Analytical Inverse kinematics Algorithm Of A 5-DOF Robot Arm

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The kinematics problem is defined as the transformation from the Cartesian space to the joint space and vice versa. Both forward and inverse kinematics solutions for the TR 4000 educational robot arm are presented. The closed form solution of the inverse kinematics problem is of utmost importance in controlling robotic manipulator. In this paper, a closed form solution to the inverse kinematics of a 5-DOF robot is presented to overcome the relatively slow iterative numerical solution. A software program developed to compute the forward and inverse kinematics of TR4000 robot arm.

Inverse velocity problem for five axis robots is investigated. The conventional method for a five axis robot is to pseudo inverse the 6x5 Jacobian matrix. Pseudo inverse of the Jacobian matrix provides a possibility to solve for approximate solutions. The spherical angles solution is provided to derive a 5x5 Jacobian matrix.

Keywords: forward kinematics, inverse kinematics, inverse velocity, Jacobian matrix.

1- Introduction:

The motion of a rigid body in three-dimensional space (3-D) has six degrees of freedom. For incompletely specified spatial motion, less than six freedoms are required. A robot with four actuated axes can guide a line in 3-D space and a five-axis machine can guide a line segment for a specified pose. Industrial applications of these types are wire cutting, grinding and polishing, arc welding, spray painting, pin insertion, etc. Many industrial robots designed for such jobs have only four or five axes.

The kinematics solution of any robot manipulator consists of two sub problems forward and inverse kinematics. Forward kinematics will determine where the robot's manipulator hand will be if all joints are known whereas inverse kinematics will calculate what each joint variable must be if the desired position and orientation of end-effector is determined. Hence Forward kinematics is defined as transformation from joint space to Cartesian space whereas Inverse kinematics is defined as transformation from Cartesian space to joint space.[1]

In many line guidance applications, velocity control of a robot is an important issue. Speed of the robot tool governs the quality of the job. For example, cutting speed of a wire or a milling machine affects the surface finish condition as well as the tool life. The travel speed of a spray-painting robot governs the uniformity of the film thickness. Velocity of a weld gun should be maintained constant for equal heat penetration along the weld seam. For these purposes, joint rates of a robot are commonly computed from a required tool velocity through the inversion of the Jacobian matrix. The Jacobian is a 6x5 matrix for a five-axis robot. It has no inverse solution because it is not a square matrix. Usually, get a least square solution from the Jacobian matrix by pseudo-inverse method. Pseudo-inverse of the Jacobian matrix provides a possibility to solve for approximate solutions. There is no exact velocity solution for five-axis robot.[2]

Pieper and Roth [3] showed that any 6DOF manipulator hose wrist axes intersect at a common point has a closed form inverse solution to system, owing to decoupling of the inverse position and inverse orientation. Paul and Mayer [4] presented the basis for all advanced manipulator control which is relationship between the Cartesian coordinates of the end-effector and manipulator joint coordinate. Dieh, Z. [5] proposed a closed form solution algorithm for the solving the inverse position and orientation problem for the robot by imposing a castrating condition to the problem and projection the tool frame on subspace of the robot. Sugimoto and Duffy [6] added two virtual axes to a five-axis robot. The robot then became a seven-axis closed loop mechanism so that it can be solved. Tsai and Morgan [7] used continuation and dual number methods respectively, to solve the position of five-axis robots. Hunt [8] applied a constraint to the tool velocity and added a dummy axis to the five-axis robot so that the matrix has inverse.

In this paper, the mathematical model and kinematical analysis of the TR 4000 educational robot manipulator is studied. So providing method to derived 5x5 Jacobian matrix based on a five parameter to give the exact velocity and acceleration solution.

2-Kinematics:

TR 4000 robot arm has five directions of motion (DOF) plus a grip movement. It is also similar to human arm from the number of joints point of view. These joints provide shoulder rotation, shoulder back and forth motion, elbow motion, wrist up and down motion, wrist rotation and gripper motion. A graphical view of all the joints was displayed in Fig.-1. TR 4000 robot has five rotational joints and a moving grip. Joint 1 represents the base and its axis of motion is Z_0. This joint provides a rotational θ_1 angular motion around Z_0 axis in X_0Y_0 plane. Joint 2 is identified as the shoulder and its axis is perpendicular to Joint 1 axis. It provides an angular motion θ_2 in X_1Y_1 plane. Z axes of Joint 3 (Elbow) and Joint 4 (Wrist) are parallel to Z_1 axis, They provide θ_3 and θ_4 angular motions in X_2Y_2 and X_3Y_3 planes respectively. Joint 5 is identified as the grip. Its Z_4 axis is vertical to Z_3 axis and it provides θ_5 angular motion in X_4Y_4 plane. Denavit-Hartenberg (D-H), representation is used to model the joints of TR 4000 Robot arm in Fig.-1. All the joints are assigned by using the principles of D-H convention. The D-H parameters are routinely noted d_i (motional distance along Z_0 axis), a_i (distance between joints along X axis), θ_i (angle around the Z axis on XY plane), α_i (angle between 2 adjacent Z axes) . These parameters describe the location of a robot link-frame *i* (a joint) from a preceding link-frame *i-1* (previous joint) through the sequence of translations and rotations.(Table-1) .

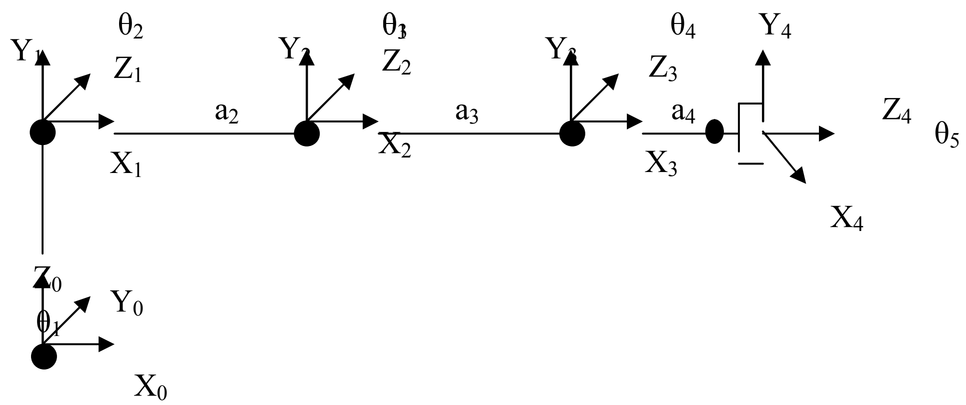

Fig.-1: Modeling of TR 4000 robotic arm

Joint	θ_i	α_i	a_i	d_i
1	θ_1	90	0	0
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0
4	θ_4	-90	a_4	0
5	θ_5	90	0	0
6	θ_6	0	0	0

Table-1:D-H parameters for TR 4000 robot arm.

Now we will consider five-degree-of-freedom robot manipulators which lose one degree of freedom in the wrist configuration, so it cannot reach the any place in space arbitrary. Simply a five-degree-of-freedom robot manipulator can be imagined as a six degree-of-freedom robot manipulator in which one joint variable is fixed as zero or some suitable value. One degree of freedom lost may appear in arm part or in wrist part of the robot manipulator. The solution of inverse kinematics, in principle, can be obtained by application of algorithm existing for six-degree-of-freedom robot manipulator. The only necessary additional step is to determine which joint can represent the one degree of freedom lost and it is not so difficult task to do so.

In most practical case, such as parts handling and assembling and spray painting. one degree of freedom in orientation may be unnecessary and so one joint of wrist construction can be omitted. We can treat five-degree-of-freedom robot manipulators but we only need to fix this joint variable of wrist as zero or some suitable data.

The homogenous transformation matrix , ${}^{i-1}T_i$, which describes the position and orientation frame i relative to frame i-1 can be formed by using:

$$
{}^{i-1}T_i(q) = \begin{bmatrix} C_{\theta i} & -S_{\theta i}C_{\alpha i} & S_{\theta i}S_{\alpha i} & a_i C_{\theta i} \\ S_{\theta i} & C_{\theta i}C_{\alpha i} & -C_{\theta i}S_{\alpha i} & a_i S_{\theta i} \\ 0 & S_{\alpha i} & C_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

...(1)

Substituting the parameters in Table-1 to obtain on transformation matrices A_1 to A_6:

$$
{}^0T_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

...(2)

$$
{}^1T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

...(3)

$$
{}^2T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

...(4)

$$
{}^3T_4 = \begin{bmatrix} C_4 & 0 & -S_4 & a_4C_4 \\ S_4 & 0 & C_4 & a_4S_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

...(5)

$$ {}^{4}T_{5} = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$
..(6)

$$ {}^{5}T_{6} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$
..(7)

<u>A: Forward kinematics</u>

Calculating the position and orientation of the end effectors with given joint angles is called Forward Kinematics analysis. Forward Kinematics equations are generated from the transformation matrixes and the forward kinematics solution of the arm is the product of these six matrices identified as ${}^{0}T_{6}$ (with respect to base). The first three columns in the matrices represent the orientation of the end effectors, whereas the last column represents the position of the end effectors.

$$ {}^{0}T_{6} = {}^{0}T_{1} * {}^{1}T_{2} * {}^{2}T_{3} * {}^{3}T_{4} * {}^{4}T_{5} * {}^{5}T_{6} $$

$$ = \begin{bmatrix} {}^{0}R_{6} & {}^{0}P_{6} \\ 0_3^T & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The orientation and position of the end effectors can be calculated in terms of joint angles shown in (8) to (19).

$n_x = C_1C_{234} C_5 - S_1S_5$
…(8)

$n_y = S_1C_{234} C_5 + C_1 S_5$
…(9)

$n_z = S_{234}C_5$
...(10)

$o_x = - C_1S_{234}$
...(11)

$o_y = - S_1 S_{234}$
..(12)

$o_z = C_{234}$
...(13)

$a_x = C_1C_{234} S_5 + S_1 C_5$
...(14)

$a_y = S_1C_{234} S_5 - C_1 C_5$ …(15)

$a_z= S_{234} S_5$ …(16)

$P_x = C_1(C_{23}a_3 + C_2a_2+C_{234} a_4)$ …(17)

$P_y= S_1(C_{23}a_3 + C_2a_2+C_{234} a_4)$ ...(18)

$P_z =S_{23}a_3 + S_2a_2+S_{234} a_4$ …(19)

B: Inverse Kinematics:

The inverse position and orientation problem demands the transformation from task space, ${}^0X_T$, to joint space coordinates, θ,

$\theta = f^{-1}({}^0X_T)$ ..(20)

In general, the inverse kinematics problem can be solved either by an algebraic, an iterative, or geometric approach. The closed form solution is very difficult and suffers from the fact that the solution does not give a clear indication on how to select the correct solution from several possible solutions for a particular arm configuration. The iteration solution of ten equations requires more computation, and it does not guarantee convergence to the correct solution, especially in the singular and degenerate cases. The geometric approach presents a better approach when the manipulator is simple but it is not suitable for computer served robots. The closed form solutions are preferable for two reasons. The first, the forward kinematics equations must be solved at a rapid rate and the second, kinematics equations in general have multiple solutions.

Inverse Kinematics analysis determines the joint angles for desired position and orientation in Cartesian space. This is more difficult problem than forward kinematics. The Transformation matrix ${}^0T_6$ will be used to calculate inverse kinematics equations. To determine the joint angles, ${}^0T_6$ matrix equation is multiplied by ${}^0T_1^{-1}$ on both sides sequentially.

$${}^0T_1^{-1} * {}^0T_6 = {}^0T_1^{-1} * {}^1T_2 * {}^2T_3 * {}^3T_4 * {}^4T_5 * {}^5T_6 \quad ...(21)$$

$$\begin{bmatrix} n_xC_1+n_yS_1 & o_xC_1+o_yS_1 & a_xC_1+a_yS_1 & P_xC_1+P_yS_1 \\ n_z & o_z & a_z & P_z \\ n_xS_1-n_yC_1 & o_xS_1-o_yC_1 & a_xC_1+a_yS_1 & P_xS_1+P_yC_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

…(22)

$$\begin{bmatrix} C_{234}C_5 & -S_{234} & C_{234}S_5 & a_4C_{234}+a_3C_{23}+a_2C_2 \\ S_{234}C_5 & C_{234} & S_{234}S_5 & a_4S_{234}+a_3S_{23}+a_2S_2 \\ -S_5 & 0 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Both matrix elements are equated to each other and the resultant θ_i values are extracted. Equating (3,4) elements of both matrices gave in Eq.(22):

$$P_xS_1 - P_yC_1 = 0$$

...(23)

$$\theta_1 = a\tan 2(P_y, P_x) \quad or\ \theta_1 = \theta_1 + 180$$

...(24)

Equating (1,4) and (2,4) elements of both matrices at Eq.(22) gives:

$$P_xC_1 + P_yS_1 = a_4C_{234} + a_3C_{23} + a_2C_2$$

...(25)

$$P_z = a_4S_{234} + a_3S_{23} + a_2S_2$$

...(26)

Rearranging and squaring Eq.(25) and Eq.(26) and adding them

$$(P_xC_1 + P_yS_1 - a_4C_{234})^2 + (P_z - a_4S_{234})^2 = a_3^2 + a_2^2 + 2*a_2*a_3 \qquad ...(27)$$

Hence,

$$C_3 = ((P_xC_1 + P_yS_1 - a_4C_{234})^2 + (P_z - a_4S_{234})^2 - a_3^2 - a_2^2)/2*a_2*a_3 \qquad ...(28)$$

$$\theta_3 = A\tan 2(S_3, C_3)$$

...(29)

Referring Eqs.(25) and (26)

$$S_2 = (C_3a_3 + a_2)(P_z - S_{234}a_4) - S_3a_3(P_xC_1 + P_yS_1 - C_{234}a_4)/(C_3a_3 + a_2)^2 + S_3^2a_3^2 \qquad ...(30)$$

$$C_2 = (C_3a_3 + a_2)(P_xC_1 + P_yS_1 - C_{234}a_4) + S_3a_3(P_z - S_{234}a_4)/(C_3a_3 + a_2)^2 + S_3^2a_3^2 \qquad ...(31)$$

$$\theta_2 = A\tan 2(S_2, C_2) \qquad ..(32)$$

Assume θ_{234}=k, then:

$$\theta_4 = \theta_{234} - \theta_3 - \theta_2$$

..(33)

Equating (3,1) and (3,2) in Eq(22),

$$S_5 = C_1n_y - S_1n_x$$

..(34)

$$C_5 = C_1a_x - S_1a_y$$

..(35)

$$\theta_5 = A\tan 2(S_5, C_5)$$

..(36)

An analysis introduced to reduce the multiple solutions in inverse kinematics part. The values of a_i^* are specified. A set of θ_i are chosen {45,45,30,-45,30} to calculate the position and orientation of the end effector $^0T_6$. A computer program do by MATLAB program used graphical user interface (GUI) to calculate both forward and inverse kinematics solution of the robot arm.

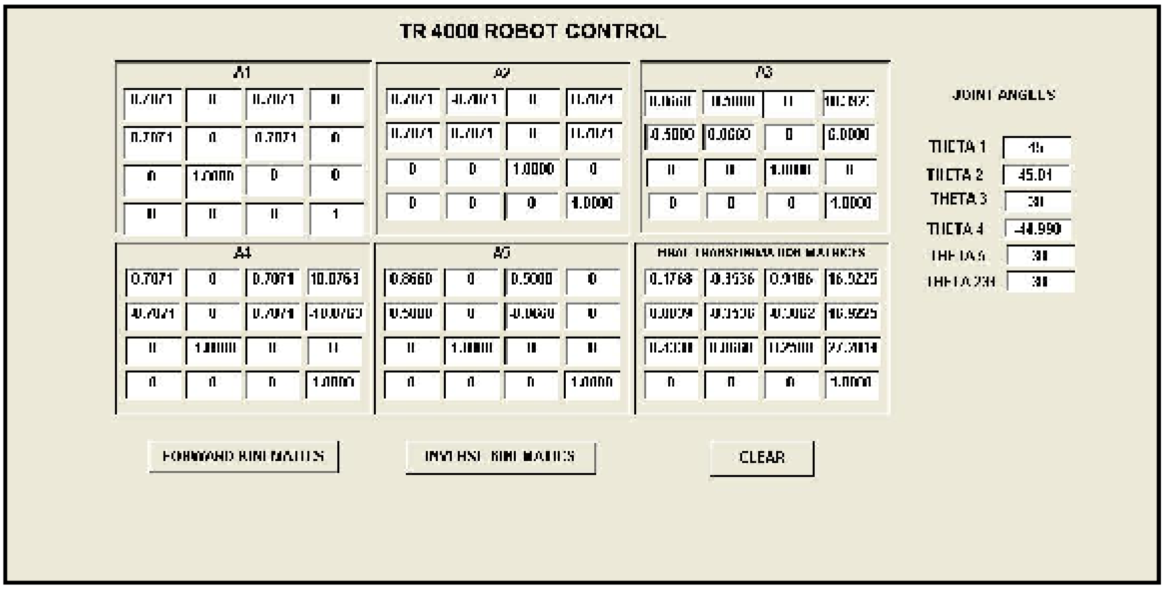

Fig-2: GUI for both forward and inverse kinematics of TR 4000 Robot

* a2=a3=12;a4=14.25ccm.

C- Inverse Velocity and Acceleration:

The forward kinematics model of a robot manipulator gives the velocity and acceleration of the end effector $\dot{X}$ *and* $\ddot{X}$ in terms of the joint velocities and acceleration $\dot{\theta}$ *and* $\ddot{\theta}$. It is written as:

$$\dot{X} = J(\theta)\,\dot{\theta} \quad ..(37)$$

$$\ddot{X} = J(\theta)\ddot{\theta} + \dot{J}(\theta)\dot{\theta} \quad ..(38)$$

Where J(θ) denotes to the (m x n) Jacobian matrix and $\dot{J}(\theta)$ denotes to the time derivative of the Jacobian matrix. We can solve the joint velocities $\dot{\theta}_i$ and joint acceleration $\ddot{\theta}_i$ by:

$$\dot{\theta} = J^{-1}(\theta)\,\dot{X} \quad ..(39)$$

$$\ddot{\theta} = J^{-1}(\theta)(\ddot{X} - \dot{J}(\theta)\,\dot{\theta}) \quad ..(40)$$

Where $J^{-1}(\theta)$ denotes to the inverse kinematics. In general, the (6x5) Jacobian matrix is non-square. In which case the inverse is undefined. In this

work, we used spherical angle representation line to derive a (5x5) Jacobian matrix. The location of a tool is commonly assigned by six parameters. These are the position (Px, Py, Pz) of the tool and the direction (Ix, Iy, Iz) of the tool. However, a line segment can be assigned with only five parameters. The tool orientation can then be represented by two spherical angles. They are the azimuth angle ψ and the latitudinal angle Φ. Then the line can be represented by five parameters (Px, Py, Pz, ψ , Φ), as shown in Fig. -3.
The angles ψ and Φ should be able to cover all the directions in the 3-D space. The relationship between the spherical angles and the direction cosines are given by:

$$\Psi = \tan^{-1}\left(\frac{I_y}{I_x}\right) + \frac{\pi}{2} - \frac{\pi}{2} sign(I_x)$$
…(41)

$$\Phi = \tan^{-1}\left(\frac{I_z}{\sqrt{I_x^2 + I_y^2}}\right)$$
…(42)

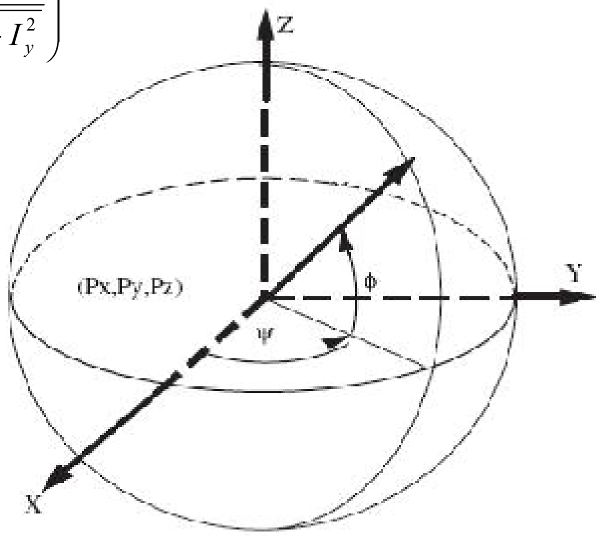

Fig-3: Line representation in a spherical angle

If the tool is to be guided by a five-axis robot, these five parameters are functions of the joint variables (θ_1, θ_2, θ_3 θ_4, θ_5). Let
$h = [P_x \ P_y \ P_z \ \Psi \ \Phi]^T$.
…(43)
Then a 5_5 Jacobian matrix can be derived from

$$\frac{dh}{dt}=\left[\frac{\partial h}{\partial\theta_1}\ \frac{\partial h}{\partial\theta_2}\ \frac{\partial h}{\partial\theta_3}\ \frac{\partial h}{\partial\theta_4}\ \frac{\partial h}{\partial\theta_5}\right]\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\\ \dot{\theta}_5\end{bmatrix}=J\,\dot{\theta}_i \qquad \text{…(44)}$$

$$[J]=\begin{bmatrix}\frac{\partial P_x}{\partial\theta_1} & \frac{\partial P_x}{\partial\theta_2} & \frac{\partial P_x}{\partial\theta_3} & \frac{\partial P_x}{\partial\theta_4} & \frac{\partial P_x}{\partial\theta_5}\\ \frac{\partial P_y}{\partial\theta_1} & \frac{\partial P_y}{\partial\theta_2} & \frac{\partial P_y}{\partial\theta_3} & \frac{\partial P_y}{\partial\theta_4} & \frac{\partial P_y}{\partial\theta_5}\\ \frac{\partial P_z}{\partial\theta_1} & \frac{\partial P_z}{\partial\theta_2} & \frac{\partial P_z}{\partial\theta_3} & \frac{\partial P_z}{\partial\theta_4} & \frac{\partial P_z}{\partial\theta_5}\\ \frac{\partial\Psi}{\partial\theta_1} & \frac{\partial\Psi}{\partial\theta_2} & \frac{\partial\Psi}{\partial\theta_3} & \frac{\partial\Psi}{\partial\theta_4} & \frac{\partial\Psi}{\partial\theta_5}\\ \frac{\partial\Phi}{\partial\theta_1} & \frac{\partial\Phi}{\partial\theta_2} & \frac{\partial\Phi}{\partial\theta_3} & \frac{\partial\Phi}{\partial\theta_4} & \frac{\partial\Phi}{\partial\theta_5}\end{bmatrix}$$

…(45)

The Jacobian matrix is a transformation matrix from the joint space to the spatial line location space. If the i-th axis of the robot is a prismatic joint, then the i-th column in Eq. (44), θ_i should be replaced by the joint off-set, d_i, and the corresponding joint rate, ω_i (in angular term), should be replaced by τ_i (in translational term).

From homogenous transformation of the end-effector ($^0T_6$), the position and orientation of the end effector are:

$$P=\begin{bmatrix}P_x\\ P_y\\ P_z\end{bmatrix}=\begin{bmatrix}C_1(a_3C_{23}+a_2C_2-d_6C_{234}S_5)-d_6S_1C_5\\ S_1(a_3C_{23}+a_2C_2-d_6C_{234}S_5)-d_6C_1C_5\\ a_3S_{23}+a_2S_2-d_6S_{234}S_5\end{bmatrix} \qquad \text{…(46)}$$

And

$$I=\begin{bmatrix}I_x\\ I_y\\ I_z\end{bmatrix}=\begin{bmatrix}R_{13}\\ R_{23}\\ R_{33}\end{bmatrix}=\begin{bmatrix}C_1C_{234}S_5+S_1C_5\\ S_1C_{234}S_5-C_1C_5\\ S_{234}S_5\end{bmatrix}$$

...(47)

The spherical angles are:

$$\Psi=\tan^{-1}\left(\frac{S_1C_{234}S_5-C_1C_5}{C_1C_{234}S_5+S_1C_5}\right)$$

...(48)

$$\Phi = \tan^{-1}\left(\frac{S_{234}S_5}{\sqrt{S_{234}^2 S_5 + C_5^2}}\right)$$
…(49)

The partial derivatives of Ψ and Φ will make a very complex Jacobian expression. We may temporarily set the reference frame at joint 1. Let P_1 and I_1 are the tool position and orientation with respect to the frame 1. From Eq.(22), we obtain:

$$P_1 = \begin{bmatrix} P_{x1} \\ P_{y1} \\ P_{z1} \end{bmatrix} = \begin{bmatrix} a_4C_{234} + a_3C_{23} + a_2C_2 \\ a_4S_{234} + a_3S_{23} + a_2S_2 \\ a_4S_{234} + a_3S_{23} + a_2S_2 \end{bmatrix}$$
…(50)

And

$$I = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} C_{234}S_5 \\ S_{234}S_5 \\ -C_5 \end{bmatrix}$$
...(51)

The spherical angles are:

$$\Psi_1 = \tan^{-1}\left(\frac{S_{234}S_5}{C_{234}S_5}\right)$$
..(52)

$$\Phi_1 = \tan^{-1}\left(\frac{-C_5}{S_5}\right)$$
..(53)

The given velocity of the tool is assigned in the fixed reference frame. It should be transferred to the frame 1 also. The contribution of joint 1 to the tool velocity should be deleted from the original tool velocity.

$$\frac{dh_1}{dt} = \begin{bmatrix} \frac{\partial h_1}{\partial \theta_2} & \frac{\partial h_1}{\partial \theta_3} & \frac{\partial h_1}{\partial \theta_4} & \frac{\partial h_1}{\partial \theta_5} \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix}$$
…(54)

$$= \begin{bmatrix} \dot{P}_x' \\ \dot{P}_y' \\ \dot{P}_z' \\ \dot{\Psi}' \\ \dot{\Phi} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -\dot{\theta}_1 P_{x1} \\ 0 \\ -\dot{\theta}_1 C_{234} \end{bmatrix}$$
…(55)

Where the $\begin{bmatrix} \dot{P}_x' & \dot{P}_y' & \dot{P}_z' & \dot{\Psi}' & \dot{\Phi}' \end{bmatrix}^T$ is the tool velocity transferred to the frame 1.
Rearranging Eq.(55)

$$\begin{bmatrix} \dot{P}_x' \\ \dot{P}_y' \\ \dot{P}_z' \\ \dot{\Psi}' \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial P_{x1}}{\partial \theta_2} & \frac{\partial P_{x1}}{\partial \theta_3} & \frac{\partial P_{x1}}{\partial \theta_4} & \frac{\partial P_{x1}}{\partial \theta_5} \\ 0 & \frac{\partial P_{y1}}{\partial \theta_2} & \frac{\partial P_{y1}}{\partial \theta_3} & \frac{\partial P_{y1}}{\partial \theta_4} & \frac{\partial P_{y1}}{\partial \theta_5} \\ -P_{x1} & \frac{\partial P_{x1}}{\partial \theta_2} & \frac{\partial P_{x1}}{\partial \theta_3} & \frac{\partial P_{x1}}{\partial \theta_4} & \frac{\partial P_{x1}}{\partial \theta_5} \\ 0 & \frac{\partial \Psi_1}{\partial \theta_2} & \frac{\partial \Psi_1}{\partial \theta_3} & \frac{\partial \Psi_1}{\partial \theta_4} & \frac{\partial \Psi_1}{\partial \theta_5} \\ -C_{234} & \frac{\partial \Phi_1}{\partial \theta_2} & \frac{\partial \Phi_1}{\partial \theta_3} & \frac{\partial \Phi_1}{\partial \theta_4} & \frac{\partial \Phi_1}{\partial \theta_5} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix} = J\dot{\theta} \qquad \ldots(56)$$

$$J = \begin{bmatrix} 0 & -(a_4S_{234}+a_3S_{23}+a_2S_2) & -(a_4S_{234}+a_3S_{23}) & -a_4S_{234} & 0 \\ 0 & (a_4C_{234}+a_3C_{23}+a_2C_2) & (a_4C_{234}+a_3C_{23}) & a_4C_{234} & 0 \\ -(a_4C_{234}+a_3C_{23}+a_2C_2) & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ -C_{234} & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(57)$$

This is the 5X5 Jacobian of the robot with respect to the frame 1, and it is no difficulty to inverse the Jacobian matrix analytically.

The joint velocities are given by:

$$\dot{\theta}_1 = \dot{P}_{z1} / P_{x1} \qquad \ldots(58)$$

$$\dot{\theta}_2 = (a_2C_{23}\dot{P}_{x1} + a_3S_{23}\dot{P}_{y1} - a_3a_4S_4\dot{\Psi}_1)/(a_3a_4S_4 - a_3a_2S_3) \qquad \ldots(59)$$

$$\dot{\theta}_3 = (-a_2C_{23}\dot{P}_{x1} - a_2C_2\dot{P}_{x1} + a_3S_{23}\dot{P}_{y1} + a_2S_2\dot{P}_{y1} - S_{34}\dot{\Psi}_1)/(a_3a_4S_4 - a_3a_2S_3) \qquad \ldots(60)$$

$$\dot{\theta}_4 = (a_2C_2\dot{P}_{x1} + a_2S_2\dot{P}_{y1} - S_{34}\dot{\Psi}_1)/(a_3a_4S_4 - a_3a_2S_3) \qquad \ldots(61)$$

$$\dot{\theta}_5 = -C_{34}\,\dot{P}_{z1} / P_{x1} + \dot{\Phi}_1 \qquad \ldots(62)$$

And the joint accelerations are given by:

$$\ddot{\theta}_1 = (\ddot{P}_{z1} - P_{y1}\dot{\theta}_1) / P_{x1} \qquad \ldots(63)$$

$$\ddot{\theta}_2 = (a_2C_{23}A + a_2S_{23}B - a_3a_4S_4\ddot{\Psi}_1)/(a_3a_4S_4 - a_3a_2S_3) \qquad \ldots(64)$$

$$\ddot{\theta}_3 = (-(a_3C_{23} + a_2C_2)A + (a_3S_{23} + a_2S_2)B - S_{34}\ddot{\Psi}_1)/(a_3a_4S_4 - a_3a_2S_3) \qquad \ldots(65)$$

$$\ddot{\theta}_4 = (a_2C_2A + a_2S_2B - S_{34}\ddot{\Psi}_1)/(a_3a_4S_4 - a_3a_2S_3) \qquad \ldots(66)$$

$$\ddot{\theta}_5 = -C_{234}\,D / P_{x1} + \ddot{\Phi}_1 - S_{234}\dot{\theta}_1 \qquad \ldots(67)$$

Where

$$A = \ddot{P}_{x1} + P_{x1}\dot{\theta}_2 + (a_4 C_{234} + a_3 C_{23})\dot{\theta}_3 + a_4 C_{234}\dot{\theta}_4$$
$$B = \ddot{P}_{y1} + P_{y1}\dot{\theta}_2 + (a_4 S_{234} + a_3 S_{23})\dot{\theta}_3 + a_4 S_{234}\dot{\theta}_4$$
$$D = \ddot{P}_{z1} - P_{y1}\dot{\theta}_1$$

3-Conclusions:

Finding closed form solution to the inverse kinematics problem is desirable since because it is required less computational time and provides accurate multiple solutions. In this paper, a closed form solution to the inverse kinematics of a 5-DOF robot is proposed. A software program was also developed to show the robot motion with respect to its mathematical analysis. Almost used pseudo inverse method to derive the 6x5 Jacobian matrix. The solution based on six freedoms inverse velocity analysis is just an approximation with a least-square error. For a five axis robot, the Jacobian is not a square matrix, and it has no inverse. In this paper, presents method to compute the joint velocity and acceleration of fives axis robot. This method based on the spherical angle of the tool to derive 5x5 Jacobian matrix for which the exact solutions are obtainable.

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