Algorithm

Consider a three-level atom in Λ formation interacting with an electric field from a laser pulse. The fundamental levels are $|2\rangle, |3\rangle$ and the upper level is $|1\rangle$. The hamiltonian is :

$$
\begin{aligned}
H &= H_0 + H_{int} \\
&= \begin{pmatrix} \hbar\omega_1 & 0 & 0 \\ 0 & \hbar\omega_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\mu_{12}E(t) & -\mu_{13}E(t) \\ \mu_{12}E(t) & 0 & 0 \\ -\mu_{13}E(t) & 0 & 0 \end{pmatrix}
\end{aligned}
$$

With the Von Neumann equation for time evolution $i\hbar\frac{\partial\rho}{\partial t} = [H, \rho]$.

$$
\begin{aligned}
i\hbar\dot{\rho_{11}} &= -\mu_{12}E(t)\rho_{21} - \mu_{12}E(t)\rho_{31} + \mu_{12}E(t)\rho_{12} + \mu_{13}E(t)\rho_{13} \\
i\hbar\dot{\rho_{22}} &= -\mu_{12}E(t)\rho_{12} + \mu_{12}E(t)\rho_{21} \\
i\hbar\dot{\rho_{33}} &= -\mu_{13}E(t)\rho_{13} + \mu_{13}E(t)\rho_{31} \\
i\hbar\dot{\rho_{12}} &= \hbar\omega_1\rho_{12} - \mu_{12}E(t)\rho_{22} - \mu_{13}E(t)\rho_{32} + \mu_{12}E(t)\rho_{11} - \rho_{12}\hbar\omega_2 \\
i\hbar\dot{\rho_{13}} &= \hbar\omega_1\rho_{13} - \mu_{12}E(t)\rho_{23} - \mu_{13}E(t)\rho_{33} + \mu_{13}E(t)\rho_{11} \\
i\hbar\dot{\rho_{23}} &= -\mu_{12}E(t)\rho_{13} + \hbar\omega_2\rho_{23} + \mu_{13}E(t)\rho_{21}
\end{aligned}
\tag{1}
$$

$E(t)$ is the electric field of the pulse laser, which can be expressed as $E(t) = \frac{1}{2}E_{env}(t)\left(e^{i\omega_L t} + e^{-i\omega_L t}\right)$. Where ω_L is the carrier wave frequency, and E_{env} is the slowly varying envelope of the laser pulse.

In rotating wave approximation, with

$$
\begin{aligned}
\rho_{12} &= \sigma_{12}e^{i\omega_L t} \\
\rho_{13} &= \sigma_{13}e^{i\omega_L t} \\
\rho_{23} &= \sigma_{23}
\end{aligned}
$$

the high frequency terms can be neglected. This approximation can also be applied to system with Zeeman sublevels.

“Flatten” or “unravel” the density matrix, for example:

$$
\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} \rho_{11} \\ \rho_{12} \\ \rho_{13} \\ \rho_{21} \\ \rho_{22} \\ \rho_{23} \\ \rho_{31} \\ \rho_{32} \\ \rho_{33} \end{pmatrix}
$$

Using the unravel version of density matrix ζ, equation eq.1 can be express as:

$$\dot{\zeta} = (A + B \times E_{env}(t))\zeta \tag{2}$$

B are the terms that cantain the dipole moments μ. And the decoherence terms can be added to A since they are time-independent.

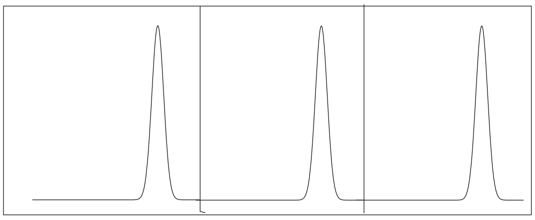

Figure 1: Pulses

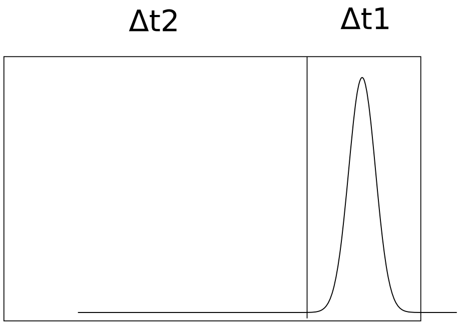

Figure 2: Pulse

The laser pulse train can be devided in to segments as fig.1 shows. Each segment contains two parts, one with the pulse, one with neglectable electric field, as fig.2 shows.

Suppose the unraveled density matrix is $\zeta(0)$ just before the pulse. Define two matrices as follow:

$$\zeta(\Delta t_1) = P'\zeta(0)$$

$$\zeta(\Delta t_1 + \Delta t_2) = P\zeta(\Delta t_1) = PP'\zeta(0)$$

Since there is no electric field in the Δt_2 period, the motion equation is $\dot{\zeta} = A\zeta$. So $\zeta(\Delta t_1 + \Delta t_2) = e^{iA\Delta t_2}\zeta(\Delta t_1)$.

$$P = e^{iA\Delta t_2}$$

P' can be solve perturbatically. Define $\zeta_n(t)$ as the nth order solution of $\dot{\zeta} = (A + BE_{env}(t))\,\zeta$ where $0 < t < \Delta t_1$. Define $F(t)_n\zeta(0) = \zeta_n(t)$ and $G_n(t)\zeta(0) = \dot{\zeta_n}(t)$. $\frac{dF_n(t)}{dt} = G_n(t)$.

If the initial condition of unravel density matrix $\zeta(0)$ is known. $\zeta_0(t) = e^{iAt}\zeta(0)$. The next order of ζ can be obtain by:

$$\zeta_{n+1}(t) = \int_0^t (A + BE_{env}(t))\,\zeta_n(t)\,dt \tag{3}$$

subtitute $\zeta_n(t)$ for $F_n(t)\zeta(0)$,eq.3 becomes:

$$F_{n+1}(t)\zeta(0) = \int_0^t (A + BE_{env}(t))\,F_n(t)\zeta(0)\,dt \tag{4}$$

remove the constant array $\zeta(0)$ which is the initial condition from both side:

$$F_{n+1}(t) = \int_0^t (A + BE_{env}(t))\,F_n(t)\,dt \tag{5}$$

$F_0(t) = e^{-iAt}$ and $F_n(0)$ is identy matrix I, eq. 5 is independent of initial condition of ζ. So $p' = \lim_{n\to\infty} F_n(\Delta t_1)$.

The change of unravel density matrix can be calculated with $PP'\zeta$. After n pulses the unravel density matrix become $(PP')^n\,\zeta$.