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KINEMATIC MODELLING AND ANALYSIS OF 5 DOF ROBOTIC ARM

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ABSTRACT

The control of a robotic arm has been a challenge since earlier days of robots. The kinematics problem is defined as the transformation from the Cartesian space to the joint space and vice versa. This paper aims to model the forward and inverse kinematics of a 5 DOF Robotic Arm for simple pick and place application. A general D-H representation of forward and inverse matrix is obtained. An analytical solution for the forward and inverse kinematics of 5 DOF robotic arm presented, to analyze the movement of arm from one point in space to another point. The 5 DOF robotic arm is a vertical articulated robot, with five revolute joints. It is a dependable and safe robotic system designed for educational purpose. This versatile system allows students to gain theoretical and practical experience in robotics, automation and control systems.

KEYWORDS: Forward Kinematics, Inverse Kinematics, Robotic Arm

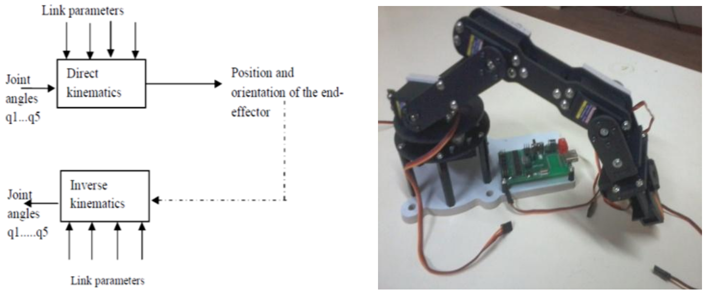
INTRODUCTION

The mathematical modeling of robot kinematics is motivated by the complexity of robotic systems, which possess highly nonlinear characteristics. Inverse kinematics modeling has been one of the main problems in robotics research. The most popular method for controlling robotic arms is still based on look-up tables that are usually designed in a manual manner [1-4].

The kinematics solution of any robot manipulator consists of two sub problems forward and inverse kinematics. Forward kinematics will determine where the robot's manipulator hand will be if all joints are known whereas inverse kinematics will calculate what each joint variable must be if the desired position and orientation of end-effector is determined. Hence Forward kinematics is defined as transformation from joint space to Cartesian space whereas Inverse kinematics is defined as transformation from Cartesian space to joint space. General methods do exist for solving forward kinematics [5-8]. The objective in this paper is to present an analytical solution for the forward and inverse kinematics of 5 DOF robotic arm, to analyze the movement of arm from one point in space to another point.

KINEMATIC MODEL OF 5 DOF ROBOTIC ARM

For the research work, 5 DOF Robotic Arm was selected. It is a vertical articulated robot, with five revolute joints. It has a stationary base, shoulder, elbow, tool pitch and tool roll. This simple block diagram indicates the relationship between direct and inverse kinematics problem as shown in Figure 1a. Figure 1b indicates robot arm links. The coordinate frame assignment is shown in Figure 2.



(a) (b)

Figure 1(a): Forward and Inverse Kinematics Model; (b) Robot Arm Links

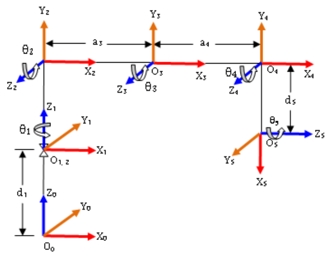


Figure 2: Coordinate Frame Assignment

FORWARD AND INVERSE KINEMATIC ANALYSIS OF 5 DOF ROBOTIC ARM

Forward Kinematic Analysis

In this study, the standard Denavit-Hartenberg (DH) [9] convention and methodology are used to derive its kinematics. Denavit-Hartenberg algorithm helps to find the position and orientation of end-effector with respect to base. Totally 20 Parameters are involved in 5- DOF robotic arm design as shown in Table1.

Table 1: D-H Parameter for 5 DOF Robotic Arm

Joint i	Type	α_i (deg)	a_i (mm)	d_i (mm)	θ_i (deg)
1	Base	0	0	86	θ_1
2	Shoulder	90	0	0	θ_2
3	Elbow	0	96	0	θ_3
4	Wrist	0	96	0	θ_4
5	Gripper	90	0	59.5	θ_5

Based on the DH convention, the transformation matrix from joint i to joint i+1 is given by:

$$^{i-1}T_i = \begin{bmatrix} C\theta i & -S\theta iC\alpha i & S\theta iS\alpha i & aiC\theta i \\ S\theta i & C\theta iC\alpha i & -C\theta iS\alpha i & aiS\theta i \\ 0 & S\alpha i & C\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where, $S\theta i=\sin\theta i$, $C\theta i=\cos\theta i$, $S\alpha i=\sin\alpha i$, $C\alpha i=\cos\alpha i$, $Sijk=\sin(\theta i+\theta j+\theta k)$, $Cijk=\sin(\theta i+\theta j+\theta k)$

$$^{0}T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$ {}^{1}T_{2} = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

$$ {}^{2}T_{3} = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 * c_3 \\ s_3 & c_3 & 0 & a_3 * s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

$$ {}^{3}T_{4} = \begin{bmatrix} c_4 & -s_4 & 0 & a_4 * c_4 \\ s_4 & c_4 & 0 & a_4 * s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

$$ {}^{4}T_{5} = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

$$ {}^{0}T_{5} = \begin{bmatrix} c_{12}c_{345} & s_{12} & c_{12}s_{345} & s_{12}d_5 + c_{12}a_4c_{34} + c_{12}a_3c_3) \\ s_{12}c_{345} & -c_{12} & s_{12}s_{345} & -c_{12}d_5 + s_{12}a_4c_{34} + s_{12}a_3c_3 \\ s_{345} & 0 & -c_{345} & a_4s_{34} + a_3s_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7} $$

$$ T_e = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8} $$

where T_e is end-effector transformation matrix.

$$ {}^{0}T_5 = T_e \tag{9} $$

Where,

$$ \left.\begin{array}{lll} n_x = c_{12}*c_{345}; & n_y = s_{12}*c_{345}; & n_z = s_{345}; \\ o_x = s_{12}; & o_y = -c_{12}; & o_z = 0; \\ a_x = c_{12}*s_{345}; & a_y = s_{12}*s_{345}; & a_z = -c_{345}; \\ \multicolumn{3}{l}{p_x = s_{12}*d_5 + c_{12}*a_4*c_{34} + c_{12}*a_3*c_3} \\ \multicolumn{3}{l}{p_y = -c_{12}*d_5 + s_{12}*a_4*c_{34} + s_{12}*a_3*c_3} \\ \multicolumn{3}{l}{p_z = a_4*s_{34} + a_3*s_3 + d_1} \end{array}\right\} \tag{10} $$

INVERSE KINEMATIC ANALYSIS

Now solving ${}^{0}T_5$ = Te by equating individual terms of both matrices, we get the inverse solution. The following equations will be used to obtain the solution for the inverse kinematics problem [4, 10].

$$ T_e = {}^{0}T_1 * {}^{1}T_2 * {}^{2}T_3 * {}^{3}T_4 * {}^{4}T_5 = {}^{0}T_5 \tag{11} $$

$$ X_1 = ({}^{0}T_1)^{-1}\, T_e = {}^{1}T_2 * {}^{2}T_3 * {}^{3}T_4 * {}^{4}T_5 = {}^{1}T_5 \tag{12} $$

X_1 =

$[c_1*n_x+s_1*n_y,\ c_1*o_x+s_1*o_y,\ c_1*a_x+s_1*a_y,\ c_1*p_x+s_1*p_y]$

$[-s^1*n^x+c^1*n^y,\ -s^1*o^x+c^1*o^y,\ -s^1*a^x+c^1*a^y,\ -s^1*p^x+c^1*p^y]$

$[n_z,\ o_z,\ a_z,\ p_z\ ]$

$[d_1*n_z,\quad d_1*o_z,\quad d_1*a_z,\quad 1+d_1*p_z\ ]$

${}^1T_5 =$

$[c_2*c_{345},\ s_2,\qquad c_2*s_{345},\ s_2*d_5+c_2*a_4*c_{34}+c_2*a_3*c_3\ ]$

$[s_2*c_{345},\ -c_2,\qquad s_2*s_{345},\ -c_2*d_5+s_2*a_4*c_{34}+s_2*a_3*c_3]$

$[s_{345},\quad 0,\qquad -c_{345},\qquad a_4*s_{34}+a_3*s_3\ ]$

$[0,\qquad 0,\qquad 0,\qquad 1\qquad]$ (13)

From Equation (13), we do not get any "θ" values.

Equation (12), will further be multiplied by $({}^2T_3)^{-1} *({}^3T_4)^{-1} *({}^4T_5)^{-1}$, to get

$X_2 = T_e*({}^0T_1)^{-1}* ({}^2T_3)^{-1} *({}^3T_4)^{-1} *({}^4T_5)^{-1} = {}^0T_1$ (14)

From the element (1, 3) of the Equation (14), we get,

$c_{345}*c_2*a_x+c_{345}*s_2*a_y+s_{345}*a_z = 0;$

$\theta_{345} = \tan^{-1}\{(-a_z)/(c_2*a_x+s_2*a_y)\}$ (15)

From the element (2, 4) of the Equation (14), we get,

$s_2*p_x-c_2*p_y = 0;$

$\theta_2 = \tan^{-1}(p_y/p_x)$ (16)

$\theta_1 = \theta_{12} - \theta_2$ (17)

Equation (11), will further be multiplied by $({}^1T_2)^{-1}$, to get

$X_3 = ({}^0T_1)^{-1} *({}^1T_2)^{-1}* Te = {}^2T_3* {}^3T_4* {}^4T_5 = {}^2T_5$ (18)

$X_3 =$

$[c_{12}*n_x+s_{12}*n_y,\quad c_{12}*o_x+s_{12}*o_y,\quad c_{12}*a_x+s_{12}*_{ay},\quad c_{12}*p_x+s_{12}*p_y]$

$[n_z,\ o_z,\ a_z,\qquad p\qquad]$

$[s_{12}*n_x-c_{12}*n_y,\quad s_{12}*o_x-c_{12}*o_y,\quad s_{12}*a_x-c_{12}*a_y,\quad s_{12}*p_x-c_{12}*p_y\ ]$

$[d_1*n_z,\quad d_1*o_z,\ d_1*a_z,\ 1+d_1*p_z]$

t25 =

$[c_{345},\ 0,\ s_{345},\ a_4*c_{34}+a_3*c_3]$

$[s_{345},\ 0,\ -c_{345},\ a_4*s_{34}+a_3*s_3\ ]$

[0, 1, 0, d]

[0, 0, 0, 1] (19)

From the element (1, 4) & (2, 4) of the Equation (19), we get

$$\left.\begin{aligned} c_{12}*p_x+s_{12}*p_y &= a_4*c_{34}+a_3*c_3 \\ p_z &= a_4*s_{34}+a_3*s_3 \end{aligned}\right\} \quad (20)$$

Squaring both side and then adding the squares gives, we get

$(c_{12}*p_x+s_{12}*p_y)^2+(p_z)^2 = (a_4*c_{34}+a_3*c_3)^2+(a_4*s_{34}+a_3*s_3)^2$

$= (a_3)^2+(a_4)^2+2*a_3*a_4*c_4$

Since, $s_3*s_{34}+c_3*c_{34} = \text{Cos}\ [(\theta_3+\theta_4) – \theta_3] = \text{Cos}\ \theta_4$

$$c_4 = \{(c_{12}*p_x+s_{12}*p_y)^2+(p_z)^2 - (a_3)^2+(a_4)^2\}/(2*a_3*a_4) \quad (21)$$

Knowing that, s_4 = , we can say that,

$$\theta_4 = \tan^{-1}(s_4/c_4) \quad \pm\sqrt{1+c_4^{\ 2}} \quad (22)$$

Now again referring to the Equation (20), we can calculate θ_3 as follows.

$c_3 = (c_{12}*p_x + s_{12}*p_y - a_4*c_{34})/a_3;$

$s_3 = (p_z - a_4*s_{34})/a_3;$

$$\theta_3 = \tan^{-1}(s_3/c_3) \quad (23)$$

$$X_4 = ({}^0T_1)^{-1} *({}^1T_2)^{-1}*({}^2T_3)^{-1}*({}^3T_4)^{-1}* \text{Te} = {}^4T_5 \quad (24)$$

From the element (1, 4) of the Equation (23), we get

$c_{34}*c_{12}*p_x+c_{34}*s_{12}*p_y+s_{34}*p_z = 0;$

$$\theta_{34} = - \tan^{-1}\{(c_{12}*p_x+s_{12}*p_y)/p_z\} \quad (25)$$

$$\theta_5 = \theta_{345} - \theta_{34} \quad (26)$$

FORWARD KINEMATIC CASE-STUDY & RESULT

For the given set of parameter, a program in MATLAB 8.0 is made. Developed model is used to determine position and orientation of end effector. For the values of θ1 = 30°, θ2 =50°, θ3 = 45°, θ4 = 25° and θ5 = 0°, results obtained is shown below.

The DH parameters of 5 DOF Robotic Arm is as follows:

a alpha1 d theta

0.0000 0.0000 86.0000 50.0000

0.0000 90.0000 0.0000 30.0000

96.0000 0.0000 0.0000 45.0000

96.0000 0.0000 0.0000 25.0000

0.0000 90.0000 59.5000 0.0000

The Final Matrix ($^0T_5$) by Matlab Program is as follows:

0.0594 0.9848 0.1632 76.0852

0.3368 -0.1736 0.9254 88.8540

0.9397 0.0000 -0.3420 244.0927

0.0000 0.0000 0.0000 1.0000

The final end-effector position is Px = 76.0852, Py = 88.8540, Pz = 244.0927.

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CONCLUSIONS

A complete analytical solution to the forward and inverse kinematics of 5 DOF Robotic arm is derived in this paper. The forward kinematic analysis of 5 DOF robotic arm is investigated. The mathematical model is prepared and solved for positioning and orienting the end effectors by preparing a programme in MATLAB 8.0. The result of the forward kinematics can be crossed checked by the analytical method of inverse kinematic model. Hence this proves the utility of the 5 DOF robotic arm as an educational tool for undergraduate robotics courses.

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