Chapter 1

Project

1.1 Mathematical Model

1.1.1 Private investor perspective

Optimization problem, maximize annuitized revenue:

$$\max_{\Omega} \sum_{t \in \mathcal{T}} \{ \, E_{sell}\left(r_d(t), t\right) - E_{buy}\left(r_c(t), t\right) - \left(AIC(C) + MC(C) + TF\right) \} \tag{1.1}$$

Where:
E_{sell} is the energy sold at the spot price.
E_{buy} is the energy bought at the spot price.
The annuitized investment cost and maintenance cost are defined as:

$$AIC = \frac{IC}{A_{t,r}}, \text{ where } A_{t,r} = \frac{1 - \frac{1}{(1+r)^t}}{r} \tag{1.2}$$

$$MC = 5\% \times AIC \tag{1.3}$$

Where
IC is the investment cost (in USD/MWh)
t is the number of years (15 years)
r is the annual interest rate (5%)
The optimization set:

$$\Omega := \{r_c(t), r_d(t), s(t), C\} \tag{1.4}$$

Are the charge rate, discharge rate, storage level and battery capacity respectively

$$\mathcal{T} := \{1, \ldots, T\} \text{ with index } t \tag{1.5}$$

Finally, TF are the fees for using the transmission network, which is a fixed value that depends on the maximum power of the battery. They are set by the ministry of energy and the current value is around $3 USD/kW.Month$

The constraints of this optimization problem are described next.
The storage level $s(t) \, \forall t \geq 2$ is:

$$s(t) = s(t-1) + \left(\eta_c r_c(t) - r_d(t)/\eta_d\right) \Delta t \tag{1.6}$$

This is, the storage level at the next step is equal to the charge minus the discharge considering the efficiency of the technology, in this case 94%.

Furthermore, the problem is bounded by the following equations:

$$0 \leq r_c(t) \leq R_c \tag{1.7}$$
$$0 \leq r_d(t) \leq R_d \tag{1.8}$$
$$0 \leq s(t) \leq C \tag{1.9}$$

1.1.2 Enhanced Frequency Response

When including EFR into the problem, the objective function is modified by adding the following term:

$$EFR_{Payment} \times P_{EFR} \times h \tag{1.10}$$

Where: $EFR_{Payment}$ is the payment the facility will receive for being available to provide the EFR service. According to National Grid last tender [1], the average value of the availability price was $12USD/MW/h$
P_{EFR} is another variable to optimize which is the amount of power the facility will provide for the EFR service.
h is the amount of hours a year the service will be provided. It was assumed 23 hours a day in order to leave a margin for maintenance.

Finally the bounded equations are modified:

$$E_{EFR} \leq s(t) \leq C - E_{EFR} \tag{1.11}$$

Where E_{EFR} is the amount of energy the battery needs in order to provide the service. Note that it is in both sides of the inequation, in order to inject or absorb power if needed. The time the service has to be provided is at least 15 minutes. Then:

$$E_{EFR} = 15/60 * P_{EFR} \tag{1.12}$$

1.1.3 Government Perspective

For doing the analysis from the government perspective we introduce some new variables. First we introduce the set of different nodes $\mathcal{N} := \{1, \ldots, N\}$. Furthermore, the optimization set Ω is now:

$$\Omega := \{p^n(t), r_c^n(t), r_d^n(t), s^n(t), k^n, \delta^n(t)\} \tag{1.13}$$

Where every variable now identifies a different node in the network, as well as the inclusion of $p^n(t)$, production of conventional generation, k^n, storage allocation and $\delta^n(t)$ voltage angle at node n.

The optimization problem is now a minimization of the production costs plus the battery investment. This is:

$$\min_{\Omega} \sum_{t \in \mathcal{T}} \left\{ \sum_{n \in \mathcal{N}} C^n(t) p^n(t) + \sum_{n \in \mathcal{N}} + (AIC(C) + MC(C) + TF) \right\} \tag{1.14}$$

Besides equations 1.6, 1.7, 1.8 and 1.9 we include the following into the constrains of the problem:

$$P^n_{\min} \leq p^n(t) \leq P^n_{\max} \tag{1.15}$$
$$-RR^n \leq p^n(t) - p^n(t-1) \leq RR^n \tag{1.16}$$

Where the first represents lower and upper limits and the latter the ramp rates of conventional generation.

At every time instant and every node, we verify the power flow equations. This is, demand plus charge of batteries and inflows must be equal to discharge of batteries, outflows, plus wind and conventional generation.

$$D^n(t) + \sum_{m \in \Theta_n} B^{nm} \left(\delta^n(t) - \delta^m(t)\right) + r_c^n(t) = W^n(t) + p^n(t) + r_d^n(t) \tag{1.17}$$

Where we denote the set Θ_n as the nodes m connected to bus n. Then B^{nm} represents the line susceptance between node n and m.

We also introduce transmission lines limits, such that:

$$-TC_{\max}^{nm} \leq B^{nm} \left(\delta^n(t) - \delta^m(t)\right) \leq TC_{max}^{nm} \tag{1.18}$$

Finally, the boundaries of the voltage angles and setting the slack bus:

$$-\pi \leq \delta^n(t) \leq \pi \tag{1.19}$$
$$\delta^{n=1}(t) = 0 \tag{1.20}$$

Bibliography

[1] National grid efr. https://www.nationalgrideso.com/balancing-services/frequency-response-services/enhanced-frequency-response-efr/. Accessed: 2019-07-15.