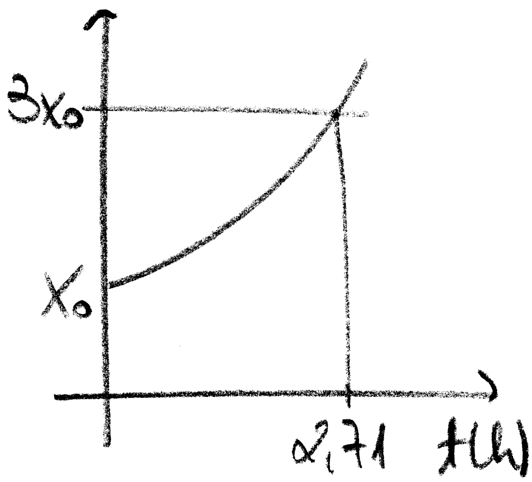
1) $x(t) = X_0 e^{\alpha t} \Rightarrow x(1) = 1{,}5\cancel{X_0} = \cancel{X_0} e^{\alpha \cdot 1} \Rightarrow$

$\Rightarrow 1{,}5 = e^{\alpha} \Rightarrow \alpha = \ln(1{,}5) \Rightarrow \alpha = 0{,}4055$

$x(t) = X_0 e^{0{,}4055t} \Rightarrow 3\cancel{X_0} = \cancel{X_0} e^{0{,}4055t} \Rightarrow$

$\Rightarrow t = \dfrac{\ln(3)}{0{,}4055} \Rightarrow \boxed{t = 2{,}71h}$



2) $x(t) = X_0 e^{\alpha t} \Rightarrow x(15) = (1 - 0{,}00043)\cancel{X_0} = \cancel{X_0} e^{\alpha \cdot 15} \Rightarrow$

$\Rightarrow 0{,}99957 = e^{\alpha \cdot 15} \Rightarrow \alpha = \dfrac{\ln(0{,}99957)}{15} \Rightarrow$

$\Rightarrow \alpha = -0{,}00002867$

$x(t) = X_0 e^{-0{,}00002867t} \Rightarrow \dfrac{1}{2}\cancel{X_0} = \cancel{X_0} e^{-0{,}00002867t} \Rightarrow$

$\Rightarrow t = \dfrac{\ln(1/2)}{-0{,}00002867} \Rightarrow \boxed{t \cong 24177 \text{ anos}}$

3) $x(t) = x_0 e^{\alpha t} \Rightarrow \frac{1}{2}\cancel{x_0} = x(5600) = \cancel{x_0} e^{\alpha 5600} \Rightarrow$

$$\Rightarrow \alpha = \frac{\ln(1/2)}{5600} \Rightarrow \alpha = -0{,}00012378$$

$$x(t) = x_0 e^{-0{,}00012378 t} \Rightarrow$$

$$\Rightarrow \frac{1}{1000}\cancel{x_0} = \cancel{x_0} e^{-0{,}00012378 t} \Rightarrow$$

$$\Rightarrow t = \frac{\ln(1/1000)}{-0{,}00012378} \Rightarrow \boxed{t \cong 55807 \text{ anos}}$$

4)

a) $\sum_{i=1}^{4} \vec{F_i} = m_1 . \ddot{w}_1(t) \Rightarrow$

$\Rightarrow -k_1 w_1(t) - k_3(w_1(t) - w_2(t)) - b_1 \dot{w}_1(t) + u(t) = m_1 \ddot{w}_1(t) \Rightarrow$

$\Rightarrow -k_1 w_1(t) - k_3 w_1(t) + k_3 w_2(t) - b_1 \dot{w}_1(t) + u(t) =$

$= m_1 \ddot{w}_1(t) \Rightarrow$

$\Rightarrow -w_1(t)(k_1 + k_3) + k_3 w_2(t) - b_1 \dot{w}_1(t) + u(t) = m_1 \ddot{w}_1(t) \Rightarrow$

$\Rightarrow \ddot{w}_1(t) = -\left(\frac{k_1+k_3}{m_1}\right) w_1(t) + \frac{k_3}{m_1} w_2(t) - \frac{b_1}{m_1} \dot{w}_1(t) + \frac{1}{m_1} u(t)$

$\sum_{i=1}^{3} \vec{F_i} = m_2 \ddot{w}_2(t) \Rightarrow$

$\Rightarrow -k_3(w_2(t) - w_1(t)) - k_2 w_2(t) - b_2 \dot{w}_2(t) = m_2 \ddot{w}_2(t) \Rightarrow$

$\Rightarrow -k_3 w_2(t) + k_3 w_1(t) - k_2 w_2(t) - b_2 \dot{w}_2(t) = m_2 \ddot{w}_2(t) \Rightarrow$

$\Rightarrow k_3 w_1(t) - (k_2 + k_3) w_2(t) - b_2 \dot{w}_2(t) = m_2 \ddot{w}_2(t) \Rightarrow$

$\Rightarrow \ddot{w}_2(t) = \frac{k_3}{m_2} w_1(t) - \left(\frac{k_2+k_3}{m_2}\right) w_2(t) - \frac{b_2}{m_2} \dot{w}_2(t)$

$$x_1(t) = w_1(t) = y_1(t)$$

$$x_2(t) = w_2(t) = y_2(t)$$

$$x_3(t) = \dot{w}_1(t) = \dot{x}_1(t)$$

$$x_4(t) = \dot{w}_2(t) = \dot{x}_2(t)$$

$$\dot{x}_3(t) = \ddot{w}_1(t) = -\left(\frac{k_1+k_3}{m_1}\right)x_1(t) + \frac{k_3}{m_1}x_2(t) - \frac{b_1}{m_1}x_3(t) + \frac{1}{m_1}u(t)$$

$$\dot{x}_4(t) = \ddot{w}_2(t) = \frac{k_3}{m_2}x_1(t) - \left(\frac{k_2+k_3}{m_2}\right)x_2(t) - \frac{b_2}{m_2}x_4(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\left(\frac{k_1+k_3}{m_1}\right) & \frac{k_3}{m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{k_3}{m_2} & -\left(\frac{k_2+k_3}{m_2}\right) & 0 & -\frac{b_2}{m_2} \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} u(t) \longrightarrow B$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \underset{\substack{\downarrow \\ D}}{0}\, u(t)$$

b) $\sum_{i=1}^{4} \vec{F_i} = m_1 \ddot{w}_1(t) \Rightarrow$

$\Rightarrow -k_1 w_1(t) - b_1(\dot{w}_1(t) - \dot{w}_2(t)) + u_1(t) = m_1 \ddot{w}_1(t) \Rightarrow$

$\Rightarrow -k_1 w_1(t) - b_1 \dot{w}_1(t) + b_1 \dot{w}_2(t) + u_1(t) = m_1 \ddot{w}_1(t) \Rightarrow$

$\Rightarrow \ddot{w}_1(t) = -\frac{k_1}{m_1} w_1(t) - \frac{b_1}{m_1} \dot{w}_1(t) + \frac{b_1}{m_1} \dot{w}_2(t) + \frac{1}{m_1} u_1(t)$

$\sum_{i=1}^{4} \vec{F_i} = m_2 \cdot \ddot{w}_2(t) \Rightarrow$

$\Rightarrow -k_2 w_2(t) - b_1(\dot{w}_2(t) - \dot{w}_1(t)) + u_2(t) = m_2 \ddot{w}_2(t) \Rightarrow$

$\Rightarrow -k_2 w_2(t) + b_1 \dot{w}_1(t) - b_1 \dot{w}_2(t) + u_2(t) = m_2 \ddot{w}_2(t) \Rightarrow$

$\Rightarrow \ddot{w}_2(t) = -\frac{k_2}{m_2} w_2(t) + \frac{b_1}{m_2} \dot{w}_1(t) - \frac{b_1}{m_2} \dot{w}_2(t) + \frac{1}{m_2} u_2(t)$

$$x_1(t) = w_1(t) = y_1(t)$$

$$x_2(t) = w_2(t) = y_2(t)$$

$$x_3(t) = \dot{w}_1(t) = \dot{x}_1(t)$$

$$x_4(t) = \dot{w}_2(t) = \dot{x}_2(t)$$

$$\dot{x}_3(t) = \ddot{w}_1(t) = -\frac{k_1}{m_1} x_1(t) - \frac{b_1}{m_1} x_3(t) + \frac{b_1}{m_1} x_4(t) + \frac{1}{m_1} u_1(t)$$

$$\dot{x}_4(t) = \ddot{w}_2(t) = -\frac{k_2}{m_2} x_2(t) + \frac{b_1}{m_2} x_3(t) - \frac{b_1}{m_2} x_4(t) + \frac{1}{m_2} u_2(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_1} & 0 & -\frac{b_1}{m_1} & \frac{b_1}{m_1} \\ 0 & -\frac{k_2}{m_2} & \frac{b_1}{m_2} & -\frac{b_1}{m_2} \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \longrightarrow B$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \underset{\displaystyle \downarrow \atop \displaystyle D}{0} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

5)

a) $i(t) = \frac{dq(t)}{dt} \Rightarrow q(t) = \int_0^t i(t)\,dt$

$v_C(t) = \frac{q(t)}{C} \Rightarrow v_C(t) = \frac{1}{C}\int_0^t i(t)\,dt \Rightarrow \int_0^t i(t)\,dt = C\,v_C(t) \Rightarrow$

$\Rightarrow i(t) = C\frac{dv_C(t)}{dt} \Rightarrow \frac{di(t)}{dt} = C\frac{d^2 v_C(t)}{dt^2}$

$v_R(t) + v_L(t) + v_C(t) = u(t) \Rightarrow$

$\Rightarrow R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)\,dt = u(t) \Rightarrow$

$\Rightarrow RC\frac{dv_C(t)}{dt} + LC\frac{d^2 v_C(t)}{dt^2} + v_C(t) = u(t) \Rightarrow$

$\Rightarrow LC\,\ddot{v}_C(t) + RC\,\dot{v}_C(t) + v_C(t) = u(t) \Rightarrow$

$\Rightarrow \boxed{\ddot{v}_C(t) = -\frac{1}{LC}v_C(t) - \frac{R}{L}\dot{v}_C(t) + \frac{1}{LC}u(t)}$

b) $M_p = 30\%$ $\quad t_s = 0{,}5\,s$ $\quad L = 200\,mH$

$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = \sqrt{\frac{[\ln(M_p)]^2}{[\ln(M_p)]^2 + \pi^2}} \Rightarrow$

$$\Rightarrow \xi = \sqrt{\frac{[\ln(0{,}3)]^2}{[\ln(0{,}3)]^2 + \pi^2}} \Rightarrow \boxed{\xi = 0{,}358}$$

$$t_s = \frac{4}{\xi w_n} \Rightarrow w_n = \frac{4}{\xi t_s} \Rightarrow w_n = \frac{4}{0{,}358 \cdot 0{,}5} \Rightarrow$$

$$\Rightarrow \boxed{w_n = 22{,}35 \text{ rad/s}}$$

$$\ddot{v}_c(t) + \frac{R}{L}\dot{v}_c(t) + \frac{1}{LC}v_c(t) = \frac{1}{LC}u(t)$$

$$\ddot{v}_c(t) + 2\xi w_n \dot{v}_c(t) + w_n^2 v_c(t) = w_n^2 u(t)$$

$$w_n^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{w_n^2 L} \Rightarrow C = \frac{1}{22{,}35^2 \cdot 200 \cdot 10^{-3}} \Rightarrow$$

$$\Rightarrow \boxed{C = 10 \text{ mF}}$$

$$2\xi w_n = \frac{R}{L} \Rightarrow R = 2\xi w_n L \Rightarrow R = 2 \cdot 0{,}358 \cdot 22{,}35 \cdot 200 \cdot 10^{-3} \Rightarrow$$

$$\Rightarrow \boxed{R = 3{,}2\,\Omega}$$

c)

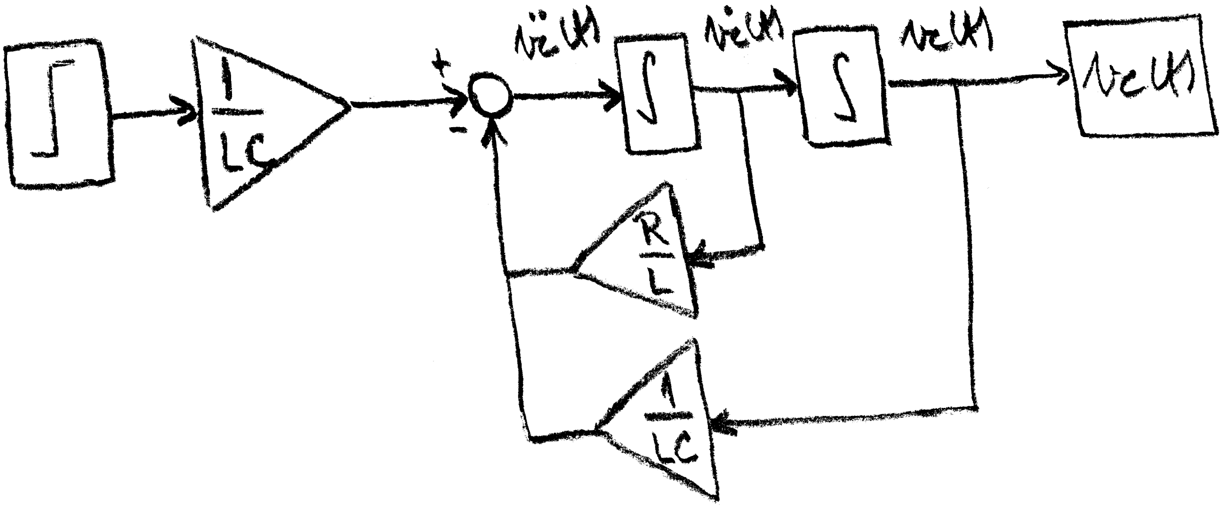
$$x_1(t) = v_c(t)$$

$$x_2(t) = \dot{v}_c(t) = \dot{x}_1(t)$$

$$\dot{x}_2(t) = \ddot{v}_c(t) = -\frac{1}{LC}x_1(t) - \frac{R}{L}x_2(t) + \frac{1}{LC}u(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix}}^{A} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix}}_{B} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{0}_{D}\, u(t)$$



d)
$$u(t) = k_p e(t) + k_d \dot{e}(t) = k_p(\overset{E}{\cancel{r(t)}} - v_c(t)) + k_d(\overset{0}{\cancel{\dot{r}(t)}} - \dot{v}_c(t)) \Rightarrow$$

$$\Rightarrow u(t) = k_p E - k_p v_c(t) - k_d \dot{v}_c(t)$$

$$\ddot{v}_c(t) + \frac{R}{L}\dot{v}_c(t) + \frac{1}{LC}v_c(t) = \frac{1}{LC}\left(k_p E - k_p v_c(t) - k_d \dot{v}_c(t)\right) \Rightarrow$$

$$\Rightarrow \ddot{v}_c(t) + \frac{R}{L}\dot{v}_c(t) + \frac{1}{LC}v_c(t) = \frac{k_p}{LC}E - \frac{k_p}{LC}v_c(t) - \frac{k_d}{LC}\dot{v}_c(t) \Rightarrow$$

$$\Rightarrow \ddot{v}_c(t) + \frac{R}{L}\dot{v}_c(t) + \frac{1}{LC}v_c(t) + \frac{k_p}{LC}v_c(t) + \frac{k_d}{LC}\dot{v}_c(t) = \frac{k_p}{LC}E \Rightarrow$$

$$\Rightarrow \ddot{v}_c(t) + \left(\frac{R}{L} + \frac{k_d}{LC}\right)\dot{v}_c(t) + \left(\frac{1}{LC} + \frac{k_p}{LC}\right)v_c(t) = \frac{k_p}{LC}E \Rightarrow$$

$$\Rightarrow \ddot{v}_c(t) + \underbrace{\left(\frac{RC + k_d}{LC}\right)}_{2\xi\omega_n}\dot{v}_c(t) + \underbrace{\left(\frac{k_p + 1}{LC}\right)}_{\omega_n^2}v_c(t) = \frac{k_p}{LC}E$$

$$\omega_n^2 = \frac{k_p + 1}{LC} \Rightarrow \boxed{k_p = LC\omega_n^2 - 1}$$

$$2\xi\omega_n = \frac{RC + k_d}{LC} \Rightarrow \boxed{k_d = C(2\xi\omega_n L - R)}$$

$M_p = 20\%$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = \sqrt{\frac{[\ln(M_p)]^2}{[\ln(M_p)]^2 + \pi^2}} \Rightarrow$$

$$\Rightarrow \xi = \sqrt{\frac{[\ln(0{,}2)]^2}{[\ln(0{,}2)]^2 + \pi^2}} \Rightarrow \boxed{\xi = 0{,}456}$$

$$t_s = \frac{4}{\xi \omega_n} \Rightarrow \omega_n = \frac{4}{\xi t_s} \Rightarrow \omega_n = \frac{4}{0{,}456 \cdot 0{,}1} \Rightarrow$$

$$\Rightarrow \boxed{\omega_n = 87{,}72 \text{ rad/s}}$$

$$k_p = LC\omega_n^2 - 1 \Rightarrow k_p = 200 \cdot 10^{-3} \cdot 10 \cdot 10^{-3} \cdot 87{,}72^2 - 1 \Rightarrow$$

$$\Rightarrow \boxed{k_p = 14{,}38}$$

$$k_d = C(2\xi\omega_n L - R) \Rightarrow$$

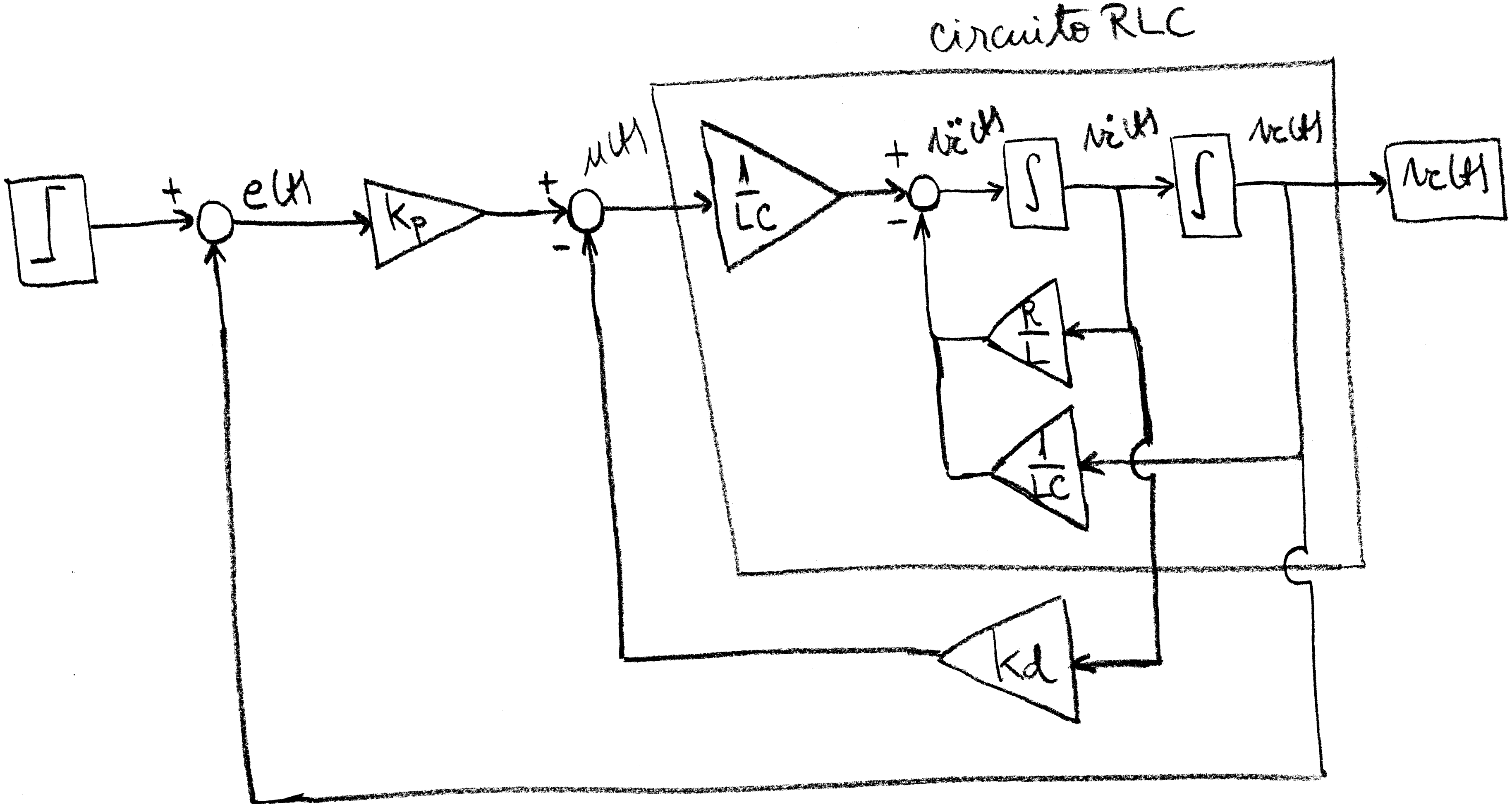
$$\Rightarrow k_d = 10 \cdot 10^{-3}(2 \cdot 0{,}456 \cdot 87{,}72 \cdot 200 \cdot 10^{-3} - 3{,}2) \Rightarrow$$

$$\Rightarrow \boxed{k_d = 0{,}128}$$

$r(t) = 0$

e) $$\ddot{v}_c(t) + \frac{R}{L}\dot{v}_c(t) + \frac{1}{LC}v_c(t) = \frac{1}{LC}\left(k_p\, e(t) - k_d\, \dot{v}_c(t)\right) \Rightarrow$$

$$\Rightarrow \ddot{v}_c(t) = -\frac{1}{LC}v_c(t) - \frac{R}{L}\dot{v}_c(t) + \frac{1}{LC}\left(k_p\, e(t) - k_d\, \dot{v}_c(t)\right)$$


circuito RLC
e(t)
Kp
u(t)
1/LC
vc''(t)
vc'(t)
vc(t)
R/L
1/LC
Kd
vc(t)
+
+
-
+
-