1 Elastomer terms

Start with Ω terms:

$$\begin{aligned}
\mathbf{\Omega} \times \mathbf{Q} &= \epsilon_{ijk}\Omega_j Q_{kl} \\
&= \epsilon_{ijk}\Omega_j \left(n_k n_l - \frac{1}{3}\delta_{kl} \right) \\
&= \epsilon_{ijk}\Omega_j n_k n_l - \frac{1}{3}\epsilon_{ijk}\Omega_j \delta_{kl} \\
&= (\mathbf{\Omega} \times \hat{\mathbf{n}}) \otimes \hat{\mathbf{n}} - \frac{1}{3}\Omega \times \mathbf{I}
\end{aligned} \tag{1}$$

Then we get:

$$\begin{aligned}
\boldsymbol{\varepsilon} : (\mathbf{\Omega} \times \mathbf{Q}) &= \varepsilon_{il}\epsilon_{ijk}\Omega_j n_k n_l - \frac{1}{3}\epsilon_{ijk}\Omega_j \delta_{kl} \\
&= \varepsilon_{il}\epsilon_{ijk}\Omega_j n_k n_l - \varepsilon_{il}\frac{1}{3}\epsilon_{ijk}\Omega_j \delta_{kl} \\
&= n_l \varepsilon_{li}\epsilon_{ijk}\Omega_j n_k - \varepsilon_{ik}\frac{1}{3}\epsilon_{ijk}\Omega_j \\
&= \hat{\mathbf{n}} \cdot \boldsymbol{\varepsilon} \cdot (\mathbf{\Omega} \times \hat{\mathbf{n}})
\end{aligned} \tag{2}$$

Additionally, we may write:

$$\begin{aligned}
\mathbf{\Omega} \times \mathbf{Q} \times \mathbf{\Omega} &= \epsilon_{mln}\epsilon_{ijk}\Omega_j n_k n_l \Omega_n - \frac{1}{3}\epsilon_{mln}\epsilon_{ijk}\Omega_j \delta_{kl}\Omega_n \\
&= \epsilon_{mln}\epsilon_{ijk}\Omega_j n_k n_l \Omega_n - \frac{1}{3}\epsilon_{nmk}\epsilon_{ijk}\Omega_j \Omega_n \\
&= \epsilon_{mln}\epsilon_{ijk}\Omega_j n_k n_l \Omega_n - \frac{1}{3}\left(\delta_{ni}\delta_{mj} - \delta_{nj}\delta_{mi}\right)\Omega_j \Omega_n \\
&= \epsilon_{mln}\epsilon_{ijk}\Omega_j n_k n_l \Omega_n - \frac{1}{3}\left(\Omega_m \Omega_i - \delta_{mi}\Omega_n \Omega_n\right) \\
&= (\mathbf{\Omega} \times \hat{\mathbf{n}}) \otimes (\mathbf{\Omega} \times \hat{\mathbf{n}}) + \frac{1}{3}\left(|\mathbf{\Omega}|^2 \mathbf{I} - \mathbf{\Omega} \otimes \mathbf{\Omega} \right)
\end{aligned} \tag{3}$$

And then taking the trace:

$$\mathrm{Tr}\left[\mathbf{\Omega} \times \mathbf{Q} \times \mathbf{\Omega}\right] = |\mathbf{\Omega} \times \hat{\mathbf{n}}|^2 + \frac{2}{3}|\mathbf{\Omega}|^2 \tag{4}$$