A Stochastic Optimization Framework for Anticipatory Transmission Investment

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by

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Declaration of Originality

The material contained in this thesis is my own work, except where other work is appropriately referenced. Any use of the first person plural is for reasons of clarity.

Abstract

An unprecedented amount of renewable generation is to be connected to the UK grid in the coming decades, giving rise to new power flow patterns and warranting unprecedented amounts of transmission investment. However, significant uncertainty surrounds the state of the electricity system, primarily in terms of the size, location and type of new generators to be connected. These sources of uncertainty render the system planner unable to make fully informed decisions about future transmission investment.

This thesis presents a stochastic formulation for the transmission expansion planning problem under uncertainty in future generation developments. The problem has been modelled as a multi-stage stochastic optimization problem where the expected system cost is to be minimized. Uncertainty is captured in the form of a multi-stage scenario tree that portrays a range of possible future system states and transition probabilities. A set of investment options with different upgradeability levels and construction times have been included in the formulation to capture the diverse choices present in a realistic setting, where the planner can choose to invest in an anticipatory manner. A novel multi-cut Benders decomposition scheme is used to render the model tractable for large systems with multiple scenarios and operating points. The developed tool can identify the optimal long-term investment strategy based on the triptych of economic efficiency, adequate security provision and acceptable risk.

Simulation results on test systems validate that the stochastic approach can lead to further expected cost minimization when compared to methods that ignore the planner's decision flexibility. Moreover, decisions are taken with subsequent adaptability in mind. The benefit of keeping future expansion options open is properly valued; investment paths that enable future delivery at lower costs are favoured while premature project commitment is avoided.

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Abbreviations

CBA	Cost-benefit analysis
CVaR	Conditional Value-at-Risk
DC OPF	Direct Current Optimal Power Flow
DSCOPF	Decomposed Security Constrained Optimal Power Flow
FACTS	Flexible AC transmission systems
GB	Great Britain
IEEE RTS	IEEE Reliability Test System
N-1	Single line outage
OFGEM	Office of gas and electricity markets
OPF	Optimal Power Flow
QB	Quadrature Booster
ROV	Real Options Valuation
SC	Series Compensator
SCOPF	Security-Constrained Optimal Power Flow
VaR	Value-at-Risk
VOLL	Value of lost load

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1 Introduction

1.1 Motivation

Many countries across the world have committed to a drastic decarbonisation of their economy with legally binding targets. In the UK, the government has pledged an 80% reduction in greenhouse gas emissions by 2050 [1]. In mainland Europe, the European Council has adopted a bill that dictates 20% reduction in CO_2 emissions and 20% uptake of renewables in total energy consumption in the EU by 2020 [2]. This global drive towards renewable energy sources is expected to lead to an unprecedented volume of transmission investment to accommodate the emerging flow patterns. Network expansion costs in Europe have been estimated to €182 billion by 2050. In the UK, transmission investment by 2020 is estimated to reach £17 billion, of which £8bn would be used for connecting offshore wind farms to the main system [3]. Given the very high level of transmission investment that electricity systems will experience in the near future, it is essential to ensure that decisions are taken in a cost-efficient manner and supported by a regulatory regime that encourages long-term economic efficiency.

Transmission expansions have been traditionally based on two fundamental premises. Firstly, investment plans are justified and supported from a strictly technical point of view with respect to pre-defined security standards and the need to reliably meet peak demand. Economic benefits associated with transmission expansion projects are not quantified, leaving little room for market-driven investment. Secondly, knowledge of future generation location and size is certain. As a result, planning has been limited to a strictly reactive exercise where investment is undertaken on a project-by-project basis, following request of connection by a new generator and backed by some financial commitment. Under these premises, anticipatory investment, where a transmission expansion project is undertaken beyond the current system needs, is outside the planner’s scope of activities. However, given the changing circumstances of the electricity market, there seems to be a pressing need to consider anticipatory investment as a viable activity that can lead to significant cost savings and accommodation of new generation in a timely manner. The factors contributing to this paradigm shift are summarized below.

i. Significant uncertainty surrounds the size, location and type of future generation. Power generation and transmission are complementary activities that have to be coordinated efficiently for the optimal development of the electricity system. Under the unbundling of the electricity sector, this coordination is more difficult due to the investors' freedom to choose the location and plant type that will provide maximum profitability. On the other hand, the system planner is bounded by mandate to ensure that no discrimination takes place against new entrants. This level playing field ensures competition between generators but at the same time presupposes that sufficient transmission capacity can be allocated to all market players. However, transmission capacity is a scarce resource. Real-time balance between supply and demand in the presence of transmission constraints is managed through the procurement of costly balancing services from out-of-merit generators. As a result, the connection of new generators to an already congested system leads to excessive constraint costs.

ii. This negative effect on constraint costs is amplified due to the very nature of renewable generators. Wind-rich areas are far away from the load centres and located in transmission-constrained zones. As a result, the existing infrastructure cannot support the energy export levels required in the presence of a large wind fleet. For example, in the UK, 9 out of the 17 generation zones have been deemed as having a very low opportunity level for new connections (defined as 0 GW margin for new generation connections), with the majority being in the wind-rich north of the country. Traditionally, locational use-of-system charges have provided suitable price signals to generation investors, discouraging them from developing new plants in severely congested areas. However, given that wind resource quality is the primary driver of a wind farm's profitability, investors are willing to face higher tariff charges for increased output levels. This has resulted in a growing queue of new entrants located in constrained generation zones awaiting access to the system.

iii. There is increased difficulty in establishing new power flow corridors and the development of new right-of-way power lines faces strong opposition. This has resulted in increased commissioning times for new transmission projects [5]. Combined with the fact that renewable generators have considerably shorter construction times than large fossil fuel and nuclear plants, a situation has emerged where new entrants are connecting to the system much faster than the grid can be upgraded to accommodate them [76]. The result is mounting constraint costs while transmission licensees are entrenched in time-consuming processes to secure planning permissions.

1.2 The case for anticipatory investment in the UK

So far, in the UK, the above factors have already come into play, challenging the traditional transmission planning framework. In May 2009, Ofgem announced its intention to grant derogations from the transmission system security standards in order to facilitate generation projects connecting to the grid and accelerate their access dates [6]. This was initially adopted as a temporary solution, but has since then been implemented as an enduring practice. Thus a 'connect and manage' approach is being followed, under which new generators are able to connect in transmission-constrained areas and export their power to the grid, prior to the necessary reinforcement works being completed. As of February 2011, the System Operator had submitted connection offers to 73 prospective generation projects, advancing their connection date by an average of 5 years. Overall, it is envisaged that up to 19 GW of offshore wind, 1.8 GW of onshore wind and 4.5 GW of other types will be allowed to connect to the GB grid through the 'connect and manage' access regime by 2020 [7]. Although substantial transmission investment is taking place to increase the transfer capability of the network, it is believed that the system will be unable to fully accommodate the power flows that will arise from these new connections. Significant wind energy will have to be curtailed and replaced with out-of-merit local generation to balance the system. This will lead to considerable operational costs due to congestion, which under the current regime are socialized across all network users. The System Operator has claimed that the incremental constraint costs for GB under 'connect-and-manage' could be between £300m and £1bn up to the year 2017/18, depending on the contracted background. This is a significant cost that could prove much larger than the associated carbon offset benefits, leading to a conflict between cost efficiency and uptake of low carbon sources of energy.

This high level of potential constraint cost warrants further transmission investment to be undertaken in anticipation of new connections, on the ground of economic efficiency. The optimal transmission capacity will be determined on a cost-benefit basis and depend on the relative magnitude of the marginal constraint costs and transmission investment. However, the likelihood that all prospective generation projects will indeed be commissioned is low. In the absence of firm financial commitments, generation developers are able to declare their interest for connection without any penalty in the event that the new plant is not built. As a result, the planner is at a severe information disadvantage on how to reinforce the system in a cost-efficient manner. In view of this increased uncertainty, deterministic planning approaches become irrelevant and a move towards anticipatory decision rules is essential. However, under the existing transmission investment regulatory framework, RPI-X, forward-looking investment to alleviate potential future congestion has been beyond the scope of network licensees.

In recognition of the uncertain electricity market landscape, the growing queue of potential new entrants and the network's inability to effectively accommodate them, a number of reviews were recently launched to determine the ability of the current regulatory frameworks to provide the necessary incentives for the long-term efficient connection of new generators. Project TransmiT [8] is an on-going review of network use of system charges, focusing on identifying suitable market arrangements that can facilitate the timely connection of new generation. In addition, a review of the codified security standards (SQSS) is under way to quantify the benefits of adopting probabilistic security criteria against the established deterministic approach. It is envisaged that defining transfer capabilities closer to the operating timescales can allow fuller utilization of existing assets and in some cases alleviate the need for construction of new power lines with little impact on system reliability. This way, a larger part of the available wind energy will be accommodated prior to major infrastructure works being completed.

1.2.1 RIIO regulatory framework

Of most interest is the extensive review of the existing transmission investment and revenue regulatory regime that was recently conducted by Ofgem under the name RPI-X@20. The review identified points of improvement to the status quo that will enable network companies to meet the challenges of delivering the transmission projects required for an efficient low-carbon energy sector. The result of this consultation was a decision document [20] published in October 2010 that initiated the move from RPI-X to a new regulatory regime known as RIIO (Revenue = Incentives + Innovation + Output) [9] that will come in effect from April 2013.

The main aim of the RIIO model is to ensure that transmission licensees play a more central role in the delivery of a sustainable energy sector and provide value for money network services to existing and future consumers. In view of the large generation additions to be carried out in the future, it encourages network companies to move from a case-by-case planning role, to a more strategic long-term action plan. Most importantly, network companies are now expected to take a proactive stance; understand and anticipate the changing needs of consumers of network services and respond appropriately[1]. Under the new arrangements, network companies can choose to undertake proactive network investment based on their view on future generation developments, signalling a shift from past reactive investment frameworks based on strict interpretation of the SQSS standards. Central to RIIO is the requirement for submission of a robust business plan where the planners are to outline their view of the future and justify their proposed investment plan. As outlined by Ofgem [9], the submitted business plans should aim to demonstrate:

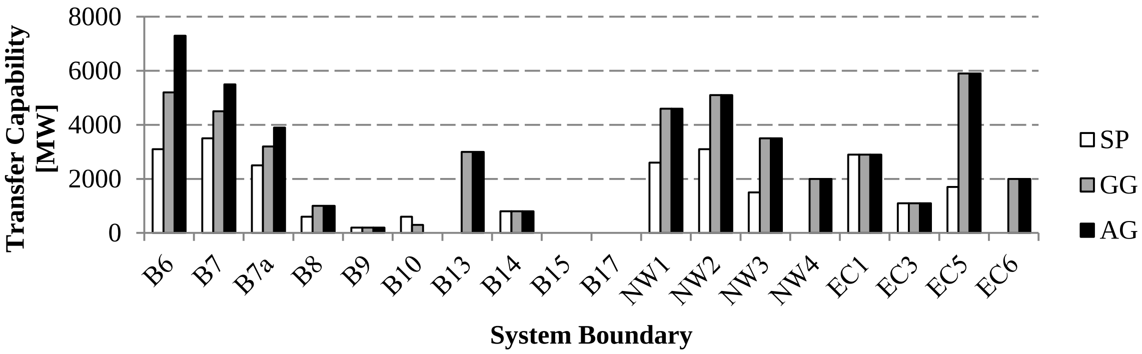
i. **Consideration of all available options:** The regulator has explicitly stated that evidence of examining alternative delivery plans should be included, focusing on the potential evolution trajectories of the system. The network company must demonstrate that it has considered potential deviations from the envisaged future and justify why the chosen plan was deemed the most suitable.

ii. **Consideration of the longer term:** the company must demonstrate how its proposals for the current eight-year control period sit within a longer-term strategy. It is important to show that the implications of the current proposals to future control periods have been considered and a holistic approach has been adopted. Commitments that impact the long-term evolution of the grid should not be made now if there is sufficient reason to believe that a more informed decision could be taken in the future. This is a novel concept, defined by Ofgem as the ‘value of keeping options open’, according to which network companies are warned to avoid premature lock-in to particular investment paths that will limit the network’s adaptability to alternative scenarios.

In view of these regulatory changes, the business plan submitted by National Grid [60], the primary transmission licensee, for the RIIO price control period 2013-2021, has identified three scenarios describing the possible future generation evolution; Slow Progression (SP), Gone Green (GG) and Accelerated Growth (AG) as defined in [11]. These scenarios present a coherent view of generation additions and closures until 2030 and their main difference lies in the rate of wind generation deployment. Under Gone Green, 40 GW of new generation is to be added to the system until the year 2030 with very significant wind fleet additions totalling 23 GW, 8 GW of new gas-fired plants and the construction of 3 GW of nuclear. At the same time, 25 GW of generation is to be decommissioned due to the Large Combustion Plant Directive and nuclear plant closures. This scenario is consistent with the government’s

[1] Par. 2.2, [20]

decarbonisation target, achieving 15% energy from renewables and 34% in emissions reduction by 2020. Despite the fact that National Grid deems highly unlikely that this scenario will materialize exactly as envisaged, it constitutes its base-case upon which all transmission recommendations are made. The capital expenditure to accommodate new entrants (local generation connections and wider works) in the first price control period under this scenario is £6.3bn. However, considerable deviation from this number is envisaged should another scenario materialize, with an upper bound of £7.1bn under Accelerated Growth and a lower bound of £5.1bn under the Slow Progression future. This is due to the variability in the required system boundary transfer capabilities under the different scenarios as shown in Figure 1-1, highlighting the significant impact of uncertainty on the planning process.



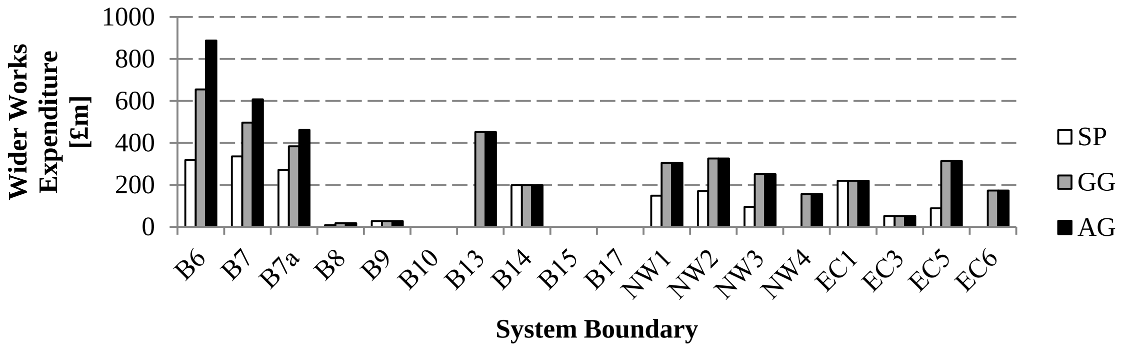


Figure 1-1: System boundary transfer capabilities (top) and the corresponding wider works expenditure (bottom) for the three scenarios under RIIO-T1. Source [60]

The most prominent feature of the planner’s approach is the adoption of a deterministic view of the future, despite the high uncertainty surrounding future developments. The proposed investment plan is based on a framework that does not explicitly consider the costs experienced under alternative scenario realizations. In addition, the cost of recourse actions that would have to be undertaken is not quantified and the option of adopting a ‘wait and see’ stance, until more information is available, cannot be properly valued. As a result, the danger of premature commitment to suboptimal capital projects is very real and well understood by the regulator. As identified by Ofgem [12], the greatest weakness in the submitted business plan is the failure to assess the possibility of alternative delivery plans to the baseline, stressing the need for sensitivity analysis with respect to alternative uncertainty realizations. In addition, Ofgem has commented on the importance of providing evidence of benchmarking against alternative reinforcement projects and considering options that may become available in the future, instead of opting for a 'jumping straight to solutions' approach[2].

The reluctance of network companies to fully encompass the new mode of thinking made possible through RIIO is partly expected, since it requires a fundamental shift to their traditional planning model; from a specific transmission investment plan to be followed over the horizon and targeted to a specific contracted background, to a more strategic evaluation of available investment opportunities under a range of potential futures. Identifying the optimal investment strategy and providing supporting evidence of optimality is a major task that requires the use of novel concepts absent from current approaches. However, there is a clear gap of appropriate tools to assist cost-benefit based planning under uncertainty. The work presented in this thesis aims to fulfill this need and derive a transparent methodology for identifying efficient transmission investment decisions under uncertainty.

1.3 Decision-making under uncertainty

1.3.1 Cost-benefit analysis in a probabilistic framework

Investment decisions can be taken on a cost-benefit basis by weighting between the cost of investment and the potential future constraints cost. This type of analysis can be incorporated in a probabilistic framework once the planner can characterize possible system evolution trajectories through a consistent scenario set, as shown in Figure 1-2. In this case, the planner's objective becomes the identification of investment decisions that will lead to the maximization of the expected net benefit.

[2] [12], p. 20

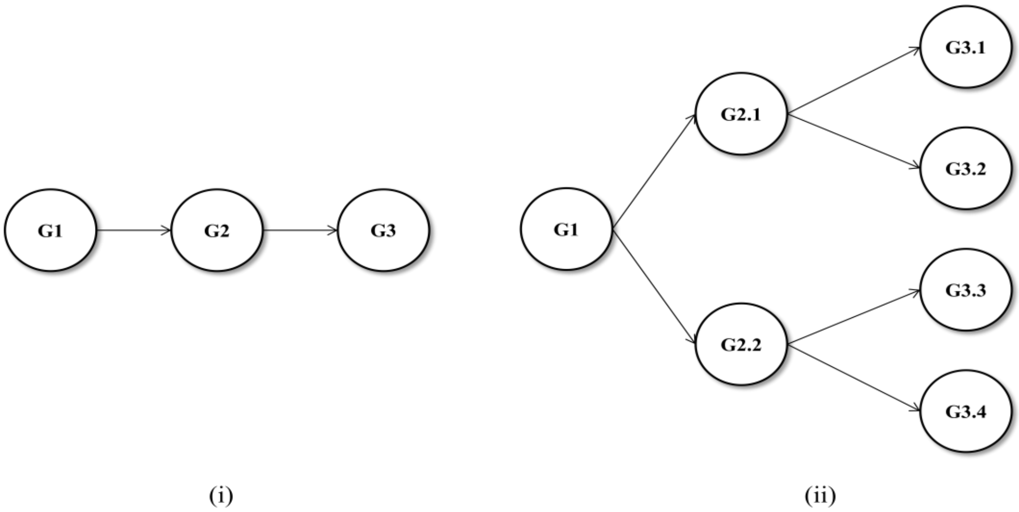


Figure 1-2: Illustration of a (i) deterministic and a (ii) probabilistic description of the future system generation profile.

The need for scenario development that enables the coherent description of uncertainty has already been recognized in the UK, where the move from a deterministic description of the future to a scenario-based approach is already a reality. As mentioned before, the primary transmission licensee, National Grid, has developed three future generation scenarios to reflect different levels of deployment of renewable generation [11]. The fact that scenarios have been embedded in the planning process shows that novel probabilistic approaches are seen as essential to effectively respond to the increased uncertainty that characterizes the electricity market's changing landscape. However, the proper incorporation of scenarios in a consistent cost-benefit framework has yet to occur, with planners choosing to optimize the network solely against the most probable future. In this research, we develop a model that identifies the optimal anticipatory investment decisions that maximize the expected net benefit over a range of scenarios. Under the proposed approach, uncertainty is fully integrated to the decision process and all envisaged futures are considered.

1.3.2 Modelling of decision flexibility

The most important claim of this thesis is that a simple extension towards probabilistic planning is not enough to identify the optimal anticipatory decisions. A parameter that needs to be appropriately considered in this new cost-benefit framework is the planner's ability to dynamically adjust his plans to the unfolding scenario realization. In the context of transmission investment, the concept of flexibility is two-fold. On the one hand, there is flexibility with respect to time. A decision maker can choose whether to go ahead with a project now or adopt a 'wait-and-see' stance for the purpose of making a commitment once the future can be more firmly known. Similarly, there can be flexibility embedded in an investment opportunity itself. This is particularly true in the case of transmission projects that are subject to significant economies of scale and consist of several distinct phases. For example, the planner may choose to undertake preliminary planning actions or engineering works, but can always choose to abandon the project if evidence shows that the reinforcement is unnecessary. Some capital costs will already be sunk, but a large portion can be salvaged with good reactive management. In a different example, the planner may choose to proceed with an upgradeable project whose capacity can be enhanced in the future. The "upgrade" decision must be modelled on a flexible basis as conditional to the unfolding uncertainty. Failure to account for this managerial flexibility results in a self-limiting analysis framework that systematically undervalues investment opportunities able to provide future adaptability.

In the face of a certain future generation background, it is logical to express a transmission expansion plan in the form of sequential decisions that ensure timely connections of new entrants subject to the security standards. However, when faced with an uncertain future, decisions have to be taken with subsequent adaptability in mind and the concept of following a one-dimensional plan is rendered obsolete. In its place, the concept of a flexible expansion strategy emerges, where the transition to a new state can be treated as a trigger event that differentiates the proposed investment plan. Under this paradigm, decisions to be taken now do not simply constitute the first stage commitment of a sequential plan, but sit within a larger strategy that considers the cost of future adjustments to a range of scenario realizations. The concept of embedding decision flexibility in a decision making process is shown graphically in Figure 1-3 with the aid of decision trees consisting of circular chance (system state) nodes and rectangular decision nodes.

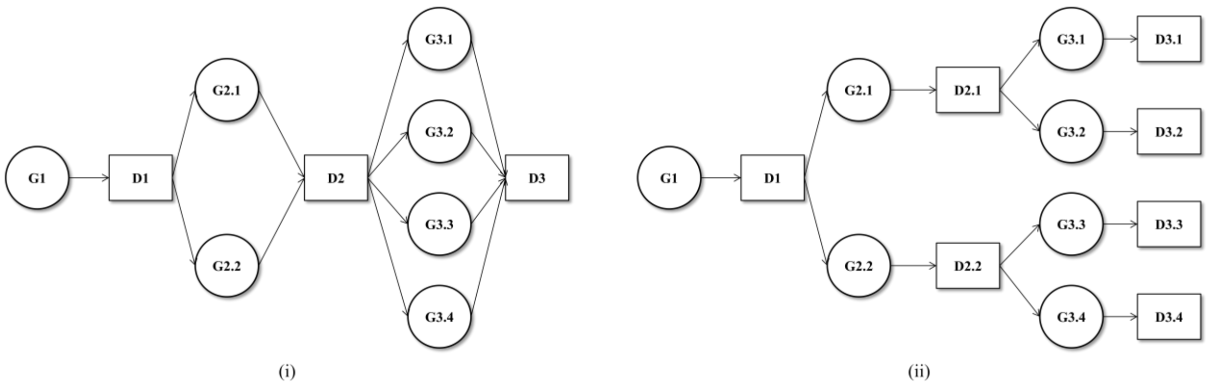


Figure 1-3: Probabilistic decision frameworks (i) ignoring flexibility and (ii) considering flexibility.

1.3.3 Risk management

A final claim of this thesis is that risk management is an important activity that should be embedded within the proposed anticipatory investment framework. Risk is an indispensable part of any business activity that should be properly managed through hedging strategies. Until now, risk aversion in the context of network planning has been limited to the eventuality of equipment failures and addressed through redundancy. In the cost-benefit based transmission planning context, risk is defined as the possibility of facing excessive constraint costs due to adverse scenarios materializing. In light of this, identifying an investment strategy that leads to expected system cost minimization is not enough. Besides cost efficiency, acceptable risk exposure is an equally important characteristic of a robust action plan.

1.4 Research goals

The purpose of this thesis is to investigate the technical and economic aspects of transmission planning under uncertainty. The main idea is the shift from a deterministic plan in the face of uncertainty, to an optimal strategy that properly values future adaptability. In view of the complexity of modern power systems and the large number of investment opportunities, all the afore-mentioned concepts have to be incorporated in an optimization framework where the optimum solution can be identified. This is achieved with the aid of mathematical programming, where we integrate the various components of transmission investment under uncertainty; economy, reliability, flexibility and risk, in a decomposed Mixed Integer Linear Programming (MILP) formulation. The final model constitutes a stochastic decision-making tool that identifies the optimal investment strategy with respect to uncertain future generation sizing and siting. These tasks pose challenges in both the modelling as well as the computational performance aspects. For this reason, significant efforts have been made to achieve a tractable formulation capable of addressing the problem of transmission planning in realistic systems with a large number of nodes and lines. The ultimate purpose of the developed model is to assist in the decision-making process in the current highly uncertain environment and provide evidence of optimality consistent with the concept of long-term economic efficiency while ensuring adequate reliability provision and acceptable risk exposure.

The tool is aimed at transmission planners and regulators and has a number of potential practical uses within the energy system planning process. More specifically, it can aid regulators in evaluating investment need cases and determining whether the benefit of current and future users has been properly considered. Similarly, the tool can be used by planners to demonstrate how specific investment proposals form part of a long-term strategy. Given that the chosen decision criterion is the minimization of an expected system cost, solution optimality can be highly sensitive to the choice of transition probabilities. Although these numbers will be a product of expert opinion and supplementary modelling and constitute a highly-informed view on the future, undertaking sensitivity analysis around them is an essential part of making good use of the presented tool.

1.5 Main research questions and tasks

In this section the main research question are defined, while outlining the relevant undertaken tasks. The central research questions of this thesis are:

RQ1. *How can the traditional security-driven transmission expansion problem formulation be transformed to address anticipatory investment under uncertainty on a cost-benefit basis?*

When placed in an economic efficiency setting, the objective of a regulated system planner is maximization of social welfare, or equivalently, minimization of system costs; the sum of investment and constraint costs. Such deterministic formulations already exist (see [13] for a comprehensive overview) but are inadequate to deal with the problem of uncertainty. In order to address the above question, a number of tasks concerning the modelling of investment and operation must be tackled:

T1.1. *Accommodate the multitude of investment opportunities in a cost-benefit analysis framework while capturing their diverse economic and technical characteristics.*

Two important aspects that are absent from existing transmission expansion models are economies of scale and the delay between investment and asset commissioning. Capturing the economies of scale present in transmission projects is essential in modelling both timing and sizing flexibility. The chosen approach is to express capital costs in terms of fixed and variable components. The planner can choose to incur some fixed costs for the ability to invest in a specific project in the future. On the other hand, incurred variable costs depend on the eventual reinforcement size, with each option limited to a maximum capacity addition. Even though in reality this cost segregation cannot always be explicitly defined, the main idea lies in the compounding nature of fund commitment and the arising flexibility. The decision-maker does not have to treat projects on a 'take-it-or-leave-it' basis, but can be subjected to only a part of the cost for the benefit of making future expansion options available. The eventual timing of full commitment is conditional and at the planner's discretion. The other important parameter is build times. Regarding transmission investment as an instantaneous process renders the problem of optimal timing void. However, construction delays are at the heart of the trade-off between congestion and investment when uncertainty is involved. For this reason, delay parameters have been included for all candidate projects and the delayed commissioning is modelled through the introduction of appropriate optimization constraints. The above considerations result in a cost-benefit framework able to capture all the characteristics that become important in the face of uncertainty.

T1.2. *Develop a model to identify the optimal investment strategy under uncertainty considering decision flexibility.*

Stochastic programming is a method that can appropriately model decision flexibility and is widely used for tackling decision-making problems under uncertainty. Most applications of stochastic programming to the electricity sector are limited to two-stage problems, where 'here-and-now' decisions are taken subject to the ability of subsequent recourse once uncertainty has been resolved. Given that long-term value for money is one of the key driving concepts and that the current uncertain climate promises for continuous changes in the contracted generation background, it is essential to express uncertainty in the form of a multi-stage scenario tree that portrays all possible system states and transition probabilities. Thus, the transmission investment problem is modelled as a multi-stage stochastic optimization problem, where the expected system cost is to be minimised. The program's final solution is not in the form of 'now-or-never' sequential investment decisions but is rather formulated as a coherent strategy, where the transitions to different system states are treated as trigger events that differentiate the proposed investment plan by taking advantage of the new information made available at each investment stage.

T1.3 *Ensure that the proposed stochastic framework can be applied to a large system while coping with the computational load of multiple scenarios.*

The stochastic model leads to large-scale mixed integer-linear problems that are hard to solve with commercial optimizers. This is amplified by the fact that one of the main motivations for this research is the prospective connection of intermittent wind generators. As a result, an expanded list of diverse demand and wind operating points must be considered for an accurate valuation of year-round costs, increasing the problem size even further. For this reason, alternative solution methods are explored. We test three solution approaches with an example case study involving the 24 bus IEEE-RTS. The first is a direct solution with a commercial optimizer, which exhibits severe limitations in terms of CPU time and memory usage. The second is a standard Benders decomposition algorithm, splitting the original problem in an investment and an operational subproblem. Given the complexity of the problem, the appended supporting hyperplanes prove to be inadequate in leading to convergence in satisfactory times. The third is a variant of the Benders decomposition algorithm, known as multi-cut, which appends multiple highly-parameterised hyperplanes to the relaxed master problem. This method proves to be highly suitable for tackling large multi-stage problems with significant saving in computational effort. A further important consideration is the incorporation of security constraints. This is tackled through a sensitivity analysis approach where the relaxed problem ignoring security constraints is initially solved. A screening algorithm identifies binding contingencies which are then appended to the problem, significantly reducing problem size. The developed solution strategy proves to be highly efficient and with the aid of parallel computing, allows the fast simulation of large systems with limited memory requirements.

RQ2. *What is the benefit of moving from a non-flexible to a flexible stochastic transmission investment framework?*

The idea of quantifying the value of flexibility has been expressed in the past (see for example [15]). It can be defined as the difference in expected system cost between the optimum investment strategy and the optimum investment plan (ignoring flexibility). By definition, the value of flexibility is always positive, since the solution space of the inflexible model is a subset of the expanded solution space made available through the stochastic formulation. By studying test systems, we show that this value can be significant, indicating strong evidence for the benefits of considering flexibility in the planning process.

RQ3. *Does ignoring decision flexibility lead to sub-optimal decisions in the present and underestimate the value of keeping future options open?*

We show that ignoring flexibility does not only lead to an overestimation of expected system costs, but can also drive the planner to unnecessarily commit to projects that limit future adaptability. The reason for this sub-optimality is the one-dimensional nature of inflexible investment plans that fail to appropriately value alternative courses of action once some critical uncertainty has been resolved. The inflexible planner has a 'jump to solutions' attitude seeking a plan that performs well on average but remains blind to future adaptability. On the other hand, the flexible planner takes advantage of the gradual resolution of uncertainty and remains well aware of the shortcomings of premature commitment. The above concepts are illustrated by studying test systems and analysing investment behaviour under the two paradigms.

RQ4. *How does the planner's risk attitude influence his investment decisions?*

Risk-neutral problem formulations can expose the system to unfavourable scenario realizations and exceedingly high constraint costs. Suitable risk constraints have to be included to mitigate such possibilities and ensure that the potential constraint costs are acceptable with respect to some confidence level.

T4.1. *Define a suitable risk measure for expressing the risk-averse behaviour of a system planner.*

Risk is an integral part of every investment activity. In the present context, risk is associated to the constraints cost variability. A variety of risk measures exist, but no widely-accepted measure has been established for the quantification of economic risk faced by the system planner. We propose that Conditional Value-at-Risk (CVaR) is a suitable risk measure for the task at hand.

T4.2 *Incorporate the risk-averse nature of the system planner in the stochastic transmission expansion model.*

Following the recent works of Rockafellar and Uryasev [56], we show how CVaR constraints can be incorporated to the stochastic transmission expansion problem formulation, without compromising problem convexity and linearity.

T4.3 *Alleviate the computational burden associated with risk constraints.*

Stochastic problems lead to very large formulations that warrant the use of decomposition methods for their solution in reasonable times. However, the incorporation of risk constraints results in inter-scenario coupling that severely limits the level of decomposition that can take place. We propose a novel implementation of multi-cut Benders decomposition where the decision variables used to model the original risk constraints are partitioned into auxiliary variables approximating the contribution of each individual operating point to the global risk measure. This approach enables us to employ an efficient solution strategy where all operation subproblems are solved independently, greatly improving computational performance.

1.6 Original research contributions

This work has made significant contributions in the area of transmission planning under generation uncertainty, on both a conceptual and a methodological level.

The conceptual contribution focuses on developing a cost-benefit framework capable of tackling the problem of optimal anticipatory investment. This topic has thus far received limited attention but is increasingly becoming more relevant. In this research we have aimed to identify the concepts that become important in the transmission under uncertainty paradigm and incorporate them in an integrated optimization framework. More specifically, the features of the developed model include:

- **Differentiation between fixed and variable investment costs.** This allows the modelling of economies of scale and the sizing flexibility embedded in candidate projects, which can be conditionally exercised in the future, according to the unfolding uncertainty realization.
- **Modelling of delays between investment decisions and asset commissioning.** Although this concept is often disregarded in deterministic formulations, it constitutes one of the key parameters in determining the optimal timing of investment and allows the valuation of adopting a ‘wait and see’ stance until uncertainty is partially resolved.
- **Accommodation of N-1 security constraints.** Reliability considerations are one of the main drivers of transmission investment and thus critical to be properly modelled in our cost-benefit framework.
- **Provision of corrective security through the optimal placement of FACTS devices.** In addition to traditional line reinforcements, the developed model can accommodate investment in quadrature boosters. These flow control devices can play a significant role in the current uncertainty setting due to their ability to re-direct power over more favourable routes and provide corrective security in the event of an outage, leading to a more efficient utilization of existing assets. In conjunction with their fast commissioning

time, FACTS can constitute a very versatile solution to managing uncertainty by postponing investment until more informed decisions can be made.

- **Modelling of decision flexibility.** In this research we illustrate how decision flexibility can be modelled using stochastic programming and show that the explicit consideration of scenario-specific recourse actions is an essential element of an anticipatory investment framework which has a major impact on the optimal ‘here and now’ investment decision.
- **Inclusion of a risk constraint on constraints cost.** In the context of anticipatory investment, it is essential to model the planner’s risk aversion towards the eventuality of exceedingly high constraints cost. By incorporating suitable risk constraints and performing a sensitivity analysis around the level of risk aversion the planner can arrive at a family of solutions and quantify the expected cost of hedging against unfavourable outcomes.

In terms of methodological contributions, the model presented in Chapter 4 is the first published model utilizing multi-stage stochastic optimization to tackle transmission investment under exogenous generation uncertainty while incorporating decision flexibility. In addition, it one of the few models to accommodate commissioning delays in the dynamic expansion decision process, allowing for the formal valuation of anticipatory investment decisions. Moreover, a node-variable approach has been employed to formulate the problem, constituting a significant improvement on traditional scenario-variable formulations that are more practical to implement but involve the use of redundant decision variables and constraints. Furthermore, a novel technique combining a contingency screening module and a multi-cut Benders decomposition scheme is employed to alleviate the model’s severe computational load. The result is a novel MILP model that can identify the optimal investment strategy under uncertainty while allowing for investment in both traditional capacity reinforcements and quadrature boosters.

An additional contribution is the incorporation of CVaR constraints to the stochastic transmission investment problem related to future constraint costs. The resulting large problem is decomposed using a novel scheme that allows independent processing of operation subproblems, significantly reducing solution times and allowing the accommodation of risk constraints while considering a large number of scenarios.

1.7 Thesis Structure

This thesis is organised in six chapters with Chapter 2 describing the general concepts of transmission investment under uncertainty, Chapters 3, 4 and 5 containing the technical aspects of the undertaken work and Chapter 6 summarizing the main findings. The relevant literature reviews are contained within each chapter.

Chapter 2 contains the main concepts upon which the subsequent work is developed. The concept of economic-driven and anticipatory transmission investment is analyzed and the importance of considering decision flexibility is discussed. We approach an example anticipatory investment valuation problem through a flexible Real Options Valuation technique and contrast it to static methods, illustrating the sub-optimality of non-flexible planning. Finally, a review of existing approaches to tackling transmission investment under uncertainty is undertaken and the need for a novel framework is highlighted.

Chapter 3 shows the deterministic transmission investment problem formulation along with techniques for the inclusion of optimal placement of FACTS devices and security constraints. A screening module is used to effectively reduce the number of contingencies considered. Moreover, a multi-cut Benders decomposition scheme is used to ameliorate the model's computational performance and render it suitable for the simulation of large systems. Finally, an extended case study on the IEEE RTS is undertaken to highlight the new model features and quantify the computational benefits of the proposed solution strategy.

Chapter 4 presents the multi-stage stochastic investment problem in its compact node-variable form. Uncertainty is expressed in the form of a scenario tree describing future generation evolution. Through the modelling of construction delays, economies of scale and decision flexibility we capture the principal characteristics of anticipatory transmission expansion. We highlight the benefits of considering flexibility in the decision process and show how the proposed stochastic framework can evaluate the benefit of adopting a 'wait-and-see' stance. A case study on a small system is presented to illustrate how a non-flexible planner can arrive at suboptimal solutions due to the undervaluation of projects with embedded upgradeability options. A further case study on the IEEE RTS is undertaken involving locational and sizing uncertainty of future wind generation to be installed in the system. This large modelling task allows us to quantify the benefits of using the proposed decomposition techniques.

Chapter 5 presents the risk-constrained stochastic formulation. Risk aversion towards excessive congestion has been modelled through a CVaR risk constraint that limits the planner's exposure to adverse scenario realizations to acceptable levels. A case study on the IEEE RTS illustrates how 'here and now' decisions are altered depending on this risk attitude and how a sensitivity analysis can be undertaken to identify the family of optimal solutions for varying levels of risk-averseness.

Chapter 6 summarizes the achievements of this thesis, presents the main conclusions and identifies some future work to be undertaken.

2 Theoretical Background

Abstract

This chapter presents the main concepts of transmission investment on a cost-benefit basis and how they can be extended to the case of anticipatory investment. The notions of managerial flexibility and uncertainty resolution are introduced and their importance highlighted through a case study involving the construction of an interconnector under generation uncertainty. By comparing the optimal investment decision identified through the traditional Net Present Value technique and Real Options Valuation, we quantify the benefit of shifting towards flexible decision frameworks. A review of optimization methods for tackling transmission expansion under uncertainty is undertaken. Techniques relying on deterministic scenario analysis are found to be inadequate for the task at hand. Moreover, multi-stage stochastic optimization models developed in the past fail to consider various aspects of transmission planning that become important under generation uncertainty.

2.1 Introduction

In electricity systems, the transmission network is the key interface that allows buyers and sellers to interact. Adequate transmission is one of the prerequisites for an efficiently functioning and competitive market. In most unbundled energy systems, the planner is a regulated monopoly mandated to carry out transmission investment with the aim of maximizing social welfare, providing non-discriminatory access to all network users and facilitating competition among participants. Lack of investment will constrain access of merit plants, giving rise to constraint costs and may increase the use of market power, while over-investment will incur higher cost to the customers that finance the network.

2.1.1 Cost-benefit based transmission investment

Historically, network design had been driven by the need to meet peak demand with sufficient reliability. In systems dominated by high capacity value thermal generators, this approach has led to economically efficient solutions. However, under high penetration of intermittent sources of energy, that have a much lower capacity value, accommodating peak flows during high demand seizes to be the primary investment driver. Ensuring that enough capacity exists to accommodate the simultaneous peaking output of wind farms makes no economic sense in the absence of appropriate energy storage technology. As a result, transmission investment is undertaken on a cost-benefit basis where the solution that minimizes total costs is pursued. The trade-off between investment and operational costs is illustrated in Figure 2-1. As more capacity is added, investment costs increase while it becomes possible to dispatch the system in a more economic way, leading to reduced operational costs. In the absence of sufficient transmission investment, congestion becomes a considerable burden as out-of-merit generators are dispatched in place of more economic units, giving rise to costs of constraints. Finding the right balance that minimizes the overall system costs including cost of transmission investment, cost of constraints and cost of unserved load constitutes the *transmission expansion problem* (TEP). Under the assumption of inelastic demand, this is equivalent to a social welfare maximization. Note that even though welfare is expressed in purely monetary terms, it is possible to optimise non-financial attributes, such as the quality of service or environmental impact, by using appropriate estimates (e.g. penalise demand curtailment using the Value of Lost Load).

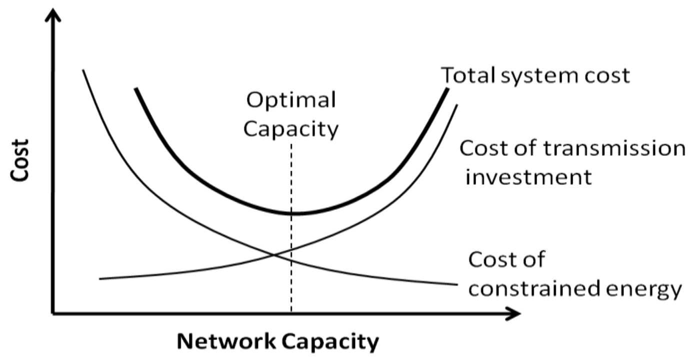


Figure 2-1: Trade-off between capital and operational costs in transmission investment

TEP on a cost-benefit basis has been a widely discussed topic in literature and has traditionally been based on the premise of system cost minimization under a deterministic view of the future. When equipped with perfect foresight, the planner can decide on network investment timing and sizing reinforcements in a straightforward way, accommodating new plants as they become operational. Capital projects are undertaken in a reactive fashion to the users' needs and the main planning challenge is delivering the optimal investment schedule in a timely and economic manner. However, in view of the increased uncertainty of the type, size and location of future generation and the rapid deployment of renewable sources of energy under the government's pressing targets for decarbonisation, this paradigm is no longer relevant. Private generation investors proceed with new plant development independently, without prior co-ordination with the system planner, and demand market access despite the network's inability to fully accommodate the arising flows. Under this uncertainty, the planner can choose to remain reactive and proceed with connections as new information is being revealed or invest in an anticipatory manner [16].

2.1.2 Reactive and anticipatory investment under uncertainty

The stochastic generation background renders the planner unable to make fully informed decisions on the optimal timing and sizing of transmission investment. With respect to timing, when faced with a prospective generation connection, the planner has two choices. The first one is to remain reactive and take decisions on transmission investment only after ensuring that the assets are set to materialize and the undertaken capital expenditure is fully justified by the envisaged future system state. Under this approach, the system will face the corresponding cost of constraints from the time the generator is commissioned until the relevant reinforcements have been carried out. The second choice is to anticipate the connection of new generation by undertaking transmission investment pre-emptively. As a result, the transmission system is able to accommodate the prospective generation faster, leading to significant savings in terms of constraint costs. However, there is a risk of sub-optimal investment and sunk costs in case that the envisaged scenario does not materialize.

Optimal reinforcement sizing is also a non-straightforward decision. In some cases, undertaking capital expenditure beyond what might appear sufficient for the short-term future can prove beneficial in the long-run. This is because transmission projects have large fixed costs (e.g. right-of-way, environmental and engineering studies etc.) and are thus subject to significant economies of scale. It follows that proceeding with sequential marginal capacity increases to accommodate new entrants on a one-to-one basis can be a very cost-inefficient practice. Undertaking large fixed costs in advance for the option of delivering additional capacity at a lower cost in the future can prove to be an attractive opportunity. On the other hand, anticipatory investment of this nature might prove unneeded depending on the eventual scenario realization, resulting in stranded costs.

From the above it follows that the introduction of generation uncertainty leads to a complex interplay of long lead times, scale economies and constraint costs. Most importantly, the notion of valuing subsequent adaptability of present commitments is introduced. These concepts lead to a much different problem formulation than the traditional TEP and new modelling approaches are required capable of capturing all transmission investment characteristics that become important when dealing with an uncertain generation background. In an extended review of existing TEP models [13], Lattore et al. identified the lack of published work focusing on future system uncertainty and its increasing relevance to the practical planning reality following deregulation of electricity sectors worldwide. In the past, the inadequacy of deterministic planning in the face of a highly uncertain future had already been recognised and theoretical frameworks debating the most appropriate decision rules developed [17]. In addition, the benefits of considering flexibility and adaptability as well as shifting from sequential long-term plans to strategies able to cope with unfavourable unfolding of uncertainty have also been well documented [18].

However, efforts to develop a rigorous cost-benefit framework able to properly model timing and sizing flexibility under uncertainty have been limited to investment appraisal tools suitable for the valuation of individual candidate projects. Traditional non-flexible valuation techniques define the principal value of an investment project as the difference in net benefit between proceeding with the investment and choosing a “do nothing” approach. This core value is calculated on the assumption that a commitment is made at the start of the forecast period to a particular plan which is to be followed without adjustment [81]. In order to properly value the timing and sizing options inherently embedded in most investment projects, it is essential to consider all alternative courses of actions that can be taken, choosing the most appropriate recourse action as new information is being revealed. Real Options Valuation is a tool that is capable of capturing this option value and is increasingly gaining attention as a suitable valuation method under uncertainty.

2.2 Valuing a transmission project under uncertainty

In this section two different risk-neutral approaches for valuing a transmission project are presented and discussed. Both decision frameworks rely on the notion of discounted cash-flow analysis to determine the value of a project under uncertainty, with their main difference being the treatment of decision flexibility. On one hand, we have Net Present Value (NPV) decision rules stating that the candidate project maximizing the expected net benefit should be chosen. Despite its long-standing status as a standard investment appraisal tool, NPV has been severely criticized for its static nature (see for example [24]). On the other hand we have Real Options techniques which can take advantage of the flexibility embedded within investment opportunities. We showcase the difference between the two methods through a simple project valuation case study and discuss the importance of modelling decision flexibility in the context of anticipatory transmission investment.

2.2.1 Case Study description

The case study that follows is intended to capture a typical investment problem faced by a regulated system planner aiming for system cost (investment and constraint costs) minimization under generation uncertainty. It has been reduced in scope to illustrate more comprehensibly the treatment of decision flexibility in NPV and ROV.

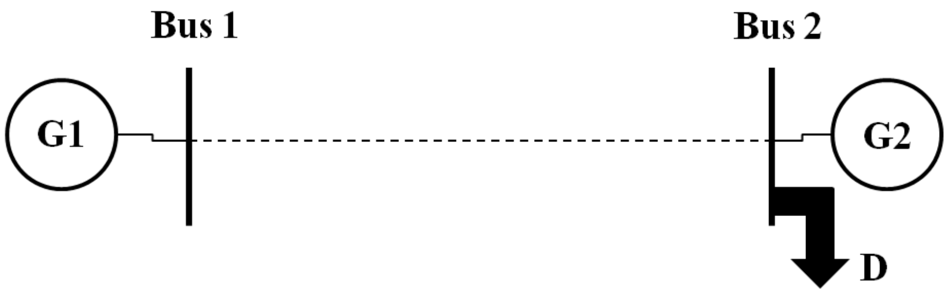


Figure 2-2: Case study two bus-bar system.

As seen in Figure 2-2, a two busbar system is to be connected with a transmission link. Generator G2 can adequately supply local demand D. However, cheap renewable generation is to be gradually connected to Bus 1, displacing part of G2 in the merit order. The case study duration spans three five-year stages (or epochs), giving a planning horizon of 15 years. Uncertainty lies in the new plant's size evolution. The prospective generation development consists of two consecutive phases; both phases are 400MW giving a potential maximum capacity of 800MW at the end of the planning horizon. We assume that the planner has an informed view of the future and using expert opinion as well as forecasts on the potential future profitability of the plant is able to assign probabilities to the various events that may occur [74]. These are captured in a three-stage scenario tree consisting of 7 possible system states and 4 scenario paths, shown in Figure 2-3. The first stage represents the current state of the system, where the cheap generation is yet to be connected ($X_1^1 = 0$ MW). In the second epoch, there is a 0.7 probability that the first phase target of 400MW is reached and a 0.3 probability that construction falls behind schedule. In that case, second phase development is postponed beyond the planning horizon and the probability of successfully commissioning the first phase target in the last epoch is 0.4. On the other hand, successful first phase commissioning can lead to the second phase being completed by the third epoch with a probability of 0.7.

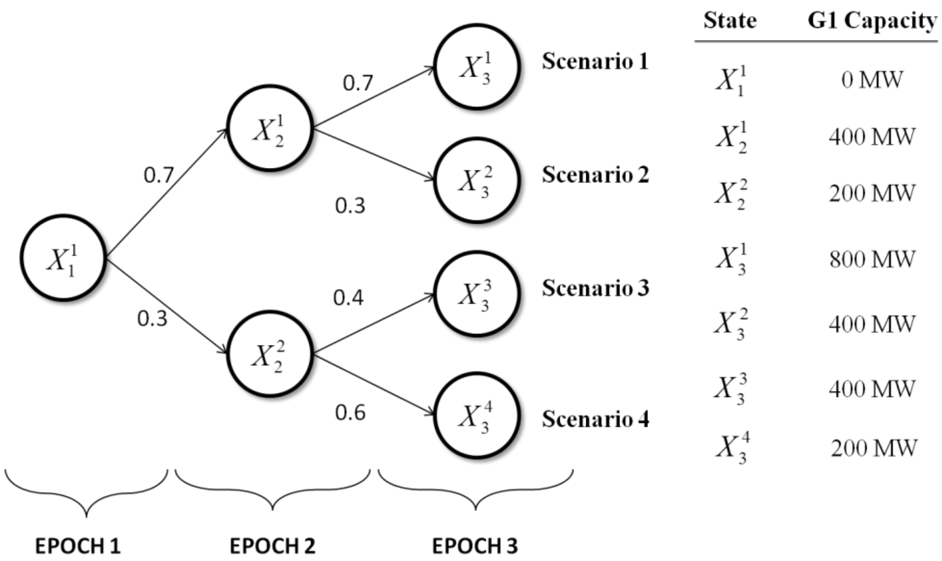


Figure 2-3: Scenario tree for transmission project valuation case study.

Demand D is assumed to always be above 800 MW and the cost of constraining G1 is £30 £/MWh. This cost is assumed to stay constant over the horizon. Each epoch spans five years consisting of 8,760 hours and the potential constraint costs experienced at each system state when foregoing link investment can be calculated as:

$$CC(X_e^j) = \sum_{e=1}^{N_E} r_e^O \cdot 8760\left(30 \cdot X_e^j\right)$$

Where r_e^O is the discount factor related to epoch *e*, and X_e^j is the capacity of G1, as defined in Figure 2-3. Assuming an annual discount rate of 5%, the calculated constraint costs are presented in Table 2-1. The expected constraint costs over the horizon is $ECC^* = £798.973m$

	Scenarios			
	1	**2**	**3**	**4**
Epoch 1	0	0	0	0
Epoch 2	384.949	384.949	192.474	192.474
Epoch 3	586.740	293.370	293.370	146.685
Total Cost	971.689	678.319	485.844	339.159
Expected Cost	798.973			

Table 2-1: Constraint costs (in £m) when foregoing link investment.

In the presented case study, the planner can choose between two candidate projects; one candidate that is certain to be sufficient in the medium-term and an upgradeable project with an extension option. More precisely the candidates are:

(1) A 400MW link with a capital annual cost of £12.5m/yr.

(2) An 800MW link with a capital cost of £18.5m/yr. However, the link's effective capacity is limited to 400MW until additional system reinforcements can be carried out. This project gives the option of full capacity utilization at an additional annual investment cost of £12.5m.

Both candidates have a construction delay of 5 years. The investment cost relating to the 15 year duration of the case study (total cost) have been calculated in Table 2-2 assuming an annual discounting rate of 5%. Projects 1 and 2 start construction on epoch 1 and are commissioned in epoch 2 while the Project 2 extension starts construction on epoch 2 and is operational for the five years of epoch 3.

Project	Total Cost	Capacity
Project 1	£135.04m	400 MW
Project 2	£202.56m	400 MW
Project 2 + extension	£281.278m	800 MW

Table 2-2: Transmission investment options

In the following sections we show how project value is calculated in an NPV and an ROV framework and how they can arrive at different answers as to which project is most cost-efficient and should be pursued.

2.2.2 Net Present Value

In general, NPV can be described as the difference between discounted cash inflows and discounted cash outflows. It is a valuation technique to analyze the profitability of an investment project. Once applied to the candidate investment plans, the candidate with the highest expected NPV should be selected. In the context of transmission planning, the NPV of a project is the difference between the expected constraint costs experienced in the absence of investment and the expected system cost (investment and constraint costs) when choosing that project. As a result, choosing the best project is a straightforward minimization problem of the form:

$$\max_{H_i}\left\{NPV(H_i)\right\} \qquad (3.1)$$

Where the NPV of project H_i is defined as the difference between ECC^* and the time-discounted and probability-weighted sum of investment and constraint costs given by (3.2):

$$NPV(H_i) = ECC^* - \sum_{e=1}^{N_E} \left\{ r_e^I I_e(H_i) + r_e^O \sum_{j=1}^{J_e} p_e^j O_e(H_i, X_e^j) \right\} \quad (3.2)$$

Where r_e^I and r_e^O are the multi-year investment and operation discounting factors for epoch e, $I(H_i)$ is the annual investment cost of project H_i and $O_e(H_i, X_e^j)$ is the annual constraint cost at epoch e under project H_i given that system state j materializes with the corresponding probability of p_e^j. These calculations can also be shown graphically by using a standard decision tree consisting of square decision nodes and circular system state (chance) nodes. Investment costs are displayed above the corresponding decision node while constraint costs (if non-zero) are displayed above system state nodes.

As can be seen in Figure 2-4, if Project 1 is commissioned, constraint costs are experienced only under scenario 1 which involves successful commissioning of both phases of the prospective generator. The resulting Net Present Value of Project 1 is $ECC^* - £135.04m + 0.49(£278.793m) = £798.973m - £278.793m = £520.180m$.

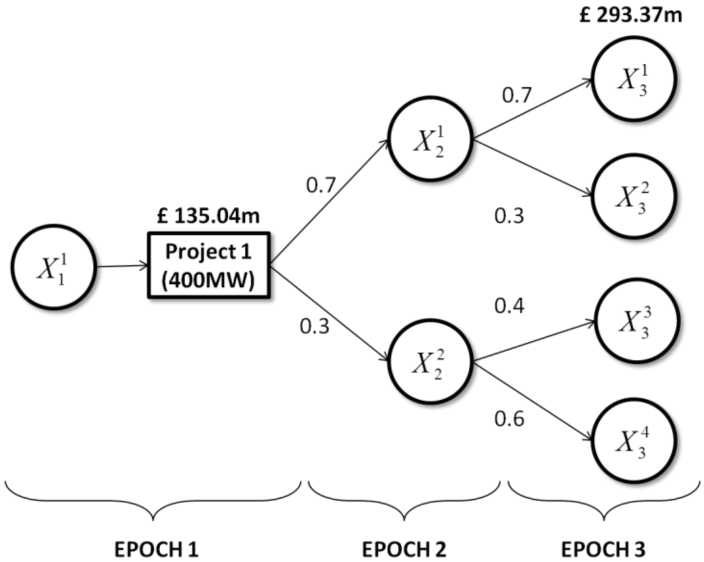


Figure 2-4: NPV analysis for Project 1.

Performing calculations in a similar fashion, the Net Present Value of Project 2 without the extension (Figure 2-5) is £452.659m, while including the extension (Figure 2-6) leads to an NPV of £517.693m. Since going ahead with the extension leads to further cost minimization, the NPV of Project 2 is £517.693m.

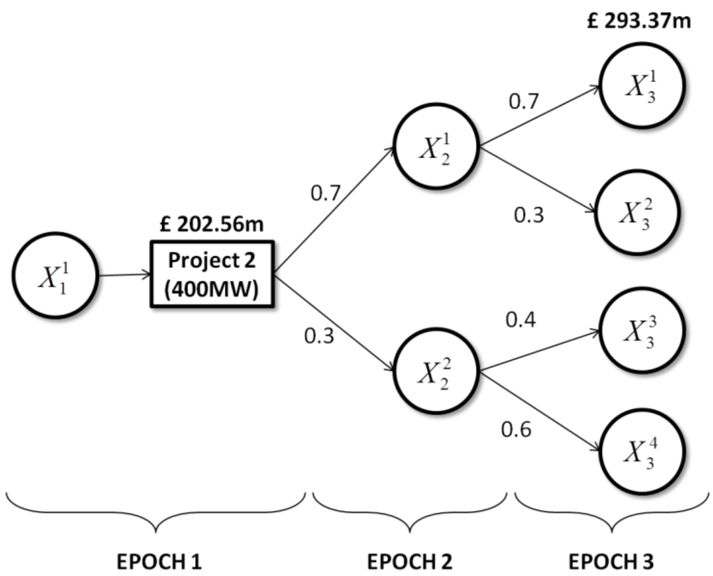


Figure 2-5: NPV analysis for Project 2 without extension.

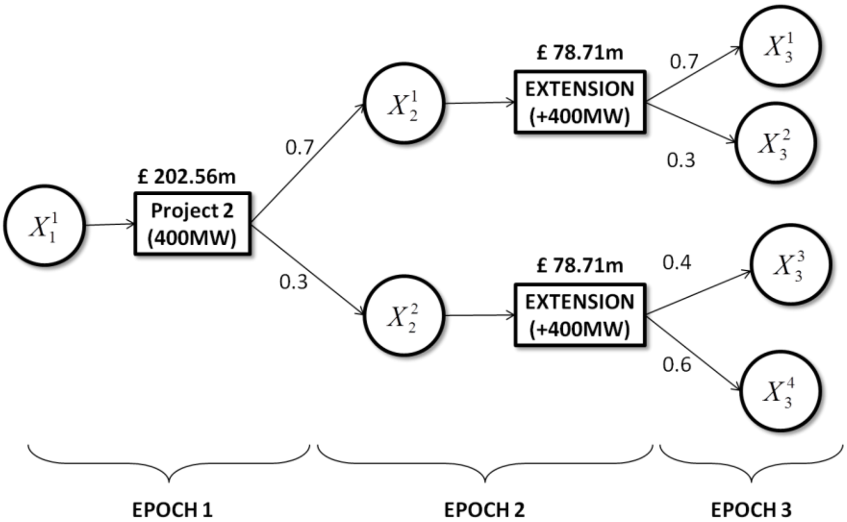


Figure 2-6: NPV analysis for Project 2 with extension.

The results are summarized in Table 2-3.

Project	Capital Cost	Expected Constraint Cost	Total Expected Cost	NPV
1	£135.042m	£143.751m	£278.793m	£520.180m
2	£281.278m	£0m	£281.278m	£517.695m

Table 2-3: Net Present Values of candidate projects using traditional NPV.

According to the undertaken analysis, Project 1 minimizes system costs further and should be the chosen candidate. However, in our approach we have completely disregarded the planner's ability to adjust the investment plan according to the unfolding scenario realization.

NPV remains indifferent to the eventual system state transitions, foregoing useful information that should be used to improve project valuation accuracy. The planner is committed to a sequence of pre-determined decisions made at the start of the forecast horizon and followed without adjustment. The value of a candidate plan can be severely underestimated due to this inflexibility. An extension to the traditional NPV technique is required for the accommodation of flexibility in the decision process and this is achieved through Real Options Valuation.

2.2.3 Real Options Valuation

The strength of ROV lies in capturing the inherent flexibility in the decision making process and yielding a decision strategy that dynamically adapts to the unfolding uncertainty. In contrast to NPV, it can appropriately value time and project flexibility by regarding investment commitments as conditional and allowing the best strategy to be chosen by considering:

i. The impact of new information concerning the requirements of the system.
ii. The extent to which a decision taken now facilitates or limits future adaptability.

In this case study, flexibility lies in the upgradeability of Project 2 which has not been properly valued with the static NPV method. By recognizing the fact that system state transitions can materially inform the decision process, the decision maker can choose whether or not to proceed with the extension on a conditional basis. As in the static NPV framework, the project to be chosen is the one that leads to maximization of the net benefit. The way to perform a multi-stage ROV analysis is known as backward induction [79] and can be summarized as follows:

1. Construct the full decision tree incorporating flexibility (i.e. scenario branching for each candidate investment decision).
2. Determine capital and constraint costs at each node.
3. Starting from the rightmost node (last epoch) and moving to the left (first epoch), identify the decisions that maximize the backwards accumulated expected net benefit at each scenario branching.

The application of ROV analysis to the extended decision tree is shown in Figure 2-7, where optimal decisions have been highlighted grey. Results are summarized in Table 2-4.

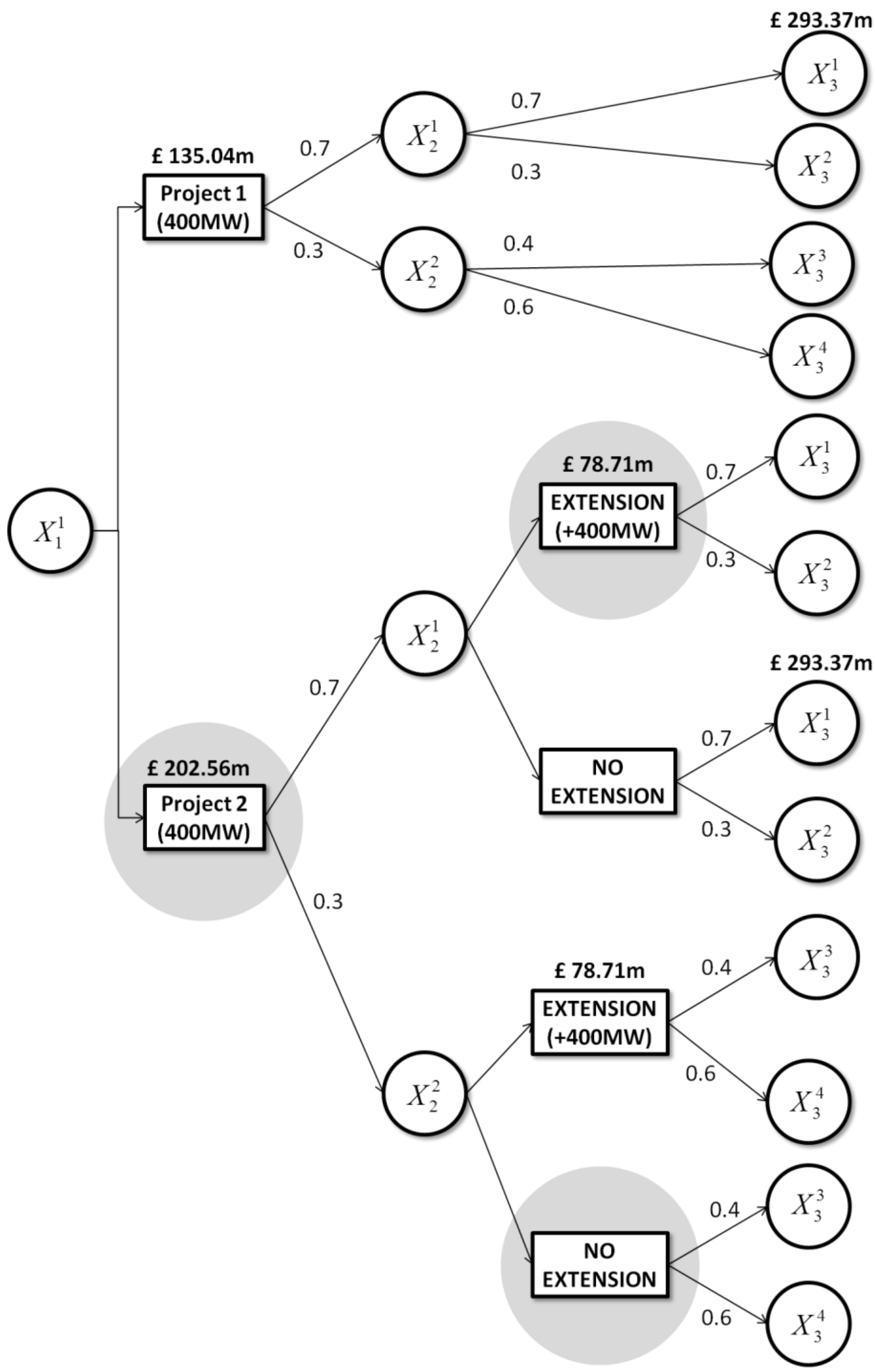


Figure 2-7: ROV analysis for candidate projects.

Project	Expected Capital Cost	Expected Constraint Cost	Expected Total Cost	Extended NPV
1	£135.042m	£143.751m	£278.793m	£520.180m
2	£257.664m	£0m	£257.664m	£541.309

Table 2-4: Extended Net Present Values of candidate projects using ROV.

As can be seen above, the optimum investment strategy is to initially commit to Project 2. If G1 is commissioned with 400MW in the second stage (transition to X_2^1 realizes), the planner should proceed with the extension. If only 200MW are commissioned (transition to X_2^2 realizes), signifying a downside trend, the planner should not upgrade the project. In summary, the differences of ROV to NPV are:

i. The final decision is in the form of a strategy instead of a static plan.
ii. Managerial flexibility is modelled through investment conditionality, leading to a more accurate project valuation, where the embedded expansion option is exercised optimally.
iii. Most importantly, in the presented case study, a different conclusion is reached concerning the first stage decision.

Project	Static NPV	Extended NPV	Option Value
1	£520.180m	£520.180m	£0m
2	£517.695m	£541.309	£23.614m

Table 2-5: Project valuation difference between NPV and ROV techniques.

As can be seen in Table 2-5, static valuation results in a material underestimation of the net benefit of Project 2 by £23.614m. This difference between the static and extended NPV of the project is known as the option value of transmission [81]. It can be defined as the change in net benefit between committing to a fixed schedule for the entire planning horizon and committing to a pre-determined decision for the first stage, but choosing the optimum alternative during subsequent stages as new information is revealed. This option value can be interpreted as the benefit of considering flexibility in the decision process. It is important to highlight that since the deviation from a fixed course of action is a right but not an obligation, the value of flexibility is always non-negative.

2.2.4 Discussion

The basic principle of flexible decision frameworks is to take advantage of the inter-temporal resolution of uncertainty present in all dynamic systems under uncertainty [25] This way, it is possible to shift from 'now-or-never' sequential decisions to an optimal strategy that is optimally prepositioned to the different possible outcomes and takes into account the planner's managerial flexibility to adapt to the eventual scenario realization. For an investment project to have significant option value, three criteria must be fulfilled. We analyze them and explain how the current landscape of the energy sector meets these conditions, necessitating the inclusion of flexibility in investment appraisal and transmission expansion models.

2.2.4.1 Learning

Without learning, the state of knowledge about the future remains unchanged. Under this condition, there is no value to delaying decisions as no uncertainty is resolved over time. When a stochastic process is characterized by learning, the difference between the expected value of the random variable and its conditional expected value given the preceding parent state must be different to zero. Generation investment is based on expected profitability and influenced by factors such as the regulatory framework, use of system charges and investment costs, which may be beyond the decision maker's immediate control but are directly observable and can be monitored. Given that transmission investment is a dynamic process, it can be materially informed by the evolution of these parameters over the planning horizon. In addition, since generation investment consists of distinct stages (e.g. planning permission acquisition, construction, commissioning), key trigger events can be identified and used in the decision process as informed indicators for subsequent state transitions. For example, consider that a prospective wind farm has been granted planning permission. Given the above, the expected value of its size in the future is higher than its expected value calculated in the absence of this knowledge. Part of the uncertainty has been resolved, rendering some scenarios more probable while other outcomes deemed credible at first may be rendered obsolete on the updated information.

2.2.4.2 Flexibility

The flexibility of a transmission project is two-fold; timing flexibility and sizing flexibility.

We define timing flexibility as the ability to undertake a particular investment when its value is maximized. In contrast to some types of financial options that have an expiration date, real-world investments do not face such limitations as the system planner has no exogenous time restrictions. An investment project can be carried out optimally with respect to time so as to maximize its net benefits, subject to the available information and the planner's risk profile.

Sizing flexibility is embedded in the technical and economic nature of transmission projects that are largely characterized by economies of scale [26] and upgradeability. The system planner can choose to bear some large upfront costs and invest in a large project beyond the current needs of the system for the option of utilizing its full potential in the future. Transmission capacity reinforcement decisions frequently involve a range of candidates with different embedded upgradeability options. For example, a large 400kV line may be constructed but may warrant the upgrade of a number of substations for the utilization of its full transfer capability. These upgrades can be carried out on a conditional basis if the need arises. Similarly, a new double-circuit line can be constructed with only one side strung. The planner can decide to upgrade it in the future at a fraction of the cost of commissioning a new line. Another example can relate to undergrounding of cables in areas with severe environmental constraints. A larger tunnel than currently needed can be constructed, so that it may house a cable of a larger rating and diameter in the future.

2.2.4.3 Irreversibility

The energy sector is characterised by investment in assets with high capital costs and long lifetimes spanning several decades. The majority of these assets has very low or zero salvage value and thus it is critical to ensure that committed funds provide long-lasting value for money. Another important parameter is that large capital projects are characterised by considerable pre-construction costs due to extended interactions with planning processes. In a recent report by KEMA investigating capital expenditure on new project proposals in GB [28], pre-construction costs are identified to be of the same order of magnitude as constructing and commissioning the project. It follows that a significant portion of funds have to be sunk to a project from the very first stages, rendering optioneering on a practical level an expensive exercise. This highlights the importance of being confident that an undertaken project will deliver substantial gains in the long-run while avoiding projects that lock-in irreversible future investment paths with uncertain long-term benefits. Investing in assets that may eventually be stranded or under-utilized leads to severe welfare loss.

Clearly, all the above factors characterise investment decisions in the energy sector today, leading to a very material value of flexibility and necessitating its consideration in transmission investment planning.

2.2.5 ROV applications to electricity networks

The benefits of flexible decision frameworks such as ROV have been well documented in the academic literature. A theoretical framework for applying multi-stage real options to transmission investment using binomial up-side down-side trees to model future load growth is presented in [29]. The application of real options to value time flexibility in a merchant transmission investment project is illustrated on a simple interconnector project in [30]. A realistic case study on a potential Norway-Germany link under price differential uncertainty is undertaken in [31]. Besides traditional asset appraisal, ROV has also been applied to investment in FACTS devices as illustrated in [32].

In the same vein, some regulators around the world have recently realized the shortcomings of strictly deterministic NPV methods and have turned to new approaches, setting a regulatory precedence for consideration of real option value in investment appraisal. The valuation of flexibility has thus far been applied through Real Options Valuation in New Zealand and Australia. More precisely, the New Zealand Electricity Commission adopted a new scenario-based framework for investment appraisal in 2005 [33]. According to these arrangements, a proposed project is deemed to pass the optimality test if its net market benefit is positive and greater than that offered by its alternatives. Central to the net benefit definition is the value of any material real options associated to the project. A similar investment test was implemented in Australia in 2010 [34], where option value is recognised as a potential market benefit. In addition, the UK energy regulator, Ofgem, has recognised the benefits of adopting a real options approach in the face of an uncertain electricity sector. The belief that flexible valuation techniques are set to become even more important is solidified in a recent consultation on the application of ROV to the energy sector [10], seeking out views on its potential inclusion in the regulatory investment appraisal process.

It is important to note that in most applications of ROV, a limited range of pre-defined candidate projects is included in the analysis. Inclusion of the possibility for investment in multiple transmission lines would entail the construction of a very large decision tree in order to evaluate all the possible courses of action. For the analysis of realistic systems with multiple buses, lines and investment opportunities under different scenarios, the feasible solution space increases vastly and results in a very large problem that cannot be solved by the presented backwards induction technique. As a result, the development of a cost-benefit framework able to model decision flexibility and ensure system-wide optimality necessitates the shift from individual project valuation methods to systematic optimization techniques. However, as is pointed out by Joskow [35], most of the existing literature on transmission investment optimization pays little attention to the inherent stochastic features of the unbundled electricity market that make real options valuable. There is a clear gap of appropriate optimization tools to accommodate anticipatory transmission investment decisions while considering flexibility. In the following section we undertake a review of existing optimization methods and their approach in dealing with various sources of uncertainty and modelling flexibility.

2.3 Optimization methods for decision-making under uncertainty

In its general form, transmission expansion planning is a stochastic mixed-integer non-linear decision problem. However, modelling it in all its complexity can lead to intractability, even for small systems. For this reason, most existing formulations adopt a range of simplifications. The investment process is usually modelled as a deterministic one-stage decision problem where the future is taken to be firmly known a priori and not influenced by exogenous sources of uncertainty that exist in a real planning setting. Under this assumption, TEP can be formulated as a simple optimization problem and different methods can be employed for its solution including linear programming, mixed integer linear programming [27] and heuristic methods such as genetic algorithms [83] and simulated annealing [84].

However, in reality, there is a wide range of uncertainties affecting the planning process. Inaccuracies in long-term load forecasting, copper price fluctuations in international commodity markets affecting transmission investment costs and ambiguous environmental constraints subject to continuous governmental reviews render the planner unable to make decisions with perfect foresight. In addition, the unbundling of the electricity sector has resulted in even more uncertainty surrounding the transmission planning process, primarily related to the future generation developments. Planning models that adopt a deterministic view of the future are no longer relevant in such a market landscape and stochastic approaches accommodating uncertainty have to be employed. Generally, uncertainties can be classified in two categories [17]:

- *Random uncertainties* can be modelled as random variables whose probability distribution can be derived from past observations. An example of a random uncertainty is load variability, which can be modelled using historic data.
- *Non-random uncertainties* relate to the evolution of parameters that cannot be significantly informed from past observations and are not repeatable. An example of a non-random uncertainty is the potential development of a new generation plant.

Different approaches are needed for different types of uncertainty. As identified in [85], the stochastic approaches that have been used for transmission expansion planning under uncertainty can be divided in four broad categories; probabilistic load flow, probabilistic reliability criteria, fuzzy decision making and scenario techniques.

Probabilistic load flow and probabilistic reliability criteria .are well-suited for repeatable random uncertainties primarily related to system loading variability and are thus mostly used for expansion planning cases where the main concern is reliability and service quality. The uncertain parameters are modelled as random variables with underlying probability distributions based on historic data and future predictions. Monte Carlo methods can be used to compute the probability distribution of output variables, such as future load flows, and identify the optimum network reinforcements to accommodate them [90], [91]. Fuzzy decision making focuses on the uncertainty related to imprecise, ambiguous and incomplete data that may characterize the investment process in an unbundled electricity sector. Ambiguity may concern environmental constraints imposed by the government or the maximum allowable capital spending over a number of years [86] dictated by the government or regulator.

Scenario techniques are well-suited for dealing with non-random uncertainties such as the generation evolution in future years. Since these events cannot be adequately described by historic probability distribution functions, a scenario set is developed to describe the possible states of the system. A scenario can be defined as a coherent description of the future and is usually the product of expert opinion, industry surveys and analysis of the underlying market dynamics (e.g. evolution of fuel prices, government subsidies for particular types of plants). In this case, the planner's objective is to identify the optimal expansion plan that minimizes the expected system cost over all scenarios and a suitable problem formulation necessitates the use of stochastic programming. However, due to the increased complexity involved, several simplifications are usually adopted to reduce problem size and arrive at informed decisions using deterministic approaches. Two such methods used by planners to tackle transmission investment under uncertainty are the optimization based on the most probable eventuality and scenario analysis.

2.3.1 Most probable scenario optimization

In this approach, the optimum expansion plan is determined on the basis of the most probable forecast of the uncertain parameters. A deterministic cost-benefit optimization is performed to determine the optimal investment plan with respect to that particular scenario and all other realizations are ignored. This method has some very attractive characteristics such as ease of implementation and severely reduced problem size. However, by disregarding all other plausible scenarios, uncertainty is essentially ignored, leaving the system severely exposed to alternative realizations. Despite its shortcomings, this approach is widely used by transmission planners across the world due to its straightforward nature and clearly defined objective along the lines of transmission investment methodologies used prior to the unbundling of the electricity sector. This is reflected in National Grid's business plan [60] where this approach has been adopted, determining the optimal transmission investment while considering only the most probable eventuality.

2.3.2 Scenario analysis

As described in [69] and [70], this approach consists of two stages. Initially, the optimal transmission expansion for each individual scenario is determined, obtaining a set of solutions corresponding to the different realizations. Based on this set, a range of analytical techniques can be used to identify "robust" decisions that are common across all scenarios. In the case of a risk-averse planner, this approach can be extended to quantify the impact of alternative scenario realizations and the contingent investment required to cope with unforeseen events. Zhao et al. [73] have developed a static TEP method that explicitly considers the cost of adapting a specific network design after an unfavourable scenario has materialized. In their model, uncertainty lies in various system parameters such as fuel prices, load growth and generation additions. The main planning objective is to ensure that the Expected Energy Not Served (curtailed energy) is kept below some predefined threshold value. Initially, the optimum investment plan i for each individual scenario i is identified. Then each plan i, is checked whether it satisfies the predefined planning objective under each scenario $j \neq i$. If the objective is indeed satisfied, then the adaptation cost of plan i under scenario j is defined as zero. In the opposite case, the adaptation cost is equal to the additional investment that will have to be undertaken to ensure that planning objectives are met. Following the quantification of adaptation costs of all plans under all scenarios, the plan with the minimum maximum adaptation cost is selected as the final expansion plan to be implemented.

The basic limitation of the scenario analysis approach is that scenario-wide optimality cannot be guaranteed when utilizing decisions "tailor-made" for different scenarios [69]. As a result, this technique is only suitable only for preliminary analysis purposes aiding the planner to identify the transmission lines likely to need reinforcement and gain an understanding of the investment patterns necessitated by the different realizations.

Both methods presented above attempt to identify the optimal expansion plan by considering scenarios in isolation. In order to determine a single plan that performs well on average across all possible realizations, stochastic optimization methods must be employed.

2.3.3 Stochastic optimization

In the past, stochastic optimization has been successfully used in the context of transmission planning under uncertainty. Depending on the number of time stages present in the scenario tree being considered, stochastic formulations can be two-stage or multi-stage. In the case of two-stage stochastic planning, a range of scenarios describe the possible deviation from the current state of the system. Naturally, if the construction delay between decisions and asset commissioning is considered, the planner cannot differentiate his investment behaviour for the different realizations. As a result, the planner is to decide on the optimal investment decisions to be taken in the first stage and quantify the operational costs in the second stage, when uncertainty materializes. The objective of such models is the identification of investment decisions that minimize the expected system cost across both stages.

An example two-stage model is presented in [87] where scenarios are used to capture uncertainty in generation costs and demand levels. Recourse actions are limited to the optimal re-dispatch of generation subject to the first stage expansion plan. A very similar modelling approach is used by Alvarez et al. [76] who employ stochastic programming to address uncertainty in future loading conditions and available transmission capacity. In another example, Carrion et al. [88] utilize two-stage stochastic programming to optimize network expansions against a set of scenarios characterizing intentional outages due to deliberate attacks to the electricity system. In this model, the objective function is expressed as the sum of investment costs and expected load shedding costs. In the first stage, transmission investment is undertaken. In the second stage, a disruption is realized and the system operator aims to minimize curtailed demand subject to the expanded network topology by optimally re-dispatching generation.

Multi-stage stochastic programming approaches consider the dynamics of uncertainty over successive planning periods through a multi-stage scenario tree. In contrast to two-stage models that are concerned solely with the expansion projects to be undertaken in the present, multi-stage models identify the optimal investment decisions to be taken successively over the planning horizon. In general, the multi-stage stochastic problem is solved in two variants, considering or not flexibility with respect to the transmission investment decisions. The case that does not consider flexibility assumes that the optimal expansion plan is a set of investment decisions to be followed statically over the future years without the ability for dynamic adjustment. Under this paradigm, optimal decisions are identified on the premise that the planner is not able to deviate from the suggested plan and take contingent actions depending on the scenario realization. The number of transmission expansion models considering the dynamic evolution of uncertainty has thus far been very limited. In a recent publication, Akbari et al. [89] consider a multi-year operation period and identify the optimal investment plan under future load growth uncertainty. The Monte Carlo simulation method has been applied to generate different loading scenarios and construct a multi-stage scenario tree. Stochastic programming is used to model the problem and investment decisions are taken on a non-flexible basis, presented as a set of deterministic actions that minimize expected system cost over the horizon subject to an N-1 security constraint. Recourse is only modelled for generation re-dispatch, and thus the objective function is the minimization of the sum of investment costs and expected operational costs.

Incorporating decision flexibility in dynamic transmission expansion planning when faced with uncertainty is an issue of paramount importance which has thus far been largely ignored by existing transmission expansion models. In order to include flexibility in the decision process, it is essential to move away from the concept of a static investment plan and instead identify the optimal investment strategy that encapsulates a range of contingent courses of action that can be taken corresponding to the possible paths of uncertainty evolution. In terms of the 'here and now' investment, focus is placed on the most flexible decision, with the ability to adapt to possible future changes at the minimum expected cost.

This concept of scenario-dependent investment recourse has been applied in the past in a limited number of power system investment models. Gorenstin et al. [69] describe a methodology to determine the optimum generation expansion strategy under load uncertainty. Multi-stage stochastic programming with recourse is used to model the planning problem. The planner's objective is to pinpoint the generation capacity additions that will result in the minimization of expected investment and operational costs. Investment in interconnectors is also possible to increase the transmission capacity between different system areas. However, a simple transportation model is used to model power flows, ignoring Kirchoff's second law. Under this assumption, the model is limited to radial network applications, while ignoring transmission investment that would be required due to loop flows [78]. In order to manage the large computational load, Benders decomposition is employed to decompose the original problem in a mixed-integer multi-stage investment master problem and several operational subproblems, each one associated to a particular scenario. The model is applied to a radial network representation of the Brazilian South-Southeastern interconnected system and a scenario tree consisting of 16 scenarios over five stages is used to capture uncertainty in the annual demand growth. The idea of modelling investment flexibility has also been recently used by Weijde and Hobbs [15]. They develop a three-stage stochastic model to represent the interaction between a proactive transmission planner and generation investors in the GB electricity system. A simplified 7 node representation of the electricity system has been used and loop flows are ignored. Primary uncertainties lie in fuel prices, demand growth and renewable targets set by the government and are captured through a scenario set describing 6 alternative equiprobable futures. In the first stage, the transmission and generator planners decide on the investment to be undertaken. The corresponding network reinforcements and plant additions are to be commissioned with a one stage delay. The same decision process is followed in the second stage. In the final stage, the only decisions made are on the optimal generation dispatch. Investment decisions that are taken in the first stage are common across all scenarios, while second-stage decisions are differentiated according to the uncertainty realization. The combined transmission and generation expansion problem is solved as a single optimization problem that aims to minimize total expected costs of electricity generation, generation investment and transmission investment. This corresponds to a social welfare maximization formulation which assumes a perfect alignment of objectives between the planner and generators. It is important to note that generation investment decisions are not taken on the basis of profit maximization but are rather modelled as the generation market's equilibrium response to the network reinforcements and the uncertainty realization.

Although the models described above constitute a good starting point for developing a cost-benefit based framework for anticipatory investment, they have a number of significant shortcomings that severely limits their applicability. A primary weakness is related to the use of radial networks and power flow transportation models. This approach ignores loop flows and thus renders the simulation of realistic meshed networks impossible. Loop flows can have a significant impact on the required level of investment due to the passive nature of transmission networks and the power flow netting effect. Another important failing is that transmission reliability considerations are completely overlooked. Realistic systems operate on the basis of N-1 transmission security constraints and generation is dispatched in preventive mode. These modelling simplifications are made for the sake of reducing computational complexity, but are bound to lead to a severe underestimation of the required investment levels [78]. In addition, none of the above models consider lines with embedded upgradeability options and the effect of scale economies, which are of strategic importance in long-term system planning under uncertainty. Furthermore, the possibility to invest in flow control devices that can provide corrective security and defer investment is not considered. Moreover, the planner’s risk averseness towards excessive operational costs that may result from postponing investment is not explicitly modelled. Thus we conclude that there is no existing literature that meets all the requirements mentioned above: consideration of exogenous generation uncertainty, modelling of timing and sizing flexibility, transmission security with optimal allocation of preventive and corrective control measures and risk-averseness.

In the following chapters we illustrate how the above requirements can be incorporated in a multi-stage stochastic transmission expansion model with investment recourse and materially impact investment decisions. The increased computational load is managed through a novel technique combining problem decomposition and a contingency screening algorithm, allowing the modelling of large meshed systems.

2.4 Conclusions

In this section we identified the importance of modelling flexibility in the transmission expansion under uncertainty problem. The concept of valuing the option value of transmission through using Real Options Valuation was introduced and contrasted to traditional static decision frameworks. Finally, a review of existing methods for tackling transmission expansion under uncertainty was undertaken. Published models that consider scenario-specific recourse decisions present a range of limitations towards modelling large realistic systems, highlighting the gap for appropriate tools able to inform anticipatory investment decisions.

3 Deterministic Transmission Expansion Planning

Abstract

This chapter presents the main concepts of cost-benefit based deterministic transmission expansion planning. We show how economies of scale and the lead time between investment and commissioning of transmission assets can be incorporated in the problem formulation. Moreover, we illustrate the use of a contingency screening module that enables the computationally efficient accommodation of N-1 security constraints. Optimal quadrature booster (QB) placement is included in the model through the use of power injection techniques, allowing for the provision of corrective control. Issues related to the computational burden caused by the large size of the multi-stage formulation are addressed through the use of a multi-cut Benders decomposition scheme. The result is a deterministic planning tool that will serve as the basis for developing the stochastic transmission expansion model. A case study on the IEEE RTS illustrates the computational benefits of the approach.

3.1 Nomenclature

3.1.1 Sets

$\Omega_E = \{1..N_E\}$	Set of all epochs.
$\Omega_T = \{1..N_T\}$	Set of all demand periods.
$\Omega_N = \{1..N_N\}$	Set of all system nodes.
$\Omega_G = \{1..N_G\}$	Set of all generation units.
$\Omega_L = \{1..N_L\}$	Set of all transmission lines.
$\Omega_{W_l} = \{1..N_{W_l}\}$	Set of expansion options for line *l*.

3.1.2 Input Variables

$p_{e,g}^{\max}$	Maximum stable generation for unit *g* in epoch *e*.	MW
$p_{e,t,g}^{*}$	Unconstrained dispatch power output of unit *g* for operating point *(e,t)*.	MW
h_g	Short-run marginal cost of unit g.	£/MWh
o_g	Offer price of unit *g*.	£/MWh
b_g	Bid price of unit *g*.	£/MWh
$d_{t,n}$	Demand at node n in period *t*.	MW
x_l	Reactance of transmission line *l*.	p.u.
r	Interest rate.	
$\gamma_{l,w}$	Annuitized fixed investment cost for line *l*, option *w*.	£/(MW.yr)
$c_{l,w}$	Annuitized variable investment cost for line *l*, option *w*.	£/(MW.km.yr)
c_l^{QB}	Annuitized investment cost for quadrature booster on line *l*.	£/yr
$k_{l,w}$	Build time for line *l*, option *w*.	
k_l^{QB}	Build time for candidate quadrature booster on line *l*.	

$F_{l,w}^{\max}$	Maximum capacity provided by expansion option *w* for line *l*.	MW
F_l^0	Initial capacity for line *l*.	MW
$\psi_l^{\min}$	Minimum phase shift angle provided by quadrature booster on line *l*.	rad
$\psi_l^{\max}$	Maximum phase shift angle provided by quadrature booster on line *l*.	rad
$\xi_l^{\min}$	Minimum reactance change provided by series compensator on line *l*.	p.u.
$\xi_l^{\max}$	Maximum reactance change setting provided by quadrature booster on line *l*.	p.u.
u_l	Sending bus for line *l*.	
v_l	Receiving bus for line *l*.	
χ_l	Length of line *l*.	km
B	Bus-to-generation incidence matrix of size $N_N \times N_G$. $B_{n,g} = 1$ if generator *g* connects to bus *n* $B_{n,g} = 0$ otherwise	
I	Bus-to-line incidence matrix of size $N_N \times N_L$. $I_{n,l} = 1$ if the receiving bus for line *l* is *n* ($v_l = n$) $I_{n,l} = -1$ if the sending bus for line *l* is *n* ($u_l = n$) $I_{n,l} = 0$ otherwise	
τ_t	Time duration of demand period *t*.	hours
r_e^I	Cumulative discount factor for investment cost in epoch *e*.	
r_e^O	Cumulative discount factor for operation cost in epoch *e*.	
Γ	Value of lost load.	£/MWh

3.1.3 Decision Variables

$f_{e,l,w}^{inv}$	Transmission capacity to be built for line *l* using option *w* in	MW

	epoch *e*.	
$F^{inv}_{e,l}$	State variable of aggregate capacity added up to epoch *e* to line *l*.	MW
$\beta_{e,l,w}$	Binary variable representing the choice of expansion option *w* for line *l* at epoch *e*.	
$qb_{e,l}$	Binary variable representing the installation of a quadrature booster on line *l* in epoch *e*.	
$QB_{e,l}$	State variable representing the installation of a quadrature booster on line *l* up to epoch *e*.	
$p_{e,t,g}$	Output of unit g for operating point *(e,t)*.	MW
$p^{+}_{e,t,g}$	Constrained-on output of unit *g* for operating point *(e,t)*.	MW
$p^{-}_{e,t,g}$	Constrained-off output of unit *g* for operating point *(e,t)*.	MW
$f_{e,t,l}$	Power flow in line *l* for operating point *(e,t)*.	MW
$\theta_{e,t,n}$	Bus angle at node *n* for operating point *(e,t)*.	rad
$p^{QB}_{e,t,l}$	Power injection due to quadrature booster on line *l* for operating point *(e,t)*.	MW
$p^{SC}_{e,t,l}$	Power injection due to series compensator on line *l* for operating point *(e,t)*.	MW
$d^{*}_{e,t,n}$	Curtailed demand at bus *n* for operating point *(e,t)*.	MW
$f^{C}_{c,e,t,l}$	Post-fault power flow in line *l* for operating point *(e,t)* when line *c* is in outage.	MW
$\theta^{C}_{c,e,t,n}$	Post-fault bus angle at node *n* for operating point *(e,t)* when line *c* is in outage.	rad
$p^{QB^C}_{c,e,t,l}$	Post-fault quadrature booster power injection for operating point *(e,t)* over line *l* when line *c* is in outage.	MW
$p^{SC^C}_{c,e,t,l}$	Post-fault series compensator power injection for operating point *(e,t)* over line *l* when line *c* is in outage.	MW

3.2 Introduction

In this section we showcase the mathematic formulation of the deterministic transmission expansion problem. The static problem formulation is first developed in order to demonstrate the basic concepts and the trade-off between transmission investment and operational costs. This is followed by the multi-stage (or dynamic) transmission expansion formulation. The maximum capacity of generation units is modelled as a multi-stage input variable over discrete epochs to simulate new plant connections. The objective is to identify the optimum investment plan minimizing system costs over the given horizon. The traditional dynamic formulation is extended to include several candidate projects of varying fixed and variable costs to capture the economies of scale present. Each project has an associated construction delay, necessitating investment decisions before new plants become operational. A deterministic N-1 security criterion is also implemented through the use of a contingency screening module that identifies binding line outages. A multi-cut Benders decomposition scheme is proposed to tackle the severe computational load of the resulting large MILP formulation. Finally, a case study on the IEEE RTS system is solved using the developed model.

3.3 Modelling assumptions

Given that the purpose of transmission expansion models is to inform the investment process rather than constitute a definitive technical planning tool, a series of assumptions are made to reduce complexity and bring the problem to a more tractable form. The resulting simplifications that can be drawn are essential in reducing the computational load and allow for problem decomposition. These are as follows:

i. A simplification is made in calculating power flows, by using a linear DC approximation as described in [36]. Reactive power and line resistance are ignored, while voltages are assumed to be very close to their nominal values. As a result, power flow equations become linear, suitable for solution by linear programming techniques.

ii. Another important assumption is on time-decoupled operation. Electrical load is a continuously varying quantity that must be matched by generation subject to a range of complex technical constraints related to both steady state and dynamic stability. Such dynamic constraints are related to generation dispatch and include ramp-rate limits and minimum up and down times of generating units. Given that planning timescales are very different to operational timescales, it is possible to ignore the constraints that couple operational decisions between sequential snapshots. In the absence of time-coupling, similar demand conditions can be aggregated in a single block with a corresponding increase in time duration. The system loading profile is thus modelled using a load duration curve. This way it is possible to represent the original demand data in a small number of snapshots, reducing problem size. Most importantly, such a formulation lends itself to decomposition since each demand block period can be solved independently.

iii. Inclusion of minimum stable generation limits involves binary decision variables to represent a unit's online/offline state, which significantly increases solution times. Although such constraints are essential in unit commitment algorithms where the real-time optimal dispatch is determined, they can be safely ignored in a long-term planning analysis.

iv. All capacity additions are modelled as upgrades to existing right-of-ways. However, in many cases transmission expansion is not only a question of reinforcing existing links but adaptation of network topology with the establishment of new transmission corridors. The class of problems where network topology is optimized by investing in new candidate right-of-ways is known as synthesis transmission planning. Modelling new candidate branches involves binary variables and non-linear constraints and thus proves problematic due to the non-convexities introduced. Some methods successfully used in the past to address the static synthesis problem are Benders hierarchical decomposition in [37], heuristic methods in [38] and disjunctive modelling in [39]. However, the developed applications have been limited to static planning on small test systems due to the high computational burden of topology optimization.

All problem formulations presented in this thesis assume a system with a bilateral energy market and a separate balancing market, similar to the market arrangements in GB. In such a framework, constrained-off generators pay their bid price (b_g) to reduce their production level, while constrained-on generators are paid their offer price (o_g) to increase output. The sum of these payments is known as the cost of constraints or congestion cost.

In general, the problem of transmission expansion planning can be divided in two categories; static and dynamic. In the static case, the system planner seeks the optimal network investment for a single “test year” in the future. The optimal time to undertake the investments is not considered and the problem focuses on the type and size of reinforcements that will lead to minimization of system costs. In the dynamic case, multiple time stages are considered and the planner’s objective is to identify the optimal sequence of decisions over the modelled horizon.

3.4 Static Transmission Expansion Planning

For simplicity, we first illustrate the static transmission expansion problem.

3.4.1 Unconstrained dispatch model

Generally, the first step in solving the transmission investment problem is to determine the system’s economic unconstrained dispatch. Generators are dispatched in order of ascending short-run marginal cost until the target demand level is met and constraints due to line capacities are ignored. This way, we can effectively define the underlying bilateral contractual positions between generators and suppliers and determine the Final Physical Notification (FPN) that each unit is to submit to the system operator. The problem is formulated as follows:

$$\min_{p^*}\left\{\sum_{t=1}^{N_T}\tau_t\left(\sum_{g=1}^{N_G}\left(p^*_{t,g}h_g\right)\right)\right\} \tag{3.1}$$

The objective function (3.1) to be minimized is the generation dispatch cost.

$$0 \le p^*_{t,g} \le p_g^{\max} \qquad \forall t, g \tag{3.2}$$

At all operating points, dispatch levels must be between zero and the generators’ installed capacity as shown in (3.2). Equation (3.3) defines how power is distributed over the network according to the DCOPF formulation [36]. Power flow and bus angle decision variables can take both positive and negative values.

$$f^*_{t,l} = \frac{\theta^*_{t,u_l} - \theta^*_{t,v_l}}{x_l} \qquad \forall t, l \tag{3.3}$$

Equation (3.4) is the system balance equation that enforces the first Kirchhoff law. The sum of power injections to a node are set equal to the local demand level. Curtailed demand $d^*_{t,n}$ is used as a slack variable to ensure that operation is feasible even in cases of inadequate generation capacity.

$$\sum_{g=1}^{N_G} B_{n,g}\, p^*_{t,g} + \sum_{l=1}^{N_L} I_{n,l}\, f^*_{t,l} = d_{t,n} \qquad \forall t,n \tag{3.4}$$

The output values of interest for the unconstrained dispatch problem are the dispatch levels $p^*_{t,g}$ that will form the basis to subsequently determine how much each unit will have to deviate from its FPN, being constrained on /off in order to satisfy network constraints.

3.4.2 Static transmission expansion model

The transmission planning problem deals with the balance between capital and operation costs. Accordingly, the objective function (3.6) to be minimized consists of two terms; the transmission investment cost modelled as a linear function of capacity additions and operation cost, defined as the sum of constraint costs and unserved load cost. Load curtailment is economically penalized using the Value of Lost Load Γ, which for the purposes of this research has been considered fixed at 30,000£/MWh. Investment cost c_l for each line is expressed on an annual basis and the objective function constitutes the yearly system cost.

$$\min_{f^{inv}, p^+, p^-, d^*} \left\{ \sum_{l=1}^{N_L} \left(f_l^{inv} c_l \chi_l \right) + \sum_{t=1}^{N_T} \tau_t \left(\sum_{g=1}^{N_G} \left(p^+_{t,g} o_g - p^-_{t,g} b_g \right) + \sum_{n=1}^{N_N} d^*_{t,n} \Gamma \right) \right\} \tag{3.6}$$

Equation (3.7) defines the power output of each unit in terms of the unconstrained dispatch solution $p^*_{t,g}$. This way, the optimal amount of generation to be constrained on ($p^+_{t,g}$) and off ($p^-_{t,g}$) is determined.

$$p_{t,g} = p^*_{t,g} + p^+_{t,g} - p^-_{t,g} \qquad \forall t,g \tag{3.7}$$

Constraint (3.8) ensures that generation output for each operating point, following balancing, is within the allowable limits.

$$0 \le p_{t,g} \le p_g^{\max} \qquad \forall t,g \tag{3.8}$$

Equation (3.9) defines how power is distributed over the network according to the DC OPF formulation.

$$f_{t,l} = \frac{\theta_{t,u_l} - \theta_{t,v_l}}{x_l} \qquad \forall t,l \tag{3.9}$$

Constraint (3.10) is the system balance equation that enforces the first Kirchhoff law while taking into account curtailed demand.

$$\sum_{g=1}^{N_G} B_{n,g}\, p_{t,g} + \sum_{l=1}^{N_L} I_{n,l}\, f_{t,l} = d_{t,n} + d_{t,n}^{*} \qquad \forall t, n \tag{3.10}$$

Constraint (3.11) constitutes the complicating constraint between investment and operation. Transmission line power flows are limited by the capacity additions to the existing network.

$$-\left(f_l^{inv} + F_l^0\right) \leq f_{t,l} \leq f_l^{inv} + F_l^0 \qquad \forall t, l \tag{3.11}$$

3.4.3 Inclusion of economies of scale and optioneering

In general, all capital project costs faced by an investor can be sub-divided in two main categories: fixed costs and variable costs. Fixed costs are defined as independent of project output, while variable costs vary with output levels. Typical formulations of TEP rely on modelling capital costs solely on a variable cost basis, expressed in terms of (£/MW·km·year). Such an approach assumes that investment cost depends exclusively on the length and capacity of the candidate line. However, transmission projects are characterized by significant economies of scale due to the large fixed costs involved (i.e. independent of capacity). Obtaining right-of-way on a new corridor, carrying out the appropriate environmental and engineering assessments, mobilisation and labour form a significant part of investment costs. In light of these issues, it is evident that variable cost modelling fails to fully capture the underlying process. There is great value in extending the formulation to include both cost components. We define fixed transmission investment costs as the necessary sunk costs to commence construction of a project. It is a linear function of route length and expressed in (£/km·year). On the other hand, variable costs depend on both the effective capacity addition and route length as stated before. In a static deterministic decision setting, the proposed cost component differentiation does not change much in terms of formulation structure. It is when dealing with long-term investment under uncertainty that the full effect for this approach becomes evident, where fixed costs can be undertaken now for the non-binding option of future capacity additions.

Another simplifying assumption that often takes place in typical TEP formulations is the absence of differentiation between alternative investment options. In practice, the planner can choose between a range of solutions to best meet the given technical, environmental and planning constraints. For example, in the case of marginal capacity additions, the planner may choose a low-cost approach of re-conductoring and re-tensioning a line to increase its thermal limit instead of a more costly voltage up-rate [41]. In the case of larger reinforcements, the planner may choose between building a single-circuit or a double-circuit line. It is important that the transmission planning framework can accommodate such optioneering decisions. For this reason, in the developed model we include multiple candidate options, each having a different cost profile, as follows.

Assuming that there are $w_l = 1..N_{w_l}$ candidate projects for line l, each project is characterized by the corresponding fixed ($\gamma_{l,w}$) and variable ($c_{l,w}$) cost components while providing up to $F_{l,w}^{\max}$ of additional transmission capacity. Decision to incur the fixed costs associated with project w_l is modelled through the binary variable:

$$\beta_{l,w} \in \{0,1\} \quad \forall l, w \tag{3.14}$$

According to its value, the upper bound of additional transmission capacity is determined:

$$F_l^{inv} \leq \sum_{w=1}^{N_{W_l}} \beta_{l,w} F_{l,w}^{\max} \quad \forall l \tag{3.15}$$

Finally, the project-specific fixed and variable cost components are included in the investment cost term of the objective function as in (3.16).

$$\min_{f^{inv},\beta,p^+,p^-,d^*} \left\{ \sum_{l=1}^{N_L} \sum_{w=1}^{N_{W_l}} \left(f_{l,w}^{inv} c_{l,w} + \beta_{l,w} \gamma_{l,w} \right) \chi_l + \sum_{t=1}^{N_T} \tau_t \left(\sum_{g=1}^{N_G} \left(p_{t,g}^+ o_g - p_{t,g}^- b_g \right) + \sum_{n=1}^{N_N} d_{t,n}^* \Gamma \right) \right\} \tag{3.16}$$

3.5 Dynamic transmission investment model

The dynamic transmission investment model is suitable for long-term system planning, where system attributes evolve over the long-term. In this research, we are concerned with the evolution of generation over time. To capture these dynamic transitions, it is essential to break down the time horizon in multiple discrete stages (in this thesis also referred to as epochs). This way, generation additions can be expressed as a series of future system states. Each epoch is a multi-year period over which it is assumed that the system's demand and generation profile remains unchanged. Investment decisions are made at the start of each epoch to accommodate the addition of new generation additions that occur. Under this paradigm, the objective is to identify the series of sequential investment decisions that minimize total system costs across the multi-stage horizon.

3.5.1 Mathematical formulation

To capture the dynamic addition of generation, it is essential to partition the investment horizon in distinct periods. Their progression is sampled at discrete time intervals known as epochs. Each epoch e spans the years y_e^* to y_e^{**}. In the presented formulation, the evolving system parameter is the maximum stable level of the generation fleet $p_{e,g}^{\max}$. Contrary to the static formulation, it is a function of epochs, allowing the simulation of partial or full commissioning and decommissioning of generation.

The mathematical formulation is very similar to the static case. The main difference is that an extra index $e = 1..N_E$ is added to all operation decision variables to allow independent operation over multiple epochs, meaning there are $N_E N_T$ operating points to be considered. This also holds true for investment decision variables, which are aggregated to give the effective capacity additions made available at each stage. The multi-epoch transmission investment problem for a horizon of N_E epochs can be formulated as follows:

$$\min_{f^{inv},\beta,p^+,p^-,d^*} \left\{ \sum_{e=1}^{N_E} \left[r_e^I \sum_{l=1}^{N_L} \sum_{w=1}^{N_{W_l}} \left(f_{e,l,wl}^{inv} c_l + \beta_{e,l,w} \gamma_{l,w} \right) \chi_l + r_e^O \sum_{t=1}^{N_T} \tau_t \left(\sum_{g=1}^{N_G} \left(p_{e,t,g}^+ o_g - p_{e,t,g}^- b_g \right) + \sum_{n=1}^{N_N} d_{e,t,n}^* \Gamma \right) \right] \right\} \quad (3.17)$$

The cumulative discounting factor for capital costs is defined by equation (3.18), accounting for the fact that annual capital payments are to be made from the year of commissioning y_e^* until the final year of the horizon $y_{N_E}^{**}$. This is on the assumption that the assets' lifetime is greater than the study period.

$$r_e^I = \sum_{i=y_e^*}^{y_{N_E}^{**}} \frac{1}{(1+r)^{i-1}} \quad (3.18)$$

In the case of operation costs, the cumulative discounting factor is defined by equation (3.19) and reflects the fact that costs relating to epoch *e* are paid for the years y_e^* to y_e^{**} i.e. for the duration of epoch *e*.

$$r_e^O = \sum_{i=y_e^*}^{y_e^{**}} \frac{1}{(1+r)^{i-1}} \quad (3.19)$$

The discount rate *r* is assumed to stay constant over the entire horizon. For realistic case studies, it is prudent to move beyond this assumption and undertake sensitivity analysis around the discount rate evolution under the different scenarios. The modification to the algorithm to take account of varying discount rates is straightforward.

Equation (3.20) defines the power output of each unit in terms of the unconstrained dispatch solution $p^{*}_{e,t,g}$. This way, the optimal amount of generation to be constrained on ($p^{+}_{e,t,g}$) and off ($p^{-}_{e,t,g}$) is determined.

$$p_{e,t,g} = p^{*}_{e,t,g} + p^{+}_{e,t,g} - p^{-}_{e,t,g} \quad \forall e,t,g \tag{3.20}$$

Constraint (3.21) ensures that generation output for each operating point, following balancing, is within the allowable limits, as defined by the generation capacity available at epoch *e*.

$$0 \le p_{e,t,g} \le p^{\max}_{e,g} \quad \forall e,t,g \tag{3.21}$$

Equation (3.22) defines how power is distributed over the network.

$$f_{e,t,l} = \frac{\theta_{e,t,u_l} - \theta_{e,t,v_l}}{x_l} \quad \forall e,t,l \tag{3.22}$$

Constraint (3.23) is the system balance equation.

$$\sum_{g=1}^{N_G} B_{n,g}\, p_{e,t,g} + \sum_{l=1}^{N_L} I_{n,l}\, f_{e,t,l} = d_{t,n} + d^{*}_{e,t,n} \quad \forall e,t,n \tag{3.23}$$

Constraint (3.26) constitutes the complicating constraint between investment and operation. Transmission line power flows are limited by the aggregate capacity additions up to the current epoch calculated in equation (3.24) and (3.25). $f^{inv}_{e,l,w}$ is the transmission capacity to be built for line *l* using option *w* in epoch *e*, while $F^{inv}_{e,l}$ is the state variable representing the sum of all capacity additions up to epoch *e* from all candidate projects.

$$F^{inv}_{e,l} = \sum_{w=1}^{N_W} \sum_{j=1}^{e} f^{inv}_{j,l,w} \quad \forall e,l \tag{3.24}$$

$$\sum_{i=1}^{e} f^{inv}_{i,l,w} \le \sum_{j=1}^{e} \beta_{j,l,w} F^{\max}_{l,w} \quad \forall e,l,w \tag{3.25}$$

$$-\left(F^{inv}_{e,l} + F^{0}_{l}\right) \le f_{e,t,l} \le F^{inv}_{e,l} + F^{0}_{l} \quad \forall e,t,l \tag{3.26}$$

3.5.2 Modelling of transmission build times

An important simplification of the traditional transmission expansion formulations is that transmission investment is regarded as an instantaneous process. However, in practice, project construction and commissioning can take several years to complete from the time the decision was taken. Particularly in the case of transmission investment under uncertainty, ignoring the delay between decision and commissioning renders the problem of optimal timing void and prohibits the formal valuation of anticipatory investment. Inclusion of build times will allow us to better model the underlying physical reality of the investment process and provide us with a more accurate cost-benefit framework for determining optimal decision timing. Due to the discrete nature of the multi-stage TEP, we express build times in terms of epochs. As a result, a particular project w on line l can be modelled as having a delay of $k_{l,w}$ epochs. A project that is built in the first epoch and is defined with $k_{l,w}=1$ becomes operational in epoch 2, while the planner incurs the corresponding fixed investment costs beginning from epoch 1, when the investment commitment was made. Through equation (3.27), state variables $F_{e,l}^{inv}$ now define the total transmission capacity available at each epoch while taking into account the corresponding build time of each project.

$$F_{e,l}^{inv} = \sum_{w=1}^{N_W} \sum_{j=1}^{e-k_{l,w}} f_{j,l,w}^{inv} \quad \forall e,l \qquad (3.27)$$

3.6 Modelling of FACTS devices

Until now we have defined the potential investment decisions with traditional transmission assets in mind; investing in capacity additions optimally accommodate the emerging flow patterns that lead to system cost minimization. However, apart from the transmission lines, there are devices that allow for congestion management through power flow manipulation. These are known as Flexible AC Transmission Systems (FACTS) and can achieve congestion management through controlling the parameters that dictate power flow levels, namely voltage angles and line reactances.

Quadrature Boosters (QB) and Series Compensators (SC) are devices whose role is set to become crucial over the coming decades as existing assets are stressed beyond their designed capacities. Strategic positioning of FACTS devices in the grid can lead to fuller utilization of existing transmission assets, thus deferring the need for immediate reinforcements. Due to their controllability, improved asset utilization can be achieved for the entire operational spectrum, including post-fault conditions. This can lead to a significant reduction of operating costs, improve system security through corrective control and reduce or defer investment in transmission assets. For these reasons, it is important to ensure that operation and investment in FACTS devices can be accommodated in our model. In this Section we illustrate how these devices can be effectively incorporated in our formulation without compromising problem linearity.

3.6.1 Mathematical Formulation

The main idea of FACTS devices is to direct power flows by controlling network parameters; voltage angle and line reactance. QBs control the voltage angles across a transmission line while SCs control the line's reactance. The straightforward approach to modelling these devices is to add extra decision variable terms in the line flow equations (3.22) to account for the controllability of those parameters. The power flow over a line l equipped with a SC can thus be modelled as:

$$f_{e,t,l} = \frac{\theta_{e,t,u_l} - \theta_{e,t,v_l}}{x_l + \xi_{e,t,l}} \quad \forall e,t,l \tag{3.28}$$

Where the reactance change $\xi_{e,t,l}$ is physically limited according to the device's specifications:

$$\xi_l^{\min} \le \xi_{e,t,l} \le \xi_l^{\max} \ \forall e,t,l \tag{3.29}$$

Similarly, the power flow over a line l equipped with a SC can be modelled as:

$$f_{e,t,l} = \frac{\theta_{e,t,u_l} - \theta_{e,t,v_l} + \psi_{e,t,l}}{x_l} \quad \forall e,t,l \tag{3.30}$$

Where the phase angle due to the quadrature booster $\psi_{e,t,l}$ is physically limited within a certain range as in:

$$\psi_l^{\min} \le \psi_{e,t,l} \le \psi_{\max}^{l} \quad \forall e,t,l \tag{3.31}$$

However, this approach results in a non-linear formulation for TCSC operation, as seen in equation (3.28), greatly increasing the complexity of the problem and forcing us to move towards more computationally intensive solution methods.

3.6.2 Power Injection Model

An alternative method that can be used to model FACTS devices is the power injection model, which interprets power flow controllability due to the shunt and series converters as real power node injections [42]. Employing this method, we can express the optimization constraints describing FACTS operation in linear form.

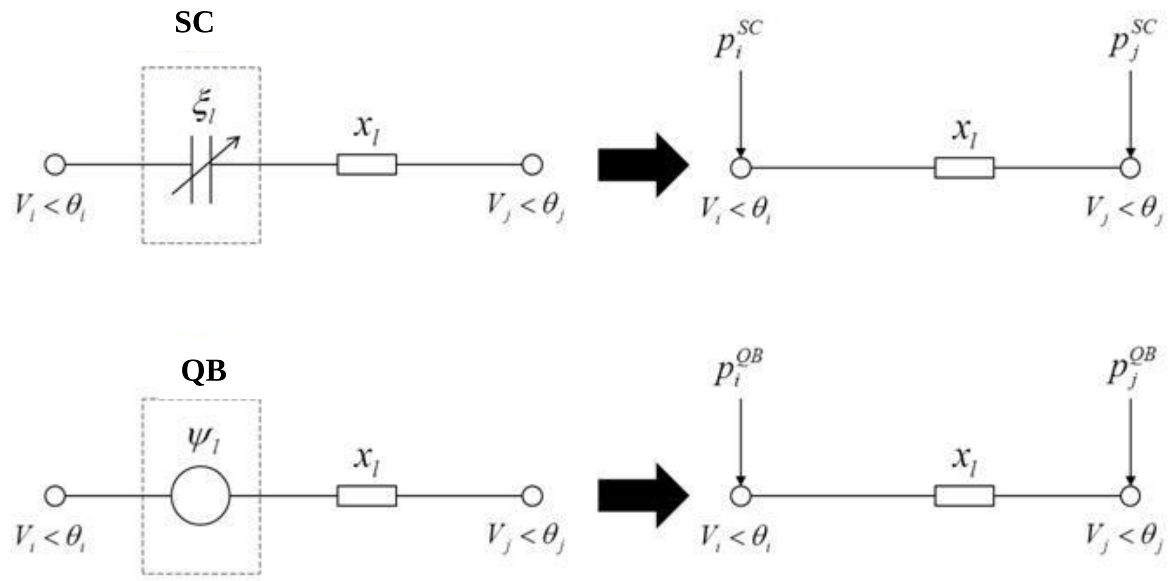


Figure 3-1: FACTS power injection model.

Referring to Figure 3-1, on the left side we can see the equivalent circuits for a series compensator and a quadrature booster. On the right side, we show the corresponding power injections. According to [42], the power injections at operating point (e,t) due to a SC installed in line l linking nodes u_l and v_l are defined as:

$$p^{SC}_{e,t,u_l} = -p^{SC}_{e,t,v_l} = \frac{-x_l}{x_l\left(x_l - \xi_{e,t,l}\right)}\left(\theta_{e,t,u_l} - \theta_{e,t,v_l}\right) \;\; \forall e,t,l \tag{3.32}$$

the power injections at operating point (e,t) due to a QB installed in line l are defined as:

$$p^{QB}_{e,t,u_l} = -p^{QB}_{e,t,v_l} = -\frac{\psi_{e,t,l}}{x_l} \;\; \forall e,t,l \tag{3.33}$$

The system balance equation (3.23) has to be modified to (3.34) in order to accommodate the power injections due to the FACTS devices. Note that power injections are defined in terms of lines l and not in terms of start and destination nodes u_l and v_l. Multiplication of the line-defined injection by the incidence matrix I results in the desired node-specific terms.

Similarly, (3.26) is modified to (3.35) in order to include the power injection terms in the power flow constraints. Note that power injection decision variables can be positive or negative.

$$\sum_{g=1}^{N_G} B_{n,g}\, p_{e,t,g} + \sum_{l=1}^{N_L} I_{n,l}\left(f_{e,t,l} + p_{e,t,l}^{QB} + p_{e,t,l}^{SC}\right) = d_{t,n} + d_{e,t,n}^{*} \quad \forall e,t,n \tag{3.34}$$

$$-\left(F_{e,l}^{inv} + F_l^{0}\right) \leq f_{e,t,l} + p_{e,t,l}^{QB} + p_{e,t,l}^{SC} \leq F_{e,l}^{inv} + F_l^{0} \quad \forall e,t,l \tag{3.35}$$

However, in the above approach, the power injection due to TCSC devices (3.32) is still a function of both the controllable reactance $\xi_{e,t,l}$ and the node phase angles, resulting in a non-linear constraint. Wang et al. [43] have suggested a method to preserve linearity. Instead of introducing the FACTS control parameters as independent decision variables in the formulation, it is possible to predefine the feasible range of the power injections given that the control parameters are physically bounded within a certain range as defined by (3.29) and (3.31). This way we can calculate the minimum and maximum values of the associated power injections as follows.

For the SC:

$$-\frac{\xi_l^{\max}}{x_l\,(x_l - \xi_l^{\max})}\left(\theta_{e,t,u_l} - \theta_{e,t,v_l}\right) \leq p_{e,t,l}^{SC} \leq -\frac{\xi_l^{\min}}{x_l\,(x_l - \xi_l^{\min})}\left(\theta_{e,t,u_l} - \theta_{e,t,v_l}\right) \quad \forall e,t,l \tag{3.35}$$

For the QB:

$$-\frac{\psi_l^{\max}}{x_l} \leq p_{e,t,l}^{QB} \leq -\frac{\psi_l^{\min}}{x_l} \qquad \forall e,t,l \tag{3.36}$$

With the above formulation, we can introduce the nodal power injections as the sole decision variables and succeed in keeping the formulation linear. It is still possible to calculate the FACTS devices' parameters $\xi_{e,t,l}^{*}$ and $\psi_{e,t,l}^{*}$, after the optimal injections $p_{e,t,l}^{QB^*}$ and $p_{e,t,l}^{SC^*}$ have been calculated as follows:

$$\xi_{e,t,l}^{*} = \frac{x_l^2\, p_{e,t,l}^{SC^*}}{-\left(\theta_{e,t,l}^{*} - \theta_{e,t,l}^{*}\right) + \xi_{e,t,l}\, p_{e,t,l}^{SC^*}} \tag{3.37}$$

$$\psi_{e,t,l}^{*} = -p_{e,t,l}^{QB^*} x_l \tag{3.38}$$

Thus, using this method we are able to simplify the problem and gain a significant computational advantage over methods employed in the past that relied on decomposition [44] or non-linear techniques to approach this non-linear problem.

3.6.3 Optimal quadrature booster placement

In this section we show how investment decisions on quadrature boosters can be included in the problem formulation. Investment in series compensators is not considered as it would entail a non-linear constraint arising from the multiplication of the series compensator investment decision variable with the bus angle difference. For this reason, series compensator controllability is only optimized for existing devices already installed in the system. In order to incorporate investment decisions on the placement of quadrature boosters, we introduce the binary investment variable $qb_{e,l}$ and the corresponding state variables $QB_{e,l}$. Integer $k_{e,l}^{QB}$ is the build time for quadrature boosters.

$$qb_{e,l} \in \{0,1\} \quad \forall e,t,l \tag{3.39}$$

$$QB_{e,l} = \sum_{j=1}^{e-k_l^{QB}} qb_{e,l} \quad \forall e,l \tag{3.40}$$

Only one device can be installed in each line throughout the horizon as declared in constraints (3.41).

$$QB_{e,l} \leq 1 \tag{3.41}$$

Equation (3.36) is modified as follows:

$$-\frac{\psi_l^{\max} QB_{e,l}}{x_l} \leq p_{e,t,l}^{QB} \leq -\frac{\psi_l^{\min} QB_{e,l}}{x_l} \tag{3.42}$$

As seen above, when $QB_{e,l} = 1$, indicating the installation of a quadrature booster on line l at epoch e, the corresponding power injection can be optimally controlled. In the case that $QB_{e,l} = 0$, power injection $p_{e,t,l}^{QB}$ is limited to zero. The objective function (3.17) is modified to include the relevant capital expenditure term, defined in terms of the investment annual cost c_l^{QB}:

$$\min_{\substack{f^{inv},\beta,qb,\\ p^+,p^-,d^*}} \left\{ \sum_{e=1}^{N_E} \left[r_e^I \sum_{l=1}^{N_L} \left(\sum_{w=1}^{N_{W_l}} \left(f_{e,l,wl}^{inv} c_l + \beta_{e,l,w} \gamma_{l,w} \right) \chi_l + qb_{e,l} c_l^{QB} \right) + r_e^O \sum_{t=1}^{N_T} \tau_t \left(\sum_{g=1}^{N_G} \left(p_{e,t,g}^+ o_g - p_{e,t,g}^- b_g \right) + \sum_{n=1}^{N_N} d_{e,t,n}^* \Gamma \right) \right] \right\} \tag{3.43}$$

The above constitutes the first QB optimal placement formulation using the Power Injection Model. Comparisons with the traditional formulation on test systems confirms that the proposed model exhibits superior computational behaviour, with virtually no result discrepancy.

3.7 N-1 Transmission Planning

Long-term transmission planning is not limited to identifying the investment plan that leads to system cost minimization, but also determining the right amount of redundancy to be invested into the system to secure feasible operation against disturbances. The exact security rules applied to determine the required level of redundancy varies from system to system according to the codified security standards that define which contingencies are deemed credible and should not threaten system integrity. There are different definitions of operation feasibility under these credible contingencies, usually involving a measure of loss of load that is deemed acceptable as well as respect of transmission lines' thermal limits. In practice, excessive power transfers can cause overheating, further equipment failures and even trigger wide-spread cascading outages. In general, security criteria can be classified as deterministic or probabilistic. Deterministic rules state that the electricity network must be able to withstand the loss of one (i.e. N-1) or two circuits (i.e. N-2) without causing any other line overloads or resulting in curtailed demand. Probabilistic rules recognize that the probability of a line outage is increased significantly in adverse weather conditions (e.g. thunderstorms, high winds etc.) and thus utilize this correlation to define the condition-specific optimal operation subject to a chance-defined constraint.

In terms of capital costs, stricter security standards result in increased transmission investment driven by the need for redundancy during post-fault operation without violating lines' thermal limits. In terms of operation costs, generators may need to be dispatched outside of the merit order to bring the system to a more robust operating point while giving rise to additional constraint costs. This is because generating units can only provide *preventive security*; their output cannot deviate from pre-fault levels as there is no time for generation re-dispatch following a contingency. As a result, the base-case dispatch resulting in minimum balancing costs while respecting security constraints must be determined. On the other hand, FACTS devices can provide *corrective security* by independently adjusting their parameters to the post-fault optimum value for each contingency. This operational flexibility can result in significant cost savings and reduce transmission investment levels by diverting power flows to alleviate line overloading. In practice, there is a wide arsenal of post-fault ancillary services that can be used to secure system operation such as generation intertripping and dispatch of fast-response and reserve generation. However, all these services have significant utilization costs and are beyond the scope of the present transmission planning cost-benefit analysis.

The problem of security-constrained operation under deterministic rules can be tackled through an extension of the standard OPF problem; determine the optimum economic dispatch while ensuring that operation is feasible (i.e. no curtailed demand and no line overloads) under all credible contingencies. Security-Constrained Optimal Power Flow (SCOPF) algorithms constitute a valuable tool and are used by Transmission System Operators for network operation, planning and pricing tasks. In the absence of a universal definition of system security many different implementations of SCOPF exist. For example, at NYISO, post-contingency power flows are computed through the use of Power Transfer Distribution Factors and Generation Shift Factors, as defined in [36], and incorporated in the pre-fault OPF. This way the number of decision variables does not increase but a very large number of potentially redundant constraints are introduced. PJM adopts a different approach where post-contingency power transfers are not examined. Instead, an extensive security analysis with respect to all credible contingencies is carried out beforehand to determine new limits on branch flows and phase angle differences among buses during intact operation, replacing their original limits. However such heuristic methods are more suited towards operational timescales and not transmission planning, where the network topology is subject to change. Another prominent method is variable duplication where an additional set of variables and constraints is introduced to represent operation under each contingency. Coupling between pre and post-fault variables exists due to preventive generation dispatch. By definition, this method results in a very large problem formulation but is suitable for decomposition and ranking techniques to reduce the eventual computational load. A comprehensive overview of different SCOPF formulations and decomposition techniques can be found in [45].

In this model we implement the deterministic N-1 security criterion, which is the operational standard in GB as defined by the SQSS [92]. All contingencies involving a single line outage are deemed as credible events that the system must be secured against. The variable duplication SCOPF method is presented and then improved through a decomposition method that involves contingency screening.

3.7.1 Security-Constrained OPF

The presented SCOPF formulation involves the introduction of a contingency index $c = 1..N_L$ to represent operation under the different line outage events. Extra variables and constraints are added to the base case formulation to ensure that operation is feasible under all credible disturbances. For example, decision variable $f^{C}_{c,e,t,l}$ is the power flow in line l at operating point (e,t) while line c is in outage. In a similar manner, new variables are introduced for bus angles and power injections due to FACTS devices. Constraints (3.44) – (3.51) constitute the post-fault operational constraints. They can be appended to the non-secure transmission investment problem, to determine the optimal N-1 secure investment decisions.

By definition, when $c = l$, the flow over the faulty line is zero. Similarly, FACTS devices on that line are disabled.

$$f^{C}_{c,e,t,l} = 0 \qquad \forall e,t \tag{3.44}$$

$$p^{SC^C}_{c,e,t,l} = 0 \qquad \forall e,t \tag{3.45}$$

$$p^{QB^C}_{c,e,t,l} = 0 \qquad \forall e,t \tag{3.46}$$

For the case that $c \neq l$, the power flow is defined according to the DC OPF formulation:

$$f^{C}_{c,e,t,l} = \frac{\theta^{C}_{c,e,t,u_l} - \theta^{C}_{c,e,t,v_l}}{x_l} \qquad \forall e,t \tag{3.47}$$

Similarly, for the case that $c \neq l$, FACTS devices can provide corrective security and thus the corresponding power injections variables ($p^{QB^C}_{c,e,t,l}$ and $p^{SC^C}_{c,e,t,l}$) can be optimized independently from their intact operation counterparts according to (3.48) and (3.49).

$$-\frac{\xi_l^{\max}}{x_l(x_l - \xi_l^{\max})}\left(\theta^{C}_{c,e,t,u_l} - \theta^{C}_{c,e,t,v_l}\right) \leq p^{SC^C}_{c,e,t,l} \leq -\frac{\xi_l^{\min}}{x_l(x_l - \xi_l^{\min})}\left(\theta^{C}_{c,e,t,u_l} - \theta^{C}_{c,e,t,v_l}\right) \quad \forall e,t \tag{3.48}$$

$$-\frac{\psi_l^{\max} QB_{e,l}}{x_l} \leq p^{QB^C}_{e,t,l} \leq -\frac{\psi_l^{\min} QB_{e,l}}{x_l} \qquad \forall e,t \tag{3.49}$$

In addition, post-fault power flows including FACTS power injections must not overload transmission lines and are bounded by the available transmission capacity as defined below.

For all $c,l \in \Omega_L$, if $(c = l)$ then:

$$-\left(F_{e,l}^{inv} + F_l^{0}\right) \le f_{c,e,t,l}^{C} + p_{c,e,t,l}^{QB^C} + p_{c,e,t,l}^{SC^C} \le \left(F_{e,l}^{inv} + F_l^{0}\right) \qquad \forall e,t \tag{3.50}$$

Generators can provide only preventive security, meaning that their output level cannot deviate from the pre-fault dispatch level $p_{e,t,g}$. The system balance equation for operating point (c,e,t) is given by:

$$\sum_{g=1}^{N_G} B_{n,g}\, p_{e,t,g} + \sum_{l=1}^{N_L} I_{n,l}\left(f_{c,e,t,l}^{C} + p_{c,e,t,l}^{QB^C} + p_{c,e,t,l}^{SC^C}\right) = d_{t,n} \qquad \forall c,e,t,n \tag{3.51}$$

It is important to note the addition of SCOPF constraints (3.44)-(3.51) creates a very significant computational bottleneck due to the increase of operating conditions to be considered from $N_E N_T$ to $(N_L + 1) N_E N_T$. Considering the fact that OPF computation time increases non-linearly with the number of contingencies considered [46], indicates that SCOPF modelling can result in tremendous scaling issues. In the following section a technique known as Decomposed Security Constrained OPF (DSCOPF) is employed to significantly reduce the computational burden of computing the N-1 secure dispatch.

3.7.2 Decomposed Security-Constrained OPF

The idea behind DSCOPF is that, in most cases, only a few contingencies are binding and require a re-dispatch of the system compared to the base case. Post-fault constraints related to non-binding contingencies can be considered redundant. Consequently, by removing all constraints related to the non-binding contingencies from the exhaustive SCOPF formulation, we can considerably reduce the problem size. DSCOPF is an iterative method relying on a contingency screening module.

3.7.2.1 Contingency Screening module

The purpose of the contingency screening module is to filter all credible contingencies and determine whether they require generation re-dispatch or additional transmission reinforcements. In terms of the mathematical formulation, this relates to a violation of the system balance constraint subject to the optimal corrective actions. To ensure problem feasibility, slack variables are used. At each node, they can be positive ($d_{c,e,t,n}^{+}$) or negative $d_{c,e,t,n}^{-}$), signifying curtailed demand or excessive power transfers that cannot be accommodated. Contingency classification is achieved through quantification of these slack variables and comparison with a threshold value Q (set close to zero) as shown in equation (3.52). Objective function values above Q signify that the contingency *(c,e,t)* is binding.

$$\sum_{n=1}^{N_N}\left(d_{c,e,t,n}^{c+} + d_{c,e,t,n}^{c-}\right) \leq Q \tag{3.52}$$

The objective function for each operating point (c,e,t) is as follows.

$$\min_{d^{c+},d^{c-}}\left\{\sum_{n=1}^{N_N}\left(d_{c,e,t,n}^{c+} + d_{c,e,t,n}^{c-}\right)\right\} \tag{3.53}$$

subject to the standard SCOPF constraints (3.44)-(3.50). The system balance equation (3.51) is modified to (3.54) in order to incorporate the afore-mentioned slack variables.

$$\sum_{g=1}^{N_G} B_{n,g}\, p_{e,t,g} + \sum_{l=1}^{N_L} I_{n,l}\left(f_{c,e,t,l}^{C} + p_{c,e,t,l}^{QB^C} + p_{c,e,t,l}^{SC^C}\right) = d_{t,n} + d_{c,e,t,n}^{c+} - d_{c,e,t,n}^{c-} \quad \forall c,e,t,n \tag{3.54}$$

The solution strategy is as follows:

Step 1. Contingency screening iteration index *i=1*.

Step 2. Initialize the list of binding contingencies as empty: $K_{c,e,t}^{i} = 0 \quad \forall c,e,t$

Step 3. Solve the transmission expansion problem while also adding explicit N-1 security constraints (3.44)-(3.51) for all the binding contingencies detected in the previous iteration *i-1* ($K_{c,e,t}^{i-1} = 1$). By definition, if *i=1* then no binding contingencies are considered. The optimal investment vectors ($\overline{F^{inv}}$ and $\overline{QB}$) and the optimal generation dispatch decision vector ($\overline{p}$) are determined.

Step 4. Determine membership status of each post-fault operating point *(c,e,t)* to the list of binding contingencies according to (3.52): if binding, $\left(K_{c,e,t}^{i} = 1\right)$, otherwise $\left(K_{c,e,t}^{i} = 0\right)$.

Step 5. If $\sum_{\forall(c,e,t)} K_{c,e,t}^{i} = 0$, meaning that no binding contingencies were detected in this iteration, go to **Step 8**. If $\sum_{\forall(c,e,t)} K_{c,e,t}^{i} > 0$, meaning that some binding contingencies were detected, go to **Step 6**.

Step 6. $K_{c,e,t}^{i} = \max\left(K_{c,e,t}^{1},...,K_{c,e,t}^{i}\right)$, meaning that if post-fault operating point *(c,e,t)* was found to be binding in some previous iteration and found to be non-binding in the current iteration *i*, its status is reinstated as binding.

Step 7. Update the contingency screening iteration index *i=i+1* and go to **Step 2**.

Step 8. END.

The above iterative process is repeated until no credible contingency results in violation of the system balance equation subject to the provided investment and dispatch solution. It is important to note that once a binding contingency *(c,e,t)* has been classified as binding, it keeps being re-added to the list in subsequent iterations until the optimal secure solution has been found. This memory process ensures that at each iteration, all potentially binding contingencies are considered in the transmission expansion problem. The worst case scenario is that all contingencies are progressively added to the list and the transmission expansion problem is eventually solved while considering all credible contingencies. However, experience shows that this algorithm usually converges within the first two iterations. Thus, in most cases, the problem constructed in the second iteration is equivalent to the exhaustive security-constrained formulation. The strength of this technique lies in the significant reduction of optimization constraints and results in much faster solution times. In addition, the contingency screening module is a fully parallelizable process where each operating point (c,e,t) can be screened independently. Thus its effect on the problem solution time is minimal.

3.8 Benders decomposition

One downside of the dynamic transmission expansion formulation is that it involves numerous variables for both operational and investment decisions that grow with the size of the system and simulation horizon. Thus, the problem formulation presented in the previous section is unable to handle the modelling of large realistic systems. This scaling effect is amplified when including multiple scenarios to tackle transmission investment under uncertainty as shown in Chapters 4 and 5. One way to address this issue is to use a decomposition technique where the original problem is partitioned in smaller chunks and is more easily manageable. Decomposition algorithms are iterative processes that involve the successive addition of variables (also known as column generation), such as Dantzig-Wolfe decomposition, or the successive addition of constraints (also known as row generation), such as Benders decomposition.

Many problems in power system planning exhibit a structure suitable for Benders decomposition. Reference [49] is a good summary of the advent of Benders decomposition applications to problems typically faced in deregulated power systems. Applications range from the security-constrained unit commitment problem [58], to solving multi-period optimal power flow [93], to maintenance coordination and scheduling [50]. Benders has also been successfully applied to transmission planning problems in the past. Pereira et al. [63] were one of the first to identify the natural decoupling between investment and operation and explore the application of mathematical decomposition techniques to its solution. Since then several researchers have employed this approach. Shrestha and Fonseka [59] develop a congestion-driven transmission expansion model where Benders decomposition is used to decouple investment and system operation. The transmission shadow prices generated from the generation dispatch subproblem are used to provide network expansion signals. Romero and Monticelli [37] address the static expansion problem using hierarchical Benders decomposition where three levels of increasing accuracy are employed. A trial solution by using a simple transportation model (only Kirchoff's first law represented) is initially obtained and more accurate solutions are progressively identified by successively switching to more detailed power flow models. S. Binato at al. [51] address the static expansion problem using Benders decomposition employing a disjunctive formulation to relax integer variables for new line additions. Applications on a 46 bus-bar system, representing the reduced south-eastern Brazilian transmission grid confirm the computational benefits of the approach. Dehghan et al. [61] employ Benders decomposition to the multistage expansion problem by minimizing investment and constraint costs subject to N-1 security constraints. The proposed model is applied to a six bus-bar system. Tor et al. [62] present a similar multi-year planning model and apply it to a reduced model of the Turkish power system, including sensitivity studies on future load growth and fuel prices.

3.8.1 General formulation

Benders decomposition is a decomposition technique developed by J.F Benders [57] for mixed integer optimization problems exhibiting a special structure that involves complicating variables. The principle of this technique is to take advantage of the problem structure and split the large original problem into a master and a subproblem. The master problem is solved while approximating the subproblem's optimal value. The master's optimal solution of the complicating variable constitutes a trial value that is passed to the subproblem. The subproblem is then solved with respect to the proposed trial value and the dual variable of the coupling constraint[3] is used to construct a linear constraint (also known as Benders cut) that is appended to the master problem. The set of the appended Benders cuts constitutes a linear piecewise representation of the subproblem. This process is repeated in an iterative manner where additional Benders cuts are added until the master's subproblem approximation accurately represents the subproblem. Generally, problems suitable for Benders decomposition have the following structure:

$$z = \min\left\{c^T x + d^T y\right\} \tag{3.55}$$

s.t.

$$Ax \geq b \tag{3.56}$$

$$Fx + Ey \geq h \tag{3.57}$$

$$x \geq 0, y \geq 0 \tag{3.58}$$

The objective function is a linear combination of the two decision vectors *x* and *y*, which are coupled through equation (3.57), while the complicating variable is x. We proceed by showing how this general problem can be solved using Benders decomposition.

Nomenclature

v	Iteration index.
α	Variable approximating the subproblem value.
$x^{(v)}$	Master problem trial decision for iteration *v*.
$\omega(x^{(v)})$	Subproblem optimal value at iteration *v* for the trial decision x^v.
$\lambda^{(v)}$	Subproblem dual variable associated with trial decision x^v.
$z_{upper}^{(v)}$	Upper bound of optimal solution of the original problem.
$z_{lower}^{(v)}$	Lower bound of optimal solution of the original problem.
ε	Convergence tolerance.

[3] The coupling constraint is the constraint in the subproblem that includes the complicating variable, thus coupling the trial value to the subproblem objective function.

Benders Decomposition algorithm

The original problem is split into a master (3.59) and a subproblem (3.60). The solution strategy is as follows:

Step 1. Iteration index *v=1*.

Step 2. The master problem is solved subject to all appended Benders cuts (none for *v=1*) and the optimal complicating variable $x^{(v)}$ is identified.

Step 3. The subproblem is solved subject to the trial complicating variable $x^{(v)}$.

Step 4. An optimality check (3.64) takes place by evaluating the difference between the lower and upper bounds of the problem, $z_{lower}^{(v)}$ and $z_{upper}^{(v)}$. If the bounds have converged, the master's approximation of the subproblem value is accurate and the algorithm goes to **Step 7**.

Step 5. If convergence has not been achieved, the Benders cut (3.61) related to the current iteration is constructed and appended to the master problem.

Step 6. Iteration index *v=v+1*. The algorithm returns to **Step 2**.

Step 7. End

Master problem

The master problem is independent of non-complicating variables and approximates the subproblem value using the scalar variable α. Solving the master problem in the absence of any Benders cuts gives the lower bound of the original problem. As Benders cuts are being added to the master problem, the subproblem approximation is built up from below [54] until $z^{(v)}$ is equal to the value of the original problem.

$$z^{(v)} = \min\left\{c^T x + \alpha\right\} \tag{3.59}$$

s.t.

$$Ax \geq b$$

$$x \geq 0$$

Subproblem

The subproblem contains all non-complicating variables y and utilizes the master's trial solution $x^{(v)}$ as an input variable in the coupling constraint. The coupling constraint's dual variable $\lambda^{(v)}$ can be interpreted as the value change in the subproblem's objective function $\omega(x^{(v)})$ following a unit change in the trial solution x^{v}.

$$\omega(x^{(v)}) = \min\left\{d^{T} y\right\} \tag{3.60}$$

s.t.

$$Ey \geq h - Fx^{(v)} \ : \lambda^{(v)}$$

$$y \geq 0$$

Benders Cut

The Benders cut to be appended to the master problem at iteration *v* is given by equation (3.61). It is a linear constraint that utilizes the dual variable $\lambda^{(v)}$ and the trial solution $x^{(v)}$ to approximate the subproblem value.

$$\alpha \geq \omega(x^{(v)}) - (x - x^{(v)})^{T} F^{T} \lambda^{(v)} \tag{3.61}$$

Lower Bound

At iteration *v*, the master problem is a relaxed version of the original problem since it includes a limited number of all the potential Benders cuts that accurately describe the subproblem. As a result, the optimal value of the master problem is a lower bound to the optimal value of the original problem.

$$z_{lower}^{(v)} = c^{T} x^{(v)} + \alpha \tag{3.62}$$

Upper Bound

At iteration *v*, the subproblem constitutes a more constrained version of the original problem because the complicating variable is fixed at the non-optimal trial value $x^{(v)}$. As a result, the value of the original problem at iteration *v* is an upper bound to the true optimal value of the original problem.

$$z_{upper}^{(v)} = c^{T} x^{(v)} + \omega(x^{(v)}) \tag{3.63}$$

Convergence Criterion

The Benders decomposition algorithm is terminated subject to the convergence criterion (3.64), where ε is a small positive number. When this holds true, the master's approximation to the subproblem value α is very close to the subproblem optimal value $\omega(x^{(v)})$ and the trial solution $x^{(v)}$ is the optimal solution to the original problem.

$$z_{upper}^{(v)} - z_{lower}^{(v)} \leq \varepsilon \tag{3.64}$$

It is important to note that the general form of Benders decomposition includes an additional step ensuring that decisions $x^{(v)}$ result in a feasible subproblem solution. In the case that the subproblem is infeasible, instead of an optimality cut, a suitable feasibility cut is generated and appended to the master problem. However, through the use of slack variables in the sub problem, problem feasibility can be guaranteed and feasibility cuts are not necessary as long as an infeasibility measure is passed to the master problem through the dual variables.

The transmission planning problem structure lends itself to Benders decomposition due to the distinct separation between capital and operation costs, with the complicating variables being the transmission investment decisions. In the context of the above formulation, x is the investment decision vector, y is the operation decision vector (including dispatch, power flow decision variables etc.) and the original objective function is a linear combination of capital and operation costs as in (3.55). In addition, investment and operation decisions are coupled through the power flow constraints as in (3.57). Following this analogy, the above Benders decomposition scheme can be used to solve the transmission expansion problem. The original problem is split into a master problem that models only investment variables and a subproblem that models only operation variables having implemented the trial expansion plan suggested by the master. Through the iterative algorithm, the master investment problem informs the subproblem about capacity addition decisions, while the operation subproblem informs the master about the need for additional capacity through the Benders cuts.

3.8.1.1 Benefits of Benders decomposition

The benefits of employing Benders decomposition to the TEP can be summarized as follows:

i. **Problem de-scaling.** The computational complexity of linear problems grows non-linearly with the addition of variables and constraints. By decomposing the original problem in a large number of small-scale programs, complexity is reduced significantly. Thus, studies involving large systems that would otherwise have prohibitive memory and CPU time requirements become manageable.

ii. **Parallel computing.** In the absence of inter-temporal constraints related to the system operation, it is possible to solve several operational subproblems in parallel, gaining a very substantial computational advantage.

3.8.2 Benders Decomposition in Transmission Expansion Planning

In this section we show how Benders decomposition can be implemented to the security-constrained dynamic transmission investment problem. A novel multi-cut formulation is also presented that allows much faster convergence through appending multiple highly-parameterised hyperplanes to the relaxed master problem.

As previously stated, Benders decomposition is an iterative process. The objective function of the master problem is derived by substituting the operational cost component of the original problem with an estimate. At each iteration *v*, the master problem determines all investment decisions while ignoring operational constraints. In turn, all the operational sub-problems are solved subject to these decisions. The Lagrangian multipliers of the sub-problems' coupling constraints are then used to build the Benders cut which is appended to the master problem. This process continues until convergence is reached. In the case of security-constrained transmission planning, the implemented algorithm utilizing the contingency screening module to implement the N-1 security constraint is summarized graphically in the flowchart shown in Figure 3-2. The solution strategy can be described as follows:

Step 1. Contingency screening iteration *i=1*.

Step 2. $K^{i}_{c,e,t} = 0$ for all operating points *(c,e,t)*.

Step 3. Benders iteration index $v = 1$

Step 4. Discard all appended Benders cuts from the master problem.

Step 5. Solve the master problem (3.65) - (3.71) including all appended Benders cuts.

Step 6. Solve the operation subproblem (3.73) – (3.95) subject to all binding contingencies included in $\overline{K}^{i-1}$ utilizing the master problem trial investment decisions $F^{inv(v)}_{e,l}$ and $QB^{(v)}_{e,l}$. By definition, if *i=1* then no binding contingencies are considered.

Step 7. Check the convergence criterion (3.96). If false, construct the relevant Benders cut (3.72), append it to the master problem, update the Benders iteration index as $v = v + 1$ and go to **Step 5**.

Step 8. Screen all operating points *(c,e,t)* for binding contingencies subject to the optimal investment decisions $F_{e,l}^{inv^{(v)}}$ and $QB_{e,l}^{(v)}$ and the optimal generation dispatch $p_{e,t,g}^{(v)}$. Determine membership status of each post-fault operating point *(c,e,t)* to the list of binding contingencies according to (3.52): if binding, $\left(K_{c,e,t}^{i} = 1\right)$, otherwise $\left(K_{c,e,t}^{i} = 0\right)$.

Step 9. If $\sum_{\forall(c,e,t)} K_{c,e,t}^{i} = 0$, meaning that no binding contingencies were detected in this iteration, go to **Step 12**. If $\sum_{\forall(c,et)} K_{c,e,t}^{i} > 0$, meaning that some binding contingencies were detected, go to **Step 10**.

Step 10. $K_{c,e,t}^{i} = \max\left(K_{c,e,t}^{1},..., K_{c,e,t}^{i}\right), \forall c, e, t$, meaning that if post-fault operating point *(c,e,t)* was found to be binding in some previous iteration and found to be non-binding in the current iteration *i*, its status is reinstated as binding.

Step 11. Update the contingency screening iteration index *i=i+1* and go to **Step 2**.

Step 12. END

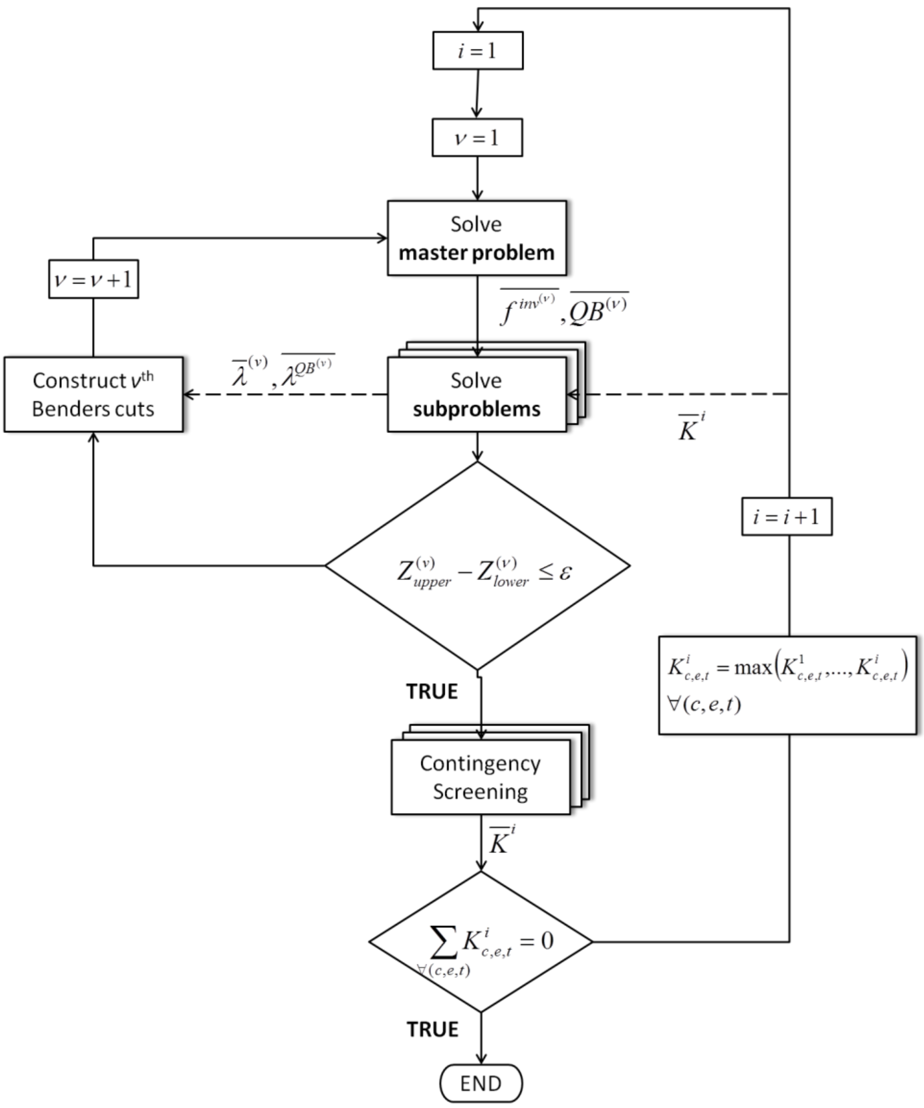


Figure 3-2: Benders decomposition and contingency screening algorithm flowchart. Stacked blocks indicate parallel implementation.

3.8.2.1 Master problem

Equation (3.65) is the master problem objective function at iteration ν. It includes investment costs related to line reinforcements and quadrature boosters, while the operating cost component of the original objective function (3.43) has been replaced with the estimate α .

$$\min_{f^{inv},\beta,qb,\alpha} \left\{ \sum_{e=1}^{N_E} \left[r_e^I \sum_{l=1}^{N_L} \left(\sum_{w=1}^{N_{W_l}} \left(f_{e,l,w}^{inv} c_{l,w} + \beta_{e,l,w} \gamma_{l,w} \right) \chi_l + qb_{e,l} c_l^{QB} \right) \right] + \alpha \right\} \tag{3.65}$$

The optimization constraints (3.66) – (3.71) that are included in the master problem deal only with the investment variables, ignoring all operational constraints. Thus the master problem is a relaxed version of the original formulation subject to the appended Benders cuts.

$$\beta_{e,l,w} \in \{0,1\} \qquad \forall e,l,w \tag{3.66}$$

$$qb_{e,l} \in \{0,1\} \qquad \forall e,l \tag{3.67}$$

$$F_{e,l}^{inv} = \sum_{w=1}^{N_{W_l}} \sum_{j=1}^{e-k_{l,w}} f_{j,l,w}^{inv} \qquad \forall e,l \tag{3.68}$$

$$\sum_{i=1}^{e} f_{i,l,w}^{inv} \le \sum_{j=1}^{e} \beta_{j,l,w} F_{l,w}^{\max} \qquad \forall e,l,w \tag{3.69}$$

$$QB_{e,l} = \sum_{j=1}^{e-k_l^{QB}} qb_{e,l} \qquad \forall e,l \tag{3.70}$$

$$QB_{e,l} \le 1 \qquad \forall e,l \tag{3.71}$$

Constraint (3.66) and (3.67) define the binary investment decision variables. Equation (3.68) is used to calculate the total capacity that has been added to line *l* up to epoch *e* while considering build times. In other words, the state variable $F_{e,l}^{inv}$ represents the additional capacity that has been commissioned up to epoch *e*. Constraint (3.69) states that the maximum capacity that can be added to line *l* using project *w* in epoch *e* is bounded by the maximum capacity that can be provided by project *w* (given that it has been selected in the current or previous epochs) and the capacity already installed under this project in the previous epochs. Equation (3.70) determines whether a QB has been commissioned in epoch *e* , while considering the corresponding construction delay and constraint (3.71) states that a single QB can be installed in each line over the horizon. The Benders cut appended to the master problem at iteration v is formulated in terms of the investment state variables and the corresponding dual variables received from the subproblem as shown in (3.72). The superscript *(v-1)* is used to identify the optimal variable value of iteration *v-1*.

$$\alpha \ge \left[\begin{array}{c} \sum_{e=1}^{N_E} \sum_{t=1}^{N_T} \omega_{e,t}^{(v-1)} \\ \sum_{e=1}^{N_E} \sum_{t=1}^{N_T} \sum_{l=1}^{N_L} \lambda_{e,t,l}^{(v-1)} \left(F_{e,l}^{inv} - F_{e,l}^{inv(v-1)} \right) \\ \sum_{e=1}^{N_E} \sum_{t=1}^{N_T} \sum_{l=1}^{N_L} \lambda_{e,t,l}^{QB(v-1)} \left(QB_{e,l} - QB_{e,l}^{(v-1)} \right) \end{array} \right] \tag{3.72}$$

3.8.2.2 Operation Subproblem

The objective function of the sub-problem for operating point *(e,t)* (3.73) is the sum of constraint costs and unserved energy. Investment decisions variables $F_{e,l}^{inv^*}$ and $QB_{e,l}^{*}$ are introduced, but are forced to be identical to the trial solution provided by the master problem through equations (3.81) and (3.82). This approach is similar to [93]. The corresponding dual variables indicate the marginal change to the sub-problem objective value if these coupling constraints were relaxed.

$$\omega_{e,t}^{(\nu)} = \min_{p^+, p^-, d^*} \left\{ r_e^O \sum_{t=1}^{N_T} \tau_t \left[\sum_{g=1}^{N_G} \left(p_{e,t,g}^+ o_g - p_{e,t,g}^- b_g \right) + \sum_{n=1}^{N_N} d_{e,t,n}^* \Gamma \right] \right\} \tag{3.73}$$

Subject to

$$0 \le p_{e,t,g} \le p_{e,g}^{\max} \qquad \forall g \tag{3.74}$$

$$p_{e,t,g} = p_{e,t,g}^* + p_{e,t,g}^+ - p_{e,t,g}^- \qquad \forall g \tag{3.75}$$

$$f_{e,t,l} = \frac{\theta_{e,t,u_l} - \theta_{e,t,v_l}}{x_l} \qquad \forall l \tag{3.76}$$

$$\sum_{g=1}^{N_G} B_{n,g} p_{e,t,g} + \sum_{l=1}^{N_L} I_{n,l} \left(f_{e,t,l} + p_{e,t,l}^{QB} + p_{e,t,l}^{SC} \right) = d_{t,n} + d_{e,t,n}^* \qquad \forall n \tag{3.77}$$

$$-\left(F_{e,l}^{inv*} + F_l^0 \right) \le f_{e,t,l} + p_{e,t,l}^{QB} + p_{e,t,l}^{SC} \le \left(F_{e,l}^{inv*} + F_l^0 \right) \qquad \forall l \tag{3.78}$$

$$-\frac{\xi_l^{\max}}{x_l (x_l - \xi_l^{\max})} \left(\theta_{e,t,u_l} - \theta_{e,t,v_l} \right) \le p_{e,t,l}^{SC} \le -\frac{\xi_l^{\min}}{x_l (x_l - \xi_l^{\min})} \left(\theta_{e,t,u_l} - \theta_{e,t,v_l} \right) \qquad \forall l \tag{3.79}$$

$$-\frac{\psi_l^{\max} QB_{e,l}^*}{x_l} \le p_{e,t,l}^{QB} \le -\frac{\psi_l^{\min} QB_{e,l}^*}{x_l} \qquad \forall l \tag{3.80}$$

$$F_{e,l}^{inv*} = F_{e,l}^{inv(\nu)} : \lambda_{e,t,l}^{(\nu)} \qquad \forall l \tag{3.81}$$

$$QB_{e,l}^* = QB_{e,l}^{(\nu)} : \lambda_{e,t,l}^{QB(\nu)} \qquad \forall l \tag{3.82}$$

For all $c, l \in \Omega_L$, if $(K_{c,e,t}^{i-1} = 1)$ then:

$$\sum_{g=1}^{N_G} B_{n,g} p_{e,t,g} + \sum_{l=1}^{N_L} I_{n,l} \left(f_{c,e,t,l}^C + p_{c,e,t,l}^{QB^C} + p_{c,e,t,l}^{SC^C} \right) = d_{t,n} + d_{e,t,n}^* \qquad \forall n \tag{3.87}$$

For all $c, l \in \Omega_L$, if $(K_{c,e,t}^{i-1} = 1)$ and $(c = l)$ then:

$$f_{c,e,t,l}^C = 0 \tag{3.88}$$

$$p_{c,e,t,l}^{SC^C} = 0 \tag{3.89}$$

$$p_{c,e,t,l}^{QB^C} = 0 \qquad (3.90)$$

For all $c, l \in \Omega_L$, if $(K_{c,e,t}^{i-1} = 1)$ and $(c \neq l)$ then :

$$f_{c,e,t,l}^{C} = \frac{\theta_{c,e,t,u_l}^{C} - \theta_{c,e,t,v_l}^{C}}{x_l} \qquad (3.91)$$

$$-\left(F_{e,l}^{inv*} + F_l^{0}\right) \leq f_{c,e,t,l}^{C} + p_{c,e,t,l}^{QB^C} + p_{c,e,t,l}^{SC^C} \leq \left(F_{e,l}^{inv*} + F_l^{0}\right) \qquad (3.92)$$

$$-\frac{\xi_l^{\max}}{x_l(x_l - \xi_l^{\max})}\left(\theta_{c,e,t,u_l}^{C} - \theta_{c,e,t,v_l}^{C}\right) \leq p_{c,e,t,l}^{SC^C} \leq -\frac{\xi_l^{\min}}{x_l(x_l - \xi_l^{\min})}\left(\theta_{c,e,t,u_l}^{C} - \theta_{c,e,t,v_l}^{C}\right) \qquad (3.94)$$

$$-\frac{\psi_l^{\max} QB_{e,l}^{*}}{x_l} \leq p_{c,e,t,l}^{QB^C} \leq -\frac{\psi_l^{\min} QB_{e,l}^{*}}{x_l} \qquad (3.95)$$

Constraints (3.74)-(3.80) are the pre-fault operational constraints describing system dispatch under intact conditions. Constraints (3.87)-(3.95) are included in the formulation only in the case that operation point *(c,e,t)* has been included in the list of binding contingencies. Equation (3.87) is the post-fault system balance equation. Equations (3.88)-(3.90) state that when a line is in outage, then the flow over that line is zero and any installed FACTS devices cannot operate. Finally, constraints (3.91)-(3.95) relate to the post-fault operations of the lines that are not in outage.

3.8.2.3 Benders convergence criterion

The convergence criterion (3.96) is expressed in terms of the difference between the total operation cost as defined in (3.97) and the master problem's approximation to that cost, α. The former is defined as th e upper bound of the problem, gradually being decreased as additional capacity is invested, while the latter is the lower bound; it increases when the gradient information provided by the sub-problem indicates that no further improvement can be made to operation costs through increased investment. The threshold value μ should be a value close to 0, in order to ensure close matching between the actual and approximated operational cost. At each iteration ν, the convergence criterion (3.96) is checked. If it is true, vectors $F^{inv(\nu)}$ and $QB^{(\nu)}$ are the optimal solutions to the transmission expansion problem.

$$Z_{upper}^{(\nu)} - Z_{lower}^{(\nu)} \leq \varepsilon \qquad (3.96)$$

$$Z_{upper}^{(v)} = \sum_{e=1}^{N_E} \sum_{t=1}^{N_T} \omega_{e,t}^{(v)} \tag{3.97}$$

$$Z_{lower}^{(v)} = \alpha^{(v)} \tag{3.98}$$

3.8.3 Multi-cut Benders decomposition

As shown above, the classical formulation of Benders decomposition generates only one constraint per iteration (known as the mono-cut variant). As illustrated in equation (3.72), in case of multiple sub-problems, dual variables are summed to produce a single gradient indicator. In addition, the sub-problem is approximated through a single variable α. The problem of this approach resides in its slow convergence. It is desirable to compute a set of cuts in order to improve the representation of the subproblem within the master. The computational advantages of appending multiple constraints per iteration to deal with this drawback have been well-documented. Birge et al. [51] employ a multi-cut Benders decomposition scheme to a two-stage stochastic problem. A separate constraint per scenario is appended to the master problem, leading to convergence in fewer iterations. In this section we show a multi-cut Benders decomposition formulation for the developed transmission expansion model, where a set of $N_E N_T$ cuts is generated per iteration. The approximation of operating costs present in the master problem's objective function (3.99) is partitioned through variable duplication to one variable $\alpha_{e,t}$ per operating point *(e,t)*. Moreover, one constraint (3.100) per operating point is appended to the master problem at every iteration. This results in a tight coupling of the partitioned $\alpha_{e,t}$ to the Lagrangian multipliers of the same operating point, providing more accurate information as to how investment decision changes will impact on operational costs. This way we can improve representation of operational costs in the master problem, provide denser gradient information and achieve faster convergence at the expense of more constraints.

$$\min_{f^{inv},\beta,qb,\alpha} \left\{ \sum_{e=1}^{N_E} \left[r_e^I \sum_{l=1}^{N_L} \left(\sum_{w=1}^{N_{W_l}} \left(f_{e,l,w}^{inv^{(v)}} c_{l,w} + \beta_{e,l,w} \gamma_{l,w} \right) + qb_{e,l}^{(v)} c_l^{QB} \right) + \sum_{t=1}^{N_T} \alpha_{e,t} \right] \right\} \tag{3.99}$$

$$\alpha_{e,t} \geq \begin{bmatrix} \omega_{e,t}^{(v-1)} + \\ \sum_{l=1}^{N_L} \lambda_{e,t,l}^{(v-1)} \left(F_{e,l}^{inv} - F_{e,l}^{inv^{(v-1)}} \right) + \\ \sum_{l=1}^{N_L} \lambda_{e,t,l}^{QB^{(v-1)}} \left(QB_{e,l} - QB_{e,l}^{(v-1)} \right) \end{bmatrix} \quad \forall e,t \tag{3.100}$$

The definition for the lower bound is changed accordingly to be the sum of the optimal values $\alpha_{e,t}$ at iteration *v*.

$$Z_{lower}^{(v)} = \sum_{e=1}^{N_E} \sum_{t=1}^{N_T} \alpha_{e,t}^{(v)} \tag{3.100}$$

The computational benefits of this approach when compared to the mono-cut version are shown in section 3.9.6, where it is applied to a large case study. The novel multi-cut algorithm offers an important enhancement to the applicability of linear programming techniques in the transmission expansion problem. Faster convergence enables us to accommodate a larger number of operating points; a feature that gains importance as we move towards larger penetration of intermittent sources.

3.9 IEEE-RTS Case study

We proceed with the application of the developed security-constrained transmission expansion formulation to a case study on the 24-bus IEEE Reliability Test System, shown in Figure 3-3. The aim of the presented case study is to illustrate the features of the proposed framework while highlighting the shortcomings of alternative approaches. For this purpose, the IEEE-RTS is a suitable system choice due to its diverse and highly-meshed topology, constituting a good proxy to the reduced transmission system models that are typically used in planning studies in the UK and other jurisdictions.

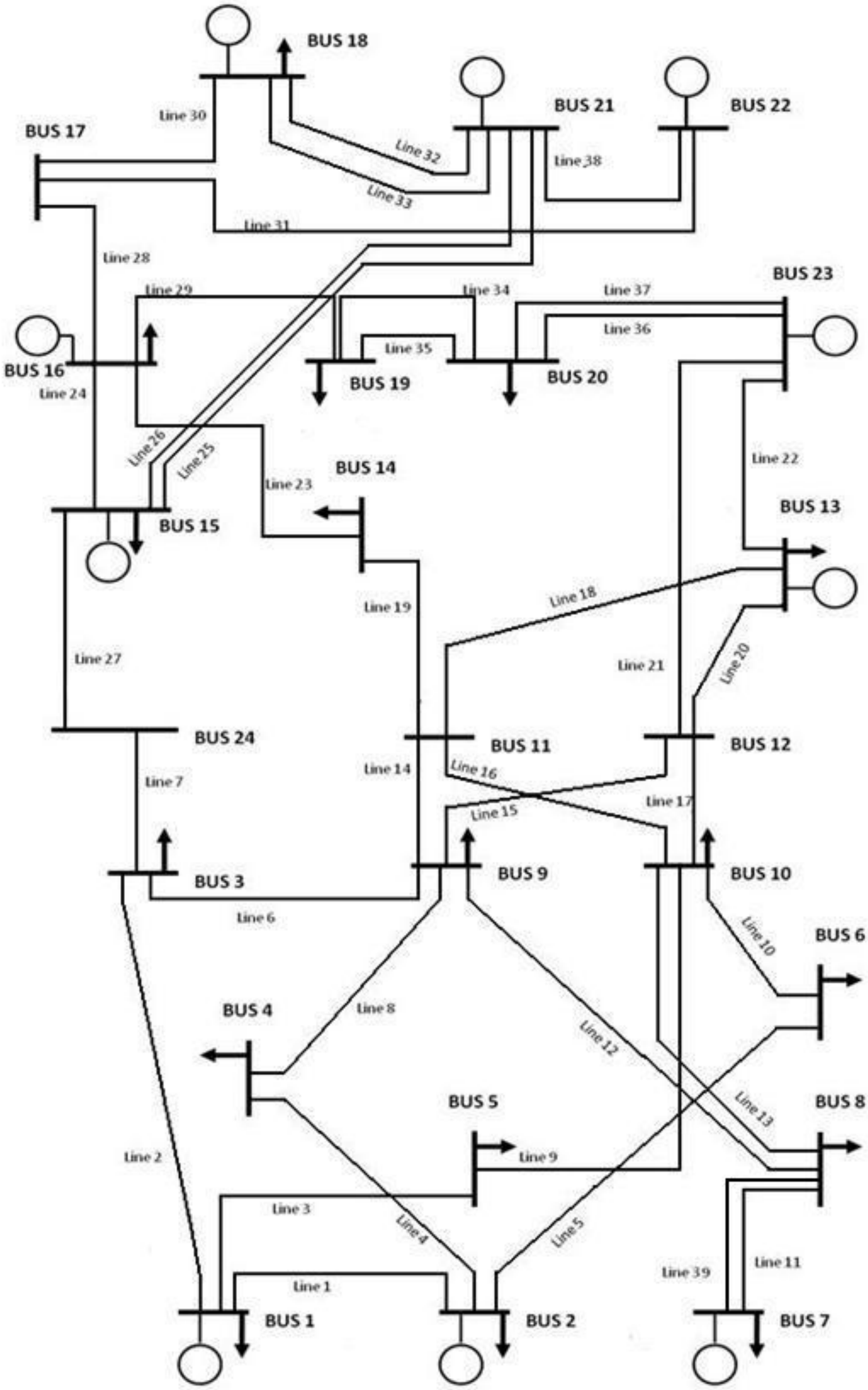


Figure 3-3: IEEE-RTS topology.

3.9.1 Case study description

The test system consists of 24 buses, 39 lines and 28 generation units whose technical and economic characteristics are shown in Table 3-1.

Gen Id	Bus	Generation Type	p^{max} [MW]	Fuel cost [£/MWh]	Offer price [£/MWh]	Bid price [£/MWh]
G1	1	Oil	20	50.01	150.03	45.01
G2	1	Oil	20	50.02	150.06	45.02
G3	1	Coal	76	30.01	90.03	27.01
G4	1	Coal	76	30.02	90.06	27.02
G5	2	Oil	20	50.03	150.09	45.03
G6	2	Oil	20	50.04	150.12	45.04
G7	2	Coal	76	30.03	90.09	27.03
G8	2	Coal	76	30.04	90.12	27.04
G9	7	Oil	100	50.05	150.15	45.05
G10	7	Oil	100	50.06	150.18	45.05
G11	7	Oil	100	50.07	150.21	45.06
G12	13	Oil	197	50.08	150.24	45.07
G13	13	Oil	197	50.09	150.27	45.08
G14	13	Oil	197	50.10	150.30	45.09
G15	15	Oil	12	50.11	150.33	45.10
G16	15	Oil	12	50.12	150.36	45.11
G17	15	Oil	12	50.13	150.39	45.12
G18	15	Oil	12	50.14	150.42	45.13
G19	15	Oil	12	50.15	150.45	45.14
G20	15	Coal	155	30.05	90.15	27.05
G21	16	Coal	155	30.06	90.18	27.05
G22	18	Nuclear	400	6.01	999.00	-100.00
G23	21	Nuclear	400	6.02	999.00	-100.00
G24	23	Coal	155	30.07	90.21	27.06
G25	23	Coal	155	30.08	90.24	27.07
G26	23	Coal	350	30.09	90.27	27.08
G27	3	Onshore Wind	0	0.00	999.00	-100.00
G28	6	Onshore Wind	0	0.00	999.00	-100.00

Table 3-1: IEEE-RTS case study generation data.

The network topology has been preserved the same as in [52] but some changes have taken place in terms of the network to make the presented analysis more straightforward; all connecting corridors are initialized with reactance of 0.02 p.u. and just enough transmission capacity to allow N-1 secure uncongested operation in the first epoch (rounded up to the nearest 50 MW). As a result, all undertaken investment is due to the new generation added in subsequent epochs. The full network information can be found in Table A-3.

As can be seen in Table 3-1, the system includes 0.8 GW of base-load nuclear plant, about 1.3 GW of mid-merit coal generators and 1GW of expensive oil generators. Total initial generation capacity sums to 3,105 MW, while system peak load is 2,850 MW. The distribution of load across the system nodes is shown in Table A-1. Buses 13, 15 and 18 are the highest loaded buses accounting for 30% of demand. In terms of operating points, 100 demand and wind snapshots have been considered covering both winter and summer periods. The detailed load and wind data can be found in Table A-2. Demand is expressed as a percentage of the system peak load level of 2,850MW. The winter period consist of 50 snapshots accounting for 6,720 hours of operation with a total of 57 hours of peak loading and 70 hours of peak wind conditions. Similarly, the summer period consists of 50 snapshots totalling 2,016 hours of operation. Maximum system load during summer is reduced to 1,900 MW, while peak wind occurs for a total of 23 hours. The average wind factor over the entire year is 30% while the average system loading factor is 66%.

The study horizon comprises of 4 five-year epochs over which a large number of wind generation is to be connected to bus 3 (G27) according to the expansion schedule shown in Table 3-2. All other system attributes such as peak demand, and existing generation capacity are considered to stay fixed across the 20 year horizon. In addition, we consider all prices to stay fixed at present levels and we discount future costs using an annual discount rate of 5%.

	Epoch 1	**Epoch 2**	**Epoch 3**	**Epoch 4**
G27 Capacity (MW)	0	250	750	1250

Table 3-2: G27 evolution over the four epochs.

The system in its initial state is severely under-equipped to deal with the very large flows that will occur following the commissioning of wind generation. The evolution of constraint costs in the absence of transmission reinforcements is shown in Table 3-3. Overall, lack of investment results in £2bn of constraint costs over the 20 year period.

	Epoch 1	**Epoch 2**	**Epoch 3**	**Epoch 4**	**Total**
Constraint Costs (£m)	0	202.4	750.8	1059.1	2012.3

Table 3-3: Constraint costs in the absence of investment.

As stated before, in the first epoch no constraints exists in the system since all transmission lines have been initialized with enough capacity to allow uncongested N-1 secure operation with respect to the existing generation background. However, in the following epochs, as wind generation is being added, link capacities are inadequate to accommodate the arising flows. This can be seen in Figure 3-4 where we compare the maximum post-fault power flows for the minimum cost generation dispatch schedule (i.e. the unconstrained dispatch) to the already installed transmission capacity. More specifically, lines 1-9 need significant reinforcements to cope with the increased power transfers. In particular, lines 2 and 6, being the primary wind power exporting links, require the highest level of investment.

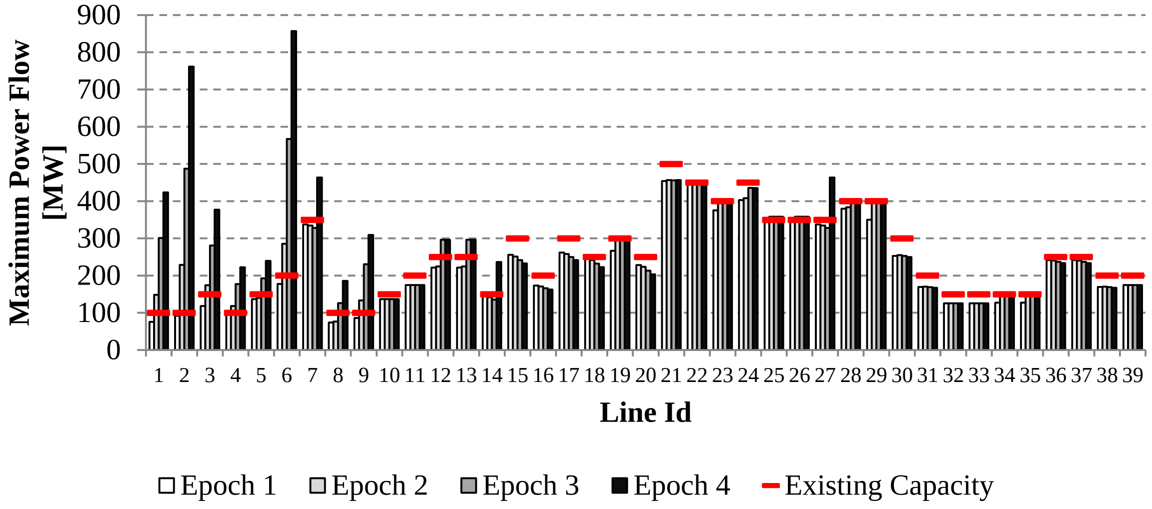


Figure 3-4: Maximum post-fault power flows for the minimum cost generation dispatch.

Due to this transmission capacity shortfall, a significant amount of wind power will have to be spilled if no investment is undertaken. As shown in Figure 3-5, in epochs 3 and 4, 53% and 65% of available wind energy is curtailed respectively. In addition, the coal plant G20 is constrained-off during high wind conditions to allow wind power exports through lines 24-27. In their place, out-of-merit coal and oil plants have to be dispatched to meet electricity demand, leading to significant constraint costs.

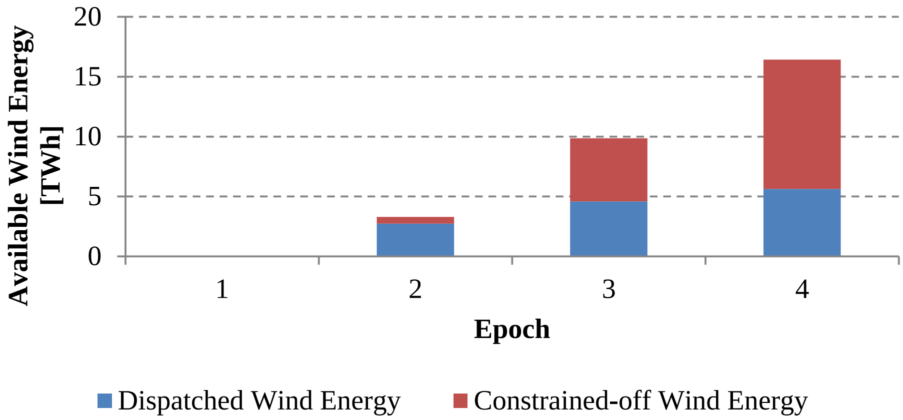


Figure 3-5: Volume of constrained-off wind energy in the absence of investment.

3.9.2 Dynamic transmission investment solution

In this section we present the optimal transmission investment solution. For each line, there are three mutually exclusive candidate expansion options (i.e. only one candidate can be built per line), whose cost and technical data are shown in Table 3-4. Mutually exclusive options were modelled by including the following constraint in the master investment problem:

$$\sum_{e=1}^{N_E}\sum_{w=1}^{N_{W_l}} \beta_{e,l,w} \leq 1 \qquad \forall l \tag{3.101}$$

All candidates have a build time of 1 epoch. Option A is a short-term solution with the lowest fixed cost but capable of providing only a 200 MW capacity reinforcement. Option B can provide a larger reinforcement up to 400 MW for higher fixed costs. Investment in option B has been limited to either a 200 MW or a 400 MW capacity addition to reflect the lumpiness of transmission investment. Similarly, option C can provide up to 800 MW for even higher fixed costs and has been limited to either a 400 MW or an 800 MW capacity addition. These discrete upgradeability options have been modelled as follows:

$$f_{e,l,w}^{inv} = n_{e,l,w} F_w^{step} \qquad \forall e,l,w \tag{3.101}$$

Where $n_{e,l,w}$ is an integer decision variable and F_w^{step} is the upgrade size (e.g. 200 for option B). The investment costs for the different options are shown in Figure 3-6, assuming a line length of 50km and that construction is initiated in the first year of the case study horizon.

Option	Maximum Capacity [MW]	Fixed Cost [£/km·year]	Variable Cost [£/MW·km·year]	Build Time [Epochs]
A	200	60,000	50	1
B	400	80,000	50	1
C	800	130,000	50	1

Table 3-4: Capacity reinforcement options.

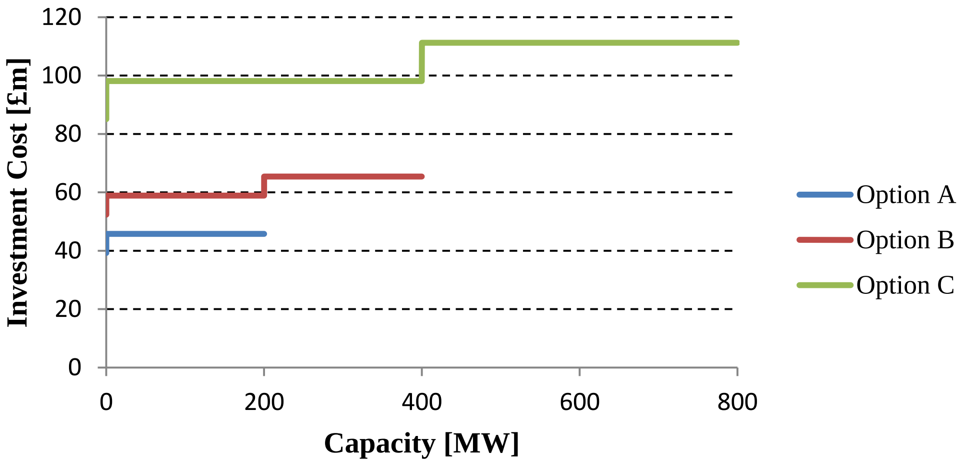


Figure 3-6: Transmission investment costs for a 50km line.

The optimal investment plan is shown in Table 3-5, where the chosen options and capacity additions undertaken at each epoch are presented.

Line Id	Epoch 1	Epoch 2	Epoch 3	Investment Cost [£m]
1	200 (A)	-	-	4.6
2	200 (B)	+ 200	-	112.4
3	-	200 (A)	-	20.9
4	-	200 (A)	-	31.7
5	-	-	200 (A)	28.2
6	200 (B)	+ 200	-	63.1
8	-	-	200 (A)	15.0
9	-	200 (A)	-	22.1

Table 3-5: Optimal investment plan. Letters in brackets indicate the chosen option for each line.

Despite the fact that no congestion exists in the first epoch, significant investment must be undertaken from the very first stage to ensure timely commissioning. As expected, the links warranting the highest level of reinforcements are lines 2 and 6 that emanate from bus 3 and export the bulk of wind energy to the rest of the system. Option B is chosen for both links in the first epoch and 200 MW are initially constructed, with further 200 MW expansions carried out in the second epoch. Moreover, line 1 is reinforced in the first epoch at a relatively small cost due to its 5km length. Note that lines 7 and 27, which also export power from bus 3, have been initialized with a large capacity of 350 MW and become binding only in the last epoch during periods of high wind output, contributing very little to congestion. For this reason, it is deemed uneconomical to upgrade them. In the second and third epochs, lines 3, 4, 5, 8 and 9 are upgraded. As can be seen in Figure 3-4, these lines face modest increases in utilization and can thus be adequately reinforced using option A. Naturally, no investment is undertaken during the last epoch, as it would materialize beyond the study horizon.

	Epoch 1	**Epoch 2**	**Epoch 3**	**Epoch 4**	**Total**
Investment Cost	168.3	86.6	43.2	0	298.1
Operation Cost	0	8.7	16.9	70.3	95.9
System Cost	168.3	95.3	60.1	70.3	**394.0**

Table 3-6: System costs (£m).

System cost data are shown in Table 3-6. The total system cost over the 20-year horizon is £394m. The total transmission investment amounts to £298.1m with the bulk capital expenditure related to reinforcements undertaken in the first epoch. Moreover, constraint costs have been significantly reduced from the no-investment levels to £95.9m, placing the net benefit of the optimal investment plan at 1,618.3m. Wind energy curtailment following transmission investment has been eliminated in epochs 2 and 3. In the last epoch, 4.5% of the available wind energy is spilled due to the inability of lines 2, 6, 7 and 27 to accommodate post-fault power flows during high wind output. However, the additional investment that would have to be undertaken to avoid this curtailment outweighs the constraint costs arising due to the out-of-merit dispatch.

3.9.3 Cost of security

If we were to determine the optimum investment plan in the absence of the N-1 security criterion, a much lower capital expenditure of £122.2m would have to be undertaken, as shown in Table 3-7. The amount of reinforcements is significantly reduced and deferred to later stages. A comparison between capacity reinforcements taken in the absence and inclusion of security constraints is shown in Figure 3-7.

Line Id	Epoch 1	Epoch 2	Epoch 3	Investment Cost [£m]
1	-	200 (A)	-	3.0
2	-	200 (A)	-	53.2
3	-	-	200 (A)	12.2
6	-	200 (B)	+ 200	40.9
9	-	-	200 (A)	12.9

Table 3-7: Optimal investment plan when ignoring the N-1 security criterion.

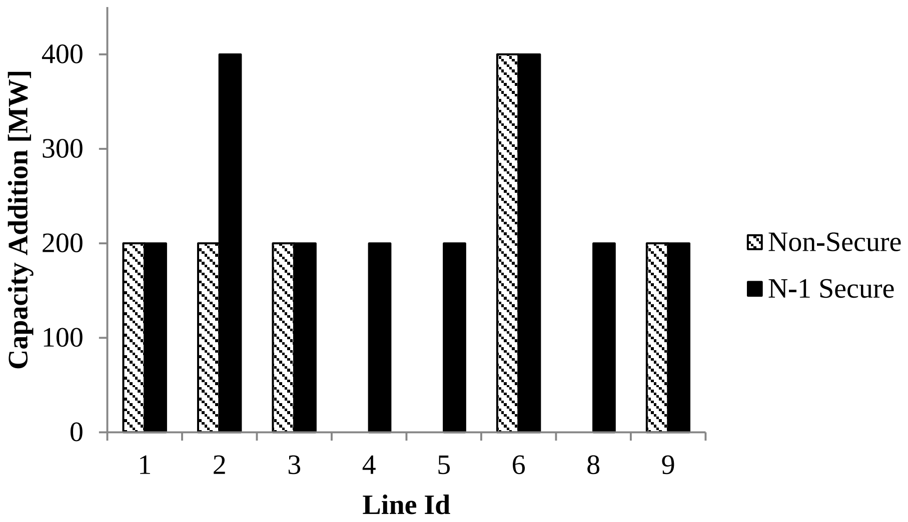


Figure 3-7: Capacity additions in the absence and inclusion of the N-1 security criterion.

Following the above plan, and ignoring security constraints, wind spilling is almost eliminated and constraint costs total £16m, leading to an overall system cost of £138.2m. The *cost of security* can be defined as the extra expenditure undertaken to enforce adequate reliability and can be computed as the system cost difference between non-secure and N-1 secure operation. It consists of the extra investment undertaken to accommodate post-fault power flows as well as the constraint costs incurred due to the dispatch of out-of-merit generation to provide preventive security. In this case study, the cost of security is £255.8m, illustrating that reliability considerations are a primary driver of transmission investment and form an integral part of the undertaken cost-benefit analysis.

3.9.4 Optimal Quadrature Booster placement

In this section we show how the optimal solution is altered when allowing investment for the optimal placement of quadrature boosters. It has been assumed that quadrature boosters can be added with a negligible construction delay (i.e. $k_l^{QB} = 0$) due to the limited planning preparation involved, thus constituting a very flexible investment opportunity that is commissioned instantaneously. Quadrature booster annuitized investment cost is set to c_l^{QB} = 300,000 £/year, meaning that the discounted capital cost of a QB commissioned in epoch 1 is about £4m. In this case study, all lines have been considered as suitable candidates for a QB addition with operating limits $\psi_l^{\min} = -30^o$ and $\psi_l^{\max} = 30^o$. The detailed investment plan is shown in Table 3-8 and system costs in Table 3-9.

Line Id	Epoch 1	Epoch 2	Epoch 3	Epoch 4	Investment Cost [£m]
1	-	200 (A)	-	-	3.0
		QB	-	-	2.6
2	-	400 (B)	-	-	76.0
3	-	200 (B)	-	-	20.9
6	-	400 (B)	-	-	42.7
7	-	QB	-	-	2.6
8	-	QB	-	-	2.6
9	-	200 (A)	-	-	22.1
24	-	QB	-	-	2.6
27	-	QB	-	-	2.6

Table 3-8: Optimal investment plan when including quadrature booster investment.

As can be seen above, a total of 5 devices have been installed at a combined cost of £13m. The operational flexibility offered by QBs significantly reduces the required level of investment in transmission assets. Most importantly, investment is deferred to the second stage. This is due to the instantaneous commissioning of FACTS devices which allows the network operator to limit second epoch constraint costs from a potential £202.4 (as shown in Table 3-3) to £11.7m through fuller utilization of the existing network assets and post-fault corrective security. A detailed example of how the installed quadrature boosters alleviate the need for preventive security provision by relieving potential overloads is included in Appendix B.

	Epoch 1	Epoch 2	Epoch 3	Epoch 4	Total
Investment Cost	0	177.7	0	0	177.7
Operation Cost	0	11.7	0	32.3	44
System Cost	0	189.4	0	32.3	**221.7**

Table 3-9: System costs (£m) when including quadrature booster investment.

Overall, the QB additions have resulted in a 40% reduction in transmission investment levels and a 54% reduction in constraint costs corresponding to very low levels of wind curtailment.

This places the net benefit of corrective security at £172.3m. All these cost savings are achieved through post-fault power flow control. Potential line overloads are avoided and the need for security-driven investment is reduced. It is thus shown that quadrature boosters can constitute very flexible investment opportunities due to their fast commissioning (relative to link reinforcements) and ability to defer investment to later periods. The need for major reinforcements may not be fully alleviated in the face of new generation additions, but can be timed more economically through the contingency management offered by QBs that allows fuller utilization of existing assets. Overall, corrective measures result in a more economic dispatch by reducing the need to engage out-of-merit plants that can provide preventive security.

In the following section we showcase the advantages of using the proposed decomposition and contingency screening scheme. All presented transmission expansion models have been programmed using FICO Xpress v7.1 [67] and solved using its branch and bound solver.

3.9.5 Computational performance of the non-decomposed formulation

The presented case study is a very large MILP optimization problem that involves more than 1 million decision variables. Its significant size is partly due to the large number of constraints associated to the N-1 security criterion as well as the inclusion of 468 binary decision variables and 468 integer variables related to investment decisions. In the absence of suitable decomposition techniques and intelligent handling of contingency constraints, the problem takes slightly less than 1 hour to solve while utilizing a very large amount of memory to construct and traverse the branch and bound tree. The dimensional properties and solution performance of the non-decomposed problem (while ignoring FACTS investment) are as follows:

Problem formulation	Objective function (£m)	CPU time (s)	CPU Memory usage (GB)	Variables	Constraints	Non-zero elements
Non-secure	138.2	57	0.18	80,128	112,186	282,538
N-1 secure (no contingency screening)	394.0	2,487	21.98	1,751,328	2,239,786	6,512,538
N-1 secure (screening)	394.0	576	5.23	1,037,201	612,513	1,700,961

Table 3-10: Dimensional properties and computational performance of non-decomposed problem formulations.

As can be seen above, ignoring N-1 security constraints results in a small problem formulation that can be easily handled by commercial solvers. However, the inclusion of an exhaustive list of all possible contingencies leads to severe dimensionality problems. More than 20 GB of memory is required for the solution of this problem. Through the use of the contingency screening module, CPU time and memory requirements are dramatically reduced. In particular, the contingency screening module identified 1,744 binding contingencies, which amounts to 11.18% of the total 15,600 credible contingencies. Through the inclusion of only these binding contingencies, the number of optimization constraints in the second screening iteration was reduced from the potential 2 million to about 600 thousand. Only two iterations of the contingency screening algorithm were required, resulting in a 75% faster solution time when compared to including the exhaustive list of contingencies. However, even greater computational benefits can be reaped when combining the concept of contingency screening with a multi-cut Benders decomposition scheme as shown in the following section.

3.9.6 Computational performance of the decomposed formulation

As mentioned before, the main advantages of problem decomposition are the reduction in problem size and the ability to solve multiple operational sub-problems in parallel. For the purposes of this case study, a Xeon 3.46GHz computer with two 6-core processors and 192GB of RAM was used. This allowed the parallel execution of 12 operational subproblems at a time, using FICO Xpress's mmjobs library that can manage distributed computing [68].

3.9.6.1 Mono-cut Benders decomposition

We first showcase the unsatisfactory convergence behaviour of the classical Benders decomposition formulation that appends a single constraint per iteration. The evolution of the upper bound for the non-secure transmission investment problem ignoring FACTS investment is shown in Figure 3-8.

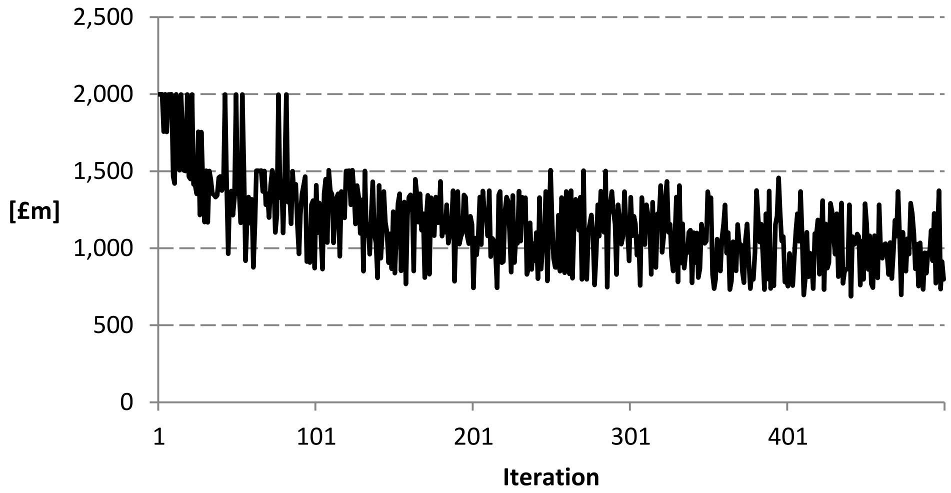


Figure 3-8: Evolution of upper bound in the mono-cut Benders decomposition formulation.

When ignoring N-1 security constraints, solution time was 40 minutes after 382 iterations. With the inclusion of binding contingencies, a gap value of 1 was maintained after 300 iterations run over the span of 4 hours, exhibiting prohibitively slow convergence behaviour. The inefficiency of one-dimensional cuts is evident in the zigzagging nature of the upper bound curve, shown in Figure 3-8, confirming the weakness of single cuts to give concrete gradient information and guide the problem to convergence within an acceptable time.

3.9.6.2 Multi-cut Benders decomposition

The multi-cut Benders decomposition scheme succeeds in obtaining convergence in a very short time while utilizing a fraction of RAM memory required under the non-decomposed problem formulation. The computational performance is summarized in Table 3-11.

Problem formulation	Objective function (£m)	Iterations	CPU time (s)	CPU Memory usage (GB)
N-1 secure (multi-cut/no screening)	394.0	17	123	0.35
N-1 secure (multi-cut + screening)	394.0	11	74	0.28

Table 3-11: Computational performance of decomposed problem formulations.

Convergence is achieved in just 15 iteration when the contingency filtering module is utilized, giving a total CPU time of 74s, a 90% reduction when compared to the decomposed formulation. The evolution of upper and lower bounds are shown in Figure 3-9.

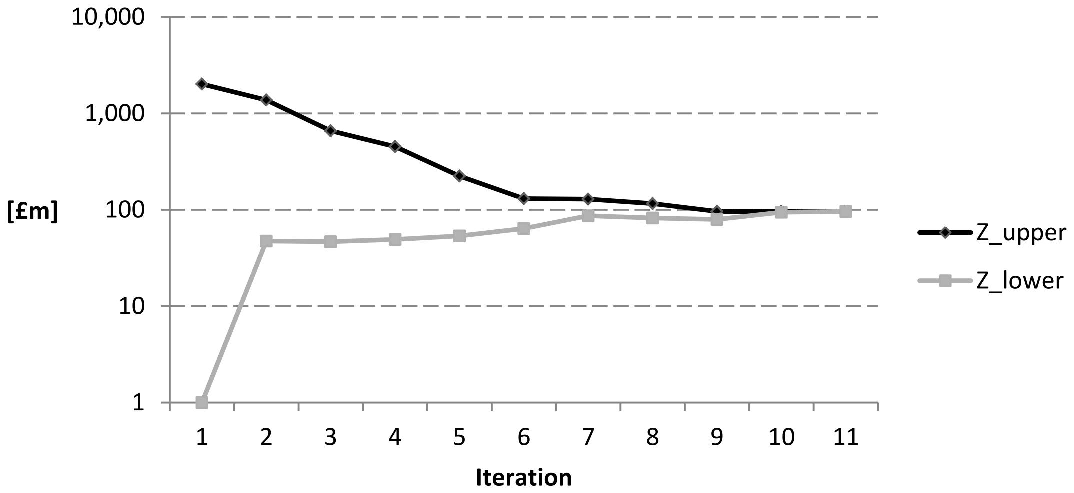


Figure 3-9: Evolution of upper and lower bounds for the N-1 secure planning problem when using the Benders multicut formulation.

We conclude that the combination of multi-cut Benders decomposition and the contingency filtering module is a very powerful approach to simulating systems with multiple operating points. It leads to very significant reduction of problem size and convergence is achieved in a small amount of iterations.

3.9.6.3 Optimal QB placement using multi-cut Benders decomposition

The proposed multi-cut decomposition scheme handles very well the problem size augmentation related to quadrature booster investment. Despite the addition of 156 binary investment variables and the large number of constraints related to the pre- and post-fault optimization of QB power injections, convergence is achieved within 15 iterations (Figure 3-10) and a CPU time of less than 5 minutes. Memory usage is also very limited. The importance of this achievement becomes clear when compared with the computational performance of the non-decomposed implementation. The optimizer requires excessive amounts of RAM and solution times are slow.

Problem formulation	Objective function (£m)	Iterations	CPU time (s)	CPU Memory usage (GB)
N-1 secure + QB (no Benders / no contingency screening)	221.7	-	3,800	32.45
N-1 secure + QB (multi-cut + screening)	221.7	15	215	0.43

Table 3-12: Computational performance for the optimal QB placement problem.

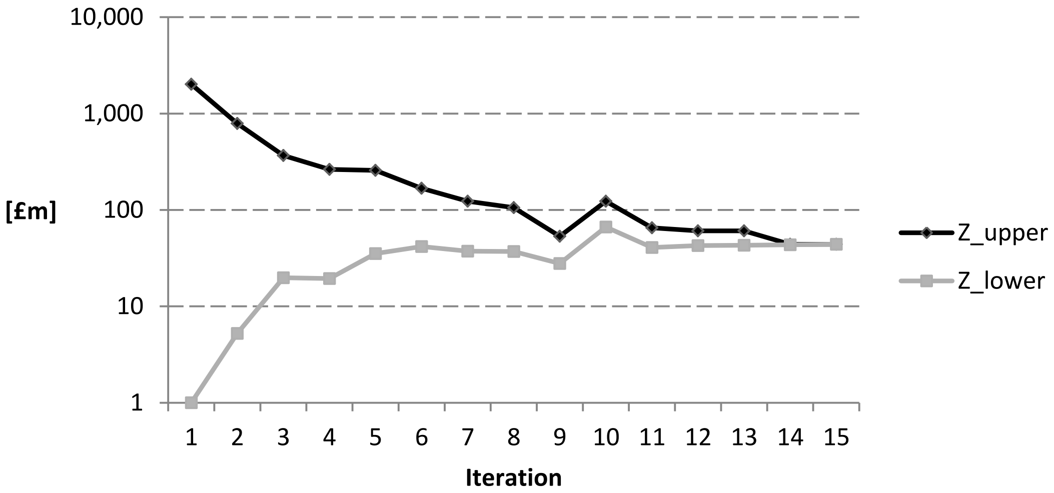


Figure 3-10: Evolution of upper and lower bounds for the N-1 secure planning problem (including QB investment) when using the Benders multicut formulation.

3.10 Concluding remarks

In Chapter 3 the deterministic multi-stage transmission investment formulation has been developed. It has been enriched with several key features that enable the development of a framework suitable for accommodating anticipatory investment decisions under uncertainty in the following chapters; upgradeable projects with fixed and variable costs, build times and optimal QB placement providing corrective security. In addition, N-1 security constraints have been accommodated through the use of an intelligent contingency screening module that identifies binding line outages. Finally, multi-cut Benders decomposition has been used to tackle the large MILP problem. We have shown how the presented dynamic transmission expansion model can be used to pinpoint the optimum investment plan through a case study on the IEEE-RTS. An additional case study allowing for the optimal placement of QBs illustrated how the operational flexibility of FACTS devices can defer investment. The computational benefits of using a multi-cut decomposition scheme and the inclusion of only binding contingencies have been quantified, indicating that the developed model is well-suited for large-scale optimization tasks. Finally, we have shown that a large part of transmission investment is driven by security constraints and their inclusion in transmission expansion modelling is critical for a complete valuation of system costs.

4 Stochastic Transmission Expansion Planning

Abstract

The need to consider sources of uncertainty in future operating conditions has been recognized for a long time as an essential element of energy system planning. In light of this, stochastic planning tools are gradually supplementing deterministic ones and new frameworks are being developed to accommodate optimal decision-making under uncertainty. In this chapter, we develop a transmission investment framework based on the concepts of multi-stage stochastic optimization while incorporating decision flexibility. In addition, we illustrate how the use of multi-cut Benders decomposition and contingency screening can be extended to the stochastic formulation, allowing the simulation of electricity systems under a large number of uncertainty scenarios. Finally, case studies are performed on two test systems in order to quantify the benefit of modelling decision flexibility. Through these studies, we illustrate that an attempt to determine investment activity without considering the possibility for recourse can lead to sub-optimal and premature commitments that lead to significant expected welfare loss.

4.1 Nomenclature

The mathematical symbols used in this Chapter in addition to the ones described in Section 3.1 are as follows.

4.1.1 Sets

$\Omega_{\bar{S}}$	Set of all scenarios.
$\Omega_M = \{1..N_M\}$	Set of all scenario tree nodes.
N_M^e	Number of scenario tree nodes at epoch *e*.
$\varepsilon(m)$	The epoch to which tree node *m* belongs.
$\Phi_0(m)$	A time-ordered set that contains all parent nodes of tree node *m*, including *m* as the last element.
$\Phi_k(m)$	A time-ordered set that contains all parent nodes of tree node *m* from the first epoch up to epoch $\varepsilon(m)-k$.
$[\Phi_0(m)]_e$	Parent of tree node *m* at stage *e*.

4.1.2 Input Variables

$p_{m,g}^{\max}$	Maximum stable generation for unit *g* at tree node *m*.	MW
$p_{m,t,g}^{*}$	Unconstrained dispatch power output of unit *g* for operating point *(m,t)*.	MW
π_s	Probability that scenario $\bar{s}$ occurs.	
π_m	Probability that tree node *m* occurs.	

4.1.3 Decision Variables

$f_{m,l,w}^{inv}$	Transmission capacity to be built for line *l* using option *w* in decision point *m*.	MW
$F_{m,l}^{inv}$	State variable of aggregate capacity added up to decision point *m* to line *l*.	MW
$\beta_{m,l,w}$	Binary variable signifying the choice of expansion option *w* for line *l* in decision point *m*.	

$qb_{m,l}$	Binary variable signifying the construction of a quadrature booster on line *l* in decision point *m*.	
$QB_{m,l}$	State variable of quadrature booster investment on line l up to decision point *m*.	
$p_{m,t,g}$	Output of unit g for operating point *(m,t)*.	MW
$p^{+}_{m,t,g}$	Constrained-on output of unit *g* for operating point *(m,t)*.	MW
$p^{-}_{m,t,g}$	Constrained-off output of unit *g* for operating point *(m,t)*.	MW
$f_{m,t,l}$	Power flow in line *l* for operating point *(m,t)*.	MW
$\theta_{m,t,n}$	Bus angle at node *n* for operating point *(m,t)*.	rad
$p^{QB}_{m,t,l}$	Power injection due to quadrature booster on line *l* f for operating point *(m,t)*.	MW
$p^{SC}_{m,t,l}$	Power injection due to series compensator on line *l* for operating point *(m,t)*.	MW
$d^{*}_{m,t,n}$	Curtailed demand at bus *n* for operating point *(m,t)*.	MW
$f^{C}_{m,c,t,l}$	Post-fault power flow in line *l* for operating point *(m,t)* when line *c* is in outage.	MW
$\theta^{C}_{m,c,t,n}$	Post-fault bus angle at node *n* for operating point *(m,t)* when line *c* is in outage.	rad
$p^{QB^C}_{m,c,t,l}$	Post-fault quadrature booster power injection for operating point *(m,t)* when line *c* is in outage.	MW
$p^{SC^C}_{m,c,t,l}$	Post-fault series compensator power injection for operating point *(m,t)* when line *c* is in outage.	MW

4.2 Introduction

In the previous Chapter we discussed deterministic planning models where all system parameters are taken to be firmly known a priori. The system planner’s objective is to optimize the transmission network subject to future generation connections. In the presence of perfect information concerning future developments, this is a well understood and widely researched topic. However, in reality, there is a wide range of uncertainties affecting the planning process. Under this paradigm, the planner’s objective becomes the minimization of expected system costs and stochastic techniques must be employed in order to find the optimal decisions with the best expected performance. In the case of future generation uncertainty, the most intuitive and practical way of representing the possibility of alternative future realizations is through the use of a scenario tree. In this section, we formally introduce the concept of scenario trees and show how the two basic variants of stochastic programming problems, two-stage and multi-stage, can formulated in terms of the scenario tree nodes. This is a departure from traditional scenario-variable formulations that define decision points in terms of stages and scenarios and will be shown to lead to very significant computational benefits. We proceed with presenting the stochastic transmission expansion problem formulation that includes investment recourse, allowing for the flexible differentiation of decisions according to the scenario realization. The use of flexibility enables the planner to adopt a 'wait and see' when it is optimal, postponing decisions until more information becomes available. The multi-cut Benders decomposition scheme and contingency screening module are also extended to apply to the stochastic formulation. This leads to a novel model able to combine the requirements of economic-based planning, security considerations and decision flexibility. The model's output is a series of conditional investment decisions in the form of a strategy that guarantee minimization of the expected system cost.

4.3 Uncertainty characterization via scenario trees

For the purposes of this research, the uncertainty of interest is the development of generation capacities over time. This can be expressed in terms of a discrete stochastic process and a scenario tree can be used to provide a coherent representation of the possible realizations. The generation evolution can be visualised through a multi-stage scenario tree spanning N_E stages and consisting of a total of N_M nodes. At each stage *e*, there exist N_M^e nodes. Each node has a single predecessor node and can have several successor nodes. We introduce the function $\varepsilon(m)$ that returns the stage to which node *m* belongs and the set $\Phi_0(m)$ that includes all parent nodes of *m*, including *m*. Similarly, $\Phi_k(m)$ is a set that includes all parent nodes of *m* until stage $\varepsilon(m)-k$. $[\Phi_0(m)]_e$ is the parent of node *m* at stage *e*. The first node *m=1*, known as the root node, represents the initial system state. The end nodes represent all possible system states at the last stage. A scenario constitutes a possible realization over the horizon and can be considered as a unique path traversing the tree from the root node to an end node. It is denoted by the sequence $S=\{s_e\}_{e=1}^{N_E}$ of size N_E and consists of all the timeindexed traversed nodes. For example, $S = \{1,2,4\}$ refers to the scenario realization that results from traversing the nodes $1 \rightarrow 2 \rightarrow 4$. The transition probability between node m at stage $\varepsilon(m)$ and node n at stage $\varepsilon(n)$ is denoted by $p^{m,n}$. There are no state transition probabilities for the end nodes. Naturally, a scenario's probability of occurrence π_S is given by the product of all state transition probabilities along the scenario path:

$$\pi_S = \prod_{e=2}^{N_E} p^{s_{e-1},s_e}$$

And the probability of occurrence of node *m* is given by:

$$\pi_m = \prod_{e=1}^{\varepsilon(m)-1} p^{[\Phi_0(m)]_e,[\Phi_0(m)]_{e+1}}$$

By definition, the probability of occurrence of the root node *m=1* is 1. Each node *m* corresponds to a vector $\left(p_{m,g}^{\max}\right)_{g\in\Omega_G}$ representing the possible state of maximum capacities of all generators at stage $\varepsilon(m)$. An example scenario tree shown is shown below that consists of a total of $N_M = 7$ nodes comprising 4 separate scenarios.

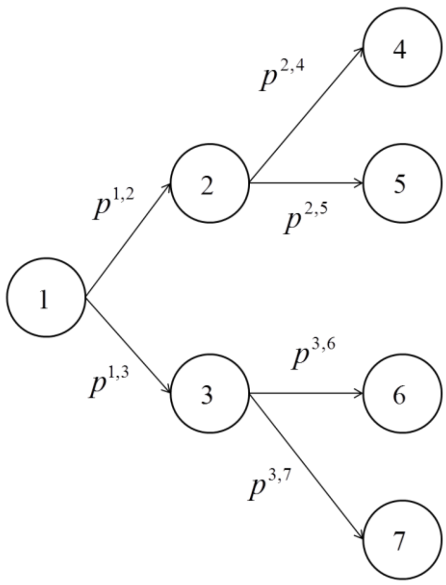


Figure 4-1: An example scenario tree.

4.4 Stochastic Programming

Stochastic programming is a framework for modelling decision problems under uncertainty that was first proposed by George Dantzig [53], where uncertain parameters are modelled as discrete random variables. Stochastic programming has been applied by several researchers in the past to the transmission expansion problem as discussed in section 2.3. Generally, stochastic programming problems based on scenario trees can be formulated using two different approaches; node-variable and scenario-variable. The latter uses variables that have been defined in terms of both scenarios and stages, resulting in significant redundancy in terms of variables and associated constraints. $N_S N_E$ decision vectors have to be defined and constrained appropriately through a set of constraints known as the non-anticipativity constraints. These constraints ensure that if the realizations of the stochastic processes are identical up to stage *k*, then the decision variables have to be identical up to stage *k*. Despite its larger size, it can prove the preferable approach to many problems due to its exploitable structure through the straightforward use of decomposition techniques [54]. For this reason, it has been the modelling method of choice in most stochastic transmission planning formulation until now [15, 69, 88, 89]. Node-variable problems are mathematically formulated so as to use variables defined in terms of the different decision points (tree nodes) along the scenario tree. Only N_M decision vectors are employed in describing the same problem and naturally results in a very compact formulation due to the exclusion of unnecessary variables and constraints. In the case of problems considering only two stages, the redundancy present in scenario-variable formulations may not prove to be an issue due to the inherent reduced size of the problem. However, in the case of multi-stage stochastic programs that consider recourse in not only the operational variables but also in investment decisions, the additional variables and non-anticipativity constraints introduced can render the problem intractable. For this reason, a node-variable formulation is employed in our stochastic model and decomposed using Benders decomposition. As shown in section 4.7.5, the computational benefits of adopting this approach are very significant.

4.4.1 Two-stage stochastic programming

In stochastic programming, uncertainty is modelled as a stochastic process which can evolve over two or more stages. The former is known as two-stage stochastic programming, where a decision is taken in the first stage so as to minimize the future expected cost in the second stage. In this section we illustrate how a two-stage stochastic transmission expansion problem can be formulated. For simplicity, we assume that investment decisions are commissioned immediately. The objective is to minimize expected investment and operational cost over the two stages. Uncertainty in generation capacities is modelled as a random process ξ_e and represented in the form of a scenario tree. In the first stage, there is a single realization of uncertainty denoted by $\xi_1(m_1)$, corresponding to the root node m_1. In the second stage, the possible realizations of the random process are denoted by $\xi_2(m_2)$ with $m_2 \in \Omega_M^2$ i.e. all tree nodes in the second stage. The decision sequence is the following:

1. In the first stage, the stochastic process ξ_1 is realized as $\xi_1(m_1)$ with probability 1 i.e. no uncertainty exists.
2. Investment decisions $x_1(m_1)$ are made.
3. Operation decisions $y_1(m_1)$ are made subject to the investment decision $x_1(m_1)$.
4. In the second stage, the stochastic process ξ_2 is realized as $\xi_2(m_2)$ with probability π_{m_2}. This corresponds to a scenario tree transition from the root node m_1 to the second-stage node m_2 and the random vector g_2, representing generation capacities, takes the values $g_2(m_2)$.
5. Investment decisions $x_2(m_2)$ are made.
6. Operation decisions $y_2(m_2)$ are made subject to the investment decisions $x_1(m_1)$ and $x_2(m_2)$. Note that the operation decisions that can be made in the second stage are independent of the operation decisions made in the first stage.

The general formulation of the two-stage stochastic problem can be expressed as the sum of investment and operational costs of the first stage and expected cost of the second stage, as follows:

$$z = \min\left\{c^T x_1(m_1) + q^T y_1(m_1) + E\left[Q_2(m_2)\right]\right\} \tag{4.1a}$$

s.t.

$$Ay_1(m_1) = d \qquad , \forall m_1 \in \Omega_M^1 \tag{4.1b}$$

$$By_1(m_1) \le g_1(m_1) \qquad , \forall m_1 \in \Omega_M^1 \tag{4.1c}$$

$$Cy_1(m_1) \le Dx_1(m_1) \qquad , \forall m_1 \in \Omega_M^1 \tag{4.1d}$$

Where E denotes the expectation operator and $Q_2(m_2)$ is the optimal value of the second stage problem for the stochastic realization $\xi_2(m_2)$. Constraint (4.1b) represents the system balance equation. Constraint (4.1c) couples operation to the scalar random vector $g_1(m_1)$

whose values correspond to the stochastic realization $\xi_1(m_1)$. Constraint (4.1d) couples operation to the investment decisions. The optimal value of each second stage problem is expressed as:

$$Q_2(m_2) = \min\{c^T x_2(m_2) + q^T y_2(m_2)\} \qquad , \forall m \in \Omega_M^2 \qquad (4.2a)$$

s.t.

$$Ay_2(m_2) = d \qquad , \forall m_2 \in \Omega_M^2 \qquad (4.2b)$$

$$By_2(m_2) \le g_2(m_2) \qquad , \forall m_2 \in \Omega_M^2 \qquad (4.2c)$$

$$Cy_2(m_2) \le D\left(x_1\left([\Phi(m_2)]_1\right) + x_2(m_2)\right) \qquad , \forall m_2 \in \Omega_M^2 \qquad (4.2d)$$

The objective function is the sum of second-stage investment and operation costs. Constraint (4.2c) couples operation to the scalar random vector $g_2(m_2)$ whose values correspond to the stochastic realization $\xi_2(m_2)$. Constraint (4.2d) couples operation to the investment undertaken in both stages. Note that $[\Phi(m_2)]_1$ is the parent of node m_2 in stage one i.e. the root node. Naturally, by expressing $E[Q_2(m_2)]$ as the probability-weighted sum of second stage problems, the original objective function (4.1a) can be re-written as:

$$z = \min\left\{ \sum_{m_1 \in \Omega_M^1} \left(c^T x_1(m_1) + q^T y_1(m_1)\right) + \sum_{m_2 \in \Omega_M^2} \pi_{m_2} \left(c^T x_2(m_2) + q^T y_2(m_2)\right) \right\} \qquad (4.3)$$

Where π_{m_2} is the probability that $\xi_2(m_2)$ realizes. The two-stage stochastic programming problem can be condensed in an even more concise form and expressed in terms of the set of all tree nodes Ω_M.

$$z = \min\left\{ \sum_{m \in \Omega_M} \pi_m \left(c^T x(m) + q^T y(m)\right) \right\} \qquad (4.4a)$$

s.t.

$$Ay(m) = d \qquad , \forall m \in \Omega_M \qquad (4.4b)$$

$$By(m) \le g(m) \qquad , \forall m \in \Omega_M \qquad (4.4c)$$

$$Cy(m) \le D\left[\sum_{k \in \Phi_0(m)} x(k) \right] \qquad , \forall m \in \Omega_M \qquad (4.4d)$$

Where for simplicity we denote $g_{\varepsilon(m)}(m)$ as $g(m)$ and similarly introduce decision vectors $y(m)$ and $x(m)$. Constraint (4.4b) states that the system balance equation must hold at all tree nodes. Constraint (4.4c) couples operation to the corresponding random vector. Constraint (4.4d) couples the operation vector *y(m)* to the investment undertaken in the scenario tree node *m* and all its parent nodes.

4.4.2 Multi-stage stochastic programming

The above concept can be easily extended to the case of multiple stages. In the case of a multi-stage stochastic program with N_E stages, uncertainty is represented by a discrete stochastic process ξ_e that can be visualised through a multi-stage scenario tree. At each stage *e,* a single realization of the stochastic process is denoted by $\xi_e(m_e)$ with $m_e \in \Omega_M^e$ .The decision sequence can be described as follows:

1. In the first stage, the stochastic process ξ_1 is realized as $\xi_1(m_1)$ with probability 1.
2. Investment decisions $x_1(m_1)$ are made.
3. Operation decisions $y_1(m_1)$ are made subject to the investment decision $x_1(m_1)$.
4. ...
5. In the last stage, the stochastic process ξ_{N_E} is realized as $\xi_{N_E}(m_{N_E})$ with probability $\pi_{m_{N_E}}$. This corresponds to a scenario tree transition from the preceding stage's parent node $\left[\Phi(m_{N_E})\right]_{N_E-1}$ to the node m_{N_E}.
6. Investment decisions $x_{N_E}(m_{N_E})$ are made.
7. Operation decisions $y_{N_E}(m_{N_E})$ are made subject to $x_{N_E}(m_{N_E})$ and the investment decisions undertaken in all parent nodes of node m_{N_E}.

The general formulation of the multi-stage stochastic problem is as follows:

$$z = \min\left\{c^T x_1(m_1) + q^T y_1(m_1) + E\left[Q_2(m_2)\right]\right\} \tag{4.5a}$$

s.t.

$$Ay_1(m_1) = d \qquad ,\forall m_1 \in \Omega_M^1 \tag{4.5b}$$

$$By_1(m_1) \le g_1(m_1) \qquad ,\forall m_1 \in \Omega_M^1 \tag{4.5c}$$

$$Cy_1(m_1) \leq Dx_1(m_1) \qquad , \forall m_1 \in \Omega_M^1 \qquad (4.5d)$$

Where $E[Q_2(m_2)]$ is the future expected cost at the second stage. The optimal value of each second stage problem $Q_2(m_2)$ is formulated as:

$$Q_2(m_2) = \min\{c^T x_2(m_2) + q^T y_2(m_2) + E[Q_3(m_3)]\} \qquad , \forall m \in \Omega_M^2 \qquad (4.6a)$$

s.t.

$$Ay_2(m_2) = d \qquad , \forall m_2 \in \Omega_M^2 \qquad (4.6b)$$

$$By_2(m_2) \leq g_2(m_2) \qquad , \forall m_2 \in \Omega_M^2 \qquad (4.6c)$$

$$Cy_2(m_2) \leq D(x_1([\Phi(m_2)]_1) + x_2(m_2)) \qquad , \forall m_2 \in \Omega_M^2 \qquad (4.6d)$$

Where $E[Q_3(m_3)]$ is the future expected cost at the third stage. Similarly, the optimal value of each third stage problem $Q_3(m_3)$ is formulated as:

$$Q_3(m_3) = \min\{c^T x_3(m_3) + q^T y_3(m_3) + E[Q_4(m_4)]\} \qquad , \forall m_3 \in \Omega_M^3 \qquad (4.7a)$$

s.t.

$$Ay_3(m_3) = d \qquad , \forall m_3 \in \Omega_M^3 \qquad (4.7b)$$

$$By_3(m_3) \leq g_3(m_3) \qquad , \forall m_3 \in \Omega_M^3 \qquad (4.7c)$$

$$Cy_3(m_3) \leq D(x_1([\Phi(m_3)]_1) + x_2([\Phi(m_3)]_2) + x_3(m_3)) \quad , \forall m_3 \in \Omega_M^3 \qquad (4.7d)$$

The same recursion principle can be applied to all the subsequent stages. The optimal values of the last stage problems are formulated as:

$$Q_{N_E}(m_{N_E}) = \min\{c^T x_{N_E}(m_{N_E}) + q^T y_{N_E}(m_{N_E})\} \qquad , \forall m_{N_E} \in \Omega_M^{N_E} \qquad (4.8a)$$

s.t.

$$Ay_{N_E}(m_{N_E}) = d \qquad , \forall m_{N_E} \in \Omega_M^{N_E} \qquad (4.8b)$$

$$By_{N_E}(m_{N_E}) \leq g_{N_E}(m_{N_E}) \qquad , \forall m_{N_E} \in \Omega_M^{N_E} \qquad (4.8c)$$

$$Cy_2(m_{N_E}) \leq D(x_1([\Phi(m_{N_E})]_1) + x_2([\Phi(m_{N_E})]_2) + \ldots + x_{N_{E_2}}(m_{N_E})) \qquad , \forall m_{N_E} \in \Omega_M^{N_E} \qquad (4.8d)$$

Constraint (4.8d) states that the operational vector $y_{N_E}(m_{N_E})$ is coupled to the investment decision taken in the current node m_{N_E} and all its preceding parent nodes. By grouping all tree nodes together, an equivalent problem can be formulated in terms of all tree nodes

$$z = \min\left\{\sum_{m\in\Omega_M}\pi_m\left(c^T x(m) + q^T y(m)\right)\right\} \tag{4.9a}$$

s.t.

$$Ay(m) = d \qquad ,\forall m \in \Omega_M \tag{4.9b}$$

$$By(m) \le g(m) \qquad ,\forall m \in \Omega_M \tag{4.9c}$$

$$Cy(m) \le D\left[\sum_{k\in\Phi_0(m)} x(k)\right] \qquad ,\forall m \in \Omega_M \tag{4.9d}$$

Thus we have shown how multi-stage stochastic programming with recourse in both investment and operation decision variables can be expressed in a node-variable form. This basic idea of expressing decisions in terms of the tree nodes can be used in order to formulate the multi-stage stochastic transmission expansion model in a similar way.

4.5 Multi-stage Stochastic Transmission Expansion Planning with Flexibility

In this section we present the multi-stage stochastic transmission expansion planning formulation. The objective is minimization of expected system cost under the uncertainty described by a multi-stage scenario tree. The main motivation of using a stochastic problem formulation is that the decision flexibility of the system planner can be modelled. The idea behind accommodating flexibility is that decisions should not be fixed, but should rather depend upon the unfolding stochastic process realization. The mathematic formulation of the stochastic planning problem is very similar to the deterministic formulation presented in Chapter 3. The main difference is that the considered operation subproblems are expanded from *(e,t)* to *(m,t)* to accommodate the different scenario tree nodes. In a similar manner, the dimension of the investment decision vectors present in the master problem is expanded from N_E to N_M. If investment flexibility was to be ignored, then all investment decisions relating to the same stage would have to be constrained to be equal. The presented formulation employs a multi-cut Benders decomposition scheme and a contingency screening module. The solution strategy is summarized as:

Step 1. Contingency screening iteration *i=1*.

Step 2. $K^{i}_{m,c,t}=0$ for all operating points *(m,c,t)*.

Step 3. Benders iteration index $v=1$

Step 4. Discard all appended Benders cuts from the master problem.

Step 5. Solve the master problem (4.10) - (4.16) including all appended Benders cuts.

Step 6. Solve the operation subproblem (4.18) – (4.35) subject to all binding contingencies included in $\overline{K}^{i-1}$ utilizing the master problem trial investment decisions $F^{inv^{(v)}}_{m,l}$ and $QB^{(v)}_{m,l}$. By definition, if *i=1* then no binding contingencies are considered.

Step 7. Check the convergence criterion (4.36). If false, construct the relevant Benders cut (4.17), append it to the master problem, update the Benders iteration index as $v=v+1$ and go to **Step 5**.

Step 8. Screen all operating points *(m,t)* for binding contingencies subject to the optimal investment decisions $F^{inv}_{m,l}$ and $QB^{(v)}_{m,l}$ and the optimal generation dispatch $p^{(v)}_{m,t,g}$ using the contingency screening module (4.40)-(4.48). Determine membership status of each post-fault operating point *(m,c,t)* to the list of binding contingencies according to (4.39): if binding, $\left(K^{i}_{m,c,t}=1\right)$, otherwise $\left(K^{i}_{m,c,t}=0\right)$.

Step 9. If $\sum_{\forall(m,c,t)}K^{i}_{m,c,t}=0$, meaning that no binding contingencies were detected in this iteration, go to **Step 12**. If $\sum_{\forall(m,c,t)}K^{i}_{m,c,t}>0$, meaning that some binding contingencies were detected, go to **Step 10**.

Step 10. $K^{i}_{m,c,t}=\max\left(K^{1}_{m,c,t},...,K^{i}_{m,c,t}\right),\ \forall m,c,t$, meaning that if post-fault operating point *(m,c,t)* was found to be binding in some previous iteration and found to be non-binding in the current iteration *i*, its status is reinstated as binding.

Step 11. Update the contingency screening iteration index *i=i+1* and go to **Step 2**.

Step 12. END

4.5.1 Master problem

Equation (4.10) is the master problem objective function, corresponding to the expected system cost. Each investment decision taken at node *m* is weighted by the node's probability of occurrence π_m. In a similar manner and according to the multi-cut Benders decomposition principle, the operating cost component has been replaced with the probability-weighted estimate $\sum_{m\in\Omega_M}\pi_m\sum_{t=1}^{N_T}\alpha_{m,t}$, progressively informed through the Benders cuts until it is equal to the optimal operating cost.

$$\min_{f^{inv},\beta,qb,\alpha}\left\{\sum_{m\in\Omega_M}\pi_m r^I_{\varepsilon(m)}\left[\sum_{l=1}^{N_L}\left(\sum_{w=1}^{N_{W_l}}\left(f^{inv}_{m,l,w}c_{l,w}+\beta_{m,l,w}\gamma_{l,w}\right)\chi_l+qb_{m,l}c^{QB}_l\right)\right]+\sum_{m\in\Omega_M}\pi_m\sum_{t=1}^{N_T}\alpha_{m,t}\right\} \quad (4.10)$$

Subject to:

$$\beta_{m,l,w}\in\{0,1\} \qquad \forall m,l,w \qquad (4.11)$$

$$qb_{m,l}\in\{0,1\} \qquad \forall m,l \qquad (4.12)$$

$$F^{inv}_{m,l}=\sum_{\phi\in\Phi_{k_{l,w}}(m)}\sum_{w=1}^{N_{W_l}}f^{inv}_{\phi,l,w} \qquad \forall m,l \qquad (4.13)$$

$$f^{inv}_{m,l,w}\le\left(\sum_{\phi\in\Phi_0(m)}\beta_{\phi,l,w}\,F^{\max}_{l,w}\right)-\left(\sum_{\phi\in\Phi_1(m)}f^{inv}_{\phi,l,w}\right) \qquad \forall m,l,w \qquad (4.14)$$

$$QB_{m,l}=\sum_{\forall\phi\in\Phi_{k^{QB}_l}(m)}qb_{\phi,l} \qquad \forall m,l \qquad (4.15)$$

$$QB_{m,l}\le 1 \qquad \forall m,l \qquad (4.16)$$

Constraints (4.13) states that the transmission capacity on line *l* that is available for operational use at tree node *m* is the sum of the investment decisions undertaken in all nodes $\Phi_0(m)$ subject to each option's build time $k_{l,w}$. Similarly, constraint (4.15) defines whether a quadrature booster has been commissioned for operational use at tree node *m*. Constraint (4.14) bounds the capacity $f^{inv}_{m,l,w}$ that can be built in line *l* using option *w* as the difference between the maximum capacity that can be added $F^{\max}_{l,w}$, depending on whether the binary variable $\beta_{m,l,w}=1$ in the current node *m* or any of its parent nodes was selected, and the investment already undertaken in all parent nodes $\Phi_1(m)$ using that option. Constraint (4.16) limits the number of quadrature boosters that can be commissioned on each line *l* to one.

Following the multi-cut Benders decomposition framework presented in section 3.8.3, $N_M N_T$ cuts are appended to the master problem per iteration as shown below. A single cut is generated for each operating point *(m,t)*. The Benders cut is formulated in terms of the trial investment solution of the previous iteration, the dual variables of the investment-to-operation coupling constraints $\lambda_{m,t,l}^{(\nu-1)}$ and $\lambda_{m,t,l}^{QB^{(\nu-1)}}$ as well as the sub-problem's optimal value $\omega_{m,t}^{(\nu-1)}$.

$$\alpha_{m,t}^{(\nu)} \geq \left[\begin{array}{c} \omega_{m,t}^{(\nu-1)} + \\ \sum_{l=1}^{N_L} \lambda_{m,t,l}^{(\nu-1)} \left(F_{m,l}^{inv} - F_{m,l}^{inv^{(\nu-1)}} \right) + \\ \sum_{l=1}^{N_L} \lambda_{m,t,l}^{QB^{(\nu-1)}} \left(QB_{m,l} - QB_{m,l}^{(\nu-1)} \right) \end{array} \right] \quad \forall m,t \tag{4.17}$$

4.5.2 Subproblem

The objective function of the operation subproblem relating to operating point (m,t) is the sum of constraint costs and the penalized unserved energy as shown below.

$$\omega_{m,t}^{(\nu)} = \min_{p^+,p^-,d^*} \left\{ r_{\varepsilon(m)}^{O} \tau_t \left[\sum_{g=1}^{N_G} \left(p_{m,t,g}^{+} o_g - p_{m,t,g}^{-} b_g \right) + \sum_{n=1}^{N_N} \left(d_{m,t,n}^{*} \Gamma \right) \right] \right\} \tag{4.18}$$

Subject to:

$$0 \leq p_{m,t,g} \leq p_{m,g}^{\max} \quad \forall g \tag{4.19}$$

$$p_{m,t,g} = p_{m,t,g}^{*} + p_{m,t,g}^{+} - p_{m,t,g}^{-} \quad \forall g \tag{4.20}$$

$$f_{m,t,l} = \frac{\theta_{m,t,u_l} - \theta_{m,t,v_l}}{x_l} \quad \forall l \tag{4.21}$$

$$\sum_{g=1}^{N_G} B_{n,g} p_{m,t,g} + \sum_{l=1}^{N_L} I_{n,l} \left(f_{m,t,l} + p_{m,t,l}^{QB} + p_{m,t,l}^{SC} \right) = d_{t,n} + d_{m,t,n}^{*} \quad \forall n \tag{4.22}$$

$$-\frac{\xi_l^{\max} SC_l}{x_l (x_l - \xi_l^{\max})} \left(\theta_{m,t,u_l} - \theta_{m,t,v_l} \right) \leq p_{m,t,l}^{SC} \leq -\frac{\xi_l^{\min} SC_l}{x_l (x_l - \xi_l^{\min})} \left(\theta_{m,t,u_l} - \theta_{m,t,v_l} \right) \quad \forall l \tag{4.23}$$

$$-\frac{\psi_l^{\max} QB_{m,l}^{*}}{x_l} \leq p_{m,t,l}^{QB} \leq -\frac{\psi_l^{\min} QB_{m,l}^{*}}{x_l} \quad \forall l \tag{4.24}$$

$$-\left(F_{m,l}^{inv*}+F_l^0\right)\leq f_{m,t,l}+p_{m,t,l}^{QB}+p_{m,t,l}^{SC}\leq\left(F_{m,l}^{inv*}+F_l^0\right)\quad\forall l \tag{4.25}$$

$$F_{m,l}^{inv*}=F_{m,l}^{inv(\nu)}:\lambda_{m,t,l}^{(\nu)}\quad\forall l \tag{4.26}$$

$$QB_{m,l}^{*}=QB_{m,l}^{(\nu)}:\lambda_{m,t,l}^{QB^{(\nu)}}\quad\forall l \tag{4.27}$$

For all $c,l\in\Omega_L$, if $(K_{m,c,t}^{i-1}=1)$ and $(c=l)$ then:

$$\sum_{g=1}^{N_G}B_{n,g}\,p_{m,t,g}+\sum_{l=1}^{N_L}I_{n,l}\left(f_{m,c,t,l}^{C}+p_{m,c,t,l}^{QB^C}+p_{m,c,t,l}^{SC^C}\right)=d_{t,n}\quad\forall n \tag{4.28}$$

For all $c,l\in\Omega_L$, if $(K_{m,c,t}^{i-1}=1)$ and $(c=l)$ then:

$$f_{m,c,t,l}^{C}=0 \tag{4.29}$$

$$p_{m,c,t,l}^{QB^C}=0 \tag{4.30}$$

$$p_{m,c,t,l}^{SC^C}=0 \tag{4.31}$$

For all $c,l\in\Omega_L$, if $(K_{m,c,t}^{i-1}=1)$ and $(c\neq l)$ then:

$$f_{m,c,t,l}^{C}=\frac{\theta_{m,c,t,l}^{C}-\theta_{m,c,t,l}^{C}}{x_l} \tag{4.32}$$

$$-\frac{\xi_l^{\max}SC_l}{x_l\,(x_l-\xi_l^{\max})}\left(\theta_{m,c,t,u_l}^{C}-\theta_{m,c,t,v_l}^{C}\right)\leq p_{m,c,t,l}^{SC^C}\leq-\frac{\xi_l^{\min}SC_l}{x_l\,(x_l-\xi_l^{\min})}\left(\theta_{m,c,t,u_l}^{C}-\theta_{m,c,t,v_l}^{C}\right) \tag{4.33}$$

$$-\frac{\psi_l^{\max}QB_{m,l}^{*}}{x_l}\leq p_{m,c,t,l}^{QB^C}\leq-\frac{\psi_l^{\min}QB_{m,l}^{*}}{x_l} \tag{4.34}$$

$$-\left(F_{m,l}^{inv^*}+F_l^0\right)\leq f_{m,c,t,l}^{C}+p_{m,c,t,l}^{QB^C}+p_{m,c,t,l}^{SC^C}\leq\left(F_{m,l}^{inv^*}+F_l^0\right) \tag{4.35}$$

Constraint (4.19) ensures that generation output for each operating point is within the allowable limits dictated by the random variable $p_{m,g}^{\max}$, whose value at each node *m* is defined through the scenario tree. Constraint (4.20) expresses the generation dispatch level $p_{m,t,g}$ in terms of an upwards ($p_{m,t,g}^{+}$) or downwards ($p_{m,t,g}^{-}$) deviation from the unconstrained optimal dispatch level $p_{m,t,g}^{*}$ that represents the underlying contractual position of generator *g* for operating point *(m,t)*. Generators required to produce more are paid their offer price while generators constrained to produce less pay their bid price. Constraint (4.21) is the DC intact system power flow equation. Constraint (4.22) is the intact system balance equation including the curtailed demand variable $d^{*}_{m,t,n}$. Constraint (4.23) bound the power injection due to a series compensator defined by the user to be installed in line *l*. Constraint (4.24) determines the upper and lower quadrature power injection levels, subject to the investment decision variable $QB^{*}_{m,l}$. Similarly, constraint (4.25) determines the upper and lower power flow limits on line *l*, subject to the investment decision variable $F^{inv*}_{m,l}$. The dual variables used to construct the Benders cuts (4.17) are obtained from equations (4.26) and (4.27) that force the subproblem investment decision variables $F^{inv*}_{m,l}$ and $QB^{*}_{m,l}$ to be equal to the trial decisions supplied by the master problem at iteration *v*. Constraints (4.28)-(4.35) relate to the N-1 secure operation and are active depending on whether the contingency screening module has identified that contingency *(m,c,t)* to be binding in the previous contingency screening iteration. Constraint (4.28) is the post-fault system balance equation. Constraints (4.29)-(4.31) force the power flow and power injections due to FACTS devices to be zero in the outaged line. Constraint (4.32) is the post-fault DC power flow equation for non-outaged lines. Constraints (4.33) and (4.34) define the limits for post-fault power injections due to installed FACTS devices. Constraint (4.35) determines the upper and lower bounds of post-faults power flows according to the subproblem investment decision variables $F^{inv*}_{m,l}$.

4.5.3 Convergence criterion

Once the optimal value for each subproblem *(m,t)* has been determined, the algorithm checks for convergence to evaluate the optimality of the trial investment decisions. The Benders convergence criterion (4.36) is expressed in terms of the difference between the total subproblem optimal operation cost as defined in (4.37) and the master problem's approximation to that cost, as defined in (4.38). The threshold value ε should be a value close to 0, in order to ensure close matching between the actual and approximated operational cost.

$$Z^{(v)}_{upper} - Z^{(v)}_{lower} \le \varepsilon \tag{4.36}$$

$$Z^{(v)}_{upper} = \sum_{\forall m \in \Omega_M} \pi_m \sum_{t=1}^{N_T} \omega^{(v)}_{m,t} \tag{4.37}$$

$$Z^{(v)}_{lower} = \sum_{\forall m \in \Omega_M} \pi_m \sum_{t=1}^{N_T} \alpha^{(v)}_{m,t} \tag{4.38}$$

4.5.4 Contingency screening module

Once convergence has been achieved, we must ensure that the optimal generation schedule $p_{m,t,g}^{(v)}$ allows for feasible N-1 secure operation. Contingency classification is achieved through quantification of the infeasibility slack variables $d_{m,c,t,n}^{c+}, d_{m,c,t,n}^{c-}$ and comparison with a threshold value Q appropriately set close to zero as shown in equation (4.25). Values above Q signify membership to the list of violating events.

$$\zeta_{m,c,t} \le Q \tag{4.39}$$

The objective function for each post-fault operating point (m,c,t) is as follows.

$$\zeta_{m,c,t} = \min\left\{\sum_{n=1}^{N_N}\left(d_{m,c,t,n}^{c+} + d_{m,c,t,n}^{c-}\right)\right\} \tag{4.40}$$

And is subject to the following constraints:

$$\sum_{g=1}^{N_G} B_{n,g}\, p_{m,t,g}^{(v)} + \sum_{l=1}^{N_L} I_{n,l}\left(f_{m,c,t,l}^{C} + p_{m,c,t,l}^{QB^C} + p_{m,c,t,l}^{SC^C}\right) = d_{t,n} + d_{m,c,t,n}^{c+} + d_{m,c,t,n}^{c-} \quad \forall n \tag{4.41}$$

For all $c, l \in \Omega_L$, if $(c = l)$ then:

$$f_{m,c,t,l}^{C} = 0 \tag{4.42}$$

$$p_{m,c,t,l}^{QB^C} = 0 \tag{4.43}$$

$$p_{m,c,t,l}^{SC^C} = 0 \tag{4.44}$$

For all $c, l \in \Omega_L$, if $(c \ne l)$ then:

$$f_{m,c,t,l}^{C} = \frac{\theta_{m,c,t,l}^{C} - \theta_{m,c,t,l}^{C}}{x_l} \tag{4.45}$$

$$-\frac{\xi_l^{\max}}{x_l(x_l - \xi_l^{\max})}\left(\theta_{m,c,t,u_l}^{C} - \theta_{m,c,t,v_l}^{C}\right) \le p_{m,c,t,l}^{SC^C} \le -\frac{\xi_l^{\min}}{x_l(x_l - \xi_l^{\min})}\left(\theta_{m,c,t,u_l}^{C} - \theta_{m,c,t,v_l}^{C}\right) \tag{4.46}$$

$$-\frac{\psi_l^{\max} QB_{m,l}^{(v)}}{x_l} \le p_{m,c,t,l}^{QB^C} \le -\frac{\psi_l^{\min} QB_{m,l}^{(v)}}{x_l} \tag{4.47}$$

$$-\left(F_{m,l}^{inv^{(v)}} + F_l^{0}\right) \le f_{m,c,t,l}^{C} + p_{m,c,t,l}^{QB^C} + p_{m,c,t,l}^{SC^C} \le \left(F_{m,l}^{inv^{(v)}} + F_l^{0}\right) \tag{4.48}$$

4.6 Three Bus-bar Case Study

4.6.1 Case study description

In this section, a case study on a three bus-bar system is presented to illustrate the differences between the non-flexible and flexible transmission expansion planning frameworks. The case study spans 4 five-year stages, giving a total planning and operation horizon of 20 years. The system topology is shown in Figure 4-2 with line and generation data presented in Table 4-1 and Table 4-2 respectively. For simplicity, only two loading conditions have been considered (Table 4-3) and N-1 security constraints have been ignored. There are 3 generators currently installed on the system with the cheapest one being G1, located at bus 1, followed by G2, located at bus 2. Bus 3 is the major load centre of the system with a peak demand of 1,050 MW and host to the most expensive generator.

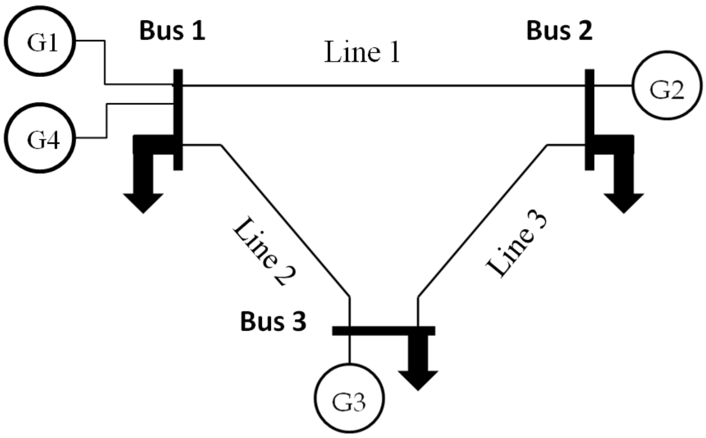


Figure 4-2: Three bus-bar system.

Generator Id	Bus	Generation Type	Capacity [MW]	Fuel cost [£/MWh]	Offer price [£/MWh]	Bid price [£/MWh]
G1	1	Coal	200	30.00	36.00	27.00
G2	2	Coal	200	35.00	42.00	31.50
G3	3	Coal	1500	40.00	48.00	36.00
G4	1	Renewable	-	10.00	50.00	-100.00

Table 4-1: Generation data for three bus-bar case study.

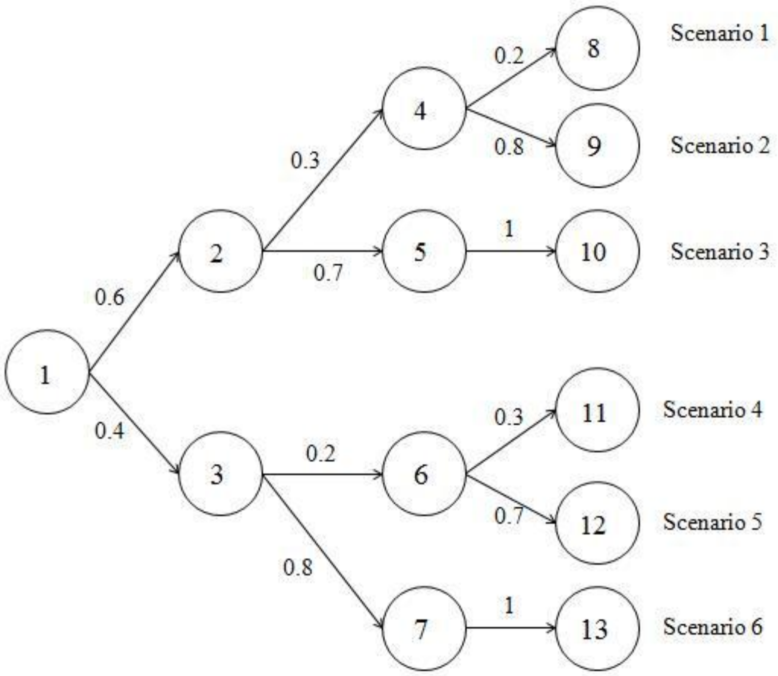
Line Id	Origin Bus	Destination Bus	Reactance [p.u.]	Length [km]	Initial Capacity [MW]
1	1	2	0.2	100	100
2	1	3	0.2	100	100
3	2	3	0.2	100	100

Table 4-2: Network data for three bus-bar case study.

Condition	Duration [hours]	Bus 1 Load [MW]	Bus 2 Load [MW]	Bus 3 Load [MW]
Peak	1,070	150	300	1050
Off-peak	7,666	105	210	735

Table 4-3: System load levels and durations.

Uncertainty lies in the capacity evolution of a new cheap renewable generator (G4) to be installed at bus 1. The capacity of G4 has been modelled as a random variable whose progression is described by the scenario tree shown in Figure 4-3. The scenario tree consists of 13 nodes and 6 possible scenario paths. Each scenario represents a different capacity deployment pattern summarized in Table 4-4. At the root node, representing the current state of the system, no capacity has been commissioned yet. At the second stage, there is a 0.6 probability for a 'high-growth' transition to node 2, where 400 MW of capacity is installed and a 0.4 probability for a 'low-growth' transition to node 3 where only 200 MW is deployed. In subsequent epochs, there is a possibility for additional additions. However, scenarios 3 and 6 are the most probable realizations, reflecting the eventuality that no further additions are made following the second epoch commissioning. Overall, the planner's predictions are biased towards scenarios involving a modest-sized generation addition. At the end of the horizon, there is only a 3.6% chance that the maximum possible 800 MW have been constructed.



Node m	$p_{m,4}^{\max}$
1	0
2	400
3	200
4	600
5	400
6	400
7	200
8	800
9	600
10	600
11	600
12	400
13	200

Figure 4-3: Scenario tree for three bus-bar case study, showing the evolution of the random variable $p_{m,4}^{\max}$ [MW].

Scenario Id	S	$\pi_{\bar{s}}$	G4 capacity [MW]			
			Epoch 1	Epoch 2	Epoch 3	Epoch 4
1	[1,2,4,8]	0.036	0	400	600	800
2	[1,2,4,9]	0.144	0	400	600	600
3	[1,2,5,10]	0.42	0	400	400	600
4	[1,3,6,11]	0.024	0	200	400	600
5	[1,3,6,12]	0.056	0	200	400	400
6	[1,3,7,13]	0.32	0	200	200	200

Table 4-4: Summary of scenarios.

The system currently has enough capacity to accommodate the arising flows, without the need to constrain any generators. However, with the commissioning of G4, much larger power flows will emerge over the network, as shown in Figure 4-4. If no investment is undertaken to reinforce the transmission system, very significant constraint costs will arise from having to replace in-merit generation with locally generated power from bus 3. The cumulative constraint costs in the absence of investment are shown in Figure 4-5. Scenario 1 that entails the commissioning of 800 MW of new generation leads to the most unfavourable realization of constraint costs and warrants the highest level of investment.

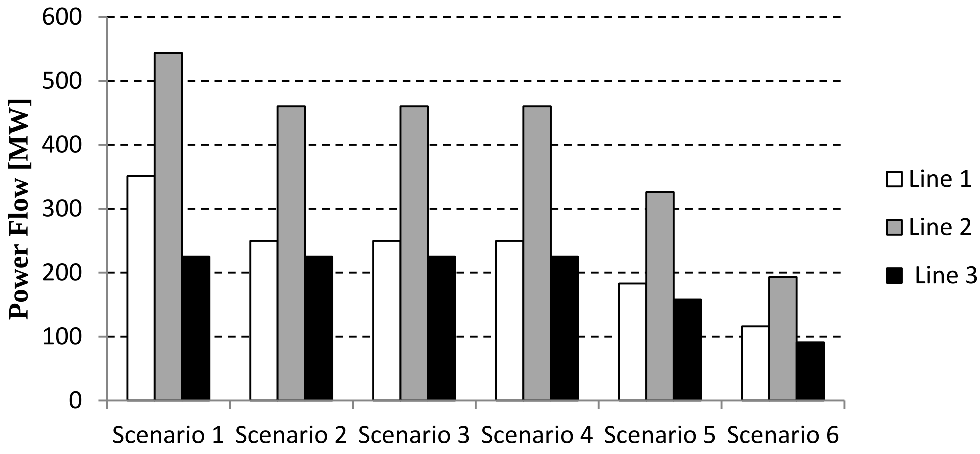


Figure 4-4: Maximum power flows under each scenario.

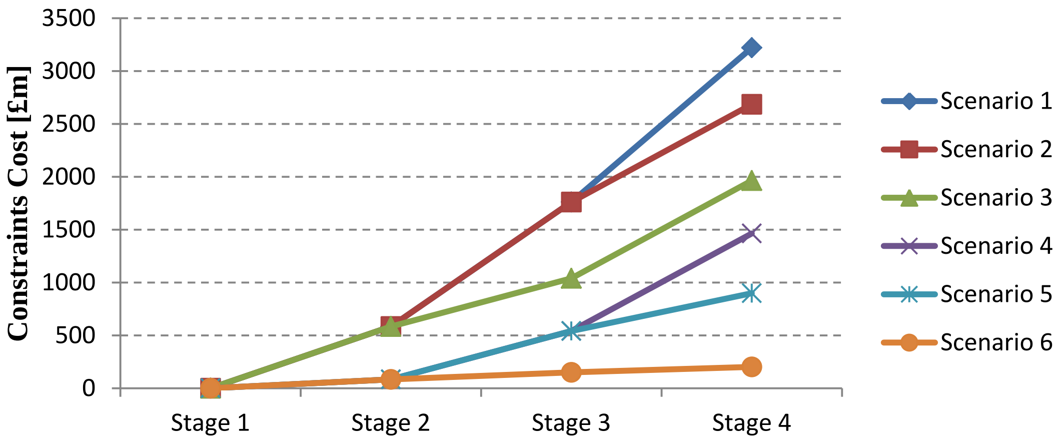


Figure 4-5: Cumulative constraints costs [£m] in the absence of transmission investment.

For each line, there are three mutually exclusive candidate expansion options (i.e. only one candidate can be built per line), whose cost and technical data are shown in Table 4-5. All candidates have a build time of 1 epoch and allow the installation of a specific number of circuits, where each circuit is represented as a 150 MW capacity reinforcement. Option A is the solution with the lowest fixed cost, capable of accommodating a single circuit. Options B and C can accommodate up to two or three circuits respectively, for higher fixed costs. The investment costs for the different options, assuming construction is initiated in the first year of the case study horizon, are shown in Figure 4-6.

w	$F_{l,w}^{\max}$ [MW]	$c_{l,w}$ [£m/km·year]	$\gamma_{l,w}$ [£/MW·km·year]	$k_{l,w}$
A	150	40,000	50	1 epoch
B	300	60,000	50	1 epoch
C	450	80,000	50	1 epoch

Table 4-5: Investment options for all lines.

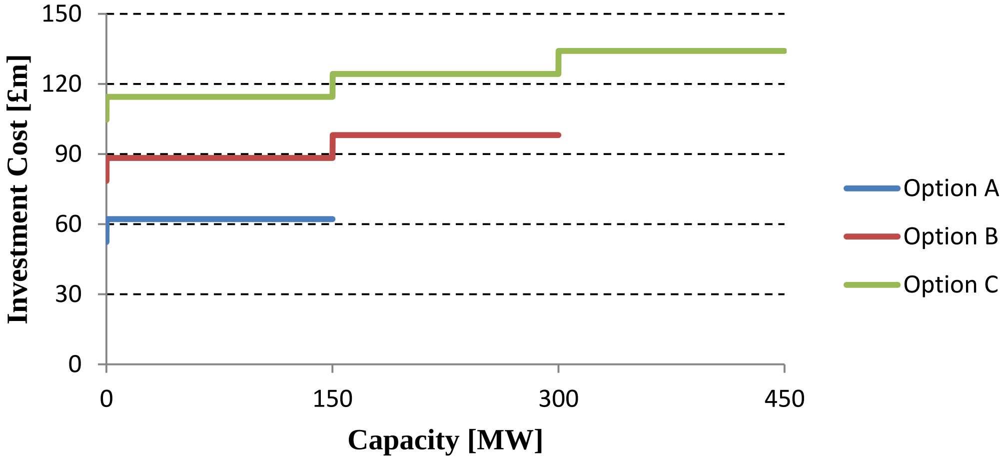


Figure 4-6: Investment cost for the three candidate options (line length = 100km).

These three candidate options can be seen as different technology paths that can be followed and once selected cannot be altered. Each path offers a different level of upgradeability and thus the ability to flexibly decide on their optimal sizing is critical. The purpose of this case study is to compare two different decision frameworks. The first is a probabilistic approach that minimizes expected system cost over the horizon but does not account for decision flexibility. It is limited by recourse actions independent of the uncertainty realization. We refer to this approach as the *non-flexible transmission expansion framework*. The second is the developed *flexible transmission expansion framework* presented in section 4.5 and utilizes investment recourse to adapt to the unfolding realization. The flexible transmission investment solution comprises a strategy where the transitions to different system states can be treated as trigger events that differentiate the proposed investment decisions. We present the two methods graphically, using decision trees to highlight the difference of treating flexibility in the decision process. The most interesting aspect of this framework comparison will relate to the first stage decisions that pinpoint the investment to be made in the present. We show that 'here and now' decisions taken on the basis of no subsequent recourse can lock in suboptimal investment paths that severely limit the system's adaptability to the future uncertainty.

4.6.2 Non-flexible transmission expansion planning

Figure 4-7 shows the non-flexible framework decision tree. The absence of flexibility limits the planner to identifying the optimal investment decision vectors $\mathbf{D_1}$, $\mathbf{D_2}$ and $\mathbf{D_3}$, which are common among all realizations. Note that since all investment options are subject to some build time, we are concerned with investment decisions only up to the third epoch. Any investment taking place in the last epoch would materialize beyond the finite horizon and is thus ignored.

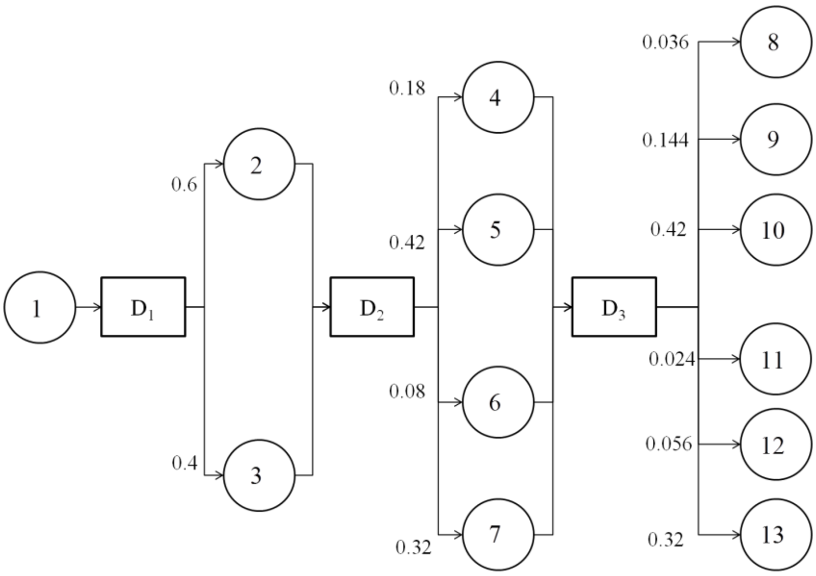


Figure 4-7: Decision tree for non-flexible transmission expansion framework.

The optimal expansion schedule under this paradigm is shown in Table 4-6. The non-flexible planner chooses to reinforce all lines from the very first epoch. The chosen options are built to their full potential (150 MW for option A and 300 MW for option B projects), rendering subsequent adjustments impossible (recall that only one option can be built per line). The transmission capacities added are sufficient to eliminate constraints under low-growth scenarios 5 and 6. However, in the event of a more extensive deployment of new generation, congestion arises due to the network's inability to export all the cheap power available. In this case, G1 and G4 have to be curtailed and the system re-balanced by engaging local resources. Naturally, scenario 1 constitutes the most adverse realization leading to a total constraint cost of £226.2m. However, due to its low probability of occurrence, this eventuality contributes very little to the expected costs and fails to tip the cost-benefit analysis in favour of larger reinforcements. The detailed system costs under each scenario when following the proposed plan are shown in Table 4-7 and Table 4-8.

Line Id	**D_1**	**D_2**	**D_3**	**Investment Cost [£m]**
1	150 (A)	-	-	62.2
2	300 (B)	-	-	98.1
3	150 (A)	-	-	62.2

Table 4-6: Optimal expansion schedule for the non-flexible decision framework.

Scenario	**Investment Cost (IC)**				**Total IC**	**Constraints Cost (CC)**				**Total CC**	**Total System Cost**
	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**		**Stage 1**	**Stage 2**	**Stage 3**	**Stage 4**		
1	222.5	0	0	0	222.5	0	0	40.4	185.8	226.2	448.7
2	222.5	0	0	0	222.5	0	0	40.4	31.7	72.1	294.6
3	222.5	0	0	0	222.5	0	0	0	31.7	31.7	254.1
4	222.5	0	0	0	222.5	0	0	0	31.7	31.7	254.1
5	222.5	0	0	0	222.5	0	0	0	0	0	222.5
6	222.5	0	0	0	222.5	0	0	0	0	0	222.5

Table 4-7: Investment, constraints and system costs (£m) under each scenario.

Investment Cost	£222.5m
Expected Constraints Cost	£32.6m
Expected System Cost	£255.0m

Table 4-8: Expected system costs.

4.6.3 Flexible transmission expansion planning

Figure 4-8 shows the stochastic framework decision tree, where $D_{[m]}$ is the investment decision vector to be chosen following a state transition to scenario tree node *m*. Following the non-anticipativity principle, scenario realizations that traverse the same states up to epoch *e* are subject to the same investment decisions until then. Thus, the optimal investment strategy consists of 4 possible vector combinations depending on the eventual realization (e.g. choose $D_{[1]} \rightarrow D_{[2]} \rightarrow D_{[4]}$ for scenarios 1 S$= [1,2,4,8]$) and 2 ($S = [1,2,4,9]$)).

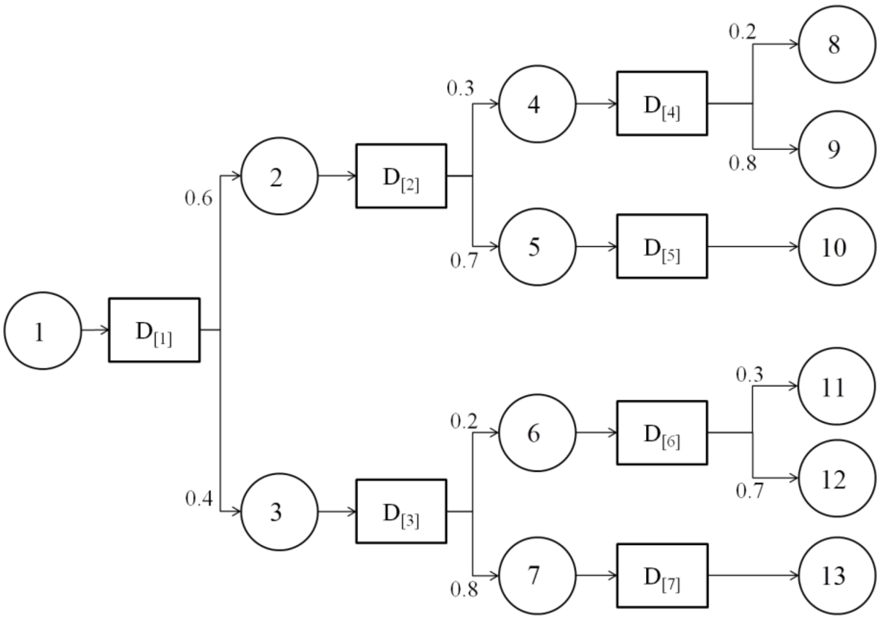


Figure 4-8: Decision tree for flexible transmission expansion framework.

The optimal expansion strategy is shown in Table 4-9. As can be seen, investment in line 3 is postponed to subsequent epochs and is undertaken only if the high growth transitions (1)→(2) or (3)→(6) occur. The planner waits for critical uncertainty to resolve and chooses to proceed with the investment only in the cases where the net benefit is maximized. In addition, we see that the flexible planner arrives at a different decision concerning line 2, which is now reinforced with 300 MW using option C. The expansion option of upgrading the line to its full potential is triggered as soon as G4 reaches 400 MW (scenario tree nodes 2 and 6), an event that indicates a high likelihood of further capacity additions. The system costs under each scenario when following the proposed expansion strategy are shown in Table 4-10 and Table 4-11.

Line Id	$\mathbf{D_{[1]}}$	$\mathbf{D_{[2]}}$	$\mathbf{D_{[3]}}$	$\mathbf{D_{[4]}}$	$\mathbf{D_{[5]}}$	$\mathbf{D_{[6]}}$	$\mathbf{D_{[7]}}$
1	150 (A)	-	-	-	-	-	-
2	300 (C)	150	-	-	-	150	-
3	-	150 (A)	-	-	-	150 (A)	-

Table 4-9: Optimal expansion strategy for the flexible decision framework.

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	186.5	47.0	0	0	233.5	0	39.4	0	49.3	88.7	322.2
2	186.5	47.0	0	0	233.5	0	39.4	0	0	39.4	272.9
3	186.5	47.0	0	0	233.5	0	39.4	0	0	39.4	272.9
4	186.5	0	27.4	0	213.9	0	0	30.9	0	30.9	244.8
5	186.5	0	27.4	0	213.9	0	0	30.9	0	30.9	244.8
6	186.5	0	0	0	186.5	0	0	0	0	0	186.5

Table 4-10: Investment, constraint and system costs (£m) under each scenario.

Expected Investment Cost	£216.8m
Expected Constraints Cost	£27.9m
Expected System Cost	£244.7m

Table 4-11: Expected system costs.

4.6.4 Discussion

The difference in expected system costs between the two frameworks is £10.3m, with the flexible stochastic model achieving a further minimization of 4%. This difference constitutes the net benefit loss when failing to consider decision flexibility and stems from the premature lock-in to a suboptimal investment path. More specifically, under the non-flexible framework, committing to an option C investment in line 2 is deemed uneconomic because it entails a £26.2m increase to the 'here and now' initial capital expenditure (fixed costs) as well as a subsequent investment of £6.4m (variable costs) to exercise the expansion option. The expected constraint costs saving due to the extra 150 MW is £29.9m, which is out-weighted by the total extra capital cost of £32.6m. However, if the expansion option is to be exercised optimally, as dictated by the flexible framework solution, it results in a positive net benefit. Thus, we conclude that modelling decision flexibility has a significant impact on the optimal 'here and now' decisions, allowing the planner to identify the optimal exercise policy for the available 'wait-and-see' and expansion options. The net benefit of these options cannot be properly valued without considering the optimal recourse action related to each system state transition.

4.7 IEEE RTS Case Study

In this section we revisit the case study examined in section 3.9 to showcase how the proposed stochastic framework can be applied to a larger system while modelling managerial flexibility and accommodating N-1 security constraints. By comparing the optimal flexible policy to the optimal sequential decisions produced by the non-flexible framework, we quantify the value of managerial flexibility and show how disregarding it can lead to premature project commitment and overestimation of expected system costs.

4.7.1 Case Study Description

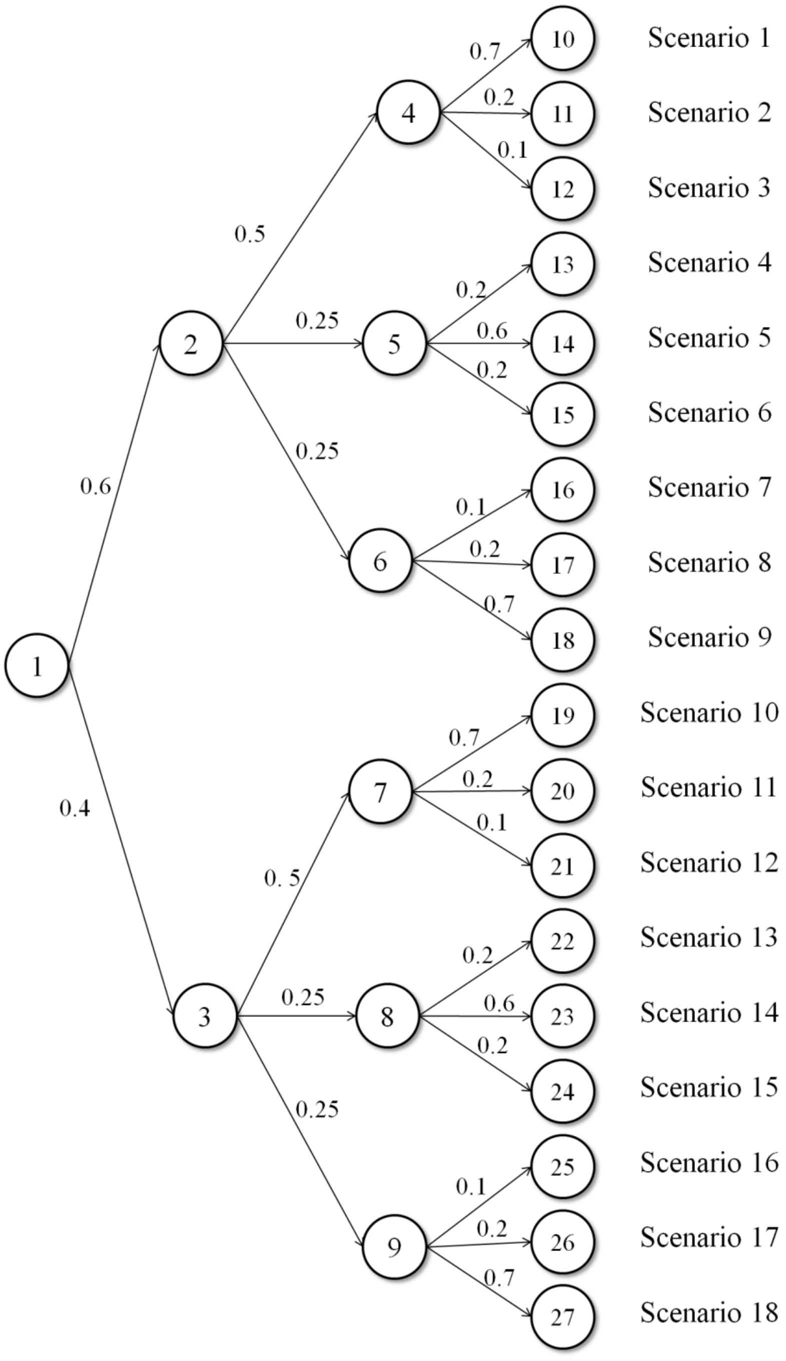
As before, the case study comprises four epochs with each epoch spanning a period of 5 years, giving a total horizon of 20 years. The system topology is as shown in Figure 3-3 and the generator data are included in Table 3-1. Year-round loading and wind conditions are represented in 100 snapshots, each one weighted by their duration τ_t as detailed in Table. In terms of transmission investment, a single reinforcement option can be chosen for each line from the three candidate projects shown in Table 3-4.

In contrast to the previous IEEE RTS study where future generation additions were taken to be firmly known a priori, the case study presented in this section involves uncertainty related to both the location and size of a new wind farm that will be connected to the system. The modelled uncertainty aims to capture some practical aspects of generation development. For example, the eventual plant location may depend on environmental or other constraints that remain undecided and are to be resolved at a certain time in the future. Moreover, once the location has been determined, construction is to be carried out in several distinct phases. In this respect, technical issues prohibiting the timely commissioning according to schedule may arise. In addition, the developer may choose to abandon further expansions to the project due to unfavourable developments in the electricity market, affecting profitability.

More specifically, in this case study there are two candidate wind farm sites, only one of which will eventually materialize. In addition, once the site has been chosen and construction begins, the development comprises of five 250 MW phases, leading to a maximum eventual capacity of 1250 MW. Successful completion of all phases is not guaranteed and commissioning may be limited to only a fraction of the final target. This location and sizing uncertainty leads to a range of possible deployment patterns that may eventually materialize, which are captured in a four-stage scenario tree shown in Figure 4-9 and summarized in Table 4-12. The tree consists of 27 nodes comprising 18 scenarios paths. It includes information on the system state transition probabilities and the possible realization values of the random variables associated to the maximum capacity of G27 (wind generation connected to bus 3) and G28 (wind generation connected to bus 6) at each stage. The scenarios that involve the highest level of wind generation commissioning are 1 and 10. Under scenario 1, a total of 1250 MW of wind is connected to bus 3. Under scenario 10, a similar amount is connected to bus 6. On the other hand, scenarios 9 and 18 reflect the eventuality of only phase 1 being constructed, resulting in a wind fleet capacity of 250 MW.

Scenario	*S*	$\pi_{\bar{s}}$	G27 capacity [MW]				G28 capacity [MW]			
			Epoch 1	Epoch 2	Epoch 3	Epoch 4	Epoch 1	Epoch 2	Epoch 3	Epoch 4
1	[1,2,4,10]	0.210	0	250	750	1250	0	0	0	0
2	[1,2,4,11]	0.060	0	250	750	1000	0	0	0	0
3	[1,2,4,12]	0.030	0	250	750	750	0	0	0	0
4	[1,2,5,13]	0.030	0	250	500	1000	0	0	0	0
5	[1,2,3,14]	0.090	0	250	500	750	0	0	0	0
6	[1,2,5,15]	0.030	0	250	500	500	0	0	0	0
7	[1,2,6,16]	0.015	0	250	250	750	0	0	0	0
8	[1,2,6,17]	0.030	0	250	250	500	0	0	0	0
9	[1,2,6,18]	0.105	0	250	250	250	0	0	0	0
10	[1,3,7,19]	0.140	0	0	0	0	0	250	750	1250
11	[1,3,7,20]	0.040	0	0	0	0	0	250	750	1000
12	[1,3,7,21]	0.020	0	0	0	0	0	250	750	750
13	[1,3,8,22]	0.020	0	0	0	0	0	250	500	1000
14	[1,3,8,23]	0.060	0	0	0	0	0	250	500	750
15	[1,3,8,24]	0.020	0	0	0	0	0	250	500	500
16	[1,3,9,25]	0.010	0	0	0	0	0	250	250	750
17	[1,3,9,26]	0.020	0	0	0	0	0	250	250	500
18	[1,3,9,27]	0.070	0	0	0	0	0	250	250	250

Table 4-12: Summary of scenarios for IEEE RTS case study.



Node m	$p_{m,27}^{\max}$	$p_{m,28}^{\max}$
1	0	0
2	250	0
3	0	250
4	750	0
5	500	0
6	250	0
7	0	750
8	0	500
9	0	250
10	1250	0
11	1000	0
12	750	0
13	1000	0
14	750	0
15	500	0
16	750	0
17	500	0
18	250	0
19	0	1250
20	0	1000
21	0	750
22	0	1000
23	0	750
24	0	500
25	0	750
26	0	500
27	0	250

Figure 4-9: Scenario tree for IEEE RTS case study.

As can be seen in the scenario tree, uncertainty lies in both the size and location of the new wind generation. However, locational uncertainty is resolved after the first stage. If the scenario tree transition (1)→(2) realizes, G27 is the project selected and the wind farm is to be constructed in the west part of the network (bus 3). If on the other hand the transition

(1)→(3) occurs, G28 is to go ahead and be constructed in the east part (bus 6). In other words, the commissioning of G27 and G28 are two mutually exclusive events that depend on the first stage scenario tree transition. Transitions in the later stages reflect the possible commissioning patterns of the selected wind farm site.

In view of the forthcoming generation addition, the system planner has to ensure that sufficient transmission capacity is installed in order to accommodate the new power flows in a timely manner. Failure to provide adequate access to market participants will result in constraint costs, wind curtailment and engagement of out-of-merit plants to balance the system. Construction delays further complicate this decision process by introducing time lag between investment and the arrival of new information. More specifically, the first stage commitments are to be made on a purely anticipatory basis since no information on the eventual location has been revealed. Subsequent decisions will be fully informed on location and future uncertainty is limited only to the wind farm sizing evolution. It follows that significant value may lie in adopting a 'wait and see' stance until locational uncertainty has been fully resolved. By deferring decisions to later periods, the planner can limit reinforcements to only the lines that will be accommodating the new flows and capital expenditure on unaffected corridors can be minimized. However, the net benefit of such a reactive approach depends on the constraint cost levels to which the planner is exposed while waiting for uncertainty to resolve. The cumulative constraints cost evolution in the absence of investment is shown in Figure 4-10.

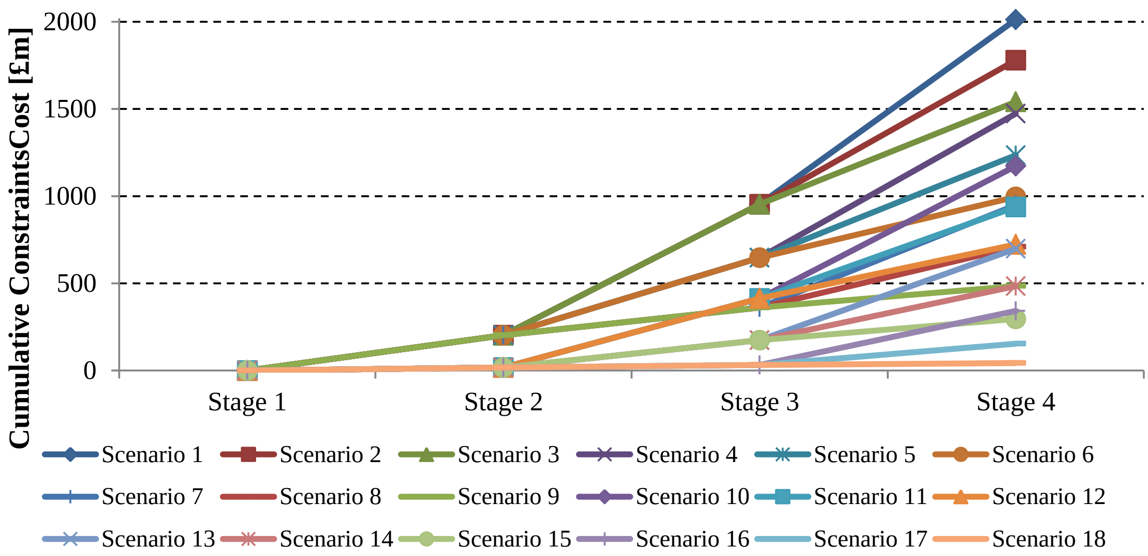


Figure 4-10: Cumulative constraints cost (£m) evolution in the absence of transmission investment.

Scenario 1, which is the most probable scenario, constitutes the most unfavourable realization leading to a potential total constraints cost of £2bn and warrants intensive capital investment. Similar costs arise for scenarios 2, 3 and 4 that involve commissioning of significant amount of wind generation in the west part of the network. In the case of scenario 10, which entails the highest wind deployment at bus 6, the potential cost of constraints total £1.2bn, illustrating that the system at its current state is better prepositioned to handle the commissioning of G28. More specifically, investment becomes necessary only once the capacity of G28 surpasses 250MW. A detailed outline of the optimal deterministic reinforcement plans associated to each scenario can be found in Appendix C and a summary of the reinforcements to be carried out at each epoch is shown in Table 4-13.

Scenario	**Lines to be upgraded**		
	Epoch 1	**Epoch 2**	**Epoch 3**
1	1,2,6	2,3,4,6,9	1,5,8
2	1,2,6	2,3,4,6,9	8
3	1,2,6	2,3,4,6,9	-
4	1,2,6	2,3,4,9	6,8
5	1,2,6	2,3,4,9	6
6	1,2,6	3,4,9	-
7	1,2,6	-	2,3,4,6,9
8	1,2,6	-	-
9	1,2,6	-	-
10	-	1,4,5,8,10	2,3,4,5,8,9,10,16
11	-	1,4,5,8,10	-
12	-	1,4,5,8,10	-
13	-	1,4,5,8,10	5,10
14	-	1,4,5,8,10	5,10
15	-	1,4,5,8,10	-
16	-	-	1,4,5,8,10
17	-	-	1,4,5,8,10
18	-	-	-

Table 4-13: Lines to be upgraded according to the optimal deterministic expansion plans.

The total system cost for each scenario when following the corresponding optimal expansion plan in shown below.

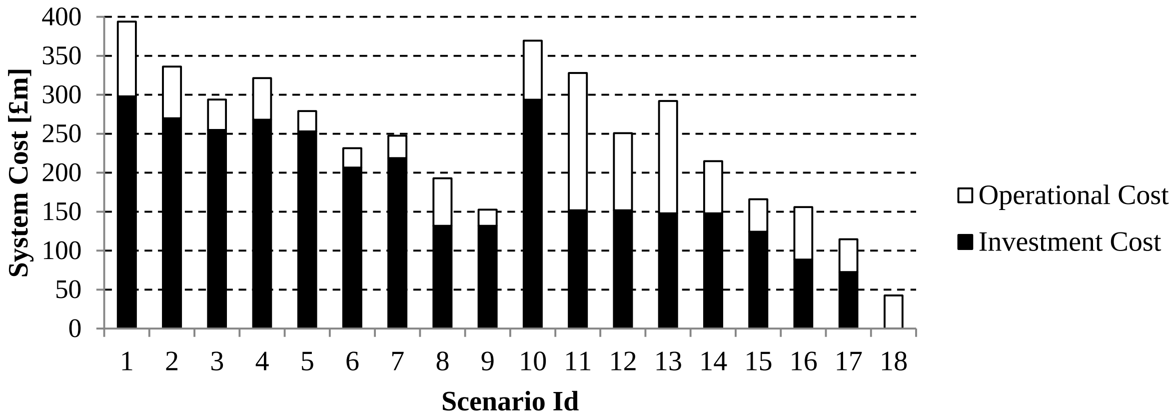


Figure 4-11: Optimal deterministic investment and operational costs for each scenario.

As can be seen in Table 4-13, under scenarios 1 to 9, first stage commitments are essential to limit congestion in subsequent epochs. Investment is mostly targeted at lines 2 and 6 which export the available wind energy to the rest of the system. In the event of high wind deployment, additional investment on neighbouring lines is needed to transfer the excess power to the load centres. On the other hand, in the case of scenarios 10 to 18, it is optimal to undertake reinforcements at later stages. Investment is focused on lines 5 and 10 that emanate from bus 6 as well as on other links that distribute power to nearby demand nodes. It is important to note that the upgrading of lines 1, 4 and 8 is common among all realizations with the exception of scenario 18 which requires no reinforcement. However, due to the differences in optimal timing and sizing, no single investment decision can be considered "robust" under all scenario realizations and no conclusive expansion plan can be drawn from a deterministic scenario analysis [82]. In order to identify the plan which is optimal "on average" for all scenarios, a stochastic optimization approach is essential. We first present the optimal non-flexible expansion schedule and then incorporate decision flexibility to identify the optimal expansion strategy.

4.7.2 Non-flexible Stochastic Transmission Expansion Planning

The optimal transmission expansion schedule under the non-flexible framework is presented in Table 4-14 and the corresponding system costs are shown in Table 4-15.

Line Id	D_1	D_2	D_3	Investment Cost [£m]
1	200 (A)	-	-	4.6
2	200 (B)	+ 200	-	112.4
3	-	200 (A)	-	20.9
4	-	200 (A)	-	31.7
5	-	400 (B)	-	69.2
6	200 (B)	+ 200	-	63.1
8	-	200 (A)	-	25.7
9	-	200 (A)	-	22.1
10	-	400 (B)	-	22.2

Table 4-14: Optimal expansion schedule for the non-flexible decision framework.

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	168.3	203.6	0	0	371.9	0	8.7	1.6	70.3	80.5	452.4
2	168.3	203.6	0	0	371.9	0	8.7	1.6	16.4	26.6	398.5
3	168.3	203.6	0	0	371.9	0	8.7	1.6	1.3	11.4	383.3
4	168.3	203.6	0	0	371.9	0	8.7	1.6	16.4	26.7	398.6
5	168.3	203.6	0	0	371.9	0	8.7	1.6	1.3	11.5	383.4
6	168.3	203.6	0	0	371.9	0	8.7	1.6	1.3	11.6	383.5
7	168.3	203.6	0	0	371.9	0	8.7	1.0	1.3	10.9	382.8
8	168.3	203.6	0	0	371.9	0	8.7	1.0	1.3	11.0	382.9
9	168.3	203.6	0	0	371.9	0	8.7	1.0	0.8	10.5	382.4
10	168.3	203.6	0	0	371.9	0	17.8	10.9	186.0	214.7	586.6
11	168.3	203.6	0	0	371.9	0	17.8	10.9	71.9	100.6	472.5
12	168.3	203.6	0	0	371.9	0	17.8	10.9	8.5	37.0	409.0
13	168.3	203.6	0	0	371.9	0	17.8	3.2	71.9	92.9	464.8
14	168.3	203.6	0	0	371.9	0	17.8	3.2	8.5	29.5	401.4
15	168.3	203.6	0	0	371.9	0	17.8	3.2	2.5	23.5	395.4
16	168.3	203.6	0	0	371.9	0	17.8	1.5	8.5	27.8	399.7
17	168.3	203.6	0	0	371.9	0	17.8	1.5	2.5	21.8	393.7
18	168.3	203.6	0	0	371.9	0	17.8	1.5	1.2	20.5	392.4

Table 4-15: Investment, constraints and system costs (£m) under each scenario.

Investment is limited to the first two stages, where commitments to projects totalling 2,600MW of capacity are made. The 'here and now' first stage decisions are targeted towards accommodating the prospective commissioning of G27. The capital expenditure associated to these commitments is £168.3m with significant reinforcements to lines 2 and 6. Subsequent investment is spread across lines accommodating both G27 and G28. In particular, both lines 5 and 10, that export power from bus 6 to the system, are reinforced though option B projects. Under the non-flexible framework, the planner anticipates the commissioning of both developments despite the fact that they are two mutually exclusive events. Of most interest are the first stage decisions, which is the implementable part of the presented schedule. They are taken on a proactive basis to ensure the system is adequately prepositioned to handle the potential commissioning of G27 in the second epoch. Despite their large capital cost, the net benefit of these commitments outweigh the option of postponing investment to later periods.

Expected Investment Cost	371.9
Expected Constraints Cost	64.7
Expected System Cost	435.6

Table 4-16: Expected system costs.

The expected system cost under the proposed expansion schedule is £435.6m. The total capital expenditure amounts to £371.9m, while there is a wide range of potential constraint costs that can occur, depending on the eventual realization. In the case of high-growth scenarios, wind has to be constrained off in period of high output due to insufficient transmission capacity. Overall, scenario 10, which entails the successful commissioning of all five phases of G28, constitutes the most adverse scenario leading to a total constraints cost of £214.7 primarily due to the curtailment of 13% of available wind energy in the last epoch.

4.7.3 Flexible Stochastic Transmission Expansion Planning

The optimal transmission investment decisions under the flexible stochastic framework are presented in Table 4-17 and system costs are shown in Table 4-18.

Line Id	$D_{[1]}$	$D_{[2]}$	$D_{[3]}$	$D_{[4]}$	$D_{[5]}$	$D_{[6]}$	$D_{[7]}$	$D_{[8]}$	$D_{[9]}$
1	-	200 (A)	200 (A)	-	-	-	-	-	-
2	200 (B)	+ 200	-	-	-	-	-	-	-
3	-	200 (A)	-	-	-	-	200 (A)	-	-
4	-	200 (A)	200 (A)	-	-	-	-	-	-
5	-	-	400 (B)	200 (A)	-	-	-	-	-
6	-	400 (B)	-	-	-	-	-	-	-
8	-	-	200 (A)	200 (A)	-	-	-	-	-
9	-	200 (A)	-	-	-	-	200 (A)	-	-
10	-	-	400 (B)	-	-	-	-	-	-

Table 4-17: Optimal expansion strategy for the flexible decision framework.

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	104.8	128.0	43.2	0	276.0	0	65.2	17.0	70.3	152.5	428.5
2	104.8	128.0	43.2	0	276.0	0	65.2	17.0	16.4	98.6	374.6
3	104.8	128.0	43.2	0	276.0	0	65.2	17.0	1.2	83.4	359.4
4	104.8	128.0	0	0	232.8	0	65.2	3.5	57.3	126.0	358.8
5	104.8	128.0	0	0	232.8	0	65.2	3.5	13.3	82.0	314.8
6	104.8	128.0	0	0	232.8	0	65.2	3.5	2.8	71.5	304.3
7	104.8	128.0	0	0	232.8	0	65.2	1.0	13.3	79.6	312.4
8	104.8	128.0	0	0	232.8	0	65.2	1.0	2.8	69.0	301.8
9	104.8	128.0	0	0	232.8	0	65.2	1.0	0.8	67.1	299.9
10	104.8	151.7	25.1	0	281.6	0	17.8	44.9	186.0	248.7	530.3
11	104.8	151.7	25.1	0	281.6	0	17.8	44.9	71.9	134.5	416.1
12	104.8	151.7	25.1	0	281.6	0	17.8	44.9	8.5	71.2	352.8
13	104.8	151.7	0	0	256.6	0	17.8	11.4	112.1	141.3	397.8
14	104.8	151.7	0	0	256.6	0	17.8	11.4	35.2	64.3	320.9
15	104.8	151.7	0	0	256.6	0	17.8	11.4	8.9	38.1	294.7
16	104.8	151.7	0	0	256.6	0	17.8	1.5	35.2	54.5	311.1
17	104.8	151.7	0	0	256.6	0	17.8	1.5	8.9	28.3	284.9
18	104.8	151.7	0	0	256.6	0	17.8	1.5	1.2	20.5	277.1

Table 4-18: Investment, constraint and system costs (£m) under each scenario.

Unlike the non-flexible probabilistic case, the only 'here and now' investment decision undertaken is on line 2, where option B is chosen. In the second epoch, there is a great differentiation between the chosen reinforcements depending on the resolution of locational uncertainty. In the case of a (1)→(2) transition, significant investment takes place in line 6 to accommodate the arising flows due to G27 and line 2 is upgraded by an extra 200MW. This is coupled with investment in lines 1, 3, 4 and 9 which enables the propagation of the power carried by line 2 to the rest of the system. In the case of a (1)→(3) transition, line 2 is not upgraded further and investment is focused on lines 5 and 10, both reinforced by 400 MW. In addition, the capacity of lines 1, 4 and 8 is increased in order to further propagate the power carried by line 5. Note that investment in option A for line 1 is carried out under both system state transitions, but has now been deferred to the second epoch. This is because the effective utilization of this extra capacity is linked to the upgrade of adjacent lines. More specifically, in the absence of a first-stage investment in line 6, the possible power that can flow over line 2 is reduced, leaving no scope for a line 1 upgrade in the first epoch.

In the third epoch, additional investment occurs only for high-growth transitions which open the possibility of very large generation additions. It is important to note that in the event that 1000 MW or 1250 MW of new wind generation is eventually commissioned, it is not possible to fully accommodate the available wind power in periods of high output, due to a transmission capacity shortfall in the main exporting corridors. Full accommodation would entail the construction of option C projects for these lines. Instead of undertaking this large capital expenditure, utilizing balancing services of out-of-merit coal plants constitutes a lower cost solution. The cheapest plants to engage for this purpose are located in buses 1 and 2. Interestingly, power injections in those buses give rise to counter-flows in lines 2 and 5, allowing further accommodation of wind power flows, but at the same time contribute to increased power flows over lines 3, 5, 8 and 9. As a result, in case of a (2)→(4) transition, investment is undertaken in lines 5 and 8 to allow the out-of-merit dispatch of coal plants in periods of high wind output. In the case of a (3)→(7) transition, investment is targeted to lines 3 and 9.

When comparing the solution of the non-flexible and flexible decision frameworks, the main differentiations are:

— Line 6 reinforcement is deferred to the second stage and undertaken only if uncertainty is resolved in favour of G27. The planner adopts a ‘wait and see’ stance and refrains from proceeding with this investment until more information has been revealed. It is deemed more economical to incur slightly higher constraint costs in the second epoch (£65.2m in the case that G27 materializes) than prematurely commit to a project which could prove unnecessary. The non-flexible planner cannot properly value the option of investment postponement because it fails to consider investment as a conditional event. On the other hand, the flexible planner recognizes the fact that the net benefit of reinforcing line 6 is maximized only under a (1)→(2) transition and the decision is thus postponed to the second stage.

— Second stage decisions are contingent upon the resolution of locational uncertainty, with the planner significantly differentiating his investment depending on which wind farm materializes. His recourse actions are tailored to each realization, focusing on the lines that will accommodate the new power flows and foregoing investment to unaffected parts of the network.
— Third stage investment takes place only for the high-growth transitions that warrant further capacity reinforcements. Low-growth transitions may lead to system constraints (as in scenarios 4 and 11), but the experienced congestion levels do not justify further capital expenditure.

The consideration of decision flexibility results in a reduction to the expected investment cost due to the prevention of unneeded commitments. On the other hand, it leads to an increase of the expected cost of constraints. Overall, the net benefit is positive, reflected in a considerable decrease in the expected system cost as shown below.

Expected Investment Cost	£260.3m
Expected Constraints Cost	£115.7m
Expected System Cost	£376.0m

Table 4-19: Expected system costs.

In the absence of security constraints, expected system cost was found to be £110.7m, placing the cost of security at £265.3m. This 70% increase confirms that system reliability is one of the primary drivers for transmission expansion.

4.7.4 Result analysis

In this section we compute a range of different metrics that allow us to further illustrate the advantages of the flexible stochastic model over using more naive approaches that do not consider scenario-specific recourse actions or the explicit modelling of the entire uncertainty characterizing the decision process.

4.7.4.1 Expected Value of Perfect Information

The expected value of perfect information (EVPI) represents the price the planner would be willing to pay to gain access to perfect information concerning future generation developments and constitutes a proxy for the value of accurate forecasts [54]. It is a useful metric for quantifying the effect of uncertainty on system costs.

In the case that the planner had perfect foresight, knowing which scenario will occur with certainty, he could follow an optimal tailor-made plan for each realization. In Appendix C we list the optimal plan that would be optimal under each scenario, obtained by solving the corresponding deterministic problem ignoring uncertainty. The probability-weighted system cost of these plans is £277.7m, which is 26% lower than the expected system cost of the flexible stochastic solution. This system cost difference of £98.3m is the EVPI and reflects the high cost impact of the uncertainty present in this case study. For the inflexible planner, EVPI increases to £157.9m, illustrating that the inability to take advantage of managerial flexibility leads to an even greater value of acquiring an accurate forecast and an amplified impact of uncertainty.

4.7.4.2 Expected Cost of Ignoring Uncertainty

The expected cost of ignoring uncertainty (ECIU) is useful in quantifying the net benefit loss when ignoring uncertainty and basing first-stage investment decisions on a single scenario which is naively deemed certain to occur. This naïve approach can result in over or underinvestment in transmission assets, ill-conditioning the planner’s ability to effectively respond to an unfavourable realization in subsequent epochs. Thus, ECIU constitutes a proxy to the benefit of utilizing a stochastic approach over a deterministic one to determine the ‘here and now’ decisions [71]. In order to calculate ECIU, we first compute the expected system cost when first-stage decisions are fixed according to the optimal deterministic plan of each scenario. The cost of ignoring uncertainty is the difference between the acquired expected system cost and the optimal solution of the stochastic problem. The detailed results are shown in Table 4-20.

First-stage decision of scenario	Expected System Cost [£m]	Cost of Ignoring Uncertainty [£m]
1	379.4	3.4
2	379.4	3.4
3	379.4	3.4
4	379.4	3.4
5	379.4	3.4
6	434.3	58.3
7	379.4	3.4
8	434.3	58.3
9	434.3	58.3
10	398.8	22.8
11	398.8	22.8
12	398.8	22.8
13	398.8	22.8
14	398.8	22.8
15	398.8	22.8
16	398.8	22.8
17	398.8	22.8
18	398.8	22.8
Expected		20.5

Table 4-20: Costs of ignoring uncertainty.

The cost of ignoring uncertainty varies according to which scenario is used as a basis to draw the first-stage decisions. If the most probable eventuality is used (scenario 1), the cost is relatively low and stems from overinvesting in line 1 and prematurely committing to a line 6 upgrade. The highest costs occur when planning with respect to scenarios 6, 8 and 9 which correspond to a low projection of new generation additions. Under these scenarios, project A is chosen to reinforce the main exporting corridors of bus 3, significantly limiting the planner's ability for recourse in the event of high-growth transitions. It is important to note that no deterministic plan results in the optimal first-stage decisions given by the stochastic model. For this case study, the ECIU is £20.5m, underlining the unsuitability of employing deterministic scenario analysis methods.

4.7.4.3 Value of the Stochastic Solution

The value of the stochastic solution (VSS) is a measure for quantifying the benefit of using a stochastic programming approach over a deterministic one, where random variables are replaced by their expected values [54]. VSS can be computed as the decrease in expected system cost between the extended stochastic formulation (£376.0m) and the objective function of the stochastic problem when fixing first-stage decision variables to the optimal values provided by the equivalent deterministic model. The random variable expected values to be included in the deterministic formulation are shown in Table 4-21.

Generator	**Expected Capacity [MW]**			
	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**
G27	0	150	337.5	517.5
G28	0	100	225	345

Table 4-21: Expected value of future generation capacities.

The first-stage optimal solution of the corresponding deterministic model is the reinforcement of line 2 by 200 MW using project A. This results in substantial ill-conditioning of the system, by locking in a very specific investment path which may limit subsequent recourse actions. By utilizing this first-stage investment decision and solving the flexible stochastic transmission expansion problem, the expected system cost is £438.9m. As a result, the VSS is £62.9m, indicating that if the planner was to base his first-stage commitments on the expected value of future generation additions, he would be exposed to a 16.7% increase in expected costs. Solving the equivalent deterministic problem may initially appear as a practical proxy for modelling the underlying uncertainty, particularly due to the reductions in problem size and modelling complexity. However, failing to properly model uncertainty leads to the so-called ‘flaw of averages’ and negatively impacts the quality of decisions. The substantial VSS highlights the importance of taking into account the whole range of possible eventualities and fully utilizing the information available from the scenario tree, instead of relying on average values. Again, the ill-conditioning that may occur from relying on deterministic methods is evident.

4.7.4.4 Value of Flexibility

The value of decision flexibility can be defined as the expected system cost difference between the flexible and non-flexible approach. By definition, the net benefit gain of modelling decision flexibility is positive. This is because decisions are constrained only by the non-anticipativity dictated by the scenario tree instead of being forced to be identical across all realizations. For this case study, the value of flexibility is £60.6m, meaning that flexible stochastic planning results in a 13.8% reduction of expected system costs. The extra cost experienced under the inflexible planning paradigm is the result of eliminating the option to ‘wait and see’ until uncertainty is resolved while also depriving the planner from considering scenario-specific recourse actions. The value of flexibility becomes even greater when versatile congestion management measures, such as the installation of quadrature boosters, are at the planner's disposal. Due to their small construction delay, these devices can be deployed in a 'just-in-time' manner, according to the unfolding uncertainty. When considering investment in QBs, the value of flexibility increases to 29.8% of the stochastic solution. The detailed optimal expansion plans and system costs are presented in Appendix D.

The most important implication of ignoring decision flexibility is the sub-optimality of the first stage investment decisions, which is the implementable part of the solution. In the flexible stochastic problem formulation, the system planner adopts a 'wait-and-see' stance towards investing in line 6. The decision to upgrade this line is postponed to the second epoch, when locational uncertainty has been fully resolved and more informed decisions can be made. The inflexible planner resorts to a more conservative first-stage planning decision, where investment in line 6 is undertaken on a non-conditional basis. We can quantify the sub-optimality of this premature commitment by calculating the difference between the optimal flexible stochastic solution and the expected system cost that arises when committing to this first-stage decision, while allowing for scenario-specific recourse in subsequent epochs. The difference is £3.5m, representing the expected welfare loss due to the failure to consider flexibility when identifying the optimal 'here and now' decisions.

4.7.4.5 Regret Analysis

Further insight on the suitability of the proposed model can be obtained by conducting a regret analysis. The regret associated to a specific plan under a scenario realization is defined as the difference between the system cost experienced and the system cost that would have been obtained if the optimal course of action had been taken. With respect to each scenario, the best possible system cost is obtained through solving the corresponding deterministic planning problem (Table 4-21). In this analysis, we focus on the regret associated to the first-stage decisions obtained when employing different planning approaches, while allowing for scenario-specific recourse in subsequent epochs. The regret matrix (Figure 4-12) presents the regrets associated with the 21 planning approaches that have been considered. The first 18 rows relate to deterministic models (presented in Appendix C), where the planner bases his first-stage decisions on the assumption that a single specific scenario will occur. We also calculate the regret associated to the first-stage decisions obtained through the equivalent deterministic, non-flexible stochastic and flexible stochastic approaches. By taking the probability-weighted average of the regret experienced over all scenarios, we can calculate the expected regret of each planning approach.

As can be seen in the regret table, the realizations that lead to the highest regret levels are scenarios 1 and 18. The former constitutes a high-growth eventuality that requires significant reinforcements. Premature commitment in projects capable of providing only small capacity additions (project A) severely ill-condition the systems and prohibits the planner from properly accommodating the arising power flows. The latter requires no reinforcements and thus any transmission investment undertaken is unnecessary. The largest expected regret is experienced when fixing first-stage decisions to the ones obtained through the equivalent deterministic model. High expected regret is also experienced when planning deterministically for scenarios 6, 8 and 9. All these approaches lead to project A commitments for the lines exporting power from bus 3, thus leading to very high costs if high-growth scenarios materialize. The expected regret is minimized under the flexible stochastic approach, since minimizing expected costs, while modelling decision flexibility, is equivalent to minimizing the expected regret [15]. Although adopting this approach results in positive regrets under all scenarios, with the greatest regret experienced if all phases of G28 are successfully commissioned (scenario 10), it performs best on average. This highlights the superior performance of first-stage decisions provided by the proposed model.

First-stage investment decisions	Scenario Realization																		Expected Regret
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
D1	0.0	3.8	31.1	2.8	1.3	38.4	30.4	74.4	112.7	221.3	148.7	162.5	151.2	165.0	207.8	215.6	230.6	295.0	101.7
D2	0.0	3.8	31.1	2.8	1.3	38.4	30.4	74.4	112.7	221.3	148.7	162.5	151.2	165.0	207.8	215.6	230.6	295.0	101.7
D3	0.0	3.8	31.1	2.8	1.3	38.4	30.4	74.4	112.7	221.3	148.7	162.5	151.2	165.0	207.8	215.6	230.6	295.0	101.7
D4	0.0	3.8	31.1	2.8	1.3	38.4	30.4	74.4	112.7	221.3	148.7	162.5	151.2	165.0	207.8	215.6	230.6	295.0	101.7
D5	0.0	3.8	31.1	2.8	1.3	38.4	30.4	74.4	112.7	221.3	148.7	162.5	151.2	165.0	207.8	215.6	230.6	295.0	101.7
D6	279.0	185.2	100.1	108.9	23.7	0.0	47.3	30.5	64.4	184.9	112.3	126.1	114.8	128.6	171.4	179.2	194.2	258.6	156.6
D7	0.0	3.8	31.1	2.8	1.3	38.4	30.4	74.4	112.7	221.3	148.7	162.5	151.2	165.0	207.8	215.6	230.6	295.0	101.7
D8	279.0	185.2	100.1	108.9	23.7	0.0	47.3	30.5	64.4	184.9	112.3	126.1	114.8	128.6	171.4	179.2	194.2	258.6	156.6
D9	279.0	185.2	100.1	108.9	23.7	0.0	47.3	30.5	64.4	184.9	112.3	126.1	114.8	128.6	171.4	179.2	194.2	258.6	156.6
D10	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D11	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D12	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D13	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D14	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D15	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D16	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D17	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
D18	135.3	139.1	166.4	138.1	136.5	173.7	165.6	209.7	247.9	81.0	0.0	0.0	2.2	2.3	24.6	50.9	65.6	129.7	121.1
ED	318.0	224.3	139.2	148.0	62.7	39.1	86.3	69.5	103.5	137.5	64.9	78.7	82.5	82.8	105.4	131.9	146.8	211.2	161.1
NFS	0.0	3.8	31.1	2.8	1.3	38.4	30.4	74.4	112.7	221.3	148.7	162.5	151.2	165.0	207.8	215.6	230.6	295.0	101.7
FS	34.5	38.3	65.6	37.3	35.7	72.9	64.9	108.9	147.2	160.8	88.1	102.0	105.8	106.1	128.7	155.2	170.2	234.5	98.2

Figure 4-12: Regret matrix when using first stage decision dictated by deterministic (Ds, where *s* is the scenario considered), equivalent deterministic (EB), non flexible stochastic (NFS) and flexible stochastic (FS) planning. All values are in £m.

4.7.5 Computational performance

The computational performance of the non-flexible and flexible models is presented in Table 4-22. In the case of the flexible model, we also include the performance of the scenario-variable formulation. All runs have employed the contingency screening module and multi-cut Benders decomposition, where operational subproblems were run in parallel using 12 processors. Both models required only two iterations of the contingency screening module to identify the optimal N-1 secure investment and dispatch schedules.

Model	Problem formulation	Contingency screening iteration index	Benders iterations	Objective function (£m)	CPU time (s)	CPU Memory usage (GB)
Non-Flexible	Node-variable	1	7	193.2	67.3	0.31
		2	7	435.6	254.1	0.54
Flexible	Scenario-variable	1	8	110.7	613.7	1.04
		2	-	-	>43,200	>56.0
	Node-variable	1	8	110.7	86.4	0.45
		2	11	376.0	1290.2	2.35

Table 4-22: Computational performance of the stochastic transmission expansion models.

The first thing to note is the increased complexity of the flexible model. Naturally, the computational cost of identifying the optimal investment strategy involving a large number of possible recourse actions is high when compared to finding a unique optimal expansion schedule.

The most important observation concerns the remarkable computational benefits of employing the node-variable over the scenario-variable problem formulation. Although the number of benders iterations taken to converge is the same, since the two formulations are equivalent, the increased number of operating points and investment decision variables as well as the explicit inclusion of non-anticipativity constraints in the scenario-variable approach, result in considerably longer CPU times and increased memory usage. For example, in this case study, the node-variable master problem contains 3,159 binary decision variables ($N_M N_L N_W$) related to the different line investment options. In addition, at each benders iteration, 2,700 operational subproblems are solved and as many Benders cuts are appended to the master problem. On the other hand, the scenario-variable formulation decomposes in a master problem containing 8,424 binary decision variables ($N_S N_E N_L N_W$) and 7,200 subproblems. The significant benefit of the reduced problem size becomes even more apparent when taking into account the fact that problem complexity scales non-linearly with the number of constraints and decision variables considered.

As seen in Table 4-22, the node-variable model utilized a limited amount of RAM memory and converged in a total time of 1376 seconds, with the non-secure planning problem being solved in just over one minute. The scenario-variable formulation took more than 10 minutes to converge in the absence of security constraints. In the second contingency screening iteration, when constraints related to the identified binding contingencies were included in the operation subproblems, the model failed to converge within an acceptable time. The model was stopped at the 5th benders iteration, with the solver utilizing more than 56 GB of RAM to traverse the master problem's branch-and-bound tree and failing to find the optimal MILP solution after 12 hours of processing.

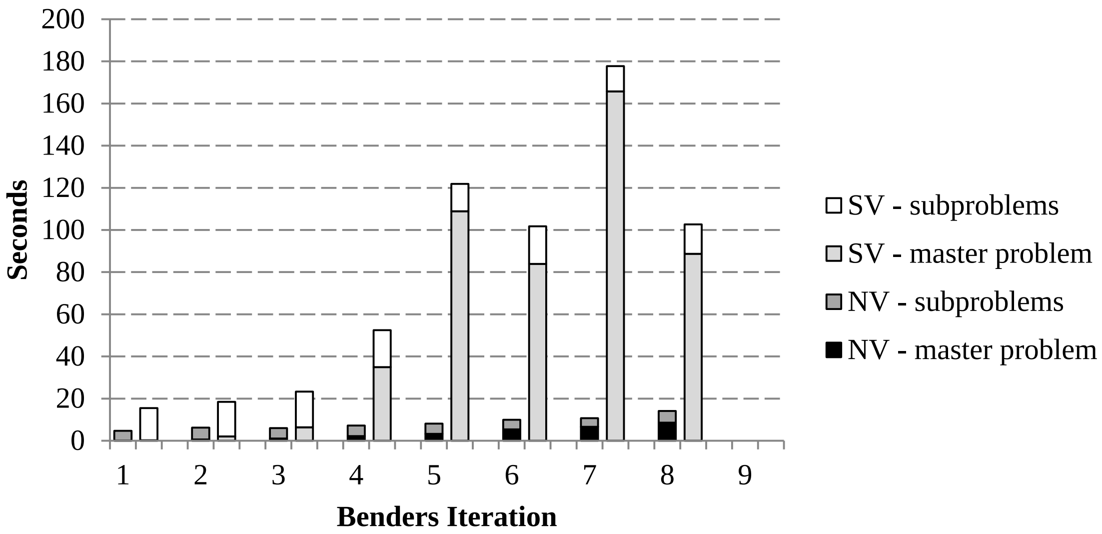


Figure 4-13: Time spent in master and subproblems when using the scenario-variable (SV) and node-variable (NV) formulations in the first contingency screening iteration.

In Figure 4-13 we show the time spent in the master and subproblems under the two problem formulations in the first contingency screening iteration (intact system planning). With respect to solving the operational subproblems, the node-variable model needed an average time of 5 seconds, while the scenario-variable model needed more than 15 seconds; a three-fold increase which is roughly equal to the $\frac{N_S N_E}{N_M} = 2.67$ ratio. As far as the master problem runtimes are concerned, the scenario-variable model clearly has a substantially inferior performance, particularly in later iterations when a larger number of Benders cuts has to be considered.

From the above observations we conclude that when dealing with stochastic MILP problems consisting of multiple stages, operating points and decision variables, the utilization of a node-variable formulation leads to superior computational behaviour and is essential for achieving convergence in acceptable times. In addition, the utilization of Benders decomposition and contingency screening significantly reduce CPU time.

4.8 Conclusions

In this section we have presented the multi-stage stochastic transmission expansion problem. Uncertainty has been expressed in the form of a discrete scenario tree representing the evolution of the generation background in the future. Under this approach, investment decisions are expressed in the form of a strategy, where the inter-temporal resolution of uncertainty is utilized to take more informed decisions. The techniques developed in the previous chapter have been incorporated in the formulation to improve computational performance. The combination of a multi-cut Benders decomposition scheme with a node-variable approach succeeds in rendering the large mixed integer linear problem tractable and allows the simulation of large systems. A case study on a three bus-bar system indicates how the lack of flexibility can prematurely lock planners into sub-optimal investment paths that lack the upgradeability required under some scenarios in the future. An additional case study is presented on the IEEE RTS, with results confirming that modelling the decision flexibility of a system planner results in further expected cost minimization than when adopting a fixed expansion schedule. This reduction highlights the strategic importance of planning with adaptability in mind. Of great interest is the difference in first-stage commitments that are taken in the absence of managerial flexibility. The stochastic planner is shown to adopt a 'wait-and-see' stance when appropriate, and proceed with projects on a conditional basis, subject to the unfolding uncertainty. On the other hand, the non-flexible planner exhibits a 'jump-to-solutions' behaviour, committing prematurely to projects that may prove to be unnecessary. Finally, the proposed method results in the minimum expected regret when compared to deterministic and inflexible approaches, illustrating that the optimal exercise of the planner's inherent flexibility can constitute a well-founded approach to coping with system uncertainties.

5 Risk-Constrained Transmission Expansion Planning

Abstract

In the face of uncertainty, the provision for limiting the impact of adverse scenarios is an indispensable part of robust decision-making. Although the optimal exercise of managerial flexibility can substantially reduce risk through contingent actions, it is only through the explicit introduction of a risk measure constraint that acceptable levels of risk exposure can be guaranteed under all realizations. In this chapter we include CVaR constraints to the flexible stochastic formulation, in order to limit the risk of experiencing excessive constraint costs. A novel node-variable formulation is developed and incorporated into the previously developed solution strategy. Computational load due to scenario-wise coupling is substantially reduced and large systems can be modelled. A case study on the IEEE RTS is undertaken in order to identify the optimal expansion strategies under different risk profiles.

5.1 Introduction

Risk is the possibility that a chosen activity or decision will lead to an undesirable outcome and is an integral part of any investment activity under uncertainty. It is essential that investors are well aware of the risk associated with their decisions and ensure that possible deviations from the expected outcome are acceptable. The stochastic transmission expansion formulation presented in Chapter 4 is risk-neutral, meaning that the planner's objective is a straightforward minimization of expected system costs. However, there are aspects of the optimal solution that the planner may find unattractive. A characteristic example of such an undesirable outcome is excessive constraint costs being experienced. Investment decisions taken in the absence of risk considerations may limit the planner's ability for recourse in the case of adverse scenarios materializing. Despite the minimization of expected system costs, the system can be left unable to adapt effectively to particular events, giving rise to a risk of high constraint costs. In many cases, the planner would wish to immunize his decisions against such an eventuality and ensure that the strategy being followed can guarantee with some degree of certainty that the realized constraint costs will always be within some allowable limit. Naturally, risk management always comes at a price, which in this context is an increase in transmission investment. By incorporating risk constraints in the developed stochastic optimization framework, it is possible to find the expansion strategy that minimizes expected system costs while bounding congestion risk according to the planner's level of risk averseness.

The planner's risk averseness has to be expressed in terms of a suitable risk measure. A risk measure is a function that associates a random variable (in this case constraints cost) with a single real number that characterizes the underlying risk. The most fundamental risk measure is the distribution variance. Other examples of risk measures may include the probability of being above a target value, or the threshold value whose probability of being surpassed is equal to a pre-defined confidence level. Risk management can be applied through the inclusion of the chosen risk measure in an optimization constraint that bounds it to be below an acceptable level. Some standard risk measures are:

5.1.1 Variance

The outcome of a decision under uncertainty can be characterized by two parameters; the expected system cost and the cost variance. A large variance indicates a high risk of experiencing unfavourable outcomes, while a small variance indicates certainty that the eventual outcome will be close to the expected value. A major disadvantage of variance-constrained optimization is that by definition, both unfavourable and favourable scenarios are penalized. Finally, due to its quadratic nature, variance constraint formulations entail severe computational complexity.

5.1.2 Shortfall probability

Shortfall probability is defined as the probability that a loss distribution function $f(x,s)$ for a portfolio x subject to uncertainties *s* (i.e. scenarios) with a cumulative probability distribution *P* goes above a pre-set tolerance level η. Mathematically, it can be expressed as:

$$SP(\eta, x) = P(s | f(x,s) < \eta)$$

Its major disadvantage lies in the fact that no information is given on the cost distribution beyond that threshold. As a result, the risk assessment scope is limited and the planner may remain blind to prohibitively adverse scenarios. In addition, the fact that the eventual risk constraint is not expressed in units of cost, but in terms of probability, necessitates the use of binary variables [54], leading to increased computational load.

5.1.3 Value-at-Risk

Value-at-Risk (VaR) is a popular risk measure used to determine what the maximum loss with respect to a specified confidence level is. Given a loss distribution function $f(x,s)$ for a portfolio *x* subject to uncertainties *s*, VaR_β can be defined for the confidence level $\beta \in [0,1]$ as being equal to the largest value η ensuring that the probability of experiencing a loss greater than η is lower than $1-\beta$. Mathematically, the VaR can be expressed as:

$$VaR_\beta(x) = \max\{\zeta : P(s | P(x,s) < \zeta) \leq 1-\beta\} \qquad (5.1)$$

where *P* is the cumulative probability distribution. In the context of transmission planning, $f(x,s)$ is the constraints cost associated to investment decisions *x* under scenario *s* and VaR_β can be interpreted as the maximum constraints cost experienced with a β level of confidence. The most important shortcoming of VaR is that no information is given about the loss distribution beyond its value, rendering it inadequate in characterizing the residual risk. This may leave the system planner exposed to events with low probability but highly unfavourable impact. An alternative risk measure incorporating a quantification of the distribution tail is needed to identify such occurrences.

5.1.4 Conditional Value-at-Risk

Conditional Value-at-Risk (CVaR), also known as expected shortfall, is a spectral risk measure defined as the expected loss of a portfolio under the condition that the losses exceed VaR. Given a loss function $f(x,s)$ for a portfolio *x* subject to uncertainties *s*, $CVaR_\beta$ can be defined for the confidence level $\beta \in [0,1]$ as the expected value of the losses beyond VaR_β. Mathematically, the CVaR for a discrete distribution can be expressed as:

$$CVaR_\beta(x) = \max\left\{VaR_\beta(x) + \frac{1}{1-\beta}E\left[\max\left(f(x,s) - VaR_\beta(x),0\right)\right]\right\} \tag{5.2}$$

Where E denotes the expectation operator and $\left(f(x,s) - VaR_\beta(x)\right)$ is the 'overshoot' of scenario *s* above VaR_β. According to Krokhmal et al. [65], equation (5.2) can be well approximated by the function:

$$\tilde{F}_\beta(x) = VaR_\beta(x) + \frac{1}{1-\beta}\sum_{s=1}^{N_s}\pi_s\left[f_s - VaR_\beta(x)\right]^+ \tag{5.3}$$

where $[\cdot]^+ = \max(\cdot,0)$ denotes the positive-part function.

The advantage of CVaR is the quantification of the residual risk beyond the VaR threshold, capturing low-probability adverse scenarios. By definition, it is a tighter measure than VaR, meaning that for a portfolio *x*, $CVaR_\beta \geq VaR_\beta$. In addition, it possesses all desirable risk measure properties as defined by Artzner et al [94]: sub-additivity, sensibility, positive homogeneity and translation invariance. Another advantage is in its implementation. It can be represented by linear relations, thus being suitable for inclusion as a risk constraint in a linear programming formulation. CVaR has been successfully used in the past in the context of generation investment to model risk-averseness of wind farm developers [80] due to uncertainty in wind variability. We will be using CVaR constraints in this research to model the planner's risk averseness towards excessive constraint costs due to underinvestment. This is the first application of risk-constrained optimisation for managing constraint costs in the context of transmission planning.

It is important to point out that defining an acceptable CVaR threshold entails taking a specific view that will largely impact investment decisions. It follows that a pessimistic or overly conservative view on the acceptable risk exposure could be used to justify very high levels of investment. Notably, this potential for overinvestment is exacerbated in jurisdictions that have an asset-based regulatory framework, such as the UK. For this reason, close regulatory oversight is essential for managing the arising agent-principal problem.

5.2 Mathematical formulation of CVaR constraints

One major drawback of CVaR constraints is that it involves scenario-wise coupling as seen in (5.3). As a result, a scenario-variable formulation is typically used [54, 66, 80, 95]. In order to incorporate the risk constraint in our node-variable model formulation and take advantage of its superior performance in dealing with multiple investment and operational decisions, a number of modifications must take place. In this section we illustrate how the typical linearized CVaR constraint can be adapted to a node-variable problem formulation. In addition, we extend the computational advantage of this approach by employing a multi-cut Benders decomposition scheme. The end result is a powerful decomposed formulation that allows individual processing of each operational subproblem and significantly relieves the computational load associated to the CVaR constraints.

5.2.1 Scenario-variable CVaR constraints

We first consider the deterministic equivalent formulation of a typical multi-stage stochastic transmission expansion problem considering generation uncertainty, where

- c : investment cost vector.
- q : operation cost vector.
- $x_{s,e}$: investment decision variables for scenario *s*, stage *e*.
- $y_{s,e}$: operation decision variables for scenario *s*, stage *e*.
- $g_{s,e}$: maximum generation capacities vector for scenario *s*, stage *e*.
- ζ : decision variable representing VaR_β.
- $\overline{C}_\beta$: user-defined maximum allowable $CVaR_\beta$.

In addition, we introduce the non-anticipativity constraint matrix of size $N_S \times N_E$, where:

- $N_{1,e} = 1, \forall e \in \Omega_E$.
- $N_{s,e} = 1$ if scenario *s* is coincident with scenario *s-1* at stage *e*.
- $N_{s,e} = 0$ otherwise.

A CVaR constraint can be imposed on operational costs, yielding the following model:

$$z = \min\left\{\sum_{s=1}^{N_S}\pi_s\sum_{e=1}^{N_E}\left(c^T x_{s,e} + q^T y_{s,e}\right)\right\} \tag{5.4a}$$

s.t.

$$Ay_{s,e} = d \qquad ,\forall s \in \Omega_s, \forall e \in \Omega_E \tag{5.4b}$$

$$By_{s,e} \le g_{s,e} \qquad ,\forall s \in \Omega_s, \forall e \in \Omega_E \tag{5.4c}$$

$$Cy_{s,e} - D\sum_{j=1}^{e} x_{s,j} \le 0 \qquad ,\forall s \in \Omega_s, \forall e \in \Omega_E \tag{5.4d}$$

$$x_{s,e} = x_{s+1,e} \qquad \forall e \in \Omega_E\ ,\ s = 1,..,N_S - 1 \text{ if } N_{s,e} = 1 \tag{5.4e}$$

$$\zeta + \frac{1}{1-\beta}\sum_{s=1}^{N_s}\pi_s\left[q^T\left(\sum_{e=1}^{N_E} y_{s,e})\right) - \zeta\right]^+ \le \overline{C_\beta} \tag{5.4f}$$

The objective function is the probability weighted sum of investment and operation costs. Constraint (5.4b) represents the system balance equation, constraint (5.4c) bounds unit dispatch levels according to the generation background realized at scenario *s*, epoch *e* and constraint (5.4d) couples power flows to the transmission investment decisions undertaken in the current and all previous stages of the corresponding scenario. (5.4e) are the non-anticipativity constraints ensuring that scenarios with a common uncertainty realization up to stage *e* are subject to the same investment up to that stage (e.g. first stage investment decisions must be constrained to be equal among all scenarios). Finally, constraint (5.4f) limits the CVaR_β of operational costs to the maximum allowable value $\overline{C_\beta}$ as in equation (5.3).

According to Rockafellar and Uryasev [56], the linearization of the CVaR constraint (5.4f) can be carried out using positive auxiliary variables z_s to represent the overshoot of operational cost above the threshold ζ (representing VaR_β if the constraint is binding) as follows:

$$\zeta + \frac{1}{1-\beta}\sum_{s=1}^{N_s}\pi_s z_s \le \overline{C_\beta} \tag{5.5}$$

$$\zeta + z_s - q^T\left(\sum_{e=1}^{N_E} y_{s,e}\right) \ge 0 \quad ,\forall s \in \Omega_s \tag{5.6}$$

The above formulation is defined in terms of scenarios and stages, constituting a scenario-variable approach. Given the computational advantages of a node-variable formulation illustrated in Section 4.7.5, we modify the above model to accommodate decisions defined in terms of each tree node *m*.

5.2.2 Node-variable CVaR constraints

For the equivalent node-variable formulation, all decision variables must be expressed in terms of scenario tree nodes. However, the CVaR constraints (5.5) and (5.6) are defined in terms of the single decision variable ζ, which is common for all scenarios and scenario-specific overshoot values z_s. We show how these constraints can be modified to accommodate node-specific decisions. We begin by introducing the following variables:

- x_m : investment decision variables for node *m*.
- y_m : operation decision variables for node *m*.
- g_m : maximum generation capacities vector for node *m*.
- ζ_m : auxiliary variables representing VaR for node *m*.
- z_m : auxiliary variables representing operational cost beyond ζ_m at node *m*.

In addition, we define the scenario-node membership matrix *H* of size $N_S \times N_M$ where:

- $H_{s,m} = 1$ if node *m* belongs to scenario *s*.
- $H_{s,m} = 0$ otherwise.

We introduce scenario-specific VaR variables ζ_s and constrain them to be equal across all scenarios through

$$\zeta - \zeta_s = 0, \forall s \in \Omega_s \tag{5.7}$$

This way, the overarching decision variable ζ can be partitioned in a set of node-specific decision variables ζ_m , while the summation across each scenario is equal to ζ .

$$\zeta_s = \sum_{m \in \Omega_M} H_{s,m} \zeta_m \tag{5.8}$$

Similarly, we define the overshoot of each scenario beyond threshold ζ in terms of the node-specific decision variables z_m as:

$$z_s = \sum_{m \in \Omega_M} H_{s,m} z_m \quad , \forall s \in \Omega_s \tag{5.9}$$

Using the above, equation (5.5) can be re-written as

$$\zeta + \frac{1}{1-\beta} \sum_{s=1}^{N_s} \sum_{m \in \Omega_M} H_{s,m} \pi_m z_m \leq \overline{C_\beta} \tag{5.10}$$

where z_m is the node-specific overshoot beyond ζ_m calculated as

$$\zeta_m + z_m - q^T y_m \geq 0 , \forall m \in \Omega_M \tag{5.11}$$

The above modifications lead us to the following node-variable formulation of the risk-constrained stochastic transmission expansion problem:

$$z = \min \left\{ \sum_{m \in \Omega_M} \pi_m \left(c^T x_m + q^T y_m \right) \right\} \tag{5.12a}$$

$$s.t. \tag{5.12b}$$

$$Ay_m = d \ , \forall m \in \Omega_M \tag{5.12c}$$

$$By_m = g_m \ , \forall m \in \Omega_M \tag{5.12d}$$

$$Cy_m - D \left[\sum_{k \in \Phi_0(m)} x_k \right] \leq 0 \ , \forall m \in \Omega_M \tag{5.12e}$$

$$\zeta + \frac{1}{1-\beta} \sum_{s=1}^{N_s} \sum_{m \in \Omega_M} H_{s,m} \pi_m z_m \leq \overline{C_\beta} \tag{5.12f}$$

$$\zeta - \sum_{m \in \Omega_M} H_{s,m} \zeta_m = 0 \ , \forall s \in \Omega_s \tag{5.12g}$$

$$\zeta_m + z_m - q^T y_m \geq 0 \ , \forall m \in \Omega_M \tag{5.12h}$$

Where constraint (5.12f) imposes the upper CVaR limit, constraint (5.12g) ensures that the summation of node-specific VaR variables ζ_m are equal among all scenarios, as in equation (5.7) and constraint (5.12h) is used to calculate the overshoot of operation costs beyond ζ_m at node *m*. Note that in the node-variable formulation, non-anticipativity of investment decisions is implicitly taken into account.

The node-variable formulation (5.12) was tested and found to have a better computational performance than the scenario-variable formulation (5.4) in terms of CPU time and memory requirements due to the reduced number of decision variables and constraints as well as the implicit consideration of non-anticipativity constraints. However, in the case of large systems and numerous scenarios, the deterministic equivalent problem can be very hard to solve directly and necessitates the application of decomposition approaches. Noting that the decision variables x_m, ζ_m and z_m are complicating variables that can be fixed to render problem (5.12) separable, we proceed with splitting the problem using Benders decomposition.

5.2.3 Benders Decomposition of CVaR constraints

Benders decomposition has been applied to CVaR constraints in the past. Bruno and Sagastizabal [95] successfully employ Benders to model risk-averse investment in gas pipelines using a two-stage stochastic model. A single investment decision is made at the first stage followed by the optimal second-stage operation. The decomposition scheme splits investment and operation, providing the subproblem with trial decisions related to the investment and the CVaR auxiliary variables. We extend this approach to multi-stage stochastic problems while employing a node-variable formulation.

The original problem (5.12) is decomposed in a single multi-stage transmission investment master problem and N_M operation subproblems.

5.2.3.1 Master problem

The master problem objective function consists of the expected investment cost and an approximation of the expected operational cost $\sum_{m \in \Omega_M} \pi_m \alpha_m$. At each iteration *v*, the master problem suggests a set of trial decision values $\left\{ x_m^{(v)}, \zeta_m^{(v)}, z_m^{(v)} \right\}$ for each subproblem associated to node *m*.

$$z = \min \left\{ \sum_{m \in \Omega_M}^{N_s} \pi_m \left(c^T x_m + \alpha_m \right) \right\} \tag{5.13a}$$

s.t.

$$\zeta + \frac{1}{1-\beta} \sum_{s=1}^{N_s} \sum_{m \in \Omega_M} H_{s,m} \pi_m z_m \leq \overline{C_\beta} \tag{5.13b}$$

$$\zeta - \sum_{m \in \Omega_M} H_{s,m} \zeta_m = 0 \text{ , } \forall s \in \Omega_s \tag{5.13c}$$

5.2.3.2 Subproblem

The subproblem objective function (5.14a) is the sum of operational costs subject to the trial investment decision $x_m^{(v)}$ and a penalty term corresponding to the infeasibility of trial CVaR variables $\zeta_m^{(v)}$ and $z_m^{(v)}$, calculated as the product of slack variable $\hat{z}_m$ with a large positive constant *M*. The optimal value of the operational subproblem associated to node *m* is given by the following linear program:

$$\omega_m = \min \left\{ q^T y_m + \hat{z}_m M \right\} \tag{5.14a}$$

s.t.

$$A y_m = d \tag{5.14b}$$

$$B y_m = g_m \tag{5.14c}$$

$$Cy_m - D\left[\sum_{k \in \Phi_0(m)} x_k^*\right] \le 0 \tag{5.14d}$$

$$\zeta_m^* + z_m^* + \hat{z}_m - q^T y_m \ge 0 \tag{5.14e}$$

$$x_m^* = x_m^{(v)} : \lambda_m^{(v)} \tag{5.14f}$$

$$\zeta_m^* = \zeta_m^{(v)} : \lambda_m^{z^{(v)}} \tag{5.14g}$$

$$z_m^* = z_m^{(v)} : \lambda_m^{z^{(v)}} \tag{5.14h}$$

Where constraint (5.14e) ensures that the operational cost is less than the trial threshold value ζ_m^* and trial overshoot value z_m^*. The slack variable $\hat{z}_m^*$ is included to ensure that the constraint remains feasible for all master problem suggestions, while the relevant infeasibility information is passed to the master problem through the dual variable $\lambda_m^{z^{(v)}}$. , impacting the coupling variables x_m, ζ_m and z_m. Constraints (5.14f)-(5.14h) equalize the decision variables to the corresponding master problem trial solution. The dual variables of these constraints are subsequently used to construct the Benders cut associated to node *m*. Note that the dual variables received from (5.14g) and (5.14h) are the same since the variables appear in the same constraint (5.14e).

5.2.3.3 Benders cut

The Benders cuts to be appended to the master problem at iteration *v* is given by equation (5.15). A single cut per tree node *m* is generated, bounding the subproblem approximation variable α_m from below.

$$\alpha_m \ge \begin{bmatrix} \omega_m^{(v-1)} + \\ \lambda_m^{(v-1)}\left(x_m - x_m^{(v-1)}\right) + \\ \lambda_m^{z^{(v-1)}}\left(\zeta_m - \zeta_m^{(v-1)}\right) + \\ \lambda_m^{z^{(v-1)}}\left(z_m - z_m^{(v-1)}\right) \end{bmatrix} \tag{5.15}$$

5.2.3.4 Convergence criterion

Convergence is reached when the difference between the upper bound (5.16) and lower bound (5.17) of the problem is smaller than a pre-defined tolerance value ε.

$$Z_{upper}^{(v)} - Z_{lower}^{(v)} \le \varepsilon \tag{5.16}$$

$$Z_{upper}^{(v)} = \sum_{m \in \Omega_M} \pi_m \omega_m \tag{5.17}$$

$$Z_{lower}^{(v)} = \sum_{m\in\Omega_M} \pi_m \alpha_m \tag{5.18}$$

Thus we have shown how a CVaR constraint can be incorporated in a decomposed node-variable stochastic problem formulation. The ideas illustrated will be applied to the stochastic transmission expansion problem presented in Chapter 4 to allow the efficient accommodation of risk constraints while enabling us to keep computational load to a manageable level.

5.3 CVaR-constrained Stochastic Transmission Expansion Planning

In this section we illustrate how the flexible stochastic transmission expansion problem can be modified to include a CVaR constraint on the constraints cost. As in Chapter 4, the Benders decomposition technique is used to split the original problem in a multi-stage investment master problem and an operation subproblem for each operating point *(m,t)*.

5.3.1.1 Master problem

The principles developed in the previous section are used to partition the global auxiliary variable ζ into individual variables $\zeta_{m,t}$ related to each node *m* and time block *t*. In addition, the decision variables $z_{m,t}$ are introduced to represent the overshoot of the operational cost of operating point *(m,t)* beyond $\zeta_{m,t}$. At each Benders iteration, the master problem produces trial values for $\zeta_{m,t}^{(v)}$ and $z_{m,t}^{(v)}$ along with investment decisions $F_{m,l}^{inv(v)}$ and $QB_{m,l}^{(v)}$ that are passed to the subproblem. Overall, the investment master problem remains the same as in Section 4.5.1 with the addition of the following constraints:

$$\zeta + \frac{1}{1-\beta}\left(\sum_{s=1}^{N_S}\sum_{m\in\Omega_M} H_{n,m}\pi_m \sum_{t=1}^{N_T} z_{m,t}\right) \le \overline{C_\beta} \tag{5.19}$$

$$\zeta - \sum_{m\in\Omega_M} H_{s,m}\sum_{t=1}^{N_T}\zeta_{m,t} = 0 \qquad ,\forall s\in\Omega_s \tag{5.20}$$

Constraint (5.19) imposes the CVaR constraint that couples the probability-weighted sum of all scenario overshoots to the global threshold decision variable ζ and the maximum allowable CVaR$_\beta$ value C_β. Equation (5.20) states that the sum of all auxiliary variables $\zeta_{m,t}$ related to each scenario *s* must be equal to the global threshold decision variable ζ .

5.3.1.2 Subproblem

The subproblem receives the trial decision variables $\zeta_{m,t}^{(v)}$, $z_{m,t}^{(v)}$, $F_{m,l}^{inv^{(v)}}$ and $QB_{m,l}^{(v)}$ from the master problem and minimizes operational costs while ensuring that the constraints cost is less than the sum of $\zeta_{m,t}^{(v)}$, $z_{m,t}^{(v)}$ and a slack variable $\hat{z}_{m,t}$. The slack variable is introduced to guarantee problem feasibility. It informs the master problem on the trial solution's infeasibility through a penalization term present in the objective function. Overall, the operational subproblem is the same as in Section 4.5.2 with the exception of the objective function which is modified to :

$$\omega_{m,t}^{(v)} = \min_{p^+,p^-,d^*,\hat{z}} \left\{ r_{\varepsilon(m)}^{O} \tau_t \left[\sum_{g=1}^{N_G} \left(p_{m,t,g}^{+} o_g - p_{m,t,g}^{-} b_g \right) + \sum_{n=1}^{N_N} d_{m,t,n}^{*} \Gamma \right] + \hat{z}_{m,t} M \right\} \tag{5.21}$$

and the addition of the following constraints:

$$z_{m,t}^{*} + \hat{z}_{m,t} \geq r_{\varepsilon(m)}^{O} \tau_t \left[\sum_{g=1}^{N_G} \left(p_{m,t,g}^{+} o_g - p_{m,t,g}^{-} b_g \right) \right] - \zeta_{m,t}^{*} \tag{5.22}$$

$$z_{m,t}^{*} = z_{m,t}^{(v)} \; z_{m,t}^{*} = z_{m,t}^{(v)} : \lambda_{m,t}^{z} \tag{5.23}$$

$$\zeta_{m,t}^{*} = \zeta_{m,t}^{(v)} \tag{5.24}$$

5.3.1.3 Benders Cuts

The Benders cut equation (4.17) is modified to include the risk-decision dual variables:

$$\alpha_{m,t} \geq \left[\begin{array}{c} \omega_{m,t}^{(v-1)} + \\ \sum_{l=1}^{N_L} \lambda_{m,t,l}^{(v-1)} \left(F_{m,l}^{inv} - F_{m,l}^{inv^{(v-1)}} \right) + \\ \sum_{l=1}^{N_L} \lambda_{m,t,l}^{QB^{(v-1)}} \left(QB_{m,l} - QB_{m,l}^{(v-1)} \right) + \\ \lambda_{m,t}^{z^{(v-1)}} \left(\zeta_{m,t} - \zeta_{m,t}^{(v-1)} \right) + \\ \lambda_{m,t}^{z^{(v-1)}} \left(z_{m,t} - z_{m,t}^{(v-1)} \right) \end{array} \right] \tag{5.25}$$

The convergence criterion (4.36) is used to check for optimality at the end of each Benders iteration. Finally, the solution strategy that combines Benders with contingency screening is kept the same as outlined in Section 4.5.

5.4 IEEE RTS case study

In this section we revisit the risk-neutral case study presented in Section 4.7 and illustrate how applying a CVaR constraint can limit the risk of excessive constraint costs and have an impact on the optimal first-stage decisions. Given that the planner's definition of acceptable risk exposure cannot always be expressed in absolute terms, it is most useful to undertake a sensitivity analysis around the level of risk-averseness, expressed in terms of an appropriate risk measure. In other words, finding the optimal solution subject to a risk measure constraint should not be viewed as an ultimate decision criterion, but rather as a means for arriving at a family of candidate solutions that are pareto-optimal for different levels of risk-averseness.

Figure 5-1 illustrates the cumulative distribution function for constraint costs when the optimal risk-neutral expansion strategy is followed. The detailed data are included in Table 4-18. Scenario 10 is the most unfavourable realization, giving $VaR_{95\%}$ and $CVaR_{95\%}$ equal to £248.7m.

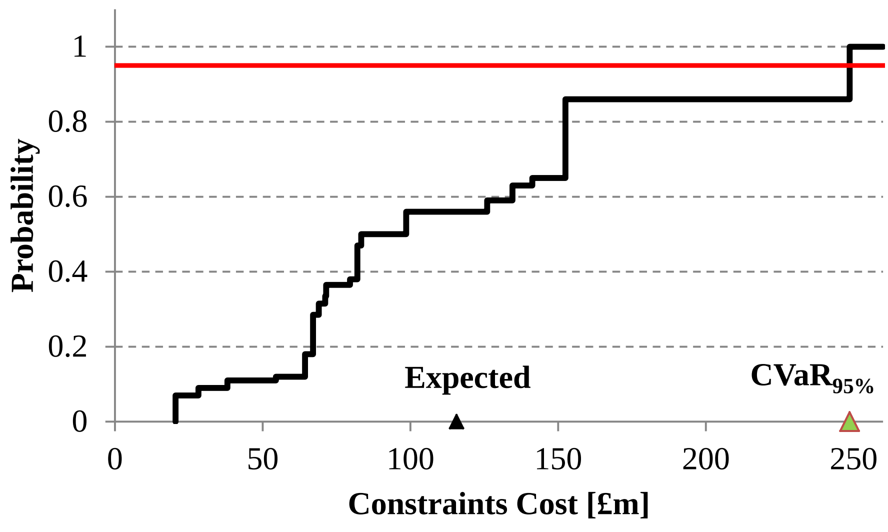


Figure 5-1: Cumulative distribution function for the risk-neutral investment solution, also indicating the level of expected constraints cost and $CVaR_{95\%}$. Red line indicates the 0.95 confidence level.

5.4.1 Risk-constrained transmission expansion planning

We proceed with showcasing the optimal investment strategy and corresponding system costs for different levels of risk-aversion. The detailed results are shown in Appendix E and summarized in Table 5-1. The cumulative distributions of constraints cost for each risk-constrained solution are shown in Figure 5-2.

C_β	Constraints Cost $VaR_{95\%}$	Constraints Cost $CVaR_{95\%}$	First-stage Investment Cost	Expected Investment Cost	Expected Constraints Cost	Expected System Cost
200	152.4	152.4	104.8	292.7	87.2	379.9
150	95.9	95.9	169.6	330.8	51.8	382.6
50	41.4	48.3	278.7	450.6	27.3	477.9
20	19.2	19.6	341.1	515.2	14	529.2

Table 5-1: System costs for different levels of risk aversion dictated by the maximum allowable $CVaR_{95\%}$ level C_β. All values in £m.

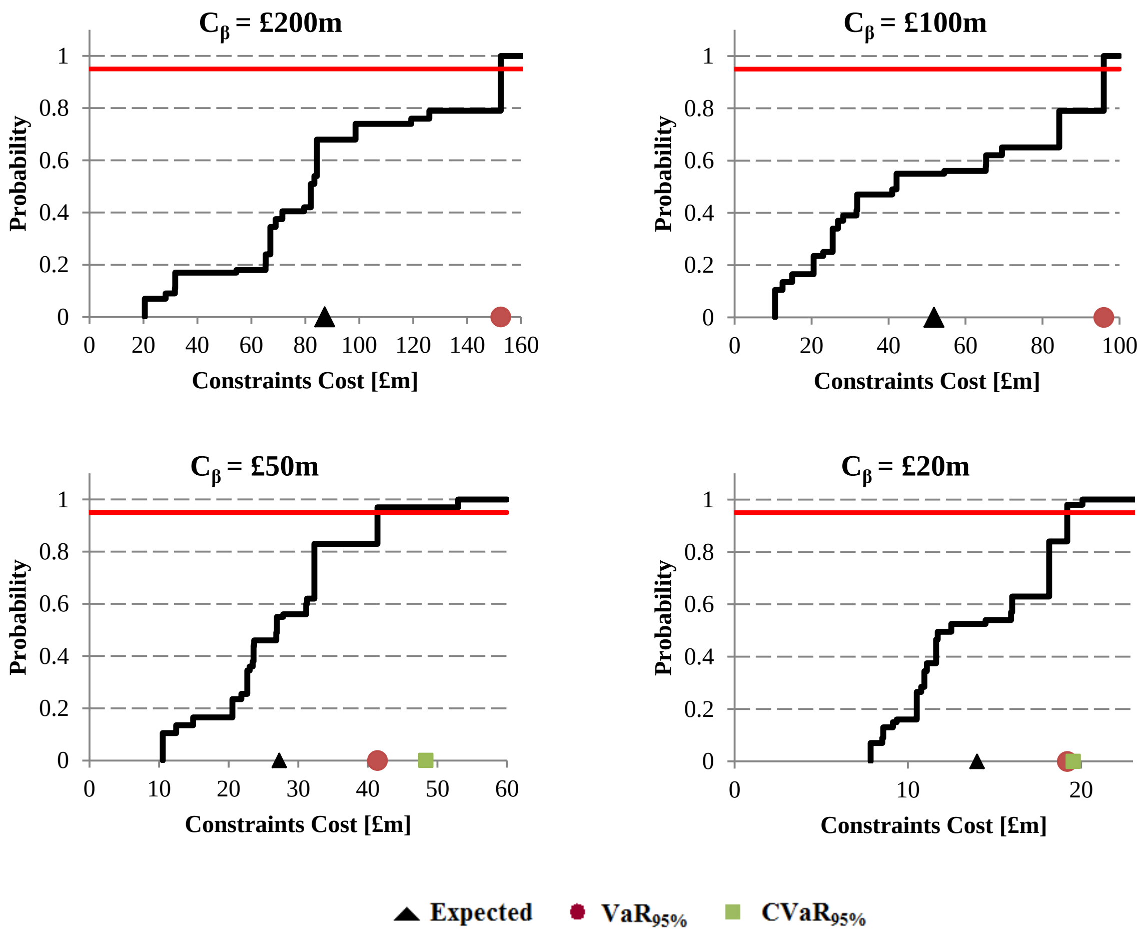


Figure 5-2: Cumulative distribution function for the risk-averse investment solutions. Red line indicates 0.95 confidence level.

For the case that C_β = £200m (Table E-1a in Appendix E), we find that the planner can ensure that the CVaR constraint is satisfied without deviating from the risk-neutral optimal first stage decisions (Table 4-9). The required reductions in congestion levels can be achieved through increased transmission investment expenditure in subsequent epochs. More specifically, project C is chosen for lines 5 and 10 in order to provide extra capacity and ensure that the commissioning of G28 does not result in excessive congestion. These lines are to be upgraded to their full potential subject to the high-growth scenario tree transitions (3)→(7) or (3)→(8). Under this expansion strategy, scenario 1 is the most adverse realization. Due to this scenario's high probability of occurrence (higher than 1-β), $VaR_{95\%}$ and $CVaR_{95\%}$ are at the same level of £152.4m. This is considerably lower than the £200m limit set by the risk constraint and is due to the lumpiness of capacity additions that prohibits the planner from building just enough capacity to exactly satisfy the risk constraint.

Further CVaR reductions impact first stage decisions, necessitating earlier asset construction and investment in projects capable of providing more capacity. The optimal expansion strategy that minimizes expected system costs while limiting the $CVaR_{95\%}$ to less than £150m involves an increase in first-stage capital expenditure by £64.8m. Lines 1 and 6 have to be reinforced from the very first-stage along with line 2, which was the sole 'here and now' commitment under the risk-neutral paradigm. This commitment enables an earlier accommodation of the power flows arising due to the potential commissioning of G27, thus constraints in the second epoch. Again, scenario 1 results in the most adverse constraint costs and leads to a $VaR_{95\%}$ and $CVaR_{95\%}$ of £96m.

In the same vein, even more conservative risk constraints can be applied. For the highly risk-averse cases that C_β = £50m and £20m, first stage investment is significantly increased to £278.7m and £341.1m respectively. In the former case, project C options are chosen for the main exporting lines 2 and 6 to allow conditional upgrading to 800 MW if uncertainty is resolved in favour of a large G27 development. Following this expansion strategy, $VaR_{95\%}$ is equal to £41.4m corresponding to the scenario 10 realization. Scenario 4 constraint costs stretch beyond this threshold level (this is due to the reduced investment undertaken in node 4), thus leading to a conditional probability-weighted tail of £48.3m. In the C_β = £20m case, an additional commitment is made to line 4, allowing a higher wind power transfer capability in the case that locational uncertainty is resolved in favour of G28. The highest constraint costs are experienced under scenarios 10 and 13, leading to a $VaR_{95\%}$ of £19.2 and $CVaR_{95\%}$ of £19.6m.

It is important to note that the presented risk-constrained framework should not be viewed as a method for drawing conclusive decisions, but rather as a vehicle to evaluate the suitability of candidate 'here and now' decisions. Performing sensitivity analysis around the level of risk-averseness enables the planner to arrive at a family of optimal 'here and now' decisions. The final choice on which lines will be reinforced will depend on the planner's risk attitude towards future constraint costs as well as his willingness to shoulder the extra capital expenditure required to ensure acceptable risk exposure. Using the above data, it is possible to pinpoint the range of possible CVaR values corresponding to each first-stage investment decision vector $D_{[1]}$. As shown in Figure 5-3, undertaking investment only on line 2 using project B leads to a minimum possible CVaR of £95.9m, which can be achieved through increased investment in subsequent epochs. This risk can be further hedged by increasing first-stage commitments. In the extreme case that the planner requires that virtually no constraint costs are experienced under all scenarios, this hedging cost reaches £236.3m.

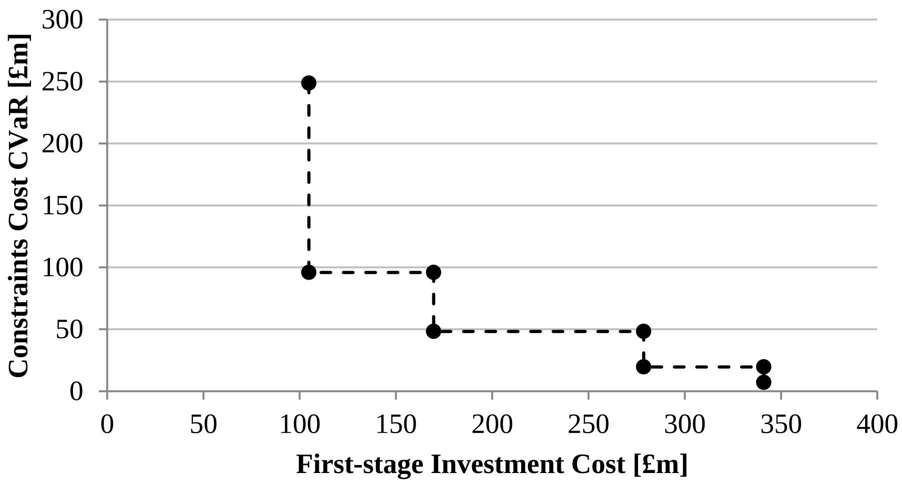


Figure 5-3: Constraints cost $CVaR_{95\%}$ versus the required investment cost to be undertaken in the first stage.

The above analysis focuses exclusively on first-stage decisions. However, it is critical for the planner to also be aware of the expected hedging cost over the entire horizon. This is defined as the difference in expected system costs between the risk-neutral and the risk-constrained solution. Using the risk analysis data it is possible to construct the efficient frontier curve showing the trade-off between expected system costs and risk exposure. In Figure 5-4 we show the efficient frontier curve. As can be seen, significant reductions to risk exposure can be achieved by facing small increases in expected system cost until $CVaR_{95\%}$=£95.9m. For a further decrease, the expected cost grows substantially due to commitment to expensive long-term projects that will ensure the ability for adequate recourse in the case of high-growth scenarios. The expected hedging cost in the cases that C_β=£50m and £20m is £101.9m and £153.2m respectively.

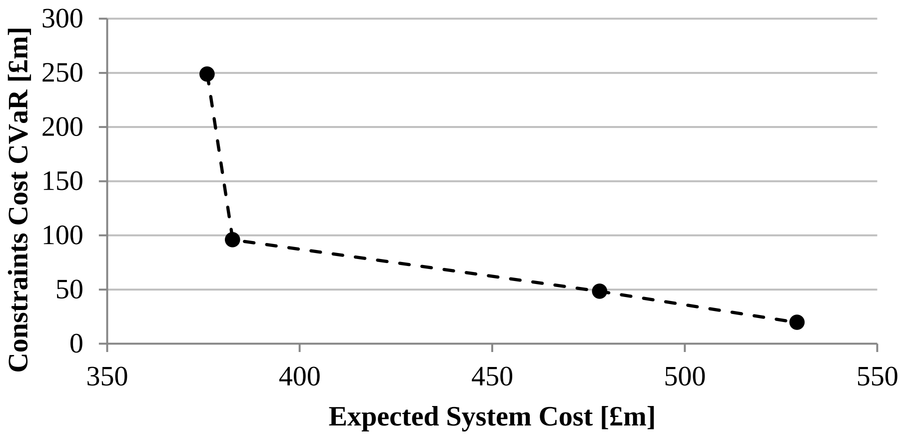


Figure 5-4: Efficient frontier.

5.4.2 Computational performance

The proposed Benders decomposition framework proved to result in reasonable convergence times and memory usage. As can be seen in Table 5-2, increasing the level of risk-averseness resulted in slower CPU time and larger memory usage. Naturally, solution of the master problem constituted the most severe computational bottleneck, with some instances requiring the processing of very large branch-and-bound trees. No significant difference was observed in solution times for the operational subproblems when compared to the risk-neutral model. All cases converged within two to three iterations of the contingency screening module.

C_β	Benders Iterations	CPU Time [s]	CPU Memory [GB]
£200m	7,10	3,953	2.63
£150m	7,12	4,587	3.71
£100m	7,12	6,325	5.89
£50m	7,11	7,207	8.25
£20m	7,13,15	10,550	9.7

Table 5-2: Computational performance of the risk-constrained transmission expansion model.

Solving the presented case studies while using a scenario-variable formulation was not possible within reasonable times. All simulations had not converged within 12 hours, while a gap value of 1 was still maintained. This further highlights the advantages of using a node-variable approach and confirms that the proposed modification to the traditional CVaR constraint formulation is well-founded.

5.5 Conclusions

In this chapter we have extended the risk-neutral stochastic transmission expansion problem to include CVaR constraints related to excessive constraint costs. A novel node-variable formulation has been developed to allow the inclusion of risk constraints in our multi-stage stochastic model. By partitioning risk decision variables in terms of operating points, a high level of problem decomposition can be achieved. The computational superiority of the proposed approach is demonstrated by simulating risk-averse planning on the IEEE RTS. A range of cases studies is carried out to perform a sensitivity analysis around the planner's levels of risk aversion. We show how the consideration of risk constraints can impact first-stage decisions and we construct the efficient frontier of expected system cost and risk.

6 Concluding remarks and further work

6.1 Summary of achievements

This thesis developed a fully integrated risk-constrained stochastic optimization framework for anticipatory network expansion planning under generation uncertainty. A number of case studies were undertaken on test systems to determine the optimal expansion strategy. This was contrasted to the concept of a static investment plan that ignores the possibility of contingent investment. Through this comparison, we quantified the value of flexibility and demonstrated that the proposed model leads to a substantial further minimization of expected system costs. In addition, the optimal decisions to be taken in the first-stage were found to be different between the flexible and non-flexible frameworks, highlighting the high differentiation between the two methods. By undertaking a regret analysis, we showed that the first-stage decisions produced by our model lead to minimization of the expected regret. We also showed that the developed model can take advantage of the inter-temporal resolution of uncertainty to determine the optimal exercise policy of embedded upgradeability options and thus accurately value the expected net benefit of highly flexible projects. Moreover, the benefit of adopting a “wait and see” stance until more information is known was quantified and in many cases, investment postponement was found to be the optimal solution.

In general, modelling of flexibility allows the planner to take decisions on the basis of subsequent adaptability, while non-flexible planning determines decisions a priori of uncertainty realization, thus undervaluing any available options on the basis of non-conditional exercise. In view of the large economies of scale of new transmission corridors to be constructed (particularly for connecting offshore wind farms and cross-border interconnectors [96]) in the near future due to the shift to renewables, it is essential that a cost-benefit framework capable of considering flexible timing and sizing options is used.

The main contributions of the presented work can be summarized as follows:

C1. *The development of a stochastic optimization modelling framework to deal with anticipatory investment decisions under generation uncertainty.*

This was developed as a multi-stage stochastic mixed-integer linear program that aims to minimize expected system costs. Uncertainty was captured using a multi-stage scenario tree to be defined by the planner on the basis of expert opinion and market modelling. Great effort was made to identify and model all economic and technical aspects that characterize anticipatory transmission investment. Decision flexibility was modelled in order to enable optimal decision-making contingent to the uncertainty realization. The possibility for multiple candidate projects with scale economies and different asset build times was also introduced through appropriate constraints. In addition, the N-1 security criterion, that is a primary driver of transmission investment was modelled. The potential for investment postponement through the optimal placement of quadrature boosters was recognised and included in the model. Overall, this resulted in a fully integrated long-term transmission investment model that can be used for identifying the optimal expansion strategy in different networks.

C2. *The inclusion of appropriate risk constraints to model the system planner's risk-averse profile.*

Risk management is an indispensable part of decision-making under uncertainty. The planner's risk-averseness towards excessive constraint costs was modelled using a CVaR constraint. This resulted in a risk-constrained stochastic transmission expansion model able to pinpoint the optimal hedging investment strategy for limiting risk exposure to the required level. The increased computational load associated with scenario-wise coupling was alleviated through a novel approach presented in Chapter 5. By performing a series of case studies on the IEEE RTS we determined the optimal investment strategy for different levels of risk-averseness. In addition, we examined how limiting the acceptable risk exposure increases the required investment levels. Furthermore, we determined the range of risk-averseness over which different first-stage decisions remain optimal. The efficient frontier between risk and expected cost was presented, able to inform the decision-maker on the pareto-optimal solutions that can be selected.

C3. *The development of stochastic modelling concepts and decomposition techniques that has allowed the optimization of large systems considering multiple stages and scenarios.*

Multi-stage stochastic MILP formulations result in problem sizes that prohibit the modelling of meshed networks with multiple expansion options. In addition, the consideration of an N-1 security criterion introduces a vast number of constraints that may easily lead to intractability. To alleviate the severe computational load, a novel solution strategy was developed based on problem decomposition principles. The stochastic problem has been modelled using a node-variable formulation and constitutes a substantial improvement on the usual scenario-variable formulations that introduce redundant decision variables and entail the use of non-anticipativity constraints. In addition, a parallelisable multi-cut Benders decomposition scheme was employed and its superior performance over the classical approach was clearly demonstrated. Moreover, very significant computational savings were demonstrated through the use of an iterative contingency screening algorithm that allows progressive consideration of binding security constraints. The combination of the above allowed the solution of large problems involving millions of constraints and several thousand binary variables in very reasonable times and with minimal RAM memory requirements.

C4. *The development of a cost-benefit framework compatible with the regulatory notions of valuing future adaptability, keeping options open and deferring investment until uncertainty is resolved.*

The developed model comprises a consistent cost-benefit based framework for network planning under uncertainty that enables the optimal use of the timing and sizing flexibility options embedded in transmission projects. By modelling the decision-maker's inherent ability to dynamically respond to the uncertainty realization, it enables the proper valuation of an investment opportunity's adaptability to future events. Moreover, by differentiating investment decisions according to the resolution of uncertainty, it allows the formal quantification of benefits that lie in deferring investment until uncertainty is partially resolved. The proposed approach leads to a further system cost minimization than non-flexible methods, highlighting its superiority in managing future uncertainty.

6.2 Further work

6.2.1 Incorporation of generation investment decisions in a bi-level structure

Generation investment is a dynamic process based on profitability maximization and influenced by a range of factors such as the regulatory framework, plant investment costs, fuel cost and use of system charges. Some of these factors are beyond the planner's control, but network charges are a direct result of transmission expansion, influencing profitability. As a result, it is prudent to take into account the effect that transmission investment has on future generation developments. Motamedi et al. [55] propose a planning framework where the reaction of generation and its possible post-expansion reactions are taken into account. The approach is based on agent-based systems and a re-evaluation of the initial plan subject to the anticipated generation response. Results obtained on a small test system indicate that ignoring this interaction can lead to quantifiable economic consequences. Other researchers [15] have modelled this problem as a Stackelberg game with the transmission planner as the leader and generators as followers, reacting to the planning commitments. Investigating how transmission investment influences generators' decisions on asset sizing and location would be essential in developing a holistic stochastic transmission planning framework.

6.2.2 Model extension to other types of uncertainty

The presented model focuses on future generation uncertainty, since it is regarded as the primary driver of system evolution in the long-term. However, in real-world planning there are many sources of uncertainty that can have a significant impact on the undertaken cost benefit analysis. The evolution of electricity demand is one of the key drivers influencing the need for transmission investment to accommodate new power flow patterns. The way it will evolve in the future becomes even more important when we take into account the current trend of encouraging electrification of transport and heating in the long-term. In addition, market prices for coal, oil and gas can be subject to considerable fluctuations over a long-term horizon. This has a direct impact on thermal units' short-run marginal costs, altering the dispatch merit order and significantly changing power flow patterns on the corridors shared with renewable generation. Another important uncertainty lies in the availability of demand-side management and energy storage technologies. Both can provide corrective security in post-fault conditions and shave off peak loads to lead to more uniform demand profiles. As a result, the need for capacity reinforcements in the future will very much depend on the penetration level of these technologies. All the above can be accommodated in the presented stochastic planning model through the use of scenario-dependent input data and straightforward modifications to the corresponding optimization constraints.

6.2.3 Modelling of a minimax regret decision criterion

Although the presented model minimizes the expected regret when compared to the optimal deterministic plans, there is great value in also identifying the investment strategy that leads to the minimization of the maximum regret experienced. This would be a suitable decision criterion in the case of a highly risk-averse planner.

6.2.4 Effect of reinforcements on line reactance

In the presented model, we have assumed that capacity reinforcements do not alter the transmission line's reactance value. A method has been proposed where line reactance change can be modelled as a linear function of capacity additions [26]. This will be an important step towards capturing the physical reality of the problem at hand.

6.2.5 Transmission losses

In the presented formulation, transmission losses have been ignored. It is possible to include them through ex-post calculations, but this does not guarantee optimality and system cost minimization for the strategy obtained in the absence of such considerations. It is suggested that losses be included in the objective function and fully considered in the cost-benefit analysis. The modelling challenge lies in the fact that losses are a non-linear function of power flow and thus a piece-wise linear approximation has to be used to keep problem linearity. Given the already large problem formulation including multiple operating points and the requirement of additional decision variables to approximate transmission losses, ensuring problem tractability will prove a difficult task.

6.2.6 Non-correlated wind

In the presented case studies, we have assumed 100% correlation of wind across the entire system. No differentiation to the wind profile exists between generation nodes. The ability to consider multi-area wind profiles is essential for the accurate cost-benefit analysis of systems with high wind penetration and a spatially diversified wind fleet. Due to the high number of possible combinations, this results in a rapid increase of possible operating points and leads to severe problems of dimensionality. It is envisaged that the presented multi-cut Benders decomposition scheme, where each operation sub-problem can be solved in parallel, is well-suited for the handling of multiple wind sources. However, additional studies will have to be undertaken to determine the exact impact on computation times, particularly to the master problem due to the addition of a large number of constraints per iteration.

6.2.7 Market design for stochastic network planning

It is possible that the current transmission investment regulatory framework would require further change to accommodate the recommendations of this research. A potential change could be a requirement on network companies to provide evidence of optimality for the proposed investment decisions with focus on how they form part of a long-term strategy (beyond the current price control) that considers future adaptability to a diverse range of scenario realizations. Investigation of appropriate incentives to encourage the proper valuation of project optionality and reward efficient anticipatory investments is another major task. There is great value in pursuing more detailed work in this area in order to provide concrete recommendations on a policy level.

Appendix A – IEEE RTS data and input parameters

In this Appendix we present the demand and wind input parameters used in the IEEE RTS case studies, as well as the technical characteristics of existing transmission lines.

Bus Id	Load sharing factor
1	3.80%
2	3.40%
3	6.30%
4	2.60%
5	2.50%
6	4.80%
7	4.40%
8	6.00%
9	6.10%
10	6.80%
11	0.00%
12	0.00%
13	9.30%
14	6.80%
15	11.10%
16	3.50%
17	0.00%
18	11.70%
19	6.40%
20	4.50%
21	0.00%
22	0.00%
23	0.00%
24	0.00%

Table A-1: Load sharing factors used in IEEE-RTS case studies.

Demand Period	Season	Time duration [h]	Load factor [p.u.]	Wind factor [p.u.]
1	Winter	558.90	56.32%	5%
2	Winter	111.80	66.88%	5%
3	Winter	914.10	75.68%	5%
4	Winter	15.10	100.00%	5%
5	Winter	186.30	70.40%	5%
6	Winter	410.90	56.32%	15%
7	Winter	82.20	66.88%	15%
8	Winter	672.10	75.68%	15%
9	Winter	11.10	100.00%	15%
10	Winter	137.00	70.40%	15%

Demand Period	Season	Time duration [h]	Load factor [p.u.]	Wind factor [p.u.]
11	Winter	287.90	56.32%	25%
12	Winter	57.60	66.88%	25%
13	Winter	470.90	75.68%	25%
14	Winter	7.80	100.00%	25%
15	Winter	96.00	70.40%	25%
16	Winter	201.90	56.32%	35%
17	Winter	40.40	66.88%	35%
18	Winter	330.10	75.68%	35%
19	Winter	5.50	100.00%	35%
20	Winter	67.30	70.40%	35%
21	Winter	174.50	56.32%	45%
22	Winter	34.90	66.88%	45%
23	Winter	285.40	75.68%	45%
24	Winter	4.70	100.00%	45%
25	Winter	58.20	70.40%	45%
26	Winter	165.60	56.32%	55%
27	Winter	33.10	66.88%	55%
28	Winter	270.90	75.68%	55%
29	Winter	4.50	100.00%	55%
30	Winter	55.20	70.40%	55%
31	Winter	102.60	56.32%	65%
32	Winter	20.50	66.88%	65%
33	Winter	167.80	75.68%	65%
34	Winter	2.80	100.00%	65%
35	Winter	34.20	70.40%	65%
36	Winter	84.60	56.32%	75%
37	Winter	16.90	66.88%	75%
38	Winter	138.40	75.68%	75%
39	Winter	2.30	100.00%	75%
40	Winter	28.20	70.40%	75%
41	Winter	91.10	56.32%	85%
42	Winter	18.20	66.88%	85%
43	Winter	149.00	75.68%	85%
44	Winter	2.50	100.00%	85%
45	Winter	30.40	70.40%	85%
46	Winter	24.60	56.32%	95%
47	Winter	4.90	66.88%	95%
48	Winter	40.20	75.68%	95%
49	Winter	0.70	100.00%	95%
50	Winter	8.20	70.40%	95%
51	Summer	156.40	44.69%	5%
52	Summer	22.30	56.87%	5%
53	Summer	234.40	66.84%	5%
54	Summer	55.70	63.14%	5%
55	Summer	67.00	58.06%	5%
56	Summer	115.00	44.69%	15%
57	Summer	16.40	56.87%	15%

Demand Period	Season	Time duration [h]	Load factor [p.u.]	Wind factor [p.u.]
58	Summer	172.40	66.84%	15%
59	Summer	41.00	63.14%	15%
60	Summer	49.30	58.06%	15%
61	Summer	80.60	44.69%	25%
62	Summer	11.50	56.87%	25%
63	Summer	120.80	66.84%	25%
64	Summer	28.70	63.14%	25%
65	Summer	34.50	58.06%	25%
66	Summer	56.50	44.69%	35%
67	Summer	8.10	56.87%	35%
68	Summer	84.70	66.84%	35%
69	Summer	20.10	63.14%	35%
70	Summer	24.20	58.06%	35%
71	Summer	48.80	44.69%	45%
72	Summer	7.00	56.87%	45%
73	Summer	73.20	66.84%	45%
74	Summer	17.40	63.14%	45%
75	Summer	20.90	58.06%	45%
76	Summer	46.30	44.69%	55%
77	Summer	6.60	56.87%	55%
78	Summer	69.50	66.84%	55%
79	Summer	16.50	63.14%	55%
80	Summer	19.90	58.06%	55%
81	Summer	28.70	44.69%	65%
82	Summer	4.10	56.87%	65%
83	Summer	43.00	66.84%	65%
84	Summer	10.20	63.14%	65%
85	Summer	12.30	58.06%	65%
86	Summer	23.70	44.69%	75%
87	Summer	3.40	56.87%	75%
88	Summer	35.50	66.84%	75%
89	Summer	8.40	63.14%	75%
90	Summer	10.10	58.06%	75%
91	Summer	25.50	44.69%	85%
92	Summer	3.60	56.87%	85%
93	Summer	38.20	66.84%	85%
94	Summer	9.10	63.14%	85%
95	Summer	10.90	58.06%	85%
96	Summer	6.90	44.69%	95%
97	Summer	1.00	56.87%	95%
98	Summer	10.30	66.84%	95%
99	Summer	2.50	63.14%	95%
100	Summer	2.90	58.06%	95%

Table A-2: Demand and wind data used in IEEE-RTS case studies.

Line Id	From bus	To bus	Reactance [p.u]	Length [km]	Initial Capacity [MW]
1	1	2	0.2	5	100
2	1	3	0.2	89	100
3	1	5	0.2	35	150
4	2	4	0.2	53	100
5	2	6	0.2	81	150
6	3	9	0.2	50	200
7	3	24	0.2	20	350
8	4	9	0.2	43	100
9	5	10	0.2	37	100
10	6	10	0.2	26	150
11	7	8	0.2	26	200
12	8	9	0.2	69	250
13	8	10	0.2	69	250
14	9	11	0.2	5	150
15	9	12	0.2	5	300
16	10	11	0.2	5	200
17	10	12	0.2	5	300
18	11	13	0.2	53	250
19	11	14	0.2	47	300
20	12	13	0.2	53	250
21	12	23	0.2	108	500
22	13	23	0.2	97	450
23	14	16	0.2	43	400
24	15	16	0.2	19	450
25	15	21	0.2	55	350
26	15	21	0.2	55	350
27	15	24	0.2	58	350
28	16	17	0.2	29	400
29	16	19	0.2	26	400
30	17	18	0.2	16	300
31	17	22	0.2	118	200
32	18	21	0.2	29	150
33	18	21	0.2	29	150
34	19	20	0.2	44	150
35	19	20	0.2	44	150
36	20	23	0.2	24	250
37	20	23	0.2	24	250
38	21	22	0.2	76	200
39	7	8	0.2	26	200

Table A-3: Network topology of IEEE-RTS.

Appendix B – Corrective security through Quadrature Boosters

In this section we show how the corrective security provided by quadrature boosters can lead to a more economic dispatch schedule due to a reduction in preventive security measures. We illustrate this through an example, considering a single operation snapshot of the IEEE RTS case study presented in section 3.9.4. We examine demand period 96 (as shown in Table A-2), when the wind availability factor is 0.95 and demand is at its lowest level, totalling 1,273 MW and consider the second epoch of the case study, when 250 MW of wind generation has been installed at bus 3 giving a maximum harvestable wind output of 237.5 MW. We show the effect of corrective security provided by quadrature boosters, by studying how the dispatch schedule can be changed to reduce the engagement of out-of-merit units providing preventive security.

We first consider the case where the optimum unconstrained dispatch schedule is used and transmission line limits are ignored. In Figure B-1 we show generation (in green colour) and load levels (in red colour) at each bus. The post-fault power flows shown are for the case when line 6 is in outage. The demand is covered by the available wind generation, nuclear generators at buses 18 and 21 and the cheapest coal units. This dispatch pattern naturally leads to zero constraint costs but results in post-fault flows in lines 1, 2, 3 and 9 being above their installed capacity. For example, due to the unavailability of line 6, line 2 has to transport the bulk of the produced wind power resulting in a severe overload beyond its thermal capacity of 100 MW. Since the unconstrained dispatch schedule is infeasible under post-fault conditions, an alternative re-dispatch that remains robust under all credible contingencies is required.

The minimum cost dispatch schedule respecting transmission line limits and all credible N-1 contingencies is shown in Figure B-2. The post-fault power flows shown are for the case when line 6 is in outage. In order to avoid line-overloading, wind power has been significantly curtailed by 132 MW. In addition, the generator at bus 21 has been partially constrained off to ensure that the power exported through lines 1, 2, 3 and 9 are within the allowable limits. In turn, the arising generation shortfall is replaced by ramping up coal plant generation at bus 2 (G8), engaging an oil unit at bus 1 (G1) and a coal unit at bus 13 (G24). This out-of-merit re-dispatch results in a non-discounted operational cost of 30,248£/hour.

Finally, we consider the case where quadrature boosters have been installed on lines 1, 7, 8, 24 and 27 according to the optimal solution shown in section 3.9.4. The optimum quadrature booster settings when line 6 is in outage are as follows:

QB on line	Power Injection [MW]	QB angle
1	-8.84	1.78^{o}
7	150	-30^{o}
8	-150	30^{o}
24	150	-30^{o}
27	-150	30^{o}

Table B-1: Optimal post-fault QB settings.

As can be seen in Figure B-3, through the post-fault power flow control offered by FACTS devices, it is possible to re-direct a larger part of the generated wind power through lines 7, 24 and 27. As a result, the available wind power can be more fully accommodated and its curtailment is reduced to 32.5 MW. The generation shortfall is can now replaced solely by G8, foregoing the need to engage expensive out-of-merit plants G1 and G24, leading to a significantly reduced operational cost of 6,165£/hour. In summary, the need for preventive security is decreased due to the availability of corrective security measures that can re-direct post-fault power flows and alleviate potential overloads.

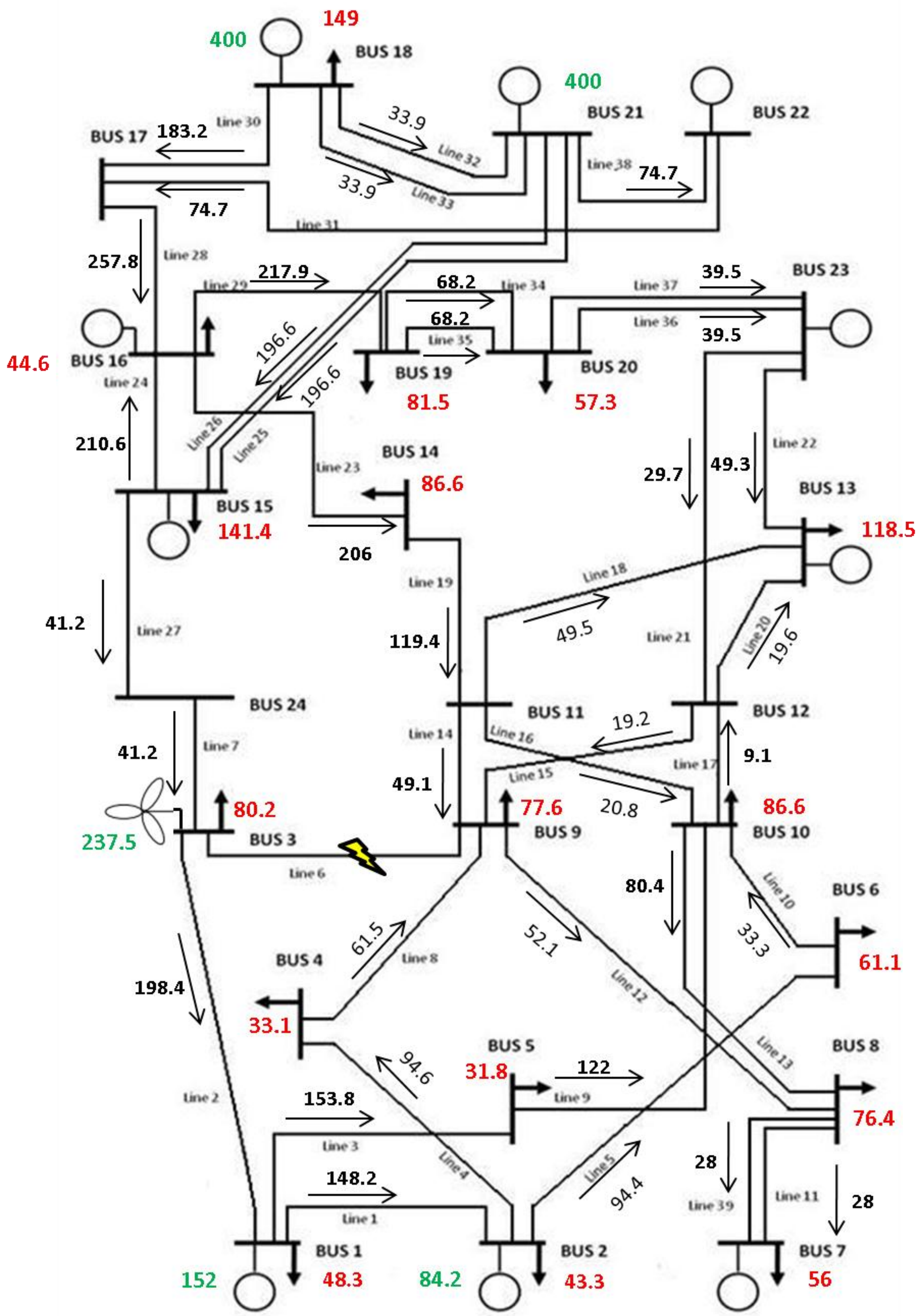


Figure B-1: Unconstrained generation dispatch and post-fault power flows (line 6 in outage). All quantities are in MW.

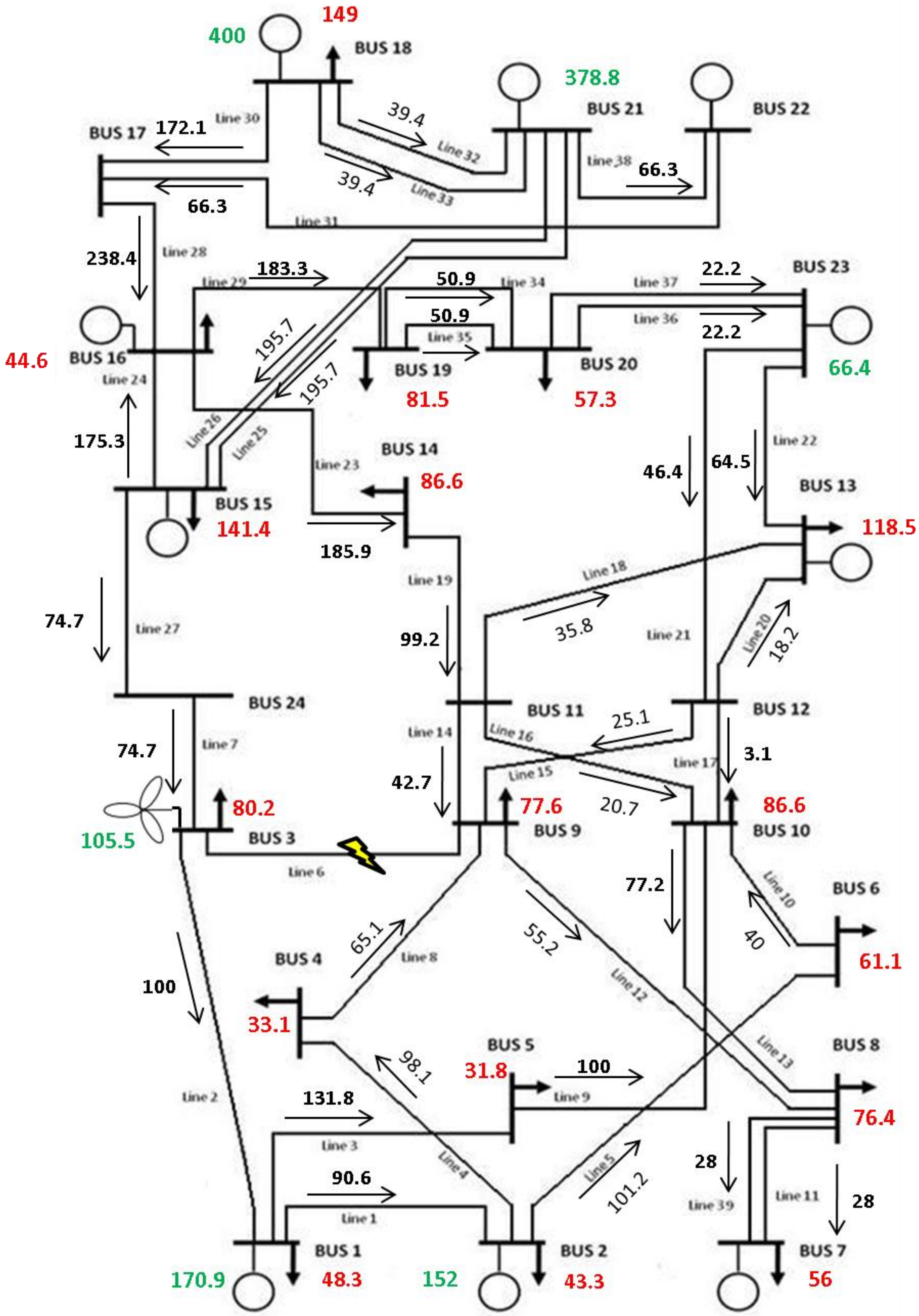


Figure B-2: Optimum generation dispatch and post-fault power flows (line 6 in outage) when no quadrature boosters have been installed. All quantities are in MW.

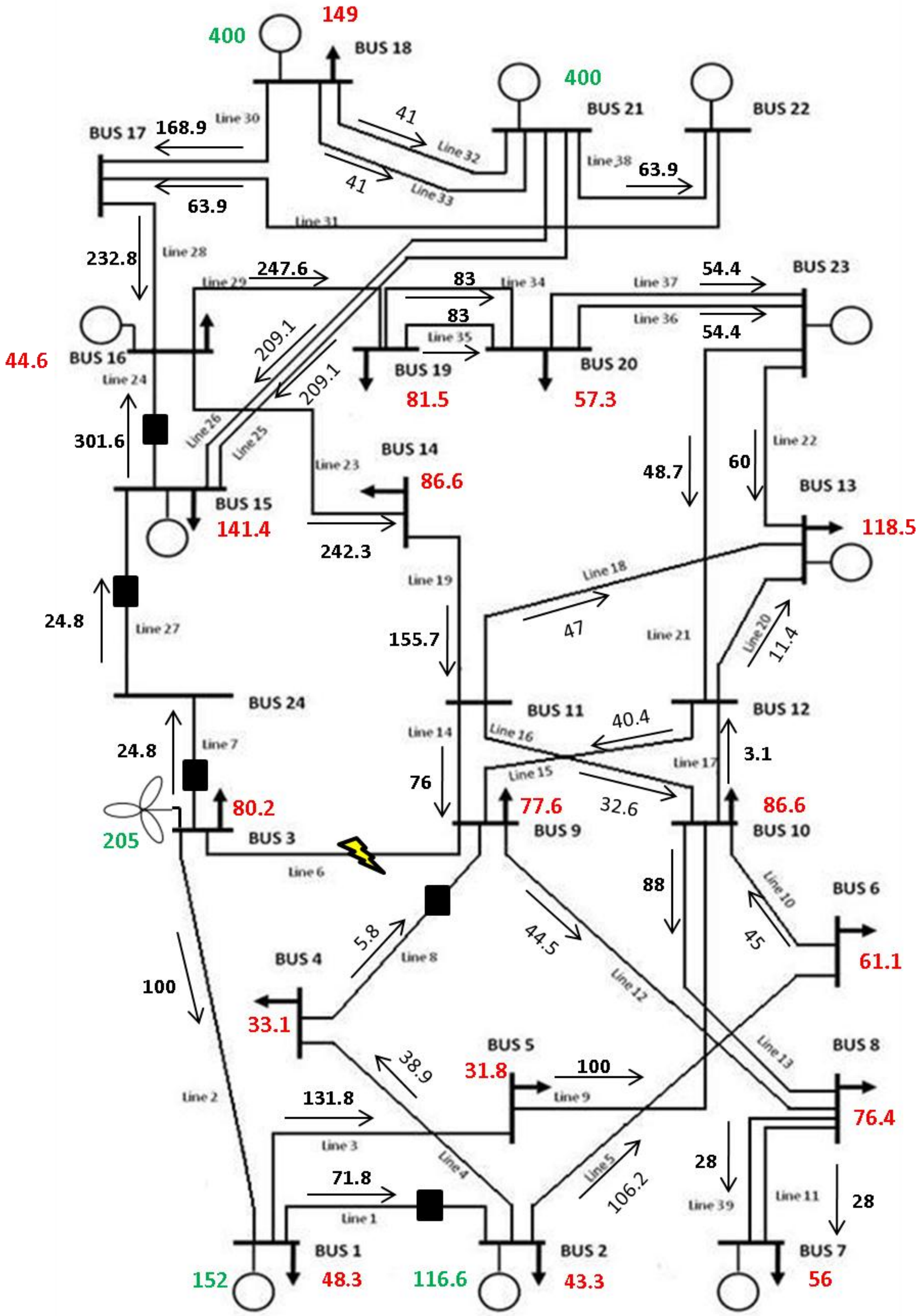


Figure B-3: Optimum generation dispatch and post-fault power flows (line 6 in outage) when five transmission lines are equipped with quadrature boosters. All quantities are in MW.

Appendix C: Optimal deterministic transmission expansion plans for the IEEE RTS case study

In this section we present the optimal deterministic plan for each scenario, along with the corresponding investment, constraint and system costs.

Line Id	D_1	D_2	D_3
1	200 (A)	-	+200
2	200 (B)	+200	
3	-	200 (A)	
4	-	200 (A)	
5	-	-	200 (A)
6	200 (B)	+200	
8	-	-	200 (A)
9	-	200 (A)	

Table C-1: Optimal investment plan for scenario 1.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (B)	+200	-
3	-	200 (A)	-
4	-	200 (A)	-
6	200 (B)	+200	-
8	-	-	200 (A)
9	-	200 (A)	-

Table C-2: Optimal investment plan for scenario 2.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (B)	+200	-
3	-	200 (A)	-
4	-	200 (A)	-
6	200 (B)	+200	-
9	-	200 (A)	-

Table C-3: Optimal investment plan for scenario 3.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (B)	+200	-
3	-	200 (A)	-
4	-	200 (A)	-
6	200 (B)	-	+200
8	-	-	200 (A)
9	-	200 (A)	-

Table C-4: Optimal investment plan for scenario 4.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (B)	+200	-
3	-	200 (A)	-
4	-	200 (A)	-
6	200 (B)	-	+200
9	-	200 (A)	-

TableC-5: Optimal investment plan for scenario 5.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (A)	-	-
3	-	200 (A)	-
4	-	200 (A)	-
6	200 (A)	-	-
9	-	200 (A)	-

Table C-6: Optimal investment plan for scenario 6.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (B)	-	+200
3	-	-	200 (A)
4	-	-	200 (A)
6	200 (B)	-	+200
9	-	-	200 (A)

Table C-7: Optimal investment plan for scenario 7.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (A)	-	-
6	200 (A)	-	-

Table C-8: Optimal investment plan for scenario 8.

Line Id	D_1	D_2	D_3
1	200 (A)	-	-
2	200 (A)	-	-
6	200 (A)	-	-

Table C-9: Optimal investment plan for scenario 9.

Line Id	D_1	D_2	D_3
1	-	400 (B)	-
2	-	-	200 (A)
3	-	-	200 (A)
4	-	200 (B)	+ 200
5	-	400 (C)	+ 400
8	-	200 (B)	+ 400
9	-	-	200 (A)
10	-	400 (C)	+ 400
16	-	-	200 (A)

Table C-10: Optimal investment plan for scenario 10.

Line Id	D_1	D_2	D_3
1	-	200 (A)	-
4	-	200 (A)	-
5	-	400 (B)	-
8	-	200 (B)	-
10	-	400 (B)	-

Table C-11: Optimal investment plan for scenario 11.

Line Id	D_1	D_2	D_3
1	-	200 (A)	-
4	-	200 (A)	-
5	-	400 (B)	-
8	-	200 (B)	-
10	-	400 (B)	-

Table C-12: Optimal investment plan for scenario 12.

Line Id	D_1	D_2	D_3
1	-	200 (A)	-
4	-	200 (A)	-
5	-	200 (B)	+200
8	-	200 (A)	-
10	-	200 (B)	+200

Table C-13: Optimal investment plan for scenario 13.

Line Id	D_1	D_2	D_3
1	-	200 (A)	-
4	-	200 (A)	-
5	-	200 (B)	+200
8	-	200 (A)	-
10	-	200 (B)	+200

Table C-14: Optimal investment plan for scenario 14.

Line Id	D_1	D_2	D_3
1	-	200 (A)	-
4	-	200 (A)	-
5	-	200 (B)	-
8	-	200 (A)	-
10	-	200 (B)	-

Table C-15: Optimal investment plan for scenario 15.

Line Id	D_1	D_2	D_3
1	-	-	200 (A)
4	-	-	200 (A)
5	-	-	400 (B)
8	-	-	200 (A)
10	-	-	400 (B)

Table C-16: Optimal investment plan for scenario 16.

Line Id	D_1	D_2	D_3
1	-	-	200 (A)
4	-	-	200 (A)
5	-	-	200 (A)
8	-	-	200 (A)
10	-	-	200 (A)

Table C-17: Optimal investment plan for scenario 17.

No investment is undertaken for scenario 18.

Following the above deterministic plans results in the following system costs for each scenario:

Scenario	Total Investment Cost	Total Constraint Costs	Total System Cost
1	298	95.9	393.9
2	269.8	66.4	336.2
3	254.9	38.9	293.8
4	268	53.4	321.4
5	253.1	26.0	279.1
6	206.6	24.8	231.4
7	218.7	28.8	247.5
8	131.9	61.0	192.9
9	131.9	20.8	152.7
10	293.6	75.9	369.5
11	151.7	176.2	327.9
12	151.7	99.1	250.8
13	147.9	144.0	291.9
14	147.9	66.9	66.8
15	124.3	41.7	166.0
16	88.4	67.4	155.8
17	72.5	42.2	114.7
18	0	42.6	42.6

Table C-18: System costs for deterministic plans. All values are in £m.

Appendix D: Stochastic transmission expansion planning with optimal QB placement

In this section we re-visit the case study presented in section 4.7 and showcase the optimal investment expansion plan when also considering investment in quadrature boosters. First we show the investment solution and resulting system costs under the non-flexible planner paradigm. This is followed by the optimal expansion strategy and resulting system costs when considering decision flexibility. It has been assumed that quadrature boosters can be installed with a negligible construction delay (i.e. $k_l^{QB} = 0$) due to the limited planning preparation involved in their deployment. All lines have been considered as potential candidates for a QB addition with an annuitized investment cost of $c_l^{QB} = 300{,}000$ £/year and operating limits $\psi_l^{\min} = -30^o$ and $\psi_l^{\max} = 30^o$.

D. 1 Non-flexible stochastic transmission expansion planning

The optimal expansion schedule is shown in Table D-1a and the corresponding system costs are shown in Table D-1b. In terms of first-stage decisions, when compared with the optimal solution is section 4.7.2, the corrective security offered by the QBs commissioned in the second stage allows the planner to defer investment in line 6 to the next epoch, leading to substantial investment cost savings. In subsequent epochs, lines that were upgraded with small capacity additions (200 MW provided by project A) are now equipped with QBs instead. This constitutes a cheaper solution and allows for the accommodation of the available wind resources by re-directing power over larger capacity corridors. Overall, the net benefit of QB investment over the horizon is £114.2m, stemming from reduced capital expenditure due to a more effective utilization of existing transmission assets.

Line Id	D_1	D_2	D_3	D_4
1	200 (A)	-	-	-
2	200 (B)	+ 200	-	-
3	-	QB	-	-
4	-	QB	-	-
5	-	200 (B)	-	-
6	-	200 (A) QB	-	-
8	-	-	QB	-
9	-	QB	-	-
10	-	400 (B)	-	-

Table D-1a: Optimal expansion schedule for the non-flexible decision framework when considering investment in quadrature boosters.

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	109.4	139.1	1.5	0	250	0	1.3	3.9	100.2	105.4	355.4
2	109.4	139.1	1.5	0	250	0	1.3	3.9	33.1	38.3	288.3
3	109.4	139.1	1.5	0	250	0	1.3	3.9	3.0	8.3	258.3
4	109.4	139.1	1.5	0	250	0	1.3	1.7	33.1	36.1	286.1
5	109.4	139.1	1.5	0	250	0	1.3	1.7	3.0	6.1	256.1
6	109.4	139.1	1.5	0	250	0	1.3	1.7	1.4	4.4	254.4
7	109.4	139.1	1.5	0	250	0	1.3	1.0	3.0	5.4	255.4
8	109.4	139.1	1.5	0	250	0	1.3	1.0	1.4	3.7	253.7
9	109.4	139.1	1.5	0	250	0	1.3	1.0	0.8	3.2	253.2
10	109.4	139.1	1.5	0	250	0	5.1	24.5	211.2	240.8	490.8
11	109.4	139.1	1.5	0	250	0	5.1	24.5	92.3	121.9	371.9
12	109.4	139.1	1.5	0	250	0	5.1	24.5	19.2	48.8	298.8
13	109.4	139.1	1.5	0	250	0	5.1	3.4	92.3	100.8	350.8
14	109.4	139.1	1.5	0	250	0	5.1	3.4	19.2	27.7	277.7
15	109.4	139.1	1.5	0	250	0	5.1	3.4	2.7	11.1	261.1
16	109.4	139.1	1.5	0	250	0	5.1	1.5	19.2	25.9	275.9
17	109.4	139.1	1.5	0	250	0	5.1	1.5	2.7	9.3	259.3
18	109.4	139.1	1.5	0	250	0	5.1	1.5	1.2	7.9	257.9
Expected					250					71.4	321.4

Table D-1b: Investment, constraint and system costs (£m) under each scenario when considering investment in quadrature boosters.

D.2 Flexible stochastic transmission expansion planning

The optimal expansion strategy is shown in Table D-2a and the corresponding system costs are shown in Table D-2b. Modelling decision flexibility in conjunction with the negligible construction delays of quadrature boosters leads to recourse actions tailored to each system state transition. As a result, the planner undertakes only the very necessary line reinforcement projects and responds to the unfolding uncertainty by deploying flow control devices. For example, under the low growth scenario 17, the second epoch investment ($D_{[3]}$) is complimented with the commissioning of three quadrature boosters in the last epoch ($D_{[26]}$) to accommodate new wind capacity in a just-in-time manner. Under scenario 18, deployment of QBs is not necessary in the last epoch. Although this instantaneous commissioning is not realistic, it reflects the fact that quadrature boosters constitute a versatile option in the planner's arsenal, allowing contingent adjustments according to the unfolding uncertainty. The modelling of decision flexibility is essential to maximize this versatility benefit. This is apparent in the £73.8m reduction of expected system costs when compared to the non-flexible expansion schedule.

Line Id	$D_{[1]}$	$D_{[2]}$	$D_{[3]}$	$D_{[4]}$	$D_{[5]}$	$D_{[7]}$	$D_{[8]}$
1	-	QB	200 (A)	200 (A)	-	-	-
2	200 (B)	+ 200	-	-	-	-	-
3	-	QB	-	-	-	QB	QB
4	-	QB	QB	-	-	200 (A)	-
5	-	-	400 (B)	QB	QB	-	-
6	-	200 (B)	-	+ 200	-	-	-
8	-	-	-	QB	-	QB	QB
9	-	QB	-	-	-	QB	QB
10	-	-	400 (B)	-	-	-	-

Line Id	$D_{[13]}$	$D_{[14]}$	$D_{[16]}$	$D_{[17]}$	$D_{[25]}$	$D_{[26]}$
1	-	-	-	-	-	-
2	-	-	-	-	-	-
3	-	-	-	-	QB	QB
4	-	-	-	-	-	-
5	-	-	QB	QB	-	-
6	-	-	-	-	-	-
8	QB	QB	QB	-	QB	QB
9	-	-	-	-	QB	QB
10	-	-	-	-	-	-

Table D-2a: Optimal expansion strategy for the flexible decision framework when considering investment in quadrature boosters. $D_{[m]}$ indicates the investment decision to be taken if the scenario tree node *m* materializes.

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	104.8	58.8	7.2	0.0	170.9	0.0	1.3	4.9	74.8	81.0	251.9
2	104.8	58.8	7.2	0.0	170.9	0.0	1.3	4.9	20.5	26.7	197.6
3	104.8	58.8	7.2	0.0	170.9	0.0	1.3	4.9	2.7	8.9	179.8
4	104.8	58.8	1.5	0.7	165.8	0.0	1.3	1.8	35.6	38.7	204.5
5	104.8	58.8	1.5	0.7	165.8	0.0	1.3	1.8	3.8	6.9	172.7
6	104.8	58.8	1.5	0.0	165.1	0.0	1.3	1.8	1.4	4.5	169.6
7	104.8	58.8	0.0	1.3	165.0	0.0	1.3	1.0	3.8	6.2	171.2
8	104.8	58.8	0.0	0.7	164.3	0.0	1.3	1.0	1.4	3.8	168.1
9	104.8	58.8	0.0	0.0	163.6	0.0	1.3	1.0	0.8	3.2	166.8
10	104.8	96.9	22.9	0.0	224.7	0.0	5.1	24.5	184.7	214.3	439.0
11	104.8	96.9	22.9	0.0	224.7	0.0	5.1	24.5	70.9	100.6	325.3
12	104.8	96.9	22.9	0.0	224.7	0.0	5.1	24.5	8.2	37.8	262.5
13	104.8	96.9	4.5	0.0	206.2	0.0	5.1	3.4	92.3	100.8	307.0
14	104.8	96.9	4.5	0.0	206.2	0.0	5.1	3.4	19.2	27.7	233.9
15	104.8	96.9	4.5	0.0	206.2	0.0	5.1	3.4	2.7	11.1	217.3
16	104.8	96.9	0.0	2.0	203.7	0.0	5.1	2.1	19.2	26.4	230.1
17	104.8	96.9	0.0	2.0	203.7	0.0	5.1	2.1	2.7	9.9	213.6
18	104.8	96.9	0.0	0.0	201.7	0.0	5.1	2.1	1.7	8.9	210.6
Expected					186.5					61.1	247.6

Table D-2b: Investment, constraint and system costs (£m) under each scenario when considering investment in quadrature boosters.

Appendix E: Risk-constrained transmission expansion planning solutions

E.1 Solution for C_β = £200m

Line Id	$D_{[1]}$	$D_{[2]}$	$D_{[3]}$	$D_{[4]}$	$D_{[5]}$	$D_{[6]}$	$D_{[7]}$	$D_{[8]}$	$D_{[9]}$
1	-	200 (A)	200 (B)	-	-	-	+ 200	+ 200	-
2	200 (B)	+ 200	-	-	-	-	-	-	-
3	-	200 (A)	-	-	-	-	200 (A)	200 (A)	-
4	-	200 (A)	200 (B)	-	-	-	+ 200	+ 200	-
5	-	-	400 (C)	200 (A)	-	-	+ 400	+ 400	-
6	-	200 (B)	-	-	-	-	-	-	-
8	-	-	200 (B)	200 (A)	-	-	+ 200	+ 200	-
9	-	200 (A)	-	-	-	-	+ 400	+ 400	-
10	-	-	400 (C)	-	-	-	-	-	-

Table E-1a: Optimal expansion strategy for the risk-constrained flexible decision framework.

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	104.8	128.0	43.2	0.0	276.0	0.0	65.2	17.0	70.3	152.5	428.5
2	104.8	128.0	43.2	0.0	276.0	0.0	65.2	17.0	16.4	98.6	374.6
3	104.8	128.0	43.2	0.0	276.0	0.0	65.2	17.0	1.2	83.4	359.4
4	104.8	128.0	0.0	0.0	232.8	0.0	65.2	3.5	57.3	126.0	358.8
5	104.8	128.0	0.0	0.0	232.8	0.0	65.2	3.5	13.3	82.0	314.8
6	104.8	128.0	0.0	0.0	232.8	0.0	65.2	3.5	2.8	71.5	304.3
7	104.8	128.0	0.0	0.0	232.8	0.0	65.2	1.0	13.3	79.6	312.4
8	104.8	128.0	0.0	0.0	232.8	0.0	65.2	1.0	2.8	69.0	301.8
9	104.8	128.0	0.0	0.0	232.8	0.0	65.2	1.0	0.8	67.1	299.9
10	104.8	214.7	40.8	0.0	360.3	0.0	17.8	44.9	21.7	84.3	444.6
11	104.8	214.7	40.8	0.0	360.3	0.0	17.8	44.9	2.7	65.3	425.6
12	104.8	214.7	40.8	0.0	360.3	0.0	17.8	44.9	2.6	65.3	425.6
13	104.8	214.7	40.8	0.0	360.3	0.0	17.8	11.4	90.1	119.3	479.6
14	104.8	214.7	40.8	0.0	360.3	0.0	17.8	11.4	2.6	31.8	392.1
15	104.8	214.7	40.8	0.0	360.3	0.0	17.8	11.4	2.5	31.7	392.0
16	104.8	214.7	0.0	0.0	319.5	0.0	17.8	1.5	35.2	54.5	374.0
17	104.8	214.7	0.0	0.0	319.5	0.0	17.8	1.5	8.9	28.3	347.8
18	104.8	214.7	0.0	0.0	319.5	0.0	17.8	1.5	1.2	20.5	340.0
Expected					292.7					87.2	379.9

Table E-1b: Investment, constraint and system costs (£m) under each scenario.

E.2 Solution for Cβ = £150m

Line Id	$D_{[1]}$	$D_{[2]}$	$D_{[3]}$	$D_{[4]}$	$D_{[5]}$	$D_{[6]}$	$D_{[7]}$	$D_{[8]}$	$D_{[9]}$
1	200 (B)	-	-	-	-	-	+ 200	+ 200	-
2	200 (B)	+ 200	-	-	-	-	-	-	-
3	-	200 (A)	-	-	-	-	200 (A)	200 (A)	-
4	-	200 (A)	200 (B)	-	-	-	+ 200	+ 200	-
5	-	-	400 (C)	200 (B)	-	-	+ 400	+ 400	-
6	200 (B)	+ 200	-	-	-	-	-	-	-
8	-	-	-	200 (B)	-	-	+ 200	-	-
9	-	200 (A)	-	-	-	-	200 (A)	200 (A)	-
10	-	-	400 (C)	-	-	-	+ 400	+ 400	-

Table E-2a: Optimal expansion strategy for the risk-constrained flexible decision framework.

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	169.6	86.6	43.2	0.0	299.4	0.0	8.7	17.0	70.3	96.0	395.3
2	169.6	86.6	43.2	0.0	299.4	0.0	8.7	17.0	16.4	42.1	341.5
3	169.6	86.6	43.2	0.0	299.4	0.0	8.7	17.0	1.2	26.9	326.2
4	169.6	86.6	0.0	0.0	256.2	0.0	8.7	3.5	57.3	69.5	325.7
5	169.6	86.6	0.0	0.0	256.2	0.0	8.7	3.5	13.3	25.5	281.7
6	169.6	86.6	0.0	0.0	256.2	0.0	8.7	3.5	2.8	15.0	271.2
7	169.6	86.6	0.0	0.0	256.2	0.0	8.7	1.0	13.3	23.0	279.2
8	169.6	86.6	0.0	0.0	256.2	0.0	8.7	1.0	2.8	12.5	268.7
9	169.6	86.6	0.0	0.0	256.2	0.0	8.7	1.0	0.8	10.5	266.7
10	169.6	210.8	40.8	0.0	421.2	0.0	17.8	44.9	21.7	84.4	505.5
11	169.6	210.8	40.8	0.0	421.2	0.0	17.8	44.9	2.7	65.4	486.5
12	169.6	210.8	40.8	0.0	421.2	0.0	17.8	44.9	2.6	65.3	486.5
13	169.6	210.8	38.6	0.0	419.0	0.0	17.8	11.4	11.8	41.0	460.0
14	169.6	210.8	38.6	0.0	419.0	0.0	17.8	11.4	2.7	31.9	450.9
15	169.6	210.8	38.6	0.0	419.0	0.0	17.8	11.4	2.5	31.7	450.7
16	169.6	210.8	0.0	0.0	380.4	0.0	17.8	1.5	35.2	54.5	434.9
17	169.6	210.8	0.0	0.0	380.4	0.0	17.8	1.5	8.9	28.2	408.7
18	169.6	210.8	0.0	0.0	380.4	0.0	17.8	1.5	1.2	20.5	400.9
Expected					330.9					51.8	382.7

Table E-2b: Investment, constraint and system costs (£m) under each scenario.

E.3 Solution for Cβ = £50m

Line Id	$D_{[1]}$	$D_{[2]}$	$D_{[3]}$	$D_{[4]}$	$D_{[5]}$	$D_{[6]}$	$D_{[7]}$	$D_{[8]}$	$D_{[9]}$
1	200 (B)	-	-	+ 200	+ 200	-	+ 200	+ 200	-
2	400 (C)	-	-	+ 400	-	-	-	-	-
3	-	200 (A)	200 (A)	-	-	-	-	-	-
4	-	200 (A)	200 (B)	-	-	-	+ 200	+200	-
5	-	-	400 (C)	200 (A)	-	-	+ 400	+ 400	-
6	400 (C)	-	-	+ 400	-	-	-	-	-
8	-	-	200 (B)	200 (A)	-	-	+ 200	+ 200	-
9	-	200 (A)	200 (A)	-	-	-	-	-	-
10	-	-	400 (C)	-	-	-	+ 400	+ 400	-
16	-	-	-	-	-	-	200 (A)	-	-

Table E-3a: Optimal expansion strategy for the risk-constrained flexible decision framework with C_{β}=£95m.

Scenario	Investment Cost (IC)				Total IC	Constraint Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	278.7	74.7	57.3	0.0	410.7	0.0	8.7	17.0	6.7	32.3	443.0
2	278.7	74.7	57.3	0.0	410.7	0.0	8.7	17.0	1.3	26.9	437.6
3	278.7	74.7	57.3	0.0	410.7	0.0	8.7	17.0	1.2	26.8	437.5
4	278.7	74.7	15.0	0.0	368.4	0.0	8.7	3.5	40.8	53.0	421.4
5	278.7	74.7	15.0	0.0	368.4	0.0	8.7	3.5	10.5	22.7	391.1
6	278.7	74.7	15.0	0.0	368.4	0.0	8.7	3.5	2.7	14.9	383.3
7	278.7	74.7	0.0	0.0	353.4	0.0	8.7	1.0	13.3	23.0	376.4
8	278.7	74.7	0.0	0.0	353.4	0.0	8.7	1.0	2.8	12.5	365.9
9	278.7	74.7	0.0	0.0	353.4	0.0	8.7	1.0	0.8	10.5	363.9
10	278.7	253.9	22.6	0.0	555.2	0.0	17.8	10.9	12.7	41.4	596.6
11	278.7	253.9	22.6	0.0	555.2	0.0	17.8	10.9	2.5	31.1	586.3
12	278.7	253.9	22.6	0.0	555.2	0.0	17.8	10.9	2.6	31.2	586.4
13	278.7	253.9	15.7	0.0	548.3	0.0	17.8	3.2	2.7	23.7	572.0
14	278.7	253.9	15.7	0.0	548.3	0.0	17.8	3.2	2.6	23.6	571.9
15	278.7	253.9	15.7	0.0	548.3	0.0	17.8	3.2	2.5	23.5	571.8
16	278.7	253.9	0.0	0.0	532.6	0.0	17.8	1.5	8.5	27.8	560.4
17	278.7	253.9	0.0	0.0	532.6	0.0	17.8	1.5	2.5	21.8	554.4
18	278.7	253.9	0.0	0.0	532.6	0.0	17.8	1.5	1.2	20.5	553.1
Expected					450.6					27.3	477.9

Table E-3b: Investment, constraint and system costs (£m) under each scenario.

E.4 Solution for Cβ = £20m

Line Id	$D_{[1]}$	$D_{[2]}$	$D_{[3]}$	$D_{[4]}$	$D_{[5]}$	$D_{[6]}$	$D_{[7]}$	$D_{[8]}$	$D_{[9]}$
1	200 (B)	-	+ 200	+ 200	-	-	-	-	-
2	400 (C)	-	-	+ 400	+ 400	-	-	-	-
3	-	200 (A)	200 (A)	-	-	-	-	-	-
4	200 (B)	-	+ 200	-	-	-	-	-	-
5	-	200 (A)	800 (C)	-	-	-	-	-	-
6	400 (C)	-	-	+ 400	+ 400	-	-	-	-
8	-	-	200 (B)	200 (A)	200 (A)	-	+ 200	-	-
9	-	200 (A)	200 (A)	-	-	-	-	-	-
10	-	-	800 (C)	-	-	-	-	-	-
14	-	-	-	200 (A)	-	-	-	-	-
16	-	-	-	-	-	-	200 (A)	-	-
25	-	-	-	-	-	-	200 (A)	-	-
26	-	-	-	-	-	-	200 (A)	-	-

Table E-4a: Optimal expansion strategy for the risk-constrained flexible decision framework

Scenario	Investment Cost (IC)				Total IC	Constraints Cost (CC)				Total CC	Total System Cost
	Stage 1	Stage 2	Stage 3	Stage 4		Stage 1	Stage 2	Stage 3	Stage 4		
1	341.1	91.5	36.0	0.0	468.6	0.0	8.7	6.1	3.4	18.2	486.8
2	341.1	91.5	36.0	0.0	468.6	0.0	8.7	6.1	1.3	16.0	484.6
3	341.1	91.5	36.0	0.0	468.6	0.0	8.7	6.1	1.2	16.0	484.6
4	341.1	91.5	28.8	0.0	461.4	0.0	8.7	1.8	2.1	12.5	473.9
5	341.1	91.5	28.8	0.0	461.4	0.0	8.7	1.8	1.2	11.6	473.0
6	341.1	91.5	28.8	0.0	461.4	0.0	8.7	1.8	1.3	11.7	473.1
7	341.1	91.5	0.0	0.0	432.6	0.0	8.7	1.0	4.8	14.5	447.1
8	341.1	91.5	0.0	0.0	432.6	0.0	8.7	1.0	1.4	11.1	443.7
9	341.1	91.5	0.0	0.0	432.6	0.0	8.7	1.0	0.8	10.5	443.1
10	341.1	236.4	47.4	0.0	624.9	0.0	5.1	3.4	10.7	19.2	644.1
11	341.1	236.4	47.4	0.0	624.9	0.0	5.1	3.4	0.1	8.6	633.5
12	341.1	236.4	47.4	0.0	624.9	0.0	5.1	3.4	0.0	8.5	633.4
13	341.1	236.4	0.0	0.0	577.5	0.0	5.1	3.2	11.8	20.1	597.6
14	341.1	236.4	0.0	0.0	577.5	0.0	5.1	3.2	2.7	11.0	588.5
15	341.1	236.4	0.0	0.0	577.5	0.0	5.1	3.2	2.5	10.8	588.3
16	341.1	236.4	0.0	0.0	577.5	0.0	5.1	1.5	2.7	9.3	586.8
17	341.1	236.4	0.0	0.0	577.5	0.0	5.1	1.5	2.5	9.1	586.6
18	341.1	236.4	0.0	0.0	577.5	0.0	5.1	1.5	1.2	7.9	585.4
Expected					515.2					14.0	529.2

Table E-4b: Investment, constraint and system costs (£m) under each scenario.

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