

# Toward a Realistic Performance Analysis of Storage Systems in Smart Grids

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**Abstract**—Energy storage devices (ESDs) have the potential to revolutionize the electricity grid by allowing the smoothing of variable-energy generator output and the time-shifting of demand away from peak times. A common approach to study the impact of ESDs on energy systems is by modeling them as electric circuits in simulations. Although recent circuit models are becoming more accurate, to obtain statistically valid results, extensive simulations need to be run. In some cases, existing datasets are not large enough to obtain statistically significant results. The impact of ESDs on energy systems has also been recently studied using analytical methods, but usually by assuming ideal ESD behavior, such as infinite ESD charging and discharging rates, and zero self-discharge. However, real-life ESDs are far from ideal. We investigate the effect of nonideal ESD behavior on system performance, presenting an analytical ESD model that retains much of the simplicity of an ideal ESD, yet captures many (though not all) nonideal behaviors for a class of ESDs that includes all battery technologies and compressed air energy storage systems. This allows us to compute performance bounds for systems with nonideal ESDs using standard teletraffic techniques. We provide performance results for five widely used ESD technologies and show that our models can closely approximate numerically computed performance bounds.

**Index Terms**—Analytical models, energy storage, performance analysis, renewable energy sources.

## I. INTRODUCTION

**B**OTH ELECTRICITY generation from renewable energy sources and electricity demand are stochastic processes. This makes them difficult to manage and control. Energy storage devices (ESDs) can smooth out variations in generation and demand, greatly simplifying grid operation. Therefore, there has been a considerable amount of work in the design of many types of ESD-based systems. Examples are off-grid networks which use renewable energy to serve their demands (stochastic energy source, stochastic demand) [1], load shaping in on-grid networks (deterministic energy source, stochastic demand) [2], and firming renewable energy sources for electricity market (stochastic energy source, deterministic demand) [3].

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The common approach to evaluate an ESD-based energy system is numerical simulation, as in [4]–[6]. Simulation methods are used to compute either active power [5] or current-voltage (I-V) values [4], [6]. I-V simulations contain more information than active power simulations about energy system operation, because they allow the monitoring of both current and voltage. However, I-V simulations are more complex, because ESDs must be modeled by (complex) electric circuits [7] that are sensitive to the choice of ESD technology [8]. There has been recently extensive improvement on the accuracy of ESD models used for simulation both for active power [5] and I-V values [6], [9].

Although quite precise, design based on simulation is feasible only when large datasets are available to allow computation of tail bounds for small target failure probabilities. Moreover, the simulation must be repeated to study the sensitivity to any single parameter, which is computationally expensive.

As an alternative to simulation, and inspired by the analogy between storing energy in an ESD and traffic buffering in a packet network, several recent papers have adopted the analytical techniques developed for teletraffic analysis to study generic ESD-based energy systems. Examples include computing the loss of power probability in a distribution network [10], using nonasymptotic Kesidis bounds for regulated traffic [11], and using a network calculus framework [12], [13].

Arguably, the technique that is best suited for the analysis of ESD-based energy systems is the network calculus approach [14]–[16] because it precisely matches the characteristics of a smart grid system, that is, nonasymptotic, nonstationary, and fluid flow. Unfortunately, the elegant mapping between teletraffic networks and smart grids makes the strong assumption of an ideal ESD, that is, one that looks like a RAM network buffer. In this paper, we show that this mapping does not hold for nonideal ESDs, bringing into question the validity of existing results. Our results show that it is critical to model nonideal ESD behavior and, in fact, our approach does this very well, specifically as follows.

- 1) We provide a simple and general model for a class of nonideal ESDs that includes battery technologies and compressed air energy storage (CAES).
- 2) We use our model to compute analytical bounds on loss of power and waste of power and show that our analytical performance bounds closely approximate numerical simulations.

- 3) We demonstrate that the choice of ESD technology as well as its nonideal behavior both significantly impact the performance of an energy system.

The remainder of the paper is organized as follows. In Section II, we review some of the most widely used ESD technologies along with their characteristics. We present the system model in Section III. We show in Section IV how to convert a complex nonideal ESD model to an ideal one. In Section V, we employ network calculus to derive performance bounds using our model. In Section VI, we show the importance of accounting for nonideal ESD behavior, evaluate our bounds, and compare the performance of different storage technologies. Finally, we conclude our paper in Section VII.

## II. BACKGROUND ON STORAGE SYSTEMS

Four ESD technologies are in common use today: mechanical, thermo-dynamic, electrochemical, and electro-magnetic, each with its own characteristics. These include the following.

- 1) *ESD Size* [ $B$  (Wh)]: This is the maximum amount of energy that can be stored in an ESD. Some ESD technologies require devices to have a maximum or minimum size. ESD size decreases over long time intervals due to deteriorations. This must be accounted for, when analyzing an ESD over a large time, e.g., in a lifetime maximization problem [19]. However, ESD size can be assumed to be fixed, if short-term performance metrics are analyzed (such as in this paper).
- 2) *Storage Charging and Discharging Rate Limit* [ $\alpha_c$  and  $\alpha_d$  (W)]: This is the limit on the charge or discharge power. Typical discharge rates are many times greater ( $10\times$  for lead-acid and  $5\times$  for lithium-ion batteries) than the charging rate.
- 3) *ESD Efficiency* ( $0 \leq \eta \leq 1$ ): Due to inherent inefficiencies, each unit of energy stored is reduced to  $\eta$  units that can be used at a later time. ESDs converting electrical energy to other forms typically have lower efficiencies.
- 4) *Self-Discharge* [ $\gamma$  (W)]: Stored energy leaks over time. The leak rate is  $\gamma$ , which can be a function of several parameters such as state of charge, storage size, and temperature, depending on the storage technology. We only study ESD technologies that have small self-discharge.
- 5) *Depth-of-Discharge (DoD)* ( $0 \leq \text{DoD} \leq 1$ ): While the entire capacity of some ESDs can be used for energy storage, battery life is extended if only a fraction of available capacity is used. There is extensive work to determine the relationship between battery lifetime and DoD, see [20].
- 6) *Temperature-Dependency*: Operating temperature can affect the charging and discharging rates and the lifetime of a battery. The operation of other ESDs, however, is nearly independent of the temperature.
- 7) *State of Health (SOH)* ( $0 \leq \text{SOH} \leq 1$ ): This factor reflects the general condition of the storage device with respect to its initial condition. Unfortunately, there is no unified definition for SOH; it is either expressed in terms of capacity fade or charging power fade. As SOH decreases (ESD is ageing), some of its imperfection parameters become more significant. For example,

TABLE I  
NOTATIONS USED IN THIS PAPER

Name	Description (units)
$T_u$	Time unit (hour)
$S(t)(S'(t))$	Input power (virtual input power) at time $t$ (W)
$D(t)(D'(t))$	Demand power (virtual demand power) at time $t$ (W)
$\bar{S}(\underline{S})$	Upper (lower) envelope on the input power (W)
$\bar{D}(\underline{D})$	Upper (lower) envelope on the demand power (W)
$B$	ESD size (Wh)
$\alpha_c(\alpha_d)$	ESD charging (discharging) power limit (W)
$\eta$	ESD efficiency
$\gamma$	ESD leakage power rate (W)
DoD	ESD depth of discharge
$l(t)$	Loss of power at time $t$ (W)
$w(t)$	Waste of power at time $t$ (W)

the charging rate, self discharge, and storage capacity are affected by SOH, while the efficiency and discharge rate are almost unchanged throughout the lifetime of an ESD.

Our model will incorporate all of the above imperfection parameters except temperature dependency and SOH. This is due to the fact that, despite the importance of these factors, it is not yet precisely known how temperature and SOH impacts an ESD. This impact is highly nonlinear and application dependent, and not yet properly modeled in spite of extensive studies [21].

This paper evaluates a class of ESD technologies that we will define precisely in the next section. This class includes as follows.

- 1) *Lead-Acid Battery (Electrochemical)*: This is a widely-used battery because of its low price, high specific power,<sup>1</sup> simple manufacturing, and its lowest self-discharge among all batteries. It has, however, a small charging rate, a low specific energy, and a limited life cycle.
- 2) *Lithium-ion Battery (Electrochemical)*: This type of battery has a high energy density, a high energy efficiency, and a low discharge rate. One of the main disadvantages of this battery is its price, which can be  $3\times$  larger than a lead-acid battery.
- 3) *Compressed Air Energy Storage (Thermo-Dynamic)*: This device stores energy in the form of compressed air in a room and releases it to rotate a turbine for electricity generation. CAESs are relatively cheap, but they have low energy and low power density. It has small efficiency, but no restriction on DoD.

Due to the diversity of ESD technologies with unique constraints and characteristics, choosing the best ESD technology for a given application is challenging. Some constraints of the above technologies are listed in Table II.

Our goal is to build a single mathematical model suitable for all ESD technologies. To do so, we first describe our system model in the next section. We summarize the important notations, which will be used in the rest of the paper, in Table I.

<sup>1</sup>Specific energy (or power) is the energy (or power) provided per unit volume or per unit mass.

TABLE II  
ESD CHARACTERISTICS IN ROOM TEMPERATURE AND AVERAGED OVER LIFETIME [2], [17], [18]

	Lead-acid	Lithium-ion	CAES
Efficiency	0.75	0.85	0.68
Charge time (=ESD size/Charge rate)	8-16h	2-4h	15 min
Discharge rate to charge rate ratio	10	5	4
Self-discharge/ day (=self-discharge rate $\times$ day / ESD size) (%)	0.3	0.1	0
DoD	0.80	0.80	1

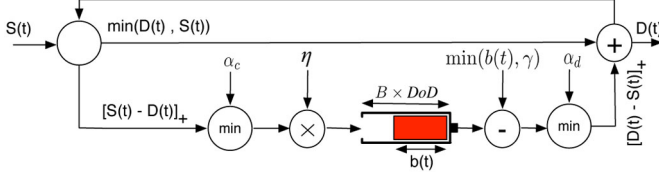


Fig. 1. System: A power source  $S$  equipped with an ESD to serve a demand power  $D$ . The physical constraints of the ESD are  $\alpha_c$ ,  $\alpha_d$ ,  $\eta$ ,  $\gamma$ , and DoD. Note:  $[x]_+ = \max\{0, x\}$  for any  $x$ .

### III. SYSTEM MODEL

Energy can be modeled as a continuous-time, fluid stochastic process. That is, at any given time, energy arrival is not a quantized value, unlike Internet packet traffic. Moreover, events in an energy system are not restricted to occur at discrete time instants (except maybe for specific applications). However, we can still assume a discrete-time model (which simplifies analysis) as long as the time granularity is chosen such that the precision is kept above an acceptable threshold. We consider a fluid-flow, but discrete-time model, where time is slotted  $t = 0, T_u, 2T_u, \dots$ , with  $T_u$  being the time unit. We consider a renewable energy source, which is meant to serve a stochastic demand, using an ESD as a backup storage (see Fig. 1) to smooth out demand or source fluctuations. Denote by  $S(t)$  and  $D(t)$ , respectively, the available power from the energy source and the demand power at time  $t$ . To simplify notation, we write  $D(s, t)$  and  $S(s, t)$  to, respectively, mean  $\sum_{\tau=s+1}^t D(\tau)$  and  $\sum_{\tau=s+1}^t S(\tau)$  (e.g.,  $S(t-1, t) = S(t)$ ).

In our system, renewable energy is preferentially used to serve the demand, i.e., without going through the ESD. If, in a given time slot  $t$ , the available renewable power is insufficient (i.e.,  $S(t) < D(t)$ ), the energy stored in ESD, if any, can be used to make up the difference. Moreover, if the available renewable power in a time slot  $t$  is larger than the demand power (i.e.,  $S(t) > D(t)$ ), then the surplus energy  $((S(t) - D(t))T_u)$  is stored in the ESD, if it is not yet full. All incoming power exceeding ESDs charging rate limit  $\alpha_c$  is dropped. Moreover, the ESD loses a fraction of  $1 - \eta$  of the total energy being stored in the ESD due to ESD efficiency. The ESD cannot be discharged faster than  $\alpha_d$ . The ESD lifetime constraint is met if only a DoD fraction  $\leq 1$  of the entire ESD is used.<sup>2</sup> Finally, The energy stored in the ESD is discarded with rate  $\gamma$  due to self-discharge. In this paper, we simply assume that the self-discharge rate  $\gamma$  is a subtractive term, independent of the state of charge and

<sup>2</sup>When converters are used, the values of  $\eta$ ,  $\alpha_c$ , and  $\alpha_d$  have to be modified to also include the imperfections of the converters.

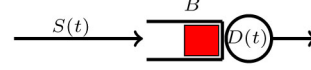


Fig. 2. Equivalent model of Fig. 1, when the ESD is ideal ( $\alpha_c, \alpha_d = \infty$ ,  $\eta, \text{DoD} = 1$ ,  $\gamma = 0$ ). For nonideal ESDs, we can still convert Fig. 1 to this figure when replacing  $S$ ,  $D$ , and  $B$  by their virtual processes  $S'$ ,  $D'$ , and  $B'$ .

scales linearly with the ESD size. Note that modeling the impact of self-discharge on the state of charge this way is not valid for other storage technologies such as flywheels and supercapacitors.

For an ideal ESD (i.e.,  $\alpha_c, \alpha_d = \infty$ ,  $\eta, \text{DoD} = 1$ ,  $\gamma = 0$ ), the complicated ESD model in Fig. 1 reduces to the simple model in Fig. 2, which is identical to a buffer in a network where the input traffic  $S$  enters a finite queue of size  $B$  with available (time-varying) service rate  $D$ . This enables elegant performance analysis of energy systems by adopting standard techniques from the rich literature on performance analysis of teletraffic networks. This is the model that has been used in recent analyses of energy systems [10], [12], [13]. However, ignoring the nonideal ESD behavior in the simple model brings into question the validity of these results. Our plan of attack, therefore, is to convert the complex system model of Fig. 1 to an equivalent simple model with appropriately modified system parameters (Fig. 2). This would allow the analytical results that assume ideal ESD behavior to be suitably transformed to take into account nonideal ESD behavior. We discuss the equivalent model in more detail in Section IV.

### IV. MODELING NONIDEAL ESDS

In this section, we show how to choose appropriate parameters so that the complex system model of Fig. 1 is converted to the ideal (simple) model of Fig. 2. The starting point for our model is the following state of charge evolution equation that is representative of a large class of ESDs including batteries and CAES. This is the class that we study

$$b(t) = \min \left( B \times \text{DoD}, \left[ \min([S(t) - D(t)]_+, \alpha_c) \eta T_u - \min([D(t) - S(t)]_+, \alpha_d) T_u - \gamma T_u + b(t - T_u) \right]_+ \right) \quad (1)$$

where  $b(t)$  is the state of charge of the storage in the system depicted in Fig. 1 and where  $[x]_+ = \max\{0, x\}$  for any  $x$ . For an ideal ESD (i.e.,  $\alpha_c, \alpha_d = \infty$ ,  $\eta, \text{DoD} = 1$ ,  $\gamma = 0$ ), (1) reduces to

$$b(t) = \min(B, [b(t - T_u) + S(t)T_u - D(t)T_u]_+) \quad (2)$$



which is equivalent to the recursive backlog process in teletraffic theory with traffic arrival  $S$ , service rate  $D(t)$  at any time  $t$ , and queue size  $B$  [22]. Unfortunately, unlike (2), (1) cannot be mapped to a backlog process in teletraffic theory. In the following, we incorporate the storage imperfections, by defining virtual power source and demand processes, to simplify the nonideal storage model. To simplify notation, we drop  $T_u$  from our formulation by assuming  $T_u = 1$ . Generalizing the formulas for any  $T_u$  is a matter of additional notations. We highlight again that the analysis of this paper is valid for any ESD technology for which the state of charge can be expressed by (1) and hence, in the following, when we mention a non-ideal ESD, we mean one for which the state of charge equation can be expressed by (1). Please see [23] for the proof.

**Lemma 1 (Equivalent Simple Queue):** Suppose that an intermittent power source  $S$  serves a stochastic demand power  $D$ . A nonideal ESD, with size  $B$  and parameters  $\alpha_c$ ,  $\alpha_d$ ,  $\eta$ ,  $\gamma$  and DoD, is used to provide power during a power shortage (i.e.,  $[D(t) - S(t)]_+$ ) and also to store the surplus power (i.e.,  $[S(t) - D(t)]_+$ ) at any time instant  $t$  (Fig. 1). The state of charge of the ESD in this model is equivalent to that of an ideal ESD model (Fig. 2), where power source  $S$ , power demand  $D$ , and ESD size  $B$  are, respectively, replaced by a virtual source  $S'$ , a virtual demand  $D'$ , and a virtual ESD size  $B'$ , given by

$$S'(t) = S(t) - (1 - \eta) [S(t) - D(t)]_+ - \eta [S(t) - D(t) - \alpha_c]_+ \quad (3)$$

$$D'(t) = D(t) - [D(t) - S(t) - \alpha_d]_+ + \gamma \quad (4)$$

$$B' = B \times \text{DoD}. \quad (5)$$

**Remark 1:** The choices of nonnegative virtual processes, which convert Figs. 1 and 2, are not unique. The specific choices we made in (3)–(5) greatly facilitate the use of this model in further analysis by separating the impact of imperfections from the original processes. In particular, (3)–(4) present the imperfections as additional or subtracting terms to the original source and demand processes.

In the next section, we will employ Lemma 1 to analyze a nonideal ESD using network calculus [14]–[16]. We use this approach because it correctly models the nonasymptotic, non-stationary, and fluid flow aspects of the smart grid. Moreover, it can be used to study a broad range of arrival processes including Markovian, self-similar, and even heavy-tailed processes [24]. The key aspect of network calculus is that it models stochastic processes as bounds on the tail distributions.

## V. USING NETWORK CALCULUS TO ANALYZE ESDS

In network calculus, stochastic processes are modeled by envelopes, which can be of several types. One of the most widely used ones is called statistical sample path envelope [25], defined next. A nondecreasing function  $\bar{A}$  is a statistical sample path upper envelope for process  $A$  with bounding function  $\bar{\varepsilon}$  if it satisfies the following at any time  $t \geq 0$  and for any  $x \geq 0$ :

$$\Pr \left\{ \max_{s \leq t} (A(s, t) - \bar{A}(t - s)) > x \right\} \leq \bar{\varepsilon}(x). \quad (6)$$

Likewise,  $\underline{A}$  is a statistical sample path lower envelope for process  $A$  with bounding function  $\underline{\varepsilon}$  if at any time  $t \geq 0$  and for any  $x \geq 0$

$$\Pr \left\{ \max_{s \leq t} (\underline{A}(t - s) - A(s, t)) > x \right\} \leq \underline{\varepsilon}(x). \quad (7)$$

Sample path envelopes for a given process can be computed either from measurement traces of that process or from an analytical model of that process (e.g., On-off Markov model). We now describe how to compute sample path envelopes from measurement traces consisting of multiple trajectories  $A^i$  indexed by  $i$ .

The lower bounding function  $\underline{\varepsilon}$  is computed as follows (a similar approach can be used to compute  $\bar{\varepsilon}$ ): construct a set  $Y$  with elements  $Y^{i,t}$  chosen at time  $t \geq 0$  corresponding to a trajectory  $A^i$  such that

$$Y^{i,t} = \max_{0 \leq s \leq t} (\underline{A}(t - s) - A^i(s, t)). \quad (8)$$

Then, from (7),  $\underline{\varepsilon}$  can be chosen to be the complimentary cumulative distribution function (CCDF) of any distribution that fits  $Y$ .

Note that for a given measurement trace, infinitely many models can be proposed by varying  $\underline{A}$  and computing the corresponding bounding function  $\underline{\varepsilon}$ . Thus, modeling a stochastic process given a measurement trace using this approach consists of three steps: 1) choose an envelope ( $\underline{A}$ ); 2) characterize the bounding function for the tail bound distribution; and 3) compute the parameters of that distribution.

### A. Existing Performance Bounds for Ideal ESDs

Two metrics are widely used to characterize energy system performance. Loss of power  $l(t)$  at time step  $t$  occurs if demand is unmet at that time from the sum of renewable power and stored energy. Waste of power  $w(t)$  at time step  $t$  is the amount of power that is wasted in that time slot due to nonideal ESD behavior or limited ESD size. The loss of power and waste of power probabilities have been computed in recent work, under the assumption of ideal ESD behavior [10], [12], [13], as follows.

**Theorem 1 (Existing Performance Bounds for Ideal ESDs [12]):** Consider the same scenario as in Lemma 1 with an ideal ESD. Suppose we are given a statistical sample path lower (upper) envelope  $\underline{S}$  ( $\bar{S}$ ) on the intermittent energy source with a bounding function  $\underline{\varepsilon}_s$  ( $\bar{\varepsilon}_s$ ). In addition, assume that there is a statistical sample path upper (lower) envelope  $\bar{D}$  ( $\underline{D}$ ) on the demand power in the sense of (6) with a bounding function  $\bar{\varepsilon}_d$  ( $\underline{\varepsilon}_d$ ). Then, the loss of power  $l(t)$  and the waste of power  $w(t)$  at time  $t$ , satisfy the following:

$$\Pr\{l(t) > 0\} \leq \bar{\varepsilon}_d \otimes \underline{\varepsilon}_s \left( B - \max_{0 \leq \tau \leq t} (\bar{D}(\tau) - \underline{S}(\tau)) \right) \quad (9)$$

$$\Pr\{w(t) > x\} \leq \underline{\varepsilon}_d \otimes \bar{\varepsilon}_s \left( B + x - \max_{0 \leq \tau \leq t} (\bar{S}(\tau) - \underline{D}(\tau)) \right) \quad (10)$$

where  $\otimes$  is the min-plus convolution operator defined as

$$f_1 \otimes f_2(u) = \min_{0 \leq s \leq u} (f_1(s) + f_2(u - s)) \quad (11)$$

for any nonnegative functions  $f_1$  and  $f_2$  and any constant  $u$ .

We will extend this theorem using Lemma 1 to derive performance bounds for nonideal ESDs in the next section.

### B. Performance Bounds for Nonideal ESDs

The waste of power in an ideal ESD at time  $t$  maps simply to the traffic loss (when an arrival is dropped because the queue is full) in a teletraffic queue of size  $B$ , with input traffic  $S(t)$ , and a service rate  $D(t)$  [see (2)]. Thus, the nonrecursive backlog formulation from [22] is used to characterize the waste of power in an ideal ESD in [12]

$$w_{ideal}(t) = \min_{0 \leq u < t} \left( \max_{u \leq s < t} ([S(s, t) - D(s, t) - BI_{t>0}]_+), \right. \\ \left. S(u, t) - D(u, t) + BI_{u>0} - BI_{t>0} \right) \quad (12)$$

where for any expression  $x$ ,  $I_x = 1$  if  $x$  is true and  $I_x = 0$ , otherwise.

The waste of power in an ideal ESD happens only due to limited ESD size. However, the total waste of power in a non-ideal ESD, consists of the power waste due to the storage imperfections  $w_{\alpha_c, \gamma, \eta}$  and the waste of power due to finite storage size  $w_B$

$$w_{nonideal}(t) = w_{\alpha_c, \gamma, \eta}(t) + w_B(t). \quad (13)$$

$w_B$  is characterized by replacing the original processes with the virtual ones (from Lemma 1) in (12) and  $w_{\alpha_c, \gamma, \eta}$  satisfies

$$w_{\alpha_c, \gamma, \eta}(t) \leq S(t) - S'(t) + \gamma \quad (14)$$

where  $S(t) - S'(t)$  accounts for all the power waste occurring due to the ESD charging rate limit ( $\alpha_c$ ), ESD inefficiency ( $\eta$ ), and depth of discharge (DoD). Equation (14) holds as an inequality, since it assumes that the self-discharge is always  $\gamma$ , meaning that the state of charge is never less than  $\gamma$ .

The loss of power event can be characterized using the concept of the deficit state of charge. The deficit state of charge  $b_d$  is the energy required to fully charge the ESD and at any time  $t$  and is given by

$$b_d(t) = \min(B, [b_d(t-1) + D(t) - S(t)]_+). \quad (15)$$

Importantly, the loss of power event occurs when the ESD state of charge crosses zero or, equivalently, when the deficit state of charge crosses  $B$ . Thus, from (15), the loss of power event is equivalent to the traffic loss in a teletraffic system in a queue with size  $B$ , traffic arrival  $D$ , and service rate  $S$  [12]

$$l_{ideal}(t) = \min_{0 \leq u < t} \left( \max_{u \leq s < t} ([D(s, t) - S(s, t) - BI_{t>0}]_+), \right. \\ \left. D(u, t) - S(u, t) + BI_{u>0} - BI_{t>0} \right). \quad (16)$$

The loss of power in an ideal ESD happens only due to an empty ESD. In a nonideal ESD, however, the loss of power can happen either due to the empty ESD,  $l_B$ , or due to the limited discharge rate of ESD,  $l_{\alpha_d}$ , which may prevent the entire demand from being met. That is

$$l_{nonideal}(t) = l_{\alpha_d}(t) + l_B(t). \quad (17)$$

$l_B$  can be computed by simply replacing the original processes with virtual ones in (16).  $l_{\alpha_d}$  is the difference between the

power shortage and the discharge rate limit (i.e.,  $[D(t) - S(t) - \alpha_d]_+$ ), which [from (4)] is equivalent to

$$l_{\alpha_d}(t) = [D(t) - S(t) - \alpha_d]_+ = D(t) - D'(t) + \gamma. \quad (18)$$

Using the above definitions, we can compute performance bounds for nonideal ESDs as (please see [23] for the proof).

*Theorem 2 (Performance Bounds for Nonideal ESDs):*

Consider the same scenario as in Lemma 1.  $S'$  and  $D'$  are, respectively, the virtual source and virtual demand as defined in Lemma 1. Suppose we are given a sample path lower (upper) envelope  $\underline{S}'$  ( $\overline{S}'$ ) on  $S'(t)$  with a bounding function  $\underline{\varepsilon}'_s$  ( $\overline{\varepsilon}'_s$ ). In addition, suppose we are given a sample path lower (upper) envelope  $\underline{D}'$  ( $\overline{D}'$ ) on  $D'$  with a bounding function  $\underline{\varepsilon}'_d$  ( $\overline{\varepsilon}'_d$ ). If for some constant  $\underline{\varepsilon}'_0$  and a function  $\overline{\varepsilon}'_0$  the following hold at any time  $t \geq 0$  and for any  $x \geq 0$ :

$$\Pr\{S'(t) - D'(t) > x\} \leq \overline{\varepsilon}'_0(x) \quad (19)$$

$$\Pr\{D'(t) - S'(t) > 0\} \leq \underline{\varepsilon}'_0 \quad (20)$$

$$\Pr\{S(t) - S'(t) + \gamma > x\} \leq \varepsilon'_w(x) \quad (21)$$

$$\Pr\{D(t) - D'(t) + \gamma > 0\} \leq \varepsilon'_l \quad (22)$$

then, the loss of power and the waste of power at time  $t$  satisfy the following:

$$\Pr\{l(t) > 0\} \leq \varepsilon'_l + \min \left( \underline{\varepsilon}'_0, \right. \\ \left. \overline{\varepsilon}'_d \otimes \underline{\varepsilon}'_s \left( B' - \max_{0 \leq \tau \leq t} (\overline{D}'(\tau) - \underline{S}'(\tau)) \right) \right) \quad (23)$$

$$\Pr\{w(t) > x\} \leq \min \left( \varepsilon'_w \otimes \overline{\varepsilon}'_0(x), \right. \\ \left. \varepsilon'_w \otimes \left( \underline{\varepsilon}'_d \otimes \overline{\varepsilon}'_s (B' + x - \max_{0 \leq \tau \leq t} (\overline{S}'(\tau) - \underline{D}'(\tau))) \right) \right). \quad (24)$$

*Remark 2:* Besides allowing the computation of performance bounds for nonideal ESDs, our paper also improves on the known results for the case of ideal ESDs in Theorem 1; it provides tighter bounds because the first terms in the minimization in two bounds of (23)–(24) (i.e.,  $\underline{\varepsilon}'_0$  and  $\varepsilon'_w \otimes \overline{\varepsilon}'_0(x)$ ) were not considered in [12]. These terms affect performance greatly when the ESD size is small. In particular, for the case of an energy system with no storage, Theorem 1 says that the probabilities of loss of power and waste of power are upper bounded by 1, whereas our theorem returns nontrivial bounds.

The performance bounds found in Theorem 2 hold for general sample path envelopes and bounding functions. In the next section, we compute bounds for the case where the source and demand processes have simple exponential tail bounds.

### C. Computing Bounds for Exponential Tail Bounds

It has been shown, in the literature of network calculus, that even simple functions can characterize several complex stochastic processes. In fact, the trivial function  $\mathcal{A}(t) = rt$  as a sample path envelope and an exponential decay rate  $ae^{-\xi t}$  as a tail bound can characterize a large class of processes called exponentially bounded burstiness processes (EBB) [26], which includes Markov modulated processes. In our work, we model a more general class of processes than EBB by using affine

functions of the form  $\mathcal{A}(t) = [a \pm rt]_+$  as envelopes with exponential decay rates as bounding functions. Specifically,

$$\underline{S}'(t) = [\rho_1 t - \sigma_1]_+; \quad \underline{\varepsilon}'_s(x) = p_1 e^{-\beta_1 x} \quad (25)$$

$$\overline{S}'(t) = \rho_2 t + \sigma_2; \quad \overline{\varepsilon}'_s(x) = p_2 e^{-\beta_2 x} \quad (26)$$

$$\underline{D}'(t) = [\rho_3 t - \sigma_3]_+; \quad \underline{\varepsilon}'_d(x) = p_3 e^{-\beta_3 x} \quad (27)$$

$$\overline{D}'(t) = \rho_4 t + \sigma_4; \quad \overline{\varepsilon}'_d(x) = p_4 e^{-\beta_4 x} \quad (28)$$

for some  $\rho_j, \sigma_j, p_j, \beta_j, (1 \leq j \leq 4)$ . In addition to our assumptions on the choices of envelopes, we also assume

$$\overline{\varepsilon}'_0(x) = p_5 e^{-\beta_5 x}; \quad \underline{\varepsilon}'_w(x) = p_6 e^{-\beta_6 x} \quad (29)$$

for some  $p_5, p_6, \beta_5, \beta_6$ . Inserting the above choices in Theorem 2, we can compute performance bounds for energy systems with nonideal ESDs.

Recall that the bounds in Theorem 2 use the convolution operation, and each convolution is a minimization over a free parameter [see (11)]. Computing this minimum is challenging for general functions. Although, for the case of exponential distributions, one can use [27, Lemma 3] to compute these convolutions, we decided to compute simpler suboptimal, albeit good, values for the free parameter in the convolution operation by choosing identical exponents in the convolving functions. With this simplification, and if, respectively,  $\rho_1 \geq \rho_4$  and  $\rho_3 \geq \rho_2$  (a stability condition), we get

$$\Pr\{l(t) > 0\} \leq \varepsilon'_l + \min\left(\varepsilon'_0, (p_1 + p_4)e^{\left\{-\frac{\beta_1 \beta_4}{\beta_1 + \beta_4}(B - \sigma_1 - \sigma_4)\right\}}\right) \quad (30)$$

and

$$\Pr\{w(t) > x\} \leq \min\left((p_6 + p_5)e^{-\frac{\beta_5 \beta_6 x}{\beta_5 + \beta_6}} \left(p_6 + (p_2 + p_3)e^{\left\{-\frac{\beta_2 \beta_3}{\beta_2 + \beta_3}(B - \sigma_2 - \sigma_3)\right\}}\right) e^{-\beta_w x}\right) \quad (31)$$

where  $\beta_w = (1/\beta_2 + 1/\beta_3 + 1/\beta_6)^{-1}$ .

#### D. Steps to Evaluate Bounds for Measurement Trace

We now describe the steps necessary to compute bounds on loss of power probability and waste of power probability using our approach. Recall that we are given measurement traces for  $S$  and  $D$  and also the physical characterizations of the ESD.

First, compute the virtual source  $S'$ , demand  $D'$ , and ESD size  $B'$  using Lemma 1. Second, compute the rates in the sample path envelopes [i.e.,  $\rho_j$  in (25)–(28)] by setting  $\rho_j (\forall j)$  to the long-term average power of its corresponding process (e.g.,  $\rho_1 = \sum_{t=1}^T S'(t)/T$ , where  $T$  is the time length of the measurement trace). Third, verify that the stability condition is met. That is, check that  $\rho_1 \geq \rho_4$  and  $\rho_3 \geq \rho_2$ , which otherwise and respectively, lead to the loss of power probability and the waste of power probability to be trivially bounded by 1. Fourth, compute the four bounding functions, with parameters  $p_j$  and  $\beta_j$ , treating  $\sigma_j$ s as free parameters, as follows: For a fixed  $\sigma_j$ , construct the set  $Y$  [see (8)] for the corresponding virtual process and envelope (e.g.,  $\overline{S}'$ ). Note that the elements

of  $Y$  must be nonnegative. Thus, set all negative elements of  $Y$  to zero. To avoid the bias of the zero elements in the fitted distribution, set  $p_j$  to the ratio of the nonzero elements of  $Y$  to the total number of elements. Then,  $\beta_j$  is the parameter of the exponential decay that is fitted to the nonzero elements of  $Y$ . Fifth, use similar steps to compute  $p_5, p_6, \beta_5$ , and  $\beta_6$ . Finally, by turning (25)–(28) to functions of the free parameter  $\sigma_j$ , minimize them over that parameter to find the optimal value of  $\sigma_j$ .

## VI. EVALUATION

In this section, we validate our analysis using the widely used wind turbine power traces from the U.S. West Coast, with a 10-min time resolution ( $T_u = 10\text{min}$ ), freely available from NREL [28]. We use this data trace to compute our analytical upper bounds on the loss of power and waste of power, following the steps described in Section V-D. We also compare our bounds with results obtained by simulating the state of charge of the ESD and computing the exact waste of power and loss of power, respectively, from (13) and (17).

We assume that this wind power is used to serve a constant demand of 0.1MW for loss of power. For evaluating the waste of power, we use a higher demand of 0.5MW with an ESD of size 10MWh, to keep the magnitude of the waste of power numerically manageable. Note that the average available power from our wind power trace is 0.6MW. We compare the performance of the five ESD technologies in Section II: lithium-ion battery, lead-acid battery, and compressed-air energy storage (CAES). The physical constraints of these technologies are as in Table II. Thus, the results of this section assume the same temperature and SOH under which the parameters in Table II are computed.

#### A. Impact of Imperfections

We use numerical simulations to investigate the impact of ESD imperfections on energy system performance. For each ESD technology, we compute the actual loss of power and the waste of power probabilities by simulating the ESD state of charge process, using (1), and computing the quantiles (i.e., CCDF) of the performance metrics. We also show the performance metrics of an ideal ESD to study the impact of imperfections for each ESD technology.

Fig. 3(a) shows the loss of power probability as a function of ESD size for each ESD technology. This graph demonstrates the substantial impact of the physical constraints; nonideal ESDs have significantly different loss of power probabilities compared to an ideal ESD for the same storage size. This vividly demonstrates the need for the analytical modeling of ESD imperfections, especially for larger storage sizes. This figure also shows the dramatic difference in behavior across ESD technologies, again, especially for larger storage sizes.

Fig. 3(a) also highlights that the impact of efficiency on the loss of power probability is negligible, because the loss probability curve for CAES (the technology with the worst efficiency among the three we studied) closely tracks that of an ideal storage. Note that the main restrictive physical imperfection of CAES is its efficiency. In contrast, the impact

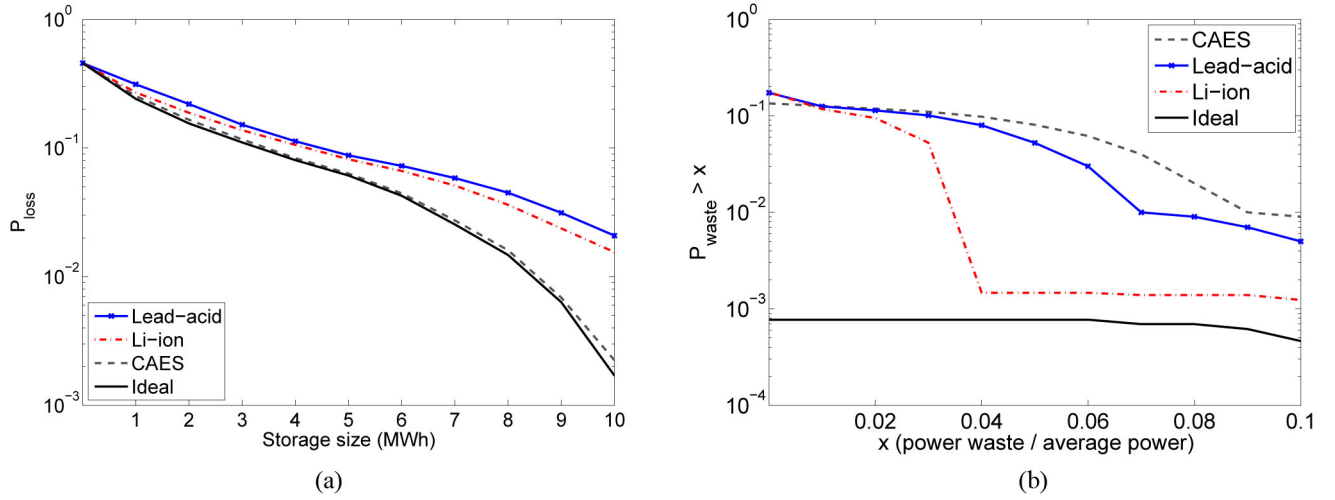


Fig. 3. Impact of ESD imperfections on  $P_{loss}$  and  $P_{waste}$  for different ESD technologies. The Y-axes are both logarithmic. (a) Loss of power probability ( $D(t) = 0.1\text{MW}$ ). (b) Waste of power probability ( $D(t) = 0.5\text{MW}$  and  $B = 10\text{MWh}$ ).

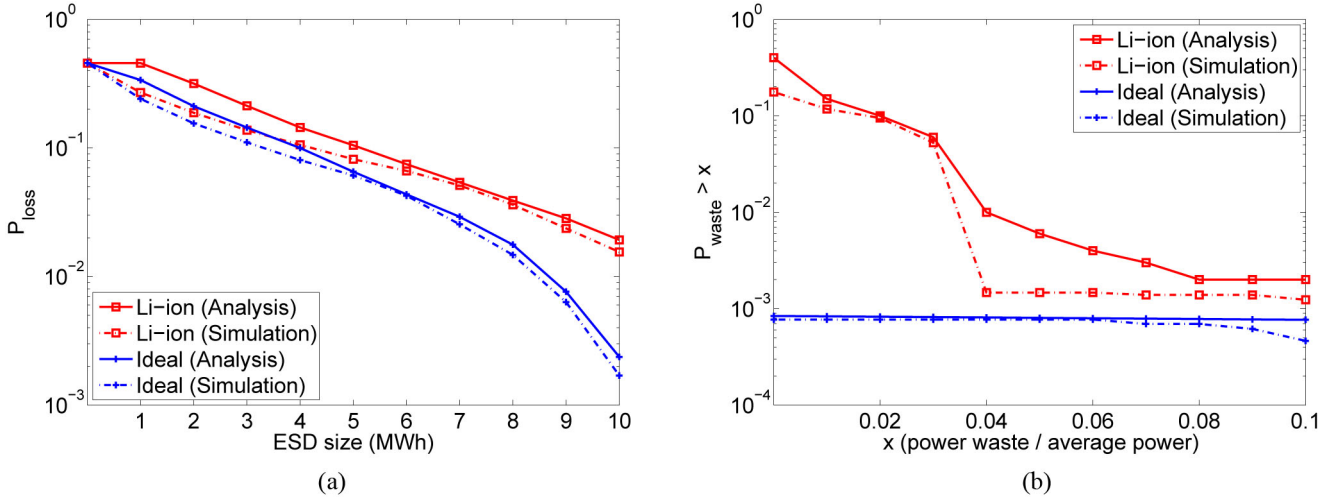


Fig. 4. Evaluating the accuracy of our analysis. (a) Loss of power probability ( $D(t) = 0.1\text{MW}$ ). (b) Waste of power probability ( $D(t) = 0.5\text{MW}$  and  $B = 10\text{MWh}$ ).

of charging rate limit and DoD on the loss probability are significant. This is inferred from the nonnegligible difference between the Li-ion and lead-acid curves with that of an ideal storage and noting that charging rate limit and DoD are the main restrictive physical constraints of these two technologies.

Fig. 3(b) shows the tail bound distribution of the waste of power probability. This graph highlights the fact that ignoring the physical constraints of ESDs drastically underestimates the required resources for all ESD technologies. Moreover, the choice of ESD technology highly impacts the waste of power. For the parameter choices we made in this example, the overall waste of power arises from ESD inefficiency, self-discharge, and limited storage size [see (13)]. The ideal storage curve captures the contribution of the limited storage size to energy waste. The difference between the ideal curve and the other curves is due to ESD inefficiency and self-discharge. We observe that the waste of power for ESD technologies in this example is mainly due to their small efficiency. Thus, Li-ion which has the largest efficiency has the smallest waste and CAES which has the smallest efficiency has the largest waste.

The sharp drops that we observe for these three technologies indicate that one of the sources of waste of power (e.g., storage inefficiency) is bounded by a threshold.

#### B. Accuracy of Analytical Performance Bounds

Section VI-A computed performance bounds through numerical simulations. Here, we evaluate the quality of our analytical bounds from Theorem 2 in Fig. 4. Though not shown here due to lack of space, we first used QQ-plots to verify that simple exponential decay functions do adequately characterize the stochastic wind power process. This allows us to use (30)–(31) as performance bounds and to compare the numerical and analytical performance bounds of the selected ESD technology.

Fig. 4(a) compares the loss of power probability and Fig. 4(b) the waste of power probability. Our analytical bounds closely match numerical simulations, especially for loss of power probabilities. This demonstrates the usefulness of our analytical approach.



## VII. CONCLUSION

By smoothing out variations in the supply and demand processes, ESDs are likely to play a significant role in all future energy systems. Sizing and choosing the best ESD technology is a critical task, that is usually accomplished using extensive simulations. Apart from time complexity, dimensioning through simulation requires large datasets which are often difficult to obtain.

An analytical approach, in contrast, requires smaller datasets and allows great flexibility in choosing design parameters. In recent work, an elegant mapping between smart grids and queuing systems in teletraffic theory has enabled the performance analysis of energy systems drawing on a large and sophisticated literature in this area [10], [12], [13]. A critical issue is that an analytical model should model nonideal ESD behavior, rather than assuming them to be identical to memory buffers in the Internet, as is the case in current work.

Our first main result is that by defining virtual source and demand processes, we can use teletraffic analysis even for a nonideal ESD. Our second main result is to extend prior work using stochastic network calculus to compute upper bounds on the loss of power and waste of power probabilities for nonideal ESDs. Using numerical simulations over a large dataset of wind power, we show that.

- 1) The storage imperfections have a significant impact on the performance of energy systems.
- 2) Our analytical bounds for nonideal ESDs are quite tight.

Our work is important in that it provides a general framework to analyze stochastic energy systems with ESDs for a class of ESD technologies whose state of charge evolution can be expressed by (1). This class includes important technologies such as batteries and CAESes. Yet, we recognize that it does not model some classes of ESD imperfections (most importantly SOH and temperature) that are still poorly understood in the literature. In future work, we plan to incorporate these into our nonideal ESD model and extend our analytical formulation to a larger class of ESDs to include other important ESD technologies such as supercapacitors and flywheels.

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