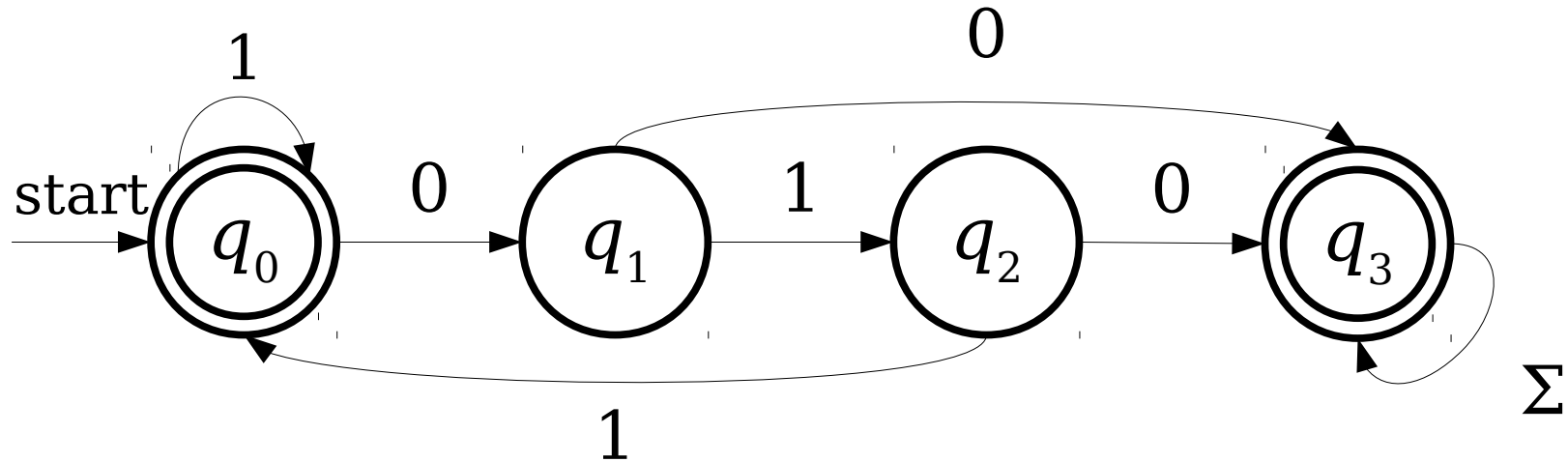


Finite Automata

Part Three

Recap from Last Time

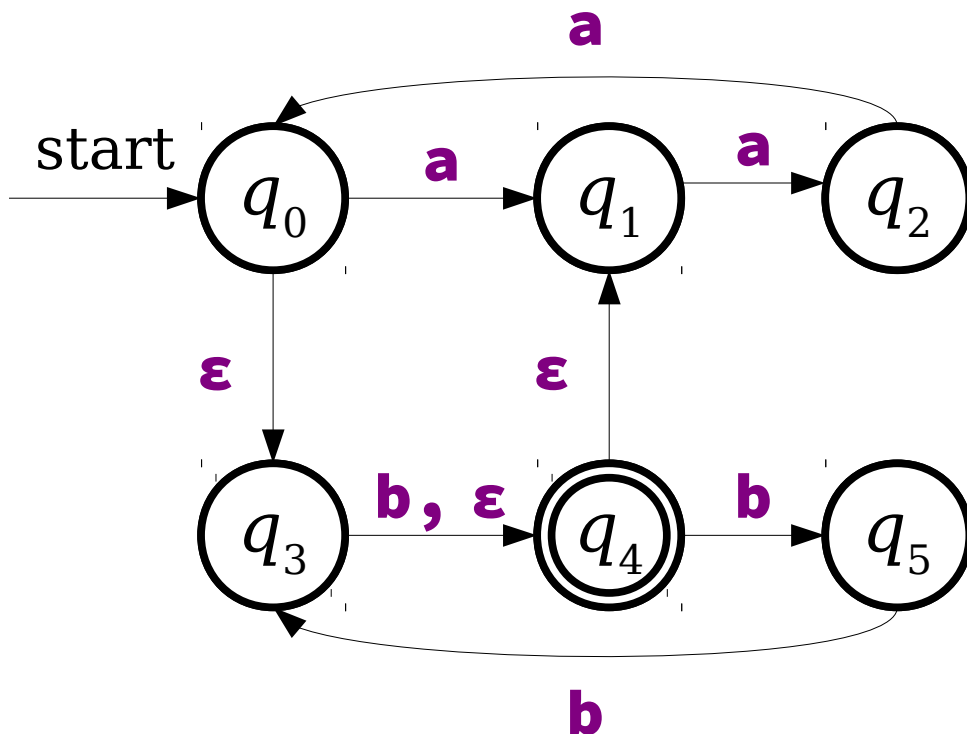
Tabular DFAs



	0	1
* q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
* q_3	q_3	q_3

ϵ -Transitions

- NFAs have a special type of transition called the **ϵ -transition**.
- An NFA may follow any number of ϵ -transitions at any time without consuming any input.



Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- The NFA accepts if *any* of the states that are active at the end are accepting states. It rejects otherwise.

Just how powerful *are* NFAs?

New Stuff!

NFAs and DFAs

- Let's just make a wild conjecture that NFAs and DFAs actually have the same power. How could we prove that?
- For all languages L , there exists an NFA that recognizes L iff there exists a DFA that recognizes L .
- **Question:** What is our proof technique for proving biconditionals? (“iff”)

NFAs and DFAs

- Forward direction:
 - For all languages L , there exists an NFA that recognizes L **if** there exists a DFA that recognizes L .
 - For all languages L , **if** there exists a DFA that recognizes L , **then** there exists an NFA that recognizes L .
- True. Why?

NFAs and DFAs

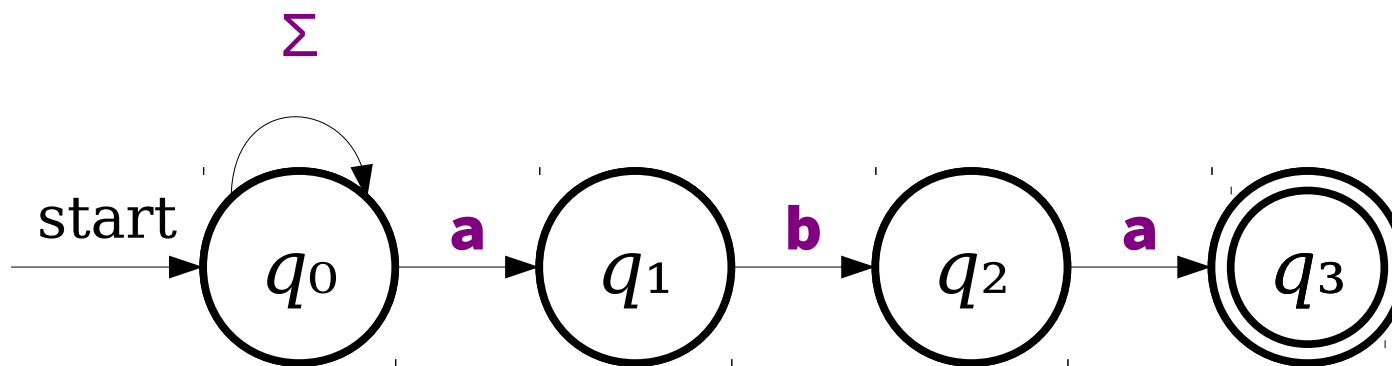
- Forward direction:
 - For all languages L , there exists an NFA that recognizes L **if** there exists a DFA that recognizes L .
 - For all languages L , **if** there exists a DFA that recognizes L , **then** there exists an NFA that recognizes L .
- True. Why?
 - Every DFA essentially already *is* an NFA!

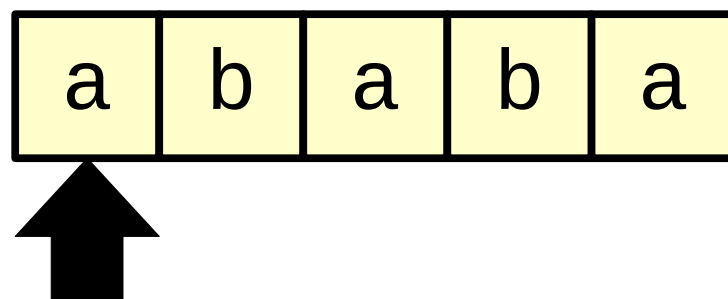
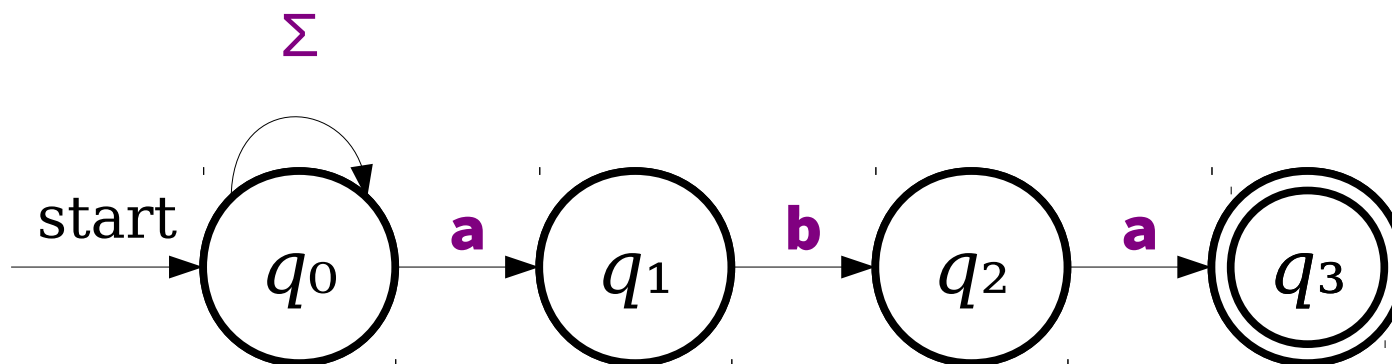
NFAs and DFAs

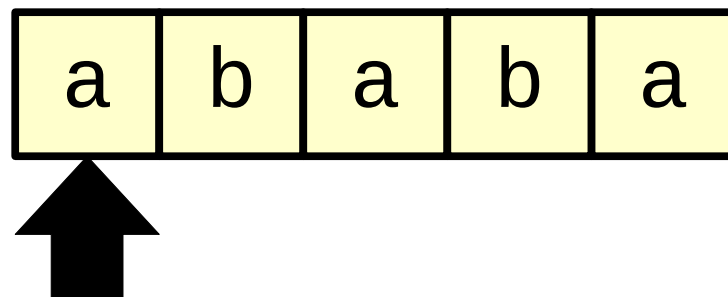
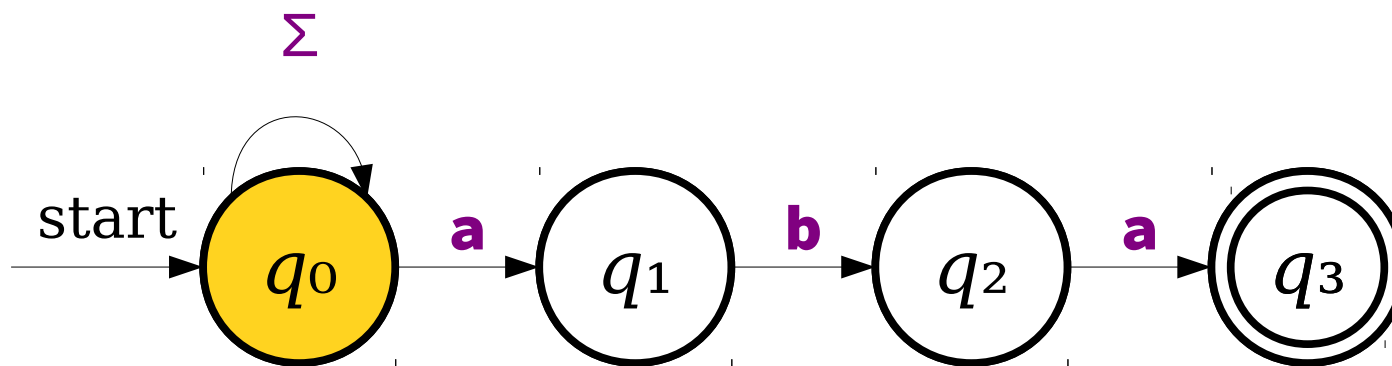
- Backward direction:
 - For all languages L , there exists an NFA that recognizes L **if** there exists a DFA that recognizes L .
 - For all languages L , **if** there exists an NFA that recognizes L , **then** there exists a DFA that recognizes L .
- True. Why?
 - *A little more complicated...*

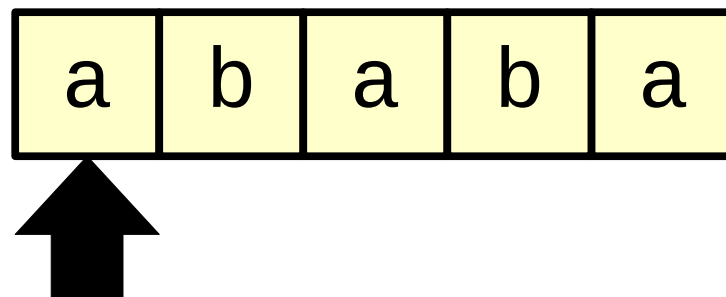
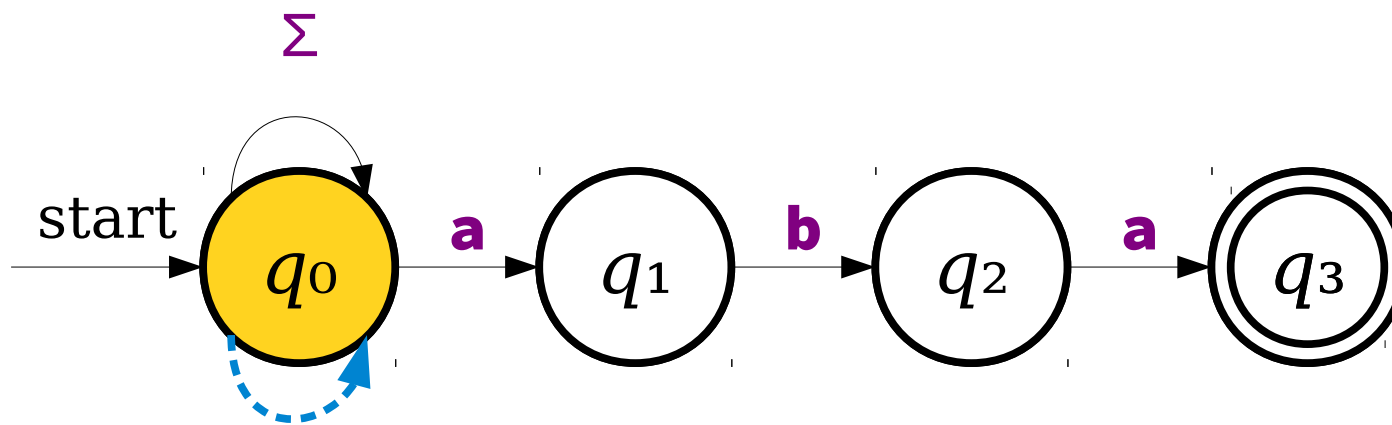
Thought Experiment:

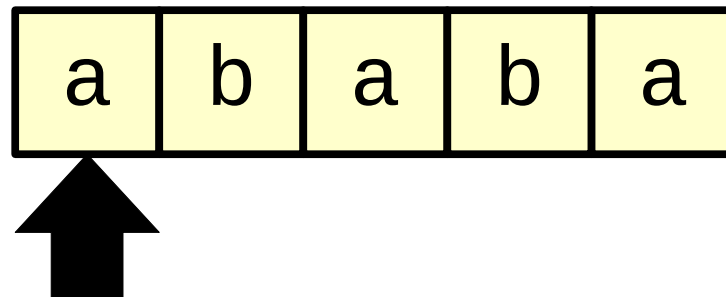
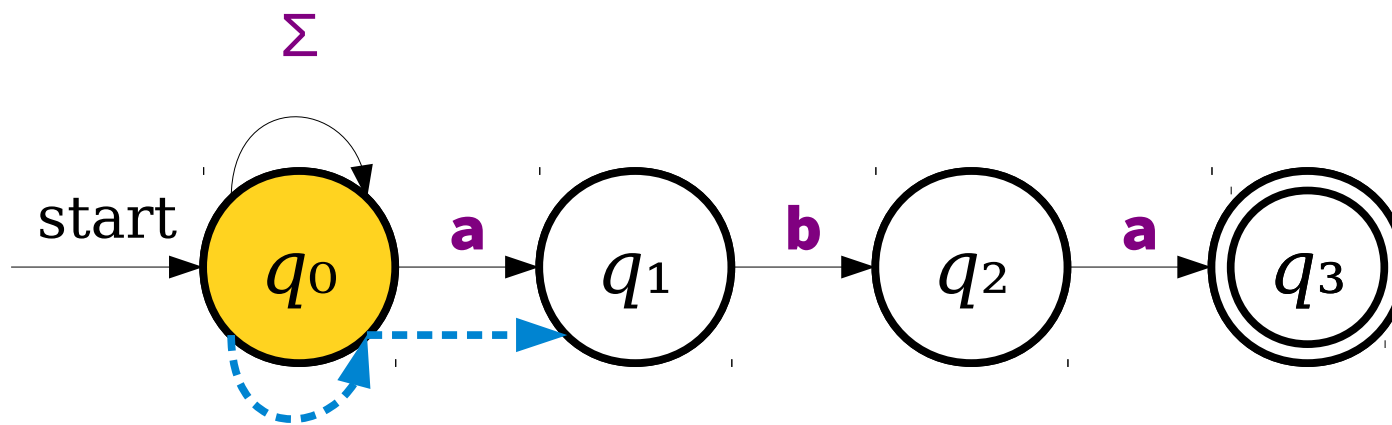
How would you simulate an NFA in software?

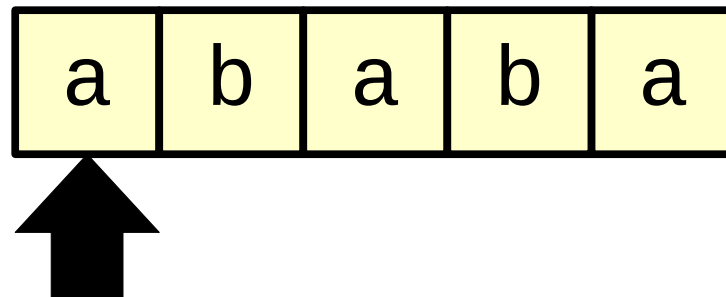
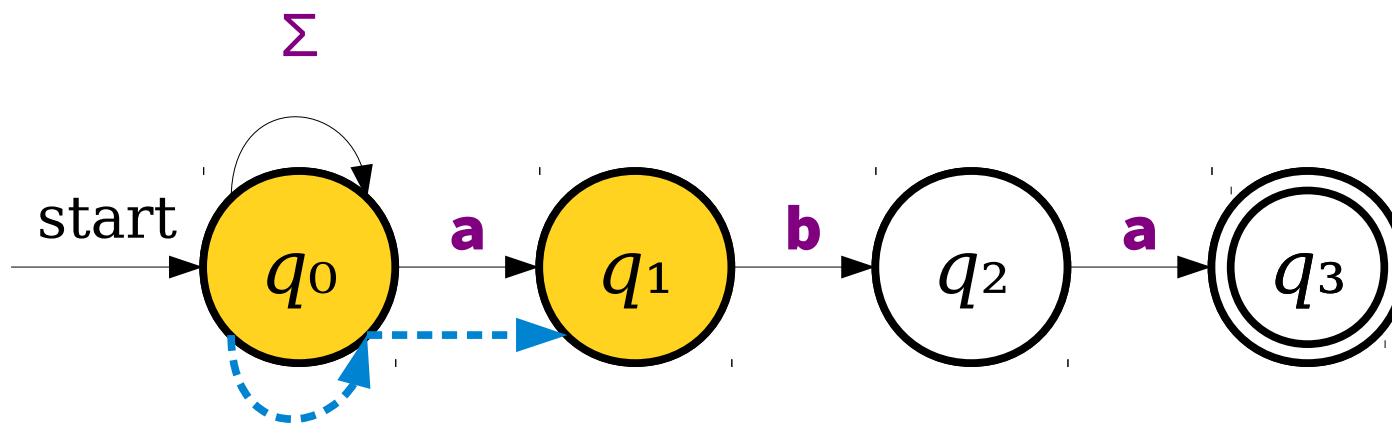


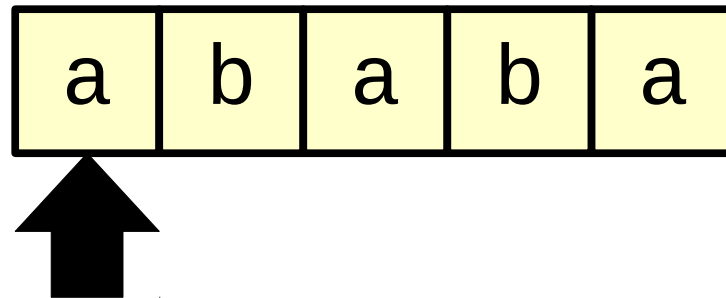
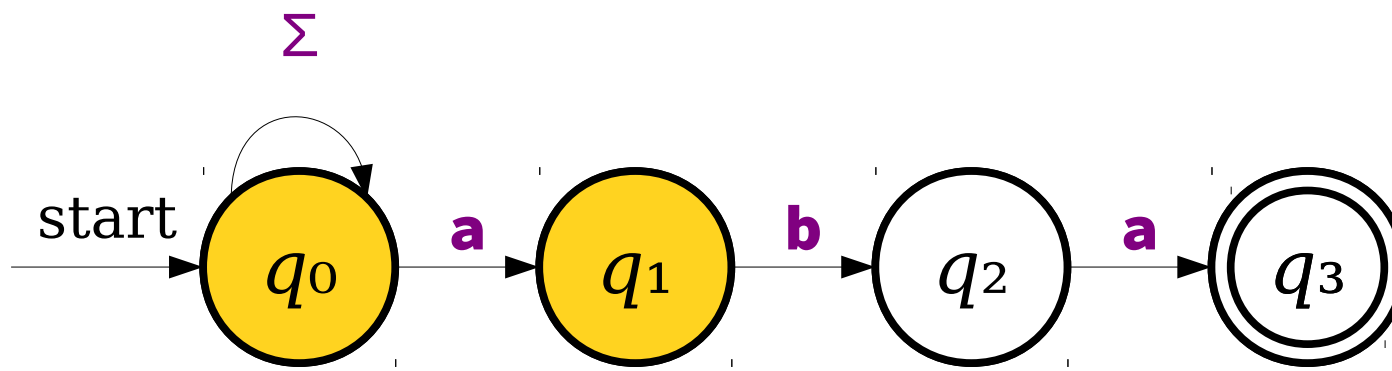


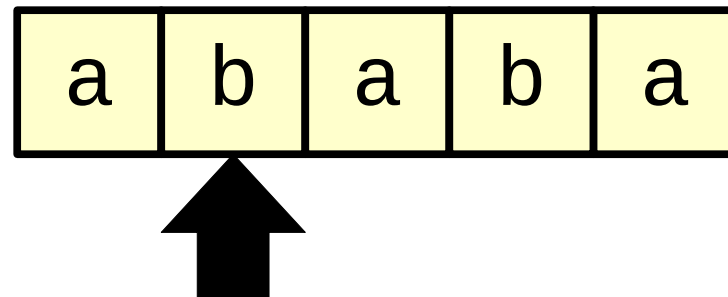
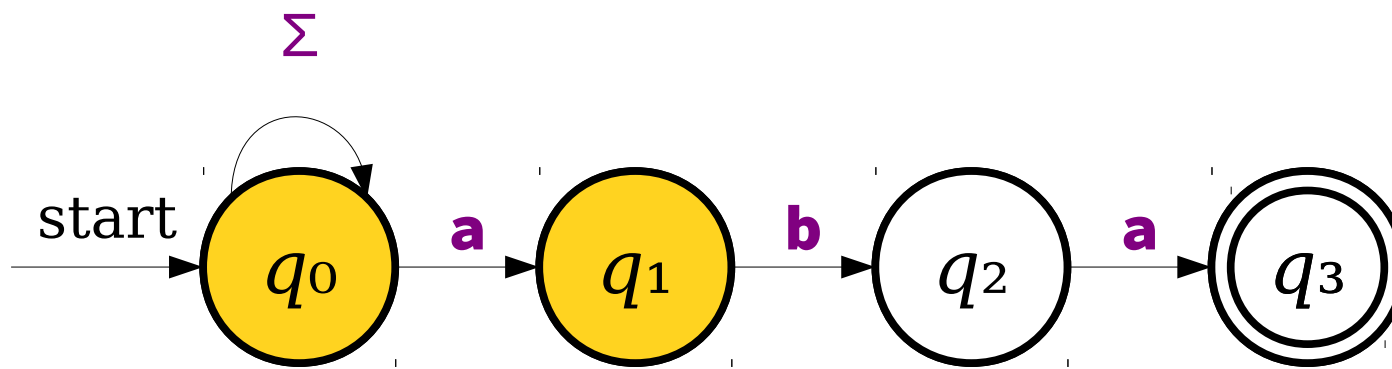


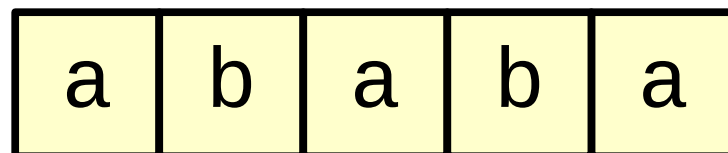
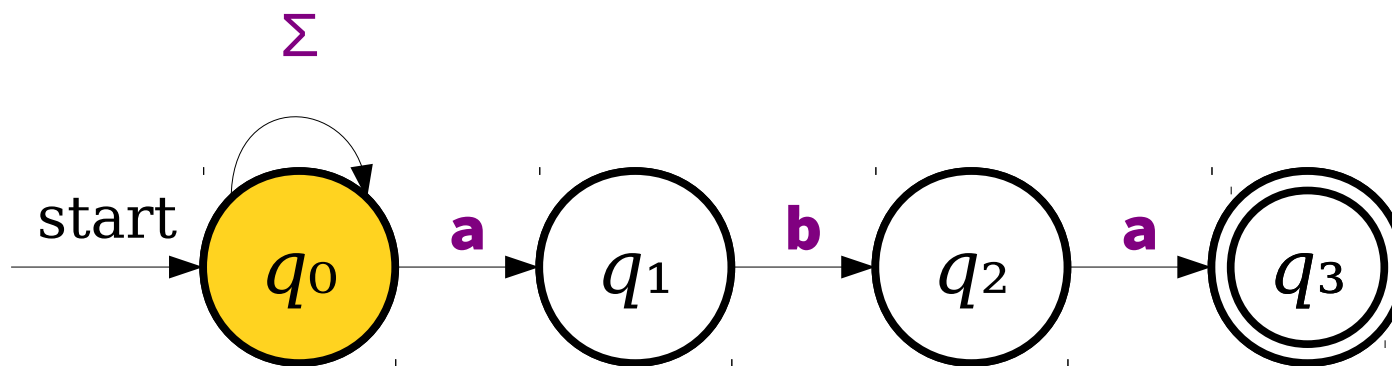


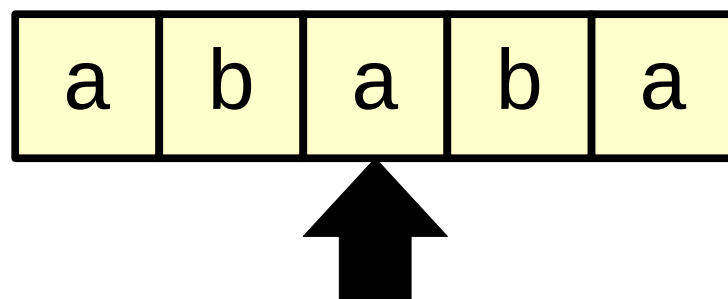
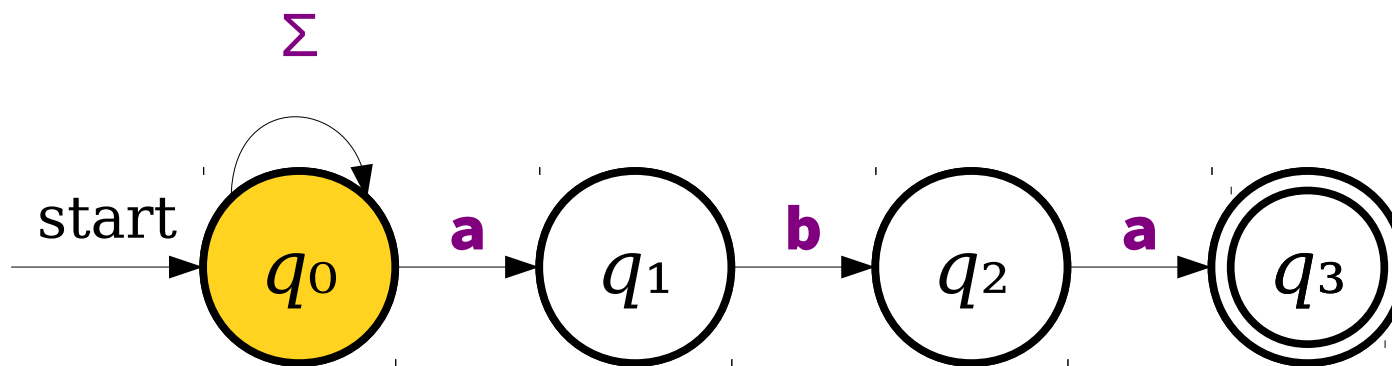


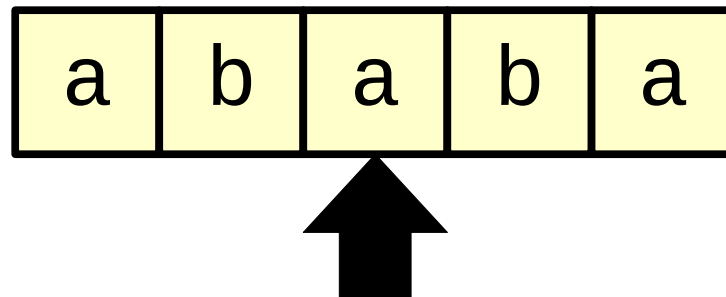
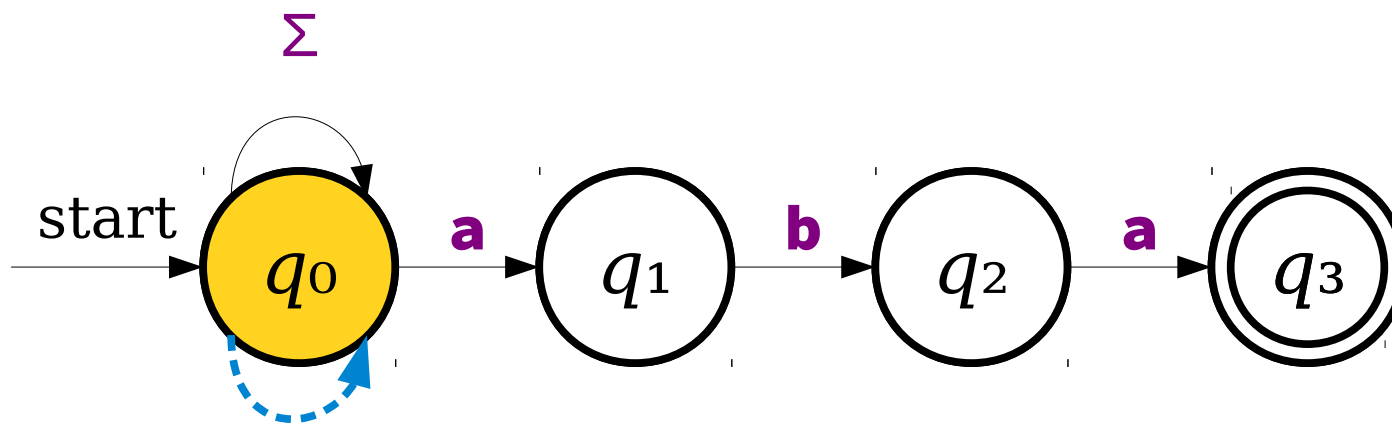


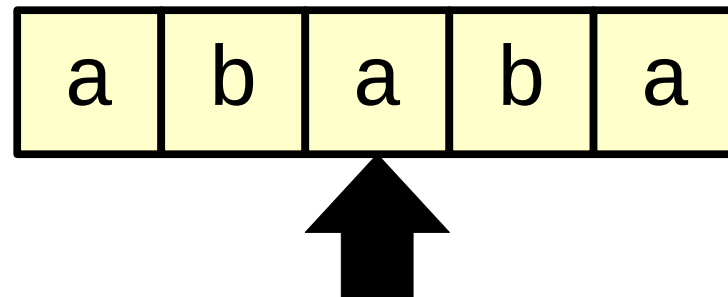
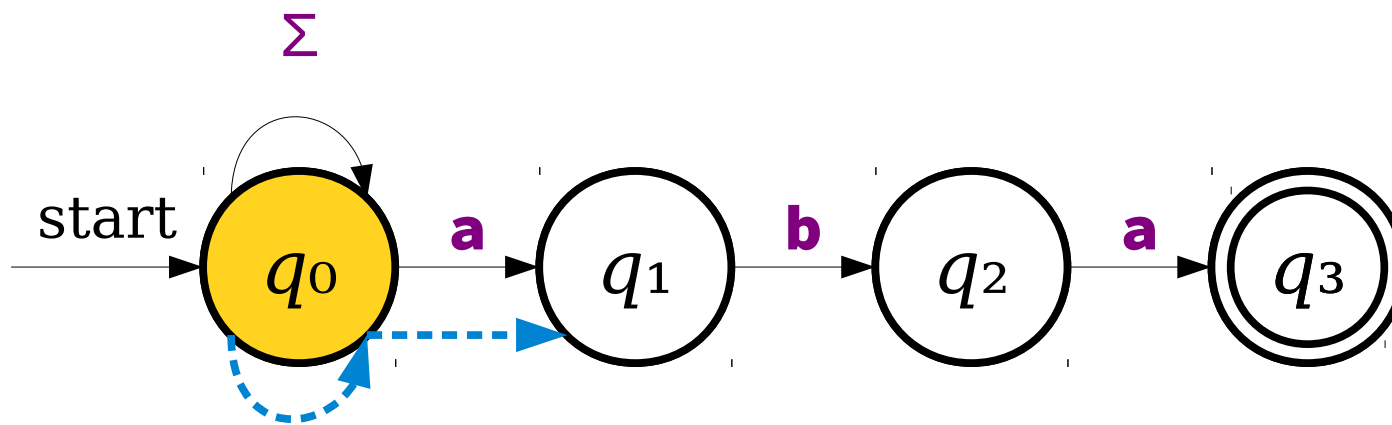


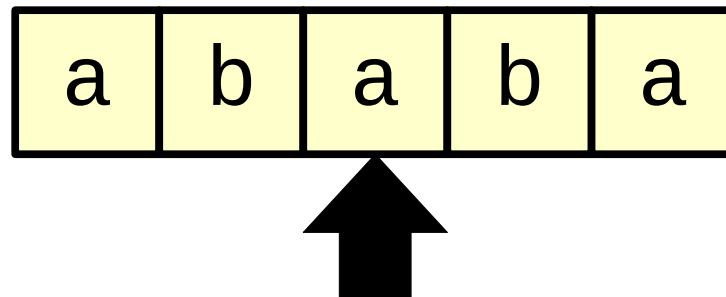
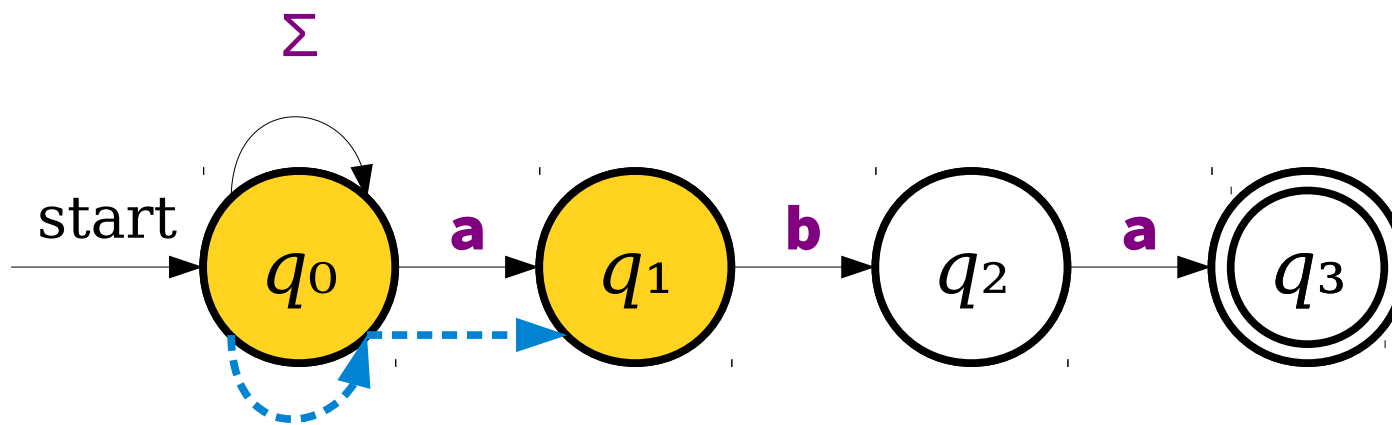


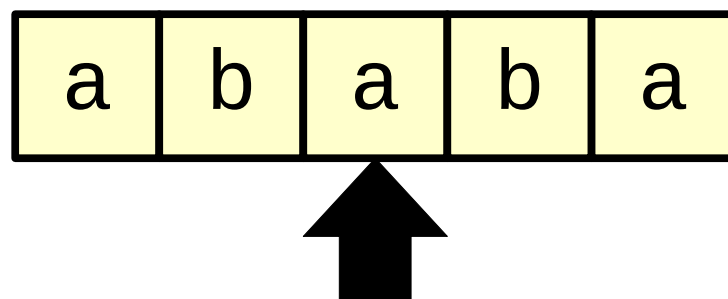
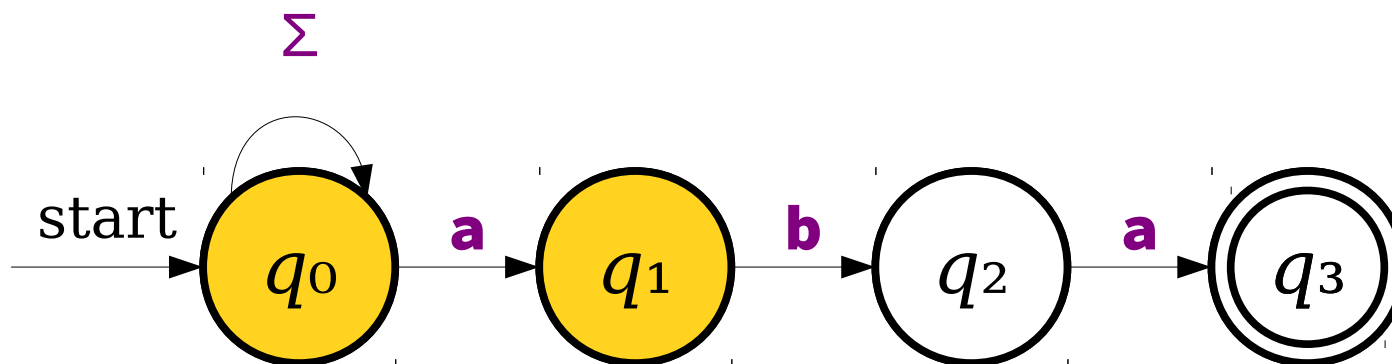


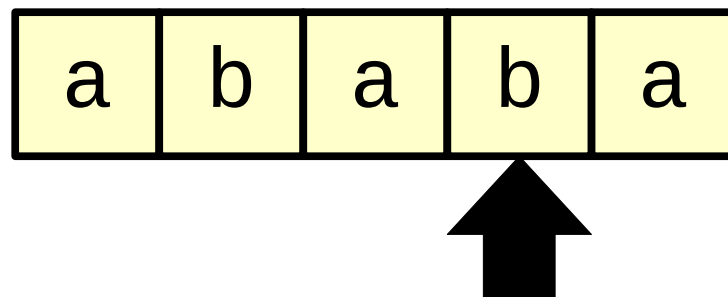
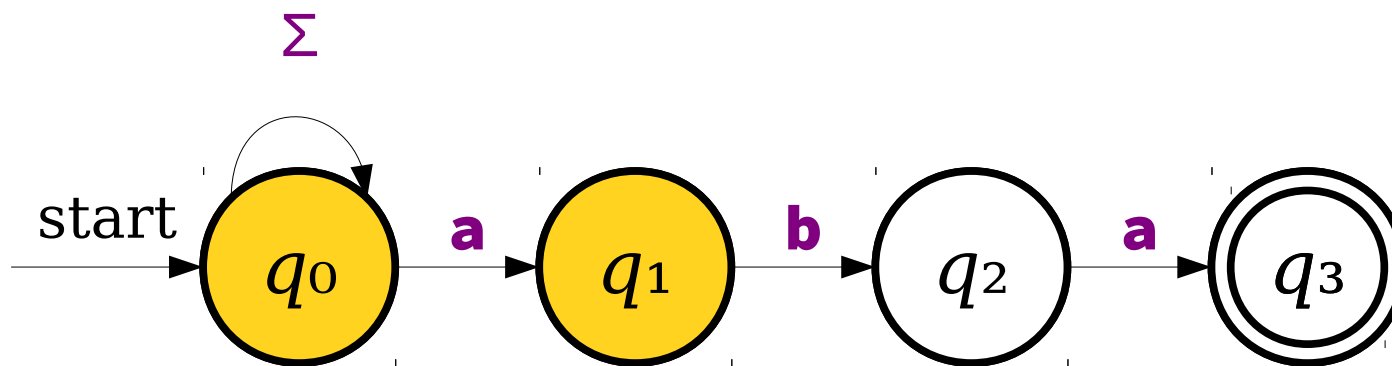


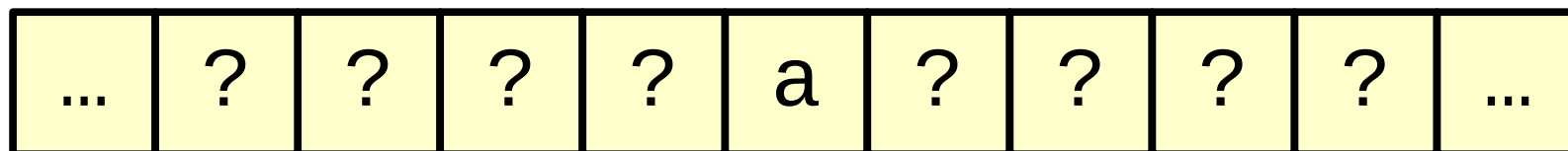
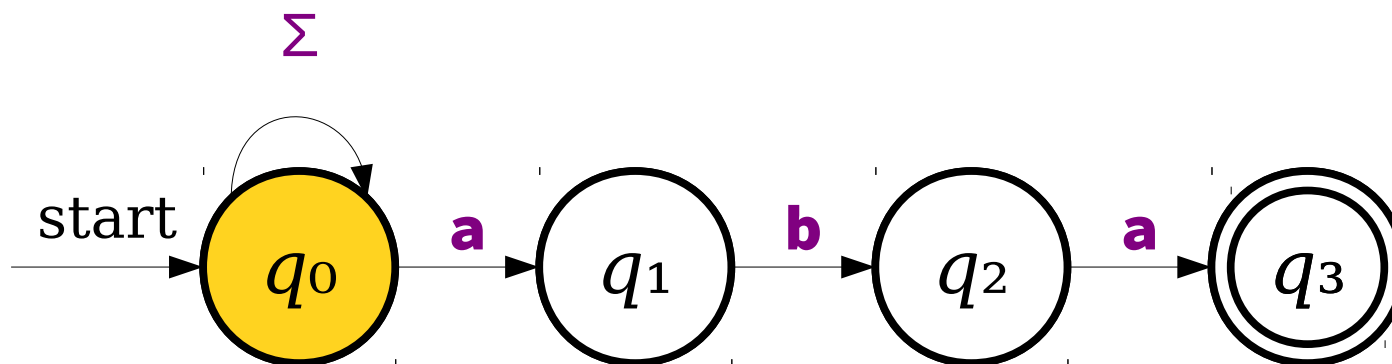


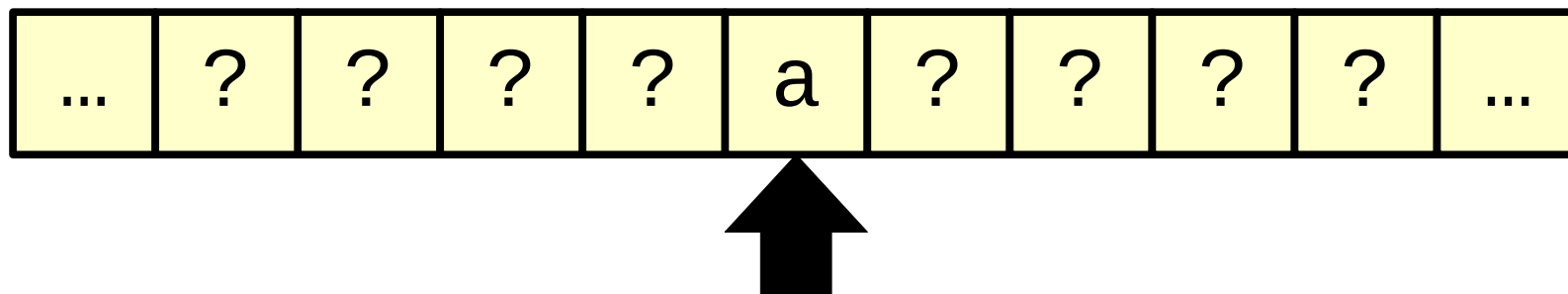
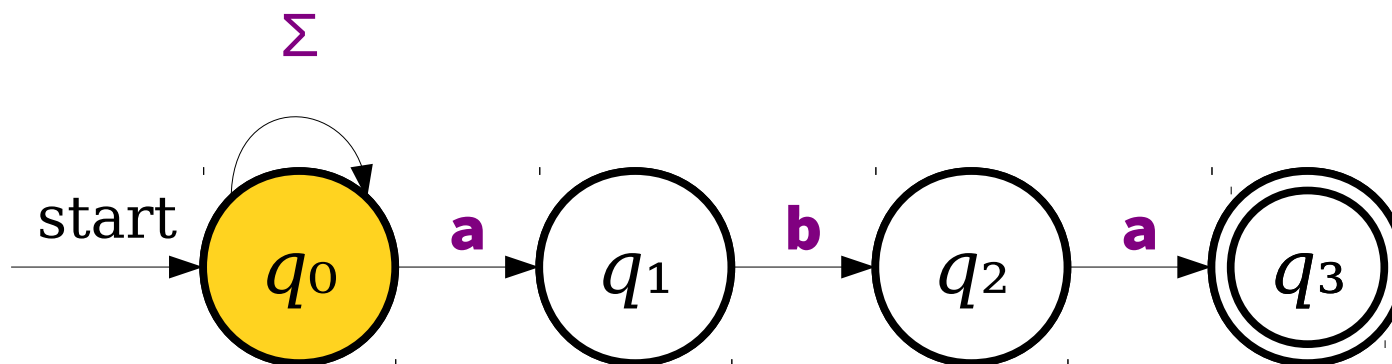


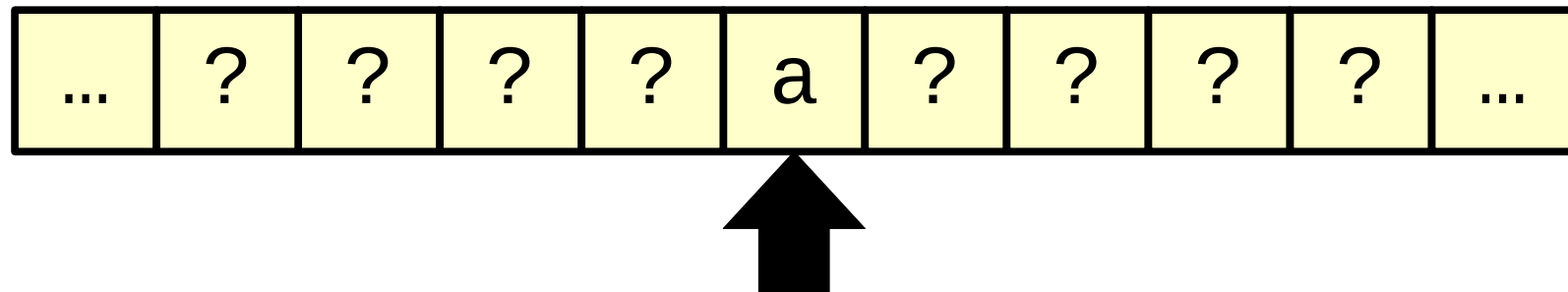
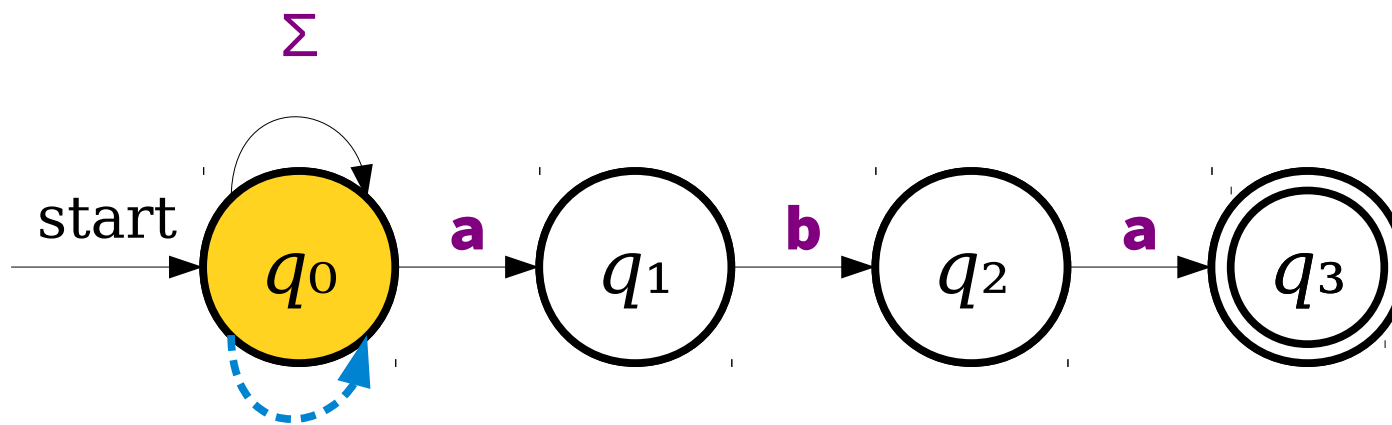


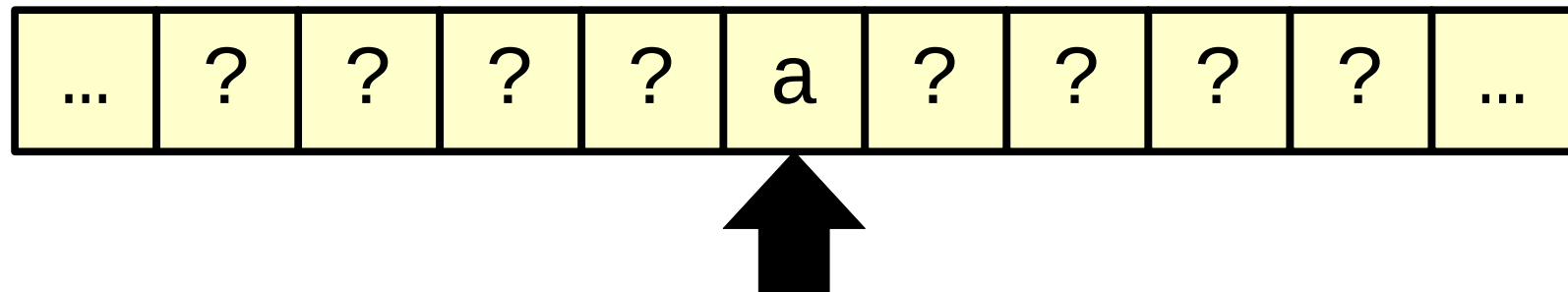
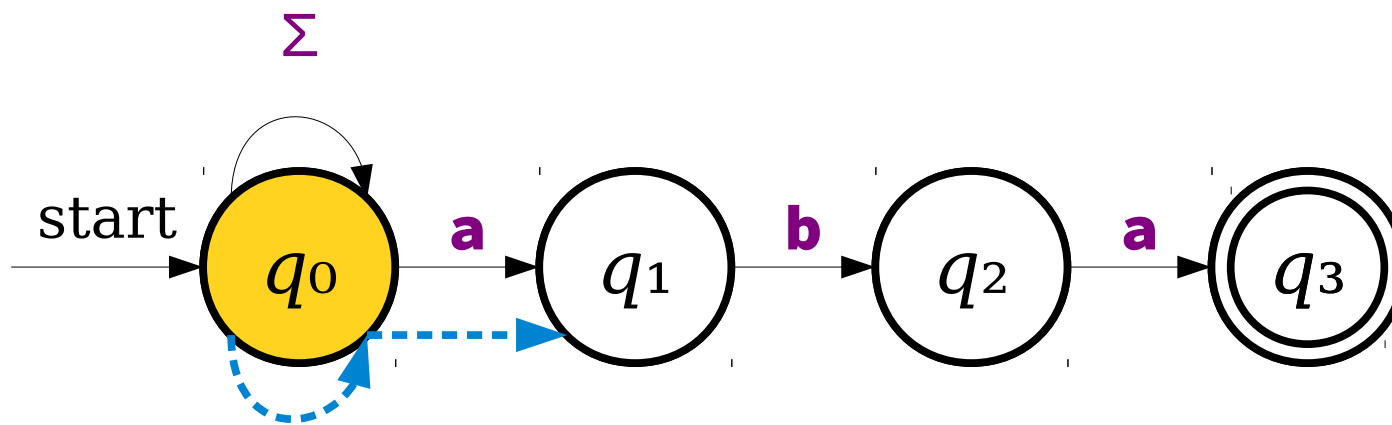


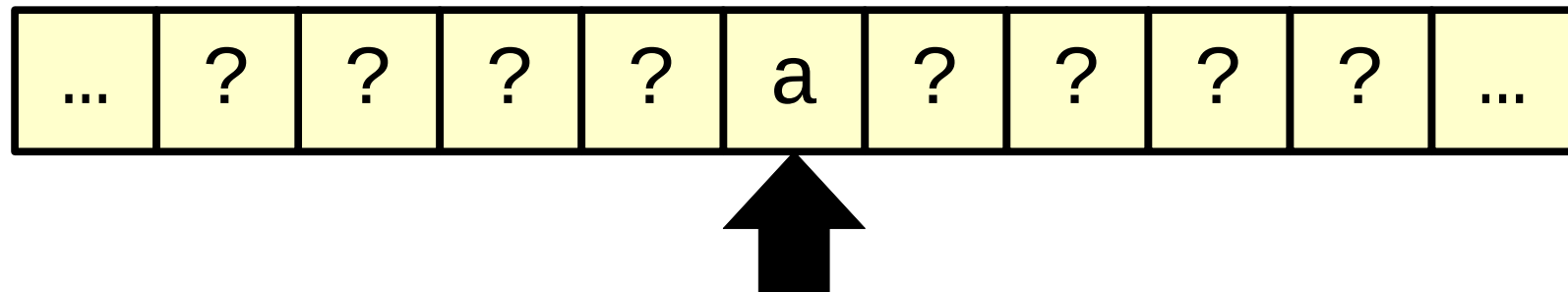
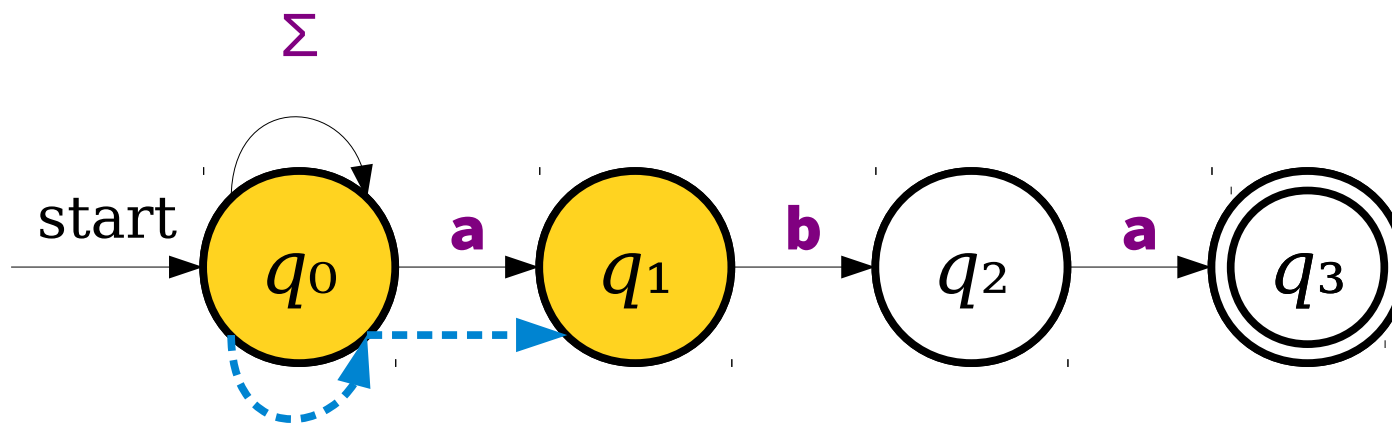


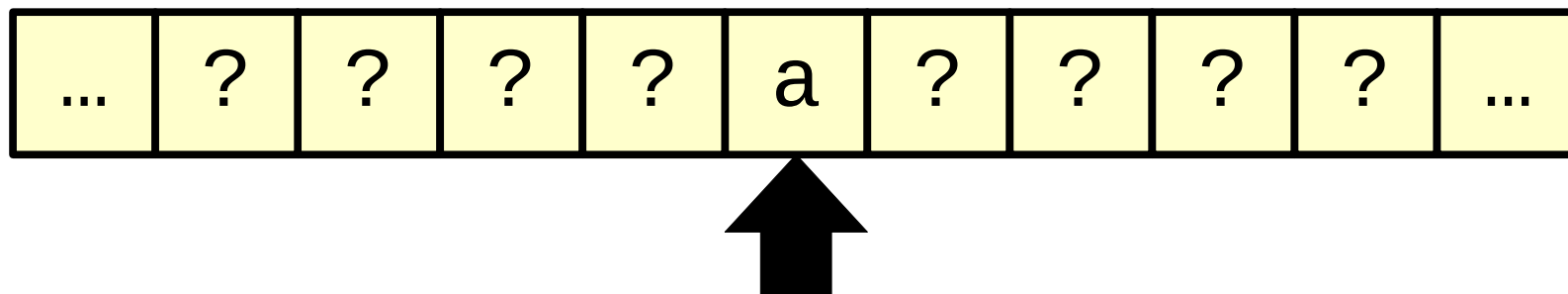
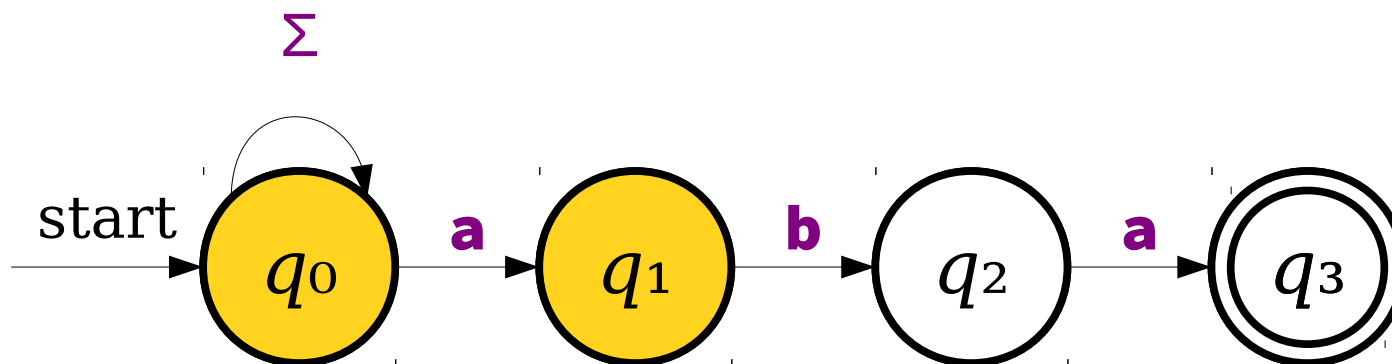


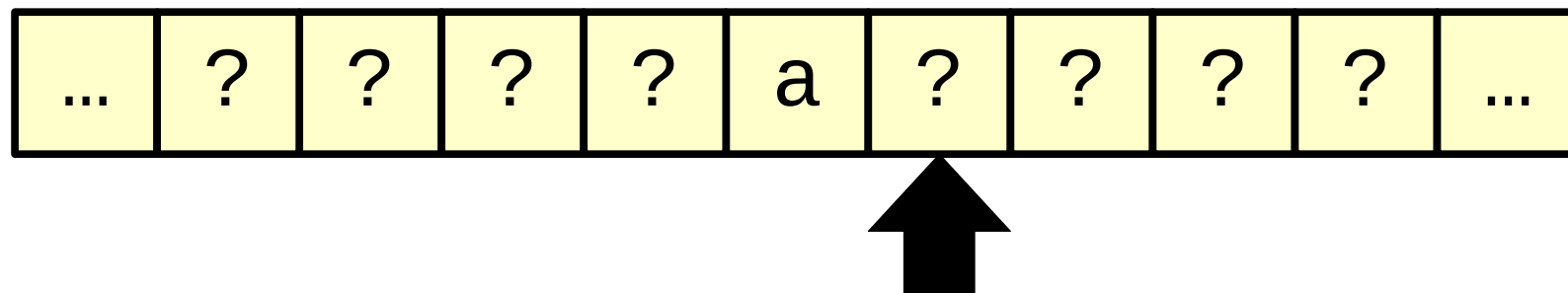
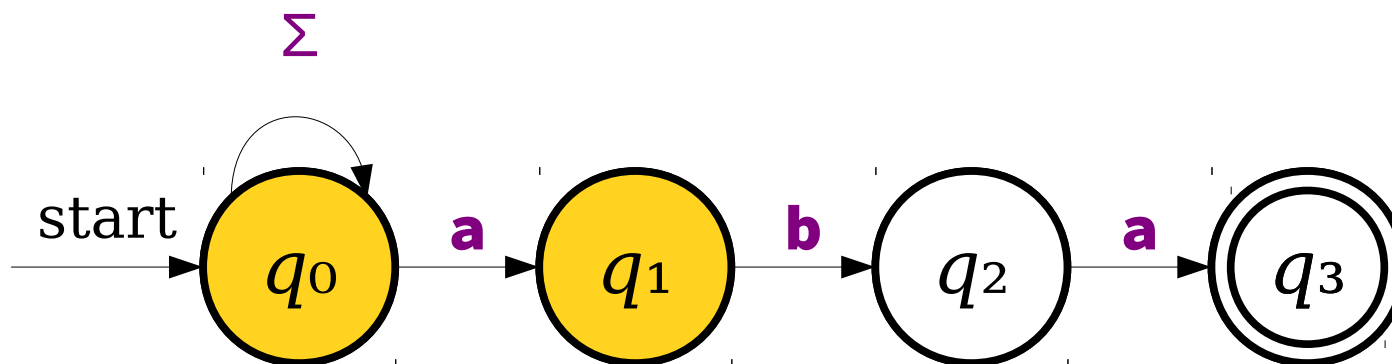


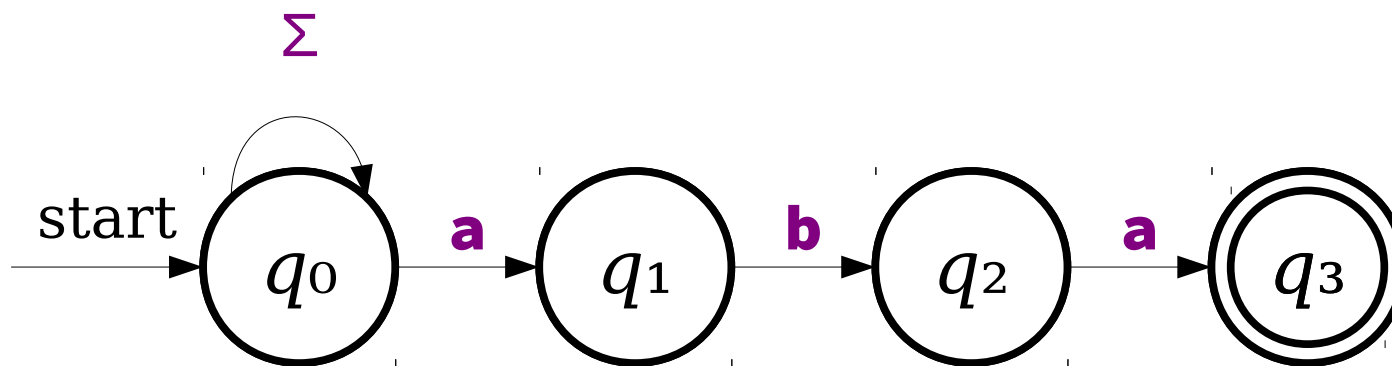




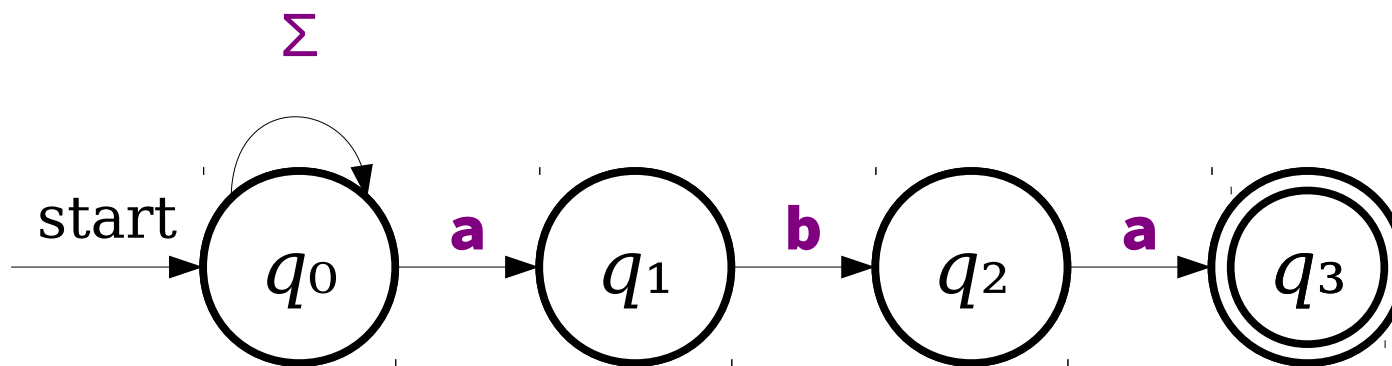




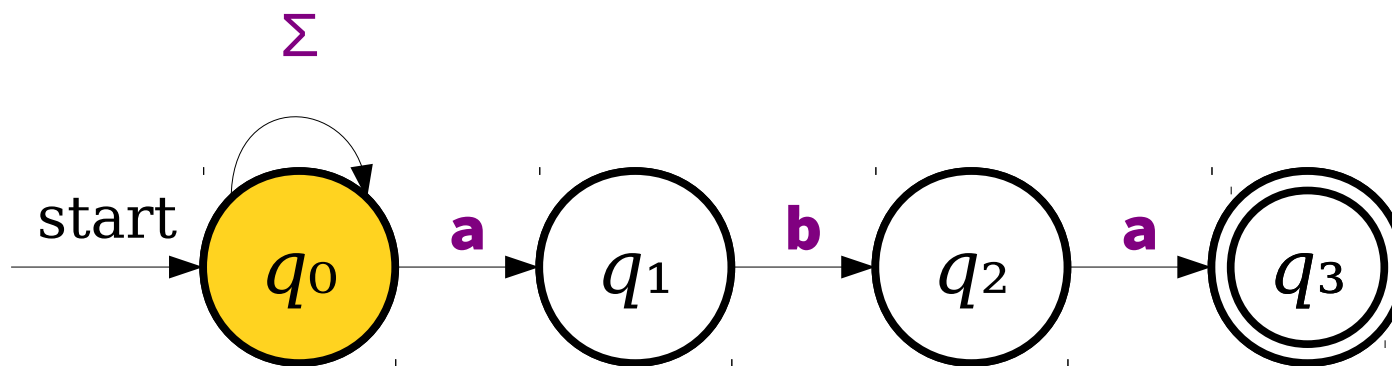




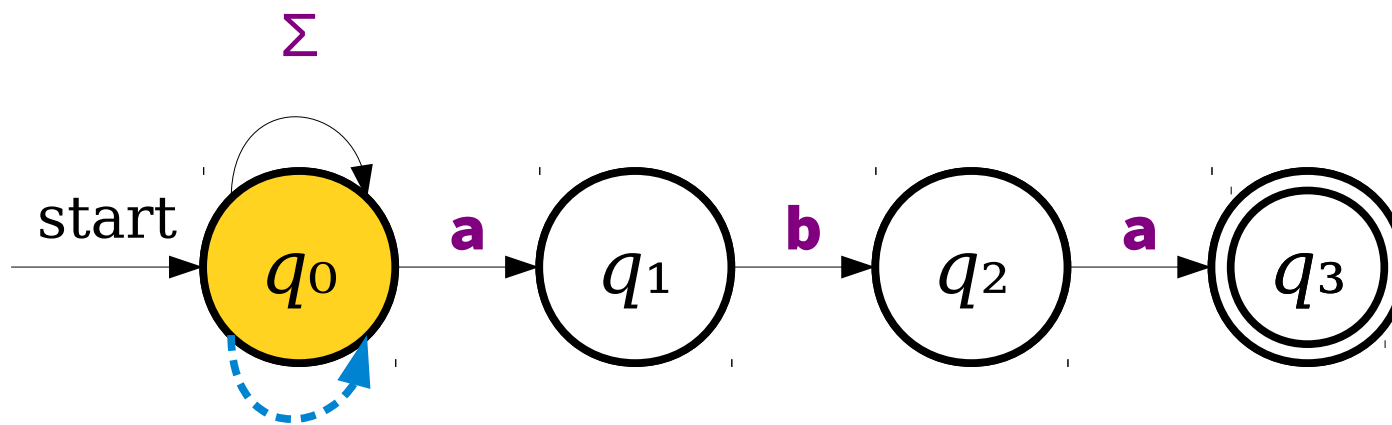
	a
$\{q_0\}$	$\{q_0, q_1\}$



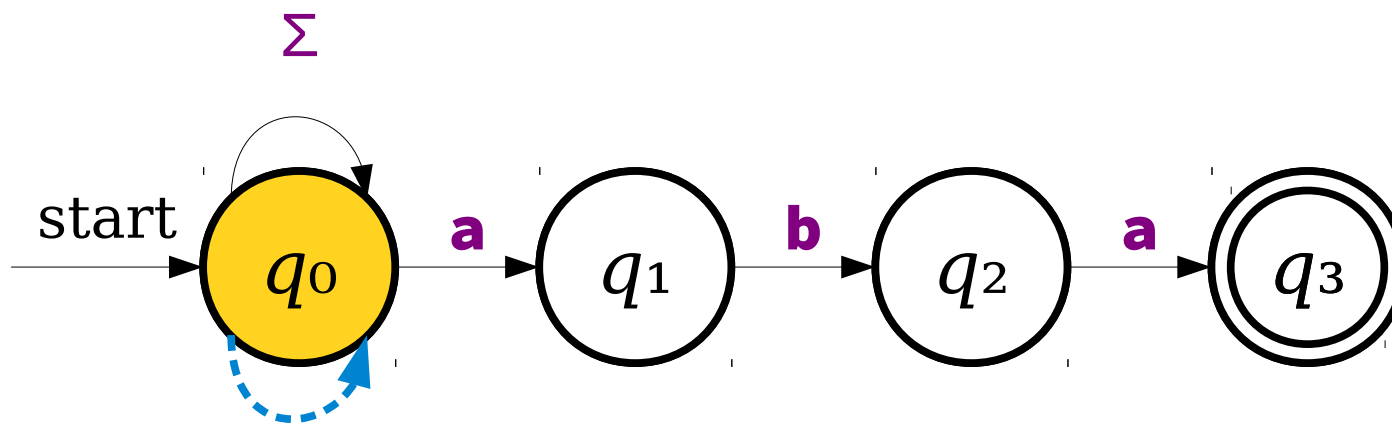
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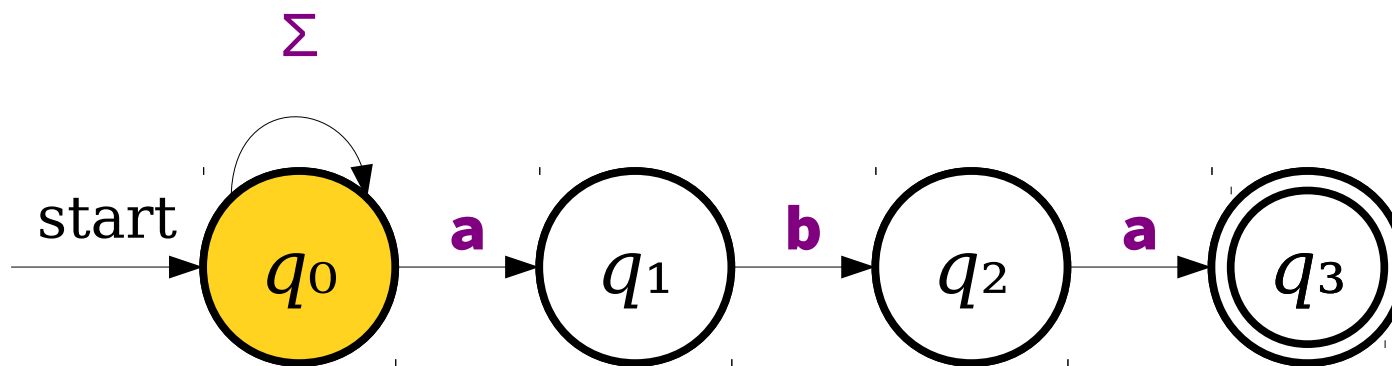
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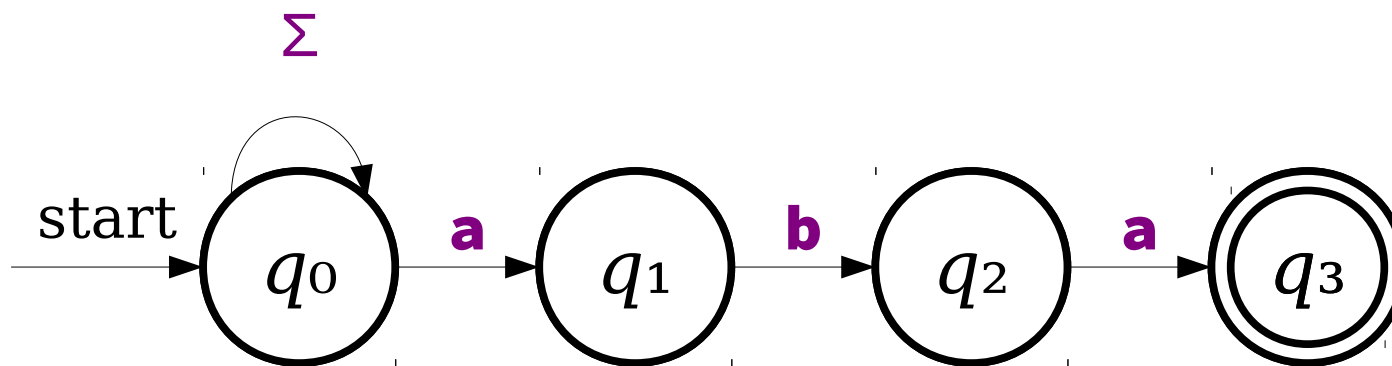
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



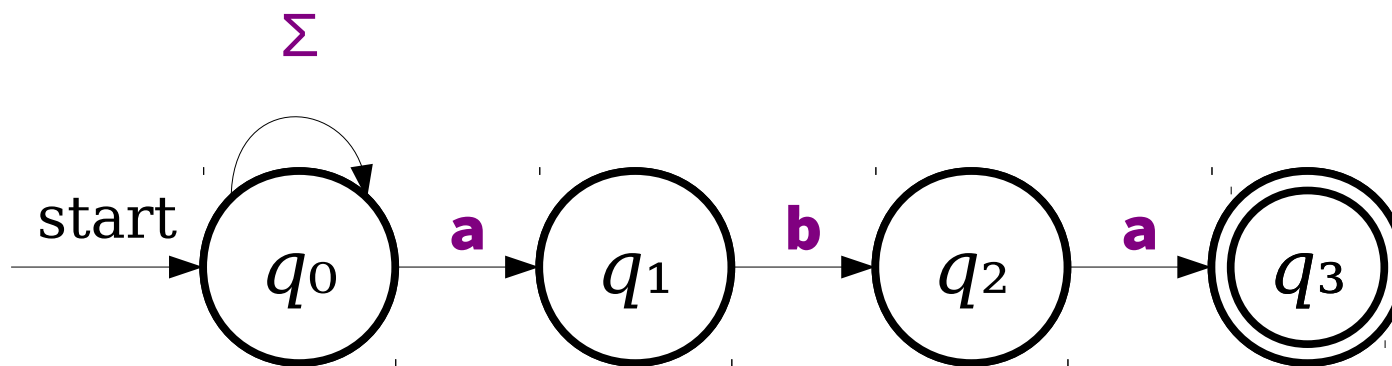
	a	b
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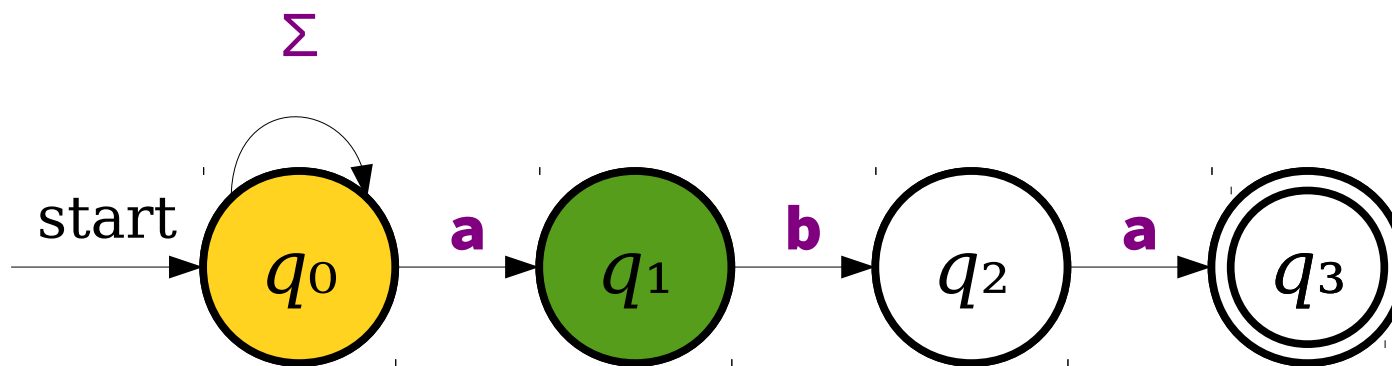
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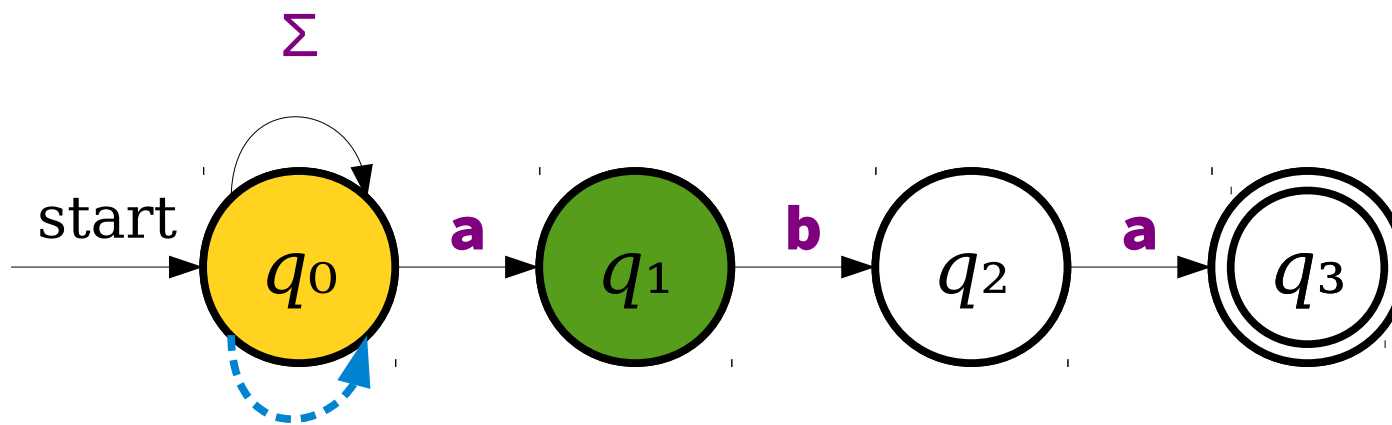
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



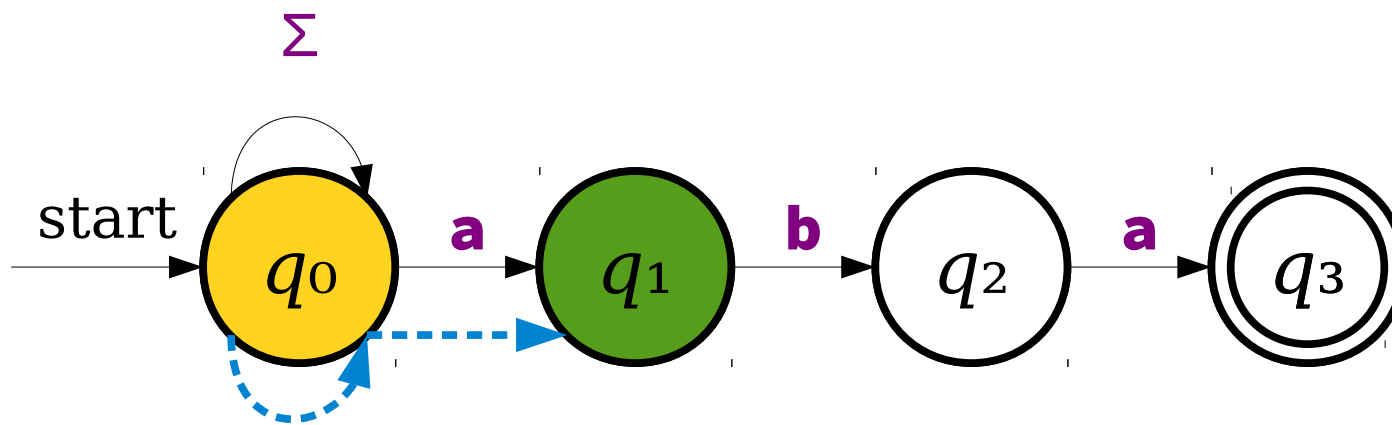
	a	b
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$\{q_0, q_1\}$		



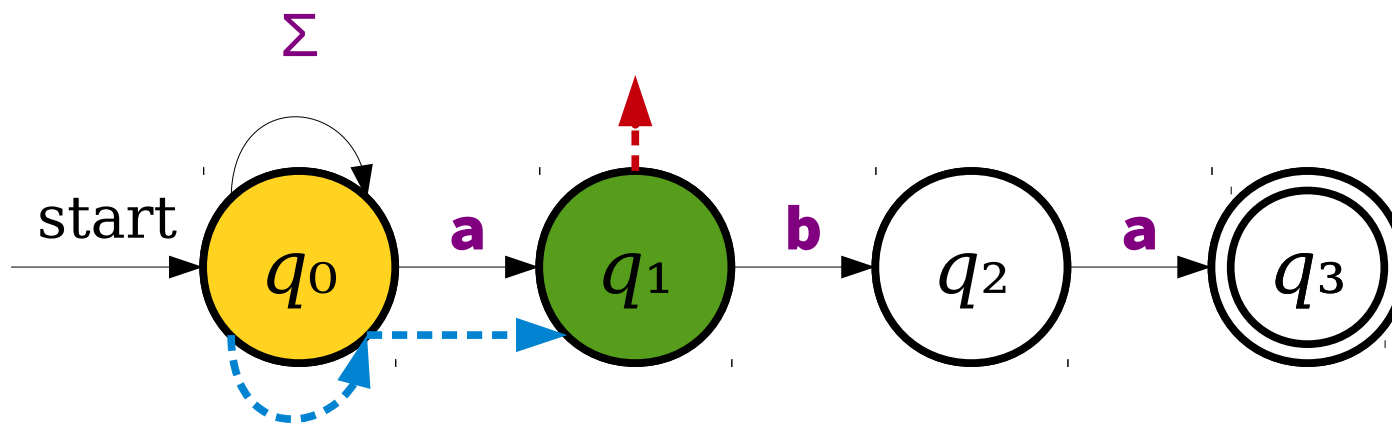
	a	b
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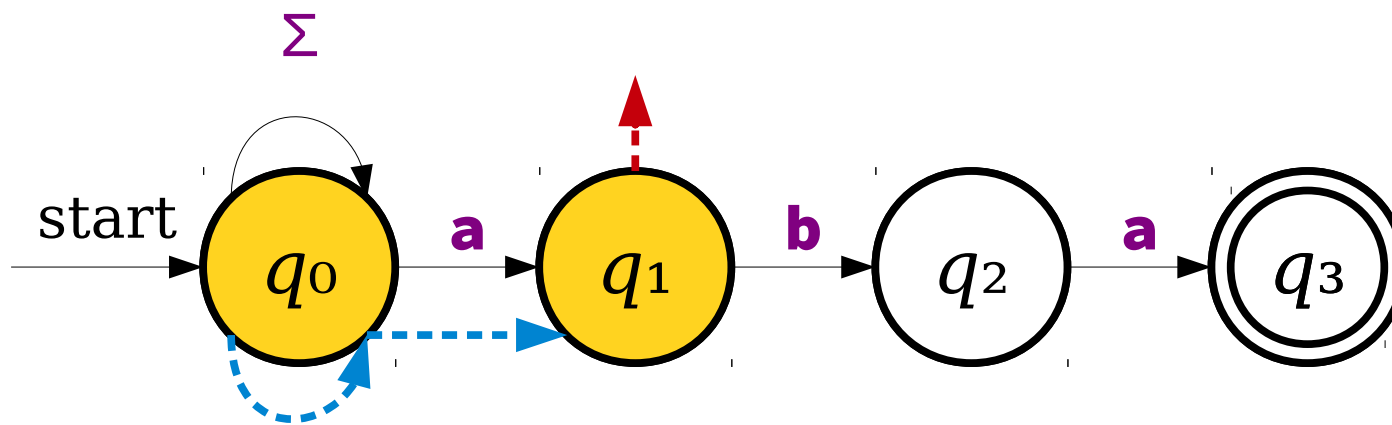
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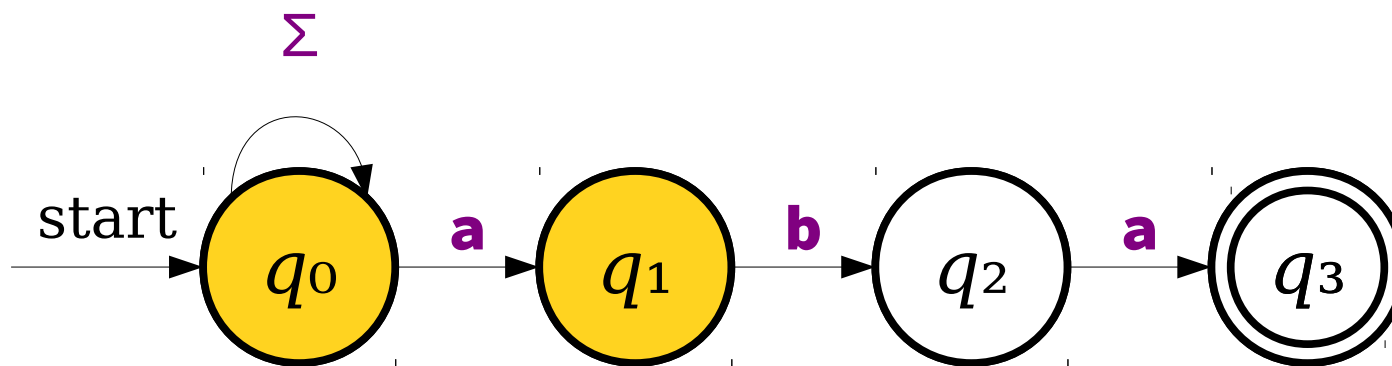
	a	b
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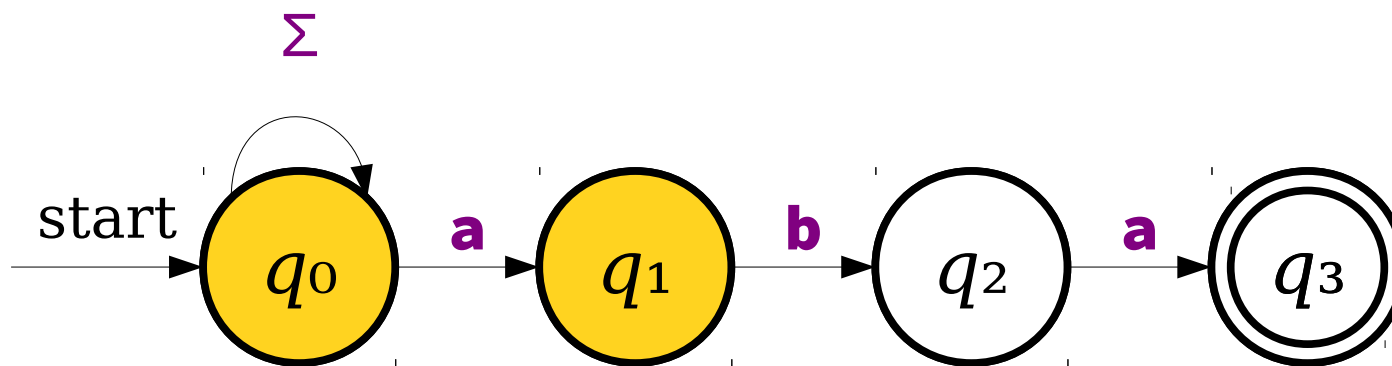
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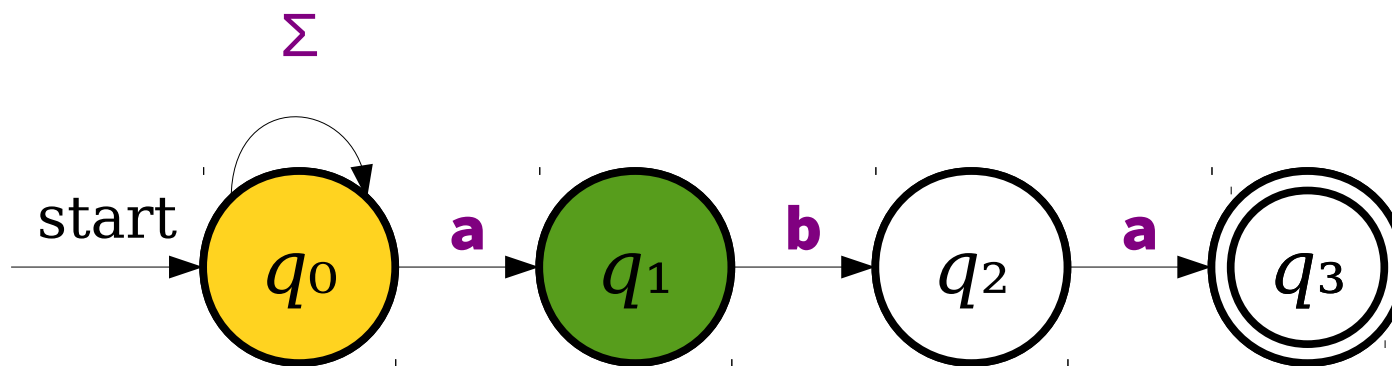
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$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
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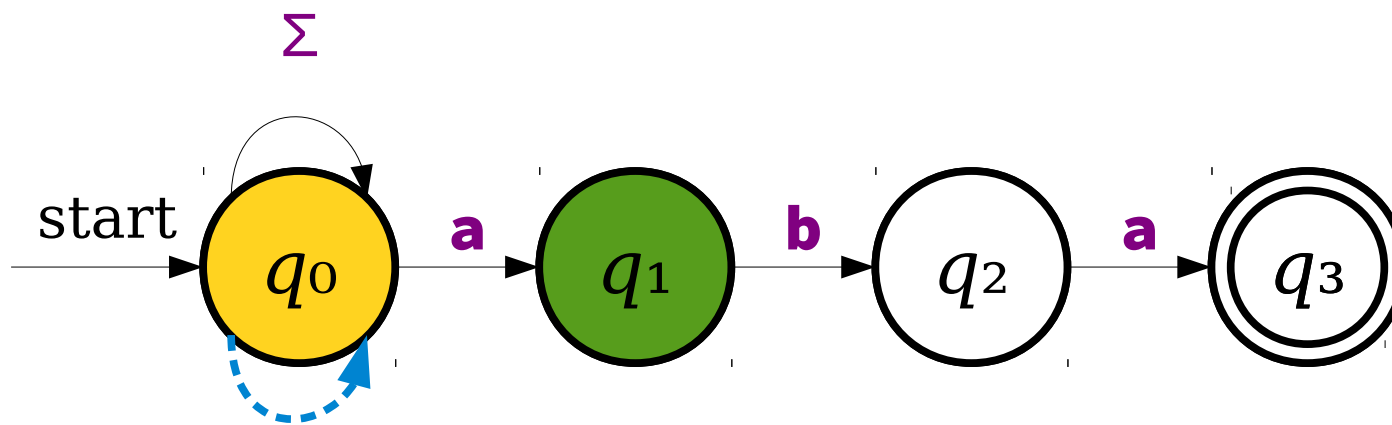
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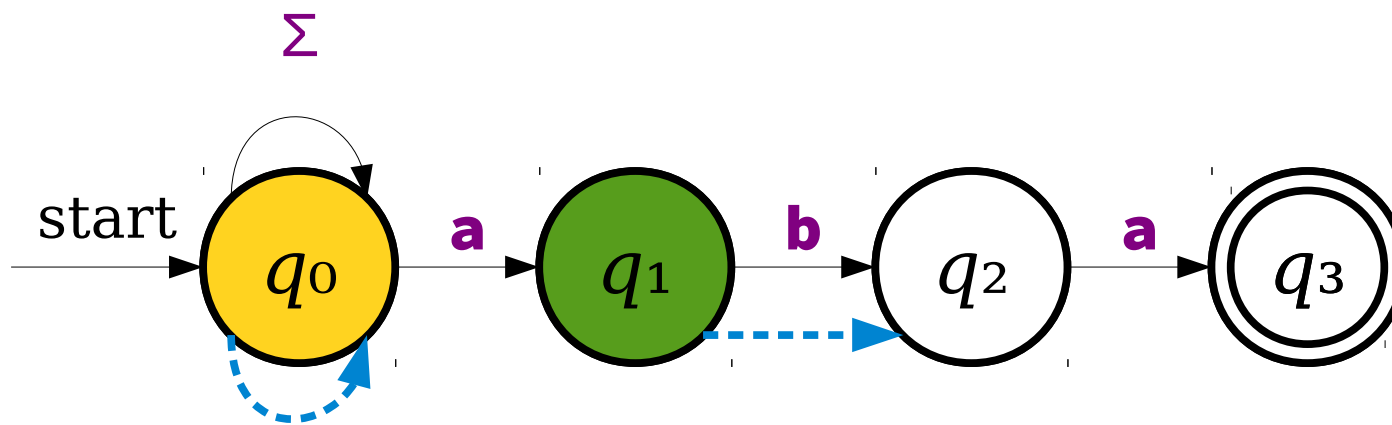
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$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



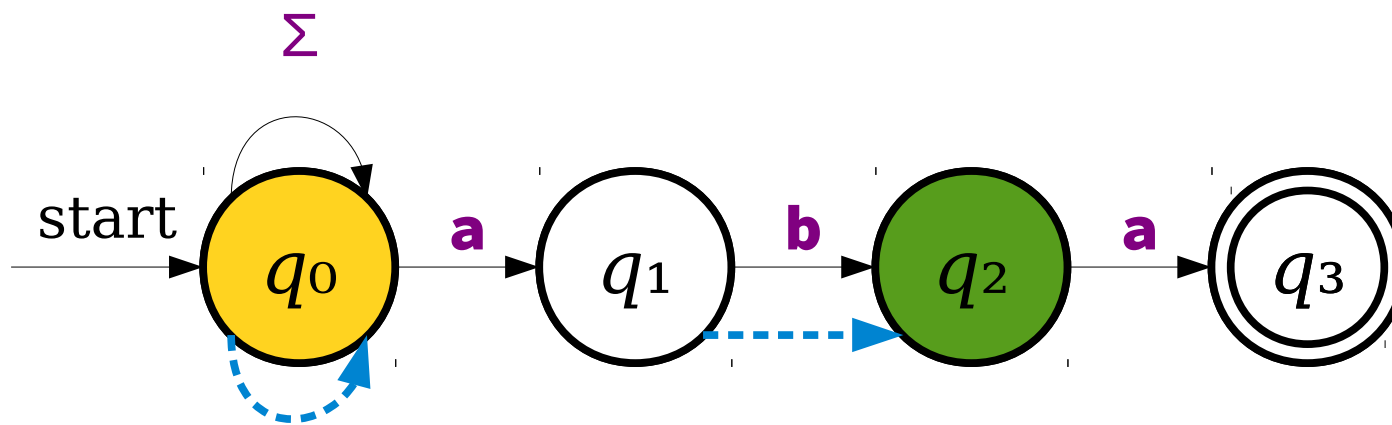
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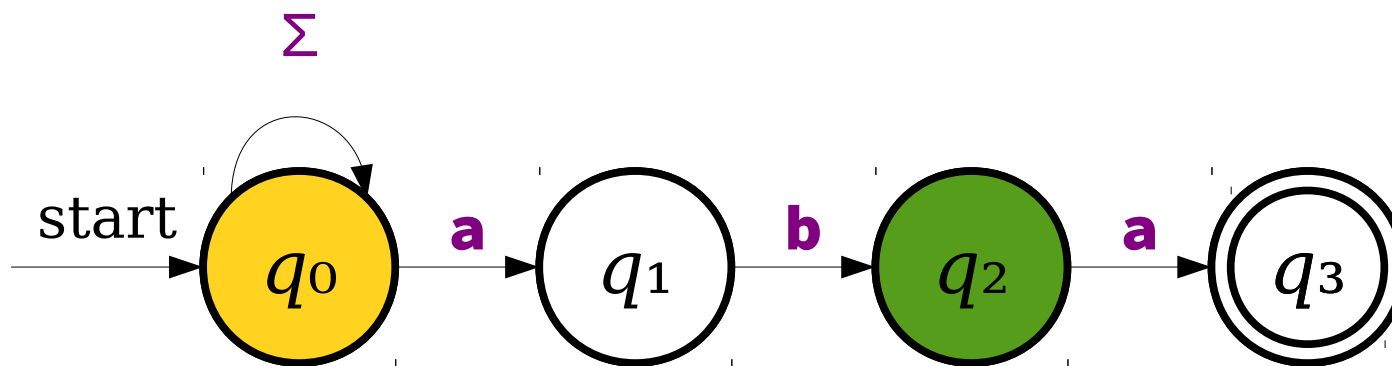
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	



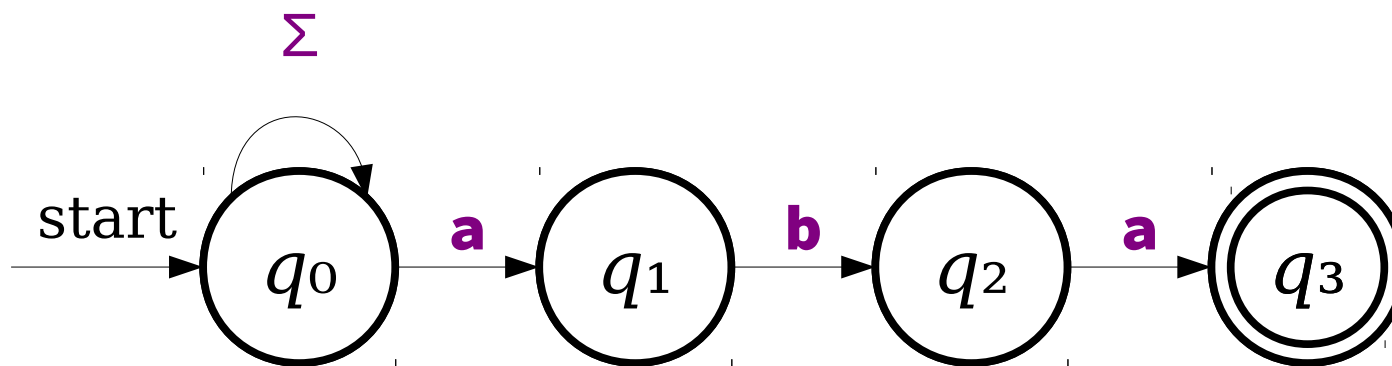
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	



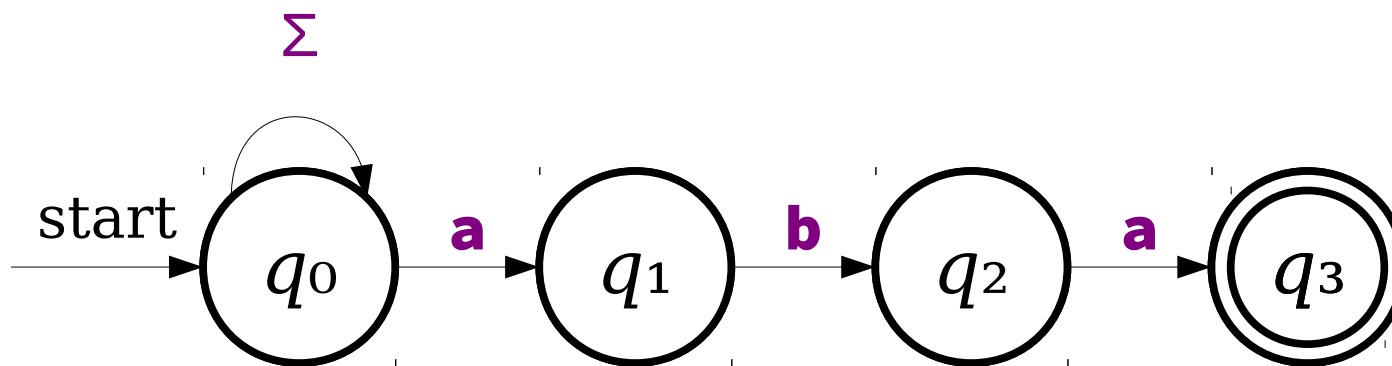
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	



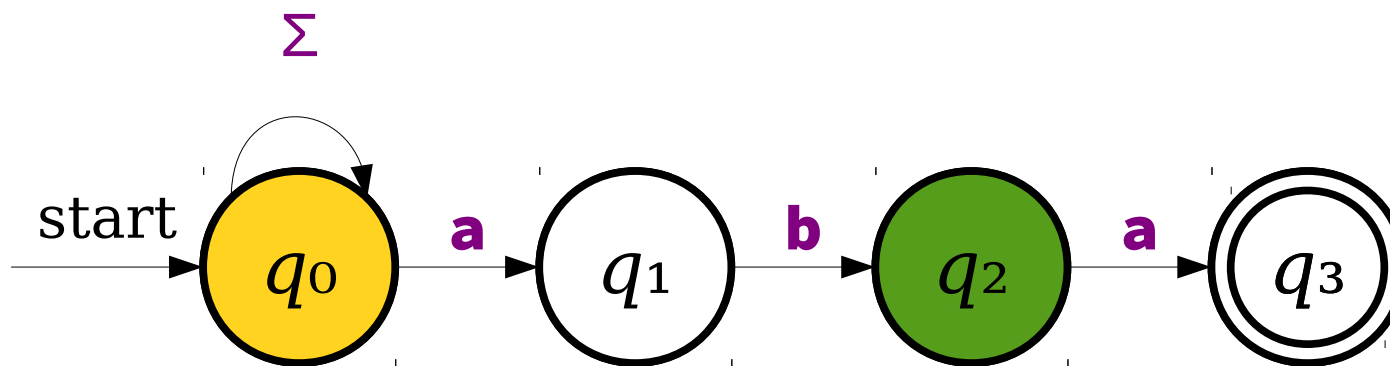
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$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



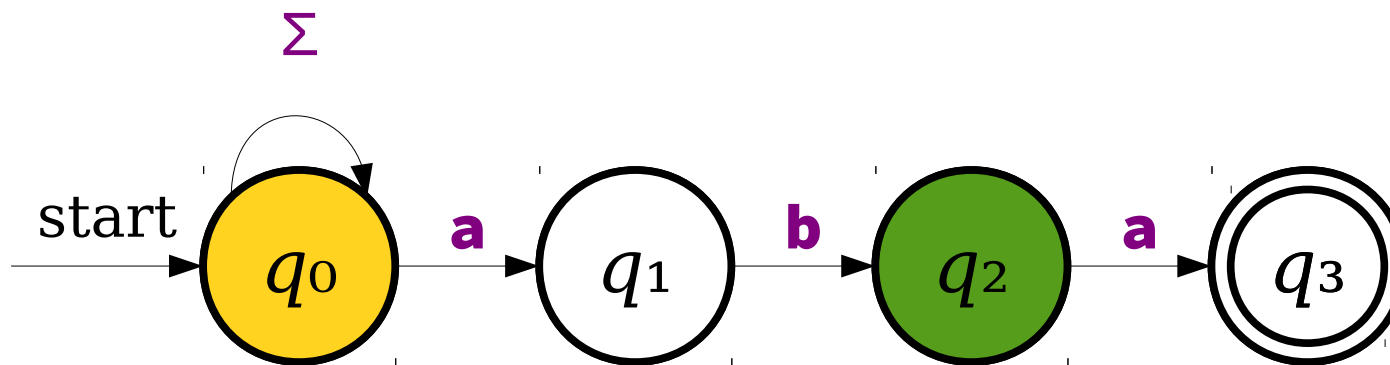
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		

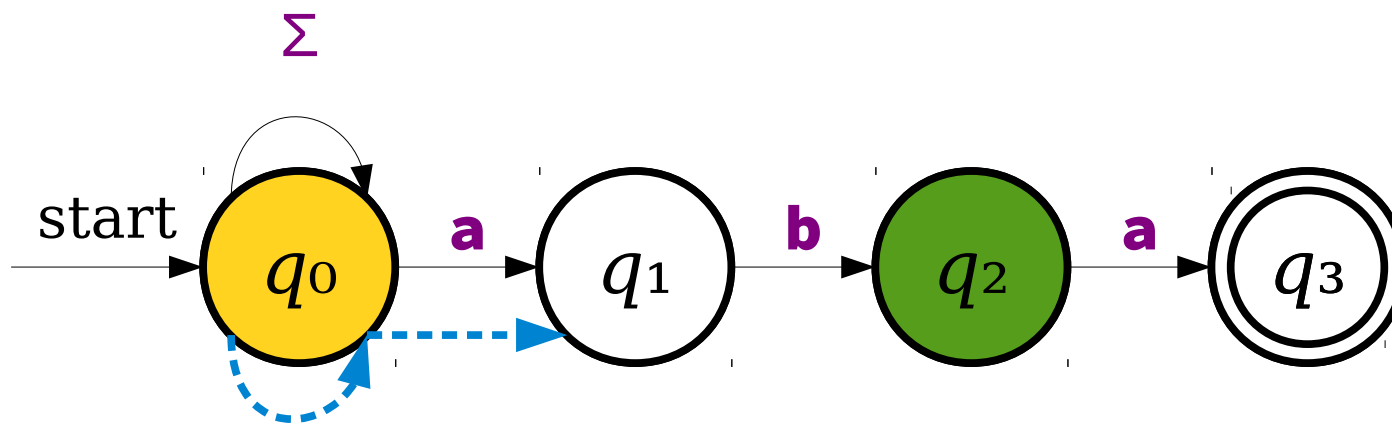


	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		

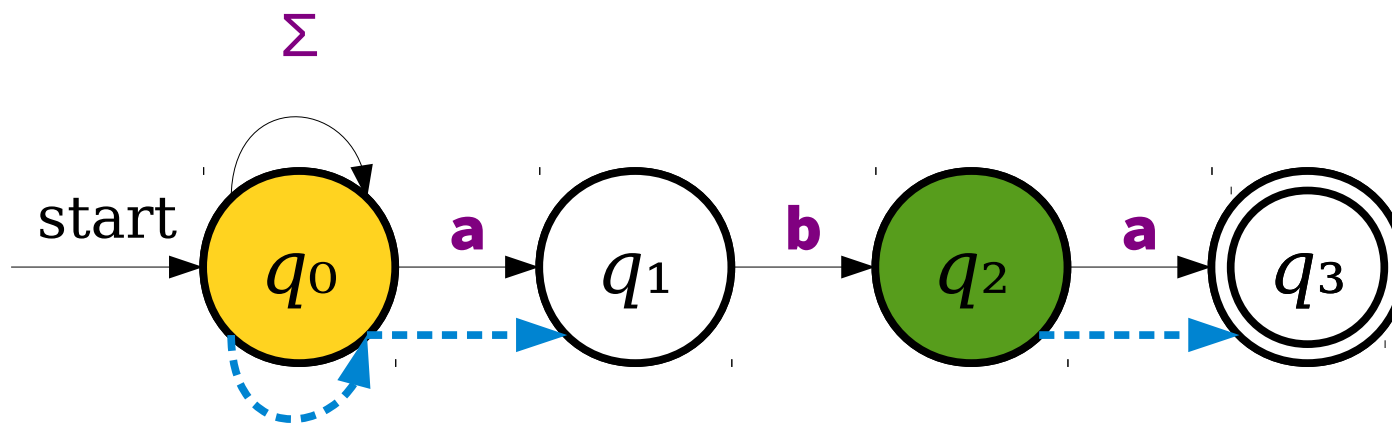
What should this row look like?

<i>A</i>	$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0\}$
<i>B</i>	$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_2\}$
<i>C</i>	$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
<i>D</i>	$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$

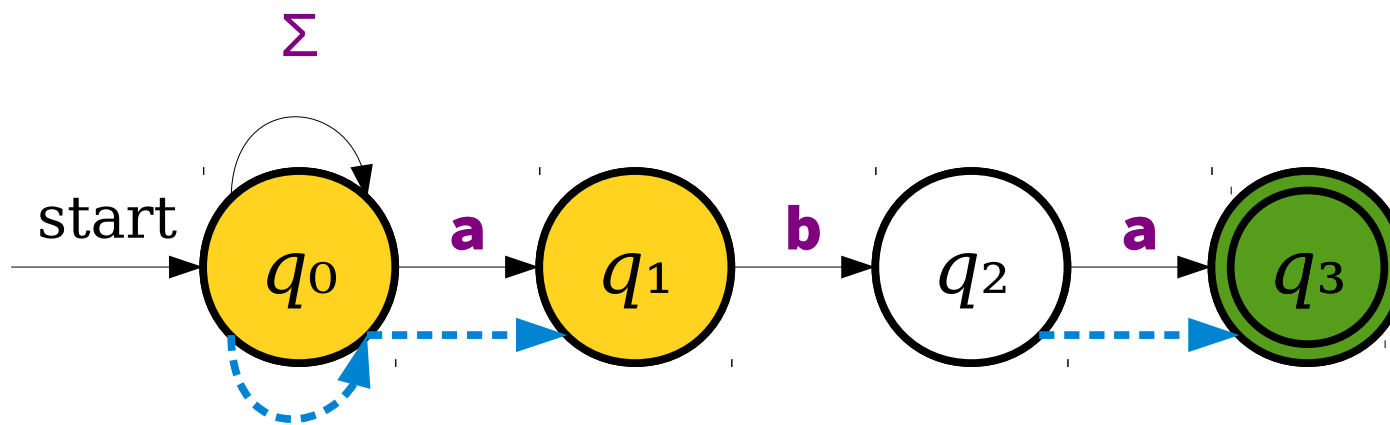
Answer at [PollEv.com/cs103](https://pollev.com/cs103) or
text **CS103** to **22333** once to join, then *A*, *B*, *C*, or *D*.



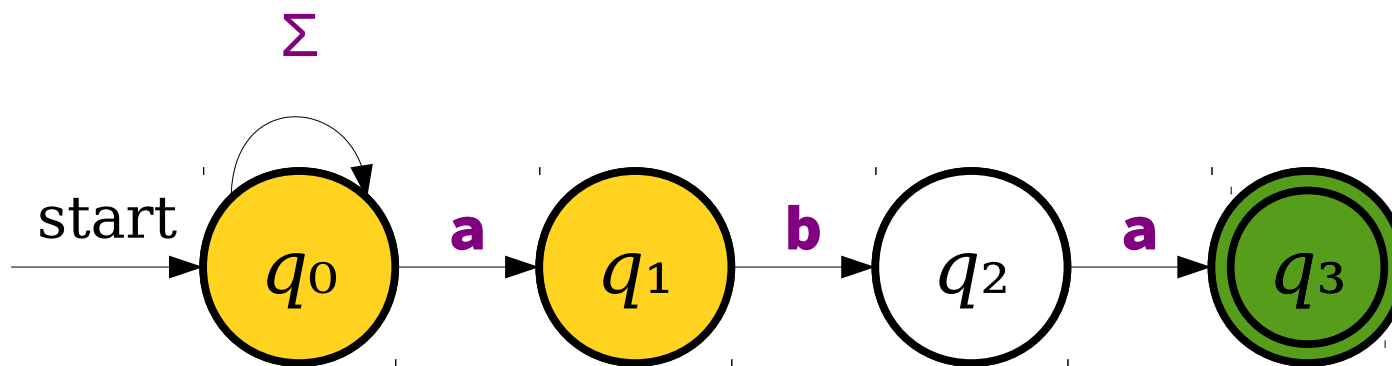
	a	b
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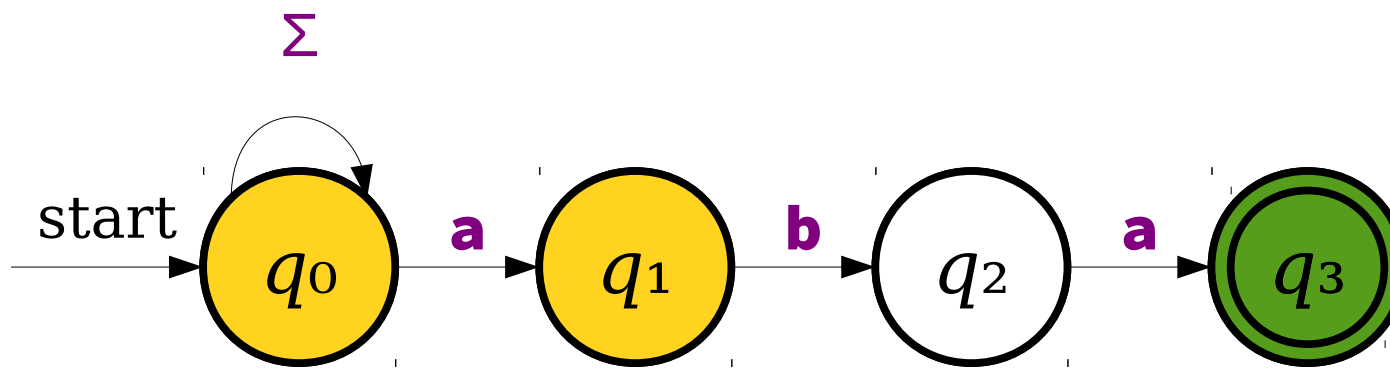
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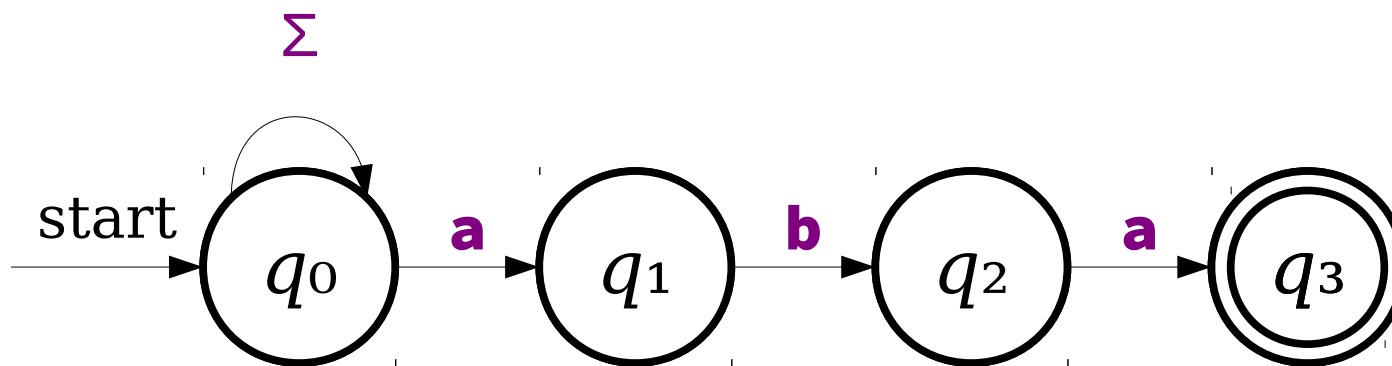
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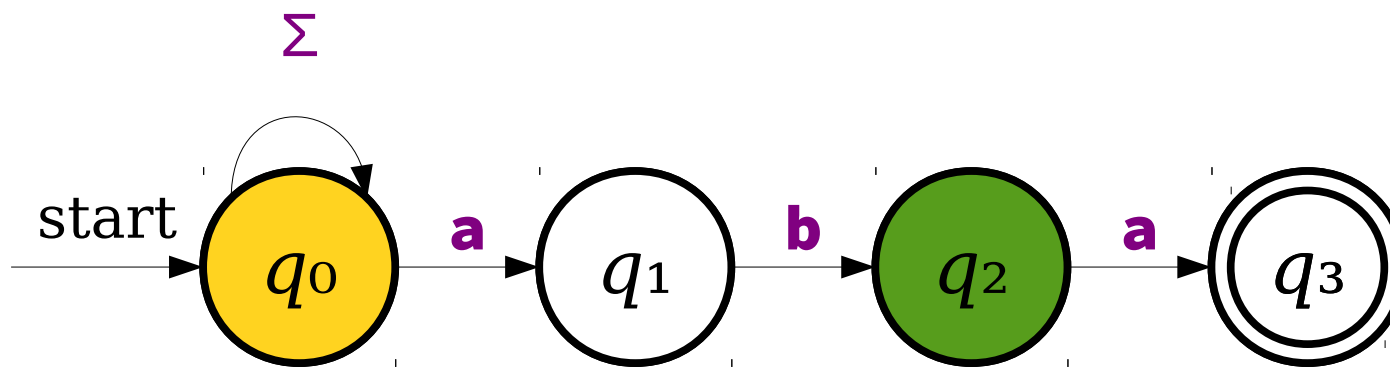
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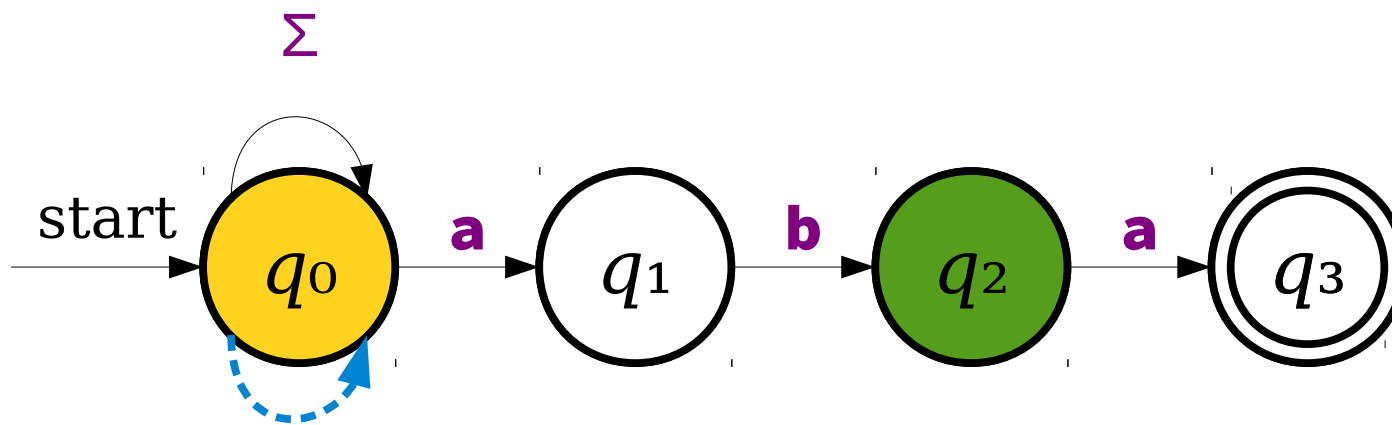
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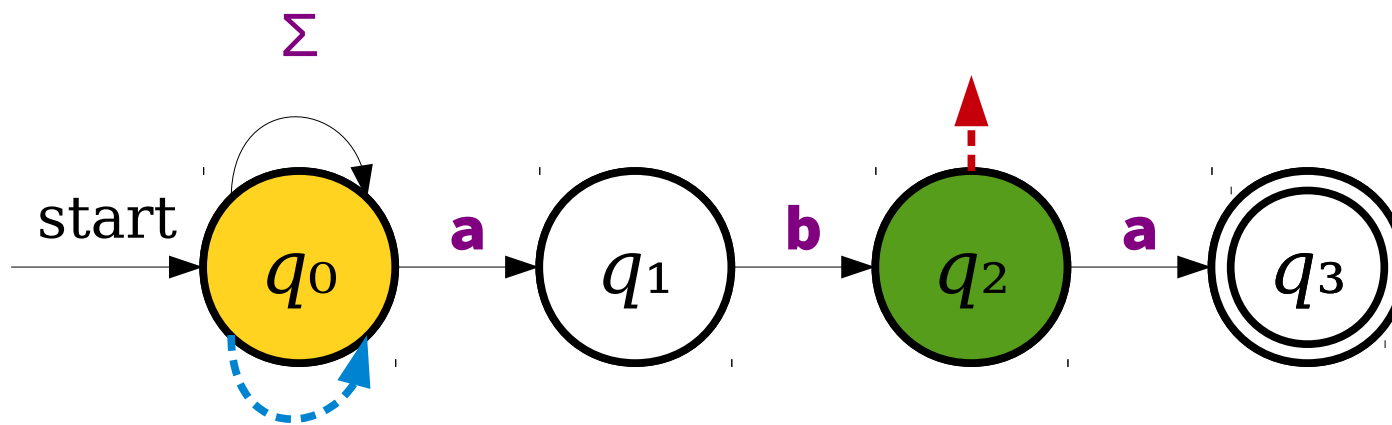
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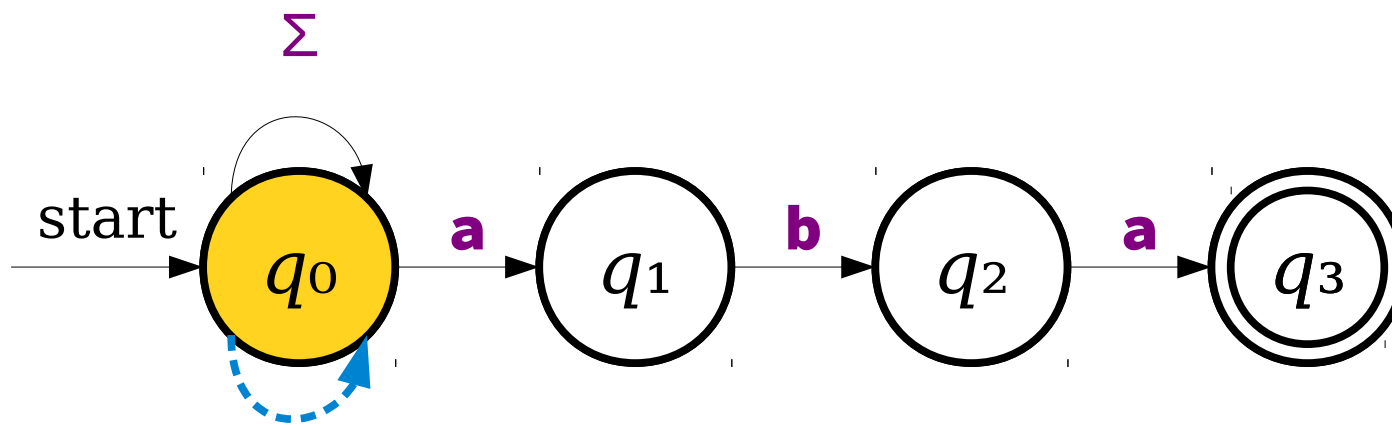
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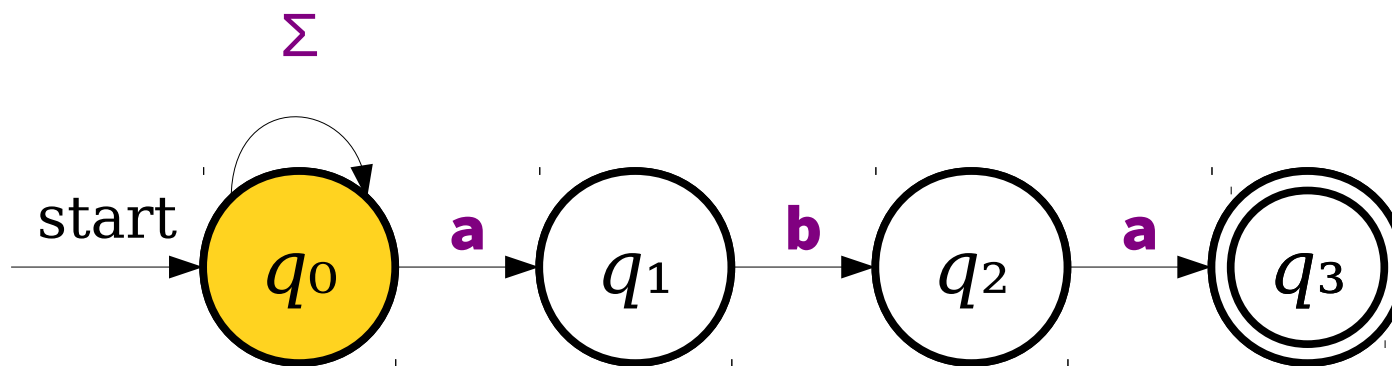
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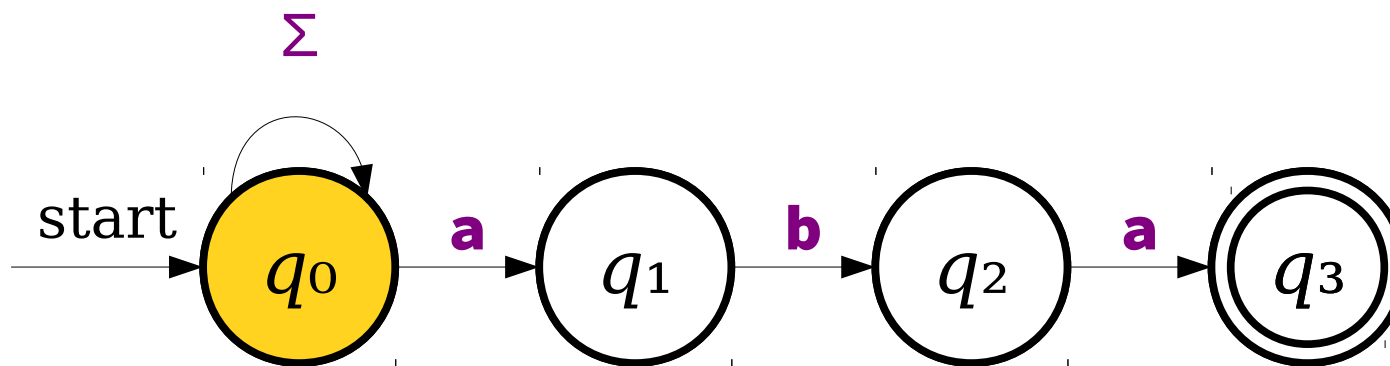
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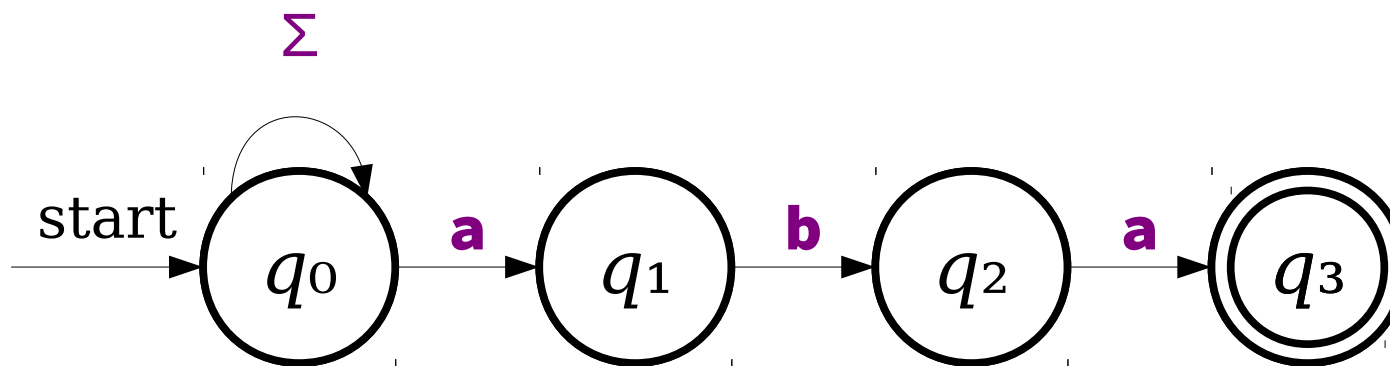
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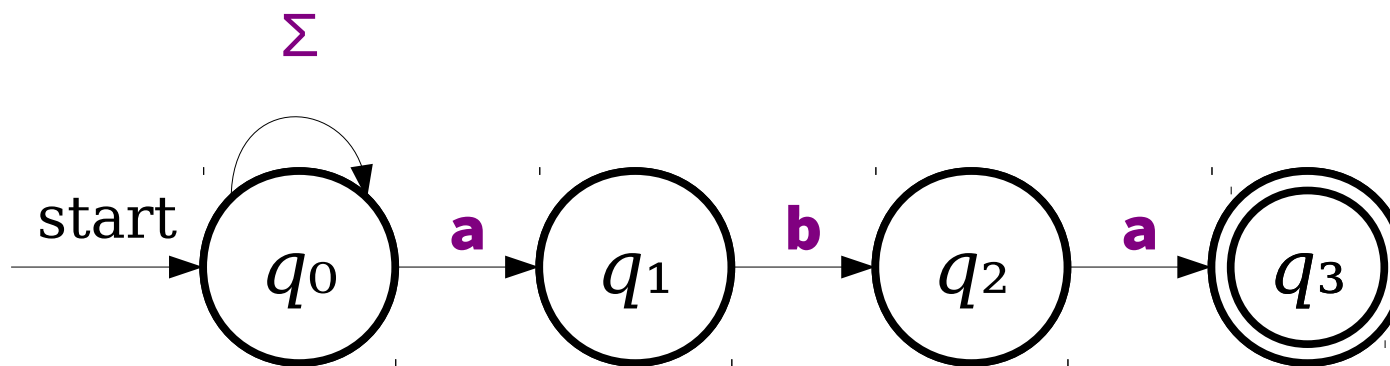
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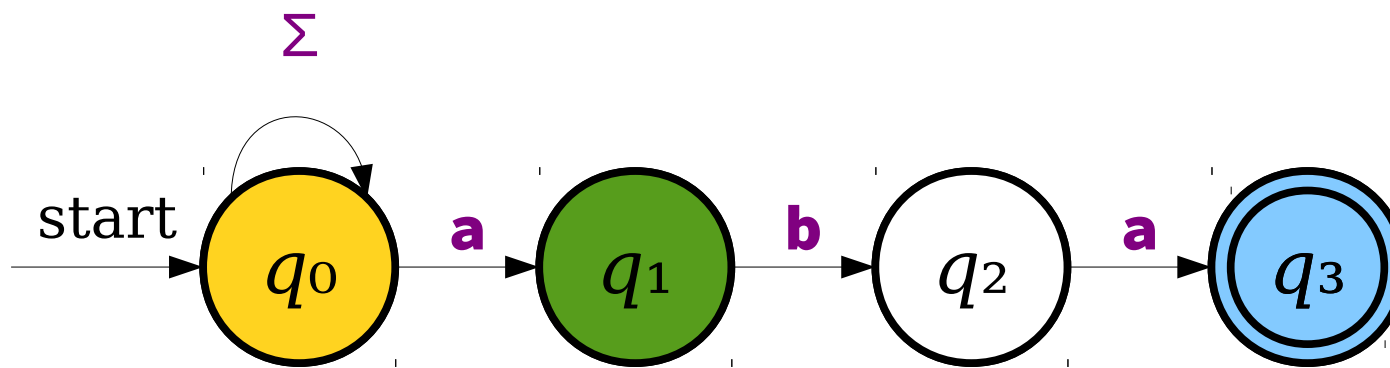
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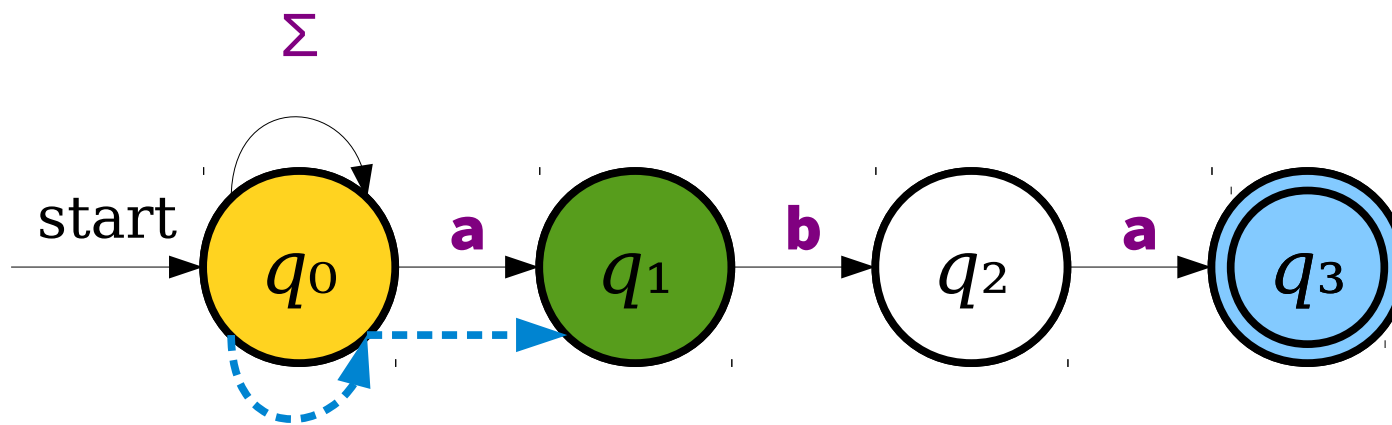
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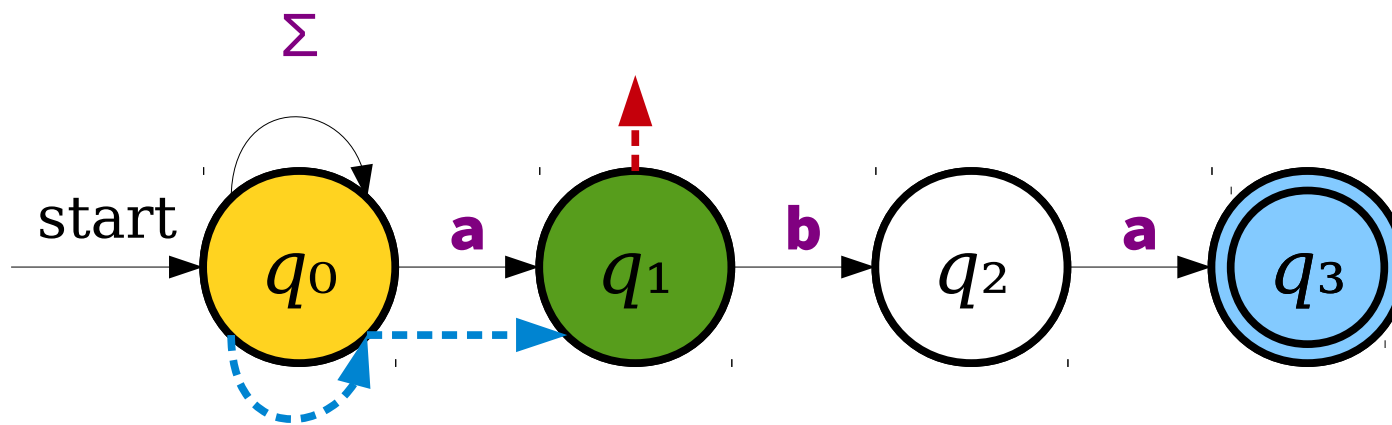
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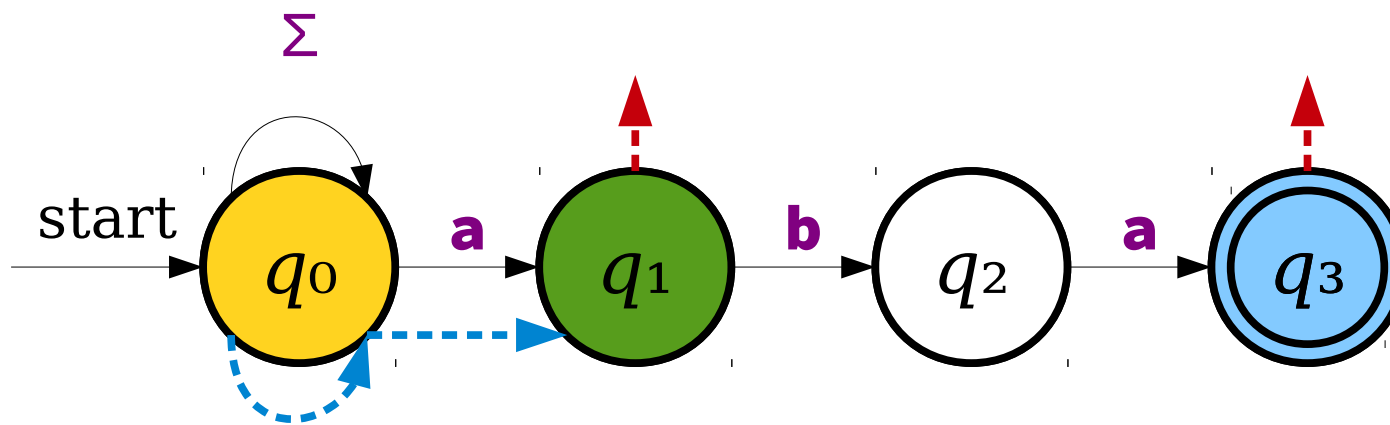
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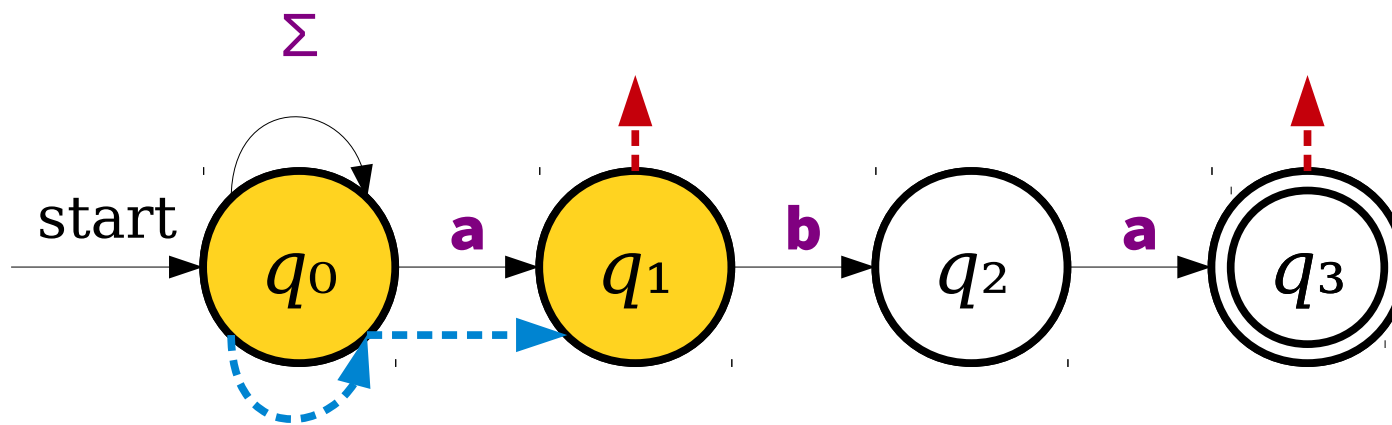
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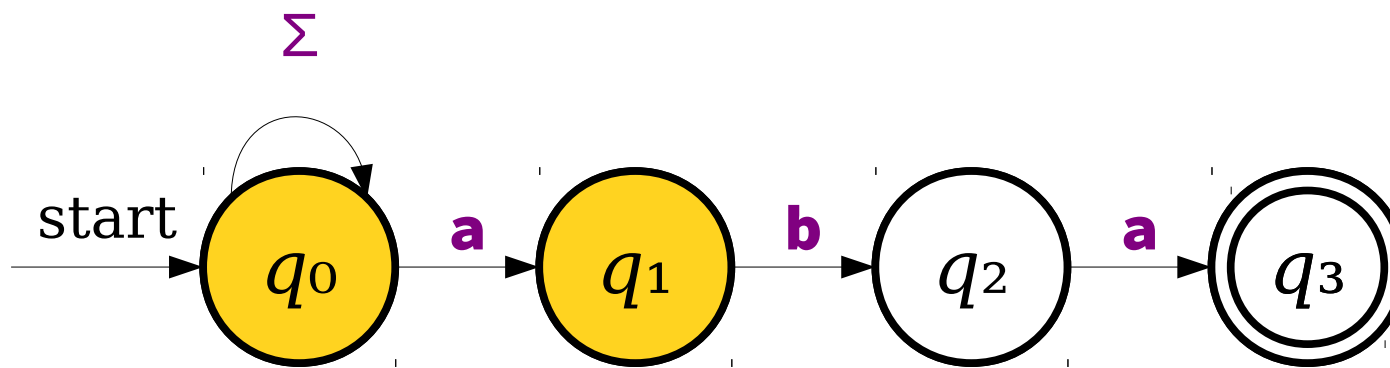
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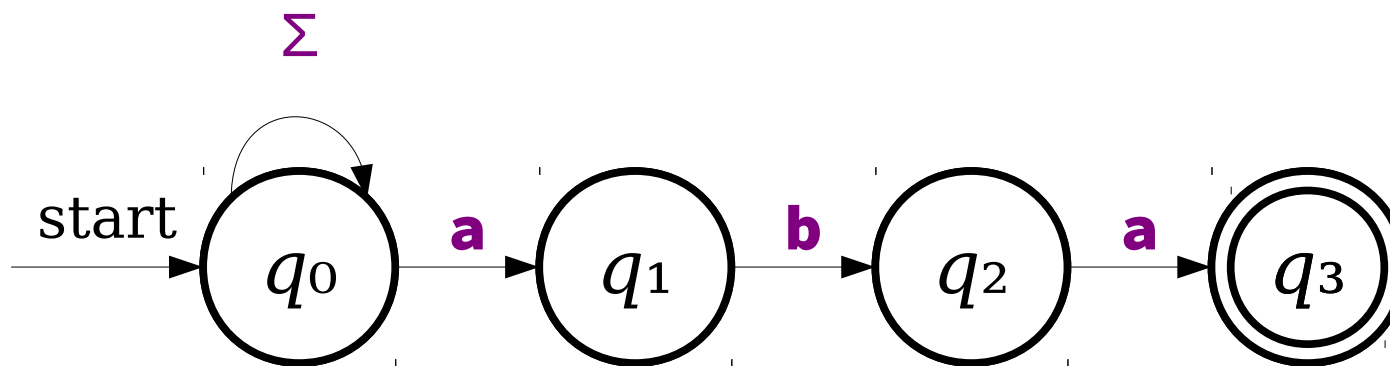
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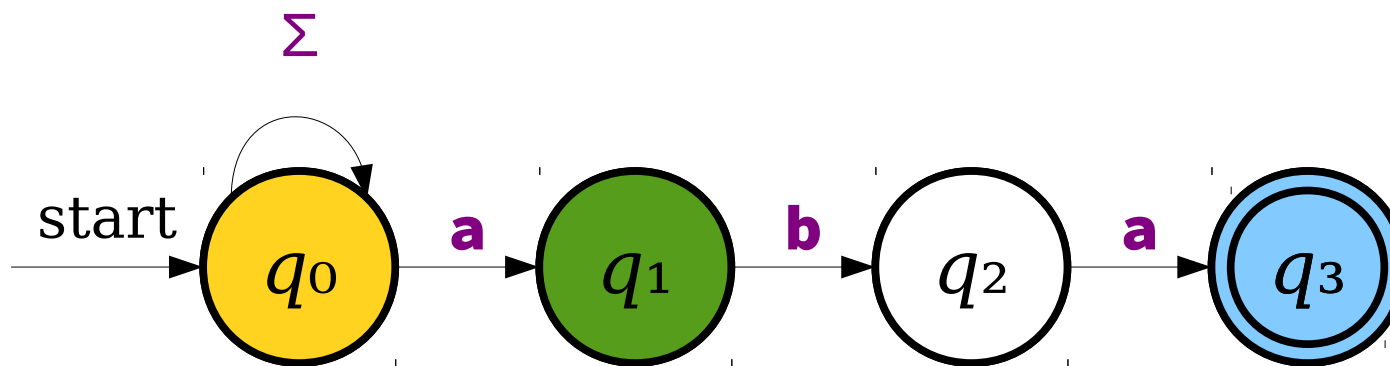
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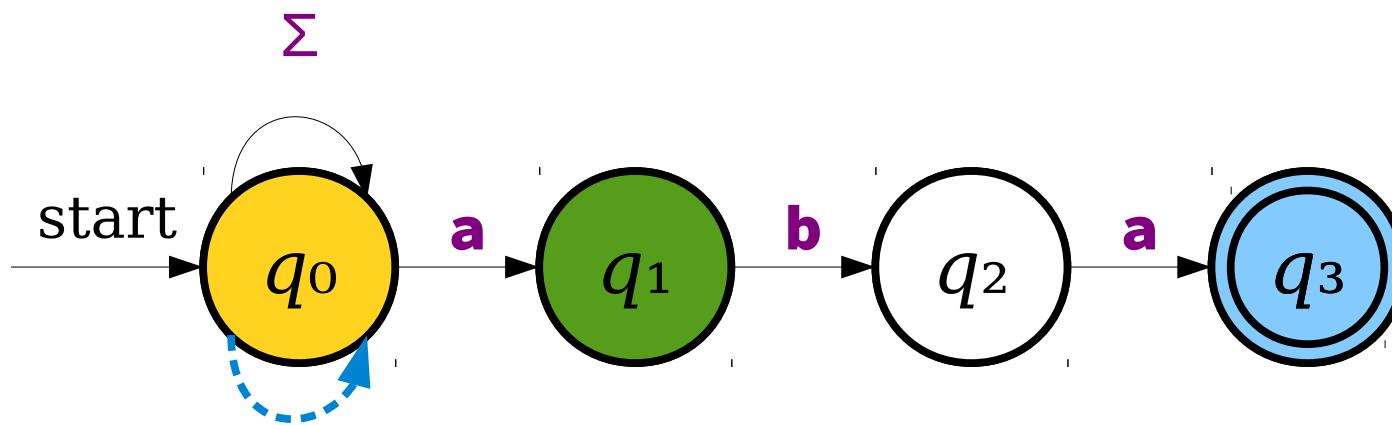
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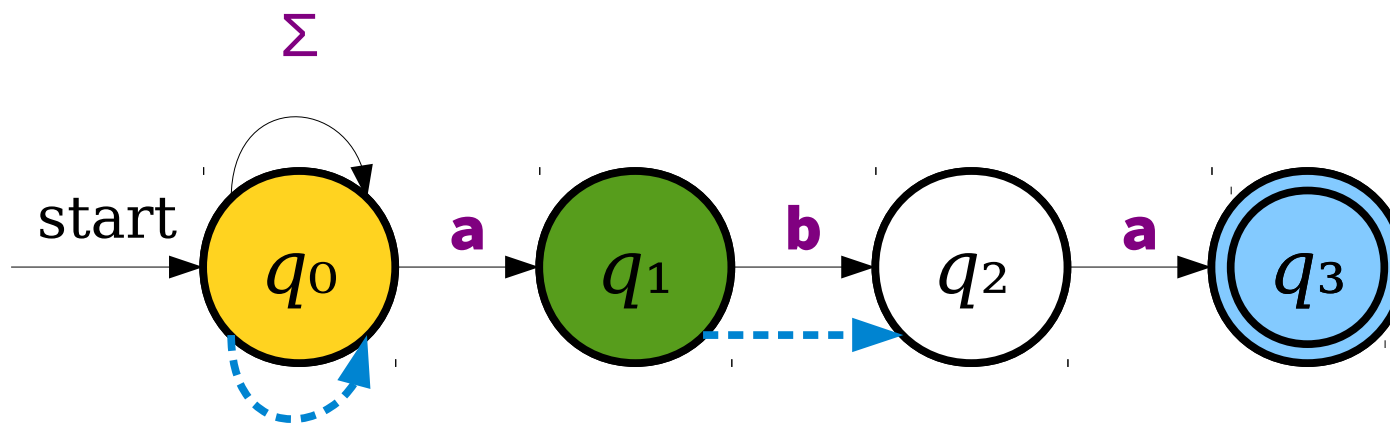
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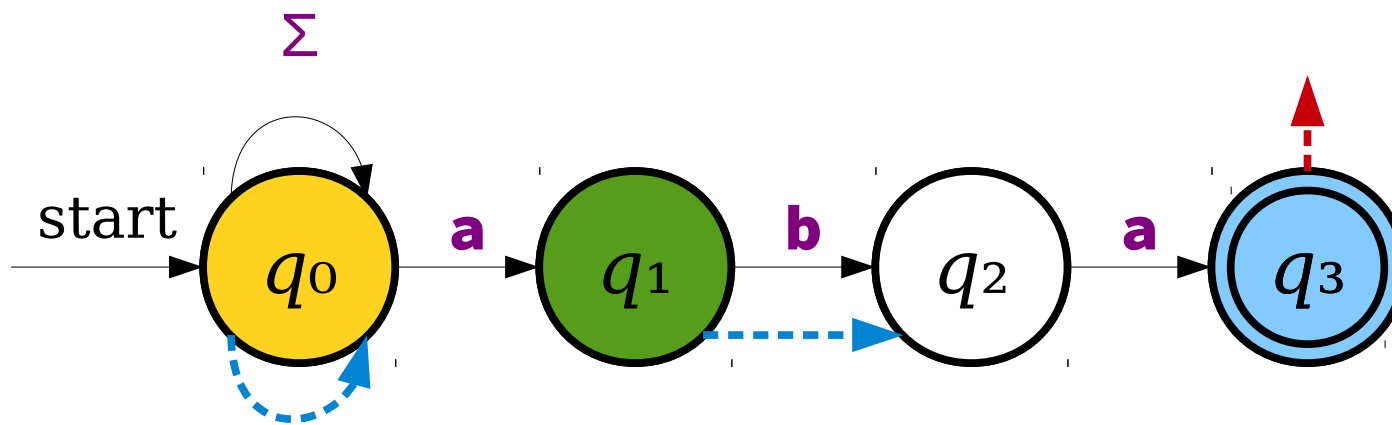
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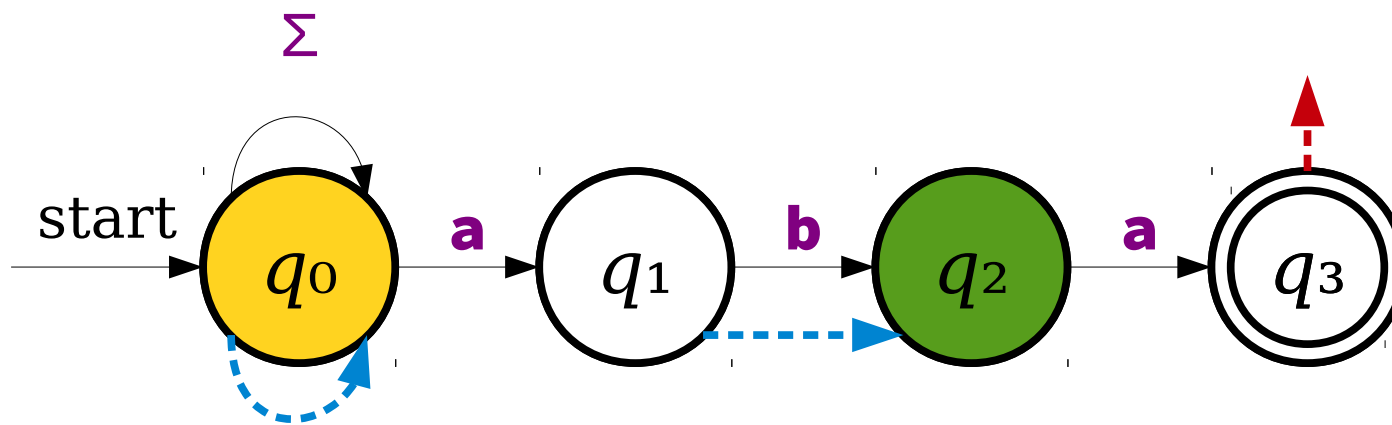
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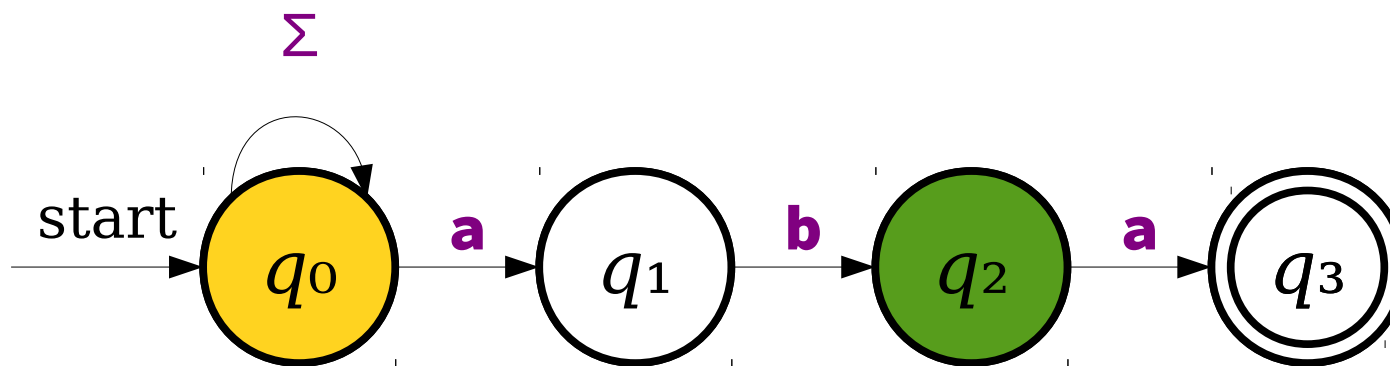
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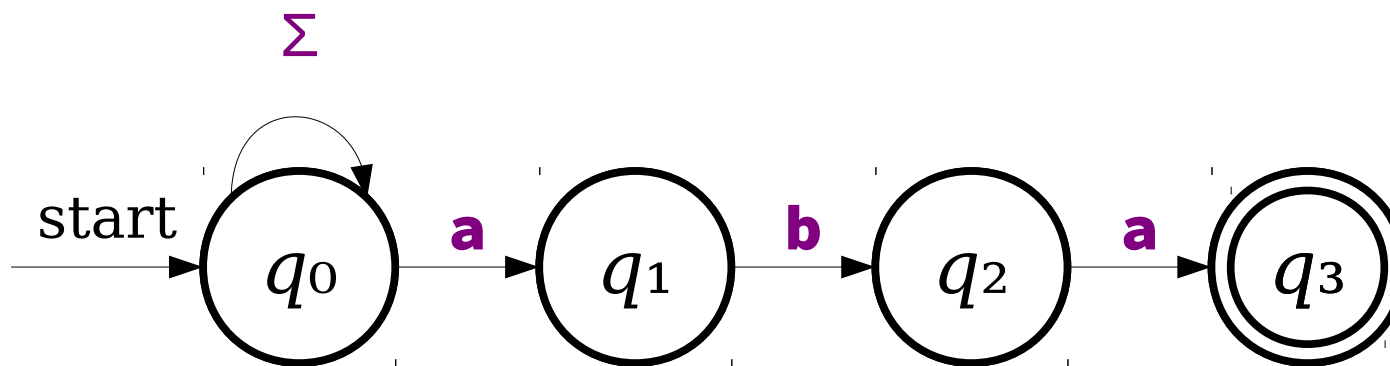
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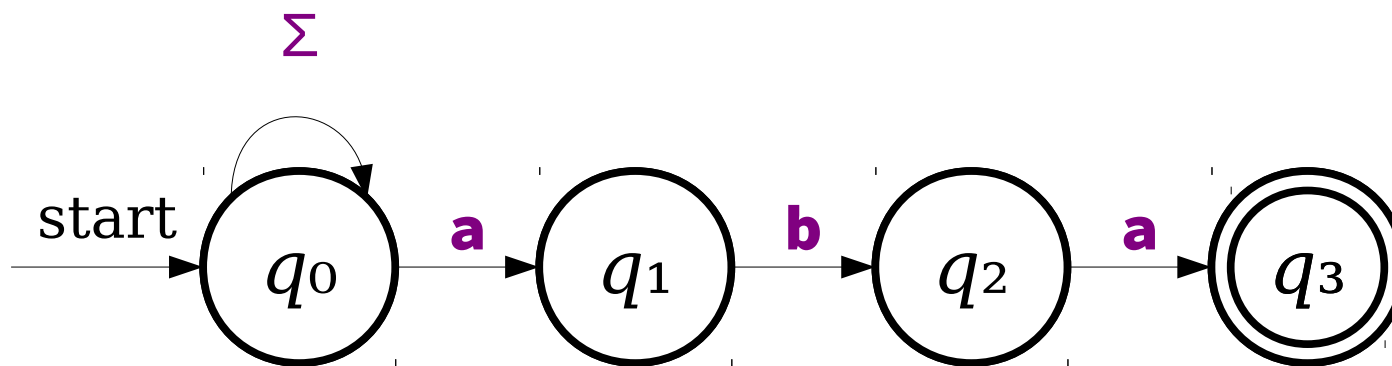
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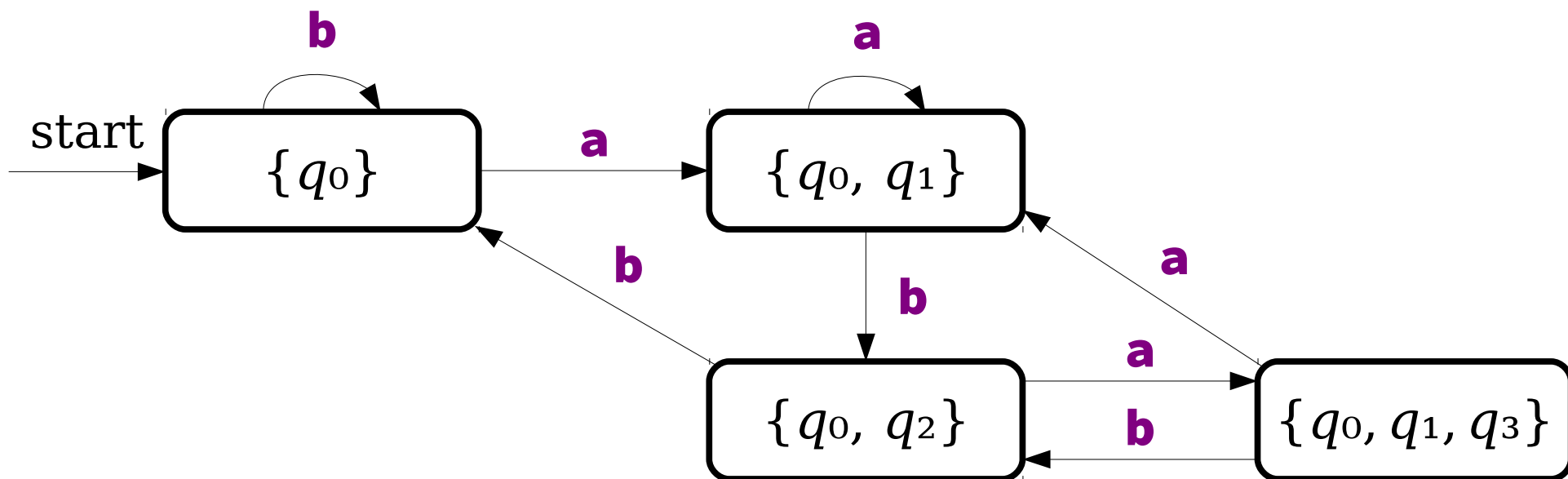
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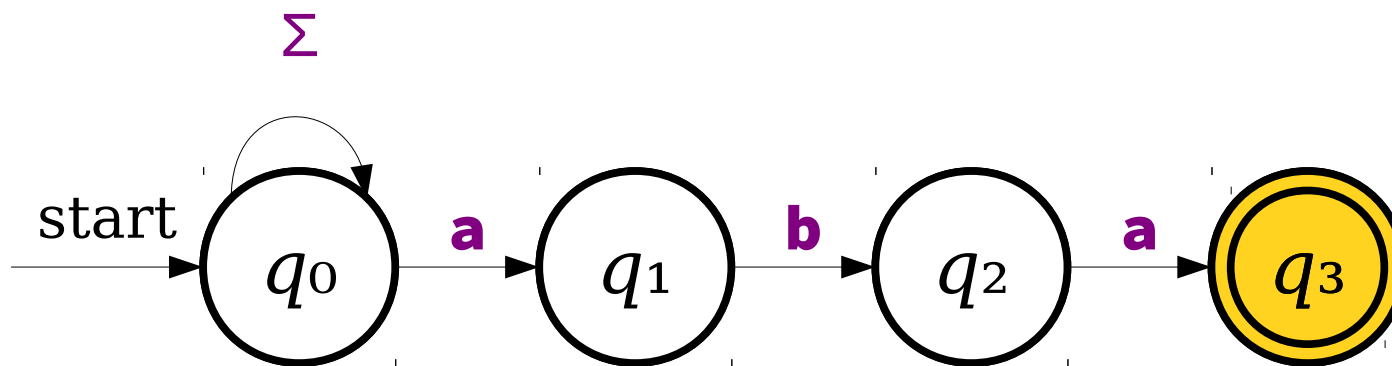


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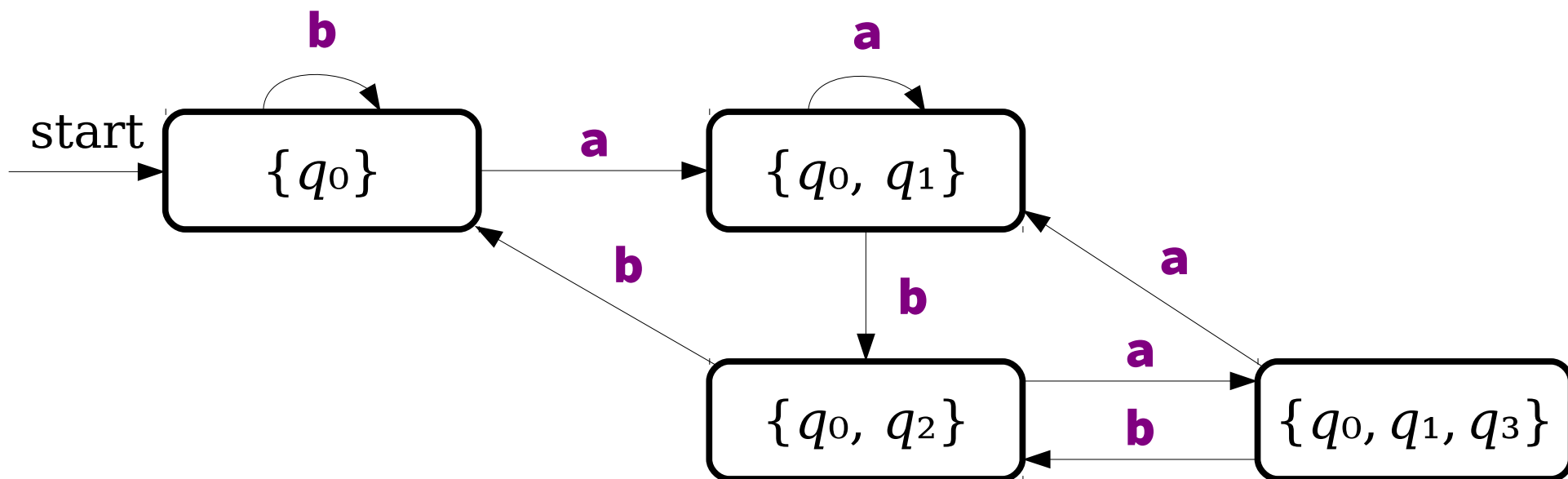


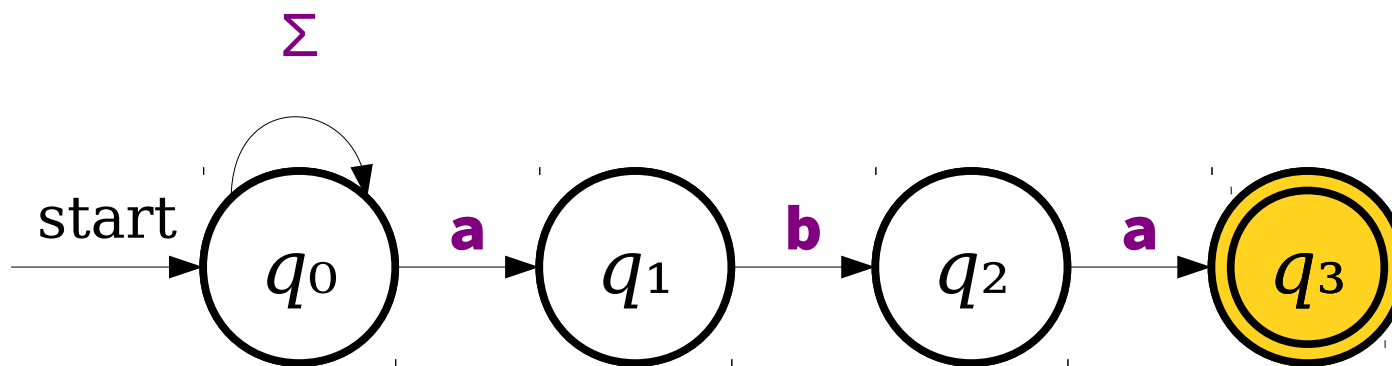
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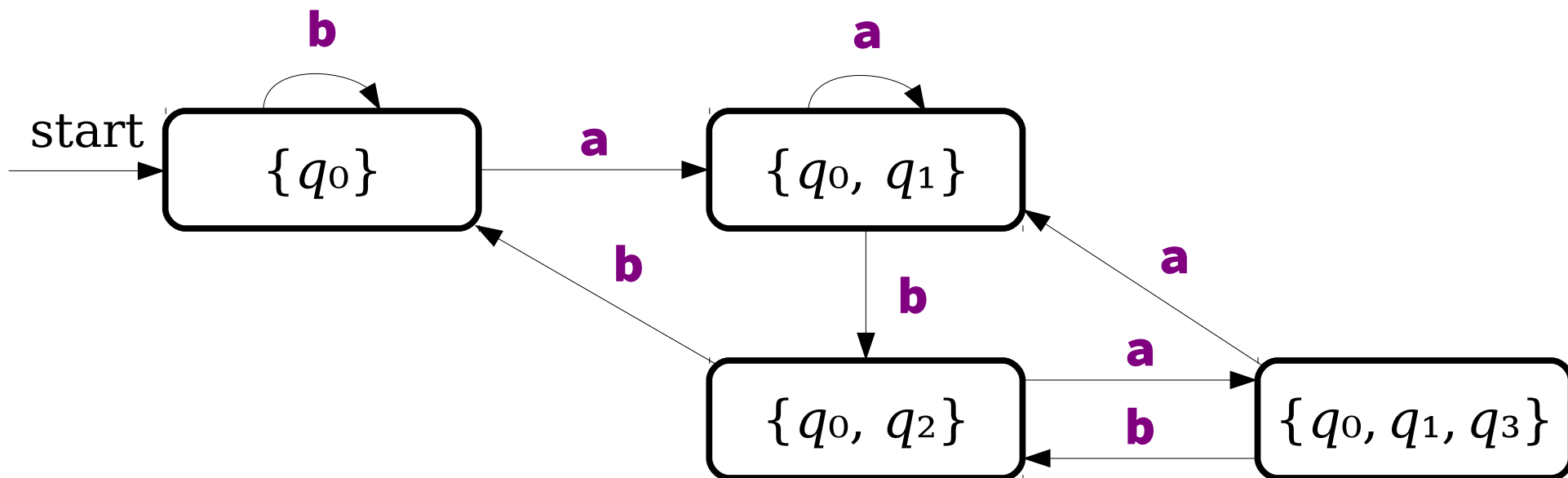


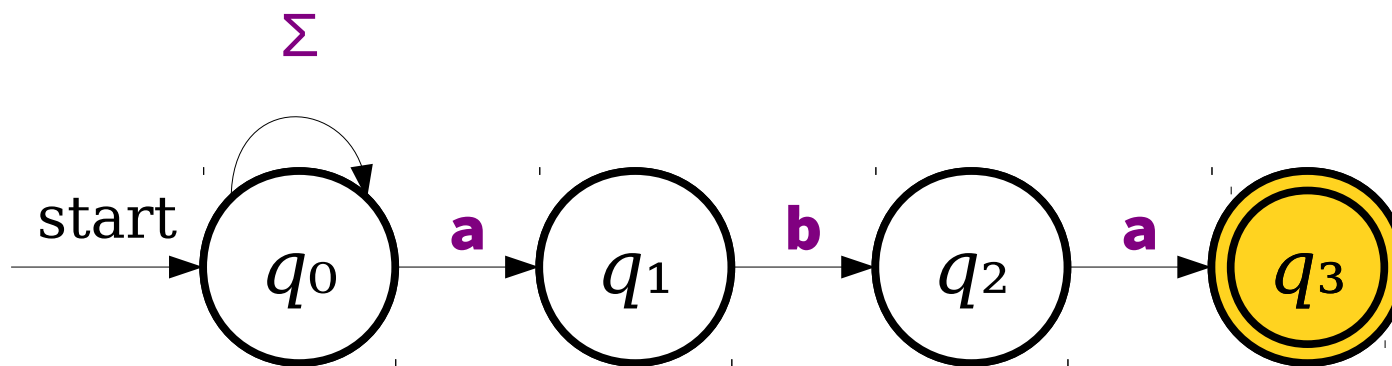
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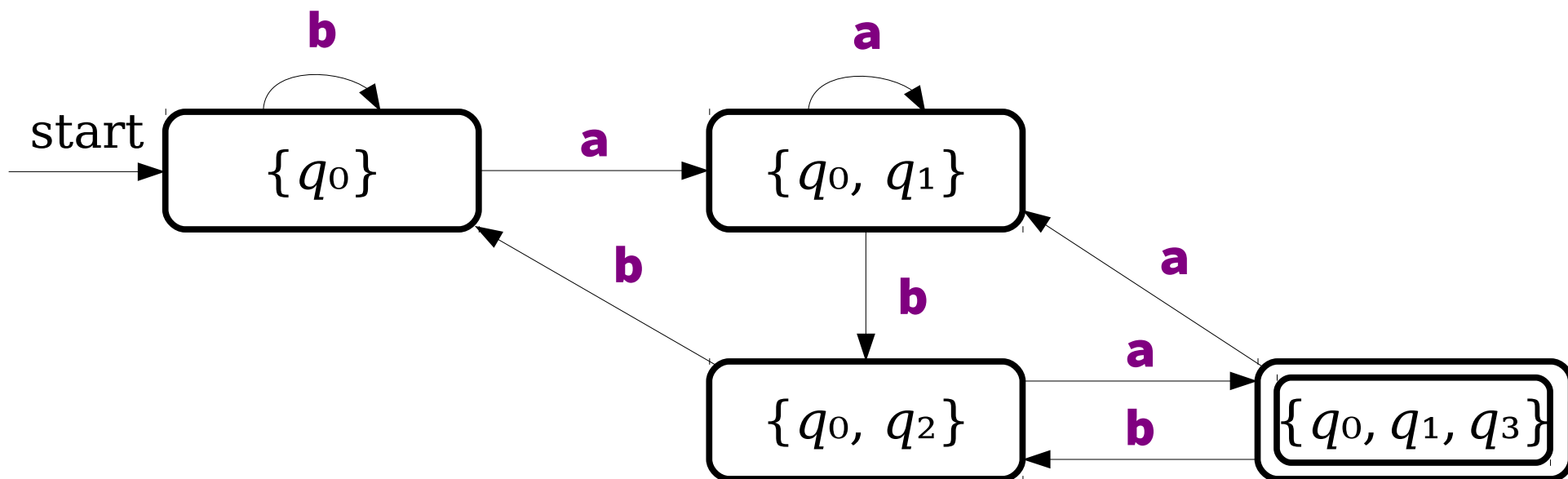


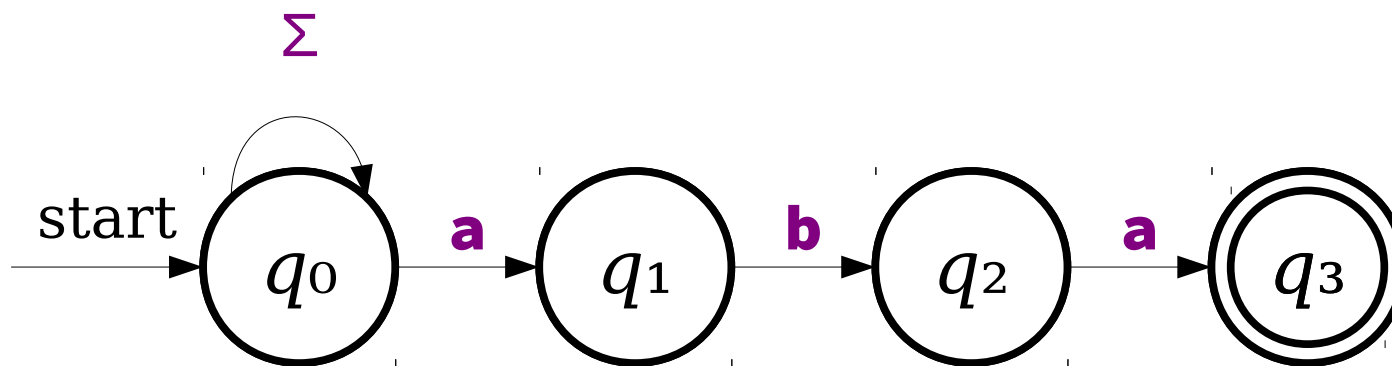
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	a	b
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	a	b
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