Aula Dúvidas 8 (19/Mar)

Folha 5, ex 4: Variáveis compatíveis e incompatíveis:

X; $r = \sqrt{x^2+y^2+z^2}$; $\sqrt{\ } = a_0 + a_1(\hat{x}-x_0) + a_2(\hat{x}-x_0)^2 + \dots$

$$\left[\hat{X}, \sqrt{\hat{X}^2+\hat{Y}^2+\hat{Z}^2}\right] = \left[\hat{X}, \sqrt{\hat{Y}^2+\hat{Z}^2} - \frac{(\hat{X}-x_0)}{\sqrt{Y^2+Z^2}} + \dots +\right] = 0$$

$$\left[\sqrt{\hat{X}^2+\hat{Y}^2+\hat{Z}^2}, \sqrt{\hat{P}_x^2+\hat{P}_y^2+\hat{P}_z^2}\right] = ? \longrightarrow \text{bem mais complicado!}$$

$$\left[f(A,\dots), g(\dots)\right] = \sum \frac{\partial f}{\partial A}\frac{\partial g}{\partial B} \dots \rightarrow \text{paper recente}$$

—— // ——

Folha 4, ex 6:

$$\hat{B} = e^{i\hat{A}} \Rightarrow \hat{B}^+ = e^{-i\hat{A}} \neq \hat{B}$$

$$\hat{B}.\hat{B}^+ = e^{i\hat{A}} e^{-i\hat{A}} = e^{i(\hat{A}-\hat{A})} = \hat{1} = \dots = \hat{B}^+\hat{B}$$

$$\hat{U} = \hat{B} \Rightarrow \hat{C} \equiv \hat{B} + \hat{B}^+ \Rightarrow \hat{C}^+ = \hat{B}^+ + \hat{B} = \hat{C}$$

$$C = U + U^+ \Rightarrow C_{mn} = U_{mn} + U_{nm}^*$$

—— // ——

$$e^{i(\hat{A}+\hat{B})} = \sum_{n=0}^{\infty} \frac{[i(\hat{A}+\hat{B})]^n}{n!} = \hat{1} + i(\hat{A}+\hat{B}) - \frac{(\hat{A}+\hat{B})^2}{2!} + \dots$$

$$\hookrightarrow e^{i(\hat{A}+\hat{B})} \neq e^{i\hat{A}} . e^{i\hat{B}}$$ será igual se $[\hat{A},\hat{B}] = 0$.

$$= \hat{1} + i(\hat{A}+\hat{B}) - \frac{1}{2}\underbrace{(\hat{A}+\hat{B})(\hat{A}+\hat{B})}$$

$$\hat{A}\hat{B} + [\hat{B},\hat{A}]$$

$$\hat{A}^2 + \hat{A}\hat{B} + \hat{B}\hat{A} + \hat{B}^2$$

$$= \hat{A}^2 + 2AB + [\hat{B},\hat{A}] + \hat{B}^2$$

se $[\hat{A},\hat{B}] = 0$

$$= \left(\hat{1} + i\hat{A} - \frac{1}{2}\hat{A}^2 + \cdots\right)\left(1 + iB - \frac{1}{2}\hat{B}^2 + \cdots\right) = e^{i\hat{A}}e^{i\hat{B}}$$

Se $\hat{A}$ é observável então $\{|u_n\rangle\}$ é base de $\mathcal{E}$, onde

$\hat{A}|u_n\rangle = a_n|u_n\rangle$.

$$\hat{B} = e^{i\hat{A}} \implies B_{mn} \equiv \langle u_m|\hat{B}|u_n\rangle$$

$$B_{mn} = \langle u_m| e^{i\hat{A}} |u_n\rangle =$$

$$= \langle u_m| \underbrace{1 + i\hat{A} - \frac{\hat{A}^2}{2} + \cdots}_{B} |u_n\rangle$$

Mas a representação de $\hat{A}$ na base $\{|u_n\rangle\}$

$\{|u_1\rangle, |u_2\rangle, \ldots\}$

é diagonal

$$\hat{A} = \begin{bmatrix} a_1 & & & \\ & a_2 & & 0 \\ & & a_3 & \\ & 0 & & \ddots \end{bmatrix}$$

$$\hat{A}^2 = \begin{bmatrix} a_1^2 & & & \\ & a_2^2 & & \\ & & a_3^2 & \\ & & & \ddots \end{bmatrix} ; \quad \hat{A}^3 = \begin{bmatrix} a_1^3 & & \\ & a_2^3 & \\ & & \ddots \end{bmatrix}$$

$$B_{mn} = \langle u_m | \, 1 + i\theta_n - \frac{\theta_n^2}{2} - i\frac{\theta_n^3}{6} \, | u_n \rangle$$

$$= \delta_{mn} \cdot e^{i\theta_n}$$

$$\Rightarrow \hat{B} = \begin{bmatrix} e^{i\theta_1} & & & 0 \\ & e^{i\theta_2} & & \\ & & e^{i\theta_3} & \\ 0 & & & \ddots \end{bmatrix}$$

$$U(t-t_0) \quad e^{-i\hat{H}(t-t_0)/\hbar} = \begin{bmatrix} e^{-iE_1\cdots} & & 0 \\ & e^{-iE_2\cdots} & \\ 0 & & \ddots \end{bmatrix} = \sum_n e^{-iE_n(t-t_0)/\hbar} c_n \cdot |\varphi_n\rangle$$

Formulação de integrais de caminho

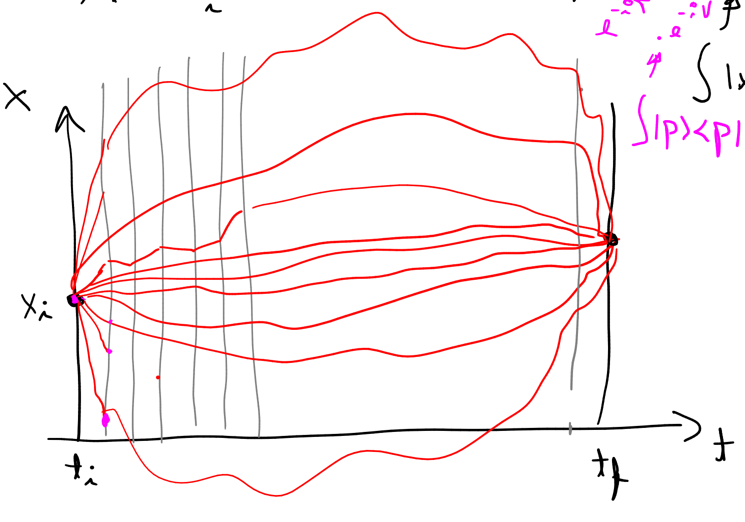
(MQ-II ou MQ-III)

$$\frac{p^2}{2m} + V(x)$$

$$\langle\psi_f| U(t-t_0) |\psi_i\rangle = \langle|\prod_i e^{-i\hat{H}\Delta t/\hbar} |\psi\rangle = \langle\psi_f| \underbrace{e^{-i\hat{H}\Delta t}}_{e^{-iT} \cdot e^{-iV}} \cdot e^{-i\hat{H}\Delta t} \cdot \ldots\ldots |\psi_i\rangle$$

$\int |x\rangle\langle x| dx$

$\int |p\rangle\langle p| dp$



$$= \int dx_1\, dp_1\, dx_2\, dp_2 \ldots \underbrace{\langle \psi_f | e^{-iT\Delta t} | p_1\rangle\langle p_1 | e^{-iV\Delta t} | x_1\rangle\langle x_1 |}\ldots\ | \psi_i \rangle$$

$$\langle 1 | 1 \rangle = e^{-S}$$

$$S \to \int dt\, \hat{\mathcal{L}}$$

Folha 4, ex 9: Evolução no tempo

$$\frac{\langle \psi | \hat{B} | \psi \rangle}{\langle \psi | \psi \rangle} = \langle \hat{B} \rangle$$ é valor esperado (ou valor médio)

$$H \longrightarrow D_H = \begin{bmatrix} E_1 & & 0 \\ & E_2 & \\ 0 & & E_3 \end{bmatrix} \Leftarrow \{ |v_1\rangle, |v_2\rangle, |v_3\rangle \}$$

$$|v_1\rangle = \sum_k b_k^1 |u_k\rangle$$

$$|v_i\rangle = \sum_{j=1}^{3} e_j^i |u_j\rangle \longrightarrow U = \begin{bmatrix} e_1^1 & e_1^2 & \cdot \\ e_2^1 & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Longrightarrow U = \begin{bmatrix} 2 & 0 & 0 \\ 0 & & \\ 0 & & \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)(\lambda^2 - 1^2) = 0$$

$$\lambda = 2 \lor \lambda^2 - 1^2 = 0 \Leftrightarrow \underline{\lambda = \pm 1}$$

$$
\begin{bmatrix}
\boxed{1} & 0 & 0 & & 0 & & & 0 \\
0 & 2 & 3 & & & & & \\
0 & 0 & 1 & & & & & \\
 & & & 0 & 3 & 4 & \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} & \\
 & 0 & & 1 & 0 & 1 & & 0 \\
 & & & 0 & 1 & 1 & & \\
 & & & & & & \ddots & \\
0 & & & 0 & & & &
\end{bmatrix}
$$

$$
\begin{bmatrix}
2a & 0 & 0 \\
0 & +1a & 0 \\
0 & 0 & -1a
\end{bmatrix}
$$

$$A = U^{-1} D_A U$$

$$
a \overbrace{\begin{bmatrix} \cdot & \cdot & \cdot \\ & \cdot & \\ \cdot & \cdot & \cdot \end{bmatrix}}^{= A}
=
\begin{bmatrix} & U^{-1} & \end{bmatrix}
\begin{bmatrix}
\lambda_1 a & & \\
 & \lambda_2 a & 0 \\
0 & & \lambda_3 a \ \ddots
\end{bmatrix}
\begin{bmatrix} & U & \end{bmatrix}
$$