In [1]:

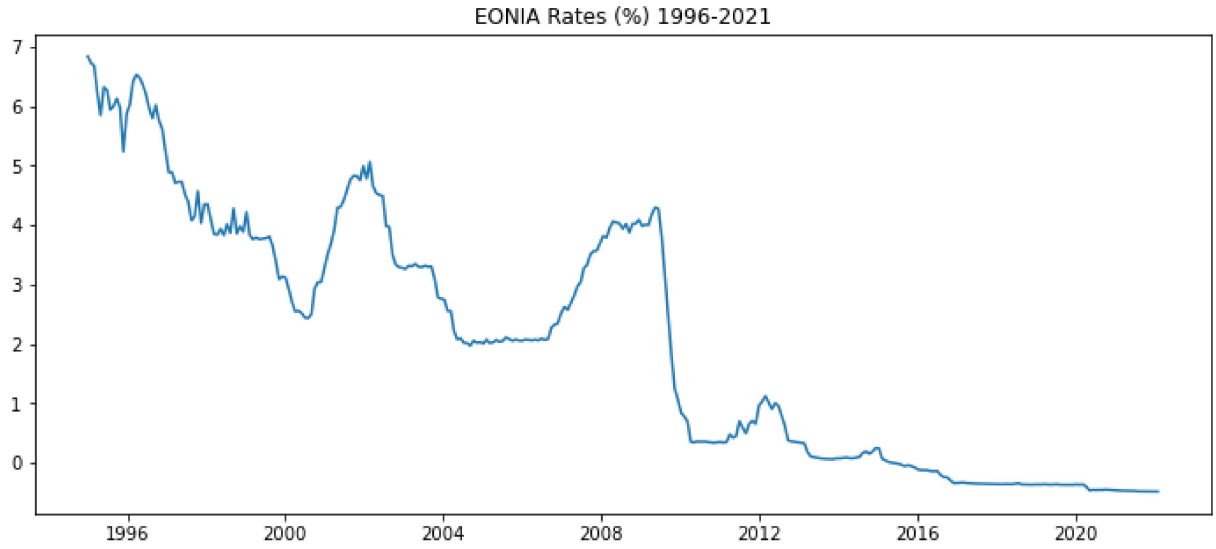
```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np
import pandas as pd
import time
import datetime
import math
import QuantLib as ql
from scipy.optimize import minimize

from initialize import *
from plotting import *
from SABR import *
from Heston import *
from mixedSABR import *
```

In [2]:

```
# Spot rates table and chart (EONIA)

rates = [-0.575, -0.557, -0.549, -0.529, -0.494]
tenors = [.25, 1, 3, 6, 12]

spot_rates = pd.DataFrame({"Tenors": tenors, "Spot Rate": rates})
spot_rates.set_index('Tenors')

display(spot_rates)

fig = plt.figure(figsize=(12,5))
eonia_dates = [datetime.date(1994, 12, 31) + datetime.timedelta(days=30*n) for n in
plt.plot(eonia_dates, eonia_rates['value'])
plt.title("EONIA Rates (%) 1996-2021")
```

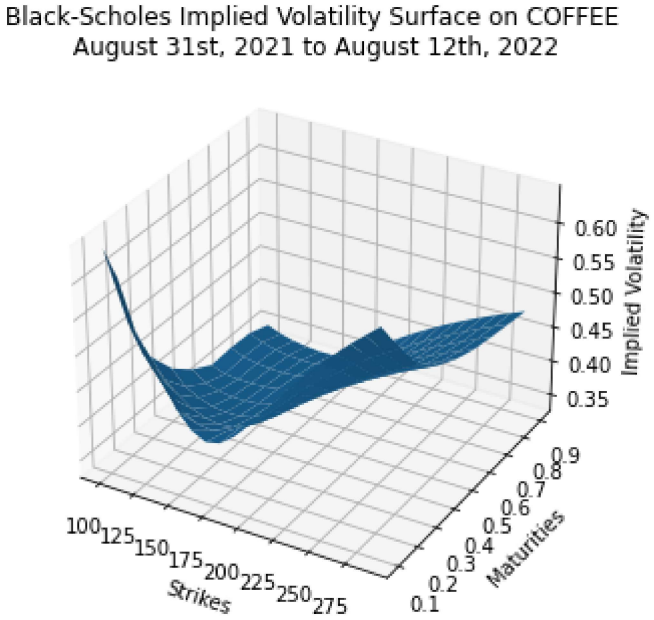
	Tenors	Spot Rate
0	0.25	-0.575
1	1.00	-0.557
2	3.00	-0.549
3	6.00	-0.529
4	12.00	-0.494

Out[2]: Text(0.5, 1.0, 'EONIA Rates (%) 1996-2021')



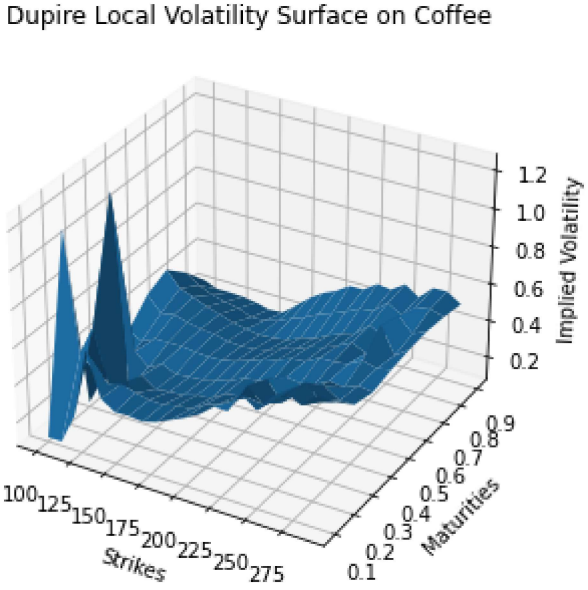
In [3]:

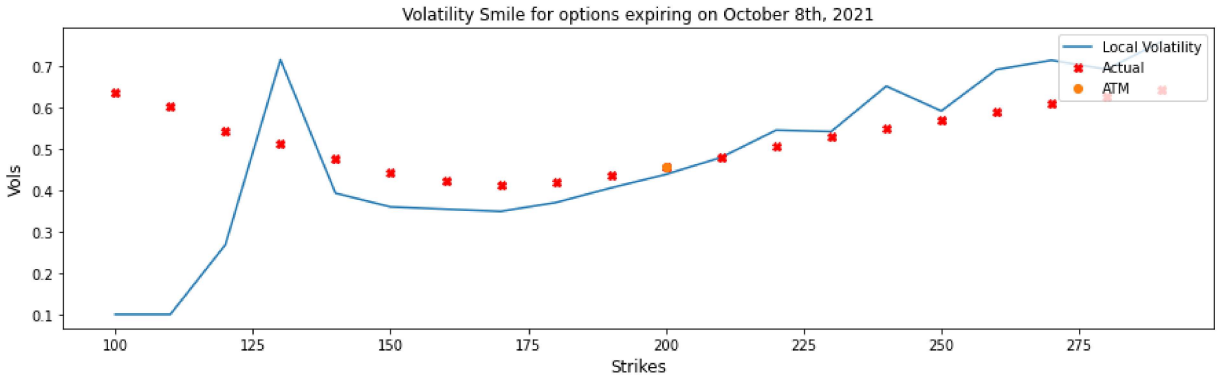
```
# BLACK VOLATILITY SURFACE

title = "Black-Scholes Implied Volatility Surface on {}\n {} to {}".format(data, tod
plot_vol_surface(vol_surface=black_var_surface, plot_strikes=strikes, funct='blackVo
```

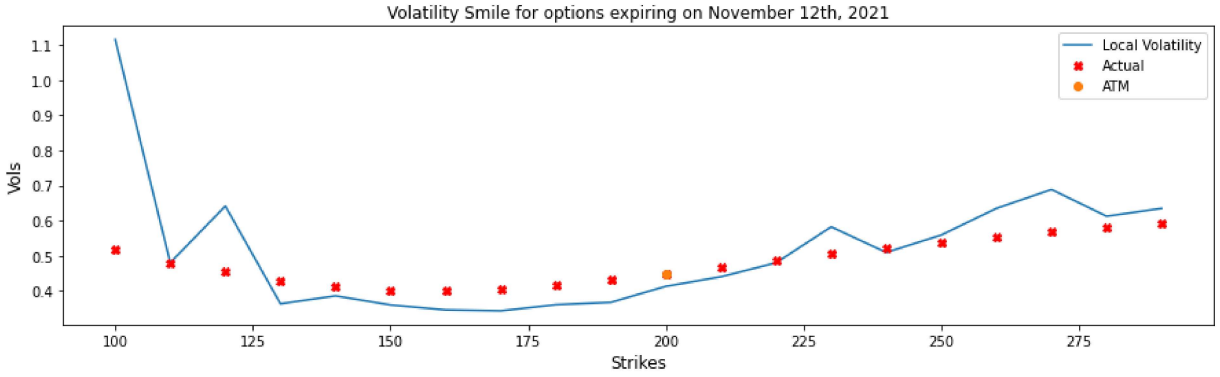


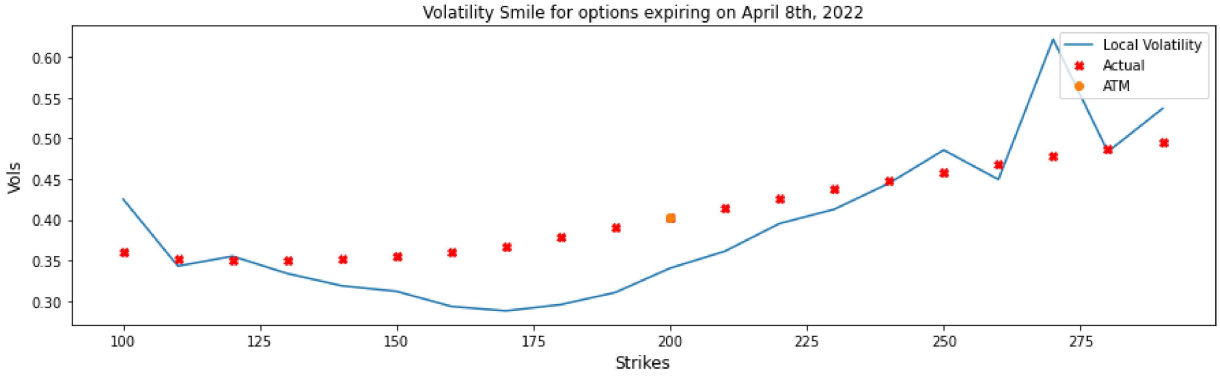
In [4]:

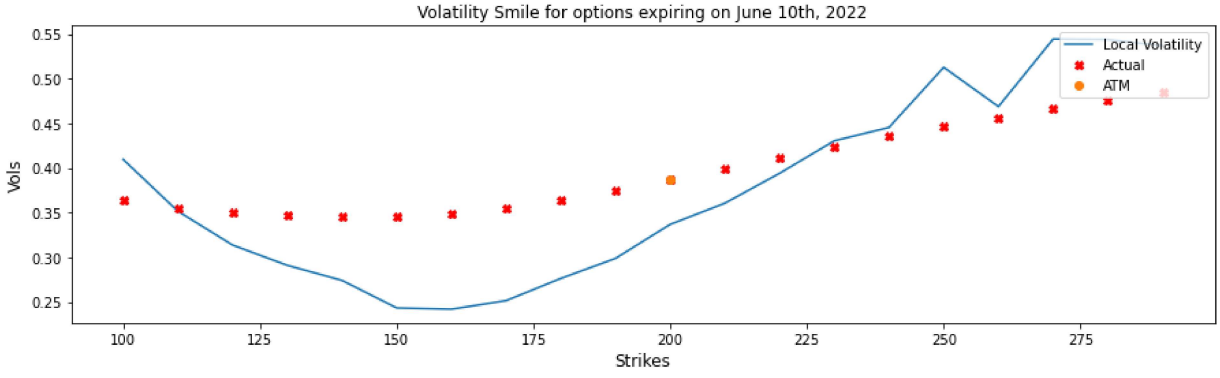
```
#DUPIRE LOCAL VOLATILITY SURFACE (NOT PLOTTABLE)

black_var_surface.setInterpolation("bicubic")
local_vol_handle = ql.BlackVolTermStructureHandle(black_var_surface)

# local_vol_surface = ql.LocalVolSurface(local_vol_handle, flat_ts, dividend_ts, spo
local_vol_surface = ql.NoExceptLocalVolSurface(local_vol_handle, flat_ts, dividend_t

# Plot the Dupire surface ...
local_vol_surface.enableExtrapolation()
plot_vol_surface(local_vol_surface, funct='localVol', title="Dupire Local Volatility

smiles_comparison(local_models=[local_vol_surface], black_volatility=False)
```

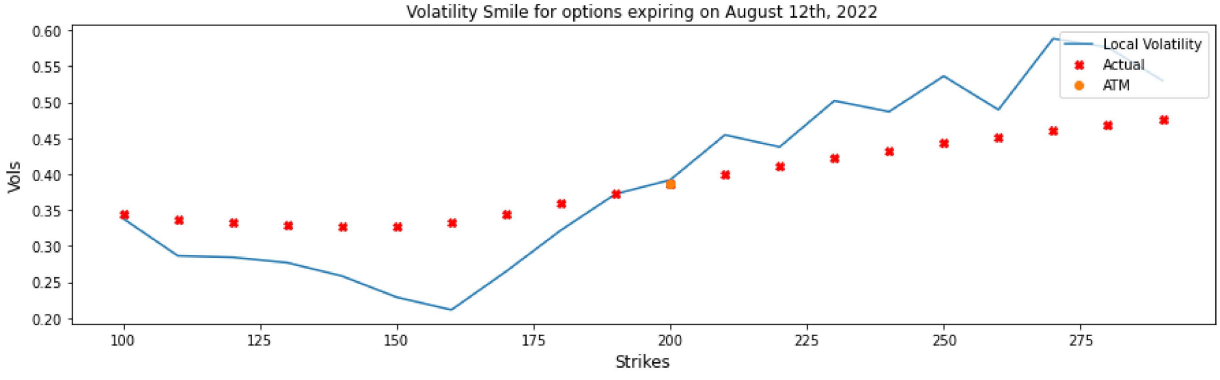












In [5]:

```
#HESTON MODEL SURFACE PLOTTING (Levenberg-Marquardt Method)
```

```
m1_params, m2_params = (None, None)

if data == "SPX":
    m1_params = (0.05,0.2,0.5,0.1,0.09)
    m2_params = (0.15,0.5,0.2,0.7,0.01)
elif data == "COFFEE":
    m1_params = (0.01,0.1,0.3,0.1,0.02)
    m2_params = (0.2,0.9,0.9,0.9,-0.19)
elif data == "OIL":
    m2_params = (0.023,0.009,1.00,0.95,0.2)
    m1_params = (0.15,0.5,0.2,0.7,0.01)
elif data == "GOLD":
    m1_params = (0.03,0.3,0.5,0.3,0.04)
    m2_params = (0.01,0.5,0.5,0.1,0.03)
else:
    m1_params = (0.03,0.3,0.5,0.3,0.04)
    m2_params = (0.01,0.5,0.5,0.1,0.03)

hestonModel1 = hestonModelSurface(m1_params, label="Heston Model 1, {}".format(data_
hestonModel2 = hestonModelSurface(m2_params, label="Heston Model 2, {}".format(data_

# Use to Calibrate first time the Heston Model
def calibrateHeston():
    def f(params):
        return hestonModelSurface(params).avgError
        # v0, kappa, theta, sigma, rho
    cons = (
            {'type': 'ineq', 'fun': lambda x: x[0] - 0.001},
            {'type': 'ineq', 'fun': lambda x: 2. - x[0]},
            {'type': 'ineq', 'fun': lambda x: x[1] - 0.001},
            {'type': 'ineq', 'fun': lambda x: 2. - x[1]},
            {'type': 'ineq', 'fun': lambda x: x[2] - 0.001},
            {'type': 'ineq', 'fun': lambda x: 2. - x[2]},
            {'type': 'ineq', 'fun': lambda x: x[3] - 0.001},
            {'type': 'ineq', 'fun': lambda x: .999 - x[3]},
            {'type': 'ineq', 'fun': lambda x: .99 - x[4]**2}
            )
    result = minimize(f, m1_params, constraints=cons, method="SLSQP", bounds=((1e-8,
    hestonModel0 = hestonModelSurface(result["x"], label="Heston Model 0, {}".format


    plot_vol_surface(hestonModel0.heston_vol_surface, title="{} Volatility Surface".
    plot_vol_surface(hestonModel1.heston_vol_surface, title="{} Volatility Surface".

init_conditions = pd.DataFrame({"theta": [m1_params[0], m2_params[0]], "kappa": [m1_
                                "sigma": [m1_params[2], m2_params[2]], "rho": [m1_pa
                                "v0": [m1_params[4], m2_params[4]]}, index = ["Model
display(init_conditions.style.set_caption("Heston Model Initial Conditions on ({})".
```

Heston Model Initial Conditions on (Coffee)

	theta	kappa	sigma	rho	v0
Model1	0.010000	0.100000	0.300000	0.100000	0.020000
Model2	0.200000	0.900000	0.900000	0.900000	-0.190000

In [6]:

```
# HESTON Surface Plotting (Model1, Model2)

for model in (hestonModel1, hestonModel2):
    plot_vol_surface(model.heston_vol_surface, title="Heston Volatility Surface for
    display(model.errors_data.style.set_caption("{} calibration results".format(mode

    fig1 = plt.figure(figsize=plot_size)
```

```
    plt.plot(model.strks, model.marketValue, label="Market Value")
    plt.plot(model.strks, model.modelValue, label="Model Value")
    plt.title('Model1: Heston surface Market vs Model Value'); plt.xlabel='strikes';
    plt.legend()
    fig2 = plt.figure(figsize=plot_size)
    plt.plot(model.strks, model.relativeError)
    plt.title('Model1: Heston surface Relative Error (%)'); plt.xlabel='strikes'; pl
    plt.legend()
```

Heston Model 1, Coffee calibration results

	Strikes	Market Value	Model Value	Relative Error (%)
0	100.000000	0.336740	0.333489	-0.965443
1	110.000000	0.655519	0.659828	0.657296
2	120.000000	1.218025	1.218920	0.073469
3	130.000000	2.131744	2.125694	-0.283803
4	140.000000	3.511885	3.528806	0.481833
5	150.000000	5.528623	5.607725	1.430781
6	160.000000	8.461670	8.548821	1.029953
7	170.000000	12.562362	12.483658	-0.626503
8	180.000000	17.671092	17.417277	-1.436324
9	190.000000	23.555020	23.228484	-1.386272
10	200.000000	30.054349	29.746388	-1.024680
11	210.000000	27.026128	26.770173	-0.947064
12	220.000000	24.402363	24.226489	-0.720728
13	230.000000	22.138513	22.028049	-0.498968
14	240.000000	20.156662	20.110653	-0.228258
15	250.000000	18.421029	18.425992	0.026945
16	260.000000	16.889495	16.936752	0.279800
17	270.000000	15.548853	15.613419	0.415246
18	280.000000	14.348527	14.432189	0.583068
19	290.000000	13.272169	13.373561	0.763947

Heston Model 1, Coffee parameters output

	Value
v0	1.230783
kappa	0.000000
theta	0.623021
sigma	0.618250
rho	0.162907
avgError	0.693019

```
No handles with labels found to put in legend.
```

Heston Model 2, Coffee calibration results

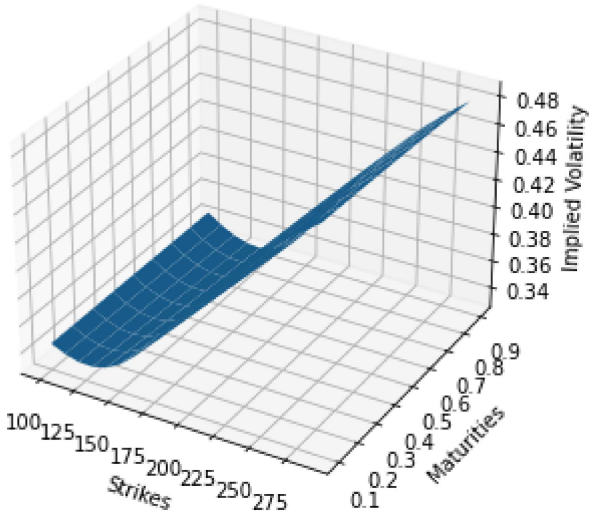
	Strikes	Market Value	Model Value	Relative Error (%)
0	100.000000	0.336740	0.333489	-0.965550
1	110.000000	0.655519	0.659827	0.657200
2	120.000000	1.218025	1.218919	0.073379
3	130.000000	2.131744	2.125693	-0.283892
4	140.000000	3.511885	3.528803	0.481740
5	150.000000	5.528623	5.607719	1.430680
6	160.000000	8.461670	8.548812	1.029845
7	170.000000	12.562362	12.483644	-0.626612
8	180.000000	17.671092	17.417259	-1.436429
9	190.000000	23.555020	23.228460	-1.386370
10	200.000000	30.054349	29.746362	-1.024769
11	210.000000	27.026128	26.770144	-0.947174
12	220.000000	24.402363	24.226457	-0.720858
13	230.000000	22.138513	22.028016	-0.499119
14	240.000000	20.156662	20.110619	-0.228430
15	250.000000	18.421029	18.425957	0.026752
16	260.000000	16.889495	16.936716	0.279587
17	270.000000	15.548853	15.613382	0.415013
18	280.000000	14.348527	14.432153	0.582815
19	290.000000	13.272169	13.373525	0.763674

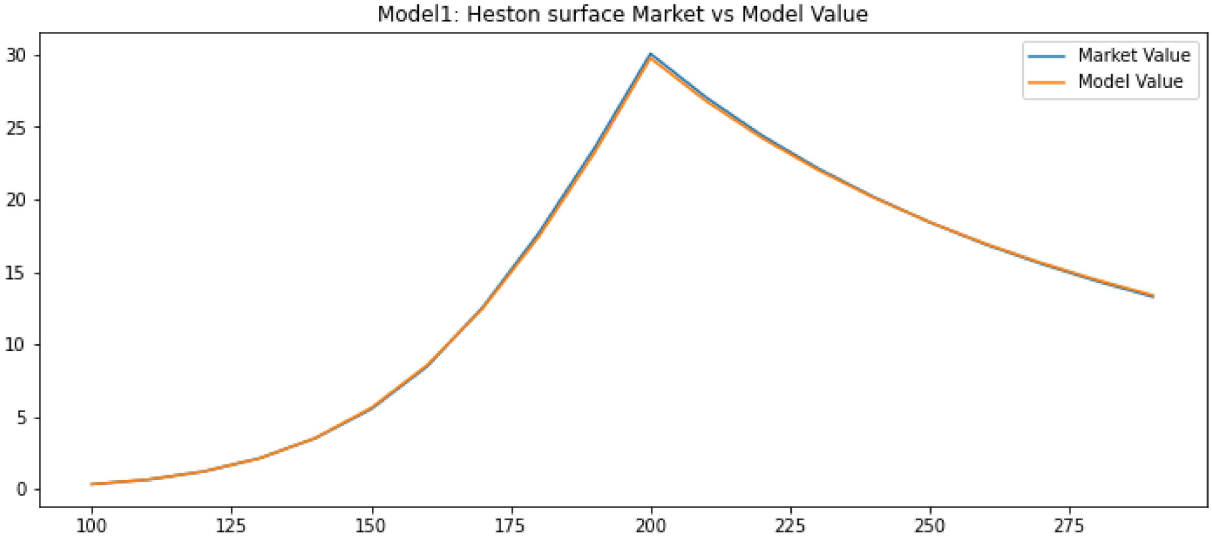
Heston Model 2, Coffee parameters output

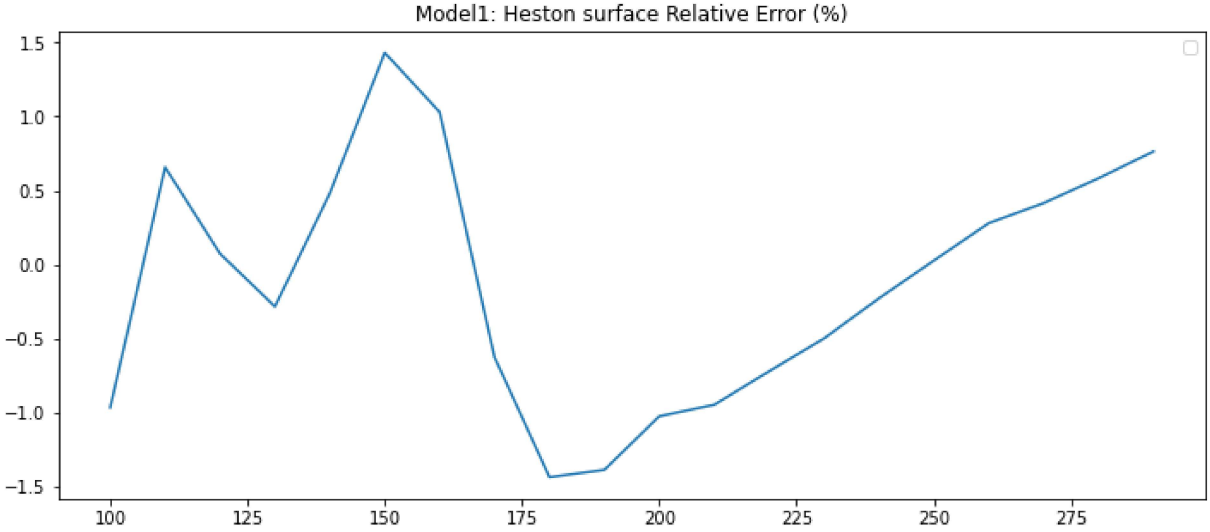
	Value
v0	0.939937
kappa	0.000000
theta	0.623019
sigma	0.618249
rho	0.162907
avgError	0.692994

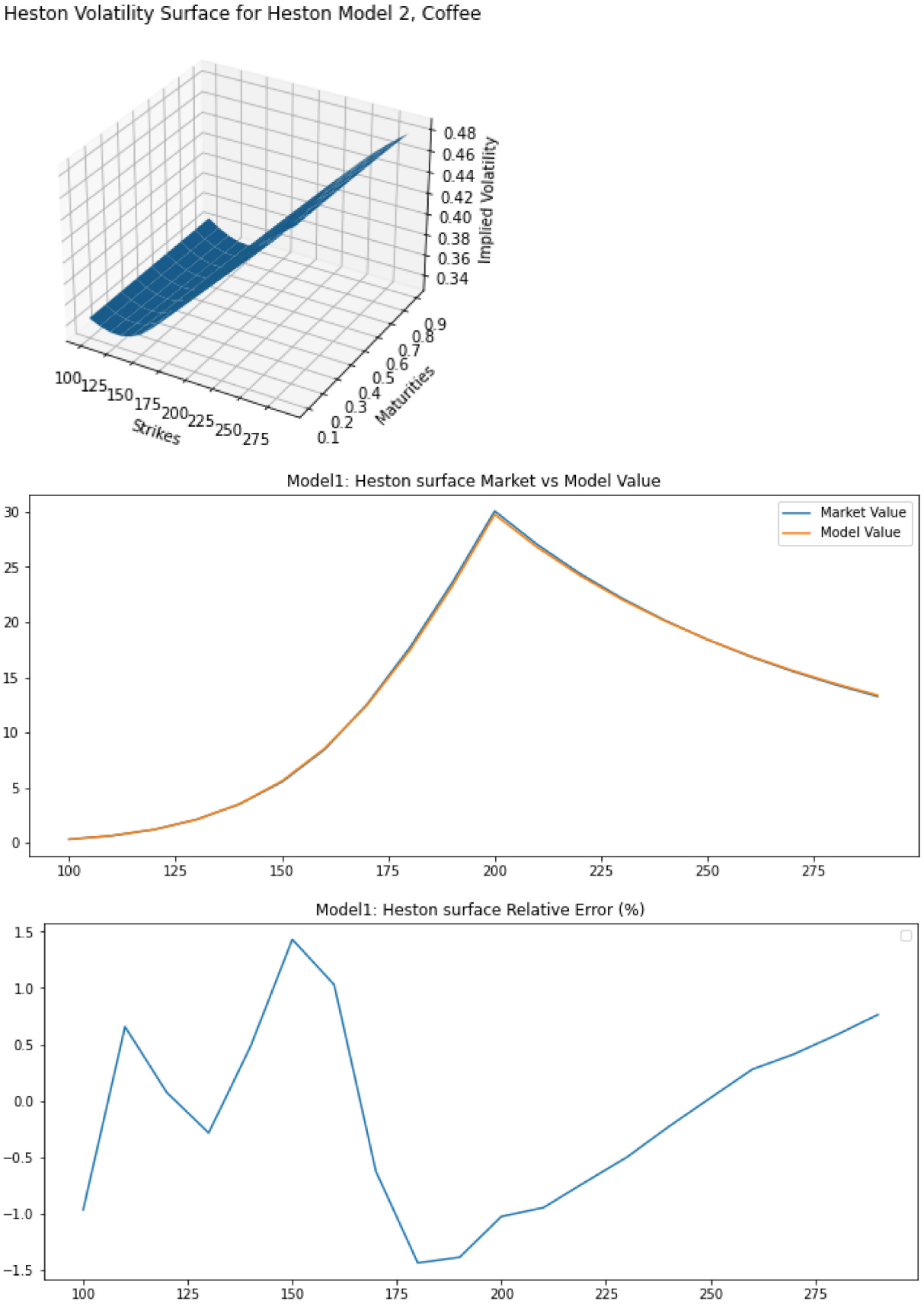
```
No handles with labels found to put in legend.
```

Heston Volatility Surface for Heston Model 1, Coffee





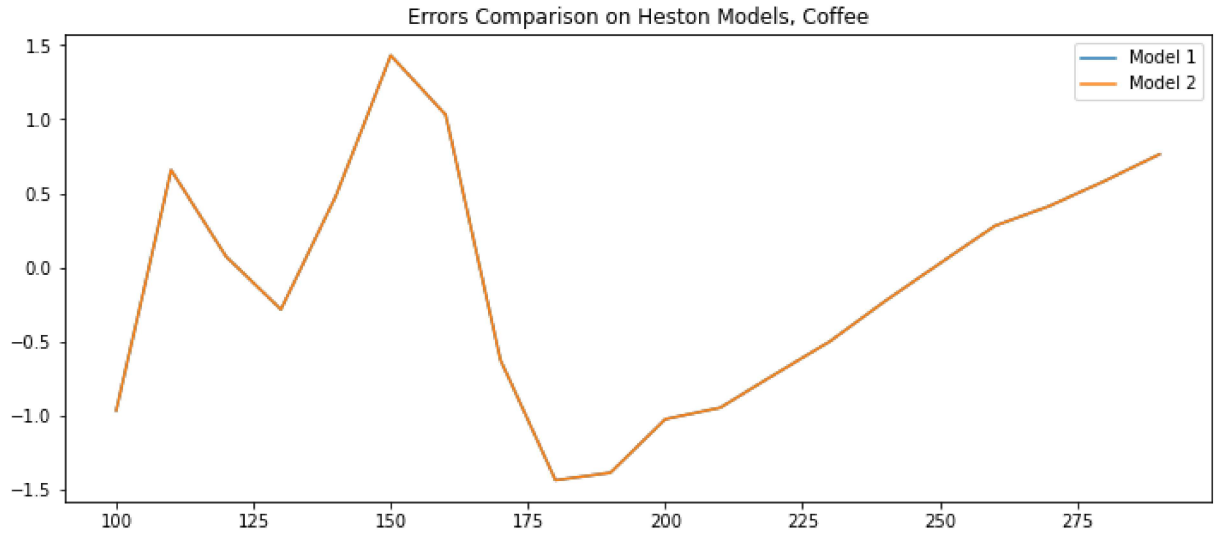


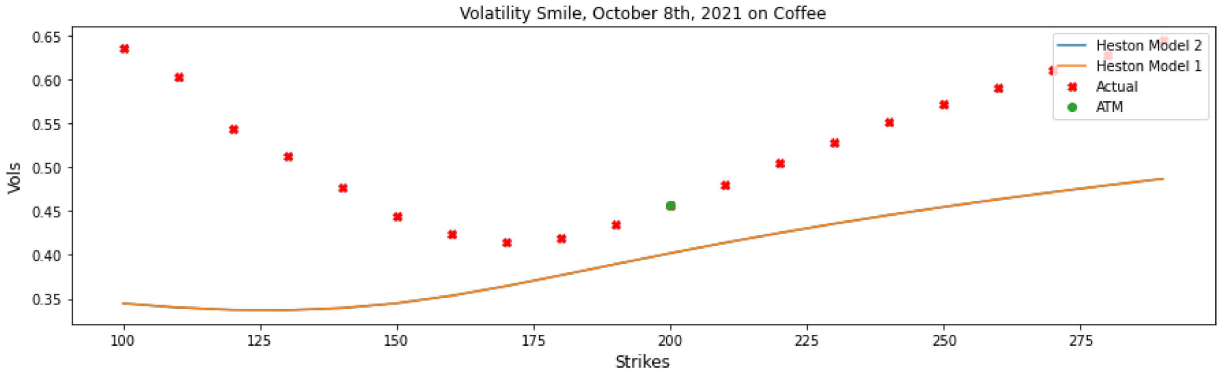


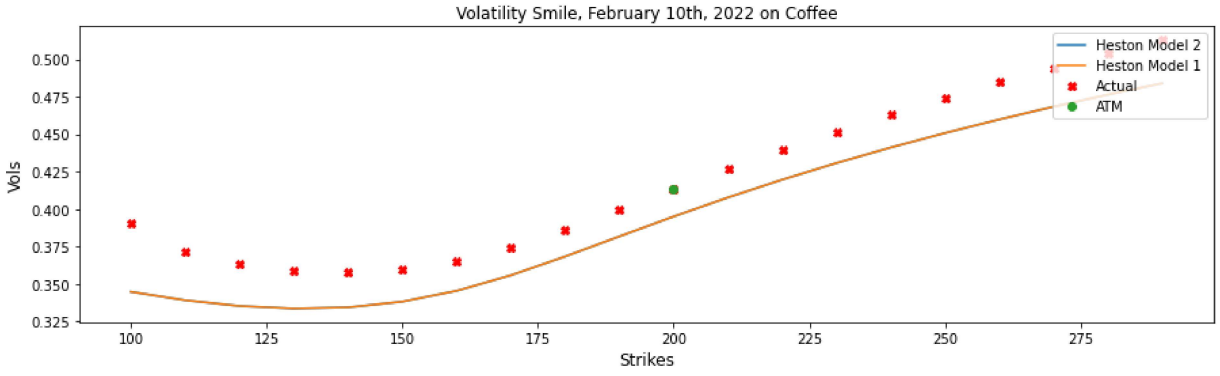
In [7]:

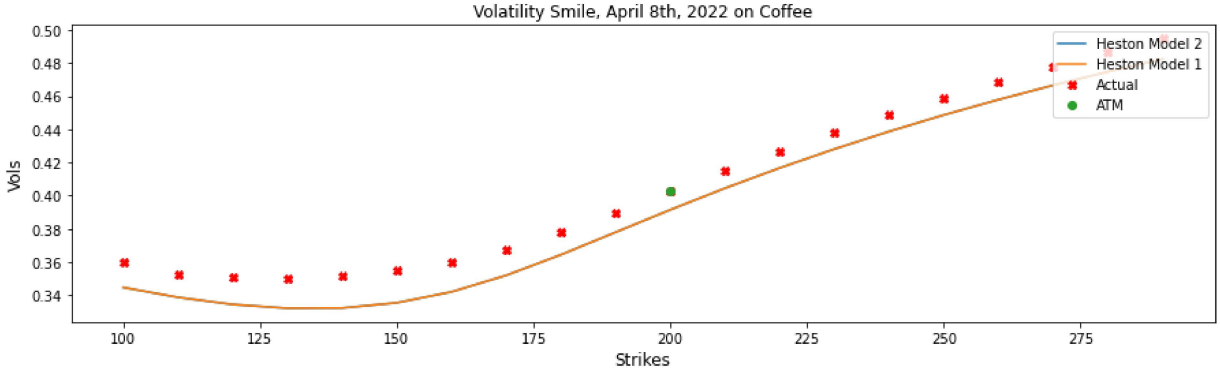
```
# Relative error comparison
plt.figure(figsize=plot_size)
plt.plot(hestonModel1.strks, hestonModel1.relativeError, label="Model 1")
plt.plot(hestonModel2.strks, hestonModel2.relativeError, label="Model 2")
plt.title("Errors Comparison on Heston Models, {}".format(data_label));
plt.legend()

# Volatility smiles comparison
tenors = [dates[round((len(dates)-1) * x)] for x in (.2, .5, .75, 1)]
```

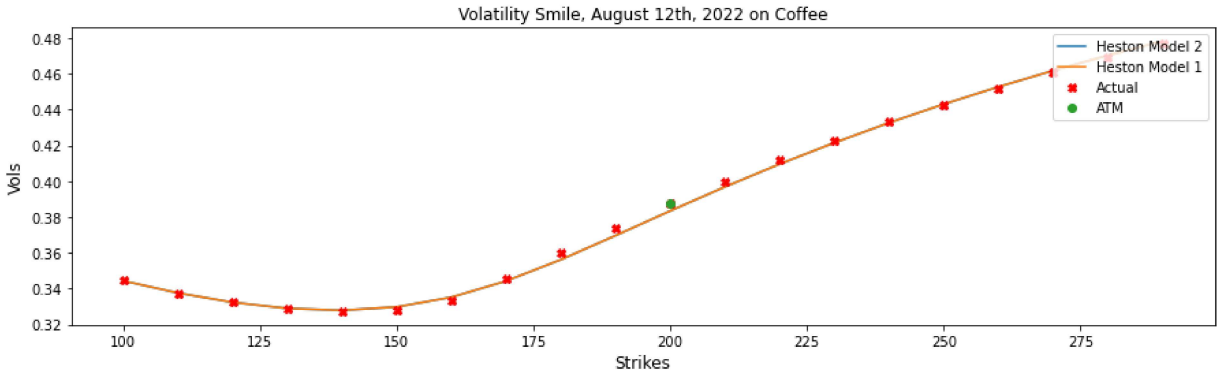
```
for tenor in tenors:
    l = [
        ([hestonModel2.heston_vol_surface.blackVol(tenor, s) for s in strikes], "Hes
        ([hestonModel1.heston_vol_surface.blackVol(tenor, s) for s in strikes], "Hes
        ]
    plot_smile(tenor, l, market=True)
```











In [8]:

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np
import pandas as pd
import time
import datetime
import math
import QuantLib as ql
from scipy.optimize import minimize

from initialize import *
from plotting import *

# CALIBRATE SABR VOLATILITY SURFACE

volMatrix = ql.Matrix(len(strikes), len(dates))

for i in range(len(vols)):
    for j in range(len(vols[i])):
        volMatrix[j][i] = vols[i][j]

black_var_surface = ql.BlackVarianceSurface(
    today, calendar, dates, strikes, volMatrix, day_count)
black_var_surface.enableExtrapolation()

def plot_vol_surface(vol_surface, plot_years=np.arange(0.1, (dates[-1] - today) / 36
    if type(vol_surface) != list:
        surfaces = [vol_surface]
    else:
        surfaces = vol_surface

    fig = plt.figure(figsize=plot_size)
    ax = fig.add_subplot(projection='3d')
    ax.set_xlabel('Strikes')
    ax.set_ylabel('Maturities')
    ax.set_zlabel('Implied Volatility')
    ax.set_title(title)
    X, Y = np.meshgrid(plot_strikes, plot_years)

    for surface in surfaces:
        method_to_call = getattr(surface, funct)

        Z = np.array([method_to_call(float(y), float(x))
                      for xr, yr in zip(X, Y)
                      for x, y in zip(xr, yr)]
                     ).reshape(len(X), len(X[0]))

        surf = ax.plot_surface(X, Y, Z, rstride=1, cstride=1, linewidth=0.3)
```

```
class SABRSmile:
    def __init__(self, date, shift=0, beta=1, method="normal", fwd=current_price, ze
        self.date = date
        self.expiryTime = round((self.date - today)/365, 6)
        self.marketVols = vols[dates.index(self.date)]
        self.shift = shift
        self.fwd = fwd
        self.forward_price = self.fwd * \
            math.exp(rate.value() * self.expiryTime)
        self.zero_rho = zero_rho
        self.alpha, self.beta, self.nu, self.rho = (
            .1, beta, 0., 0. if self.zero_rho else .1)
        self.method = method
        self.newVols = None
        self.error = None

    def initialize(self):
        # alpha, beta, nu, rho
        cons = (
            {'type': 'ineq', 'fun': lambda x: x[0] - 0.001},
            {'type': 'eq', 'fun': lambda x: x[1] - self.beta},
            {'type': 'ineq', 'fun': lambda x: x[2] - .001},
            {'type': 'ineq', 'fun': lambda x: .99 - x[3]**2},
        )

        x = self.set_init_conds()

        result = minimize(self.f, x, constraints=cons, method="SLSQP", bounds=(
            (1e-8, None), (-1, 1), (1e-8, None), (-.999, .999)))
        self.error = result['fun']
        [self.alpha, self.beta, self.nu, self.rho] = result['x']

        self.newVols = [self.vols_by_method(
            strike, self.alpha, self.beta, self.nu, self.rho) for strike in strikes]

    def set_init_conds(self):
        return [self.alpha, self.beta, self.nu, self.rho]

    def vols_by_method(self, strike, alpha, beta, nu, rho):
        if self.method == "floch-kennedy":
            return ql.sabrFlochKennedyVolatility(strike, self.forward_price, self.ex
        elif self.shift != 0:
            return ql.shiftedSabrVolatility(strike, self.forward_price, self.expiryT
        else:
            return ql.sabrVolatility(strike, self.forward_price, self.expiryTime, al

    def f(self, params):

        alpha, beta, nu, rho = params

        # beta = self.beta
        # alpha = max(alpha, 1e-8) # Avoid alpha going negative
        # nu = max(nu, 1e-8) # Avoid nu going negative
        # rho = max(rho, -0.999) if self.zero_rho==False else 0.0 # Avoid rhp going
        # rho = min(rho, 0.999) # Avoid rho going > 1.0

        vols = np.array([self.vols_by_method(
            strike, alpha, beta, nu, rho) for strike in strikes])

        self.error = ((vols - np.array(self.marketVols))**2).mean() ** .5

        return self.error


class SABRVolatilitySurface:
```

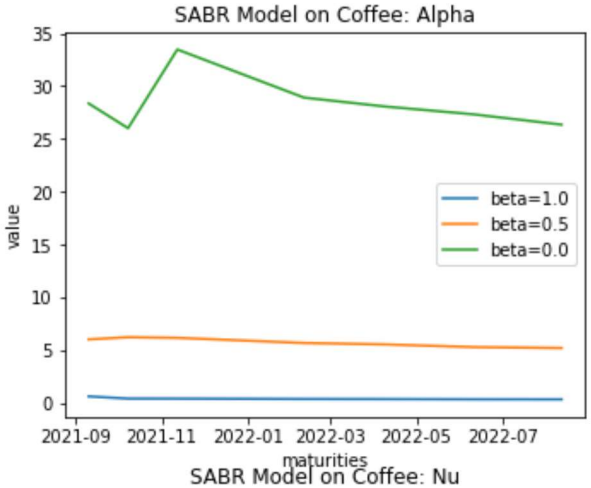
```
    def __init__(self, method="normal", beta=1, shift=0, fwd=current_price, label=""
        self.method = method
        self._beta = beta
        self.shift = shift
        self.fwd = fwd
        self.label = label
        self.zero_rho = zero_rho

        self.initialize()

    def initialize(self):
        self.vol_surface_vector, self.errors, self.smiles, self.alpha, self.beta, se
        ], [], [], [], [], [], []
        self.SABRVolMatrix, self.SABRVolDiffMatrix = (
            ql.Matrix(len(strikes), len(dates)), ql.Matrix(len(strikes), len(dates))

        for i, d in enumerate(dates):
            volSABR = SABRSmile(date=d, beta=self._beta, shift=self.shift,
                                method=self.method, fwd=self.fwd, zero_rho=self.zero
            volSABR.initialize()

            self.alpha.append(volSABR.alpha)
            self.beta.append(volSABR.beta)
            self.nu.append(volSABR.nu)
            self.rho.append(volSABR.rho)

            self.errors.append(volSABR.error)

            smile = volSABR.newVols

            self.vol_surface_vector.extend(smile)
            self.smiles.append(volSABR)

            # constructing the SABRVolatilityMatrix
            for j in range(len(smile)):
                self.SABRVolMatrix[j][i] = smile[j]
                self.SABRVolDiffMatrix[j][i] = (
                    smile[j] - vols[i][j]) / vols[i][j]

            self.vol_surface = ql.BlackVarianceSurface(
                today, calendar, dates, strikes, self.SABRVolMatrix, day_count)
            self.vol_surface.enableExtrapolation()

    def to_data(self):
        d = {'alpha': self.alpha, 'beta': self.beta,
             'nu': self.nu, 'rho': self.rho}
        return pd.DataFrame(data=d, index=dates)


# Backbone modelling for SABR
def SABR_backbone_plot(beta=1, bounds=None, shift=0, fixes=(.95, 1, 1.14, 1.24), ten
    l = []
    for i in fixes:
        vol_surface = SABRVolatilitySurface(
            method="normal", shift=current_price*shift, beta=beta, fwd=current_price
        SABR_vol_surface = ql.BlackVarianceSurface(
            today, calendar, dates, strikes, vol_surface.SABRVolMatrix, day_count)
        SABR_vol_surface.enableExtrapolation()

        l.append(([SABR_vol_surface.blackVol(tenor, s)
                   for s in strikes], "fwd = {}".format(current_price * i)))

    plot_smile(tenor, l, bounds=bounds, market=False,
               title="backbone, beta = {}, {}".format(vol_surface.beta[0], tenor))
```

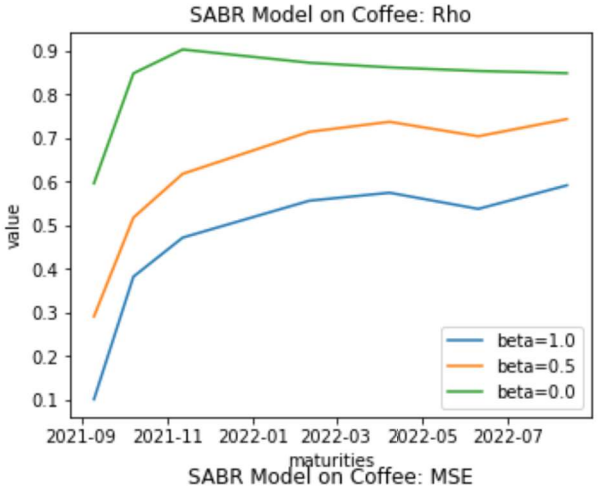
```
def SABRComparison(methods, title="", display=False):
    fig, axs = plt.subplots(2, 2, figsize=plot_size)
    plt.subplots_adjust(left=None, bottom=None, right=None,
                        top=1.5, wspace=None, hspace=None)

    for method in methods:
        lbl = "beta={}".format(method.beta[1])
        axs[0, 0].plot(maturities, method.alpha, label=lbl)
        axs[0, 0].set_title('{}: Alpha'.format(title))
        axs[0, 0].set(xlabel='maturities', ylabel='value')
        axs[0, 0].legend()
        axs[1, 0].plot(maturities, method.nu, label=lbl)
        axs[1, 0].set_title('{}: Nu'.format(title))
        axs[1, 0].set(xlabel='maturities', ylabel='value')
        axs[1, 0].legend()
        axs[0, 1].plot(maturities, method.rho, label=lbl)
        axs[0, 1].set_title('{}: Rho'.format(title))
        axs[0, 1].set(xlabel='maturities', ylabel='value')
        axs[0, 1].legend()
        axs[1, 1].plot(maturities, method.errors, label=lbl)
        axs[1, 1].set_title('{}: MSE'.format(title))
        axs[1, 1].set(xlabel='maturities', ylabel='value')
        axs[1, 1].legend()

        if display:
            method_df = method.to_data()
            display(method_df.style.set_caption("SABR, {}".format(lbl)))

        plot_vol_surface(method.vol_surface, title="{}".format(method.label))

    smiles_comparison(methods)
```

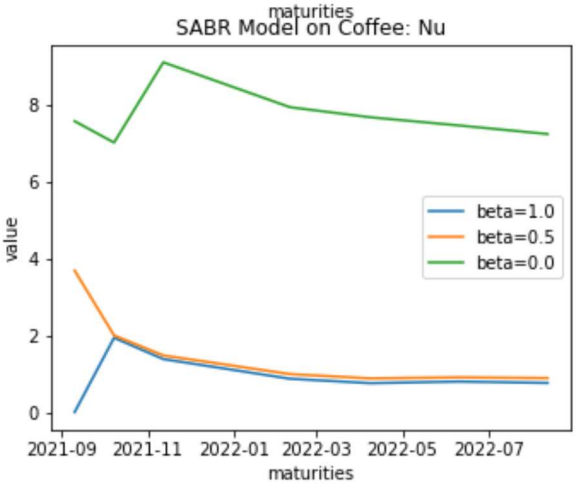
In [9]:

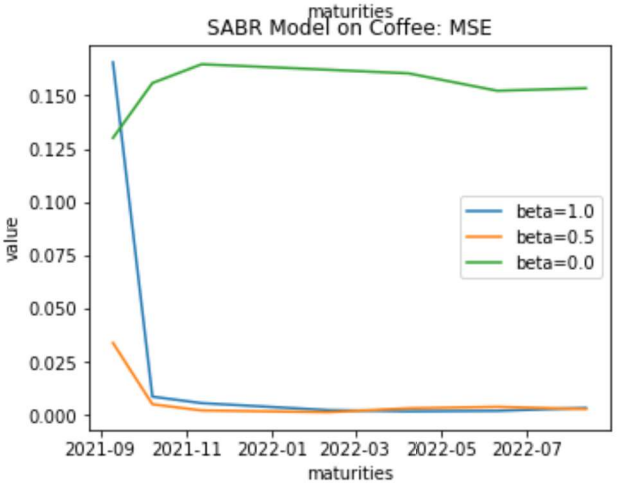
```
# SABR Volatility model

SABR_beta1 = SABRVolatilitySurface(beta=1, shift=0, label="SABR, beta=1, {}".format(
SABR_beta5 = SABRVolatilitySurface(beta=.5, shift=0, label="SABR, beta=.5 {}".format
SABR_beta0 = SABRVolatilitySurface(beta=.0, shift=0, label="Normal SABR, beta=0, {}"

SABRComparison([SABR_beta1, SABR_beta5, SABR_beta0], title="SABR Model on {}".format
```

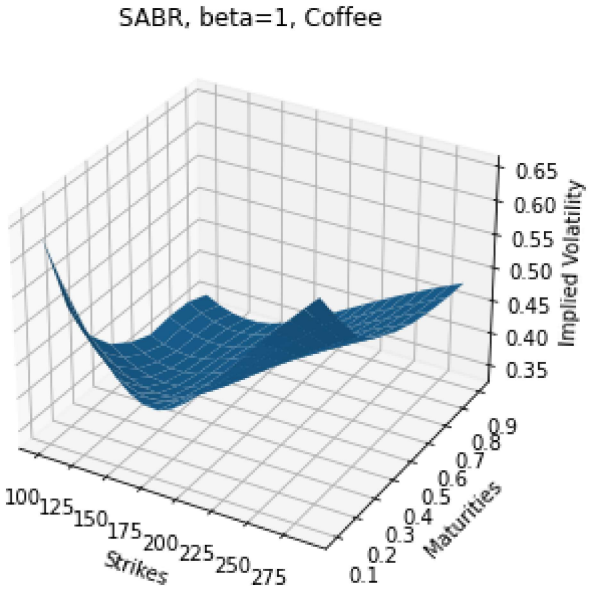
```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```


SABR Model on Coffee: Alpha
value
maturities
beta=1.0
beta=0.5
beta=0.0

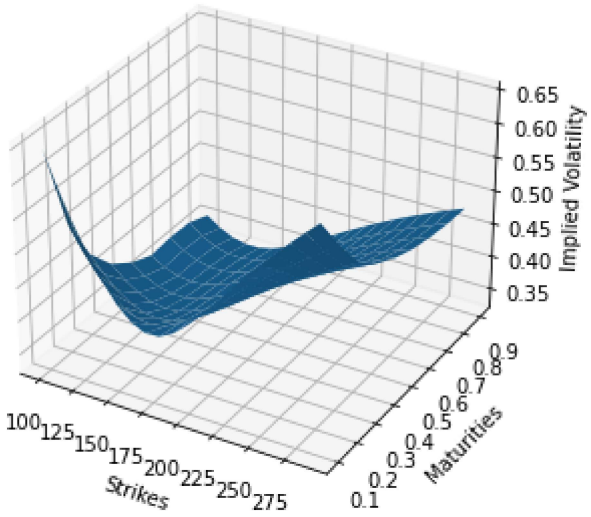

SABR Model on Coffee: Rho
value
maturities
beta=1.0
beta=0.5
beta=0.0


SABR Model on Coffee: Nu
value
maturities
beta=1.0
beta=0.5
beta=0.0

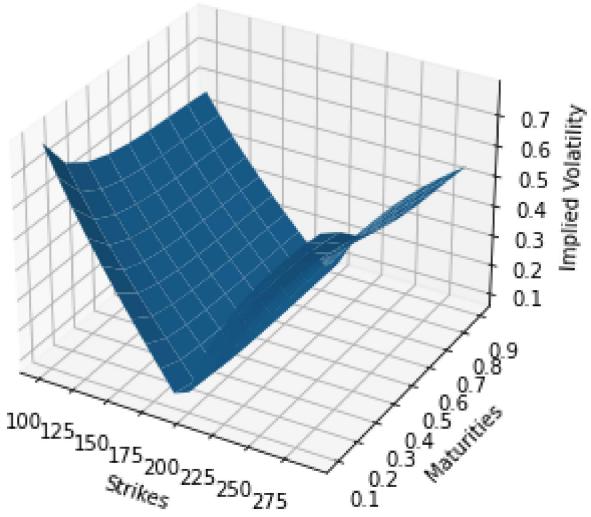

SABR Model on Coffee: MSE
value
maturities
beta=1.0
beta=0.5
beta=0.0

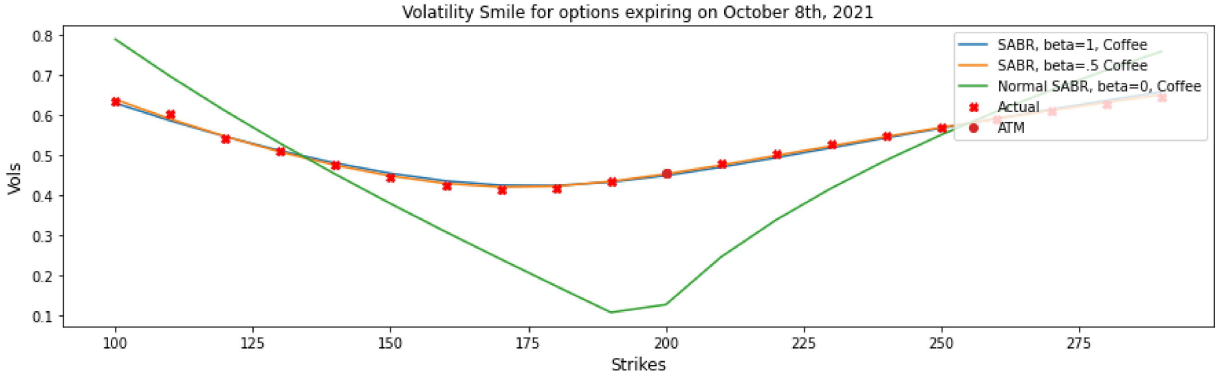

SABR, beta=1, Coffee
Implied Volatility
Strikes
Maturities

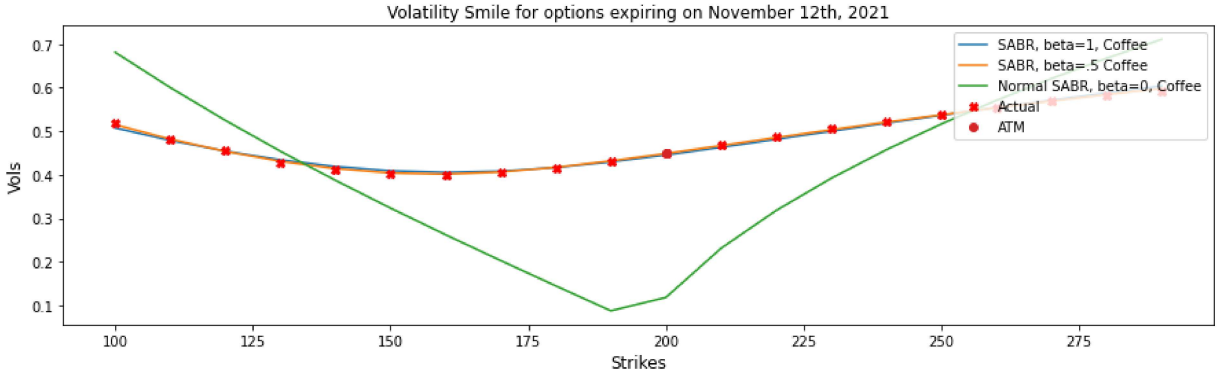
SABR, beta=.5 Coffee

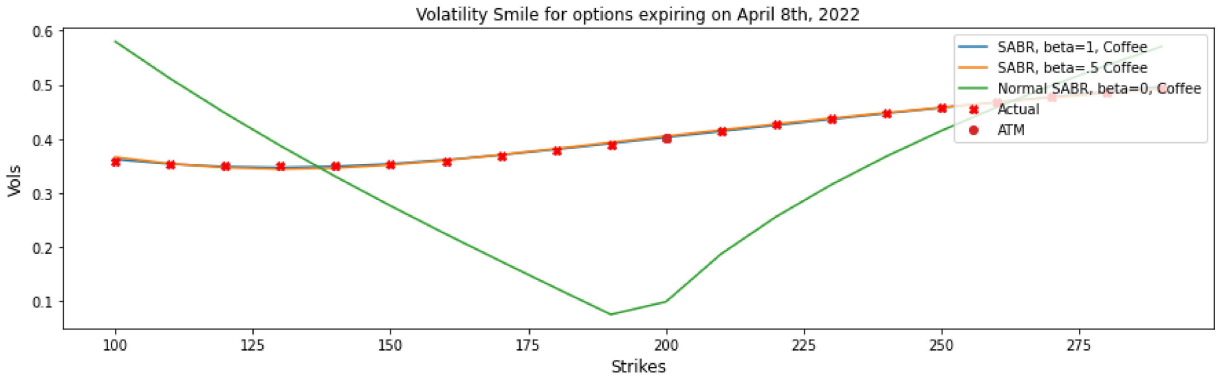


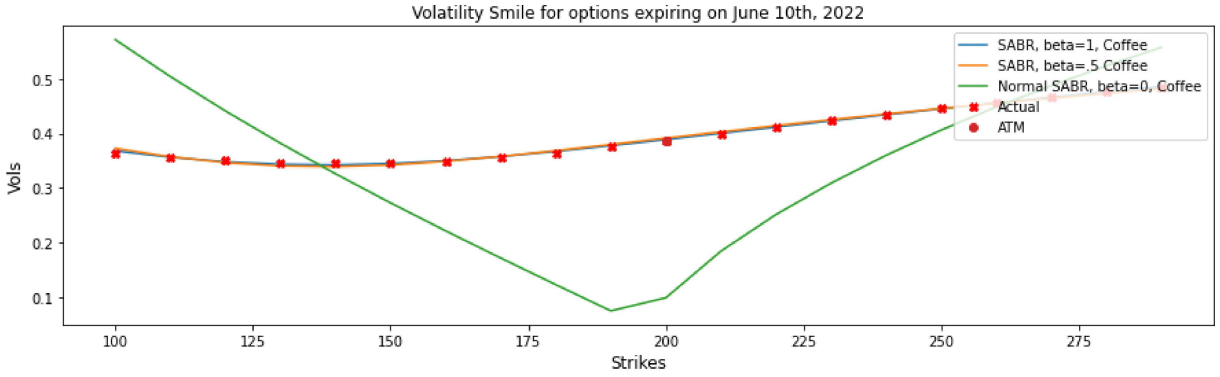
Normal SABR, beta=0, Coffee

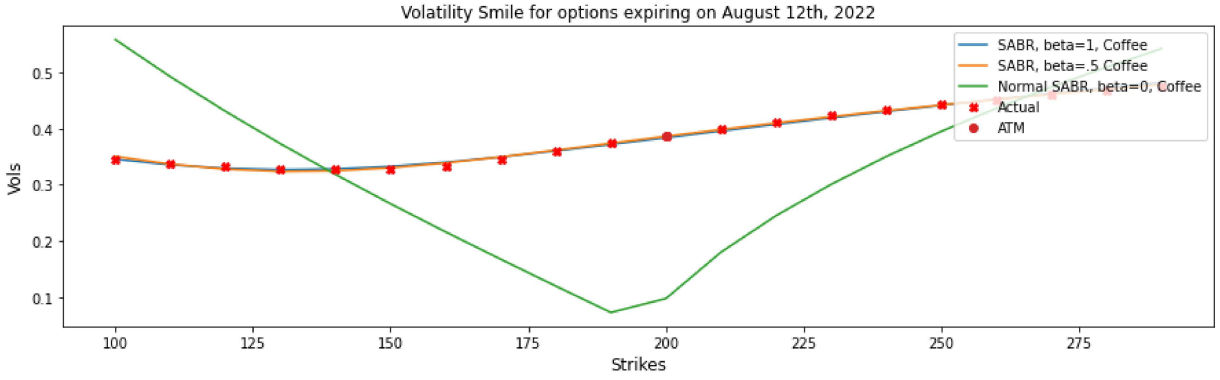








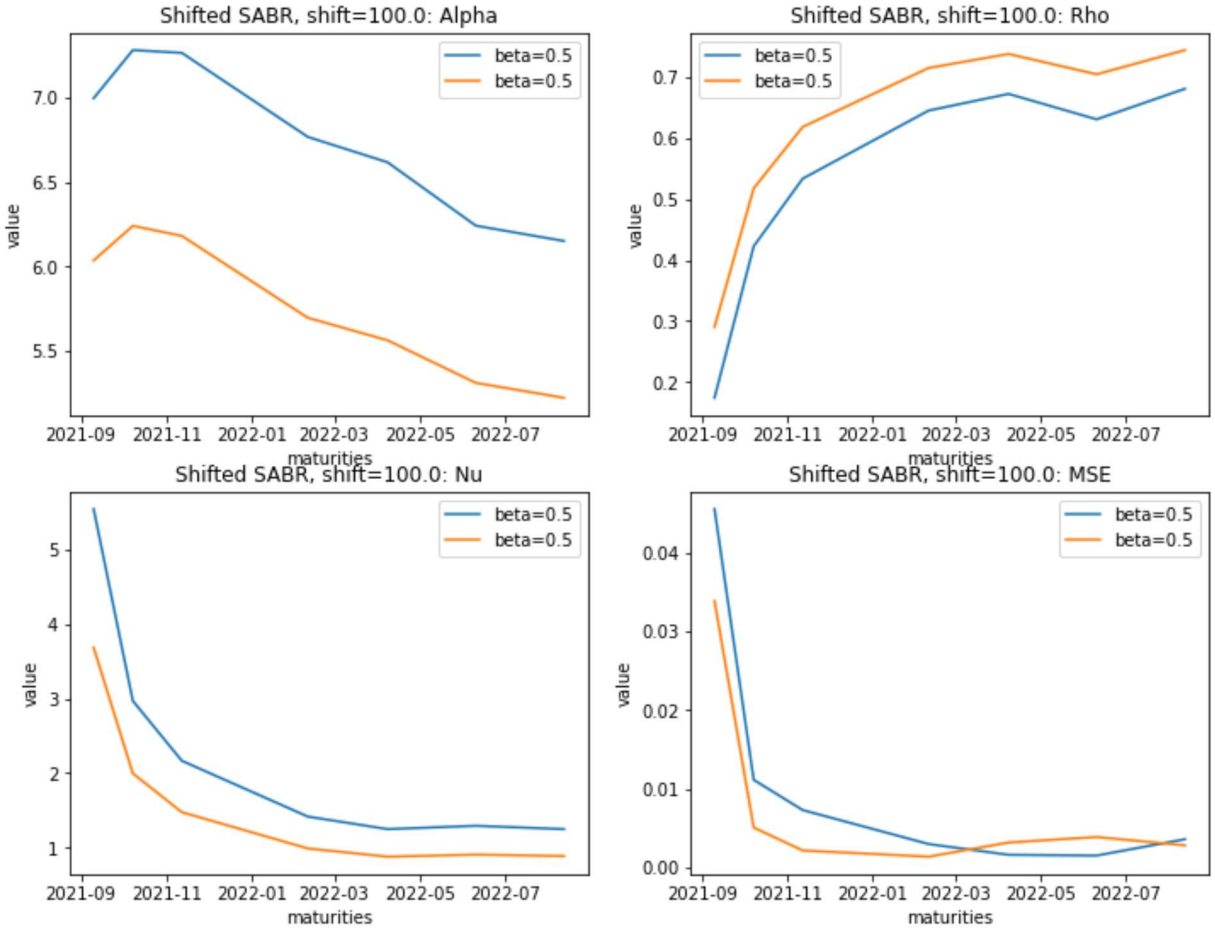




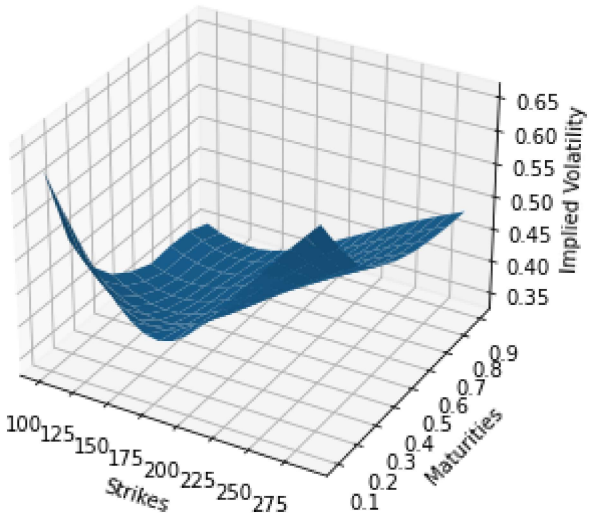
In [10]:

```
# Shifted SABR Volatility model
shft = .50 * current_price
shiftedSABR_beta1 = SABRVolatilitySurface(beta=1, shift=shft, label="Shifted SABR, b
shiftedSABR_beta5 = SABRVolatilitySurface(beta=.5, shift=shft, label="Shifted SABR,
shiftedSABR_beta0 = SABRVolatilitySurface(beta=.0, shift=shft, label="Shifted Normal

SABRComparison([shiftedSABR_beta5, SABR_beta5], title="Shifted SABR, shift={}".forma
SABRComparison([shiftedSABR_beta0, SABR_beta0], title="Shifted SABR, shift={}".forma
```

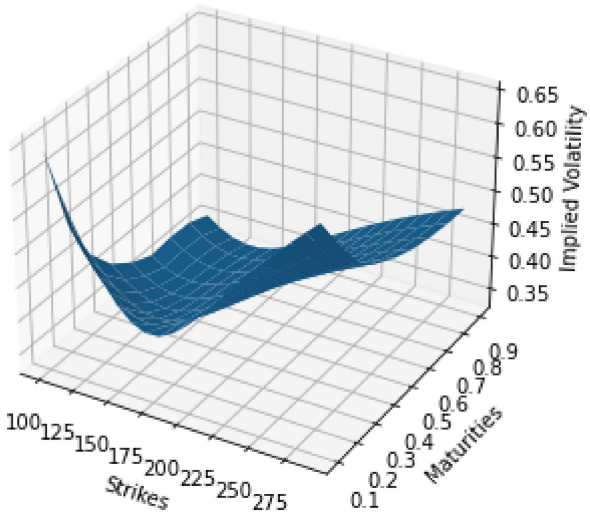
```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```

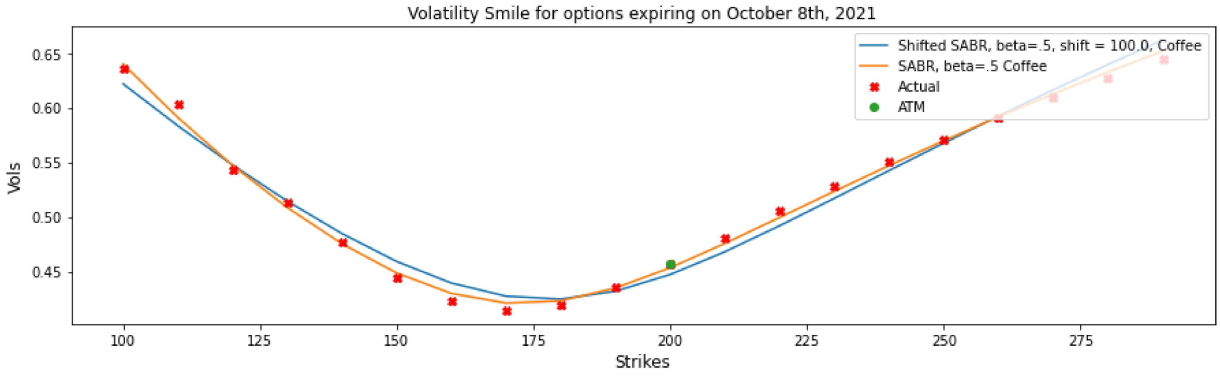


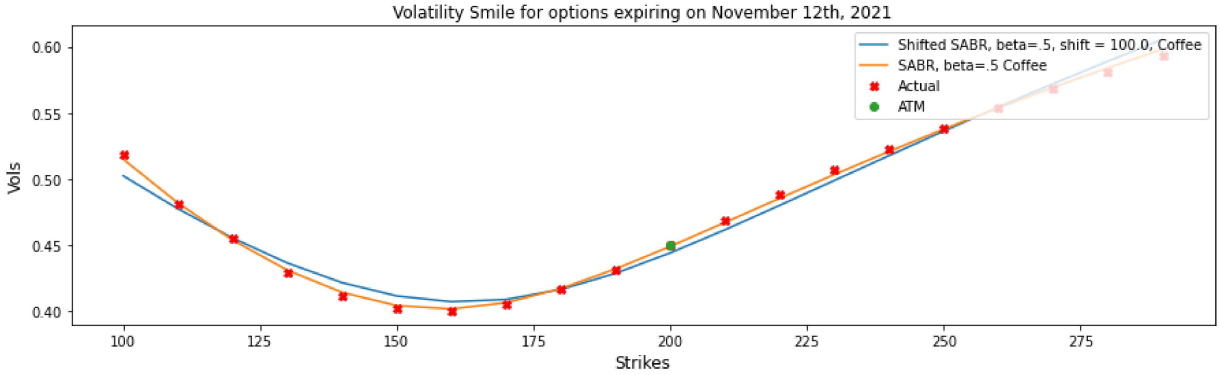
Shifted SABR, beta=.5, shift = 100.0, Coffee

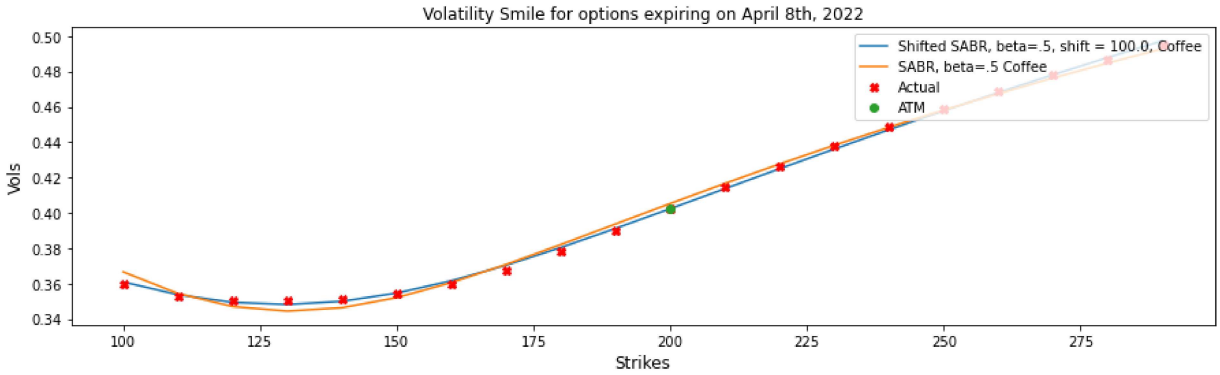


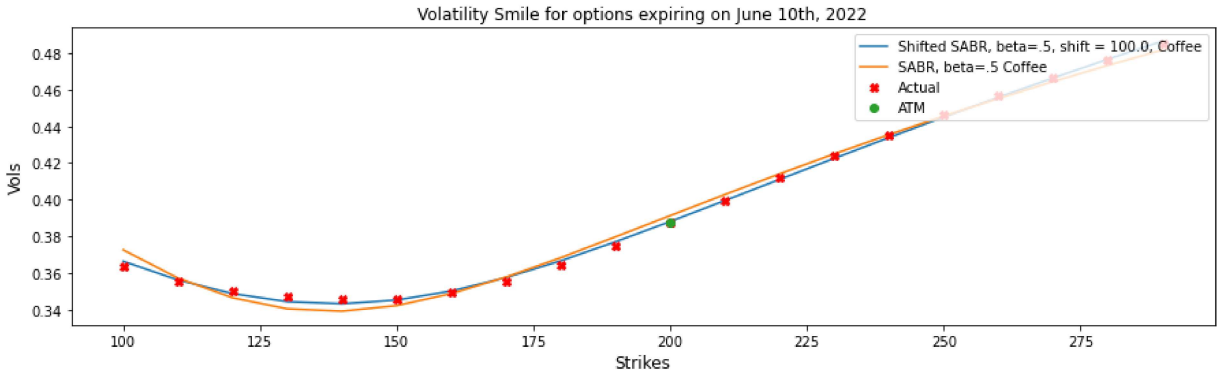
SABR, beta=.5 Coffee

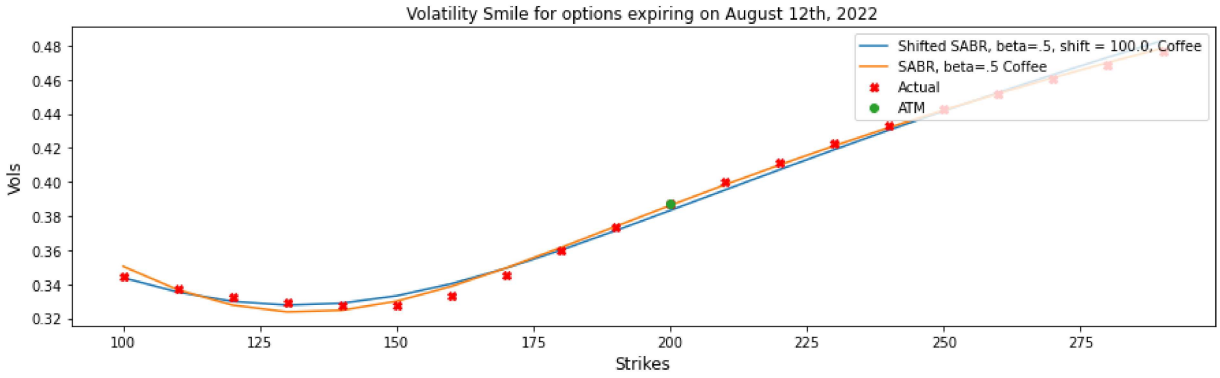


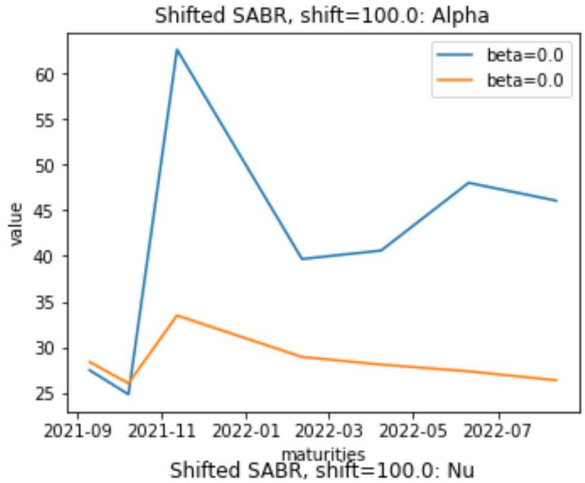


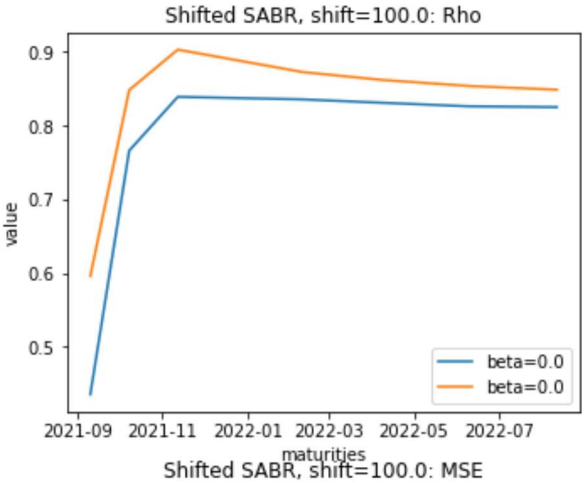


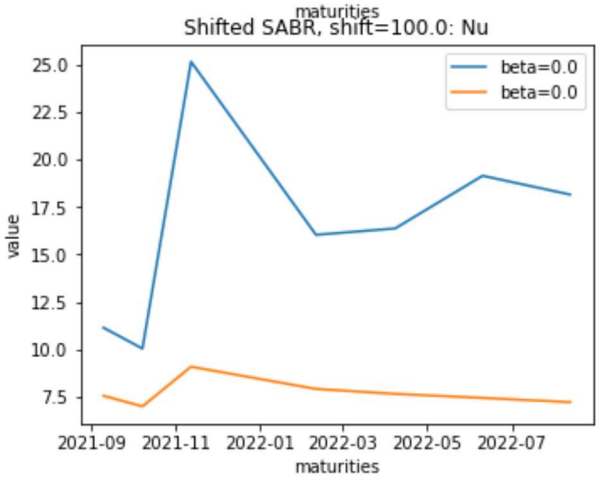


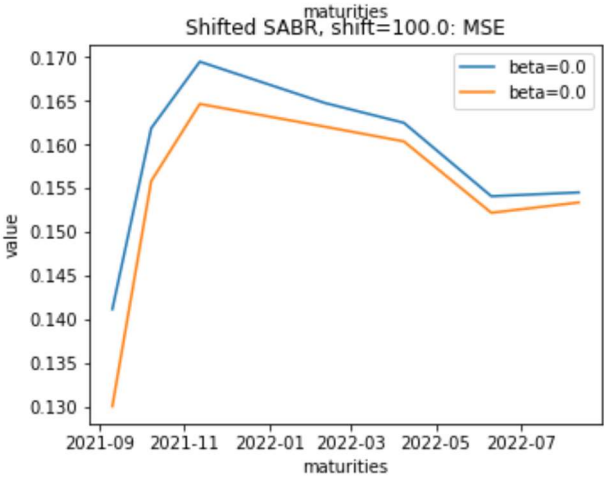

Volatility Smile for options expiring on June 10th, 2022
Shifted SABR, beta=.5, shift = 100.0, Coffee
SABR, beta=.5 Coffee
Actual
ATM
Vols
Strikes


Volatility Smile for options expiring on August 12th, 2022
Shifted SABR, beta=.5, shift = 100.0, Coffee
SABR, beta=.5 Coffee
Actual
ATM
Vols
Strikes

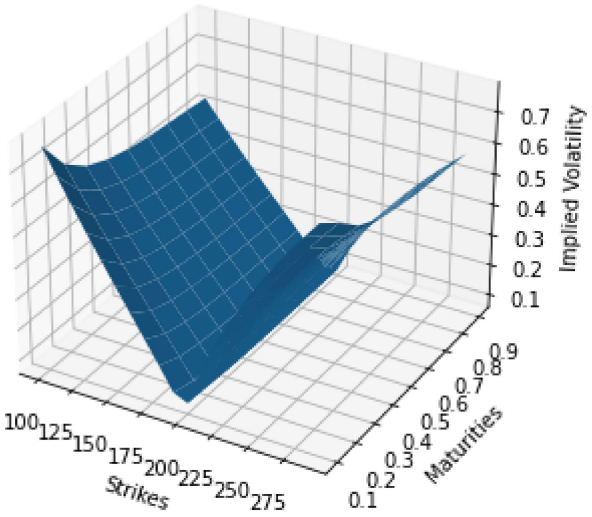

Shifted SABR, shift=100.0: Alpha
beta=0.0
beta=0.0
value
maturities


Shifted SABR, shift=100.0: Rho
beta=0.0
beta=0.0
value
maturities

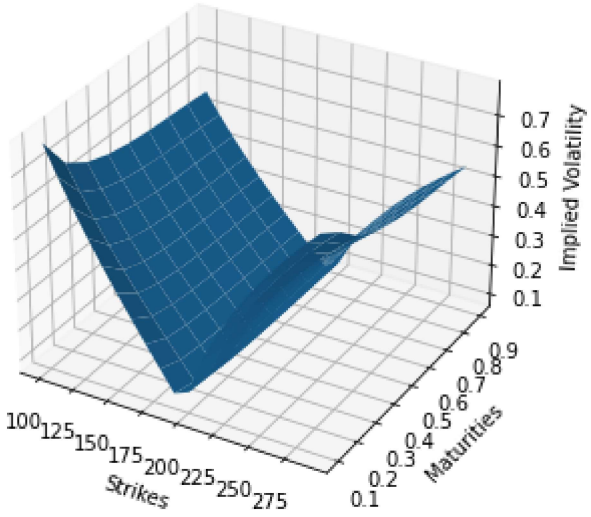

Shifted SABR, shift=100.0: Nu
beta=0.0
beta=0.0
value
maturities

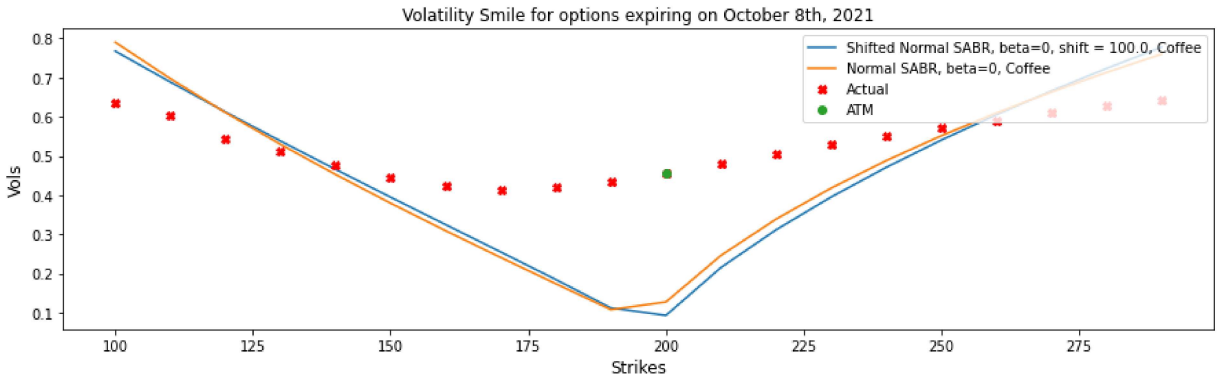

Shifted SABR, shift=100.0: MSE
beta=0.0
beta=0.0
value
maturities

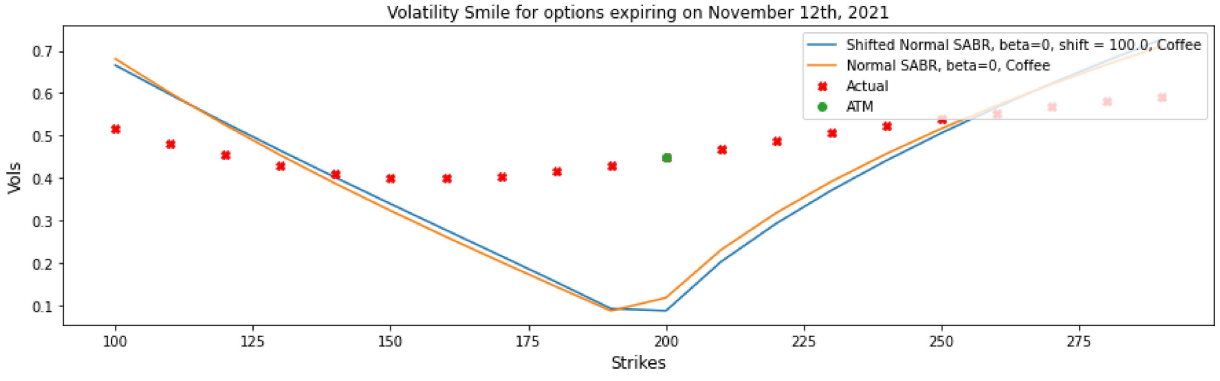
Shifted Normal SABR, beta=0, shift = 100.0, Coffee

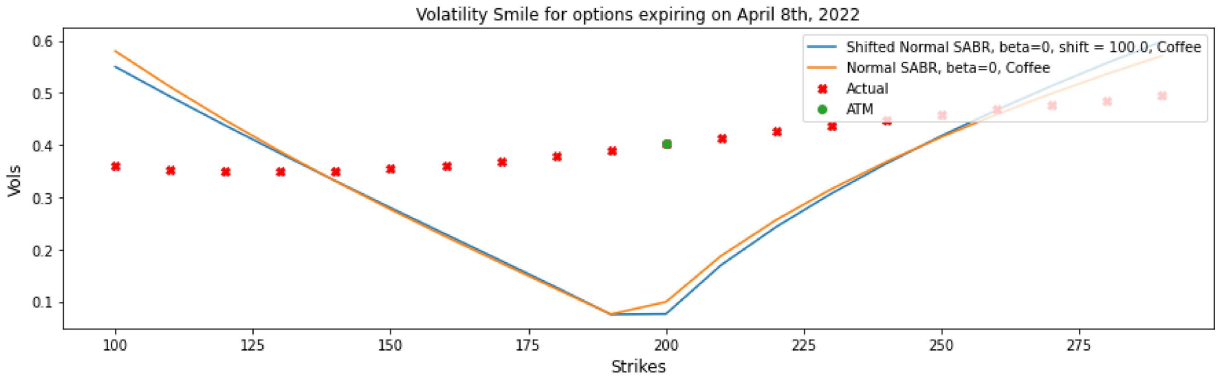


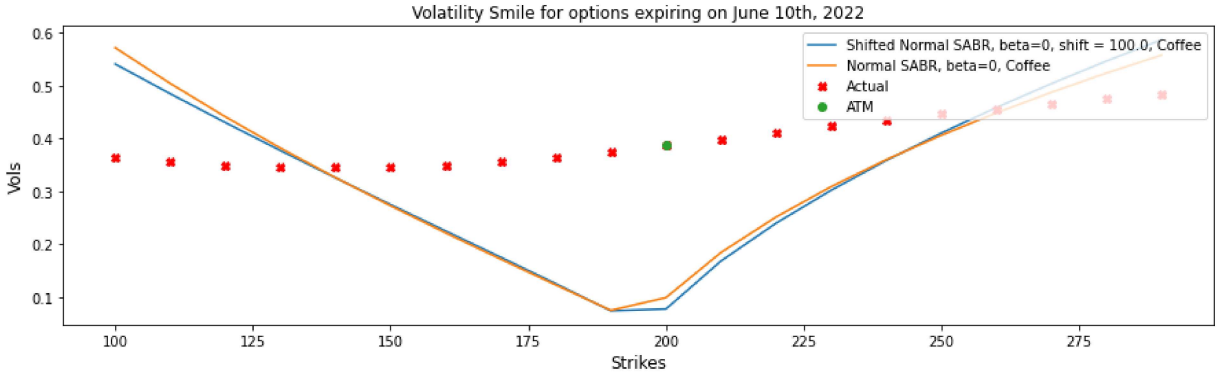
Normal SABR, beta=0, Coffee

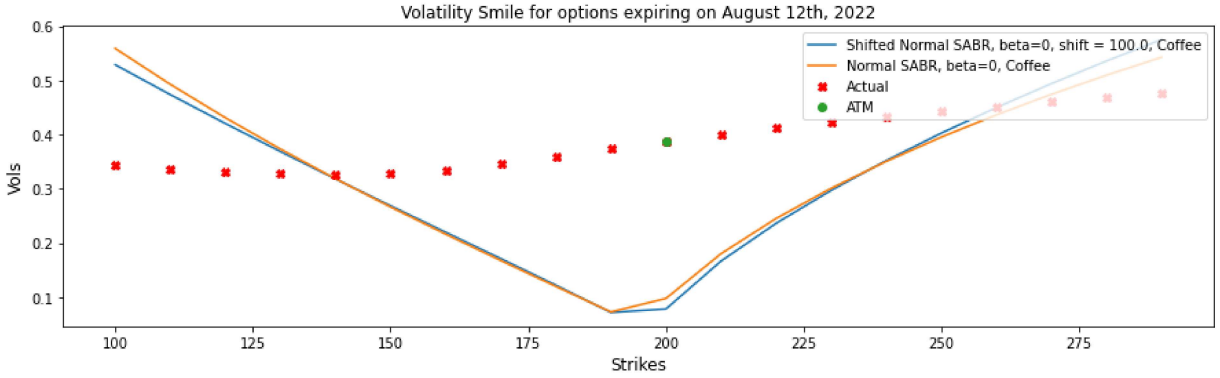








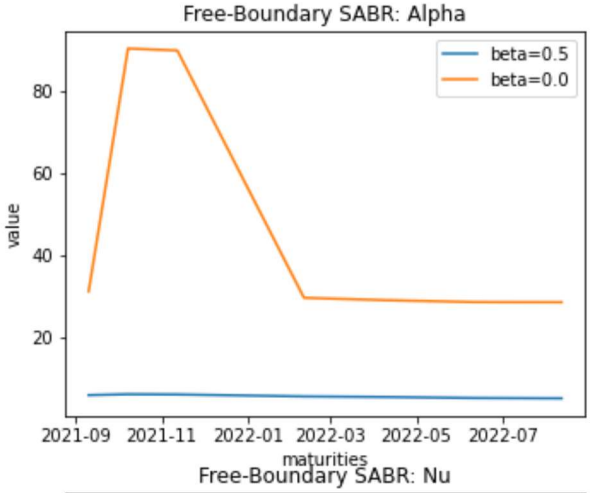


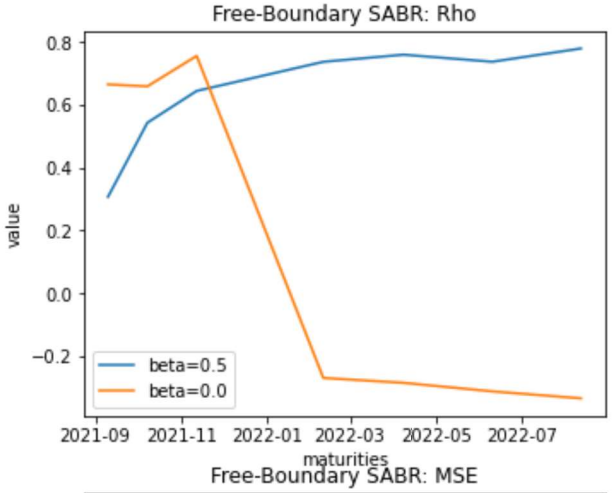


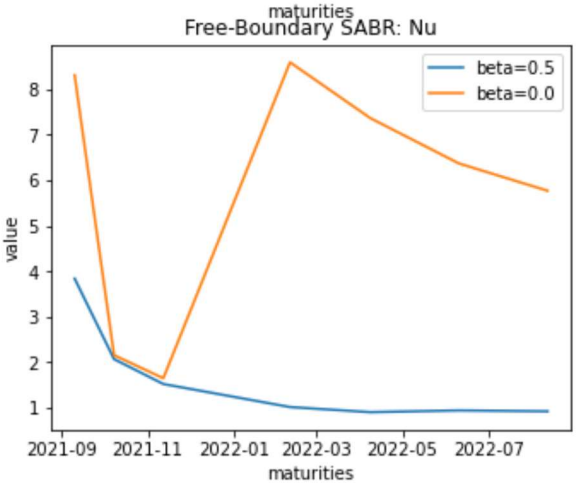
In [11]:

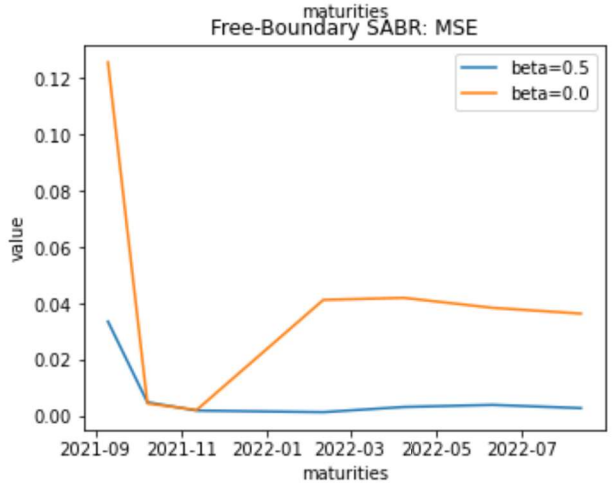
```
# Free-Boundary SABR Volatility model

freeSABR_beta5 = SABRVolatilitySurface(beta=.5, shift=0, method="floch-kennedy", lab
freeSABR_beta0 = SABRVolatilitySurface(beta=.0, shift=0, method="floch-kennedy", lab

SABRComparison([freeSABR_beta5, freeSABR_beta0], title="Free-Boundary SABR")
```

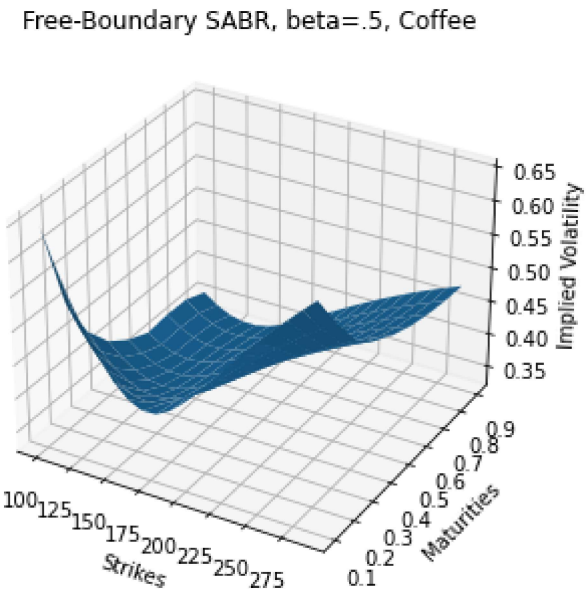
```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```


Free-Boundary SABR: Alpha
beta=0.5
beta=0.0
value
maturities
2021-09
2021-11
2022-01
2022-03
2022-05
2022-07

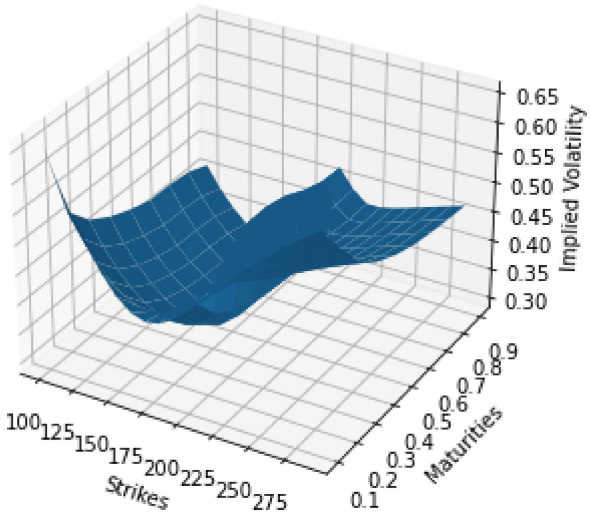

Free-Boundary SABR: Rho
beta=0.5
beta=0.0
value
maturities

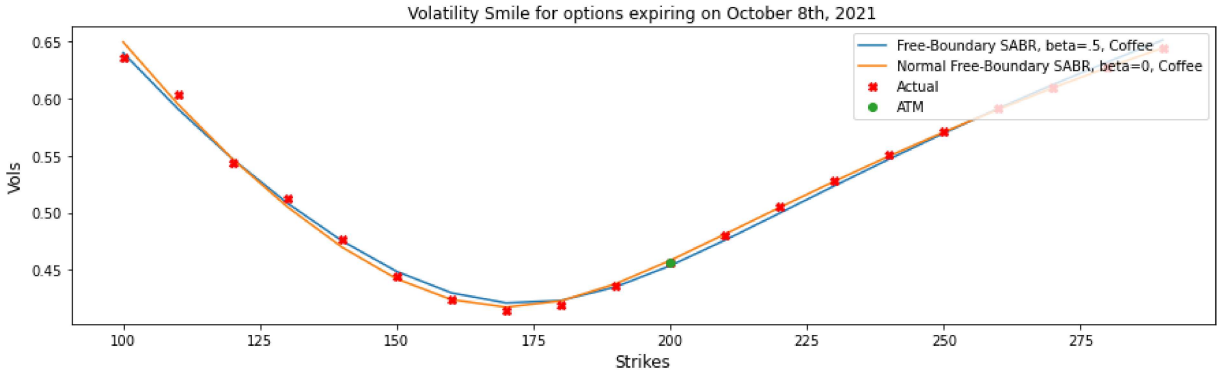

Free-Boundary SABR: Nu
beta=0.5
beta=0.0
value
maturities

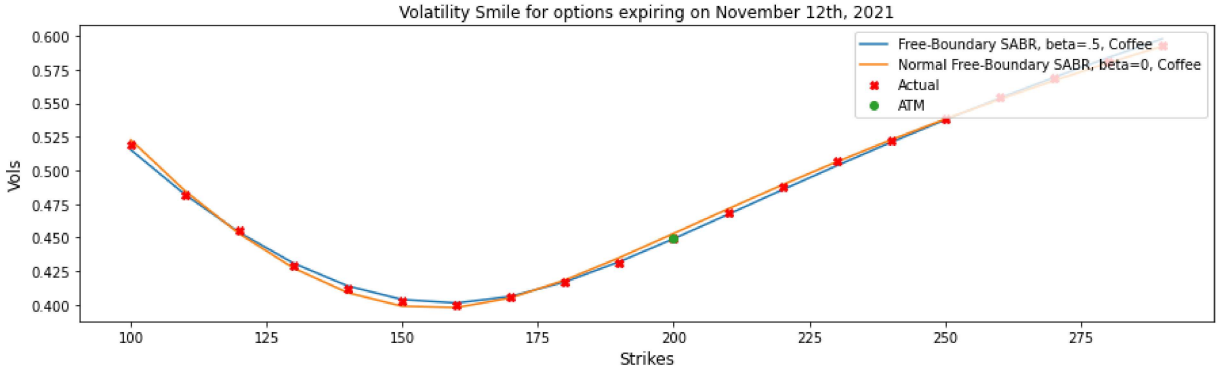

Free-Boundary SABR: MSE
beta=0.5
beta=0.0
value
maturities

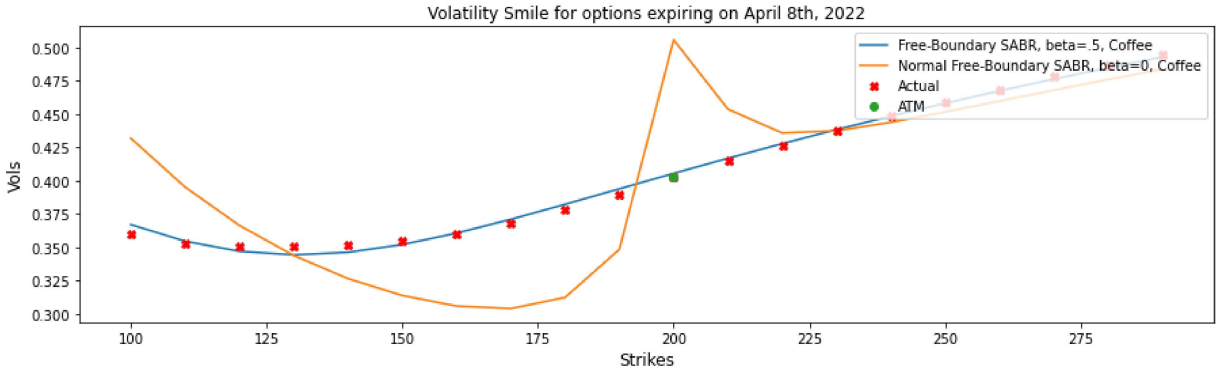

Free-Boundary SABR, beta=.5, Coffee
Implied Volatility
Strikes
Maturities

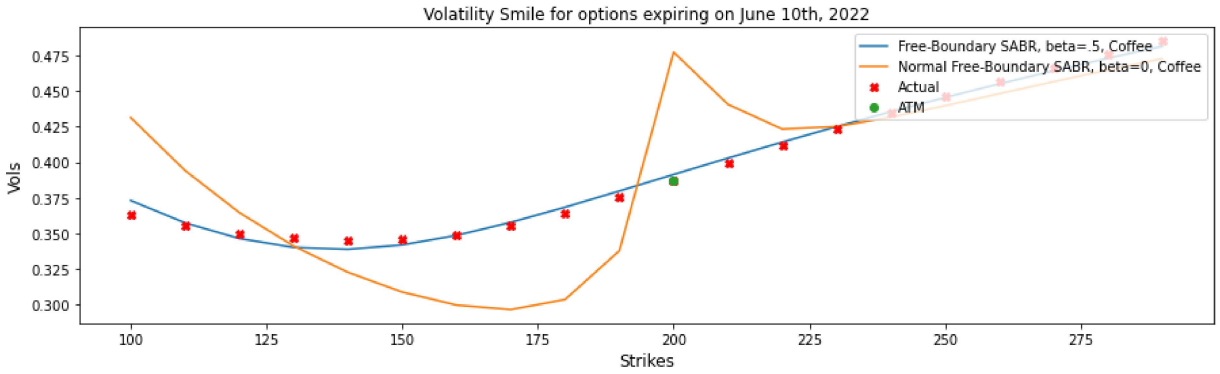
Normal Free-Boundary SABR, beta=0, Coffee

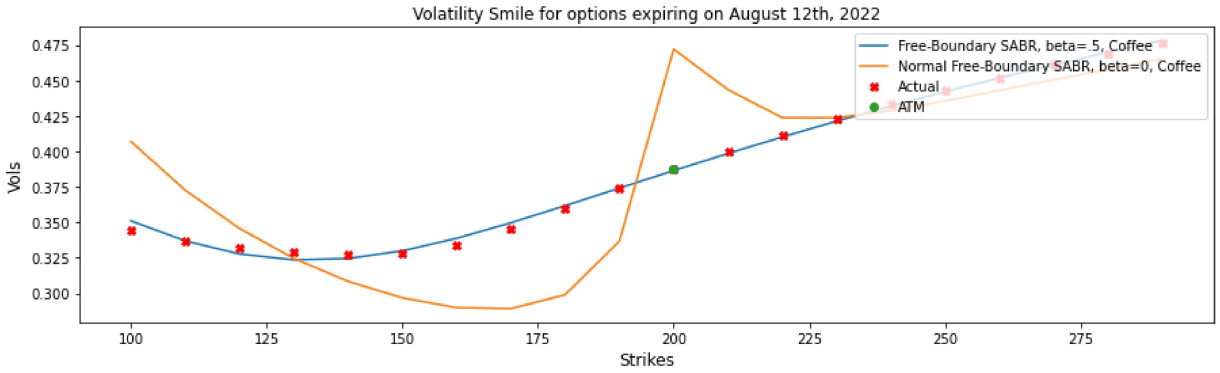










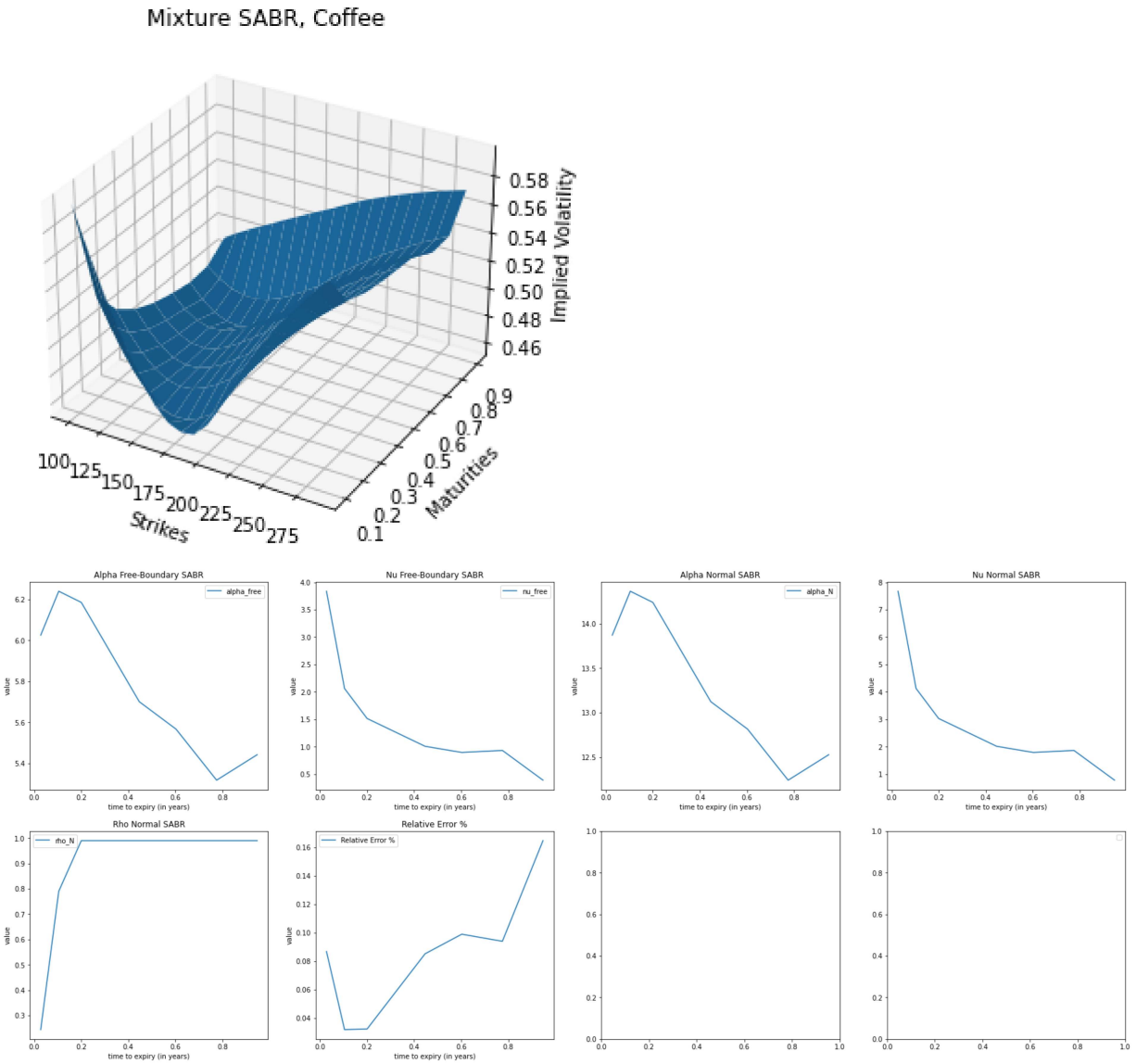


In [12]:

```
# Mixed SABR

tenors = dates[6:] if data == "SPX" else dates
mixtureSABR = MixtureSABRVolatilitySurface(dates=tenors)
display(mixtureSABR.to_data())
plot_vol_surface([mixtureSABR.vol_surface], title=mixtureSABR.label)
mixtureSABR.plot()
```

	alpha_free	beta_free	nu_free	rho_free	alpha_N	beta_N	nu_N	rho_N	MSI
September 10th, 2021	6.025622	0.5	3.837102	0.0	6.025622	0.0	7.674203	0.244219	0.08677
October 8th, 2021	6.240184	0.5	2.063050	0.0	6.240184	0.0	4.126099	0.790108	0.03182
November 12th, 2021	6.185562	0.5	1.515953	0.0	6.185562	0.0	3.031906	0.990000	0.03222
February 10th, 2022	5.700999	0.5	1.008944	0.0	5.700999	0.0	2.017888	0.990000	0.08515
April 8th, 2022	5.567359	0.5	0.896325	0.0	5.567359	0.0	1.792650	0.990000	0.09892
June 10th, 2022	5.318200	0.5	0.933453	0.0	5.318200	0.0	1.866906	0.990000	0.09392
August 12th, 2022	5.442629	0.5	0.390795	0.0	5.442629	0.0	0.781591	0.990000	0.16470

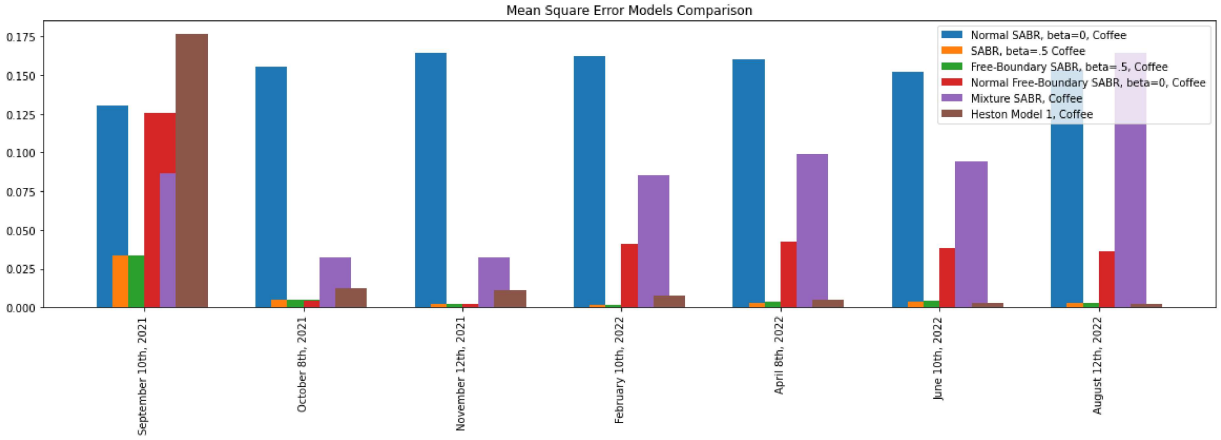
```
No handles with labels found to put in legend.
```



In [13]:

```
# VOLATILITY SMILES ERRORS COMPARISON



# fix Mixture SABR errors due to missing dates
mixtureSABR_errors = None
if data == "SPX":
    np.concatenate((np.zeros(6), mixtureSABR.errors))
else:
     mixtureSABR_errors = mixtureSABR.errors

fig = plt.figure(figsize=(20, 5), )
width = .20
plt.bar(np.arange(len(maturities)) - 2*width/2, SABR_beta0.errors, label=SABR_beta0.
plt.bar(np.arange(len(maturities)) - width/2, SABR_beta5.errors, label=SABR_beta5.la
plt.bar(np.arange(len(maturities)) , freeSABR_beta5.errors, label=freeSABR_beta5.lab
plt.bar(np.arange(len(maturities)) + width/2, freeSABR_beta0.errors, label=freeSABR_
plt.bar(np.arange(len(maturities)) + 2*width/2, mixtureSABR_errors, label=mixtureSAB
plt.bar(np.arange(len(maturities)) + 3*width/2, hestonModel1.errors, label=hestonMod
plt.xticks(np.arange(len(maturities)), dates, rotation='vertical')
plt.title("Mean Square Error Models Comparison")
plt.legend()
```

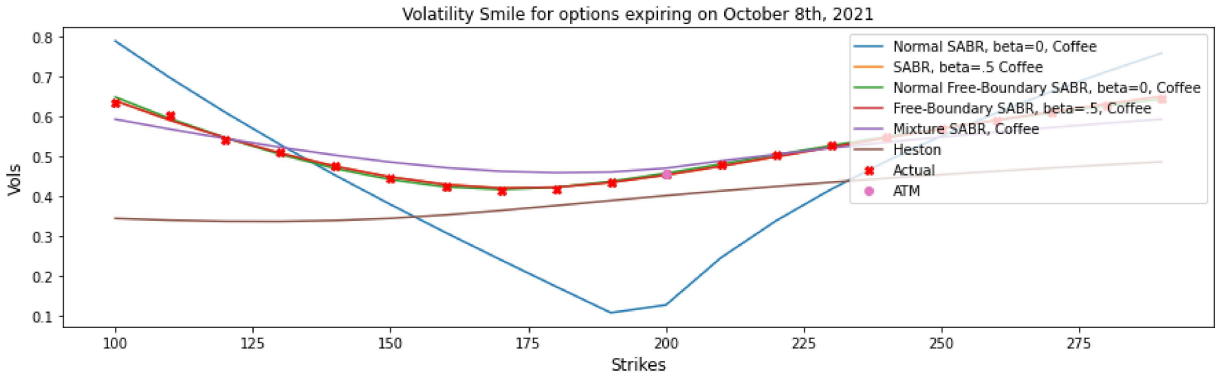
Out[13]: <matplotlib.legend.Legend at 0x26b19ce8850>

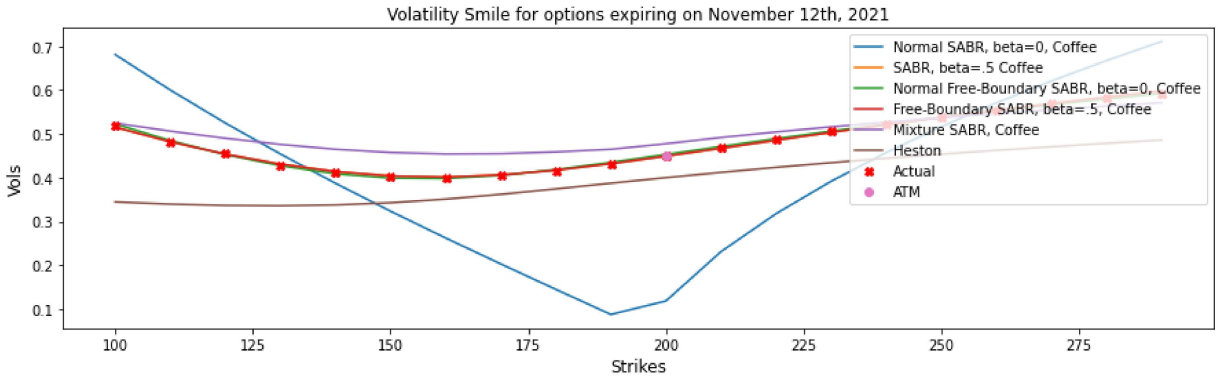


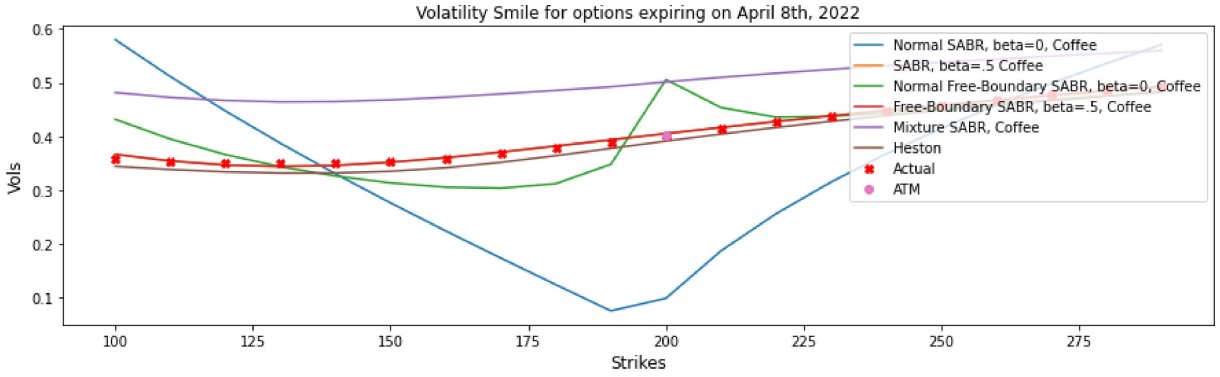
In [14]:

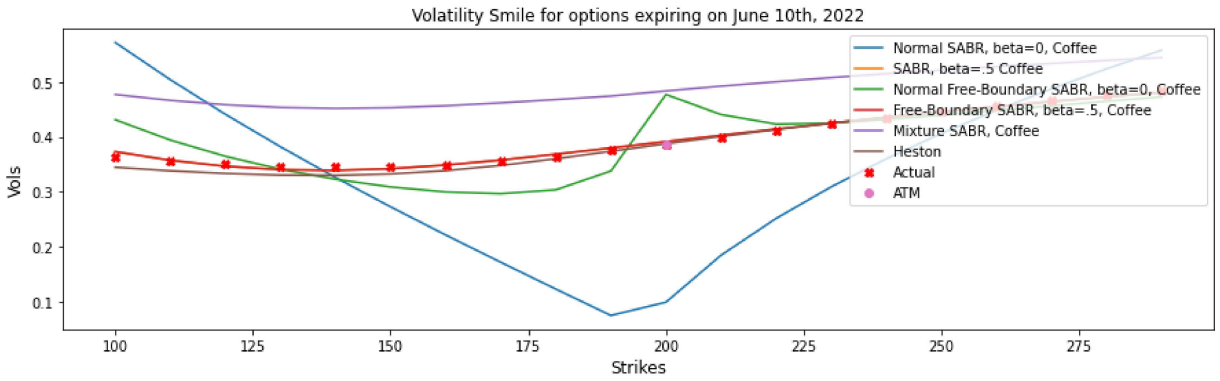
```
# Volatility Smiles Comparisons (Final)

models = (
        SABR_beta0,
        SABR_beta5,
        freeSABR_beta0,
        freeSABR_beta5,
        mixtureSABR
        )

errors_data = pd.DataFrame([np.mean(m.errors) for m in models], index=[m.label for m
display(errors_data)

smiles_comparison(models, heston_models=[hestonModel1])
```

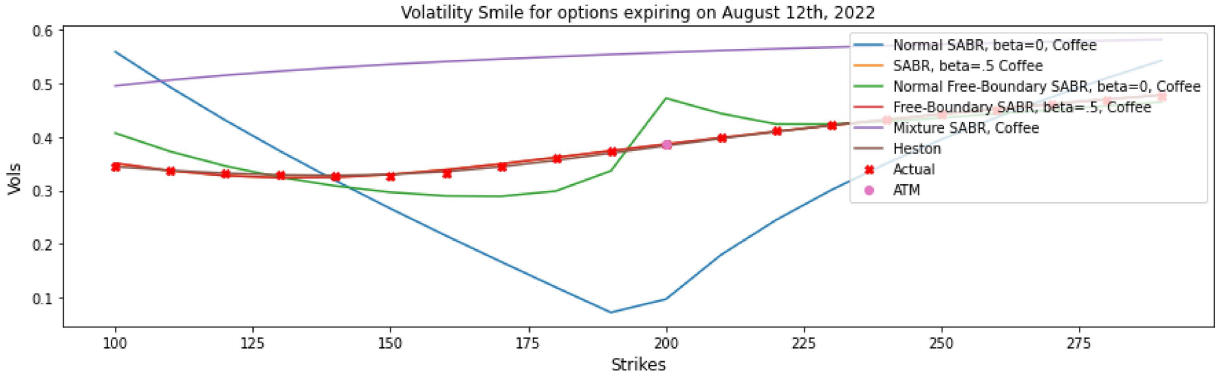
	Error
Normal SABR, beta=0, Coffee	0.154055
SABR, beta=.5 Coffee	0.007454
Normal Free-Boundary SABR, beta=0, Coffee	0.041553
Free-Boundary SABR, beta=.5, Coffee	0.007456
Mixture SABR, Coffee	0.084790









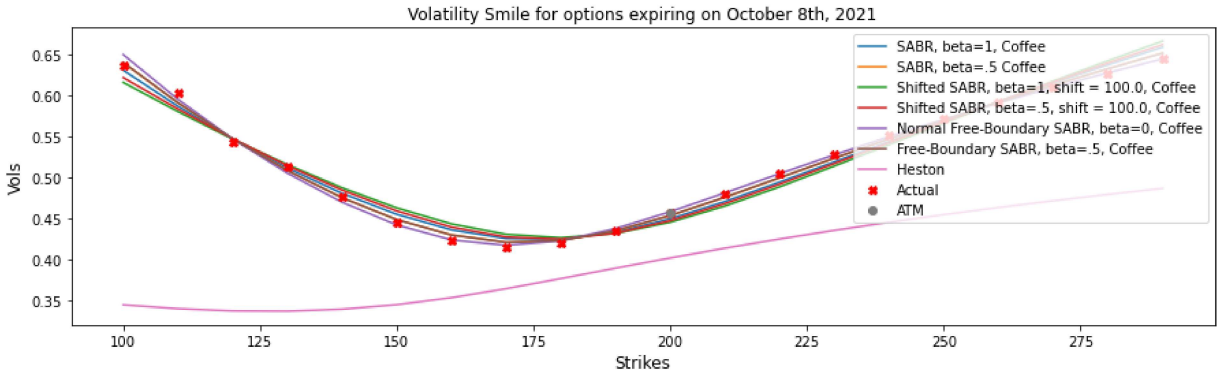


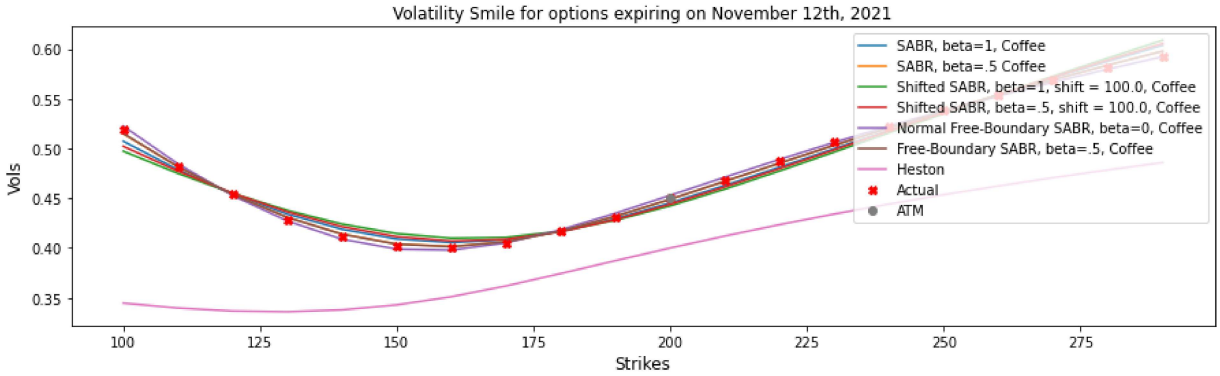
In [15]:

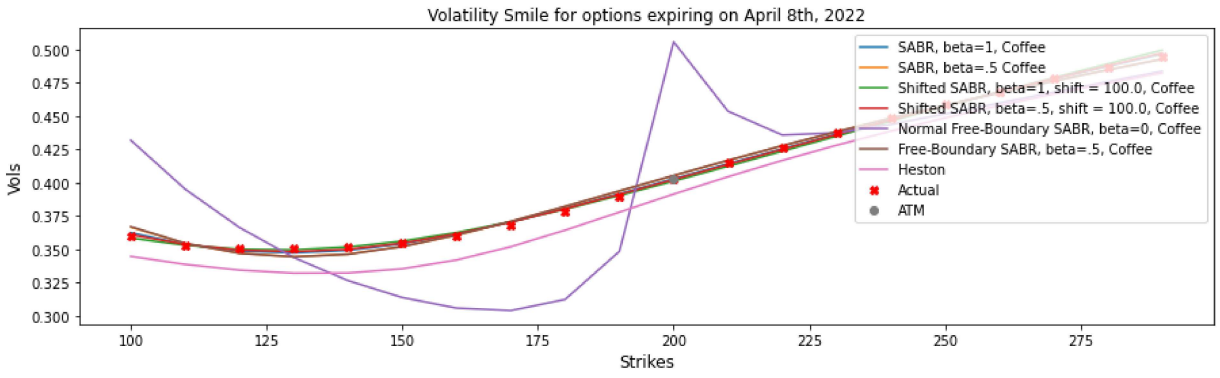
```
# Volatility Smiles Comparisons 2 (Final)

models = (
        SABR_beta1,
        SABR_beta5,
        shiftedSABR_beta1,
        shiftedSABR_beta5,
        freeSABR_beta0,
        freeSABR_beta5,
        )

errors_data = pd.DataFrame([np.mean(m.errors) for m in models], index=[m.label for m
display(errors_data)
```

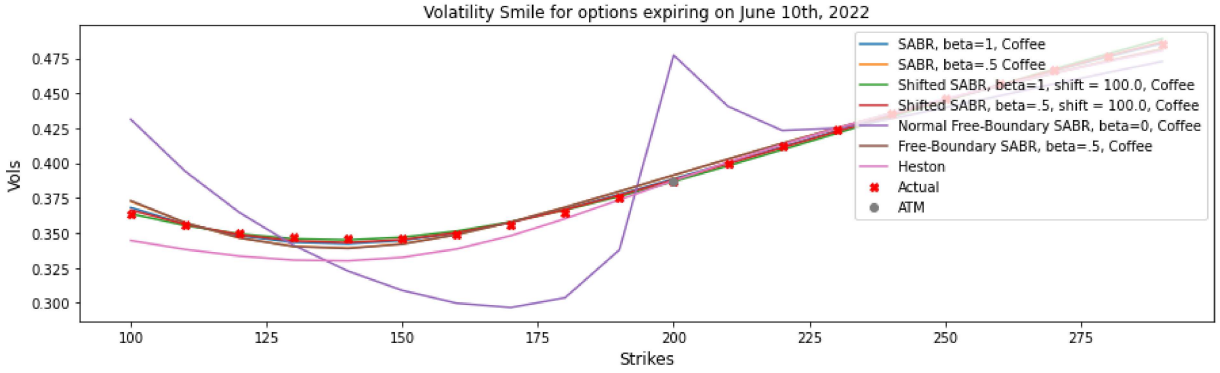
```
smiles_comparison(models, heston_models=[hestonModel1])
```

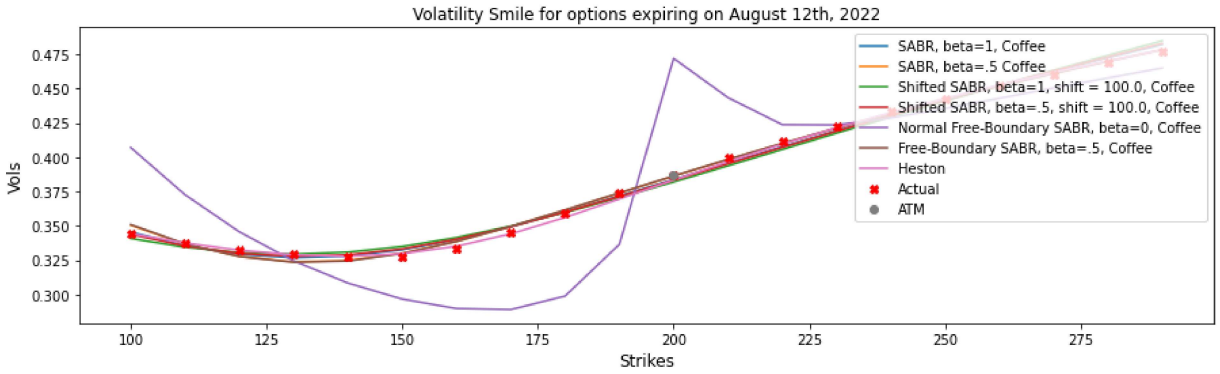
	Error
SABR, beta=1, Coffee	0.027006
SABR, beta=.5 Coffee	0.007454
Shifted SABR, beta=1, shift = 100.0, Coffee	0.028827
Shifted SABR, beta=.5, shift = 100.0, Coffee	0.010505
Normal Free-Boundary SABR, beta=0, Coffee	0.041553
Free-Boundary SABR, beta=.5, Coffee	0.007456





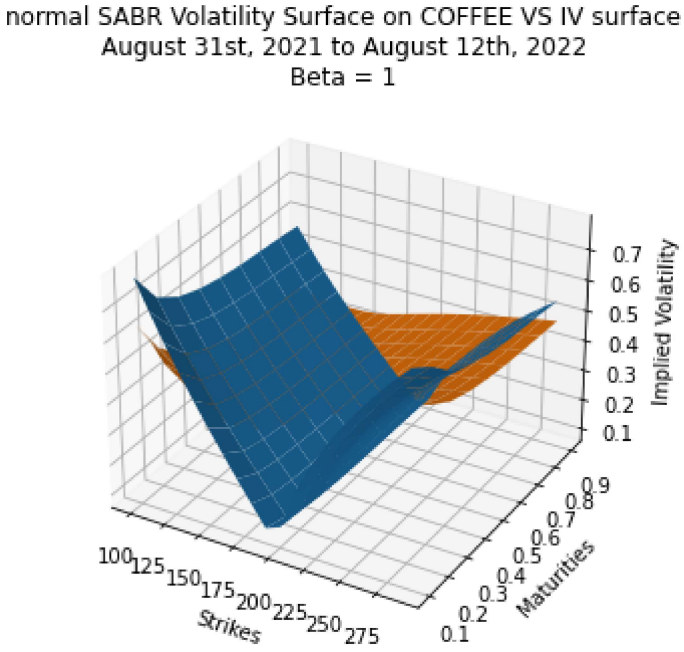




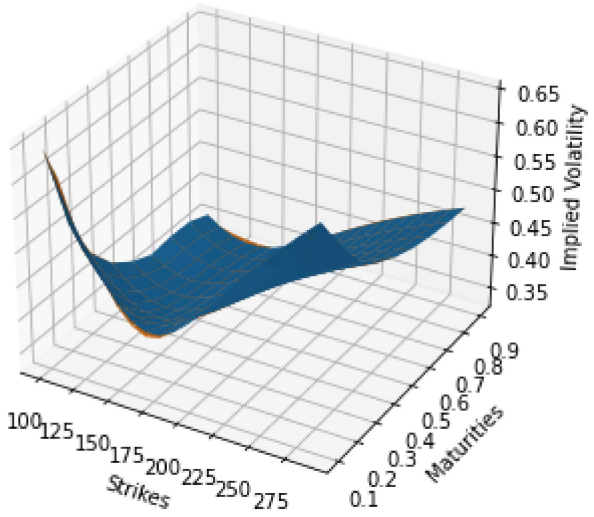


In [16]:

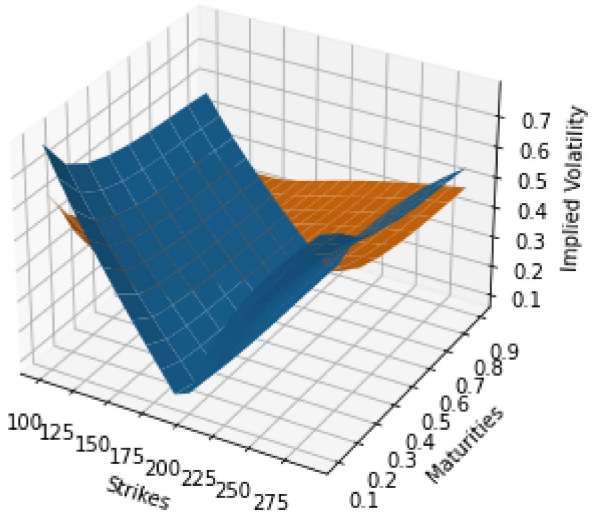
```
# Volatility Surfaces plots comparison

title = "normal SABR Volatility Surface on {} VS IV surface\n{} to {}\nBeta = 1".for
plot_vol_surface([SABR_beta0.vol_surface, black_var_surface], title=title)

title = "normal SABR Volatility Surface on {} VS IV surface\n{} to {}\nBeta = 0.5".f
plot_vol_surface([SABR_beta5.vol_surface, black_var_surface], title=title)

title = "normal SABR Volatility Surface on {} VS IV surface\n{} to {}\nBeta = 0".for
plot_vol_surface([SABR_beta0.vol_surface, black_var_surface], title=title)

title = "SABR Volatility Surface on {} VS Heston surface\n{} to {}\nBeta = 1".format
plot_vol_surface([SABR_beta1.vol_surface, hestonModel1.heston_vol_surface], title=ti

title = "IV Surface on {} vs Heston Surface\n{} to {}\nBeta = 1".format(data, today,
plot_vol_surface([black_var_surface, hestonModel1.heston_vol_surface], title=title)
```



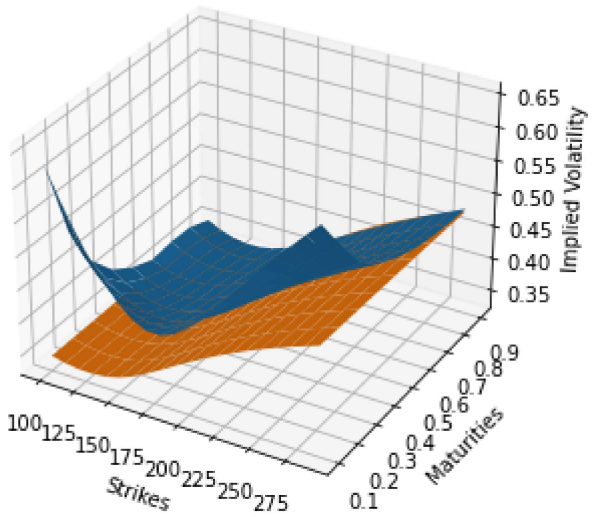
normal SABR Volatility Surface on COFFEE VS IV surface
August 31st, 2021 to August 12th, 2022
Beta = 0.5

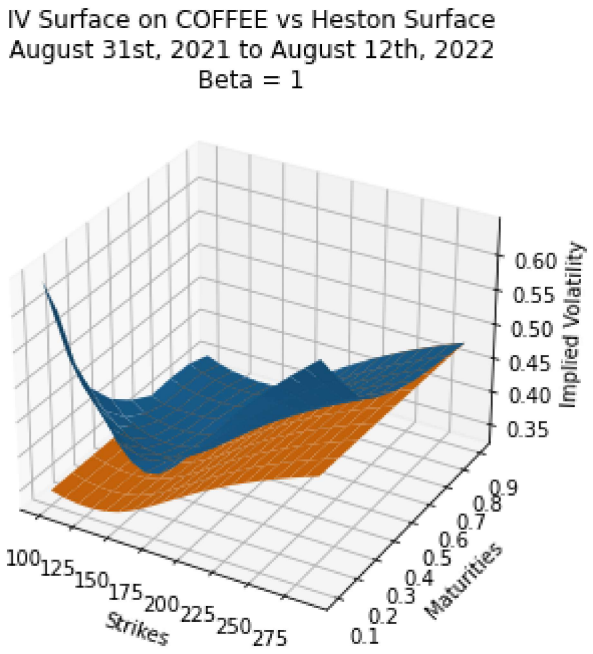


normal SABR Volatility Surface on COFFEE VS IV surface
August 31st, 2021 to August 12th, 2022
Beta = 0



SABR Volatility Surface on COFFEE VS Heston surface
August 31st, 2021 to August 12th, 2022
Beta = 1

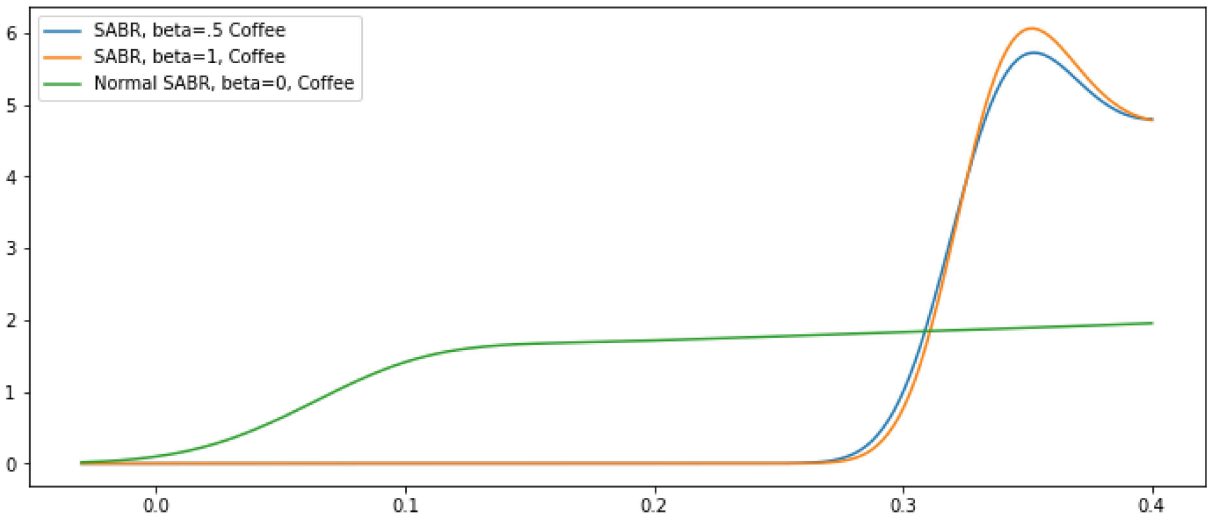




In [17]:

```
from scipy.stats import gaussian_kde
import seaborn as sns
plt.figure(figsize=plot_size)
def density(model):
    rn = []
    for i in np.arange(0, 1.5, .01):
        for j in np.arange(model.vol_surface.minStrike(), model.vol_surface.maxStrik
            rn.append(model.vol_surface.blackVol(i,j))

    density = gaussian_kde(rn)
    xs = np.linspace(-.03,.4,200)
    density.covariance_factor = lambda : .25
    density._compute_covariance()
    plt.plot(xs,density(xs), label=model.label)

    plt.legend()



density(SABR_beta5), density(SABR_beta1), density(SABR_beta0),
```

Out[17]: (None, None, None)

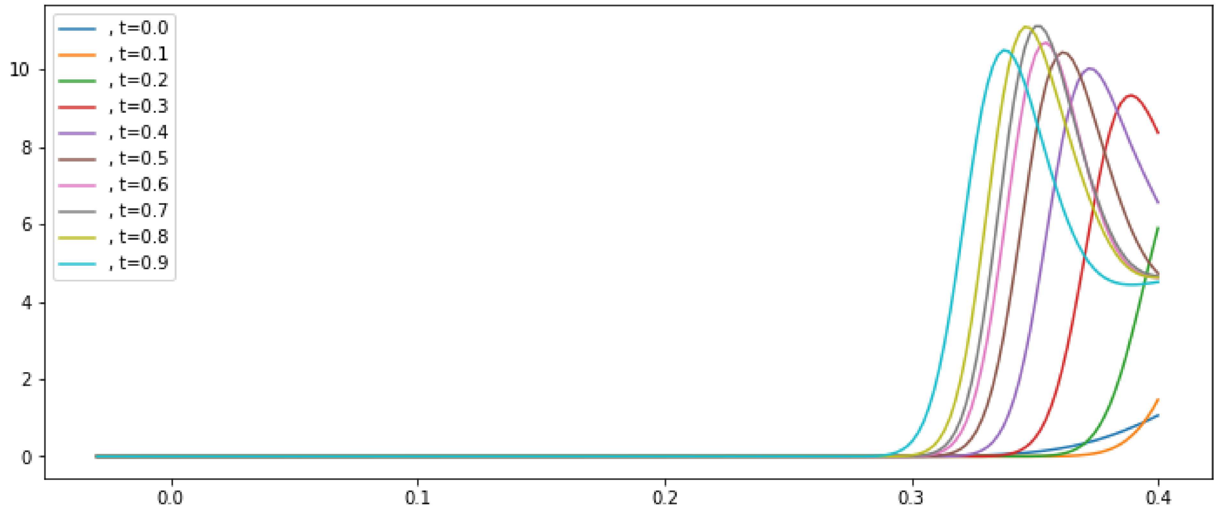


In [ ]:

```
from scipy.stats import gaussian_kde
```

```
plt.figure(figsize=plot_size)
def density(model, t):
    rn = []

    for j in np.arange(model.vol_surface.minStrike(), model.vol_surface.maxStrike(),
        rn.append(model.vol_surface.blackVol(t,j))

    density = gaussian_kde(rn)
    xs = np.linspace(-.03,.4,200)
    density.covariance_factor = lambda : .25
    density._compute_covariance()
    plt.plot(xs,density(xs), label=model.label+", t="+str(round(t, 2)))

    plt.legend()


for i in np.arange(0,1,.1):
    density(SABRVolatilitySurface(beta=.5, zero_rho=True), i)
plt.show()
for i in np.arange(0,1,.1):
    density(SABRVolatilitySurface(beta=.5, zero_rho=False), i)
```

```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```




```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```

In [ ]: