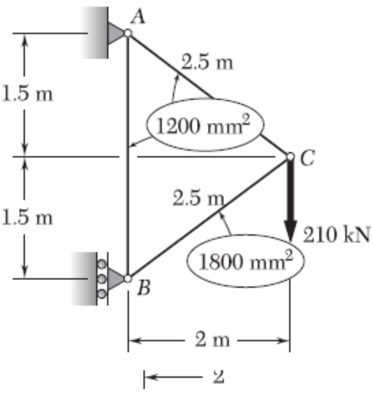
300040 Mechanics of Materials

Tutorial set 10 Solutions

- Your tutorial exercises have been marked for completeness rather than correctness – marks awarded don't necessarily indicate a correct solution.
- You should therefore check your answers against these solutions.
- These are skeletal solutions only – an ideal solution should contain greater details and description of what you are doing.
- Tutors will attempt to return tutorial exercises during the tutorial class.
- Ask during your tutorial class if there is anything you don't understand.

Question 1

The members of the truss shown are made of steel and have the cross-sectional areas shown. Use the work energy method to determine the vertical deflection of joint C caused by the application of the 210 kN load.
E = 200 GPa



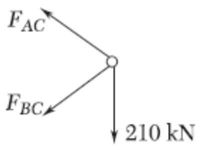
SOLUTION

Joint C $\qquad + \rightarrow \Sigma F_x = 0$

$$-\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$$

$$+\uparrow \Sigma F_y = 0$$

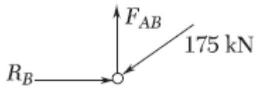
$$\frac{3}{5}F_{AC} - \frac{3}{5}F_{BC} - 210 = 0$$



Solving simultaneously

$$F_{AC} = 175 \text{ kN} \qquad F_{BC} = -175 \text{ kN}$$

Joint B $\qquad +\uparrow \Sigma F_y = 0$



$$F_{AB} - \left(\frac{3}{5}\right)(175) = 0$$

$$F_{AB} = 105 \text{ kN}$$

$$U_m = \sum \frac{F^2 L}{2EA}$$

Member	F (kN)	L(m)	$A(10^{-6}\,\text{m}^2)$	F^2L/A (N²/m)
AB	105	3.0	1200	27.5625×10^{12}
AC	175	2.5	1200	63.8021×10^{12}
BC	−175	2.5	1800	42.5347×10^{12}
				133.8993×10^{12}

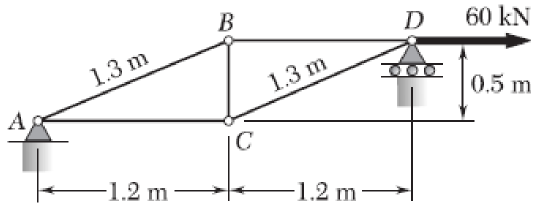
$$U_m = \frac{1}{2E}\sum\frac{F^2 L}{A} = \frac{133.8993\times10^{12}}{(2)(200\times10^{9})} = 334.75 \text{ J}$$

$$\frac{1}{2}P_m\Delta_m = U_m$$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(334.75)}{210\times10^3} = 3.19\times10^{-3}\text{ m} = 3.19 \text{ mm}$$

Question 2

Each member of the truss shown is made of steel; the cross-sectional area of the member BC is 800 mm^2 and for all other members the cross-sectional area is 400 mm^2. Use the work energy method to determine the deflection of point D caused by the 60 kN load shown.



E = 200 GPa

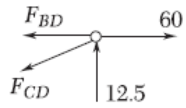
SOLUTION

Entire truss $\circlearrowleft + \; \Sigma M_A = 0$

$2.4 \; R_D - (0.5)(60) = 0 \qquad R_D = 12.5 \text{ kN}$

Joint D

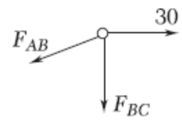
$+\uparrow \Sigma F_y = 0 \qquad 12.5 - \dfrac{0.5}{1.3} F_{CD} = 0 \qquad F_{CD} = 32.5 \text{ kN}$



$\rightarrow + \Sigma F_x = 0 \quad 60 - F_{BD} - \dfrac{1.2}{1.3} F_{CD} = 0 \qquad F_{BD} = 30 \text{ kN}$

Joint B

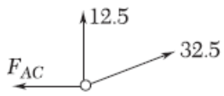
$+\rightarrow \Sigma F_x = 0 \qquad 30 - \dfrac{1.2}{1.3} F_{AB} = 0 \qquad F_{AB} = 32.5 \text{ kN}$



$+\uparrow \Sigma F_y = 0 \qquad -\dfrac{0.5}{1.3} F_{AB} + F_{BC} = 0 \qquad F_{BC} = 12.5 \text{ kN}$

Joint C

$+\rightarrow \Sigma F_x = 0 \qquad -F_{AC} + \dfrac{1.2}{1.3}(32.5) = 0 \qquad F_{AC} = 30 \text{ kN}$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F(kN)	L(m)	$A\,(10^{-6}\,\text{m}^2)$	$F^2L/A\,(\text{N}^2/\text{m})$
CD	32.5	1.3	400	3.4328×10^{12}
BD	30	1.2	400	2.7×10^{12}
AB	32.5	1.3	400	3.4328×10^{12}
BC	12.5	0.5	800	0.0977×10^{12}
AC	30	1.2	400	2.7×10^{12}
				12.3633×10^{12}

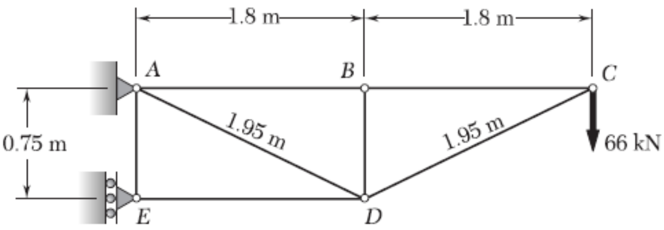
$$U = \frac{12.3633 \times 10^{12}}{(2)(200 \times 10^9)} = 30.908 \text{ J}$$

$$\frac{1}{2} P\Delta = U \qquad \Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60 \times 10^3} = 1.030 \times 10^{-3} \text{ m}$$

$$= 1.030 \text{ mm} \rightarrow$$

Question 3 *

Each member of the truss shown is made of steel and has a uniform cross-sectional area of 3125 mm². Use the work energy method to determine the vertical deflection of point C caused by the 66 kN force.



E = 200 GPa

SOLUTION

Members BD and AE are zero force members.

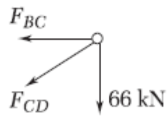
For entire truss $\circlearrowleft + \; M_A = 0$

$$0.75\,R_D - (3.6)(66) = 0$$

$$R_D = 316.8 \text{ kN}$$

For equilibrium of joint E $\qquad F_{ED} = -R_D = -316.8$ kN

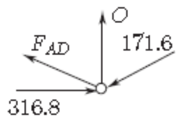
Joint C $\quad +\uparrow \Sigma F_y = 0 \qquad\qquad +\rightarrow \Sigma F_x = 0$



$$\frac{0.75}{1.95} F_{CD} - 66 = 0 \qquad\qquad -\frac{1.8}{1.95} F_{CD} - F_{BC} = 0$$

$$F_{CD} = -171.6 \text{ kN} \qquad\qquad F_{BC} = 158.4 \text{ kN}$$

Joint D $\quad +\rightarrow \Sigma F_x = 0$



$$316.8 - \frac{1.8}{1.95}(F_{AD} + 171.6) = 0$$

$$F_{AD} = 171.6 \text{ kN}$$

Joint B $\quad \Sigma F_x = 0$

$$-F_{AB} + F_{BC} = 0$$

$$F_{AB} = 158.4 \text{ kN}$$

Strain energy $\quad U_m = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} \Sigma F^2 L$

Member	F(kN)	L(m)	F^2L(kN$^2\cdot$m)
AB	158.4	1.8	45163
BC	158.4	1.8	45163
CD	−171.6	1.95	57420
DE	−316.8	1.8	180652
BD	0	0.75	0
AE	0	0.75	0
AD	171.6	1.95	57420
Σ			385818

Data: $\qquad E = 200 \times 10^9$ Pa

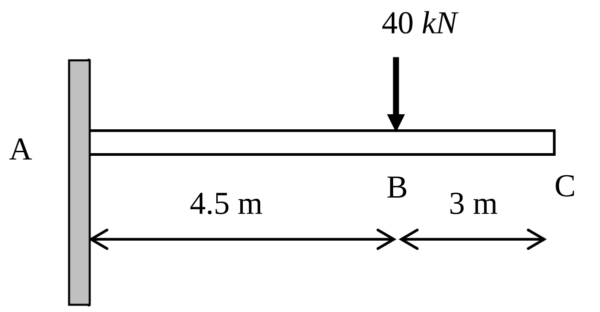
$\qquad\qquad A = 3125$ mm^2

$$U_m = \frac{385818}{(2)(200 \times 10^9)(3125 \times 10^{-6})} = 0.3086 \text{ kN}\cdot\text{m}$$

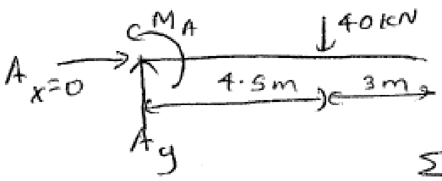
$$\frac{1}{2} P_m \Delta_m = U \qquad \Delta_m = \frac{2U_m}{P_m} = \frac{(2)(0.3086)}{66} = 0.00935 \text{ m} = 9.35 \text{ mm} \downarrow$$

Question 4

Determine the displacement of point *B* on the steel beam. $I = 104 \times 10^6\ mm^4$ and $E = 200$ GPa.



Find Reactions.



$\uparrow \sum F_y = 0 \ ; \quad A_y = 40\,kN$

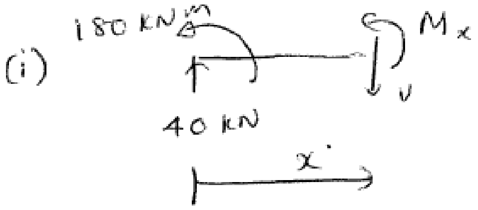
$\sum M_A = 0$

$M_A - 40 \times 4.5 = 0$

$M_A = 180\ kNm$

Find moment equations.

(i)



$0 \le x \le 4.5\,m$

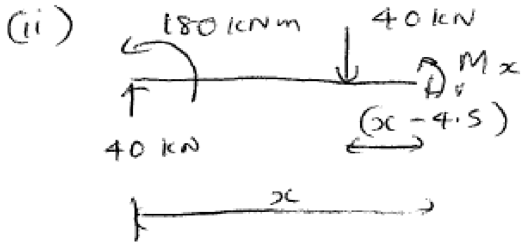
$\sum M = 0$

$M_x + 180 - 40x = 0$

$M_x = (40x - 180)\ kNm$

$M_x^2 = (1600x^2 - 14400x + 32400) \times 10^6\ (Nm)^2$

(ii)



$\sum M = 0$

$M_x + 180 + 40(x - 4.5) - 40x = 0$

$M_x = -180 - 40x + 180 + 40x$

$M_x = 0$

Strain energy $U_i = \int_0^L \frac{M^2 dx}{2EI}$

$$U_i = \int_0^{4.5} \frac{10^6 \times (40x-180)^2}{2EI} dx = \int_0^{4.5} \frac{10^6}{2EI}(1600x^2 - 14400x + 32400)dx$$

$$= \frac{10^6}{2EI}\left[\frac{1600}{3}x^3 - 7200x^2 + 32400x\right]_0^{4.5}$$

$$= \frac{10^6}{2\times200\times10^9\times104\times10^6\times10^{-12}}\left[\frac{1600}{3}(4.5)^3 - 7200(4.5)^2 + 32400\times4.5\right]$$

$$U_i = 1.168\times10^3 \text{ J.}$$

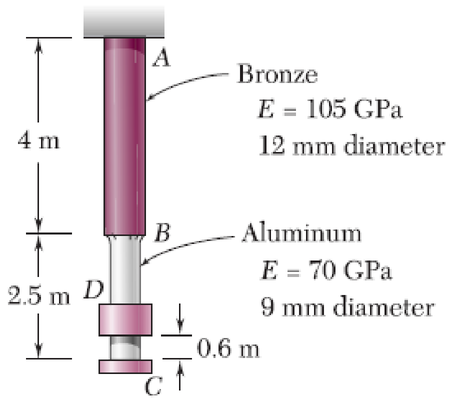
$$U_e = U_i$$

$$\frac{1}{2}\times 40\times10^3\, \Delta = 1.168\times10^3$$

$$\Delta = 58.41 \text{ mm}$$

Question 5

Collar D of mass 4kg is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC. Determine the maximum stress in the rod resulting from this impact.



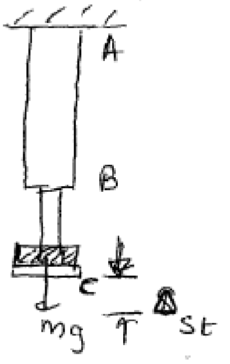
Solution.

$$A_{AB} = \pi\left(\frac{0.012}{2}\right)^2 = 1.13\times10^{-4}\, m^2$$

$$A_{BC} = \pi\left(\frac{.009}{2}\right)^2 = 6.3617\times10^{-5}\, m^2$$

$$\Delta_{max} = \Delta_{st}\left[1+\sqrt{1+\frac{2h}{\Delta_{st}}}\right]$$

Determine Δ_{st} ; $\Delta_{st} = \Delta_{AB} + \Delta_{BC}$



$P = mg$

$$\Delta_{st} = \frac{P L_{AB}}{E_{AB} A_{AB}} + \frac{P L_{BC}}{E_{BC} A_{BC}}.$$

$$= \frac{(mg) L_{AB}}{E_{AB} A_{AB}} + \frac{(mg) L_{BC}}{E_{BC} A_{BC}}$$

$$= \frac{(4 \times 9.81) \times 4}{105 \times 10^{9} \times 1.13 \times 10^{-4}} + \frac{(4 \times 9.81) \times 2.5}{70 \times 10^{9} \times 6.3617 \times 10^{-5}}$$

$$\Delta_{st} = 3.525 \times 10^{-5}\, m$$

$$\Delta_{max} = 3.525 \times 10^{-5} \left[1 + \sqrt{1 + \frac{2 \times 0.06}{3.525 \times 10^{-5}}} \right] = 0.0065392 m$$

$$\Delta_{max} = \frac{P_{max} L_{AB}}{E_{AB} A_{AB}} + \frac{P_{max} L_{BC}}{E_{BC} A_{BC}} = P_{max} \left[\frac{L_{AB}}{E_{AB} A_{AB}} + \frac{L_{BC}}{E_{BC} A_{BC}} \right]$$

$$P_{max} = \frac{\Delta_{max}}{\left[\frac{L_{AB}}{E_{AB} A_{AB}} + \frac{L_{BC}}{E_{BC} A_{BC}} \right]}$$

$$P_{max} = \frac{0.0065392}{\left[\frac{4}{105 \times 10^{9} \times 1.13 \times 10^{-4}} + \frac{2.5}{70 \times 10^{9} \times 6.3617 \times 10^{-5}} \right]}$$

$$= 7277.73 \text{ N}$$

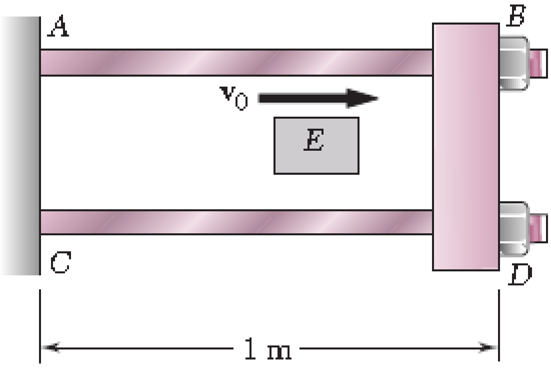
$$\sigma_{max} = \frac{P_{max}}{A_{BC}}$$

$$\sigma_{max.} = \frac{7277.73}{6.3617 \times 10^{-5}} = \underline{\underline{114.4 \text{ MPa}}}$$

Question 6

The cylindrical block E has a mass of m = 3 kg and a speed v_0 = 5 m/s when it strikes squarely the yoke BD that is attached to two 22 mm diameter rods AB and CD. The rods are made of a steel for which E = 200 GPa.

Calculate the maximum stress in the two rods as a result of the impact.



Calculate the maximum stress in the two rods as a result of the impact.

$$\boxed{\Delta = \frac{PL}{AE}}$$

$$A_{Total} = 2 \times (11 \times 10^{-3})^2 \pi$$
$$= 7.603 \times 10^{-4} \, m^2$$

$$P_{max} = \frac{AE}{L} \Delta_{max}$$

$$U_i = \frac{1}{2} \times P_{max} \times \Delta_{max}$$
$$= \frac{1}{2} \times \left(\frac{AE}{L} \Delta_{max} \right) \Delta_{max} = \frac{1}{2} \frac{AE}{L} \Delta_{max}^2$$

$$U_e = U_i$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} \frac{AE}{L} \Delta_{max}^2$$

$$\Delta_{max} = \sqrt{\frac{Lm}{AE}} \, v_0$$

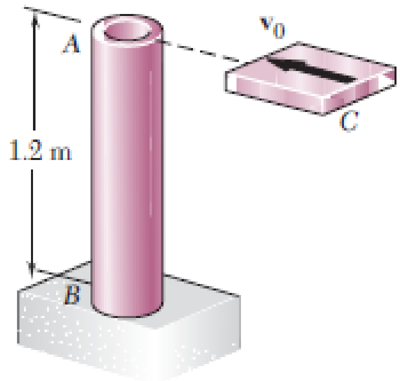
$$P_{max} = \frac{AE}{L} \left(\sqrt{\frac{Lm}{AE}} \right) v_0$$
$$= v_0 \sqrt{\frac{AEm}{L}}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{v_0}{A} \sqrt{\frac{AEm}{L}}$$
$$= \frac{5}{7.603 \times 10^{-4}} \sqrt{\frac{7.603 \times 10^{-4} \times 200 \times 10^{9} \times 3}{1}}$$
$$= 140.46 \times 10^{6} \, Pa$$

$$\underline{\underline{\sigma_{max} = 140.46 \, MP}}$$

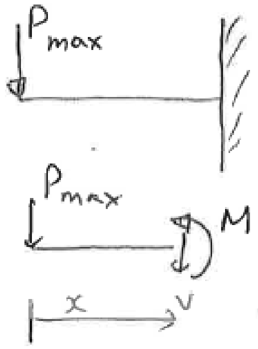
Question 7
The post AB consists of a steel pipe of 90 mm outer diameter, and 74 mm inner diameter. A 6.5 kg block C is moving horizontally with a velocity v_0 and hits the post squarely at A.

Using E = 200 GPa, determine the largest speed v_0 for which the maximum normal (bending) stress in the pipe does not exceed 165 MPa.



Solution:

$$U_e = U_i$$
$$U_e = \frac{1}{2} m v_0^2$$



$$\sum M = 0$$
$$M + Px = 0$$
$$M = P_{max} x$$
$$M^2 = P_{max}^2 x^2$$
$$U_i = \int \frac{M^2}{2EI} dx = \frac{1}{2EI} \int M^2 dx$$
$$U_i = \frac{1}{2EI} \int_0^L P_{max}^2 x^2 dx = \frac{P_{max}^2}{2EI} \int_0^L x^2 dx$$
$$U_i = \frac{P_{max}^2}{2EI} \left[\frac{1}{3} x^3\right]_0^L = \frac{P_{max}^2}{6EI} [L^3]$$

$$U_e = U_i$$
$$\frac{1}{2} m v_0^2 = \frac{P_{max}^2}{6EI} L^3$$
$$P_{max}^2 = \frac{3EImv_0^2}{L^3}$$
$$P_{max} = \sqrt{\frac{3EIm}{L^3}}\, v_0 = \sqrt{\frac{3 \times 200 \times 10^9 \times 1.749 \times 10^{-6} \times 6.5}{1.2^3}}\, v_0$$
$$P_{max} = 1986.805 v_0 \longrightarrow (1)$$
$$M_{max} = P_{max} \times 1.2 \longrightarrow (2)$$
$$\sigma_{max} = \frac{M_{max} y}{I} \Rightarrow M_{max} = \frac{\sigma_{max} I}{y}$$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$= \frac{\pi}{4}\left[(45\times10^{-3})^4 - (37\times10^{-3})^4\right] = 1.749\times10^{-6}\,m^4$$

$$M_{max} = \frac{165\times10^6 \times 1.749\times10^{-6}}{45\times10^{-3}}$$

$$= 6412\ Nm$$

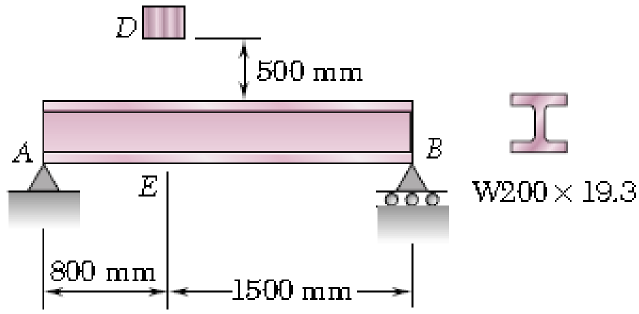
From eq^n ②; $P_{max} = \frac{M_{max}}{1.2} = \frac{6412}{1.5} = 5343\,N$

From eq^n ①; $\frac{P_{max}}{1986.805} = v_o$

$$v_o = \frac{5343}{1986.805}$$

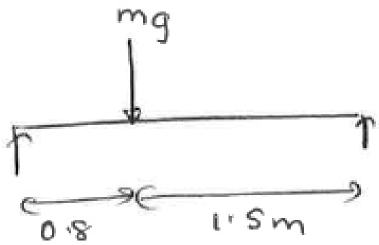
$$v_o = 2.69\ m/s$$

Question 8 *



The 25 kg block D is dropped from a height of 500 mm into the steel beam AB. Knowing that E = 200 GPa, determine
(a) the maximum deflection at point E,
(b) the maximum normal stress in the beam.

For the beam; I = 16.6x10^6 mm^4 , height = 203 mm



$$\Delta_{st} = \frac{(mg)a^2 b^2}{3EIL}$$

$$\Delta_{st} = \frac{25\times 9.81\times 0.8^2 \times 1.5^2}{3\times 200\times 10^9 \times 166\times 10^{-6}\times 2.3}$$

$$\Delta_{st} = 1.54\times 10^{-5}\ m$$

$$\Delta_{max} = \Delta_{st}\left[1+\sqrt{1+\frac{2h}{\Delta_{st}}}\right]$$

$$= 1.54\times 10^{-5}\left[1+\sqrt{1+\frac{2\times 0.5}{1.54\times 10^{-5}}}\right]$$

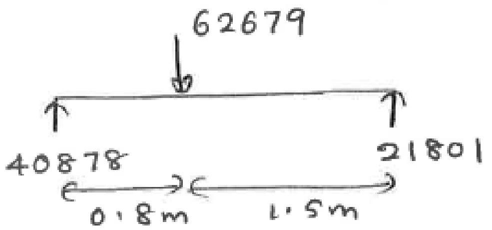
$$\Delta_{max} = 0.00394\ m$$

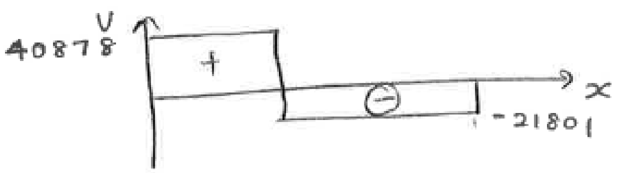
$$\Delta_{max} = \left|-\frac{P_{max}\,ba}{6EIL}\right|(L^2-b^2-a^2)\ (\text{Appendix C})$$

$$P_{max} = \frac{6EIL\Delta_{max}}{ba(L^2-b^2-a^2)}$$

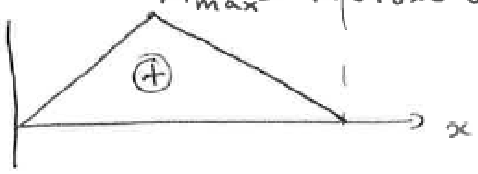
$$P_{max} = \frac{6 \times 200 \times 10^9 \times 16.6 \times 10^{-6} \times 2.3 \times 0.00394}{1.5 \times 0.8 \times (2.3^2 - 1.5^2 - 0.8^2)}$$

$$P_{max} = 62679 N$$





$M_{max} = 40878 \times 0.8 = 32702.4$ Nm



$$\sigma_{max} = \frac{My}{I}$$

$$= \frac{32702 \times 0.203/2}{16.6 \times 10^{-6}} = 199.955 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 200 \text{ MPa}$$