In [1]:

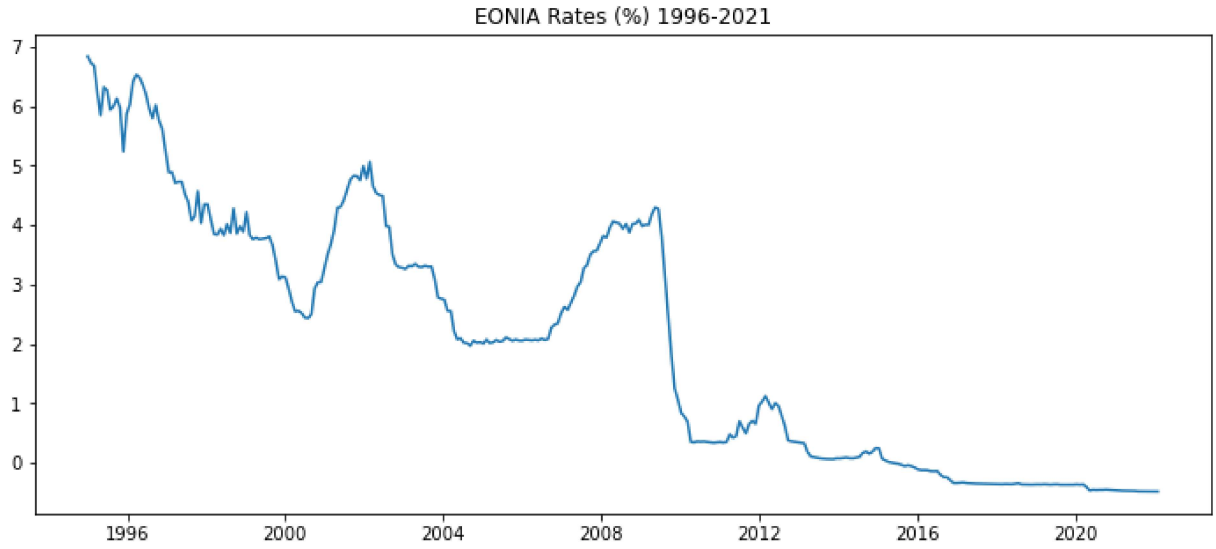
```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np
import pandas as pd
import time
import datetime
import math
import QuantLib as ql
from scipy.optimize import minimize

from initialize import *
from plotting import *
from SABR import *
from Heston import *
from mixedSABR import *
```

In [2]:

```
# Spot rates table and chart (EONIA)

rates = [-0.575, -0.557, -0.549, -0.529, -0.494]
tenors = [.25, 1, 3, 6, 12]

spot_rates = pd.DataFrame({"Tenors": tenors, "Spot Rate": rates})
spot_rates.set_index('Tenors')

display(spot_rates)

fig = plt.figure(figsize=(12,5))
eonia_dates = [datetime.date(1994, 12, 31) + datetime.timedelta(days=30*n) for n in
plt.plot(eonia_dates, eonia_rates['value'])
plt.title("EONIA Rates (%) 1996-2021")
```

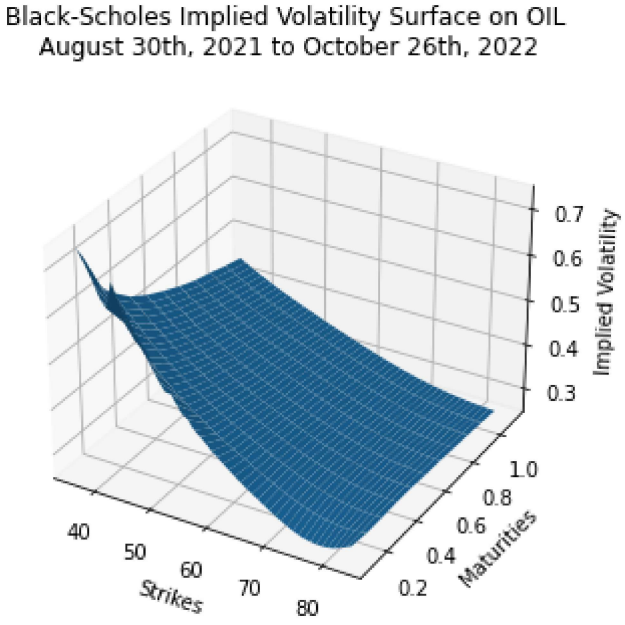
	Tenors	Spot Rate
0	0.25	-0.575
1	1.00	-0.557
2	3.00	-0.549
3	6.00	-0.529
4	12.00	-0.494

Out[2]: Text(0.5, 1.0, 'EONIA Rates (%) 1996-2021')



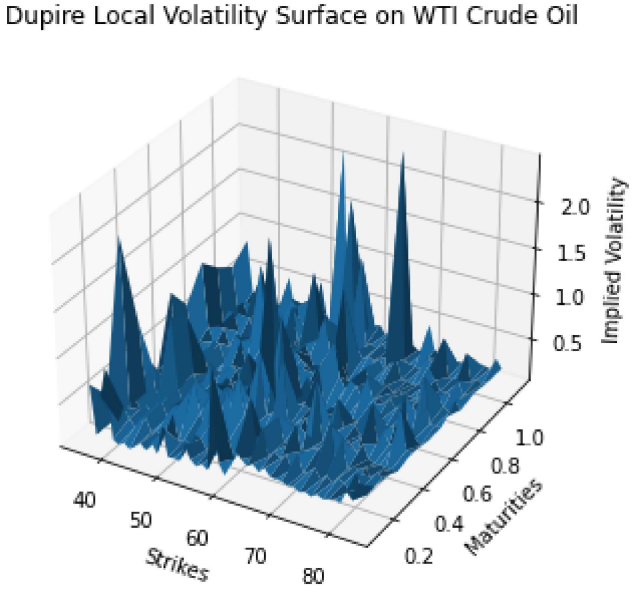
In [3]:

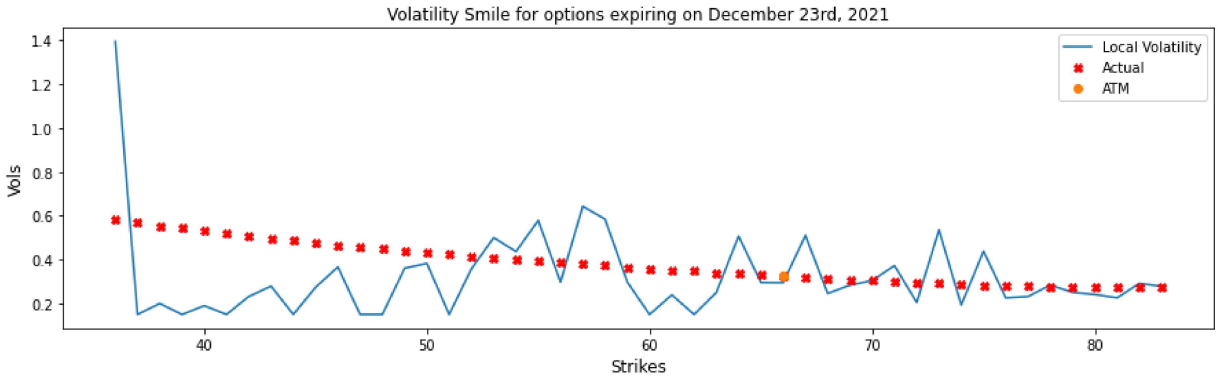
```
# BLACK VOLATILITY SURFACE

title = "Black-Scholes Implied Volatility Surface on {}\n {} to {}".format(data, tod
plot_vol_surface(vol_surface=black_var_surface, plot_strikes=strikes, funct='blackVo
```

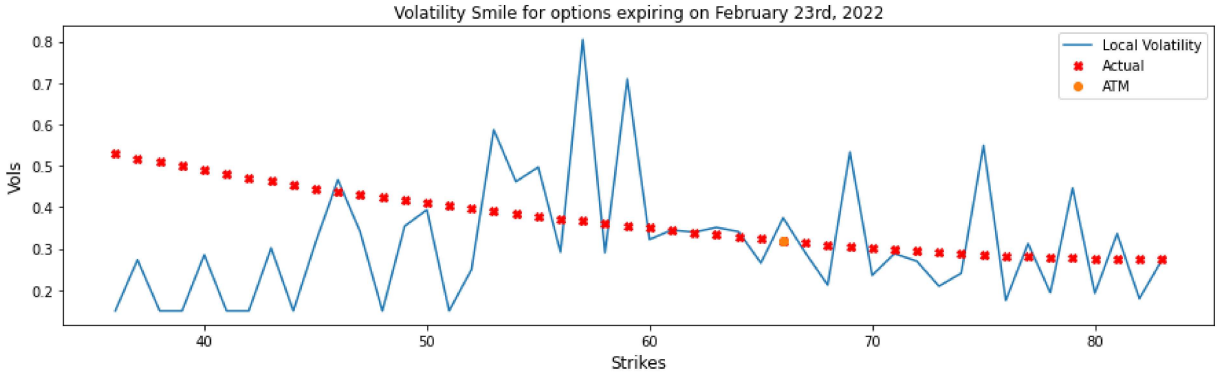


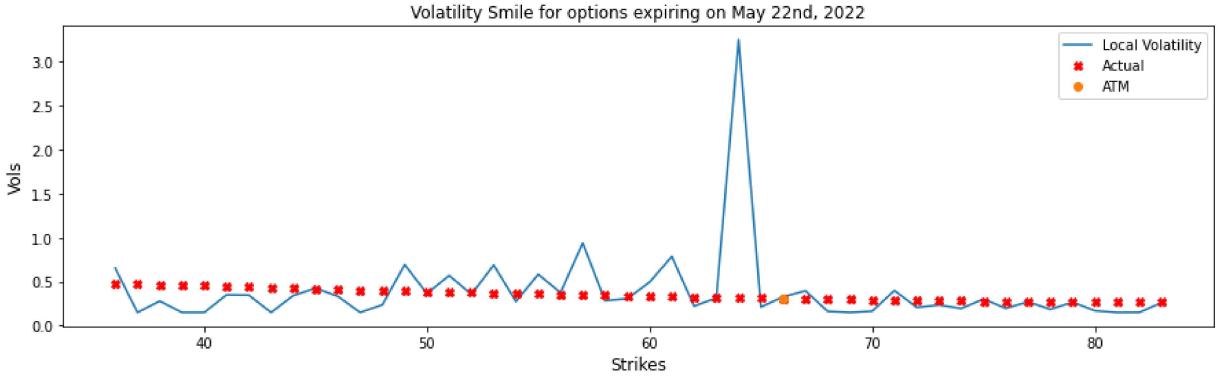
In [4]:

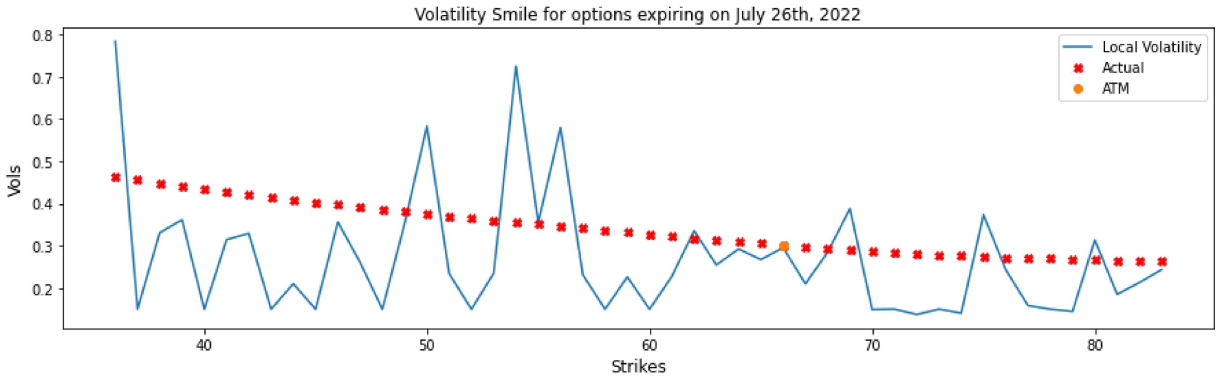
```
#DUPIRE LOCAL VOLATILITY SURFACE (NOT PLOTTABLE)

black_var_surface.setInterpolation("bicubic")
local_vol_handle = ql.BlackVolTermStructureHandle(black_var_surface)

# local_vol_surface = ql.LocalVolSurface(local_vol_handle, flat_ts, dividend_ts, spo
local_vol_surface = ql.NoExceptLocalVolSurface(local_vol_handle, flat_ts, dividend_t

# Plot the Dupire surface ...
local_vol_surface.enableExtrapolation()
plot_vol_surface(local_vol_surface, funct='localVol', title="Dupire Local Volatility

smiles_comparison(local_models=[local_vol_surface], black_volatility=False)
```

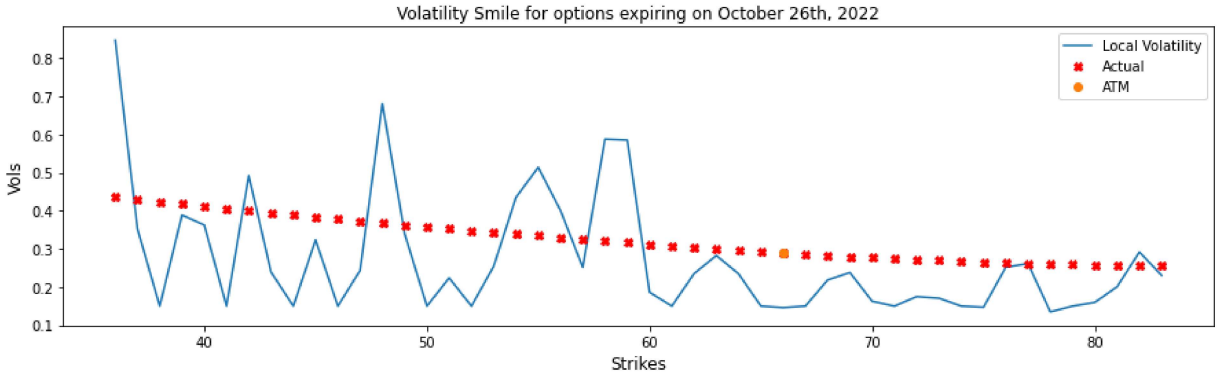












In [5]:

```
#HESTON MODEL SURFACE PLOTTING (Levenberg-Marquardt Method)
```

```
m1_params, m2_params = (None, None)
if data == "COFFEE":
    m1_params = (0.01,0.1,0.3,0.1,0.02)
    m2_params = (0.2,0.9,0.9,0.9,-0.19)
elif data == "OIL":
    m2_params = (0.023,0.009,1.00,0.95,0.2)
    m1_params = (0.15,0.5,0.2,0.7,0.01)
elif data == "GOLD":
    m1_params = (0.03,0.3,0.5,0.3,0.04)
    m2_params = (0.01,0.5,0.5,0.1,0.03)
elif data == "SILVER":
    m2_params = (0.5,0.5,1.25,0.3,0.00)
    m1_params = (0.01,0.5,0.5,0.1,0.03)


hestonModel1 = hestonModelSurface(m1_params, label="Heston Model 1, {}".format(data_
hestonModel2 = hestonModelSurface(m2_params, label="Heston Model 2, {}".format(data_

# Use to Calibrate first time the Heston Model
def calibrateHeston():
    def f(params):
        return hestonModelSurface(params).avgError
        # v0, kappa, theta, sigma, rho
    cons = (
            {'type': 'ineq', 'fun': lambda x: x[0] - 0.001},
            {'type': 'ineq', 'fun': lambda x: 2. - x[0]},
            {'type': 'ineq', 'fun': lambda x: x[1] - 0.001},
            {'type': 'ineq', 'fun': lambda x: 2. - x[1]},
            {'type': 'ineq', 'fun': lambda x: x[2] - 0.001},
            {'type': 'ineq', 'fun': lambda x: 2. - x[2]},
            {'type': 'ineq', 'fun': lambda x: x[3] - 0.001},
            {'type': 'ineq', 'fun': lambda x: .999 - x[3]},
            {'type': 'ineq', 'fun': lambda x: .99 - x[4]**2}
            )
    result = minimize(f, m1_params, constraints=cons, method="SLSQP", bounds=((1e-8,
    hestonModel0 = hestonModelSurface(result["x"], label="Heston Model 0, {}".format

    print(result["x"])

    plot_vol_surface(hestonModel0.heston_vol_surface, title="{} Volatility Surface".
    plot_vol_surface(hestonModel1.heston_vol_surface, title="{} Volatility Surface".

init_conditions = pd.DataFrame({"theta": [m1_params[0], m2_params[0]], "kappa": [m1_
                                "sigma": [m1_params[2], m2_params[2]], "rho": [m1_pa
                                "v0": [m1_params[4], m2_params[4]]}, index = ["Model
display(init_conditions.style.set_caption("Heston Model Initial Conditions on ({})".
```

Heston Model Initial Conditions on (WTI Crude Oil)

	theta	kappa	sigma	rho	v0
Model1	0.150000	0.500000	0.200000	0.700000	0.010000
Model2	0.023000	0.009000	1.000000	0.950000	0.200000

In [6]:

```
# HESTON Surface Plotting (Model1, Model2)

for model in (hestonModel1, hestonModel2):
    plot_vol_surface(model.heston_vol_surface, title="Heston Volatility Surface for
    display(model.errors_data.style.set_caption("{} calibration results".format(mode

    fig1 = plt.figure(figsize=plot_size)
    plt.plot(model.strks, model.marketValue, label="Market Value")
    plt.plot(model.strks, model.modelValue, label="Model Value")
```

```
    plt.title('Model1: Heston surface Market vs Model Value'); plt.xlabel='strikes';
    plt.legend()
    fig2 = plt.figure(figsize=plot_size)
    plt.plot(model.strks, model.relativeError)
    plt.title('Model1: Heston surface Relative Error (%)'); plt.xlabel='strikes'; pl
    plt.legend()
```

Heston Model 1, WTI Crude Oil calibration results

	Strikes	Market Value	Model Value	Relative Error (%)
0	36.000000	0.958086	1.199101	25.155870
1	37.000000	1.037192	1.312911	26.583265
2	38.000000	1.125773	1.434219	27.398577
3	39.000000	1.214808	1.563324	28.688957
4	40.000000	1.313419	1.700528	29.473371
5	41.000000	1.422697	1.846141	29.763453
6	42.000000	1.531639	2.000477	30.610180
7	43.000000	1.649683	2.163855	31.167928
8	44.000000	1.788489	2.336599	30.646502
9	45.000000	1.927791	2.519038	30.669656
10	46.000000	2.075759	2.711506	30.627235
11	47.000000	2.224970	2.914342	30.983444
12	48.000000	2.392998	3.127887	30.709959
13	49.000000	2.571337	3.352486	30.379097
14	50.000000	2.758713	3.588488	30.078368
15	51.000000	2.946000	3.836244	30.218741
16	52.000000	3.153999	4.096106	29.870250
17	53.000000	3.372193	4.368427	29.542604
18	54.000000	3.609428	4.653560	28.927910
19	55.000000	3.858208	4.951856	28.346005
20	56.000000	4.114372	5.263664	27.933599
21	57.000000	4.382562	5.589330	27.535677
22	58.000000	4.670428	5.929191	26.951766
23	59.000000	4.966610	6.283581	26.516499
24	60.000000	5.273682	6.652822	26.151370
25	61.000000	5.610139	7.037226	25.437650
26	62.000000	5.956183	7.437090	24.863361
27	63.000000	6.322759	7.852698	24.197326
28	64.000000	6.710584	8.284311	23.451412
29	65.000000	7.117514	8.732174	22.685731
30	66.000000	7.532672	9.196506	22.088221

	Strikes	Market Value	Model Value	Relative Error (%)
31	67.000000	7.987682	9.677498	21.155268
32	68.000000	8.062401	9.783134	21.342698
33	69.000000	7.544439	9.292317	23.167766
34	70.000000	7.053301	8.818551	25.027283
35	71.000000	6.595024	8.361895	26.790971
36	72.000000	6.146337	7.922370	28.895785
37	73.000000	5.727596	7.499953	30.944179
38	74.000000	5.338542	7.094581	32.893623
39	75.000000	4.958786	6.706147	35.237674
40	76.000000	4.610897	6.334496	37.380986
41	77.000000	4.282964	5.979433	39.609701
42	78.000000	3.974510	5.640718	41.922347
43	79.000000	3.698365	5.318067	43.795067
44	80.000000	3.429545	5.011157	46.117240
45	81.000000	3.183419	4.719625	48.256511
46	82.000000	2.956429	4.443076	50.285215
47	83.000000	2.740365	4.181080	52.573819

Heston Model 1, WTI Crude Oil parameters output

	Value
v0	0.403305
kappa	0.910291
theta	0.992253
sigma	-0.562644
rho	0.000000
avgError	30.688544

```
No handles with labels found to put in legend.
```

Heston Model 2, WTI Crude Oil calibration results

	Strikes	Market Value	Model Value	Relative Error (%)
0	36.000000	0.958086	1.026927	7.185248
1	37.000000	1.037192	1.120166	7.999892
2	38.000000	1.125773	1.219387	8.315483
3	39.000000	1.214808	1.324851	9.058454
4	40.000000	1.313419	1.436829	9.396061
5	41.000000	1.422697	1.555603	9.341805
6	42.000000	1.531639	1.681467	9.782146

	Strikes	Market Value	Model Value	Relative Error (%)
7	43.000000	1.649683	1.814726	10.004535
8	44.000000	1.788489	1.955700	9.349272
9	45.000000	1.927791	2.104722	9.177934
10	46.000000	2.075759	2.262142	8.979025
11	47.000000	2.224970	2.428322	9.139572
12	48.000000	2.392998	2.603646	8.802682
13	49.000000	2.571337	2.788512	8.445993
14	50.000000	2.758713	2.983339	8.142434
15	51.000000	2.946000	3.188565	8.233719
16	52.000000	3.153999	3.404651	7.947132
17	53.000000	3.372193	3.632078	7.706693
18	54.000000	3.609428	3.871350	7.256612
19	55.000000	3.858208	4.122996	6.862959
20	56.000000	4.114372	4.387566	6.639987
21	57.000000	4.382562	4.665638	6.459139
22	58.000000	4.670428	4.957810	6.153208
23	59.000000	4.966610	5.264705	6.001981
24	60.000000	5.273682	5.586968	5.940558
25	61.000000	5.610139	5.925263	5.617047
26	62.000000	5.956183	6.280268	5.441144
27	63.000000	6.322759	6.652674	5.217892
28	64.000000	6.710584	7.043177	4.956237
29	65.000000	7.117514	7.452471	4.706088
30	66.000000	7.532672	7.881236	4.627360
31	67.000000	7.987682	8.330131	4.287214
32	68.000000	8.062401	8.407597	4.281554
33	69.000000	7.544439	7.892971	4.619718
34	70.000000	7.053301	7.400161	4.917696
35	71.000000	6.595024	6.929579	5.072842
36	72.000000	6.146337	6.481532	5.453577
37	73.000000	5.727596	6.056207	5.737340
38	74.000000	5.338542	5.653655	5.902615
39	75.000000	4.958786	5.273783	6.352299
40	76.000000	4.610897	4.916345	6.624483
41	77.000000	4.282964	4.580948	6.957428
42	78.000000	3.974510	4.267052	7.360451

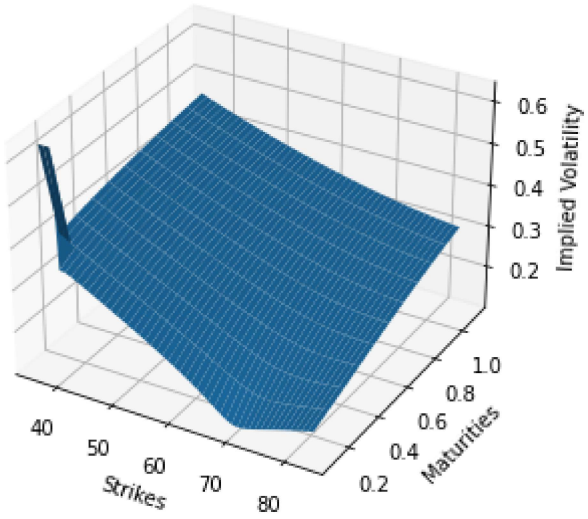
	Strikes	Market Value	Model Value	Relative Error (%)
43	79.000000	3.698365	3.973984	7.452447
44	80.000000	3.429545	3.700956	7.913910
45	81.000000	3.183419	3.447084	8.282467
46	82.000000	2.956429	3.211411	8.624644
47	83.000000	2.740365	2.992929	9.216412

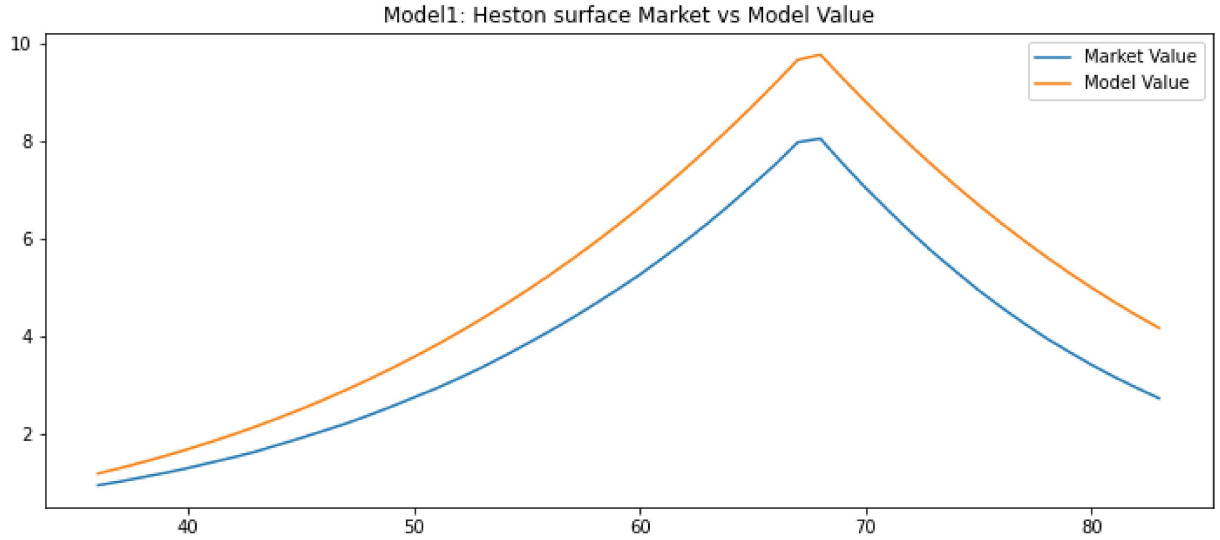
Heston Model 2, WTI Crude Oil parameters output

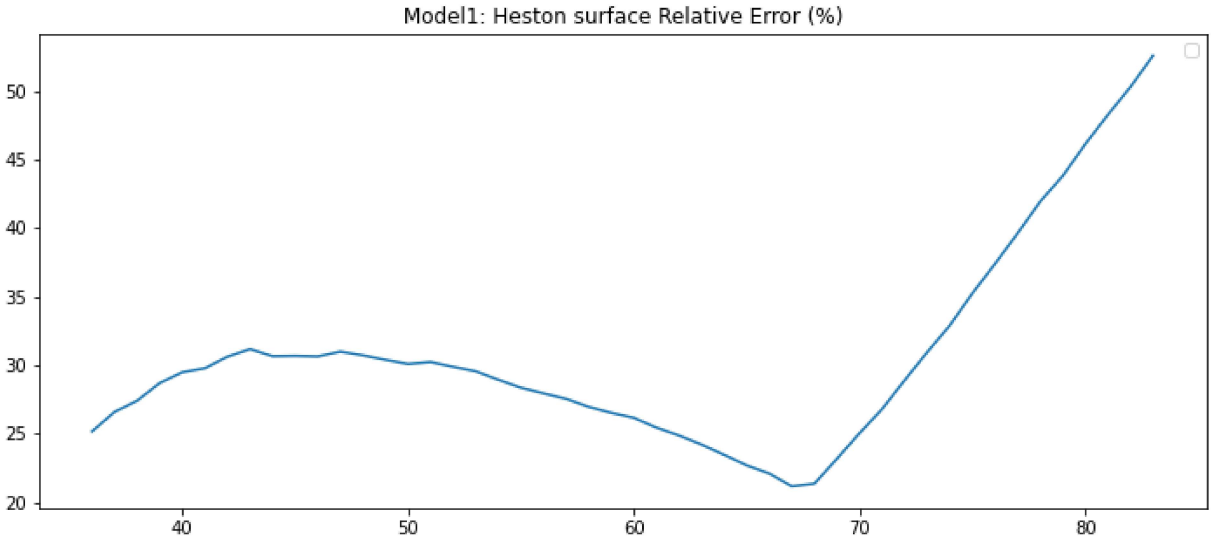
	Value
v0	4.924425
kappa	0.046090
theta	0.980092
sigma	-0.549255
rho	0.000000
avgError	7.123862

```
No handles with labels found to put in legend.
```

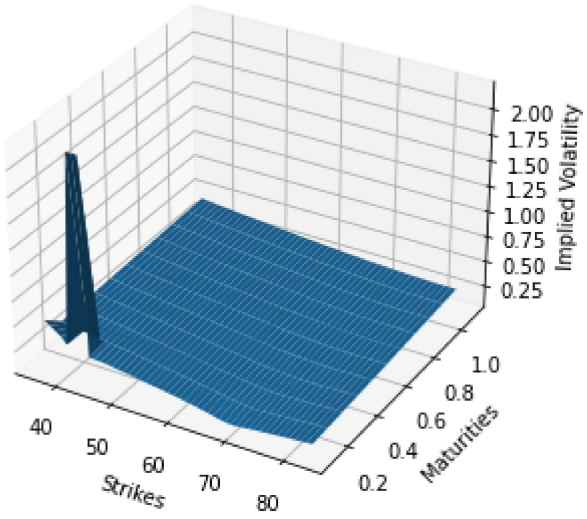
Heston Volatility Surface for Heston Model 1, WTI Crude Oil

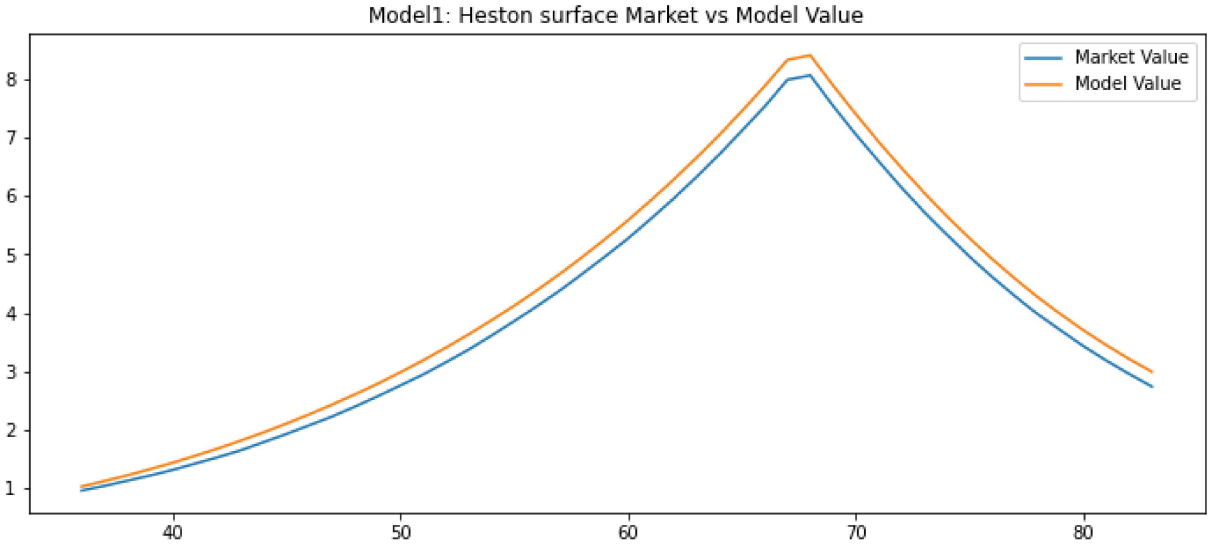


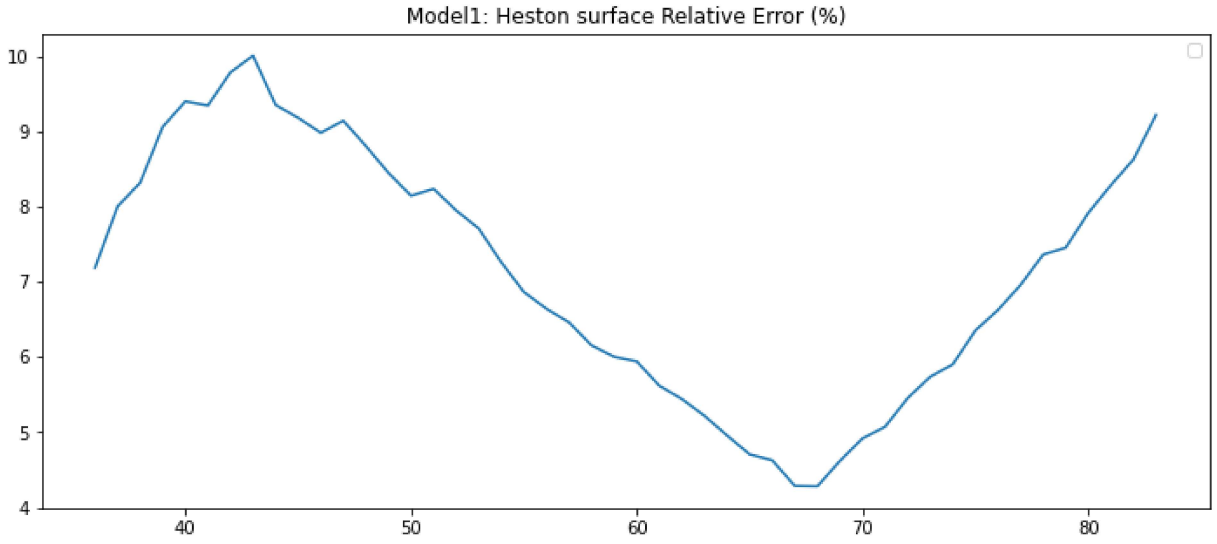




Heston Volatility Surface for Heston Model 2, WTI Crude Oil

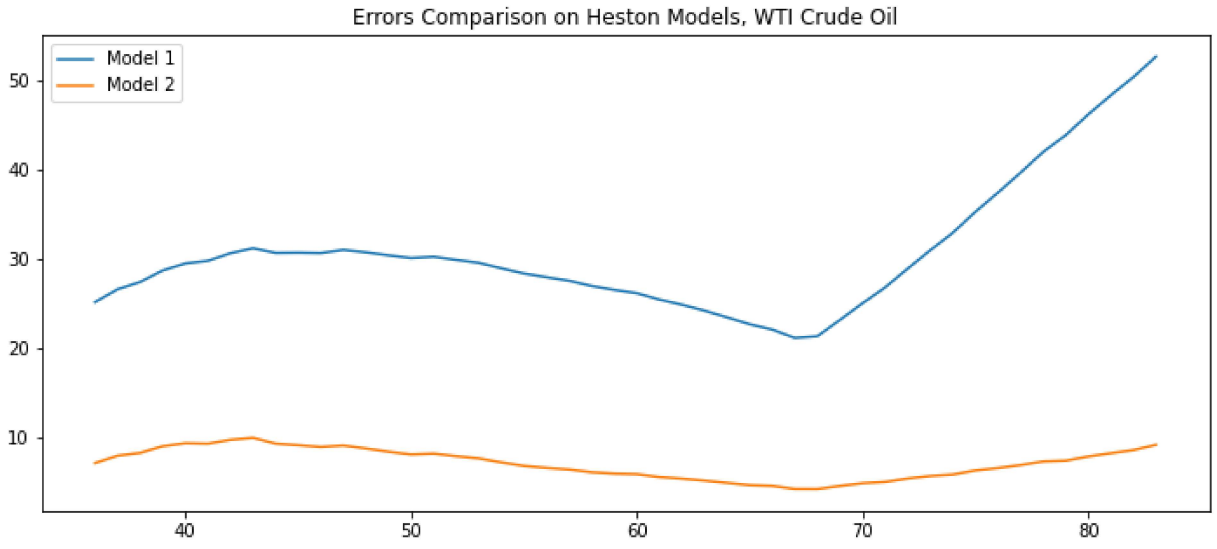


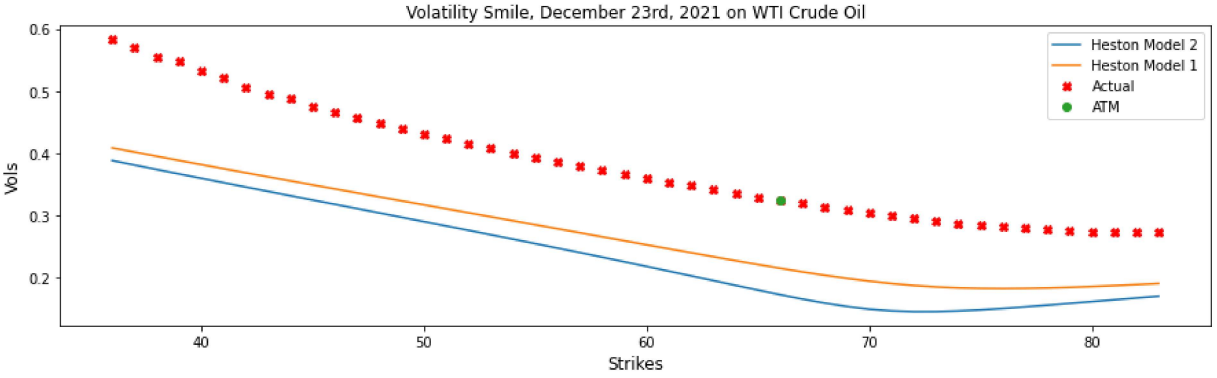


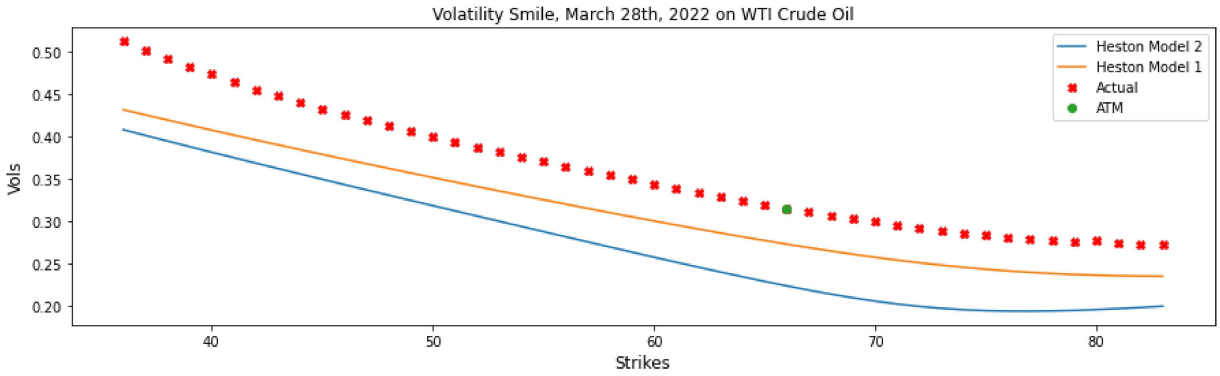


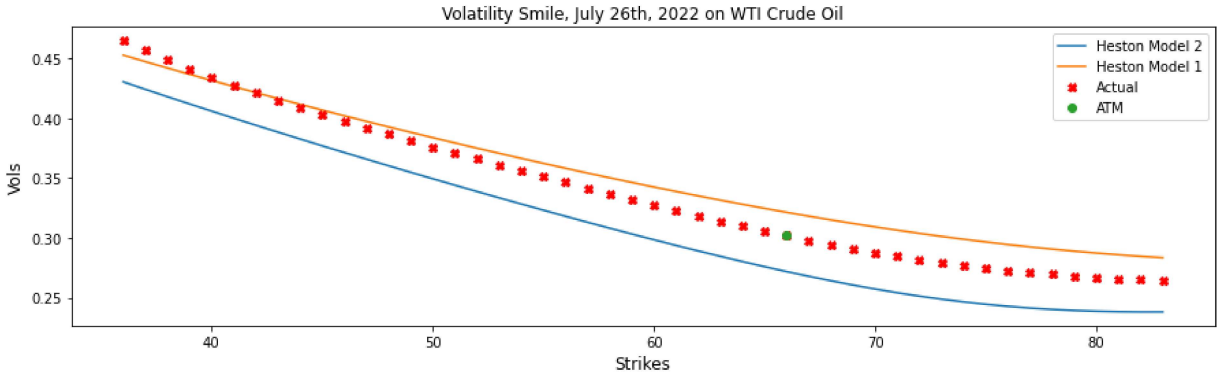
In [7]:

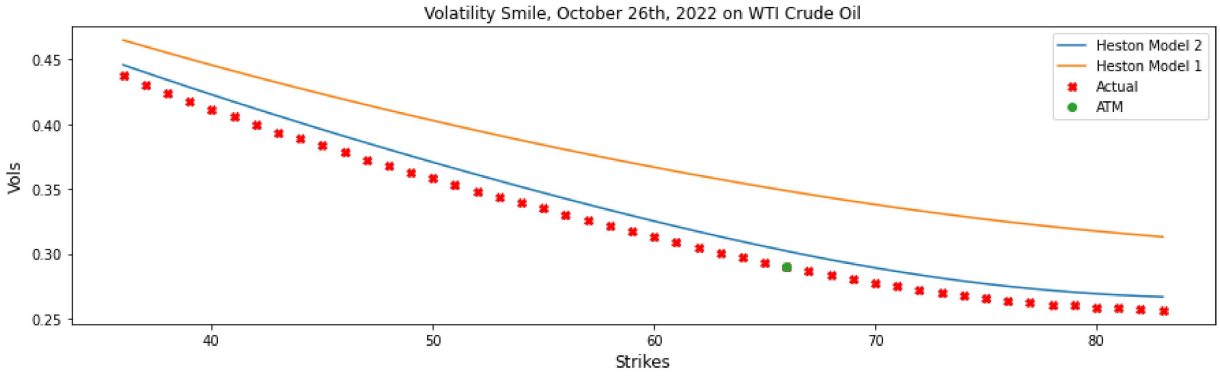
```
# Relative error comparison
plt.figure(figsize=plot_size)
plt.plot(hestonModel1.strks, hestonModel1.relativeError, label="Model 1")
plt.plot(hestonModel2.strks, hestonModel2.relativeError, label="Model 2")
plt.title("Errors Comparison on Heston Models, {}".format(data_label));
plt.legend()

# Volatility smiles comparison
tenors = [dates[round((len(dates)-1) * x)] for x in (.2, .5, .75, 1)]
for tenor in tenors:
    l = [
        ([hestonModel2.heston_vol_surface.blackVol(tenor, s) for s in strikes], "Hes
        ([hestonModel1.heston_vol_surface.blackVol(tenor, s) for s in strikes], "Hes
        ]
    plot_smile(tenor, l, market=True)
```











In [8]:

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np
import pandas as pd
import time
import datetime
import math
import QuantLib as ql
from scipy.optimize import minimize

from initialize import *
from plotting import *

# CALIBRATE SABR VOLATILITY SURFACE

volMatrix = ql.Matrix(len(strikes), len(dates))

for i in range(len(vols)):
    for j in range(len(vols[i])):
        volMatrix[j][i] = vols[i][j]

black_var_surface = ql.BlackVarianceSurface(
    today, calendar, dates, strikes, volMatrix, day_count)
black_var_surface.enableExtrapolation()


class SABRSmile:
```

```
    def __init__(self, date, marketVols, shift=0, beta=1, method="normal", strikes=s
        self.date = date
        self.expiryTime = round((self.date - today)/365, 6)
        self.marketVols = marketVols
        self.shift = shift
        self.strikes=strikes;
        self.fwd = fwd
        self.forward_price = self.fwd * \
            math.exp(rate.value() * self.expiryTime)
        self.zero_rho = zero_rho
        self.alpha, self.beta, self.nu, self.rho = (
            .1, beta, 0., 0. if self.zero_rho else .1)
        self.method = method
        self.newVols = None
        self.error = None

    def initialize(self):
        # alpha, beta, nu, rho
        cons = (
            {'type': 'ineq', 'fun': lambda x: x[0] - 0.001},
            {'type': 'eq', 'fun': lambda x: x[1] - self.beta},
            {'type': 'ineq', 'fun': lambda x: x[2] - .001},
            {'type': 'ineq', 'fun': lambda x: .99 - x[3]**2},
        )

        x = self.set_init_conds()

        result = minimize(self.f, x, constraints=cons, method="SLSQP", bounds=(
            (1e-8, None), (0, 1), (1e-8, None), (-.999, .999)))
        self.error = result['fun']
        [self.alpha, self.beta, self.nu, self.rho] = result['x']

        self.newVols = [self.vols_by_method(
            strike, self.alpha, self.beta, self.nu, self.rho) for strike in self.str

    def set_init_conds(self):
        return [self.alpha, self.beta, self.nu, self.rho]

    def vols_by_method(self, strike, alpha, beta, nu, rho):
        if self.method == "floch-kennedy":
            return ql.sabrFlochKennedyVolatility(strike, self.forward_price, self.ex
        elif self.shift != 0:
            return ql.shiftedSabrVolatility(strike, self.forward_price, self.expiryT
        else:
            return ql.sabrVolatility(strike, self.forward_price, self.expiryTime, al

    def f(self, params):

        alpha, beta, nu, rho = params

        vols = np.array([self.vols_by_method(
            strike, alpha, beta, nu, rho) for strike in self.strikes])

        self.error = ((vols - np.array(self.marketVols))**2).mean() ** .5

        return self.error


class SABRVolatilitySurface:
    def __init__(self, strks=strikes, method="normal", beta=1, shift=0, fwd=current_
        self.method = method
        self._beta = beta
        self.shift = shift
        self.strikes = strks
        self.fwd = fwd
```

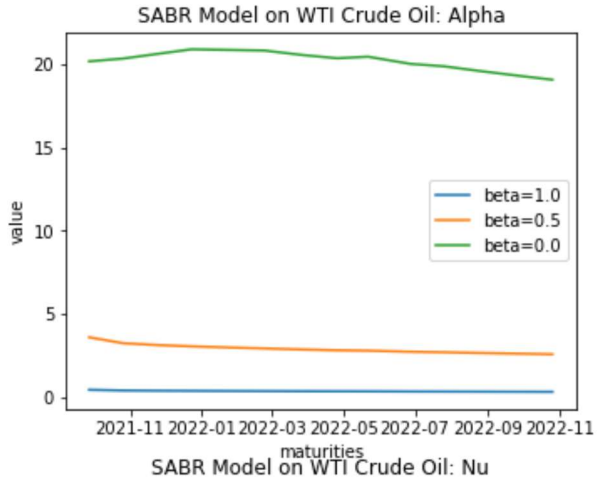
```
        self.label = label
        self.zero_rho = zero_rho

        self.initialize()

    def initialize(self):
        self.vol_surface_vector, self.errors, self.smiles, self.alpha, self.beta, se
        ], [], [], [], [], [], []
        self.SABRVolMatrix, self.SABRVolDiffMatrix = (
            ql.Matrix(len(self.strikes), len(dates)), ql.Matrix(len(self.strikes), l

        for i, d in enumerate(dates):

            v = []
            for j in range(len(self.strikes)):
                if self.strikes[i] in strikes:
                    v.append(vols[i][j])

            volSABR = SABRSmile(date=d, beta=self._beta, strikes=self.strikes, marke
                                method=self.method, fwd=self.fwd, zero_rho=self.zero
            volSABR.initialize()

            self.alpha.append(volSABR.alpha)
            self.beta.append(volSABR.beta)
            self.nu.append(volSABR.nu)
            self.rho.append(volSABR.rho)

            self.errors.append(volSABR.error)

            smile = volSABR.newVols

            self.vol_surface_vector.extend(smile)
            self.smiles.append(volSABR)

            # constructing the SABRVolatilityMatrix
            for j in range(len(smile)):
                self.SABRVolMatrix[j][i] = smile[j]
                self.SABRVolDiffMatrix[j][i] = (
                    smile[j] - v[j]) / v[j]

            self.vol_surface = ql.BlackVarianceSurface(
                today, calendar, dates, self.strikes, self.SABRVolMatrix, day_count)
            self.vol_surface.enableExtrapolation()

    def to_data(self):
        d = {'alpha': self.alpha, 'beta': self.beta,
             'nu': self.nu, 'rho': self.rho}
        return pd.DataFrame(data=d, index=dates)


# Backbone modelling for SABR
def SABR_backbone_plot(beta=1, bounds=None, shift=0, strikes=strikes, fixes=(.95, 1,
    l = []
    for i in fixes:
        vol_surface = SABRVolatilitySurface(
            method="normal", shift=current_price*shift, beta=beta, fwd=current_price
        SABR_vol_surface = ql.BlackVarianceSurface(
            today, calendar, dates, strikes, vol_surface.SABRVolMatrix, day_count)
        SABR_vol_surface.enableExtrapolation()

        l.append(([SABR_vol_surface.blackVol(tenor, s)
                 for s in strikes], "fwd = {}".format(current_price * i)))

    plot_smile(tenor, l, bounds=bounds, market=False,
               title="backbone, beta = {}, {}".format(vol_surface.beta[0], tenor))
```

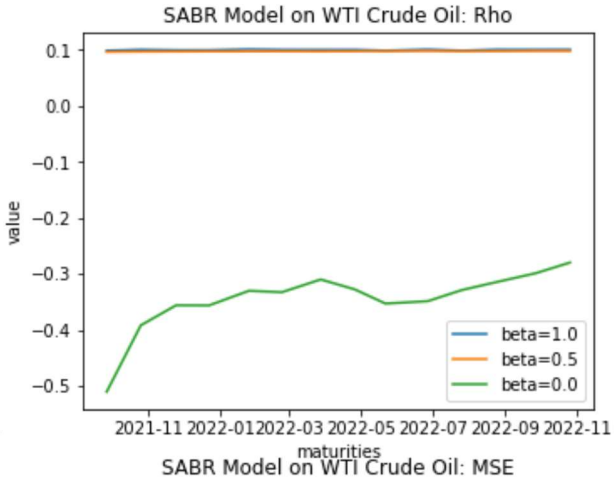
```
def SABRComparison(methods, title="", display=False):
    fig, axs = plt.subplots(2, 2, figsize=plot_size)
    plt.subplots_adjust(left=None, bottom=None, right=None,
                        top=1.5, wspace=None, hspace=None)

    for method in methods:
        lbl = "beta={}".format(method.beta[1])
        axs[0, 0].plot(maturities, method.alpha, label=lbl)
        axs[0, 0].set_title('{}: Alpha'.format(title))
        axs[0, 0].set(xlabel='maturities', ylabel='value')
        axs[0, 0].legend()
        axs[1, 0].plot(maturities, method.nu, label=lbl)
        axs[1, 0].set_title('{}: Nu'.format(title))
        axs[1, 0].set(xlabel='maturities', ylabel='value')
        axs[1, 0].legend()
        axs[0, 1].plot(maturities, method.rho, label=lbl)
        axs[0, 1].set_title('{}: Rho'.format(title))
        axs[0, 1].set(xlabel='maturities', ylabel='value')
        axs[0, 1].legend()
        axs[1, 1].plot(maturities, method.errors, label=lbl)
        axs[1, 1].set_title('{}: MSE'.format(title))
        axs[1, 1].set(xlabel='maturities', ylabel='value')
        axs[1, 1].legend()

        if display:
            method_df = method.to_data()
            display(method_df.style.set_caption("SABR, {}".format(lbl)))

        plot_vol_surface(method.vol_surface, title="{}".format(method.label))

    smiles_comparison(methods)
```

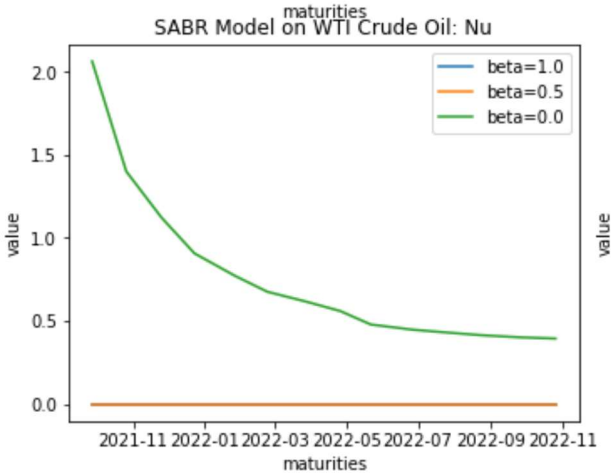
In [9]:

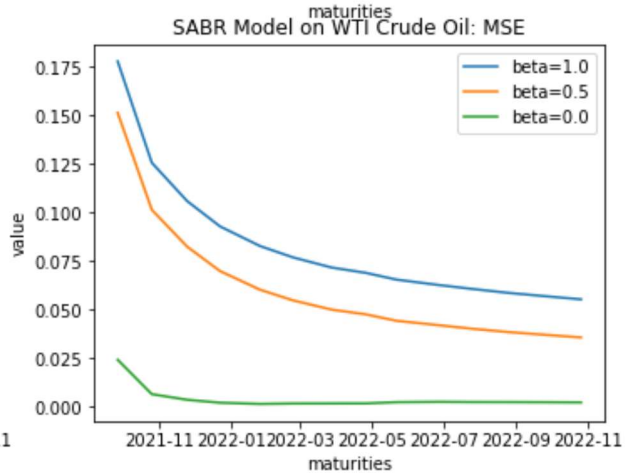
```
# SABR Volatility model

strks = strikes
SABR_beta1 = SABRVolatilitySurface(beta=1, shift=0, strks=strks, label="SABR, beta=1
SABR_beta5 = SABRVolatilitySurface(beta=.5, shift=0, strks=strks, label="SABR, beta=
SABR_beta0 = SABRVolatilitySurface(beta=.0, shift=0, strks=strks, label="Normal SABR

SABRComparison([SABR_beta1, SABR_beta5, SABR_beta0], title="SABR Model on {}".format
```

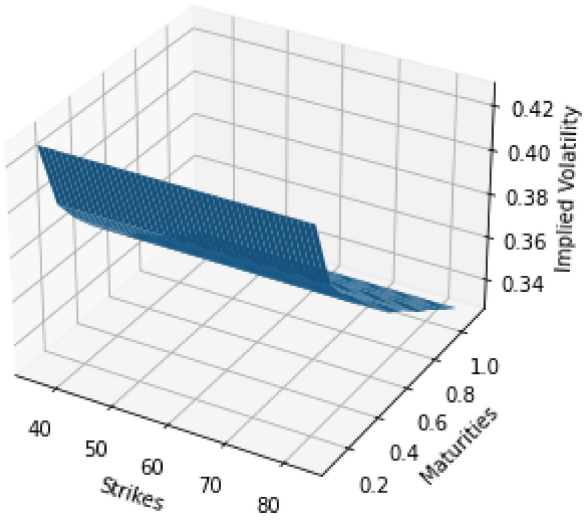
```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```


SABR Model on WTI Crude Oil: Alpha
value
maturities
beta=1.0
beta=0.5
beta=0.0
2021-11 2022-01 2022-03 2022-05 2022-07 2022-09 2022-11

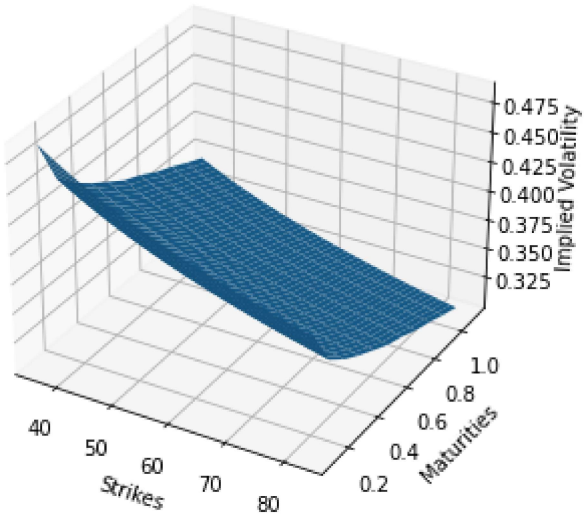

SABR Model on WTI Crude Oil: Rho
value
maturities
beta=1.0
beta=0.5
beta=0.0
2021-11 2022-01 2022-03 2022-05 2022-07 2022-09 2022-11


SABR Model on WTI Crude Oil: Nu
value
maturities
beta=1.0
beta=0.5
beta=0.0
2021-11 2022-01 2022-03 2022-05 2022-07 2022-09 2022-11

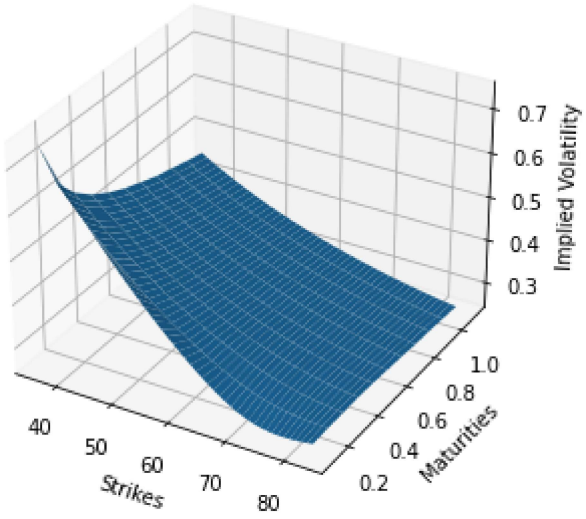

SABR Model on WTI Crude Oil: MSE
value
maturities
beta=1.0
beta=0.5
beta=0.0
2021-11 2022-01 2022-03 2022-05 2022-07 2022-09 2022-11

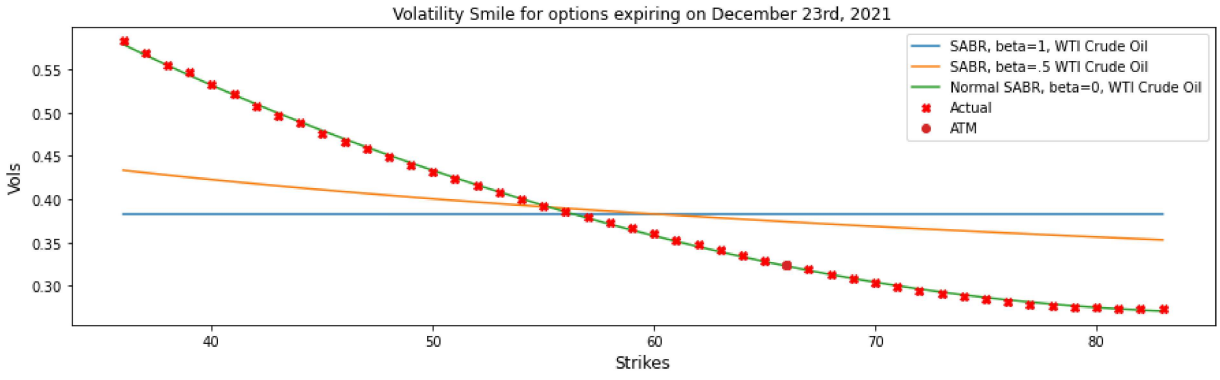

SABR, beta=1, WTI Crude Oil
Implied Volatility
Strikes
Maturities

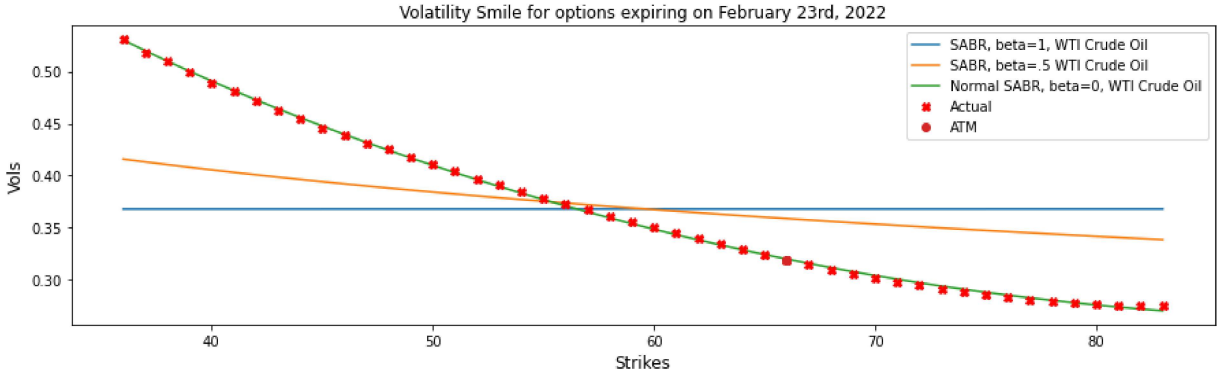
SABR, beta=.5 WTI Crude Oil

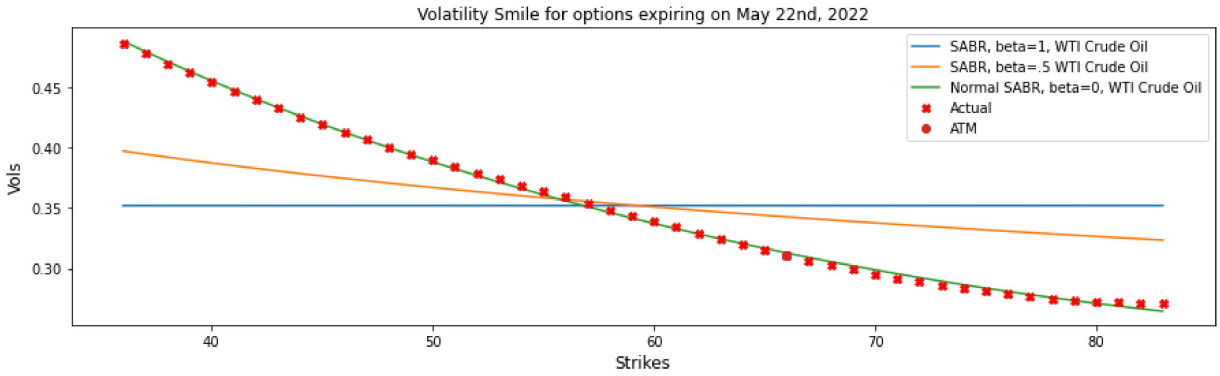


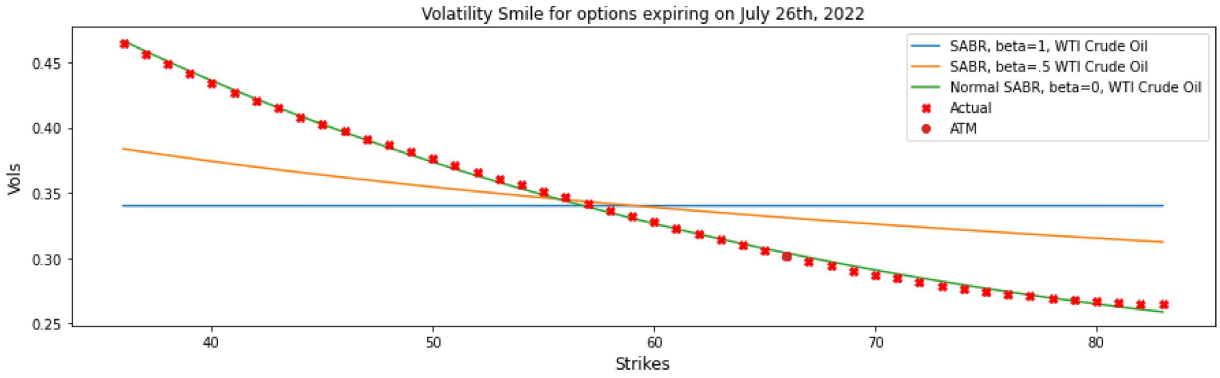
Normal SABR, beta=0, WTI Crude Oil

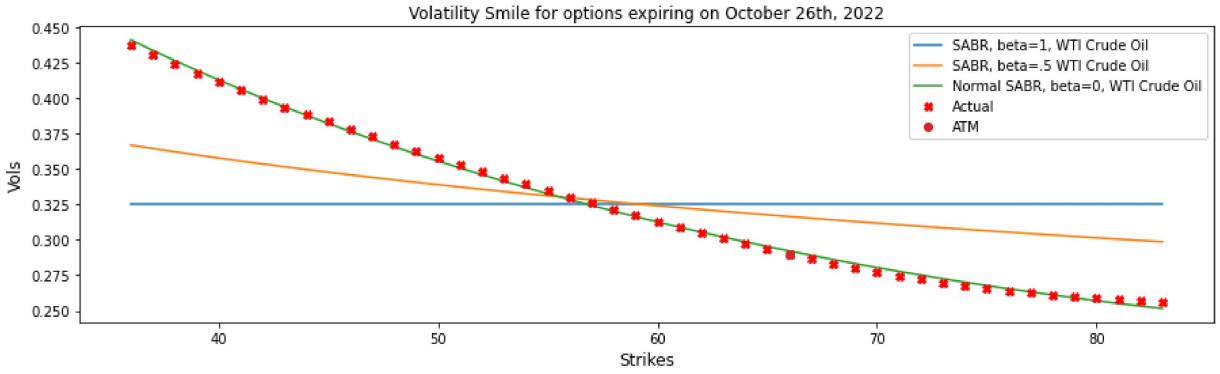






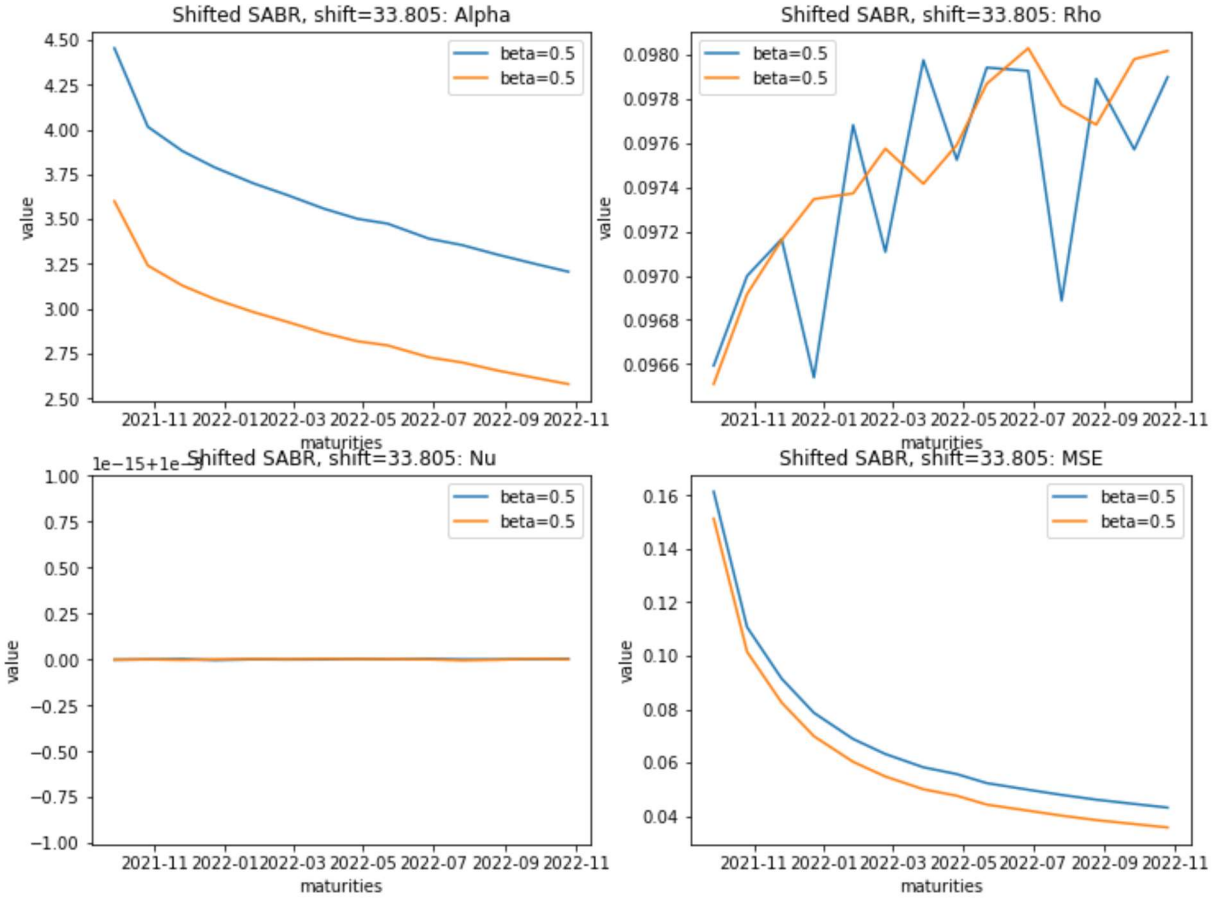




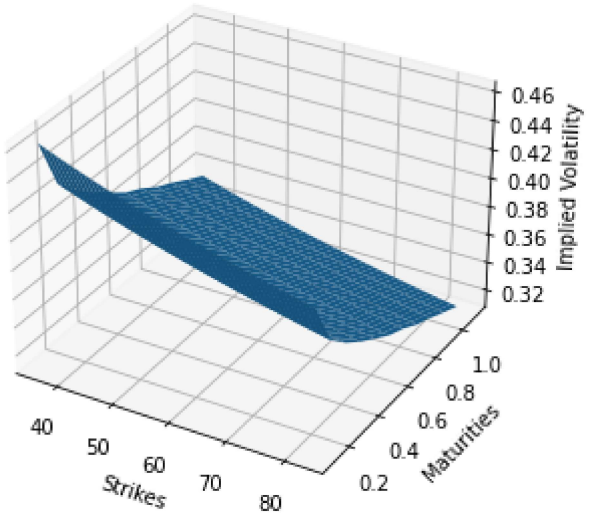


In [10]:

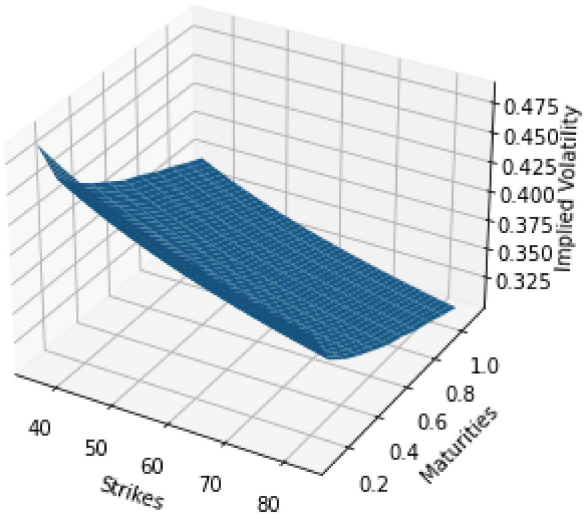
```
# Shifted SABR Volatility model

shft = .50 * current_price
shiftedSABR_beta1 = SABRVolatilitySurface(beta=1, shift=shft,strks=strikes, label="S
shiftedSABR_beta5 = SABRVolatilitySurface(beta=.5, shift=shft, strks=strikes,label="
shiftedSABR_beta0 = SABRVolatilitySurface(beta=.0, shift=shft, strks=strikes,label="

SABRComparison([shiftedSABR_beta5, SABR_beta5], title="Shifted SABR, shift={}".forma
SABRComparison([shiftedSABR_beta0, SABR_beta0], title="Shifted SABR, shift={}".forma
```

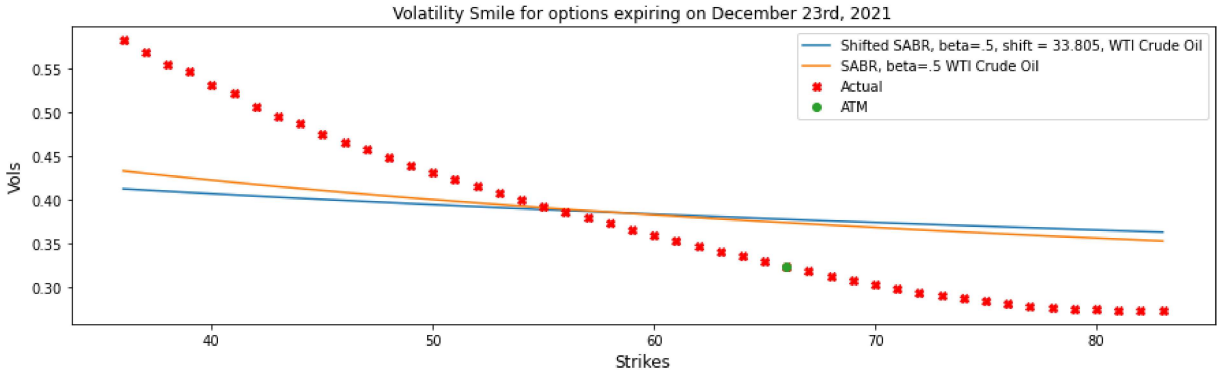


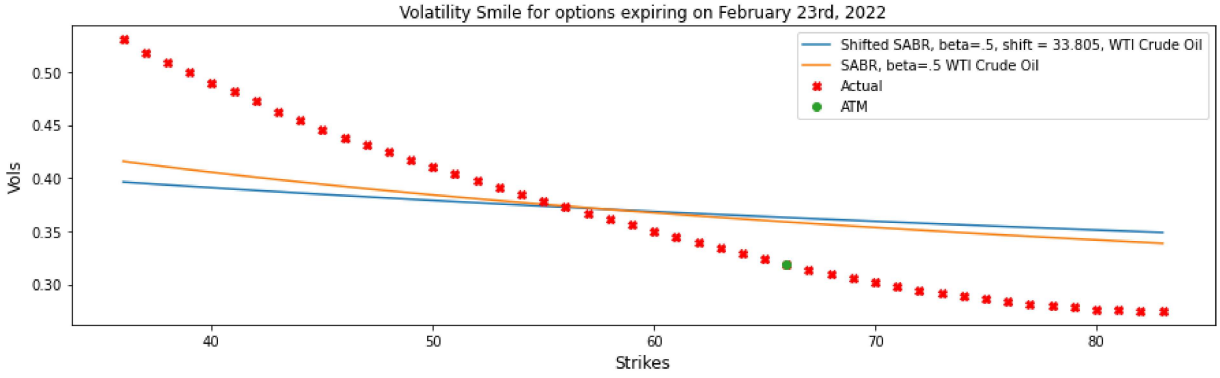
Shifted SABR, beta=.5, shift = 33.805, WTI Crude Oil

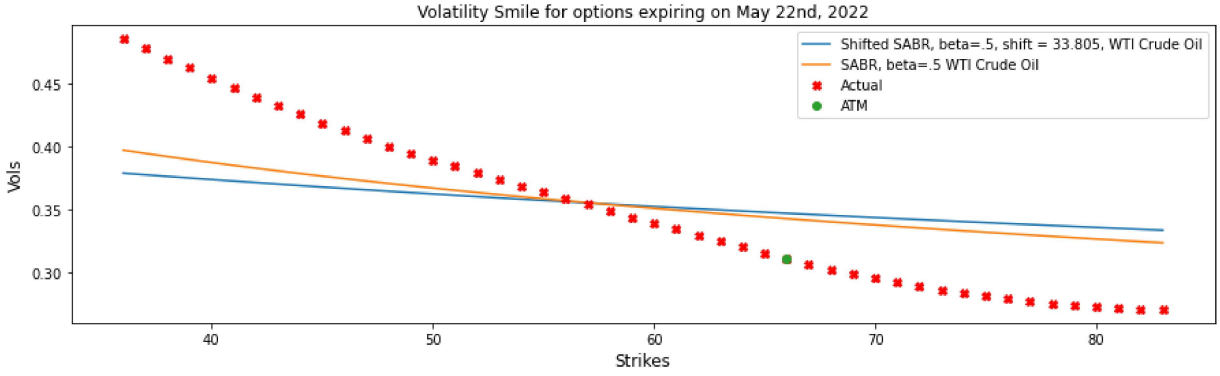


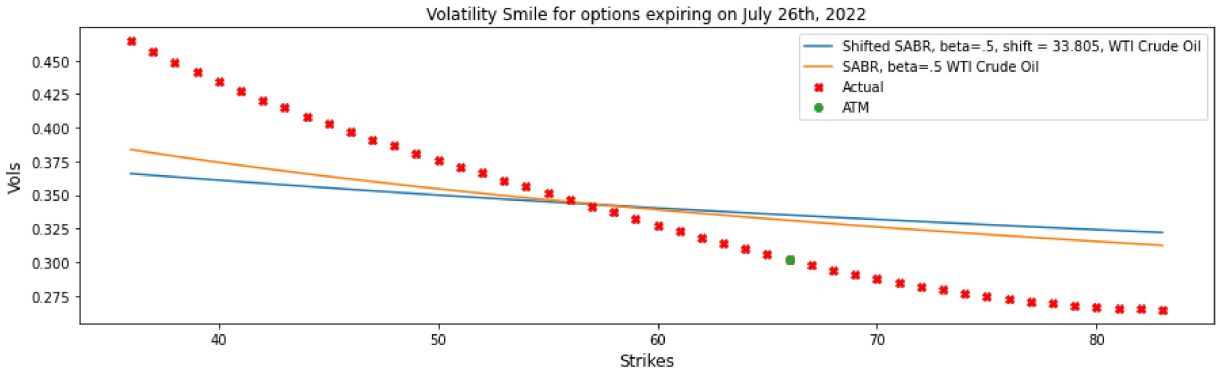
SABR, beta=.5 WTI Crude Oil

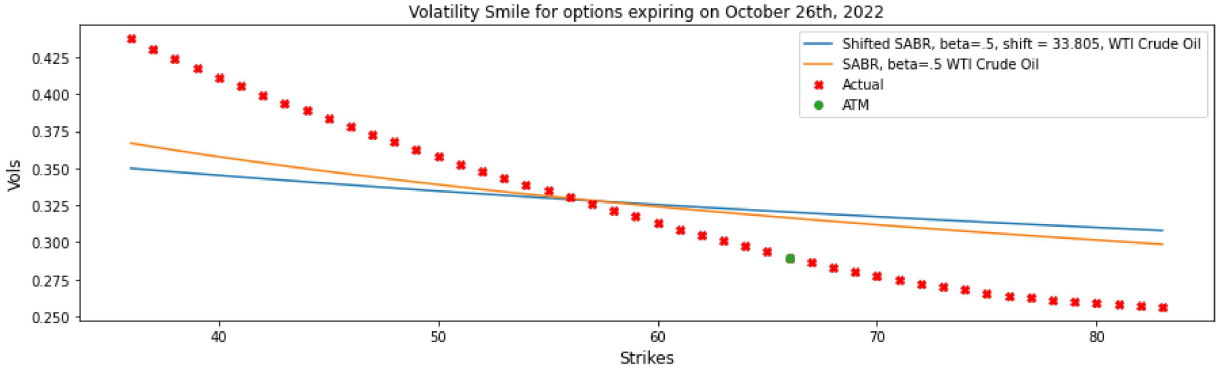


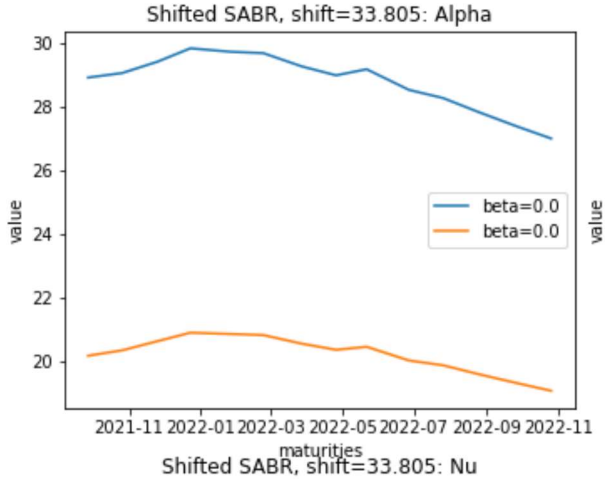


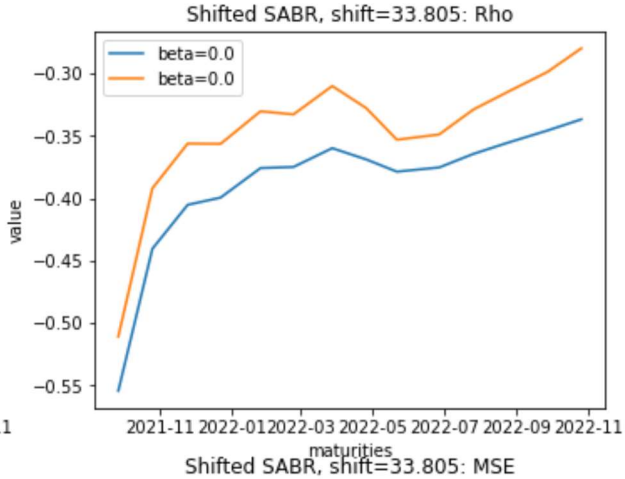


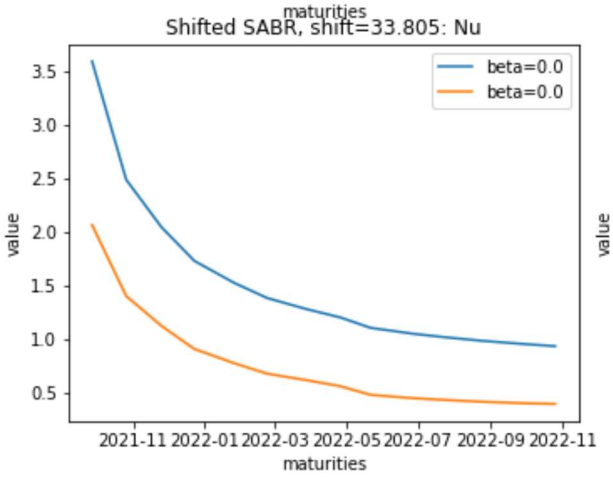


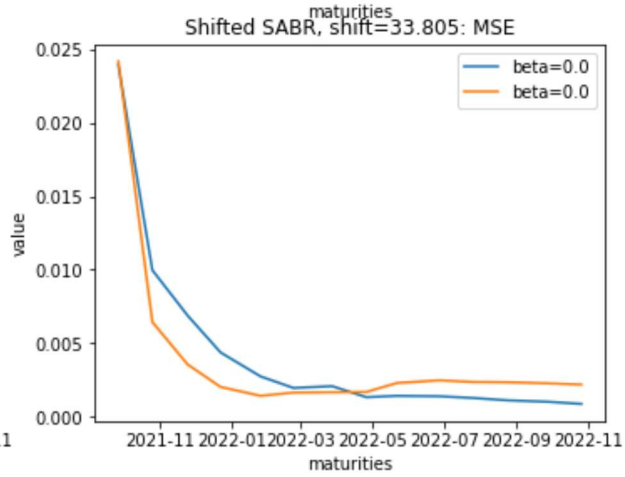

Volatility Smile for options expiring on July 26th, 2022
Shifted SABR, beta=.5, shift = 33.805, WTI Crude Oil
SABR, beta=.5 WTI Crude Oil
Actual
ATM
Vols
Strikes


Volatility Smile for options expiring on October 26th, 2022
Shifted SABR, beta=.5, shift = 33.805, WTI Crude Oil
SABR, beta=.5 WTI Crude Oil
Actual
ATM
Vols
Strikes

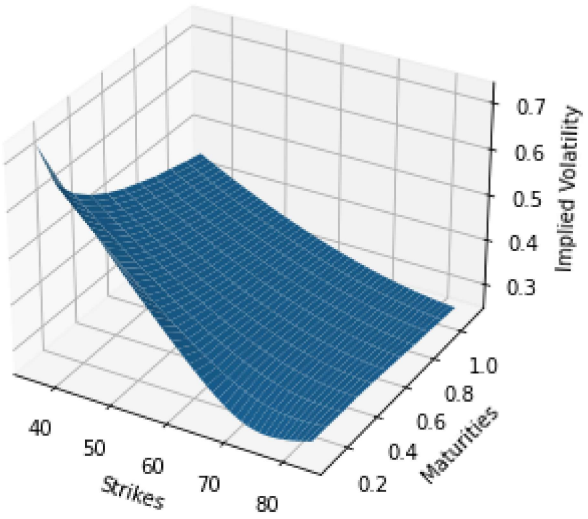

Shifted SABR, shift=33.805: Alpha
beta=0.0
beta=0.0
value
maturities


Shifted SABR, shift=33.805: Rho
beta=0.0
beta=0.0
value
maturities

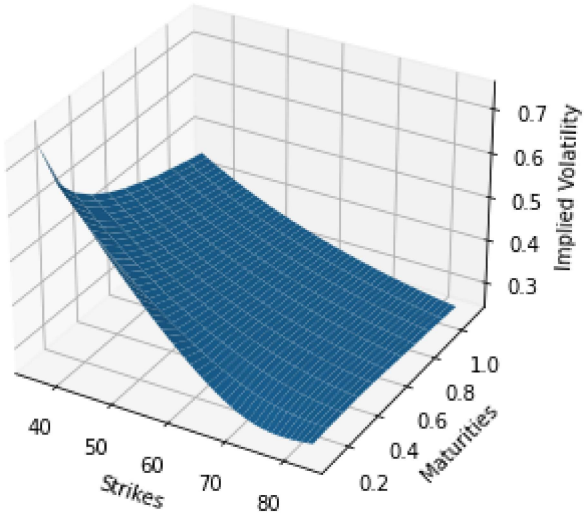

Shifted SABR, shift=33.805: Nu
beta=0.0
beta=0.0
value
maturities

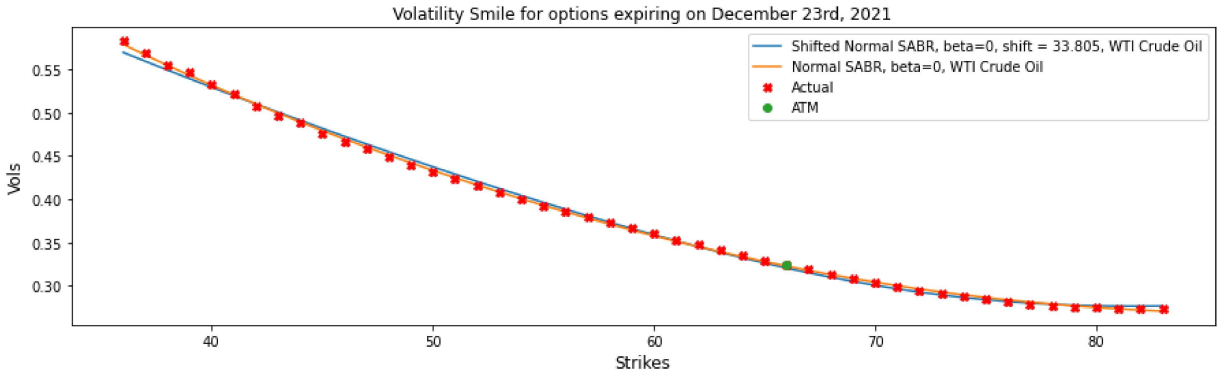

Shifted SABR, shift=33.805: MSE
beta=0.0
beta=0.0
value
maturities

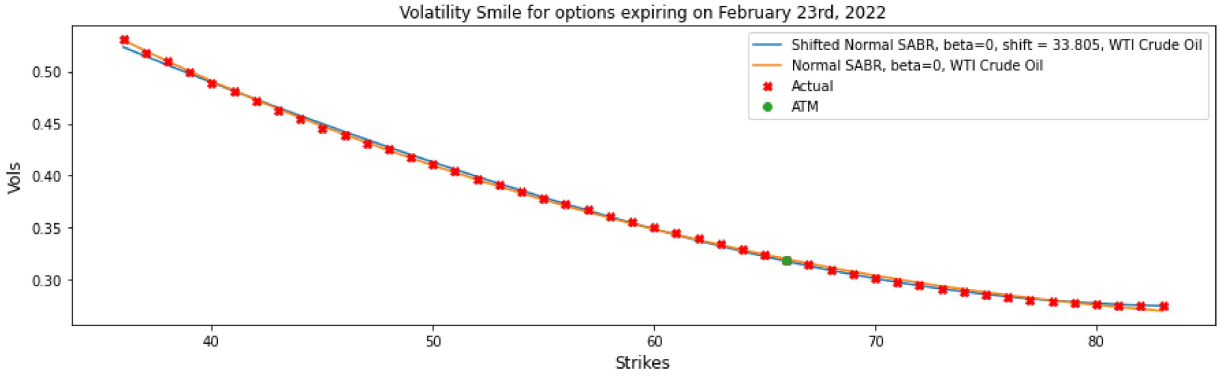
Shifted Normal SABR, beta=0, shift = 33.805, WTI Crude Oil

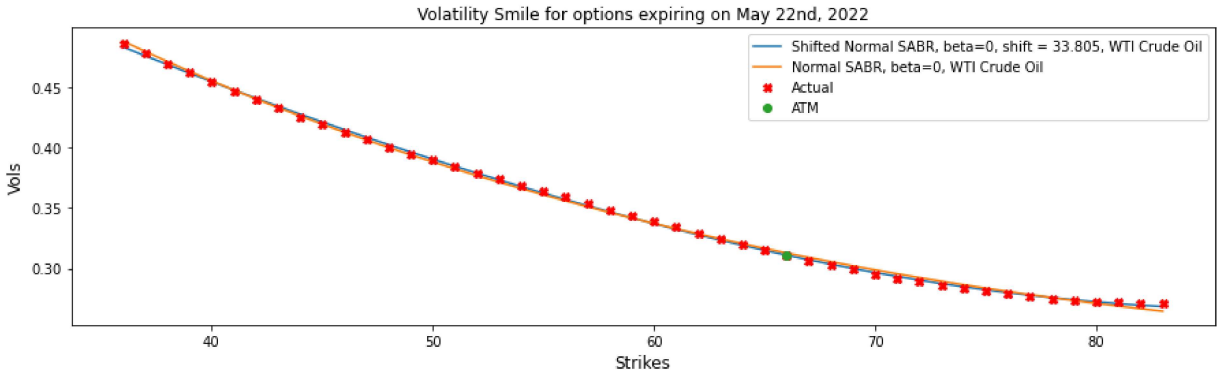


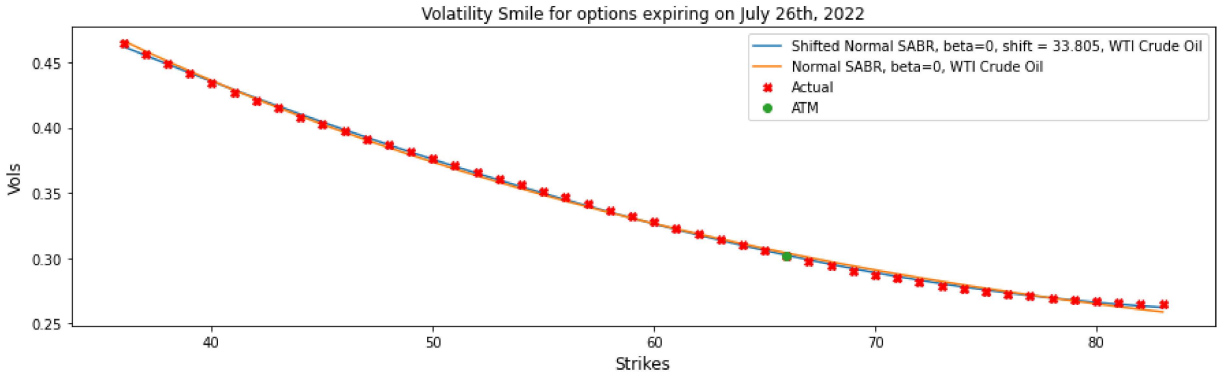
Normal SABR, beta=0, WTI Crude Oil

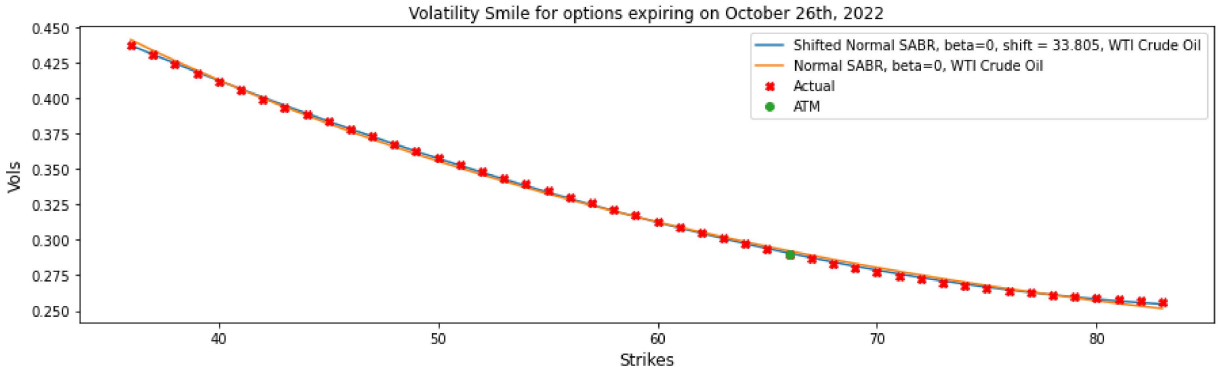






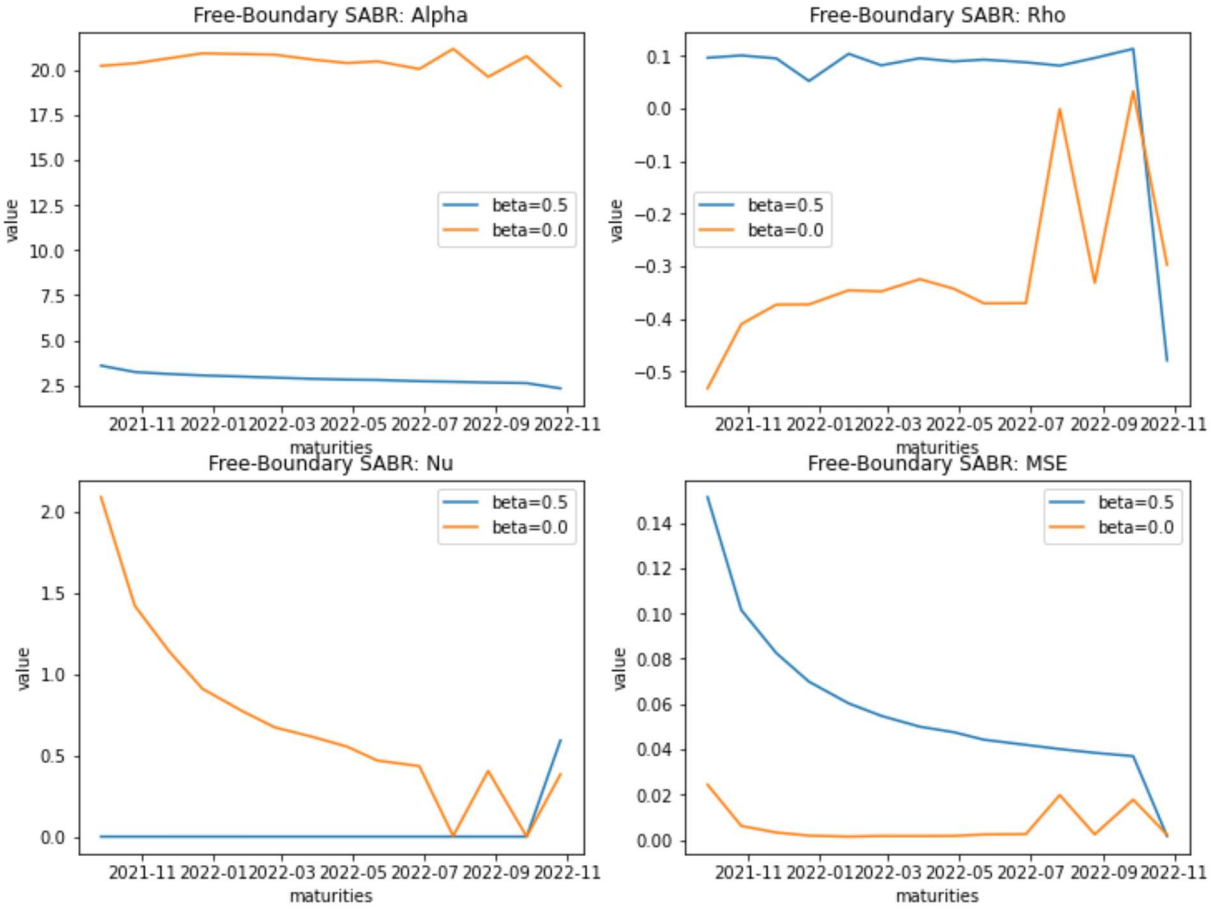




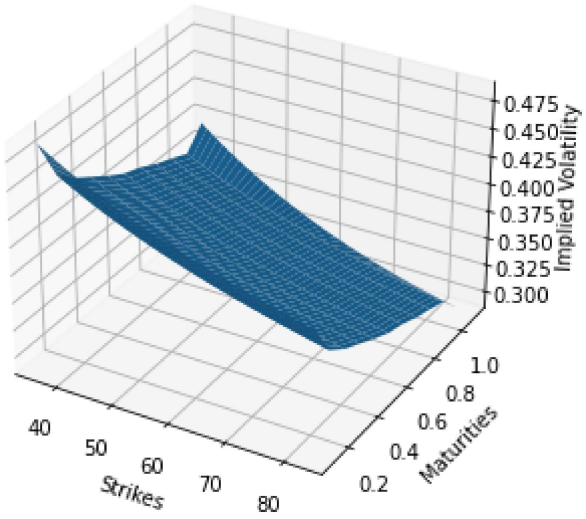


In [11]:

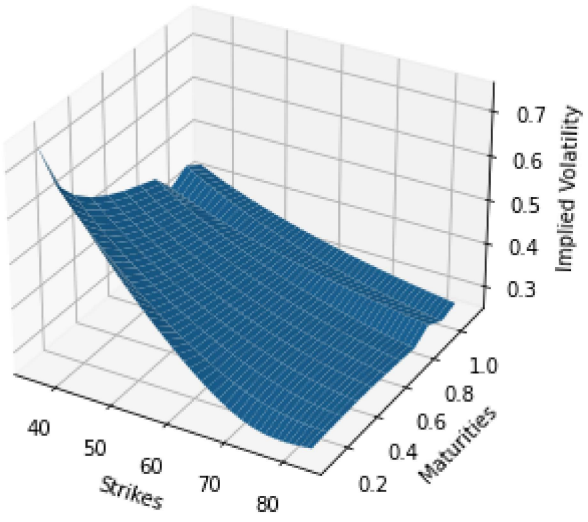
```
# Free-Boundary SABR Volatility model

freeSABR_beta5 = SABRVolatilitySurface(beta=.5, shift=0, method="floch-kennedy", str
freeSABR_beta0 = SABRVolatilitySurface(beta=.0, shift=0, method="floch-kennedy", str

SABRComparison([freeSABR_beta5, freeSABR_beta0], title="Free-Boundary SABR")
```

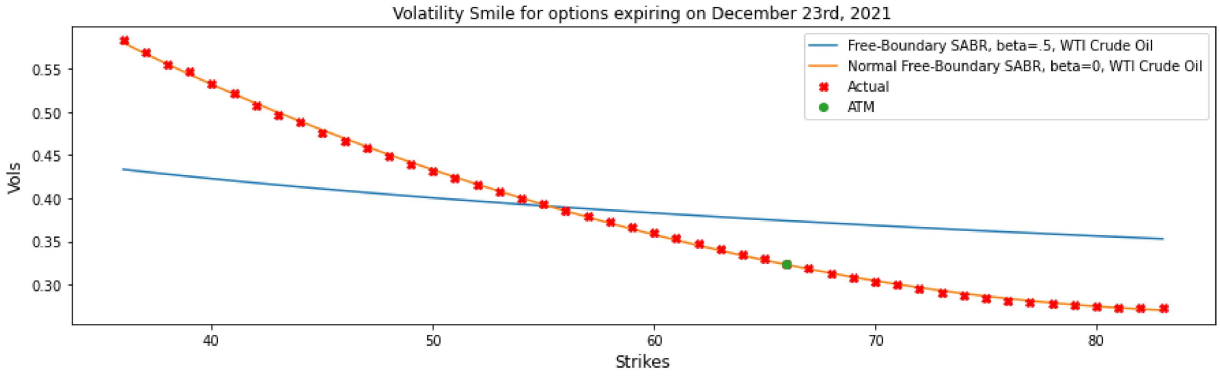


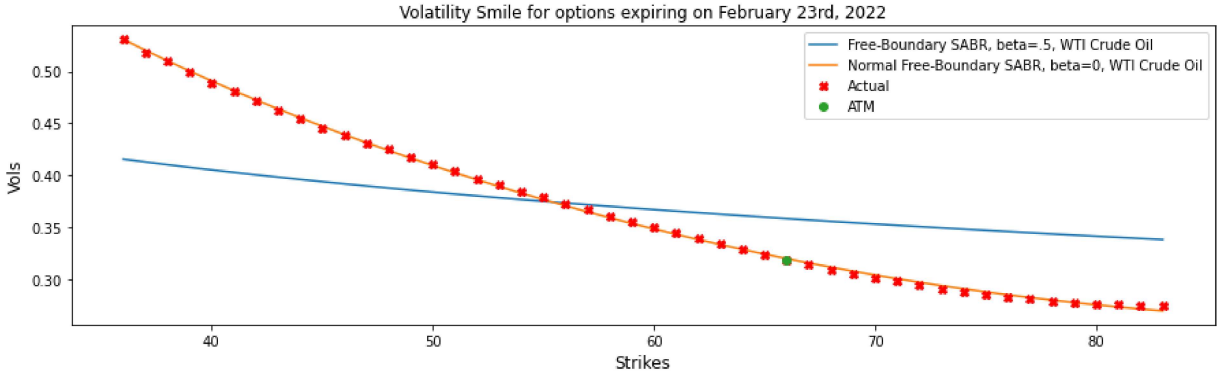
Free-Boundary SABR, beta=.5, WTI Crude Oil

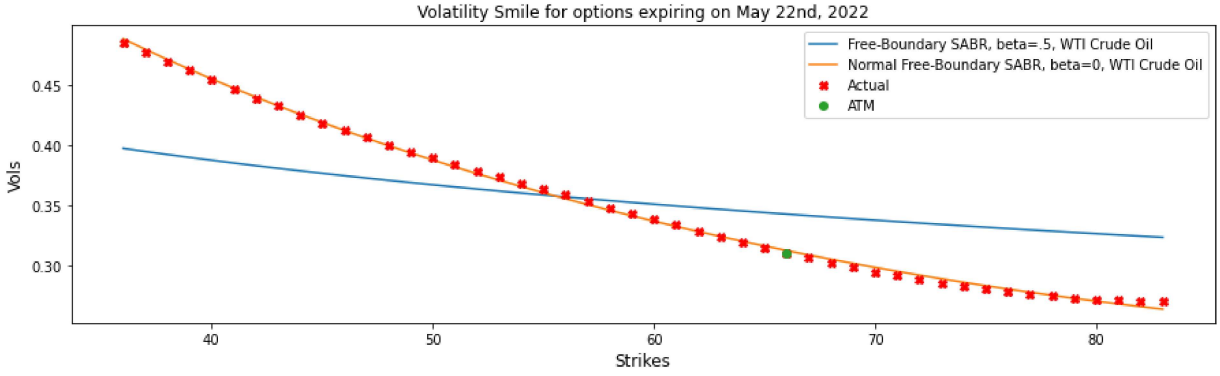


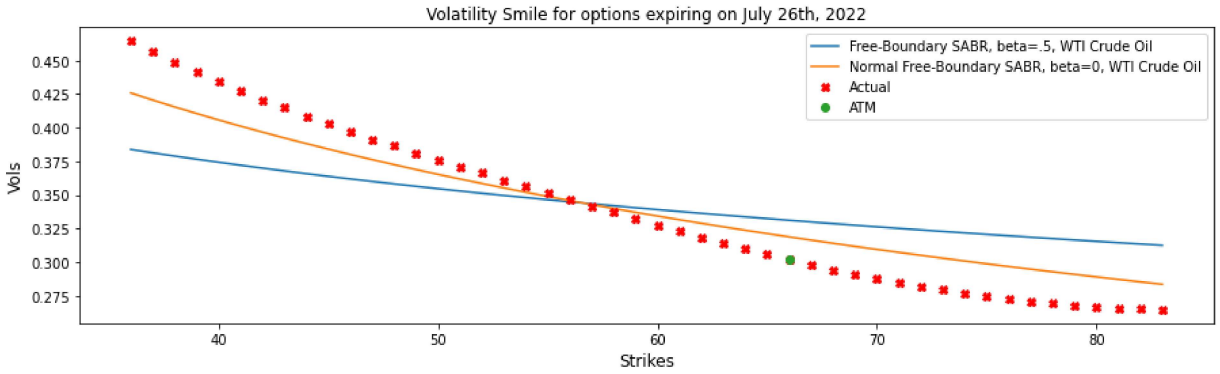
Normal Free-Boundary SABR, beta=0, WTI Crude Oil

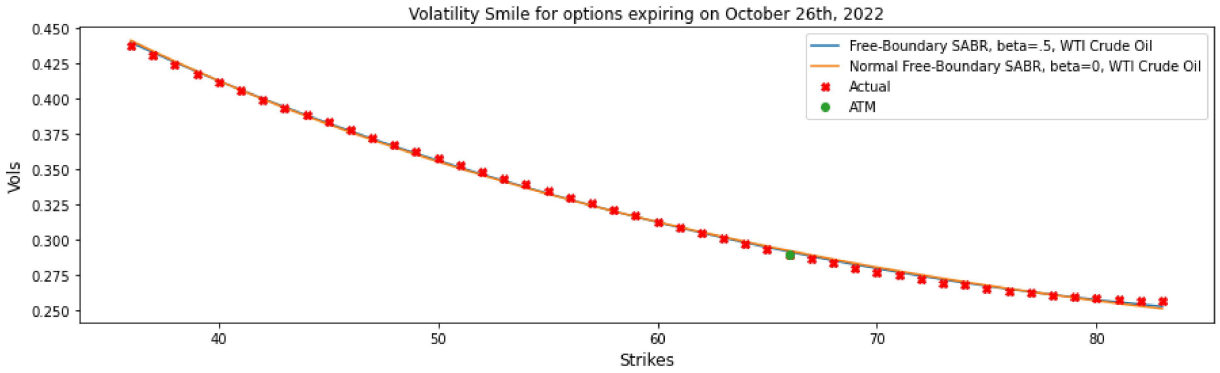












In [12]:

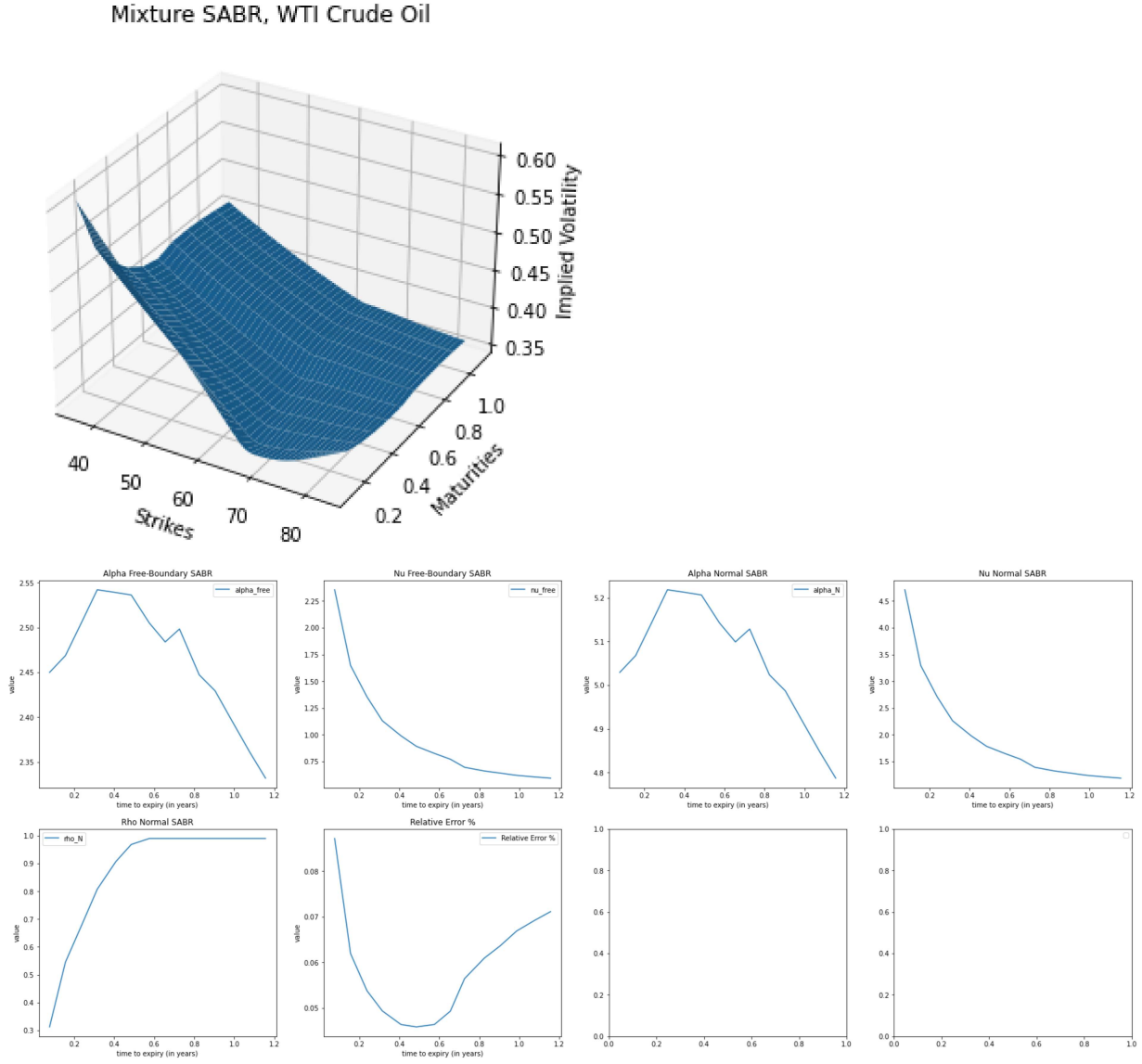
```
# Mixed SABR

mixtureSABR = MixtureSABRVolatilitySurface(dates=dates)
display(mixtureSABR.to_data())
plot_vol_surface([mixtureSABR.vol_surface], title=mixtureSABR.label)
mixtureSABR.plot()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWa
rning: Values in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```

	alpha_free	beta_free	nu_free	rho_free	alpha_N	beta_N	nu_N	rho_N	MSE
September 27th, 2021	2.449958	0.5	2.354421	0.0	2.449958	0.0	4.708842	0.312404	0.08714
October 26th, 2021	2.468698	0.5	1.646774	0.0	2.468698	0.0	3.293547	0.545531	0.06193
November 25th, 2021	2.506417	0.5	1.353955	0.0	2.506417	0.0	2.707910	0.680170	0.05378
December 23rd, 2021	2.542062	0.5	1.130503	0.0	2.542062	0.0	2.261007	0.807568	0.04936
January 26th, 2022	2.538988	0.5	0.989837	0.0	2.538988	0.0	1.979675	0.907097	0.04639
February 23rd, 2022	2.536233	0.5	0.892065	0.0	2.536233	0.0	1.784129	0.968444	0.04586
March 28th, 2022	2.505012	0.5	0.825410	0.0	2.505012	0.0	1.650820	0.990000	0.04639
April 26th, 2022	2.483907	0.5	0.771357	0.0	2.483907	0.0	1.542715	0.990000	0.04931
May 22nd, 2022	2.498308	0.5	0.695785	0.0	2.498308	0.0	1.391571	0.990000	0.05645
June 27th, 2022	2.447246	0.5	0.661259	0.0	2.447246	0.0	1.322519	0.990000	0.06093

	alpha_free	beta_free	nu_free	rho_free	alpha_N	beta_N	nu_N	rho_N	MSI
July 26th, 2022	2.429392	0.5	0.641539	0.0	2.429392	0.0	1.283077	0.990000	0.06363
August 25th, 2022	2.396771	0.5	0.620344	0.0	2.396771	0.0	1.240688	0.990000	0.06687
September 27th, 2022	2.361294	0.5	0.605586	0.0	2.361294	0.0	1.211172	0.990000	0.06922
October 26th, 2022	2.332039	0.5	0.593914	0.0	2.332039	0.0	1.187828	0.990000	0.07113

```
No handles with labels found to put in legend.
```

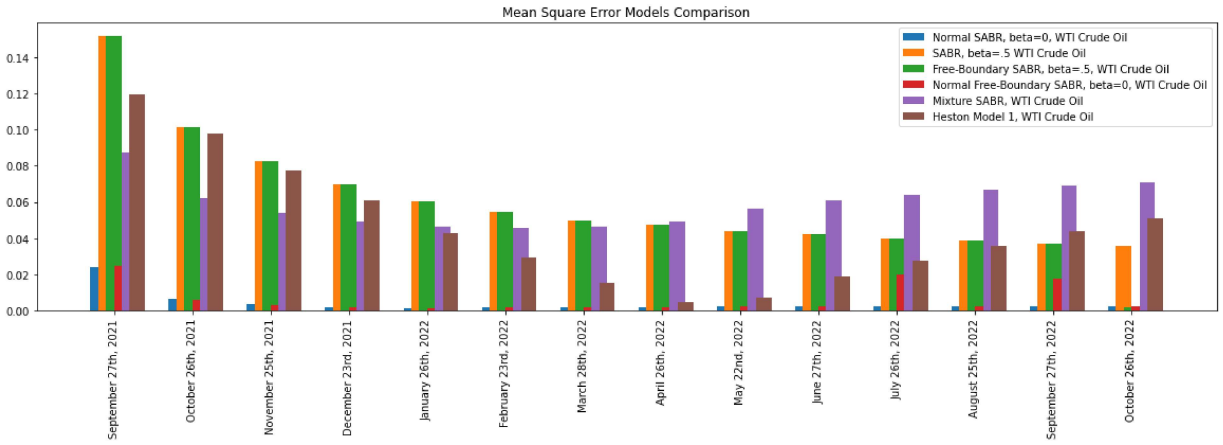


In [13]:

```
# VOLATILITY SMILES ERRORS COMPARISON

fig = plt.figure(figsize=(20, 5), )
width = .20
plt.bar(np.arange(len(maturities)) - 2*width/2, SABR_beta0.errors, label=SABR_beta0.
plt.bar(np.arange(len(maturities)) - width/2, SABR_beta5.errors, label=SABR_beta5.la
plt.bar(np.arange(len(maturities)) , freeSABR_beta5.errors, label=freeSABR_beta5.lab
plt.bar(np.arange(len(maturities)) + width/2, freeSABR_beta0.errors, label=freeSABR_
plt.bar(np.arange(len(maturities)) + 2*width/2, mixtureSABR.errors, label=mixtureSAB
plt.bar(np.arange(len(maturities)) + 3*width/2, hestonModel1.errors, label=hestonMod
plt.xticks(np.arange(len(maturities)), dates, rotation='vertical')
```

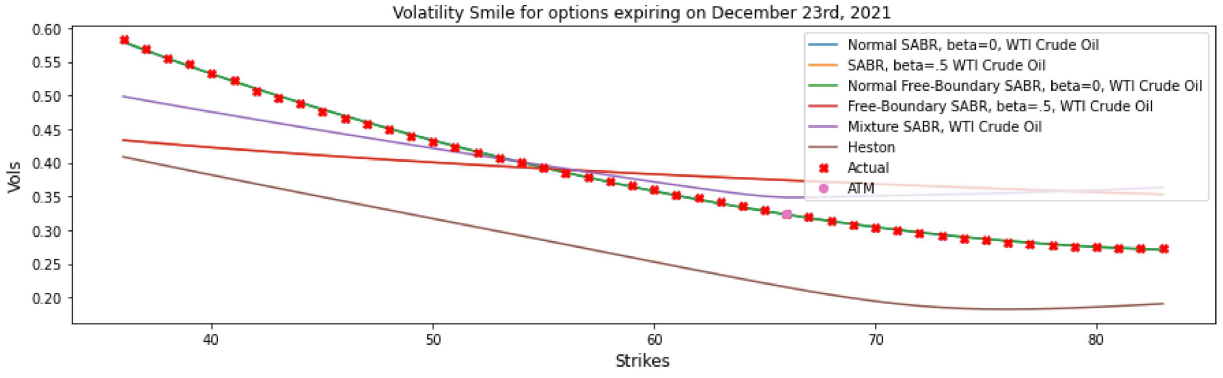
```
plt.title("Mean Square Error Models Comparison")
plt.legend()
```

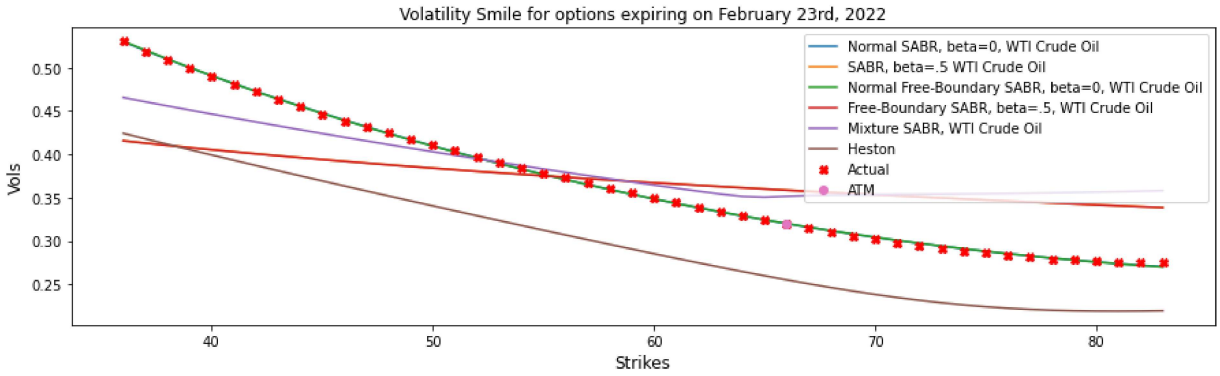
Out[13]: <matplotlib.legend.Legend at 0x14210e1c430>

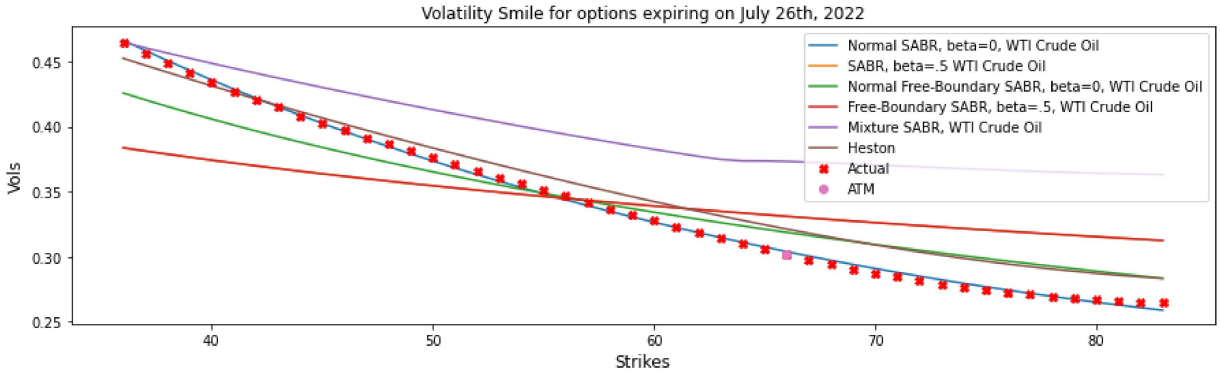


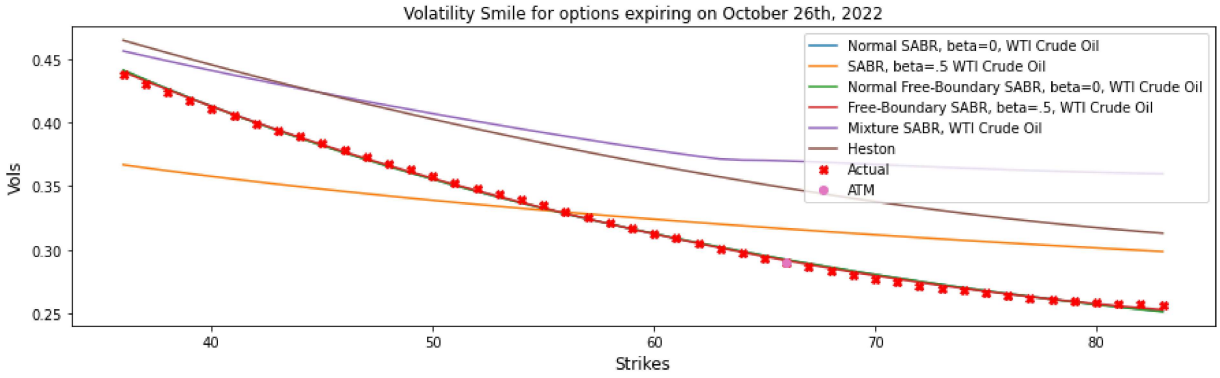
In [14]:
```
# Volatility Smiles Comparisons (Final)

models = (
        SABR_beta0,
        SABR_beta5,
        freeSABR_beta0,
        freeSABR_beta5,
        mixtureSABR
        )

errors_data = pd.DataFrame([np.mean(m.errors) for m in models], index=[m.label for m
display(errors_data)

smiles_comparison(models, heston_models=[hestonModel1])
```

	Error
Normal SABR, beta=0, WTI Crude Oil	0.004042
SABR, beta=.5 WTI Crude Oil	0.061090
Normal Free-Boundary SABR, beta=0, WTI Crude Oil	0.006413
Free-Boundary SABR, beta=.5, WTI Crude Oil	0.058661
Mixture SABR, WTI Crude Oil	0.059175





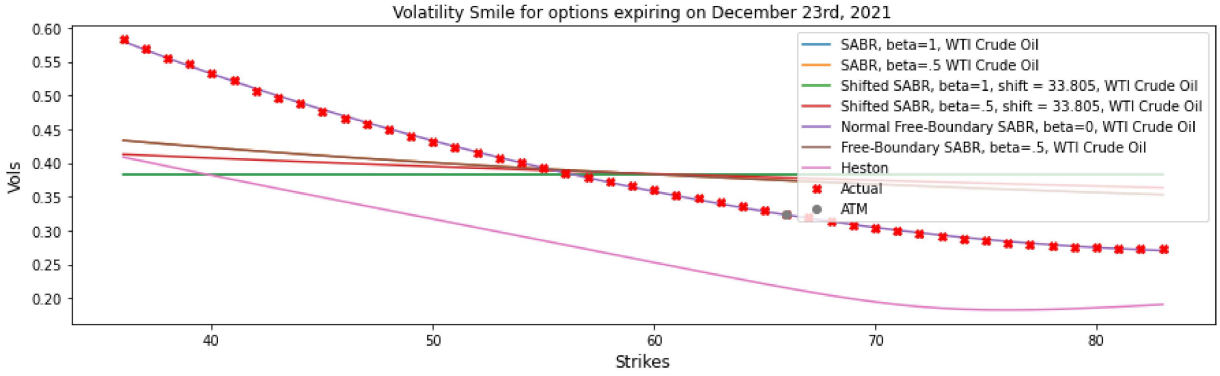


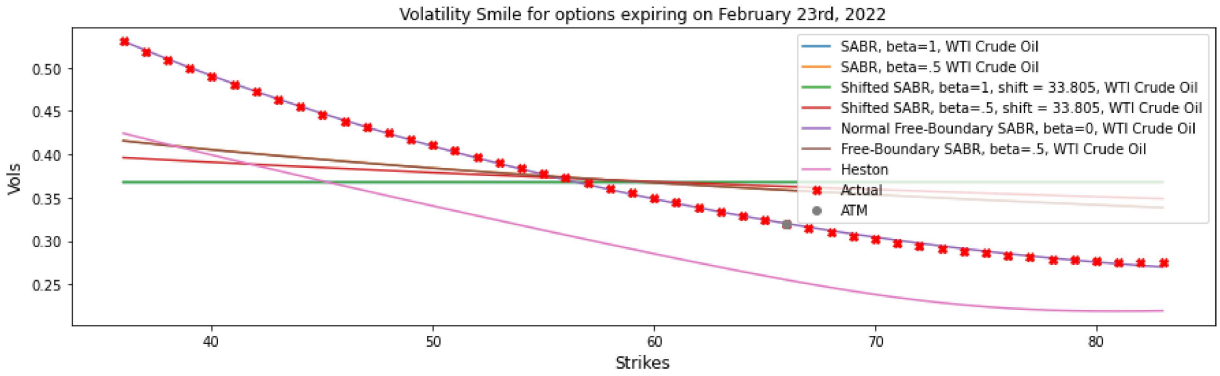


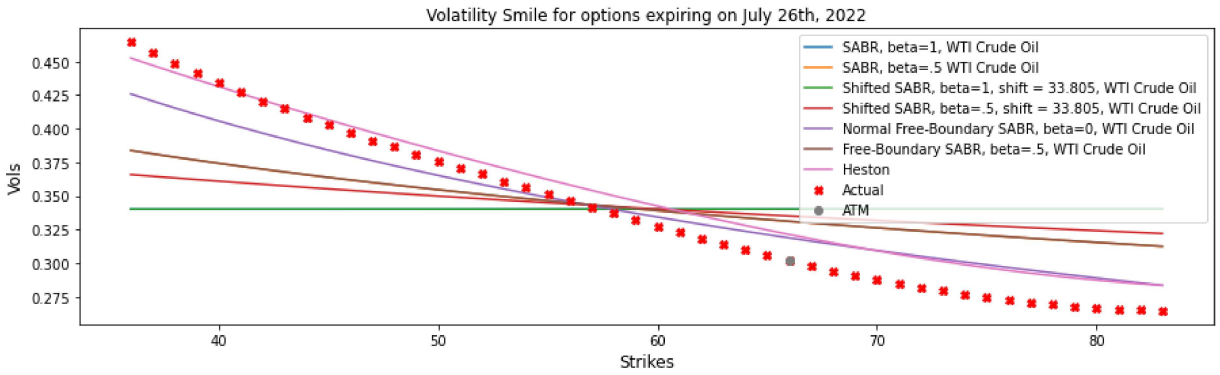
In [15]:

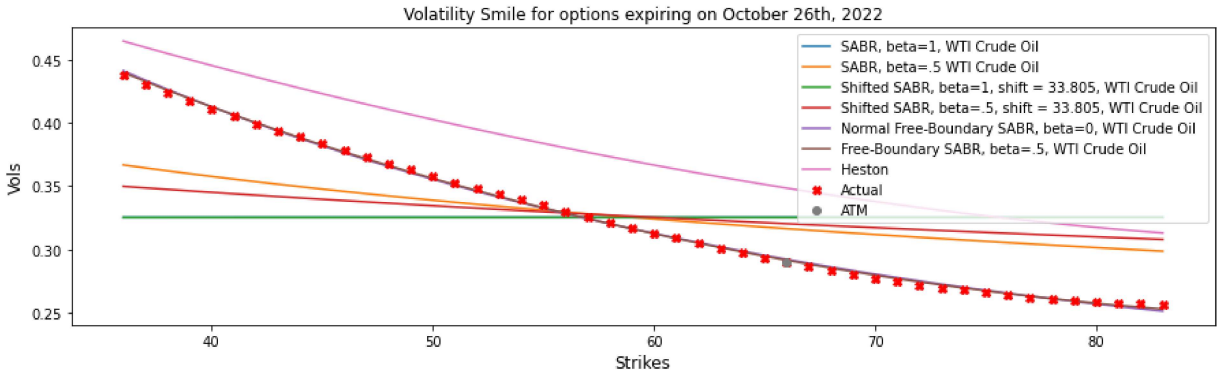
```
# Volatility Smiles Comparisons 2 (Final)

models = (
        SABR_beta1,
        SABR_beta5,
        shiftedSABR_beta1,
        shiftedSABR_beta5,
        freeSABR_beta0,
        freeSABR_beta5,
        )

errors_data = pd.DataFrame([np.mean(m.errors) for m in models], index=[m.label for m
display(errors_data)

smiles_comparison(models, heston_models=[hestonModel1])
```

	Error
SABR, beta=1, WTI Crude Oil	0.083024
SABR, beta=.5 WTI Crude Oil	0.061090
Shifted SABR, beta=1, shift = 33.805, WTI Crude Oil	0.083019
Shifted SABR, beta=.5, shift = 33.805, WTI Crude Oil	0.069423
Normal Free-Boundary SABR, beta=0, WTI Crude Oil	0.006413
Free-Boundary SABR, beta=.5, WTI Crude Oil	0.058661



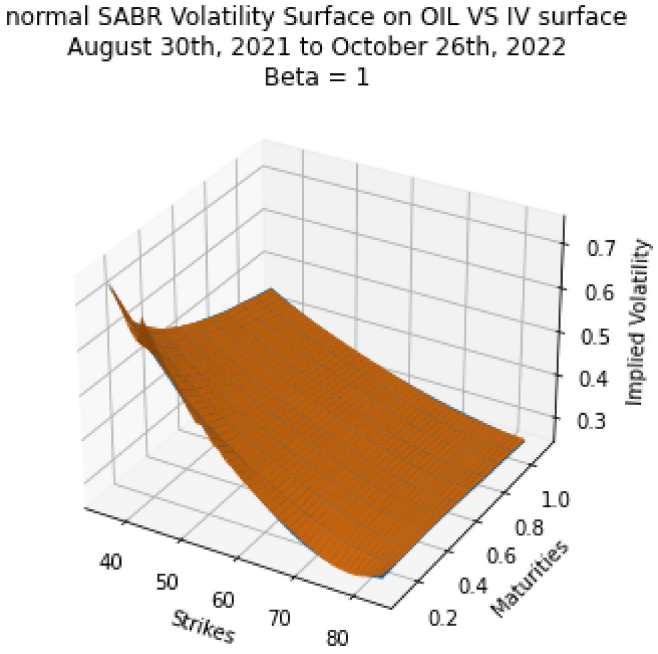




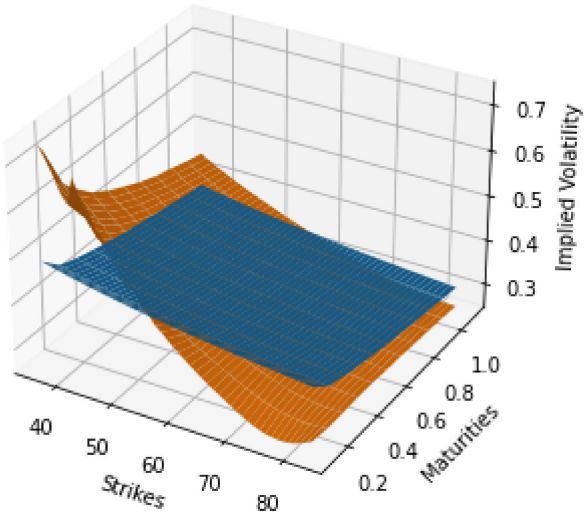


In [16]:

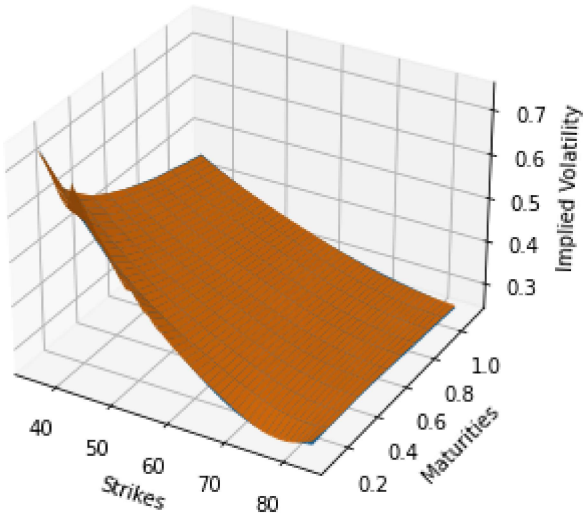
```
# Volatility Surfaces plots comparison

title = "normal SABR Volatility Surface on {} VS IV surface\n{} to {}\nBeta = 1".for
plot_vol_surface([SABR_beta0.vol_surface, black_var_surface], title=title)

title = "normal SABR Volatility Surface on {} VS IV surface\n{} to {}\nBeta = 0.5".f
plot_vol_surface([SABR_beta5.vol_surface, black_var_surface], title=title)

title = "normal SABR Volatility Surface on {} VS IV surface\n{} to {}\nBeta = 0".for
plot_vol_surface([SABR_beta0.vol_surface, black_var_surface], title=title)

title = "SABR Volatility Surface on {} VS Heston surface\n{} to {}\nBeta = 1".format
plot_vol_surface([SABR_beta1.vol_surface, hestonModel1.heston_vol_surface], title=ti

title = "IV Surface on {} vs Heston Surface\n{} to {}\nBeta = 1".format(data, today,
plot_vol_surface([black_var_surface, hestonModel1.heston_vol_surface], title=title)
```



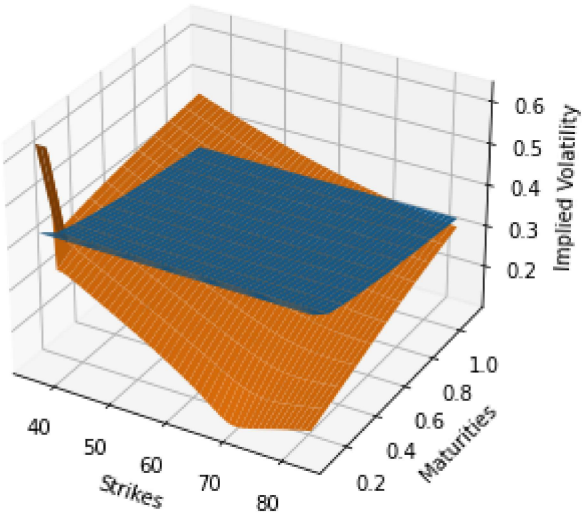
normal SABR Volatility Surface on OIL VS IV surface
August 30th, 2021 to October 26th, 2022
Beta = 0.5

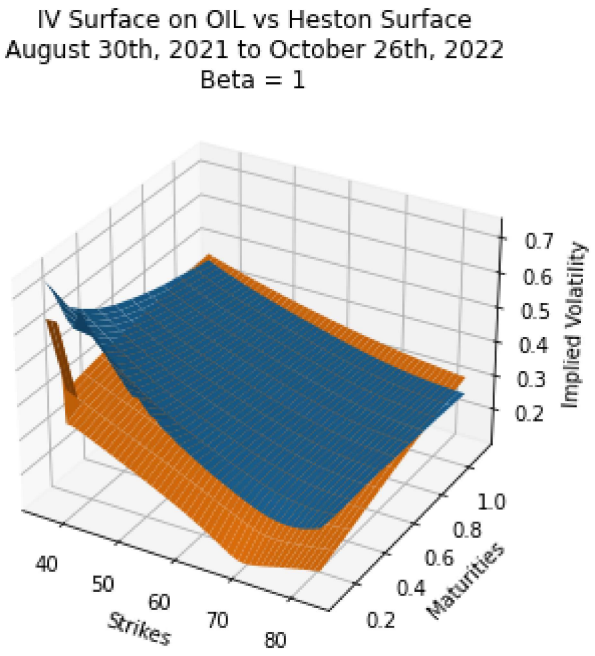


normal SABR Volatility Surface on OIL VS IV surface
August 30th, 2021 to October 26th, 2022
Beta = 0



SABR Volatility Surface on OIL VS Heston surface
August 30th, 2021 to October 26th, 2022
Beta = 1

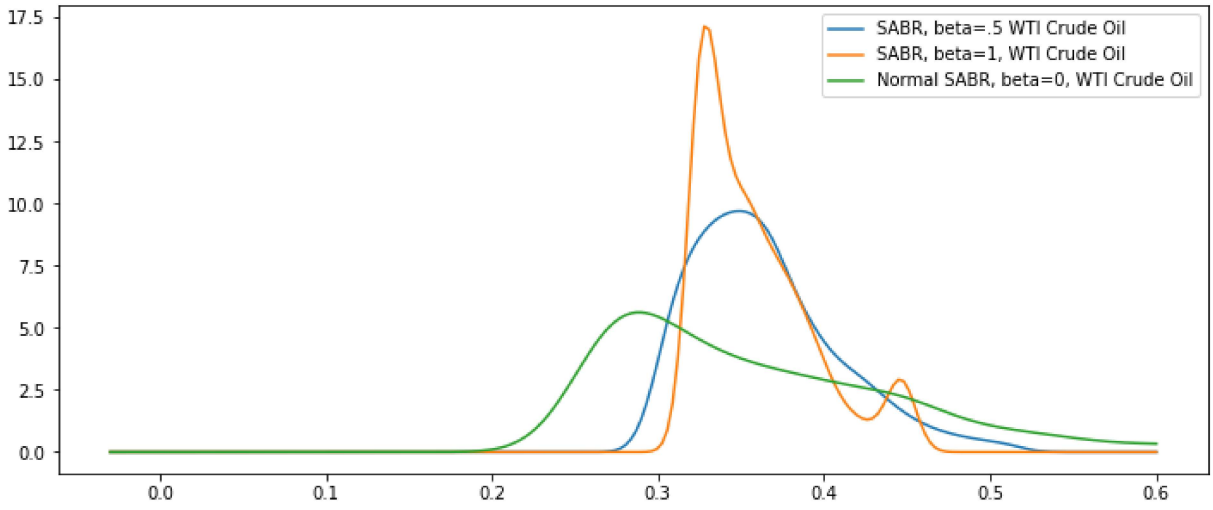




In [20]:

```
from scipy.stats import gaussian_kde
import seaborn as sns
plt.figure(figsize=plot_size)
def density(model):
    rn = []
    for i in np.arange(0, 1.5, .01):
        for j in np.arange(model.vol_surface.minStrike(), model.vol_surface.maxStrik
            rn.append(model.vol_surface.blackVol(i,j))

    density = gaussian_kde(rn)
    xs = np.linspace(-.03,.6,200)
    density.covariance_factor = lambda : .25
    density._compute_covariance()
    plt.plot(xs,density(xs), label=model.label)

    plt.legend()


density(SABR_beta5), density(SABR_beta1), density(SABR_beta0),
```

Out[20]: (None, None, None)



In [ ]: