# Title for paper submitted to International Journal of Scientific and Research Publications

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***Abstract*-** The Quantitative finance world has stepped up well since its last centuries beginnings. Since the beginning with Black’s formula in 1973, financial mathematicians, engineers, academics and practitioners have contributed to building models that more realistically identify the stochastic behavior of prices in the financial markets. Such behavior has changed drastically over the decades and since the times where volatility was assumed to be constant over time. The presence of the equity skew and volatility smiles, moving from a flatter, simpler kind, suggested the utilization of a new way of thinking, with volatility not being fixed anymore, but following a stochastic process.

This research paper wants not only to study, but also to compare the current trends in identifying forward looking implied volatility, starting with the Dupire model – fixing volatility over time and treating as a constant; the Heston model – volatility following a stochastic process with a mean reverting structure; the more modern SABR models – stochastic alpha, beta and rho with complex structures that have many variants according to specific situations. Follows an in-depth analysis and plotting of such models that will serve our understanding of their accuracy and intricacy with respect to calculating market implied volatility.

1. Introduction

T

his section is here to introduce the topics confronted by outlining the major discussions, divided by chronological order. Section 1 will do something, section 2 will follow with something else, etcetera.

Identify the constructs of a Journal – Essentially a journal consists of five major sections. The number of pages may vary depending upon the topic of research work but generally comprises up to 5 to 7 pages. These are:

1. Abstract
2. Introduction
3. Research Elaborations
4. Results or Finding
5. Conclusions

**In Introduction you can mention the introduction about your research.**

1. LITERAURE REVIEW

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3. Attend conferences, workshops and symposiums on the same fields or on related counterparts.
4. Understand the scientific terms and jargon related to your research work.
5. THEORY: A HISTORY OF VOLATILITY, FROM CONSTANT TO STOCHASTIC

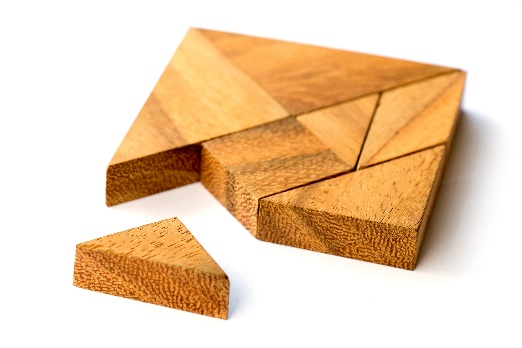
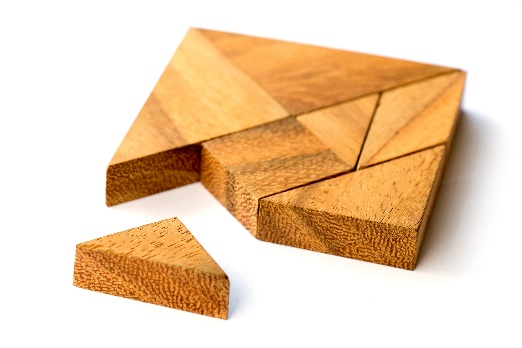
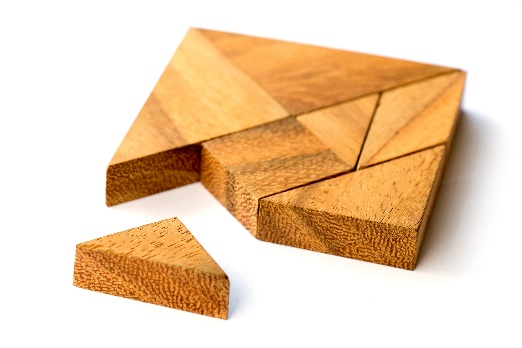
## The Black Model

In the Black-Scholes model, the premium for a call option with maturity and evaluated at time is given by the following model:

where is the asset price, is the strike price, is a constant annual risk free-rate and is the Gaussian cumulative distribution function. Now, the model assumes that changes in the underlying asset prices satisfy the following stochastic differential equation.

And are all constants, ant this is way the Black-Scholes model for pricing options is the most straight forward, easy to compute and reliable up to a certain extent. In fact, the requiring assumptions have led practitioners calculate implied volatility remarkably well up until the eighties, with a stable degree of accuracy with respect to the actual market. Following the assumptions, returns - meaning - follow a lognormal distribution with no fat nor heavy tails. Holding everything equal, we can retrieve the value of standard deviation of asset prices by simply reversing the equation and solving for . As shown below, we have that now sigma is a linear function of constants:

These assumptions worked well because of a combination of both the way that the market was behaving up to a market crash in 1987 (Black Monday) and the reliability that professionals in the field were giving to the model. As a matter of fact, professionals were not even considering the idea of dealing really with fat tailed distributions instead of Gaussians when modelling derivatives. Historically, credit to the model was mainly given to how the market volatilities were moving, and with a good reason. Here we can see the original behavior of volatility skews, which at the time, presented the famous smile shaped behavior, a long gone characteristic today.



With low computational requirements, ease of replication and application to the real financial world, the Black Scholes model has been the foundation of the entire field of mathematical finance. Having said that, as volatilities in the market started having a different, less predictable behavior, the need of something new that would integrate such new trends was needed.

## Constant Elasticity of Variance and Dupire’s Local Volatility

With the need of a more sophisticated model that would instantiate from the simpler and overused Black-Scholes-Merton model, in 1994 Dupire, alongside Derman and Kani, brought in the field a real first academic step forward to analyzing new volatility patterns. Assuming only minimal changes to the original model, they proposed a replacement of the constant with a deterministic function of time and prices , hence coming to the following stochastic differential equation:

with and r bring a constant risk-free rate. With the first paper being released, a series of debates arose to identify the strength and durability of the local volatility model. First and foremost, doubts were around the idea that there can be a single equation to match the implied volatility surface and therefore fit well for each volatility smile.

Firstly, the solution was found by Derman and Kani shortly after the release of the first paper by Dupire in 1994. They showed that the solution existed by constructing an implied binomial tree, where the local volatility is calculated at each node in time and calibrated across strikes and expirations to stock and option market data. Furthermore, Dupire then solved the entire calibration question by providing the Dupire equation. Through this discovery, the entire field of mathematical finance stepped up from calculating implied volatility straight and only from option prices to a traceable, calibrated expression for local variance. From the following Dupire equation, we solve for :

The derivation of the formula can be found either by using a probabilistic approach or by using Fokker-Plank equation. Again, the most important benefit we have when pricing using Dupire’s Local Volatility model with respect to Black-Scholes is a greater precision in matching implied skews for all strikes in a market with no smiles. Now, issues with this model arise with implied variance for longer tenors. Whereas for short term calibration, smiles are registered fine by the model, with longer maturities the effect of a smile continues which is highly unrealistic in a real-world situation. Longer maturities tend to have a flatter skew, which goes against short term scenarios. The necessity of a process that would flatten the curve enough for longer maturities to behave in a more realistic manner was very much felt. As stated by Hagan a few years after in the famous 2002 paper “Managing Smile Risk”, due to this contradiction between model and market, delta and vega hedges derived from the local volatility model can be unstable and may perform worse than naive Black-Scholes’ hedges.

## Stochastic Volatility and the Heston Model

With the rising concern of proving a good statistical match with volatility surfaces in option prices, the Heston model came in to provide the financial community with a variant that for the first time treated the volatility as a stochastic process, in contrast with the previous deterministic (Dupire) and constant (Black Scholes) predecessors. The asset price obeys a diffusion process:

where is the instantaneous expected rate of return, is the volatility of volatility , is the long term mean of the variance, is a positive constant indicating speed of mean reversion and and are Brownian motions with being their correlation, often called ‘leverage effect’. The later imposed Feller classification implies the following condition: . This is a necessary constraint in order to have a strictly positive variance .

## SABR

There are numbers of software available which

## Beyond SABR, Stochastic Local Volatility and mixed models

There are numbers of software available which

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1. CONCLUSION

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

Appendix

Appendixes, if needed, appear before the acknowledgment.

Acknowledgment

The preferred spelling of the word “acknowledgment” in American English is without an “e” after the “g.” Use the singular heading even if you have many acknowledgments.

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