# Debugging $L_3$ term

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## 1 From Jorge

$$\frac{\delta}{\delta Q_{\alpha\beta}} Q_{lk}(\partial_l Q_{ij})(\partial_k Q_{ij}) = Q_{\alpha\beta}(\partial_\alpha Q_{ij})(\partial_\beta Q_{ij}) - 2\partial_k[Q_{kl}\partial_l Q_{\alpha\beta}] \tag{1}$$

## 2 From me

$$\begin{aligned}\frac{\delta}{\delta Q_{\alpha\beta}} Q_{lk}\left(\partial_l Q_{ij}\right)\left(\partial_k Q_{ij}\right) &= \frac{\partial}{\partial Q_{\alpha\beta}} Q_{lk}\left(\partial_l Q_{ij}\right)\left(\partial_k Q_{ij}\right) - \partial_\gamma \frac{\partial}{\partial\left(\partial_\gamma Q_{\alpha\beta}\right)} Q_{lk}\left(\partial_l Q_{ij}\right)\left(\partial_k Q_{ij}\right) \\ &= \delta_{\alpha l}\delta_{\beta k}\left(\partial_l Q_{ij}\right)\left(\partial_k Q_{ij}\right) \\ &\quad - \partial_\gamma\left[Q_{lk}\delta_{\gamma l}\delta_{\alpha i}\delta_{\beta j}\left(\partial_k Q_{ij}\right) + Q_{lk}\left(\partial_l Q_{ij}\right)\delta_{\gamma k}\delta_{\alpha i}\delta_{\beta j}\right] \\ &= \left(\partial_\alpha Q_{ij}\right)\left(\partial_\beta Q_{ij}\right) - \partial_\gamma\left[Q_{\gamma k}\left(\partial_k Q_{\alpha\beta}\right) + Q_{l\gamma}\left(\partial_l Q_{\alpha\beta}\right)\right] \\ &= \left(\partial_\alpha Q_{ij}\right)\left(\partial_\beta Q_{ij}\right) - 2\partial_\gamma\left(Q_{\gamma k}\left(\partial_k Q_{\alpha\beta}\right)\right)\end{aligned} \tag{2}$$

Altogether:

$$\frac{\delta}{\delta Q_{\alpha\beta}} Q_{lk}\left(\partial_l Q_{ij}\right)\left(\partial_k Q_{ij}\right) = \left(\partial_\alpha Q_{ij}\right)\left(\partial_\beta Q_{ij}\right) - 2\partial_\gamma\left(Q_{\gamma k}\left(\partial_k Q_{\alpha\beta}\right)\right) \tag{3}$$

## 3 Periodic configuration

Consider a configuration with $\mathbf{n} = (1, \epsilon \sin x, 0)$ for some small $\epsilon$. Then for $Q = S\left(\mathbf{n}\otimes\mathbf{n} - \frac{1}{3}I\right)$ we get:

$$Q = S\begin{bmatrix} \frac{2}{3} & \epsilon\sin x & 0 \\ \epsilon\sin x & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3}\end{bmatrix} \tag{4}$$

We want to calculate this for two different $Q$ terms:

$$\left(\partial_\alpha Q_{ij}\right)\left(\partial_\beta Q_{ij}\right) \tag{5}$$

$$\left(\partial_\alpha Q_{ij}\right)\left(\partial_j Q_{i\beta}\right) \tag{6}$$