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CN - Resumo

Mínimos Quadrados

$$g(x) = a_1 g_1(x) + \dots + a_K g_K(x)$$

x	$g_1(x)$	$\dots$	$g_K(x)$	y
x_1	$g_1(x_1)$	$\dots$	$g_K(x_1)$	y_1
$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_n	$g_1(x_n)$	$\dots$	$g_K(x_n)$	y_n

	g_1	$\dots$	g_K		y
g_1	Σg_1^2	$\dots$	$\Sigma g_1 g_K$	$=$	$\Sigma g_1 y$
$\vdots$	$\vdots$		$\vdots$		$\vdots$
g_K	$\Sigma g_1 g_K$	$\dots$	Σg_K^2	$=$	$\Sigma g_K y$

Interpolação Polinomial

1) $$\begin{cases} a_0 + a_1 x_0 + a_2 x_0^2 + \dots + a_n x_0^n = y_0 \\ \vdots \\ a_0 + a_1 x_n + a_2 x_n^2 + \dots + a_n x_n^n = y_n \end{cases}$$

Exemplo: interpolação polinomial quadrática:

$$\begin{cases} a_0 + a_1 x_0 + a_2 x_0^2 = y_0 \\ a_0 + a_1 x_1 + a_2 x_1^2 = y_1 \\ a_0 + a_1 x_2 + a_2 x_2^2 = y_2 \end{cases} \rightarrow \begin{bmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

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2) Lagrange

$$P_n(x) = \sum_{k=0}^{n} y_k L_k(x), \text{ onde:}$$

$$L_k(x) = \frac{\prod_{j=0, j\neq k}^{n} (x - x_j)}{\prod_{j=0, j\neq k}^{n} (x_k - x_j)}$$

EXEMPLO: $P_3(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x)$

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \qquad j = \cancel{0}, 1, 2, 3$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \qquad j = 0, \cancel{1}, 2, 3$$

⋮

3) Newton

$1 \leq k \leq n$; $i = 0, 1, \dots, n-k$;

$$\nabla_i^k = \frac{\nabla_{i+1}^{k-1} - \nabla_i^{k-1}}{x_{i+k} - x_i} \; ; \; P_n(x) = \nabla_0^0 + \nabla_0^1(x-x_0) + \nabla_0^2(x-x_0)(x-x_1) + \dots + \nabla_0^n(x-x_0)(x-x_1)\dots(x-x_{n-1}).$$

— ERRO DE TRUNCAMENTO

$$|R_n(f;x)| \leq \frac{|x-x_0||x-x_1|\dots|x-x_n|}{(n+1)!} \max_{a \leq t \leq b} |f^{(n+1)}(t)|$$

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Integral.

1) Soma de Riemann

$$\sum_{i=1}^{n} f(c_i) \cdot \Delta x_i$$

- posicionamento de C_i = esquerda, centro, direita.

2) Trapézios (2 pontos)

$$A_T = (B+b)\frac{h}{2}$$

$$|E_{TRAP}| \leqslant \frac{(n-1) \cdot h^3}{12} \text{ MÁX } |f''(c)| \quad ; c \in [x_1, x_n]$$

$$\boxed{h = \frac{(b-a)}{(n-1)}}$$

nº de pontos

3) 1/3 de Simpson (3 pontos)

$$I = \frac{h}{3}(y_0 + 4y_1 + y_2)$$

4) 3/8 de Simpson (4 pontos)

$$I = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_3)$$

$$|E_{Simpson}| \leqslant \frac{(n-1) h^5}{180^{1/3}} \text{ MÁX } |f^{IV}(c)| \quad ; c \in [x_1 ; x_n]$$

no caso de 3/8 utilizar $\boxed{80}$

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