Aula Dúvidas (5/Fev)

Folha 1

(2.1) (b) $\quad u(\phi) = \dfrac{1}{r(t)} \quad ; \quad \dot{\phi} = \text{const} \Rightarrow \phi(t) = c.t$

$$\frac{d\,u(\phi)}{d\phi} = \frac{du}{dt}\underbrace{\frac{dt}{d\phi}}_{=1/\dot{\phi}} = \frac{d}{dt}\left(\frac{1}{r(t)}\right)\frac{1}{\dot{\phi}} = -\frac{\dot{r}(t)}{r^2 .\, \dot{\phi}(t)} = -\frac{\dot{r}(t)}{l}$$

$$\frac{d^2u}{d\phi} = \frac{d}{d\phi}\left(-\frac{\dot{r}}{l}\right) = \frac{dt}{d\phi}\frac{d}{dt}\left(-\frac{\dot{r}}{l}\right) = \frac{1}{\dot{\phi}}\cdot\left(-\frac{\ddot{r}}{l}\right) = -\frac{\ddot{r}}{l.\dot{\phi}}$$

$$= -\ddot{r}.\frac{r^2}{l^2} \Rightarrow \ddot{r} = -u^2(\phi).\, l^2.\, \frac{d^2u}{d\phi^2}$$

A eq. movim.,

$$\frac{d}{dr}\left(-\frac{\alpha}{r}\right) = \frac{\alpha}{r^2}$$

$$m\ddot{r} = -\frac{dV}{dr} + \frac{ml^2}{r^3} = -\frac{\alpha}{r^2} + \frac{ml^2}{r^3}$$

$$\Leftrightarrow -\cancel{u^2}.\,\frac{d^2u}{d\phi^2} = -\frac{\alpha}{ml^2}.\cancel{u^2} + \quad u^{\cancel{3}}$$

$$\Leftrightarrow \frac{d^2u}{d\phi^2} = \frac{\alpha}{ml^2} - u$$

$$\Leftrightarrow \frac{d^2u}{d\phi^2} + u = \frac{\alpha}{ml^2}$$

(c)

$$\Rightarrow \quad \frac{d^2}{d\phi^2}\left(u(\phi) - \frac{\alpha}{ml^2}\right) + \left(u(\phi) - \frac{\alpha}{ml^2}\right) = 0$$

$$\Leftrightarrow \quad u(\phi) - \frac{\alpha}{ml^2} = A \cdot \cos(\phi + \phi_0)$$

$$\Leftrightarrow \quad \frac{1}{r(\phi)} = A \cdot \cos(\phi + \phi_0) + \frac{\alpha}{ml^2}$$

$$\Leftrightarrow \quad r(\phi) = \frac{1}{A\cos(\phi + \phi_0) + \frac{\alpha}{ml^2}}$$

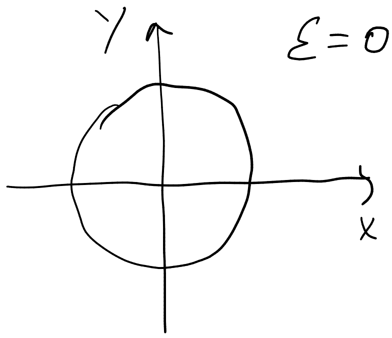
Eq. cônica

$$r(\phi) = \frac{B}{1 + \varepsilon \cos\phi}$$

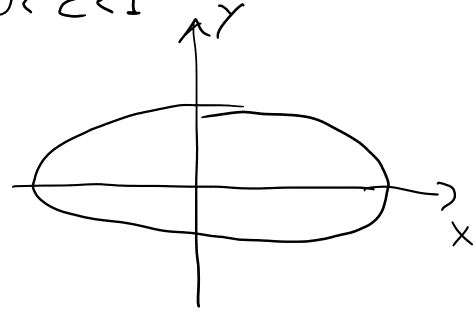
excentricidade

Tipos de órbitas possíveis:

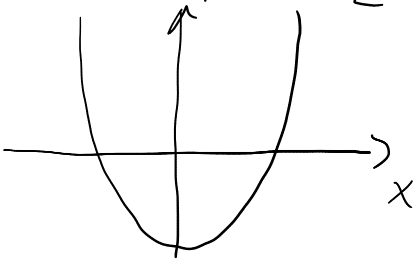
$\varepsilon = 0$



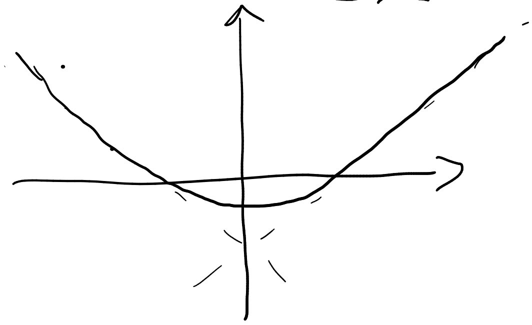
$0 < \varepsilon < 1$



$\varepsilon = 1$



$\varepsilon > 1$



(d) $\quad r(t) = r_0 \implies r(t) = r_0 + \delta(t) \quad , \quad \frac{\delta(t)}{r_0} \ll 1$

$$m\ddot{r} = -\frac{\alpha}{r^2} + \frac{ml^2}{r^3} \xRightarrow{r(t)=r_0} 0 = -\frac{\alpha}{r_0^2} + \frac{ml^2}{r_0^3}$$

$$\implies \frac{ml^2}{\alpha . r_0} = 1$$

$$\xRightarrow{r(t)=r_0+\delta(t)} m\ddot{\delta} = -\frac{\alpha}{(r_0+\delta)^2} + \frac{ml^2}{(r_0+\delta)^3}$$

$$= -\frac{\alpha}{r_0^2}\frac{1}{\left(1+\frac{\delta}{r_0}\right)^2} + \frac{ml^2}{r_0^3}\frac{1}{\left(1+\frac{\delta}{r_0}\right)^3}$$

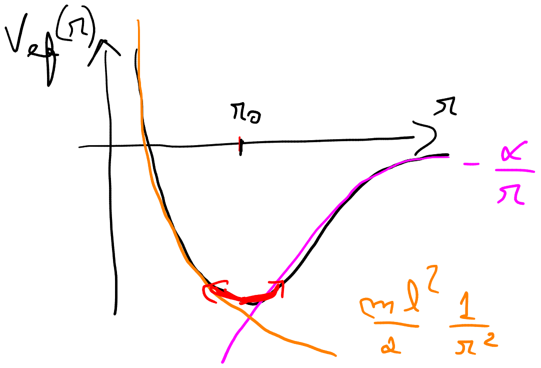
$$= -\frac{\alpha}{r_0^2}\left(1 - 2\frac{\delta}{r_0} + \dots\right) + \frac{ml^2}{r_0^3}\left(1 - 3\frac{\delta}{r_0} + \dots\right)$$

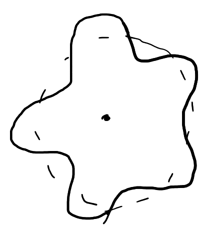
$$= \underbrace{\left(-\frac{\alpha}{r_0^2} + \frac{ml^2}{r_0^3}\right)}_{=0} + \underbrace{\left(\frac{2\alpha}{r_0^3} - \frac{3ml^2}{r_0^4}\right)}_{=\frac{\alpha}{r_0^3}\left(2 - 3\left(\frac{ml^2}{\alpha . r_0}\right)\right) = -\frac{\alpha}{r_0^3}} . \delta(t) + \dots$$

$$\frac{ml^2}{\alpha . r_0} = 1$$

$$\iff m\ddot{\delta}(t) = -\frac{\alpha}{r_0^3} . \delta(t) \iff \ddot{\delta}(t) + \frac{\alpha}{m r_0^3}\delta(t) = 0$$

$$\implies \delta(t) = \tilde{A} . \cos\left(\sqrt{\frac{\alpha}{m r_0^3}} . t + t_0\right)$$





Outro método $\Rightarrow$ expandir $V_{ef}(r)$ em torno de r_0:

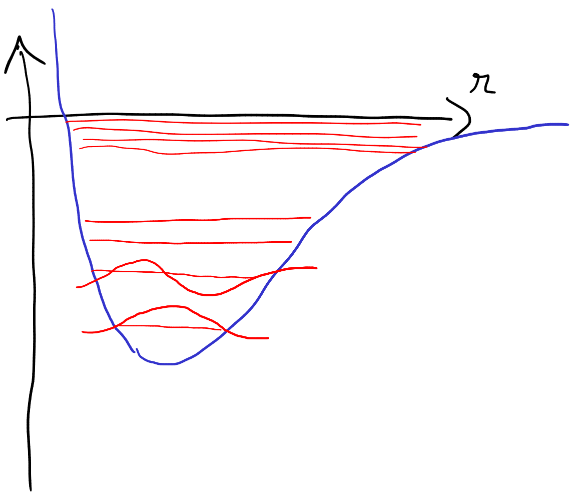
$r = r_0 + \delta$

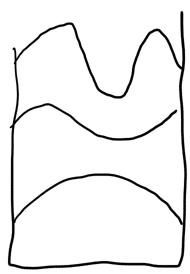
$$V_{ef}(r) = -\frac{\alpha}{r} + \frac{m l^2}{2r^2} = -\frac{\alpha}{r_0}\frac{1}{1+\frac{\delta}{r_0}} + \frac{m l^2}{2 r_0^2}\left(\frac{1}{1+\frac{\delta}{r_0}}\right)^2$$

$$= -\frac{\alpha}{r_0}\left(1 + \frac{\delta}{r_0} + \cdots\right) + \frac{m l^2}{2 r_0^2}\left(1 + 2\frac{\delta}{r_0} + \cdots\right)$$

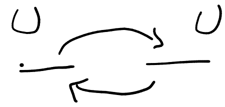
$$= -\frac{\alpha}{r_0} + \frac{m l^2}{2 r_0^2} - \cancel{\frac{\alpha}{r_0^2}\cdot\delta} + \underbrace{\cancel{\frac{m l^2}{r_0^3}}}_{\frac{\alpha r_0}{r_0^3}}\cdot\delta + \cdots = \mathcal{O}(\delta^2)$$

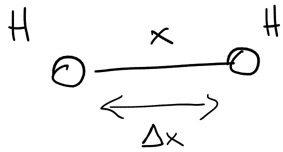
potencial harmônico

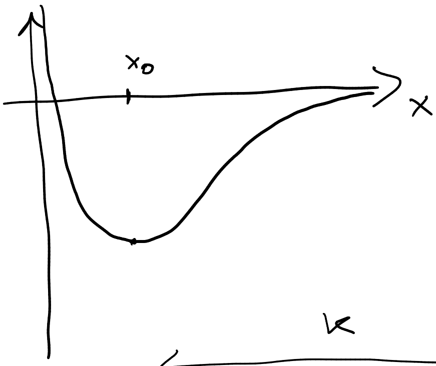




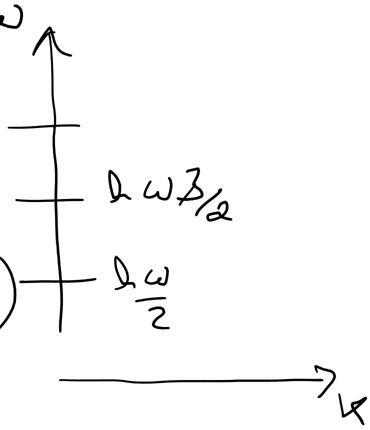
Molécula H_2^+

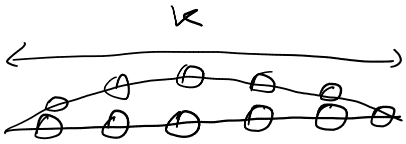


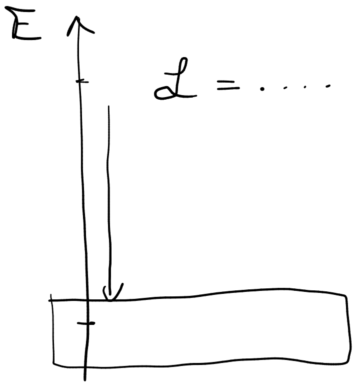




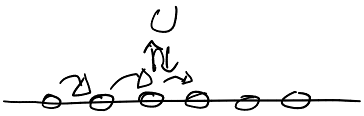
$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$







$\mathcal{L} = \dots$



$\tilde{\mathcal{L}} = \dots =$

$$\vec{r} = \sum_{i=1}^{3} c_i \cdot \vec{e}_i = c_x \vec{e}_x + c_y \vec{e}_y + c_z \vec{e}_z$$

$\psi(x) = \sum c_i \cdot \delta(x_i) =$

$=$