Debugging time evolution

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1 Checking periodically perturbed free energy

To check that the configuration force matches with the energy, we check the energy for a uniform configuration first, and then for a small perturbation. The latter check gives an estimate for the configuration force by the functional Taylor series expansion:

$$F(Q + \delta Q) = F(Q) + \left.\frac{\delta F}{\delta Q}\right|_Q : \delta Q + \mathcal{O}\left(|\delta Q|^2\right) \tag{1}$$

with

$$\frac{\delta F}{\delta Q} = \frac{\partial F}{\partial Q} - \nabla \cdot \frac{\partial F}{\partial \left(\nabla Q\right)} \tag{2}$$

Here Q is given by:

$$Q = S_0 \begin{bmatrix} \frac23 & 0 & 0 \\ 0 & -\frac13 & 0 \\ 0 & 0 & -\frac13 \end{bmatrix} \tag{3}$$

with

$$\delta Q = S_0 \begin{bmatrix} 0 & \epsilon \sin kx & 0 \\ \epsilon \sin kx & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

This corresponds (to first order) to a perturbation of the director so that the total configuration $(Q + \delta Q)$ is a uniform-S configuration with director $\mathbf{n} = (1, \epsilon \sin kx, 0)$.

Here, the nondimensional free energy is given by:

$$F = \int_\Omega \left(-\tfrac12 \alpha Q : Q + \left(\log 4\pi - \log Z + \Lambda : \left(Q + \tfrac13 I\right)\right) + \tfrac12 \left|\nabla Q\right|^2 \right) dV \tag{5}$$