48** (a) What we can see is limited by the finite age of the Universe and by the finite speed of light. We can see out only to a look-back time equal to a bit less than the age of the Universe. Every year on your birthday the Universe also gets a year older, and light now in transit arrives at Earth from still more distant objects. So the radius of the visible Universe expands at the speed of light, which is

$$\frac{dr}{dt} = c = 1\ \text{ly/yr}$$

(b) The volume of the visible section of the Universe is $\frac{4}{3}\pi r^3$, where $r$ = 13.7 billion light-years. The rate of volume increase is

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi 3r^2\frac{dr}{dt} = 4\pi r^2 c$$

$$= 4\pi\left[\left(13.7\times10^{9}\ \text{ly}\right)\left(\frac{9.4605\times10^{15}\ \text{m}}{1\ \text{ly}}\right)\right]^2\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)$$

$$= \boxed{6.34\times10^{61}\ \text{m}^3/\text{s}}$$

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**P46.49** The density of the Universe is

$$\rho = 1.20\rho_c = 1.20\left(\frac{3H^2}{8\pi G}\right)$$

Consider a remote galaxy at distance $r$. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{0.600H^2r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy-sphere system is constant as the galaxy moves to apogee distance $R$:

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R}$$

so $$\frac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \quad \text{so} \quad R = 6.00r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

## Section 46.12 Problems and Perspectives

**P46.50** (a) The Planck length is

$$L = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{(1.054\ 5\times10^{-34}\ \text{J}\cdot\text{s})(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}{(2.998\times10^8\ \text{m/s})^3}}$$
$$= \boxed{1.62\times10^{-35}\ \text{m}}$$

(b) The Planck time is given as

$$T = \frac{L}{c} = \frac{1.616\times10^{-35}\ \text{m}}{2.998\times10^8\ \text{m/s}} = \boxed{5.39\times10^{-44}\ \text{s}}$$

of the same order of magnitude as the ultrahot epoch.

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## Additional Problems

**P46.51** (a) $\pi^- + \mathrm{p} \to \Sigma^+ + \pi^0$

Total charge is 0 on the left side of the equation, +1 on the right side. $\boxed{\text{Charge is not conserved.}}$

(b) $\mu^- \to \pi^- + \nu_e$

The rest mass of the pion is larger than the rest mass of the muon. Muon lepton number is +1 on the left side of the equation, 0 on the right side. Electron lepton number is 0 on left side, +1 on right side. $\boxed{\text{Energy, muon lepton number, and electron lepton number are not conserved.}}$

(c) $\mathrm{p} \to \pi^+ + \pi^+ + \pi^-$

Baryon number is +1 on the left side of the equation, 0 on the right side. $\boxed{\text{Baryon number is not conserved.}}$

**P46.52** In $? + \mathrm{p}^+ \to \mathrm{n} + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1. So the unknown particle must be an muon-antineutrino $\boxed{\bar{\nu}_\mu}$.

***P46.53** The time of flight is given by $\Delta t = d/v$.

Since $K = \frac{1}{2}mv^2$,

$$\Delta t = \frac{d}{\sqrt{\frac{2K}{m}}} = \frac{10.0 \times 10^3 \text{ m}}{\sqrt{\frac{2(0.040\,0 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}}} = 3.61 \text{ s}$$

The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{614 \text{ s}} = 1.13 \times 10^{-3} \text{ s}^{-1}$.

Therefore we have

$$\lambda \Delta t = (1.13 \times 10^{-3} \text{ s})(3.61 \text{ s}) = 4.08 \times 10^{-3} = 0.004\,08$$

And the fraction remaining is

$$\frac{N}{N_0} = e^{-\lambda \Delta t} = e^{-0.004\,08} = 0.995\,9$$

Hence, the fraction that has decayed in this time interval is

$$1 - \frac{N}{N_0} = 0.004\,07 \quad \text{or} \quad \boxed{0.407\%}$$

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**P46.54** Let's find the minimum energy necessary for the increase in rest energy to occur.

$$\Delta E_R = (3m_e - m_e)c^2 = 2m_e c^2 = 2(0.511 \text{ eV}) = 1.02 \text{ eV}$$

This calculation may make it look like the reaction is possible. But there is more to the energy picture here than just the increase in rest energy. There is kinetic energy associated with the moving particles. Let's demand that energy be conserved for the isolated system:

$$E_i = E_f \quad \rightarrow \quad E_\gamma + m_e c^2 = 3\gamma m_e c^2 \qquad \textbf{[1]}$$

Now demand that momentum in the direction of travel of the initial photon be conserved for the isolated system:

$$p_{xi} = p_{xf} \quad \rightarrow \quad \frac{E_\gamma}{c} = 3\gamma m_e u \qquad \textbf{[2]}$$

Divide equation [1] by equation [2]:

$$\frac{E_\gamma + m_e c^2}{E_\gamma / c} = \frac{c^2}{u} \quad \rightarrow \quad \frac{E_\gamma + m_e c^2}{E_\gamma} = \frac{c}{u} = \frac{1}{\beta} \qquad \textbf{[3]}$$

where $\beta = u/c$. Multiply equation [2] by $c$ and subtract it from equation [1]:

$$E_\gamma + m_e c^2 - E_\gamma = 3\gamma m_e c^2 - 3\gamma m_e uc$$

$$\rightarrow \quad m_e c^2 = 3\gamma m_e c^2 - 3\gamma m_e uc$$

$$\rightarrow \quad 1 = 3\gamma - 3\gamma \frac{u}{c} = 3\gamma(1 - \beta)$$

Substitute for $\gamma$:

$$1 = \frac{3(1-\beta)}{\sqrt{1-u^2/c^2}} = \frac{3(1-\beta)}{\sqrt{1-\beta^2}} = 3\sqrt{\frac{1-\beta}{1+\beta}}$$

$$1+\beta = 9(1-\beta) \quad \rightarrow \quad \beta = \frac{8}{10} = 0.800$$

Substitute this value into equation [3]:

$$\frac{E_\gamma + m_e c^2}{E_\gamma} = \frac{1}{0.800}$$

$$1 + \frac{m_e c^2}{E_\gamma} = 1.25 \rightarrow \quad E_\gamma = 4m_e c^2 = 2.04 \text{ MeV}$$

Therefore, the photon arriving with 1.05 MeV of energy cannot cause this reaction.

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Let's check the assumptions. If the final particles have any velocity component perpendicular to the initial direction of travel of the photon, then they must be moving with a higher speed after the collision and the incoming photon energy would have to be larger. If any one of the particles had a different energy than the other two, then the only way to satisfy both energy and momentum conservation would be for at least two of the particles to have components of velocity perpendicular to the initial direction of motion of the photon, so again the incoming photon energy would have to be larger. Therefore, 2.04 MeV represents the *minimum* energy for the reaction to occur.

**P46.55** We find the number $N$ of neutrinos:

$$10^{46}\ \text{J} = N(6\ \text{MeV}) = N\left(6\times 1.60\times 10^{-13}\ \text{J}\right)$$
$$N = 1.0\times 10^{58}\ \text{neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times 10^{58}}{4\pi\left(1.7\times 10^5\ \text{ly}\right)^2}\left(\frac{1\ \text{ly}}{9.460\ 5\times 10^{15}\ \text{m}}\right)^2$$
$$= 3.1\times 10^{14}\ \text{m}^{-2}$$

The number passing through a body presenting 5 000 cm$^2$ = 0.50 m$^2$

is then $\left(3.1\times 10^{14}\ \dfrac{1}{\text{m}^2}\right)\left(0.50\ \text{m}^2\right) = 1.5\times 10^{14}$, or $\boxed{\sim 10^{14}}$.

**P46.56** Since the neutrino flux from the Sun reaching the Earth is 0.400 W/m$^2$, the total energy emitted per second by the Sun in neutrinos in all directions is that which would irradiate the surface of a great sphere around it, with the Earth's orbit as its equator.

$$\left(0.400\ \text{W/m}^2\right)\left(4\pi r^2\right) = \left(0.400\ \text{W/m}^2\right)\left[4\pi\left(1.496\times 10^{11}\ \text{m}\right)^2\right]$$
$$= 1.12\times 10^{23}\ \text{W}$$

In a period of $10^9$ yr, the Sun emits a total energy of $\Delta E = P\Delta t$.

$$E = \left(1.12\times 10^{23}\ \text{J/s}\right)\left(10^9\ \text{yr}\right)\left(3.156\times 10^7\ \text{s/yr}\right) = 3.55\times 10^{39}\ \text{J}$$

carried by neutrinos. This energy corresponds to an annihilated mass according to

$$E = m_\nu c^2 = 3.55\times 10^{39}\ \text{J} \quad \text{or} \quad m_\nu = 3.94\times 10^{22}\ \text{kg}.$$

Since the Sun has a mass of $1.989\times 10^{30}$ kg, this corresponds to a loss of only about $\boxed{\text{1 part in } 5\times 10^7}$ of the Sun's mass over $10^9$ yr in the form of neutrinos.

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**P46.57** In our frame of reference, Hubble's law is exemplified by $\vec{\mathbf{v}}_1 = H\vec{\mathbf{R}}_1$ and $\vec{\mathbf{v}}_2 = H\vec{\mathbf{R}}_2$.

(a) From the first equation $\vec{\mathbf{v}}_1 = H\vec{\mathbf{R}}_1$ we may form the equation $-\vec{\mathbf{v}}_1 = -H\vec{\mathbf{R}}_1$. This equation expresses Hubble's law as seen by the observer in the first galaxy cluster, as she looks at us to find our velocity relative to her (away from her) is $-\vec{\mathbf{v}}_1 = H\left(-\vec{\mathbf{R}}_1\right)$.

(b) From both equations we may form the equation $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = H\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$. This equation expresses Hubble's law as seen by the observer in the first galaxy cluster, as she looks at cluster two to find the relative velocity of cluster 2 relative to cluster 1 is $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = H\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$.

**P46.58** $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. By energy conservation,

$$m_\pi c^2 = E_\mu + E_{\bar{\nu}} = 139.6 \text{ MeV} \qquad \textbf{[1]}$$

Because we assume the antineutrino has no mass, $E_{\bar{\nu}} = p_{\bar{\nu}}c$, and by momentum conservation, $p_\mu = p_{\bar{\nu}}$; thus, we can relate the total energies of the muon and antineutrino:

$$E_\mu^2 = \left(p_\mu c\right)^2 + \left(m_\mu c^2\right)^2 = \left(p_{\bar{\nu}} c\right)^2 + \left(m_\mu c^2\right)^2 = \left(E_{\bar{\nu}}^2\right)^2 + \left(m_\mu c^2\right)^2$$

or $$E_\mu^2 - E_{\bar{\nu}}^2 = \left(m_\mu c^2\right)^2$$

and $$\left(E_\mu + E_{\bar{\nu}}\right)\left(E_\mu - E_{\bar{\nu}}\right) = \left(m_\mu c^2\right)^2. \qquad \textbf{[2]}$$

Substituting [1] into [2], we find that

$$E_\mu - E_{\bar{\nu}} = \frac{\left(m_\mu c^2\right)^2}{\left(E_\mu + E_{\bar{\nu}}\right)} = \frac{\left(m_\mu c^2\right)^2}{m_\pi c^2} \qquad \textbf{[3]}$$

Subtracting [3] from [1],

$$\left(E_\mu + E_{\bar{\nu}}\right) - \left(E_\mu - E_{\bar{\nu}}\right) = m_\pi c^2 - \frac{\left(m_\mu c^2\right)^2}{m_\pi c^2}$$

$$2E_{\bar{\nu}} = m_\pi c^2 - \frac{\left(m_\mu c^2\right)^2}{m_\pi c^2}$$

$$E_{\bar{\nu}} = \frac{\left(m_\pi c^2\right)^2 - \left(m_\mu c^2\right)^2}{2m_\pi c^2} = \frac{(139.6 \text{ MeV})^2 - (105.7 \text{ MeV})^2}{2(139.6 \text{ MeV})}$$

$$= \boxed{29.8 \text{ MeV}}$$

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**P46.59** Each particle travels in a circle, so each must experience a centripetal force:

$$\sum F = ma: \quad qvB\sin 90° = \frac{mv^2}{r} \quad \rightarrow \quad mv = qBr$$

The proton and the pion have the same momentum because they have the same magnitude of charge and travel in a circle of the same radius:

$$p_\text{p} = p_\pi = p = qBr = \left(1.60\times 10^{-19}\ \text{C}\right)(0.250\ \text{T})(1.33\ \text{m})$$
$$= 5.32\times 10^{-20}\ \text{kg}\cdot\text{m/s}$$

so

$$pc = \left(3.00\times 10^{8}\ \text{m/s}\right)\left(5.32\times 10^{-20}\ \text{kg}\cdot\text{m/s}\right)\left(\frac{1\ \text{MeV}}{1.60\times 10^{-13}\ \text{J}}\right)$$
$$= 99.8\ \text{MeV}$$

Using masses from Table 46.2, we find the total energy of the proton to be

$$E_\text{p} = \sqrt{(pc)^2 + \left(m_\text{p}c^2\right)^2} = \sqrt{(99.8\ \text{MeV})^2 + (938.3\ \text{MeV})^2}$$
$$= 944\ \text{MeV}$$

and the total energy of the pion to be

$$E_\pi = \sqrt{(pc)^2 + \left(m_\pi c^2\right)^2} = \sqrt{(99.8\ \text{MeV})^2 + (139.6\ \text{MeV})^2}$$
$$= 172\ \text{MeV}$$

The unknown particle was initially at rest; thus, $E_\text{total after} = E_\text{total before} =$ rest energy, and the rest energy of unknown particle is

$$mc^2 = 944\ \text{MeV} + 172\ \text{MeV} = 1\ 116\ \text{MeV}$$

$$\text{Mass} = \boxed{1.12\ \ \text{GeV}/c^2}$$

From Table 46.2, we see this is a $\Lambda^0$ particle.

**P46.60** Each particle travels in a circle, so each must experience a centripetal force:

$$\sum F = ma: \quad qvB\sin 90° = \frac{mv^2}{r} \quad \rightarrow \quad mv = qBr$$

The particles have the same momentum because they have the same magnitude of charge and travel in a circle of the same radius:

$$p_+ = p_- = p = eBr \quad \rightarrow \quad pc = eBrc$$

We find the total energy of the positively charged particle to be

$$E_{+,\ \text{total}} = \sqrt{(pc)^2 + \left(E_+\right)^2} = \sqrt{(qBrc)^2 + E_+^2}$$

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and the total energy of the negatively charged particle to be

$$E_{+,\text{ total}} = \sqrt{(pc)^2 + (E_-)^2} = \sqrt{(qBrc)^2 + E_-^2}$$

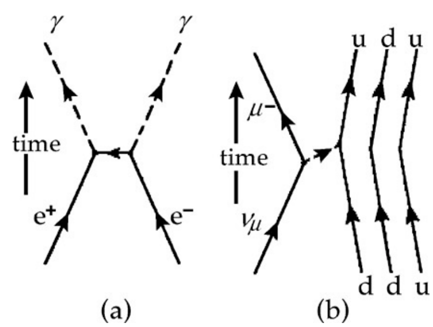
The unknown particle was initially at rest; thus, $E_{\text{total after}} = E_{\text{total before}} =$ rest energy, and the rest energy of the unknown particle is

$$mc^2 = \sqrt{(qBrc)^2 + E_+^2} + \sqrt{(qBrc)^2 + E_-^2}$$

$$\boxed{m = \frac{\sqrt{(qBrc)^2 + E_+^2} + \sqrt{(qBrc)^2 + E_-^2}}{c^2}}$$

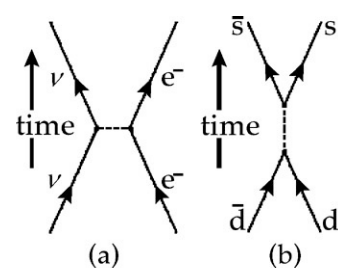
**P46.61** (a) This diagram represents $\boxed{\text{electron–positron annihilation}}$. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-.}$

(b) $\boxed{\text{A neutrino collides with a neutron, producing a proton and a muon.}}$ This is a weak interaction. The exchanged particle has charge $+e$ and is a $\boxed{W^+.}$




**ANS. FIG. P46.61**

**P46.62** (a) The Feynman diagram in ANS. FIG. P46.62 shows a neutrino scattering off an electron, and the neutrino and electron do not exchange electric charge. The neutrino has no electric charge and interacts through the weak interaction (ignoring gravity). The mediator is a $\boxed{Z^0 \text{ boson.}}$




**ANS. FIG. P46.62**

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, if the quarks have opposite color charges, no color charge. In this case

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the mediating particle could be a $\boxed{\text{photon or } Z^0 \text{ boson.}}$

Depending on the color charges of the d and $\bar{\text{d}}$ quarks, the ephemeral particle could also be a $\boxed{\text{gluon}}$, as suggested in the discussion of Figure 46.13(b).

For conservation of both energy and momentum in the collision we would expect two mediating particles; but momentum need not be strictly conserved, according to the uncertainty principle, if the particle travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in ANS. FIG. P46.62(b).

**P46.63** The expression $e^{-E/k_BT}dE$ gives the fraction of the photons that have energy between $E$ and $E + dE$. The fraction that have energy between $E$ and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}\,dE}{\int_0^\infty e^{-E/k_BT}\,dE} = \frac{\int_E^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}{\int_0^\infty e^{-E/k_BT}\left(-dE/k_BT\right)} = \frac{e^{-E/k_BT}\Big|_E^\infty}{e^{-E/k_BT}\Big|_0^\infty} = e^{-E/k_BT}$$

We require $T$ when this fraction has a value of 0.010 0 (i.e., 1.00%)

and $\quad E = 1.00 \text{ eV} = 1.60\times 10^{-19} \text{ J}.$

Thus, $\quad 0.010\ 0 = e^{-\left(1.60\times10^{-19}\ \text{J}\right)/\left(1.38\times10^{-23}\ \text{J/K}\right)T}$

or $\quad \ln(0.010\ 0) = -\dfrac{1.60\times10^{-19}\ \text{J}}{\left(1.38\times10^{-23}\ \text{J/K}\right)T} = -\dfrac{1.16\times10^{4}\ \text{K}}{T},$

giving $\quad T = 2.52\times10^{3}\ \text{K} \sim \boxed{10^3\ \text{K}}.$

**P46.64** $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1\,192.5\ \text{MeV}/c^2$ and $m_\Lambda = 1\,115.6\ \text{MeV}/c^2$.

Conservation of energy in the decay requires

$$m_\Sigma c^2 = \left(m_\Lambda c^2 + K_\Lambda\right) + E_\gamma \quad \text{or} \quad m_\Sigma c^2 = \left(m_\Lambda c^2 + \frac{p_\Lambda{}^2}{2m_\Lambda}\right) + E_\gamma$$

System momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$m_\Sigma c^2 = \left(m_\Lambda c^2 + \frac{p_\gamma{}^2}{2m_\Lambda}\right) + E_\gamma$$

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or $$m_\Sigma c^2 = \left( m_\Lambda c^2 + \frac{p_\gamma^2 c^2}{2m_\Lambda c^2} \right) + E_\gamma.$$

Recognizing that $p_\gamma c = E_\gamma$, we now have

$$1\,192.5 \text{ MeV} = 1\,115.6 \text{ MeV} + \frac{E_\gamma^2}{2(1\,115.6 \text{ MeV})} + E_\gamma$$

Solving this quadratic equation gives $E_\gamma = \boxed{74.4 \text{ MeV}}$.

**P46.65** $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \pi^+ + \mathrm{X}$

The protons each have 70.4 MeV of kinetic energy. In accord with conservation of momentum for the collision, particle X has zero momentum and thus zero kinetic energy. Conservation of system energy then requires

$$m_\mathrm{p}c^2 + m_\pi c^2 + m_\mathrm{X}c^2 = \left(m_\mathrm{p}c^2 + K_\mathrm{p}\right) + \left(m_\mathrm{p}c^2 + K_\mathrm{p}\right)$$

$$\begin{aligned} m_\mathrm{X}c^2 &= m_\mathrm{p}c^2 + 2K_\mathrm{p} - m_\pi c^2 \\ &= 938.3 \text{ MeV} + 2(70.4 \text{ MeV}) - 139.6 \text{ MeV} \\ &= 939.5 \text{ MeV} \end{aligned}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus, X is a $\boxed{\text{neutron.}}$

**P46.66** $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{n} + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve system momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy for the reaction gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$\begin{aligned} K_p &= \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3) \text{ MeV}}{2} \\ &= \boxed{70.4 \text{ MeV}} \end{aligned}$$

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## Challenge Problems

**P46.67** See the discussion of P46.19 in this volume for more details of the mathematical steps used in the following calculations.

From Table 46.2, $m_\Lambda c^2 = 1\ 115.6$ MeV, $m_p c^2 = 938.3$ MeV, and $m_\pi c^2 = 139.6$ MeV.

Since the $\Lambda^0$ is at rest, the difference between its rest energy and the rest energies of the proton and the pion is the sum of the kinetic energies of the proton and the pion.

$$K_p + K_\pi = 1\ 115.6\ \text{MeV} - 938.3\ \text{MeV} - 139.6\ \text{MeV} = 37.7\ \text{MeV}$$

Now, since $p_p = p_\pi = p$, applying conservation of relativistic energy to the decay process, we have

$$\left[\sqrt{(938.3\ \text{MeV})^2 + p^2c^2} - 938.3\ \text{MeV}\right] + \left[\sqrt{(139.6\ \text{MeV})^2 + p^2c^2} - 139.6\ \text{MeV}\right] = 37.7\ \text{MeV}$$

Solving yields

$$p_\pi c = p_p c = 100.4\ \text{MeV}$$

Then,

$$K_p = \sqrt{\left(m_p c^2\right)^2 + (100.4\ \text{MeV})^2} - m_p c^2 = \boxed{5.35\ \text{MeV}}$$

$$K_\pi = \sqrt{(139.6)^2 + (100.4\ \text{MeV})^2} - 139.6 = \boxed{32.3\ \text{MeV}}$$

**P46.68** (a) Let $E_{\text{min}}$ be the minimum total energy of the bombarding particle that is needed to induce the reaction. At this energy the product particles all move with the same velocity. The product particles are then equivalent to a single particle having mass equal to the total mass of the product particles, moving with the same velocity as each product particle. By conservation of energy:

$$E_{\text{min}} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + \left(p_3c\right)^2} \qquad \textbf{[1]}$$

By conservation of momentum, $p_3 = p_1$, so

$$\left(p_3c\right)^2 = \left(p_1c\right)^2 = E_{\text{min}}^2 - \left(m_1c^2\right)^2 \qquad \textbf{[2]}$$

Substitute [2] into [1]:

$$E_{\text{min}} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + E_{\text{min}}^2 - \left(m_1c^2\right)^2}$$

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Square both sides:

$$E_{\min}^2 + 2E_{\min}m_2c^2 + \left(m_2c^2\right)^2 = \left(m_3c^2\right)^2 + E_{\min}^2 - \left(m_1c^2\right)^2$$

$$\therefore E_{\min} = \frac{\left(m_3^2 - m_1^2 - m_2^2\right)c^2}{2m_2}$$

$$\therefore K_{\min} = E_{\min} - m_1c^2 = \frac{\left(m_3^2 - m_1^2 - m_2^2 - 2m_1m_2\right)c^2}{2m_2}$$

$$= \frac{\left[m_3^2 - \left(m_1 + m_2\right)^2\right]c^2}{2m_2}$$

Refer to Table 46.2 for the particle masses.

(b) $$K_{\min} = \frac{[4(938.3)]^2\ \text{MeV}^2/c^2 - [2(938.3)]^2\ \text{MeV}^2/c^2}{2\left(938.3\ \text{MeV}/c^2\right)} = \boxed{5.63\text{ GeV}}$$

(c) $$K_{\min} = \frac{(497.7 + 1\,115.6)^2\ \text{MeV}^2/c^2 - (139.6 + 938.3)^2\ \text{MeV}^2/c^2}{2(938.3)\ \text{MeV}/c^2}$$

$$= \boxed{768\text{ MeV}}$$

(d) $$K_{\min} = \frac{[2(938.3) + 135]^2\ \text{MeV}^2/c^2 - [2(938.3)]^2\ \text{MeV}^2/c^2}{2(938.3)\ \text{MeV}/c^2}$$

$$= \boxed{280\text{ MeV}}$$

(e) $$K_{\min} = \frac{\left(91.2\times10^3\right)^2 - \left[(938.3 + 938.3)^2\right]\ \text{MeV}^2/c^2}{2(938.3)\ \text{MeV}/c^2} = \boxed{4.43\text{ TeV}}$$

**P46.69** (a) $\Delta E = \left(m_{\text{n}} - m_{\text{p}} - m_{\text{e}}\right)c^2$

From Table 44.2 of masses of isotopes,

$$\Delta E = (1.008\ 665\ \text{u} - 1.007\ 825\ \text{u})(931.5\ \text{MeV/u})$$

$$= \boxed{0.782\text{ MeV}}$$

(b) Assuming the neutron at rest, momentum conservation for the decay process implies $p_{\text{p}} = p_{\text{e}}$. Relativistic energy for the system is conserved:

$$\sqrt{\left(m_{\text{p}}c^2\right)^2 + p_{\text{p}}^2c^2} + \sqrt{\left(m_{\text{e}}c^2\right)^2 + p_{\text{e}}^2c^2} = m_{\text{n}}c^2$$

Since $p_p = p_e = p$, we have

$$\sqrt{\left(m_{\text{p}}c^2\right)^2 + p^2c^2} = m_{\text{n}}c^2 - \sqrt{\left(m_ec^2\right)^2 + p^2c^2}$$

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$$\left(m_{\text{p}}c^2\right)^2 + \cancel{p^2c^2} = \left(m_n c^2\right)^2 - 2m_{\text{n}}c^2\sqrt{\left(m_{\text{e}}c^2\right)^2 + p^2c^2} + \left(m_{\text{e}}c^2\right)^2 + \cancel{p^2c^2}$$

$$\sqrt{\left(m_{\text{e}}c^2\right)^2 + p^2c^2} = \frac{\left(m_{\text{n}}c^2\right)^2 - \left(m_{\text{p}}c^2\right)^2 + \left(m_{\text{e}}c^2\right)^2}{2m_{\text{n}}c^2}$$

$$p^2c^2 = \left[\frac{\left(m_{\text{n}}c^2\right)^2 - \left(m_{\text{p}}c^2\right)^2 + \left(m_{\text{e}}c^2\right)^2}{2m_{\text{n}}c^2}\right]^2 - \left(m_{\text{e}}c^2\right)^2$$

Refer to Table 46.2 for the particle masses.

$$p^2c^2 = \left[\frac{(939.6\text{ MeV})^2 - (938.3\text{ MeV})^2 + (0.511\text{ MeV})^2}{2(939.6\text{ MeV})}\right]^2 - (0.511\text{ MeV})^2$$

$$pc = 1.19\text{ MeV}$$

From $p_{\text{e}}c = \gamma\, m_{\text{e}}v_{\text{e}}c,$ we find the speed of the electron:

$$\frac{\gamma\, v_{\text{e}}}{c} = \frac{p_{\text{e}}c}{m_{\text{e}}c^2} = \frac{1}{\sqrt{1-\left(v_{\text{e}}/c\right)^2}}\frac{v_{\text{e}}}{c}$$

$$1-\left(\frac{v_e}{c}\right)^2 = \left(\frac{v_e}{c}\right)^2\left(\frac{m_e c^2}{p_e c}\right)^2 \to \left(\frac{v_e}{c}\right)^2\left[1+\left(\frac{m_e c^2}{p_e c}\right)^2\right] = 1$$

$$\frac{v_{\text{e}}}{c} = \frac{1}{\sqrt{1+\left(m_e c^2/p_e c\right)^2}} = \frac{1}{\sqrt{1+\left(0.511\text{ MeV}/1.19\text{ MeV}\right)^2}}$$

$$\boxed{v_{\text{e}} = 0.919c}$$

To find the speed of the proton, a similar derivation (basically, substituting $m_{\text{p}}$ for $m_{\text{e}}$), yields

$$v_{\text{p}} = \frac{c}{\sqrt{1+\left(m_{\text{p}}c^2/p_e c\right)^2}} = \frac{2.998\times 10^8\text{ m/s}}{\sqrt{1+\left(938.3\text{ MeV}/1.19\text{ MeV}\right)^2}}$$

$$= 3.82\times 10^5\text{ m/s} = \boxed{382\text{ km/s}}$$

(c) $\boxed{\text{The electron is relativistic; the proton is not.}}$ Our criterion for answers accurate to three significant digits is that the electron is moving at more than one-tenth the speed of light and the proton at less than one-tenth the speed of light.

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**P46.70** (a) At threshold, we consider a photon and a proton colliding head-on to produce a proton and a pion at rest, according to $p+\gamma \to p+\pi^0$. Energy conservation gives

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}}+E_\gamma = m_p c^2 + m_\pi c^2$$

Momentum conservation gives $\dfrac{m_p u}{\sqrt{1-u^2/c^2}}-\dfrac{E_\gamma}{c}=0.$

Combining the equations, we have

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}}+\frac{m_p c^2}{\sqrt{1-u^2/c^2}}\frac{u}{c} = m_p c^2 + m_\pi c^2$$

$$\frac{(938.3\text{ MeV})(1+u/c)}{\sqrt{(1-u/c)(1+u/c)}} = 938.3\text{ MeV} + 135.0\text{ MeV}$$

so $\dfrac{u}{c}=0.134$

and $E_\gamma = \boxed{127\text{ MeV}}$.

(b) $\lambda_{\max}T = 2.898\text{ mm}\cdot\text{K}$

$$\lambda_{\max}=\frac{2.898\text{ mm}\cdot\text{K}}{2.73\text{ K}}=\boxed{1.06\text{ mm}}$$

(c) $E_\gamma = hf = \dfrac{hc}{\lambda}=\dfrac{1\,240\text{ eV}\cdot 10^{-9}\text{ m}}{1.06\times 10^{-3}\text{ m}}=\boxed{1.17\times 10^{-3}\text{ eV}}$

(d) In the primed reference frame, the proton is moving to the right at $\dfrac{u'}{c}=0.134$ and the photon is moving to the left with $hf' = 1.27\times 10^{8}$ eV. In the unprimed frame, $hf = 1.17\times 10^{-3}$ eV. Using the Doppler effect equation (Equation 39.10), we have for the speed of the primed frame (suppressing units)

$$1.27\times 10^{8} = \sqrt{\frac{1+v/c}{1-v/c}}\,1.17\times 10^{-3}$$

$$\frac{v}{c}=1-1.71\times 10^{-22}$$

Then the speed of the proton is given by

$$\frac{u}{c}=\frac{u'/c+v/c}{1+u'v/c^2}=\frac{0.134+1-1.71\times 10^{-22}}{1+0.134\left(1-1.71\times 10^{-22}\right)}=1-1.30\times 10^{-22}$$

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And the energy of the proton is

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}} = \frac{938.3\text{ MeV}}{\sqrt{1-\left(1-1.30\times 10^{-22}\right)^2}}$$

$$= 6.19\times 10^{10}\times 938.3\times 10^6\text{ eV} = \boxed{5.81\times 10^{19}\text{ eV}}$$

**P46.71** (a) Consider a sphere around us of radius $R$ large compared to the size of galaxy clusters. If the matter $M$ inside the sphere has the critical density, then a galaxy of mass $m$ at the surface of the sphere is moving just at escape speed $v$ according to $K+U_g=0$,

or $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0.$

The energy of the galaxy-sphere system is conserved, so this equation is true throughout the history of the Universe after the Big Bang, where $v = \frac{dR}{dt}$. Then,

$$\left(\frac{dR}{dt}\right)^2 = \frac{2GM}{R}$$

or $\frac{dR}{dt} = R^{-1/2}\sqrt{2GM}.$

integrating,

$$\int_0^R \sqrt{R}\,dR = \sqrt{2GM}\int_0^T dt$$

$$\left.\frac{R^{3/2}}{3/2}\right|_0^R = \sqrt{2GM}\,t\Big|_0^T \quad\text{gives}\quad \frac{2}{3}R^{3/2} = \sqrt{2GM}\,T$$

or $T = \frac{2}{3}\frac{R^{3/2}}{\sqrt{2GM}} = \frac{2}{3}\frac{R}{\sqrt{2GM/R}}.$

From above, $\sqrt{\frac{2GM}{R}} = v$

so $T = \frac{2}{3}\frac{R}{v}.$

Now Hubble's law says $v = HR$, so $T = \frac{2}{3}\frac{R}{HR} = \frac{2}{3H}.$

(b) $T = \frac{2}{3\left(22\times 10^{-3}\text{ m/s}\cdot\text{ly}\right)}\left(\frac{2.998\times 10^8\text{ m/s}}{1\text{ ly/yr}}\right) = \boxed{9.08\times 10^9\text{ yr}}$

$= 9.08$ billion years

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**P46.72** A photon travels the distance from the Large Magellanic Cloud to us in 170 000 years. The hypothetical massive neutrino travels the same distance in 170 000 years plus 10 seconds:

$$c(170\ 000\ \text{yr}) = v(170\ 000\ \text{yr} + 10\ \text{s})$$

$$\frac{v}{c} = \frac{170\ 000\ \text{yr}}{170\ 000\ \text{yr} + 10\ \text{s}}$$

$$= \frac{1}{1 + \{10\ \text{s}/[(1.7\times10^{5}\ \text{yr})(3.156\times10^{7}\ \text{s/yr})]\}}$$

$$= \frac{1}{1 + 1.86\times10^{-12}}$$

For the neutrino we want to evaluate $mc^2$ in $E = \gamma mc^2$:

$$mc^2 = \frac{E}{\gamma} = E\sqrt{1 - \frac{v^2}{c^2}} = 10\ \text{MeV}\sqrt{1 - \frac{1}{(1 + 1.86\times10^{-12})^2}}$$

$$= (10\ \text{MeV})\sqrt{\frac{(1 + 1.86\times10^{-12})^2 - 1}{(1 + 1.86\times10^{-12})^2}}$$

$$mc^2 \approx (10\ \text{MeV})\sqrt{\frac{2(1.86\times10^{-12})}{1}} = (10\ \text{MeV})(1.93\times10^{-6})$$

$$= 19\ \text{eV}$$

Then the upper limit on the mass is

$$m = \boxed{\frac{19\ \text{eV}}{c^2}}$$

$$m = \frac{19\ \text{eV}}{c^2}\left(\frac{\text{u}}{931.5\times10^{6}\ \text{eV}/c^2}\right) = 2.1\times10^{-8}\ \text{u}$$

**P46.73** (a) If $2N$ particles are annihilated, the energy released is $2Nmc^2$. The resulting photon momentum is $p = \frac{E}{c} = \frac{2Nmc^2}{c} = 2Nmc$. Since the momentum of the system is conserved, the rocket will have momentum $2Nmc$ directed opposite the photon momentum.

$$\boxed{p = 2Nmc}$$

(b) Consider a particle that is annihilated and gives up its rest energy $mc^2$ to another particle which also has initial rest energy $mc^2$ (but no momentum initially).

$$E^2 = p^2c^2 + (mc^2)^2$$

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Thus, $\left(2mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$.

Where $p$ is the momentum the second particle acquires as a result of the annihilation of the first particle. Thus
$4\left(mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$, $p^2 = 3\left(mc^2\right)^2$. So $p = \sqrt{3}mc$.

This process is repeated $N$ times (annihilate $\frac{N}{2}$ protons and $\frac{N}{2}$ antiprotons). Thus the total momentum acquired by the ejected particles is $\sqrt{3}Nmc$, and this momentum is imparted to the rocket.

$$\boxed{p = \sqrt{3}Nmc}$$

(c) Method (a) produces greater speed since $2Nmc > \sqrt{3}Nmc$.

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## ANSWERS TO EVEN-NUMBERED PROBLEMS

**P46.2** (a) $2.27 \times 10^{23}$ Hz; (b) $1.32 \times 10^{-15}$ m

**P46.4** $\sim 10^{3}$ Bq

**P46.6** $\sim 10^{-18}$ m

**P46.8** Baryon number conservation allows the first reaction and forbids the second.

**P46.10** $\sim 10^{-23}$ s

**P46.12** (a) See P46.12(a) for full explanation; (b) Strangeness is not conserved in the second reaction.

**P46.14** (a) $\nu_e$; (b) $\nu_\mu$; (c) $\bar{\nu}_\mu$; (d) $\nu_\mu$, $\bar{\nu}_\tau$

**P46.16** $\bar{\nu}_\mu$ and $\nu_e$

**P46.18** (a) See P46.18(a) for full explanation;
(b) $E_e = E_\gamma = 469$ MeV, $p_e = p_\gamma = 469$ MeV/$c$; (c) $v = 0.999\,999\,4c$

**P46.20** The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction. The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

**P46.22** (a) electron and muon lepton numbers; (b) electron lepton number; (c) charge and strangeness; (d) baryon number; (e) strangeness

**P46.24** (a) $B$, charge, $L_e$, and $L_\tau$; (b) $B$, charge, $L_e$, $L_\mu$, and $L_\tau$;
(c) $S$, charge, $L_e$, $L_\mu$, and $L_\tau$; (d) $B$, $S$, charge, $L_e$, $L_\mu$, and $L_\tau$;
(e) $B$, $S$, charge, $L_e$, $L_\mu$, and $L_\tau$; (f) $B$, $S$, charge, $L_e$, $L_\mu$, and $L_\tau$

**P46.26** (a) $p_{\Sigma^+} = 686$ MeV/$c$, $p_{\pi^+} = 200$ MeV/$c$; (b) 626 MeV/c;
(c) $E_{\pi^+} = 244$ MeV, $E_n = 1.13$ GeV; (d) 1.37 GeV; (e) 1.19 GeV/$c^2$;
(f) The result in part (e) is within 0.05% of the value ion Table 46.2.

**P46.28** (a) See table in P46.28(a); (b) See table in P46.28(b).

**P46.30** (a) $\Sigma^+$; (b) $\pi^-$; (c) $K^0$; (d) $\Xi^-$

**P46.32** (a) The reaction has a net of 3u, 0d, and 0s before and after; (b) The reaction has a net of 1u, 1d, and 1s before and after; (c) The reaction must net of 4u, 2d, and 0z before and after; (d) $\Lambda^0$ or $\Sigma^0$

**P46.34** $3.34 \times 10^{26}$ electrons, $9.36 \times 10^{26}$ up quarks, $8.70 \times 10^{26}$ down quarks

**P46.36** $m_u = 312$ MeV/$c^2$; $m_d = 314$ MeV/$c^2$

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**P46.38** (a) $4.30\times10^{-18}\ \text{m/s}$ ; (b) 0.892 nm/s

**P46.40** (a) $8.41\times10^{6}\ \text{kg}$ ; (b) No. It is only the fraction $4.23 \times 10^{-24}$ of the mass of the Sun.

**P46.42** $1.36\times10^{10}\ \text{yr}$

**P46.44** $3.15\times10^{-6}\ \text{W/m}^2$

**P46.46** (a) $c\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$; (b) $\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$

**P46.48** (a) See P46.48(a) for full explanation; (b) $6.34\times10^{61}\ \text{m}^3\text{/s}$

**P46.50** (a) $1.62\times10^{-35}$ m; (b) $5.39\times10^{-44}$ s

**P46.52** $\bar{\nu}_\mu$

**P46.54** See P46.54 for full explanation.

**P46.56** 1 part in $5 \times 10^7$

**P46.58** 29.8 MeV

**P46.60** $m=\dfrac{\sqrt{(qBrc)^2+E_+^2}+\sqrt{(qBrc)^2+E_-^2}}{c^2}$

**P46.62** (a) $Z^0$ boson; (b) photon or $Z^0$ boson, gluon

**P46.64** 74.4 MeV

**P46.66** 70.4 MeV

**P46.68** (a) See P46.68(a) for full explanation; (b) 5.63 GeV; (c) 768 MeV; (d) 280 MeV; (e) 4.43 TeV

**P46.70** (a) 127 MeV; (b) 1.06 mm; (c) $1.17 \times 10^{-3}$ eV; (d) $5.81 \times 10^{19}$ eV;

**P46.72** 19 eV/c²

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