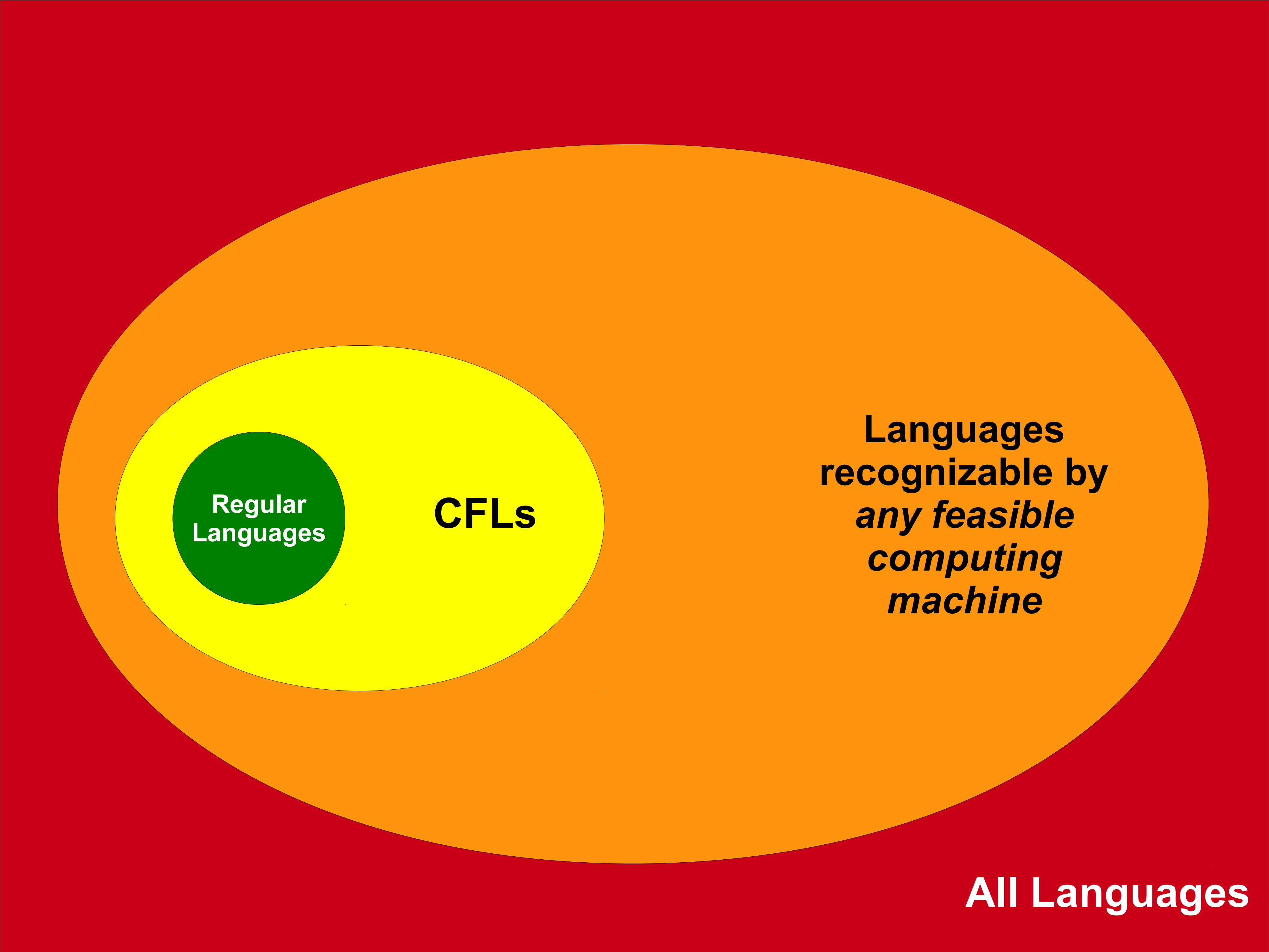
Turing Machines

Part One

What problems can we solve with a computer?



Regular Languages

CFLs

Languages recognizable by *any feasible computing machine*

All Languages

That same drawing, to scale.

All Languages

The Problem

- Finite automata accept precisely the regular languages.
- We may need unbounded memory to recognize context-free languages.
  - e.g. $\{ \mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{N} \}$ requires unbounded counting.
- How do we build an automaton with finitely many states but unbounded memory?

A Brief History Lesson

<ahem>

Technology has solved all of mankind’s problems! No more wars or sad ever!

Hilbert’s Vision

- 1900: International Congress of Mathematicians meeting in Paris
- Proposes 23 unsolved problems as the agenda for the coming years
- An important theme is not simply proving more theorems, but achieving *automation* of theorem-proving, even theorem generation.
- Humanity lives in leisure while all Truth flows effortlessly into our hands on a ticker tape!

“No one shall expel us from the Paradise that Cantor has created!” -David Hilbert

Hi there, Reality!

- Hilbert’s agenda is both a spectacular success and a spectacular failure
- Inspires some of the most impactful theoretical work in mathematical history, human history
- Brings us heroes like **Alan Turing** and **Kurt Gödel**!
- These incredible results consist of utterly demolishing all the pillars of Hilbert’s vision of automated knowledge creation, within just a few years

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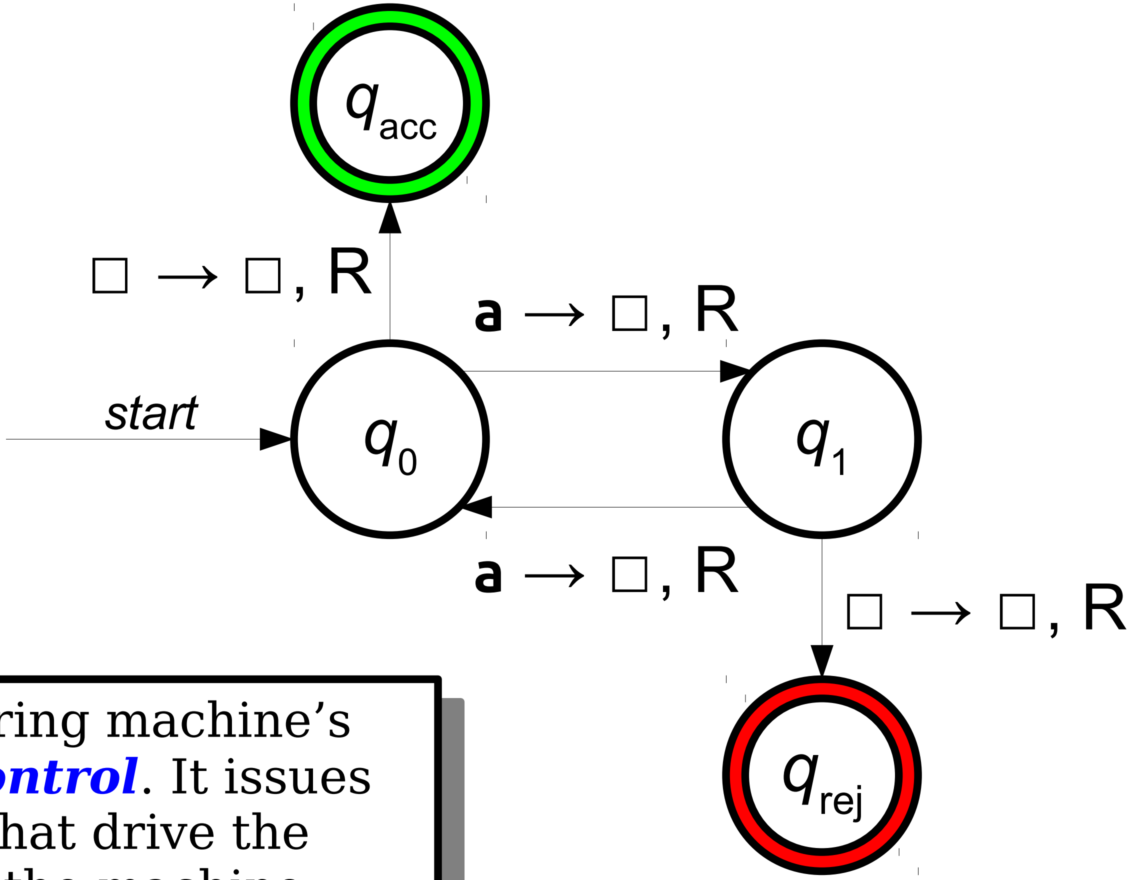
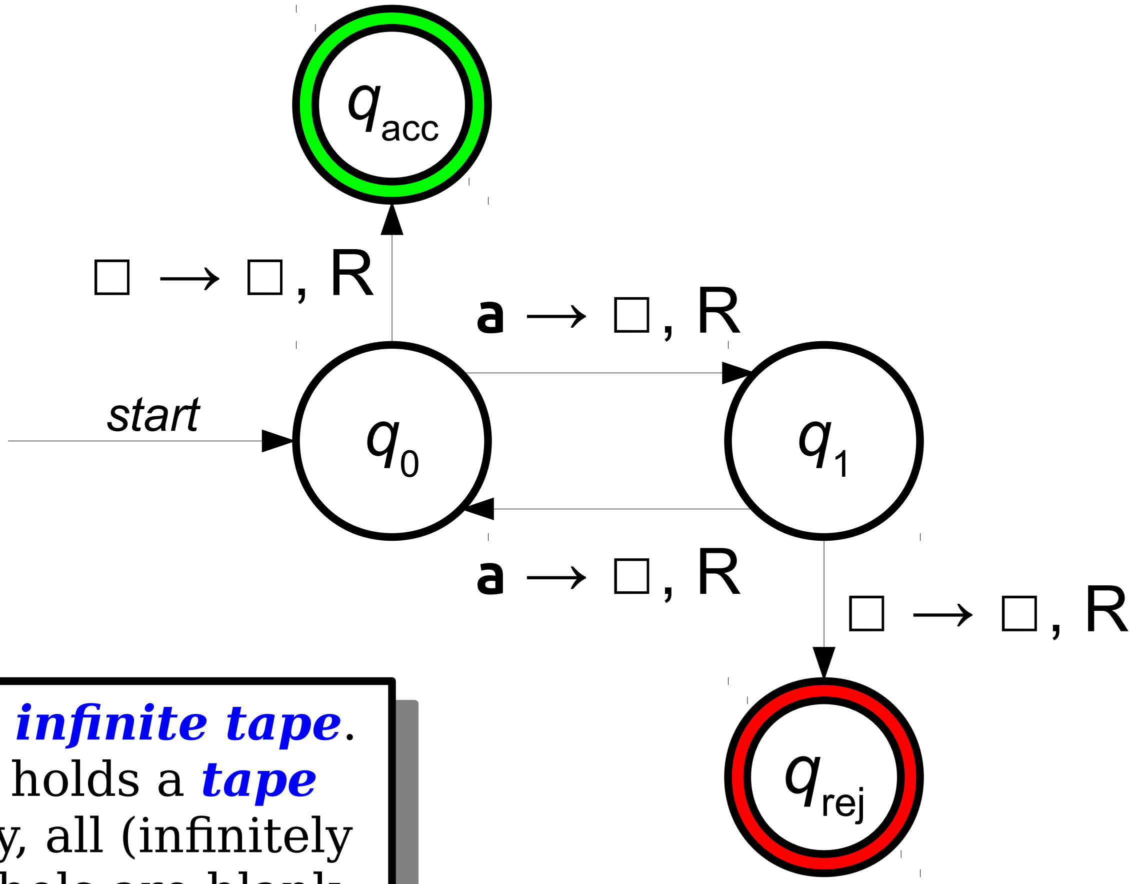
A Simple Turing Machine

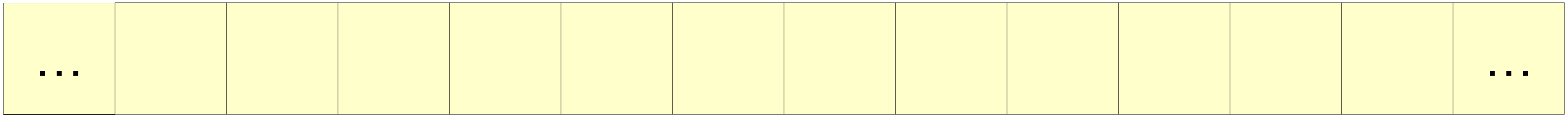


This is the Turing machine's ***finite state control***. It issues commands that drive the operation of the machine.

A Simple Turing Machine



This is the TM's ***infinite tape***. Each tape cell holds a ***tape symbol***. Initially, all (infinitely many) tape symbols are blank.



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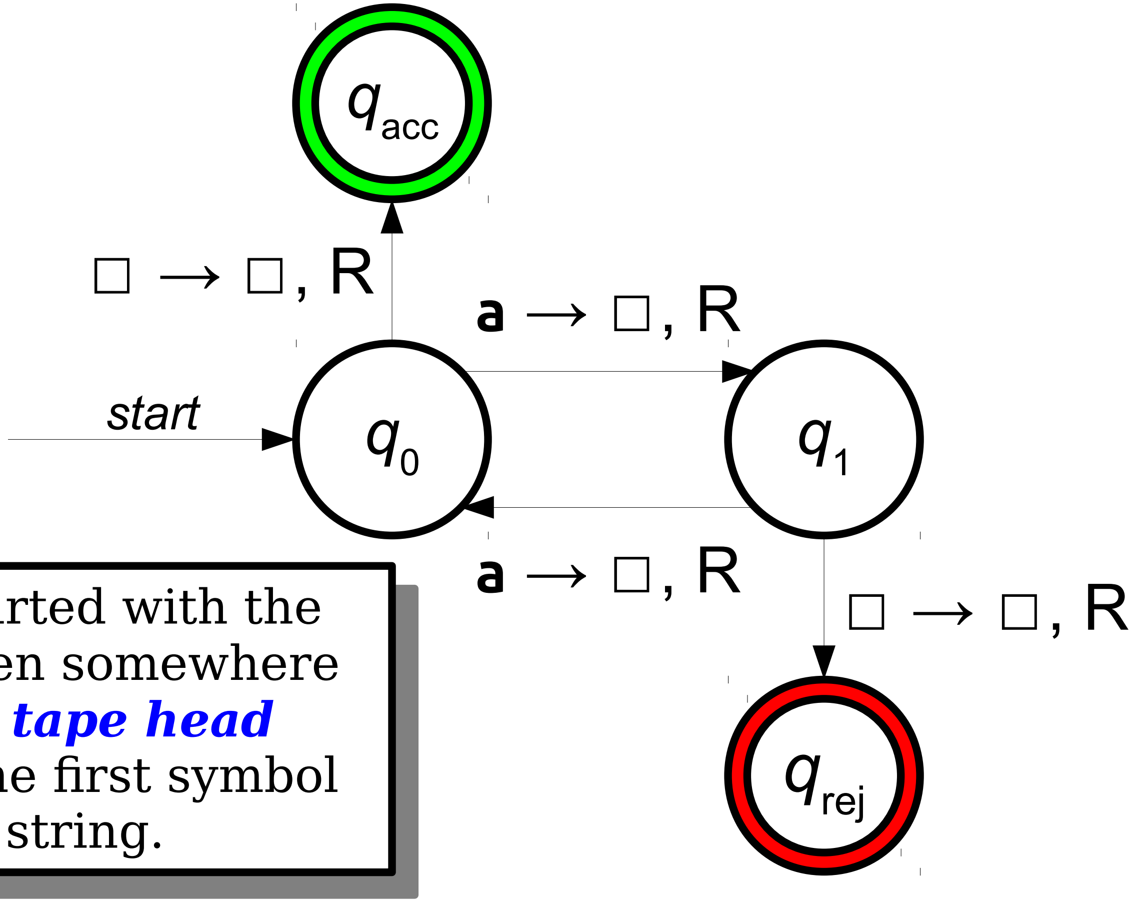
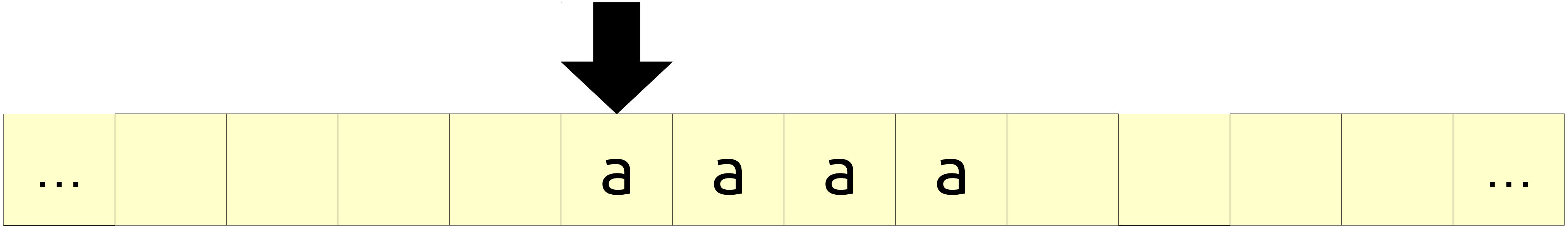


The machine is started with the ***input string*** written somewhere on the tape. The ***tape head*** initially points to the first symbol of the input string.



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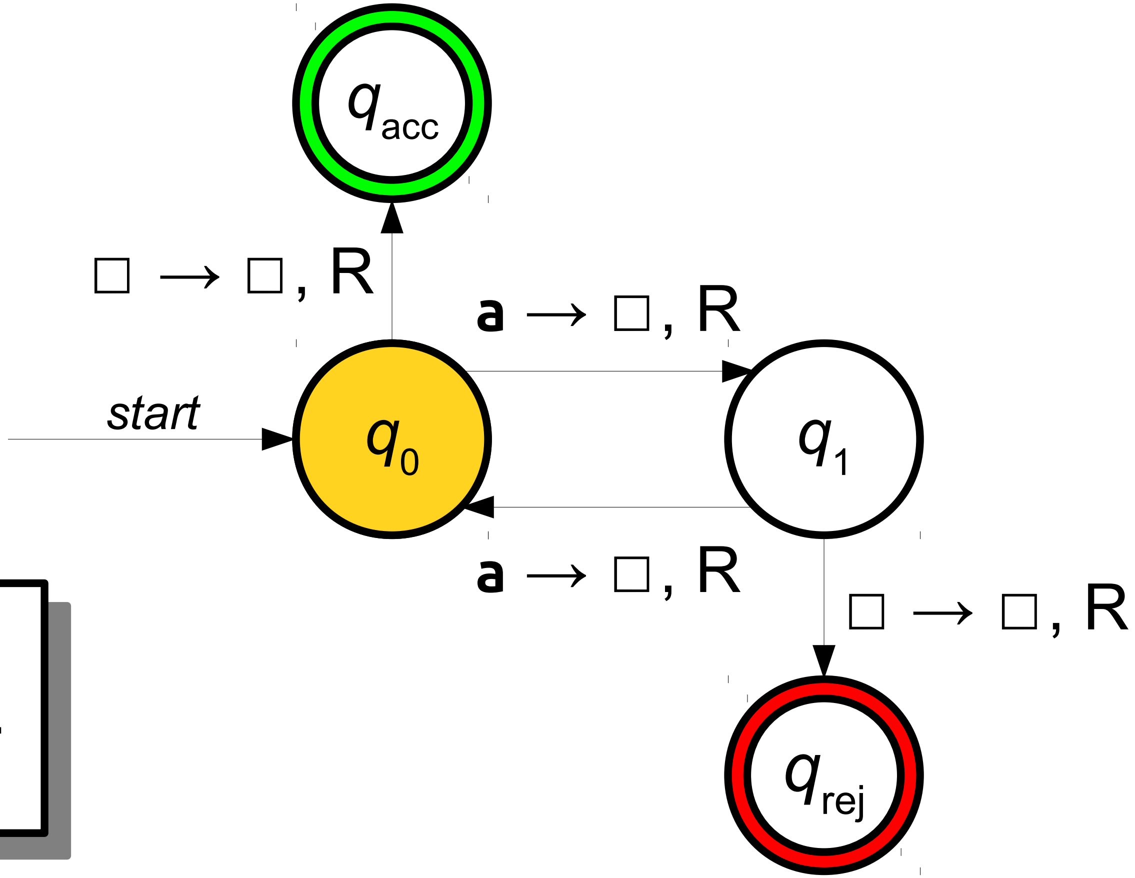
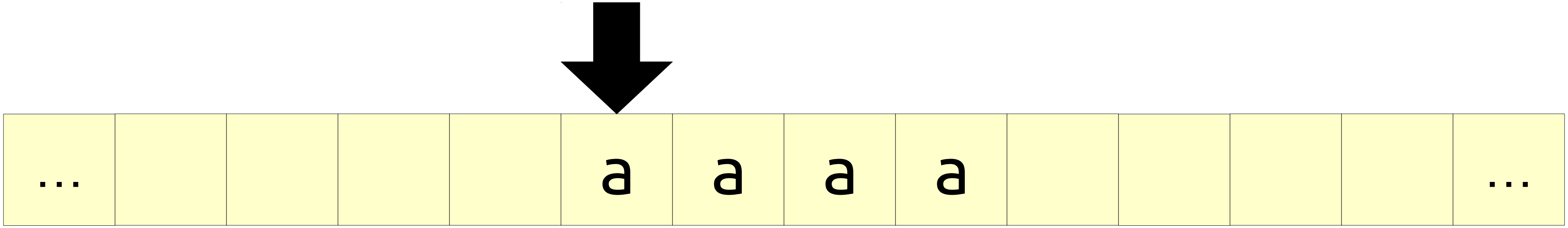


Like DFAs and NFAs, TMs begin execution in their ***start state***.



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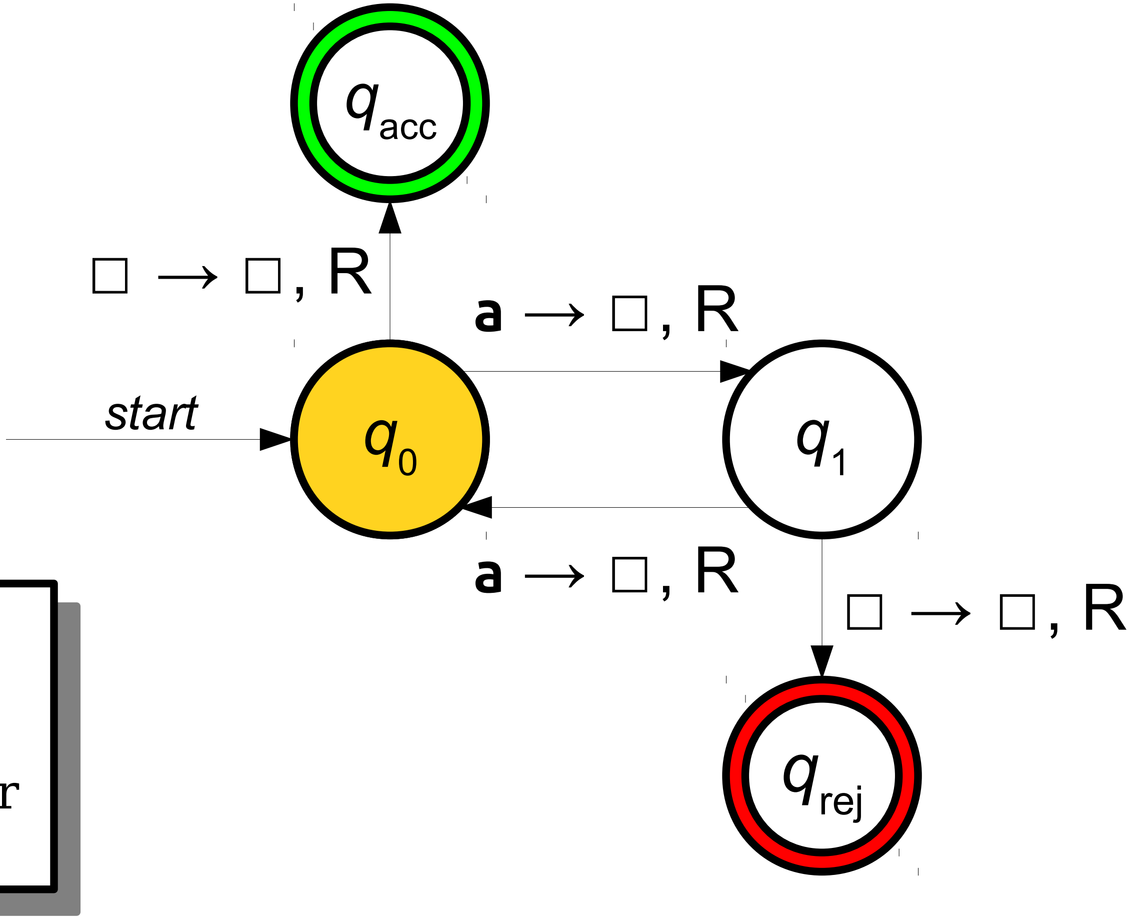
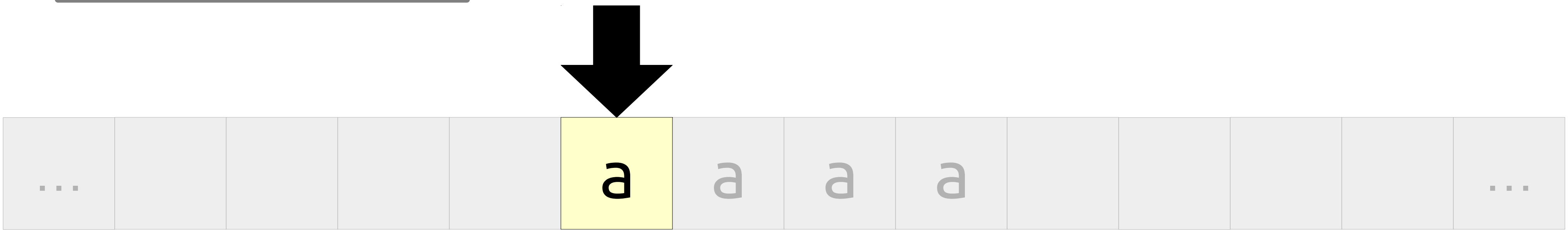


At each step, the TM only looks at the symbol immediately under the ***tape head***.



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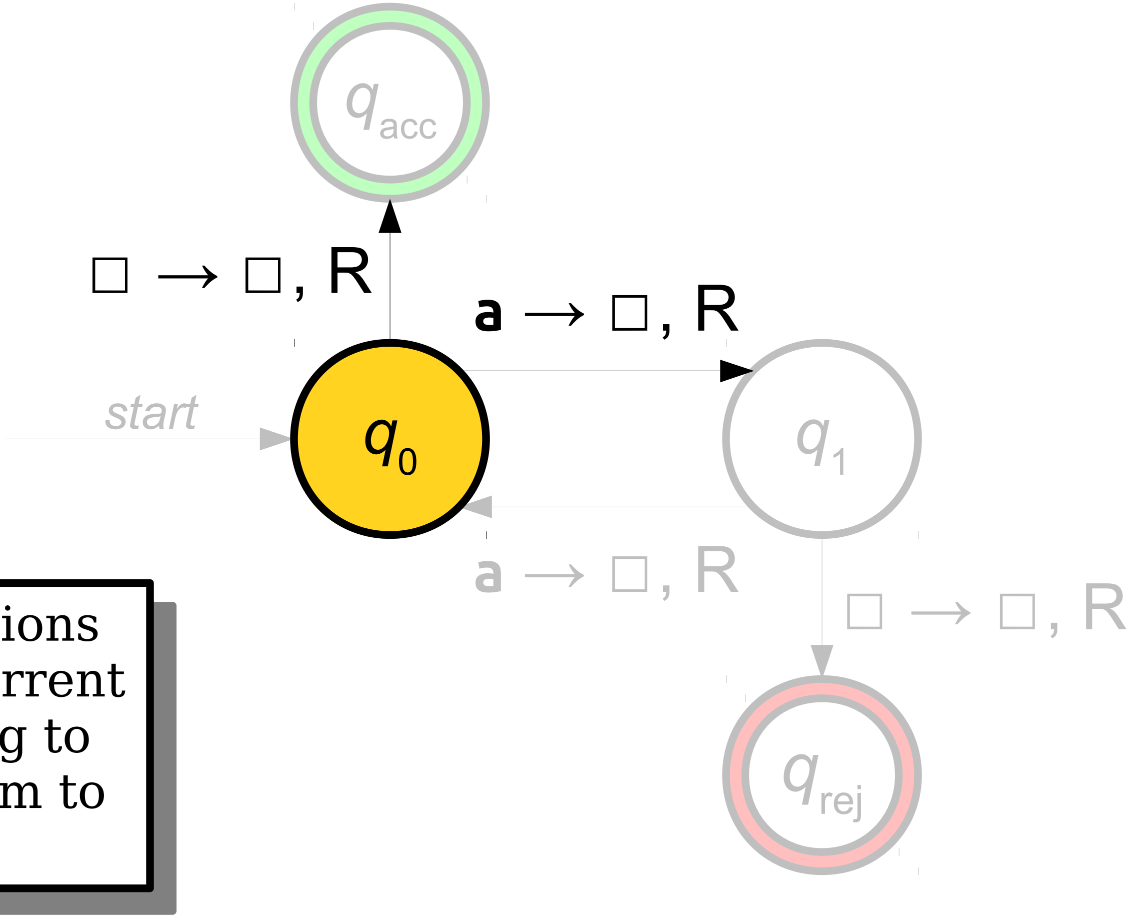
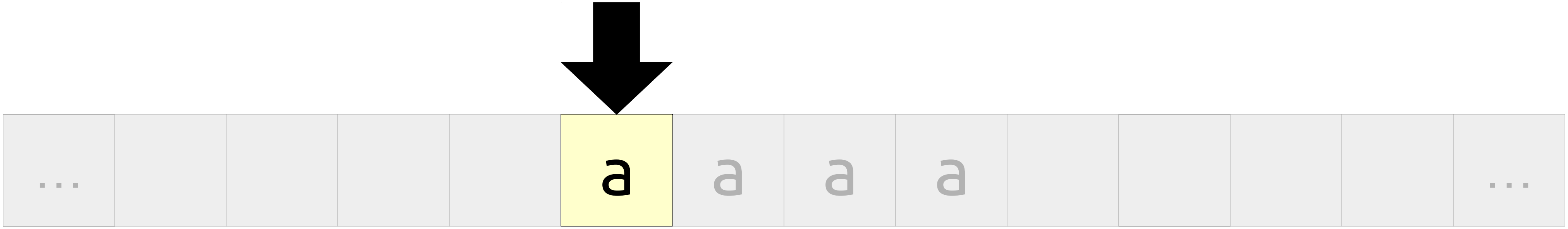


These two transitions originate at the current state. We're going to choose one of them to follow.



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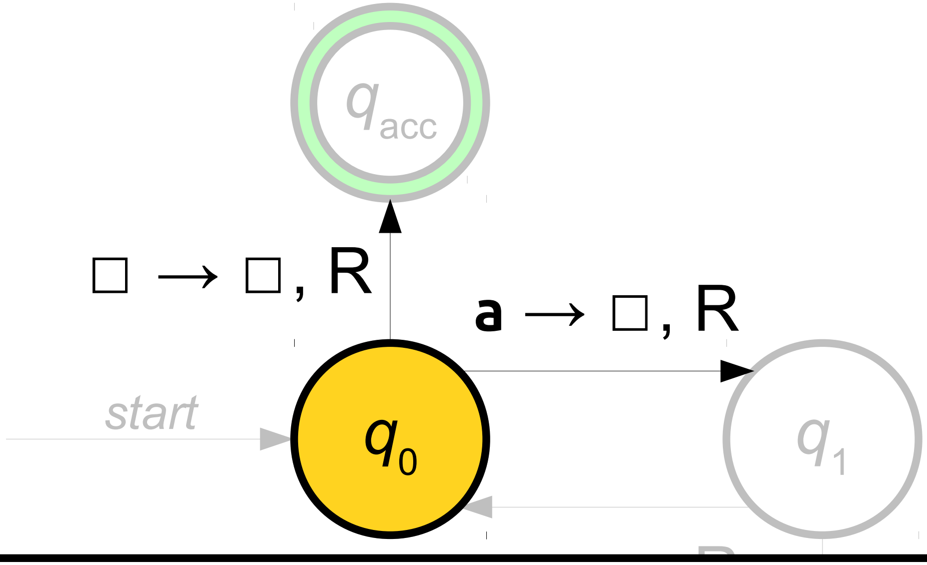
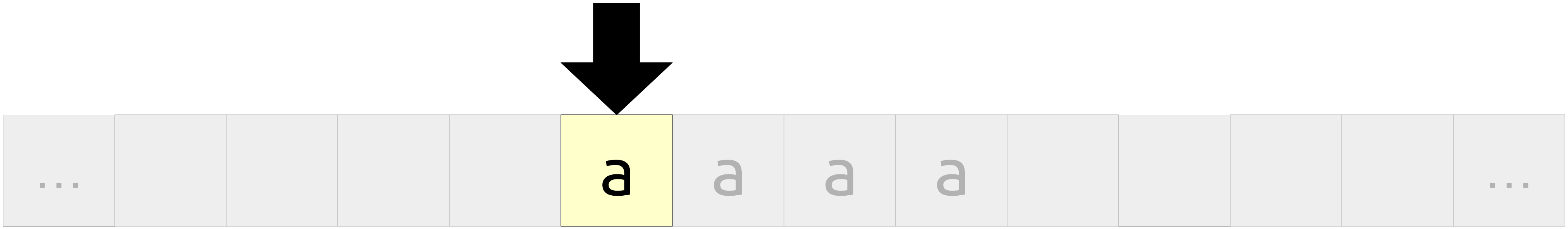


Each transition has the form

read* → *write*, *dir

and means "if symbol ***read*** is under the tape head, replace it with ***write*** and move the tape head in direction ***dir*** (L or R). The □ symbol denotes a blank cell.



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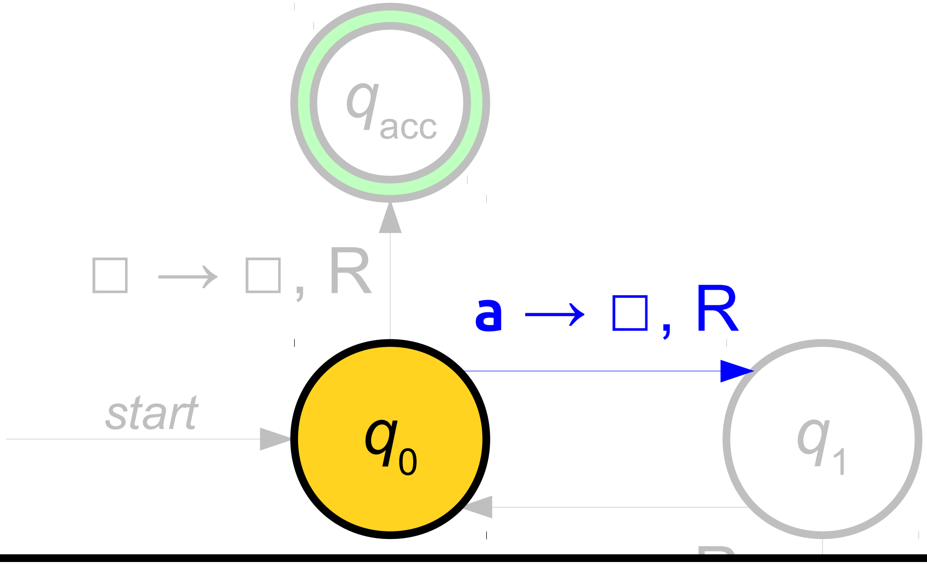

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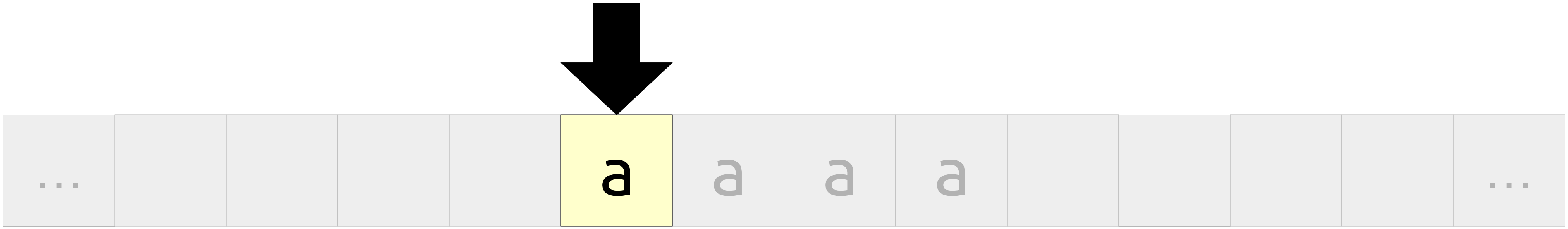

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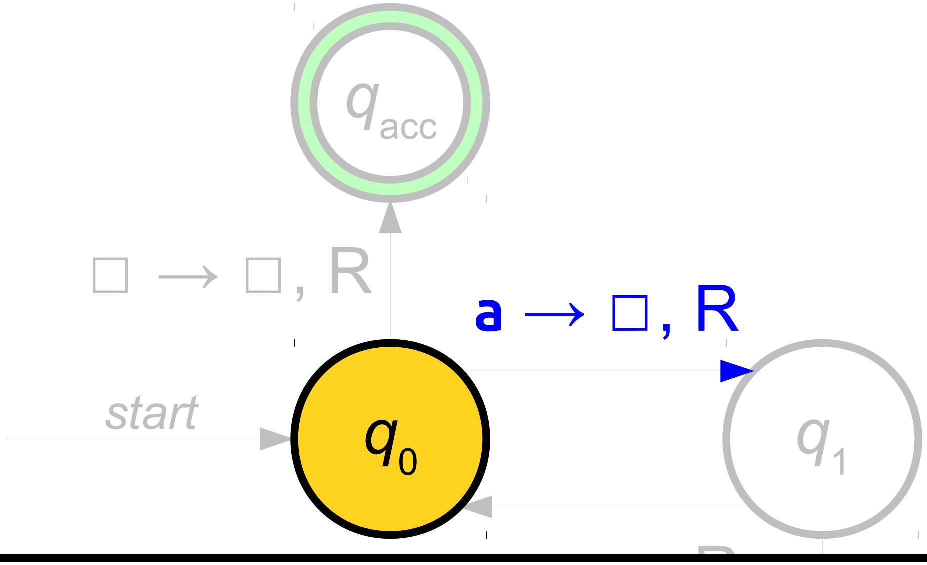

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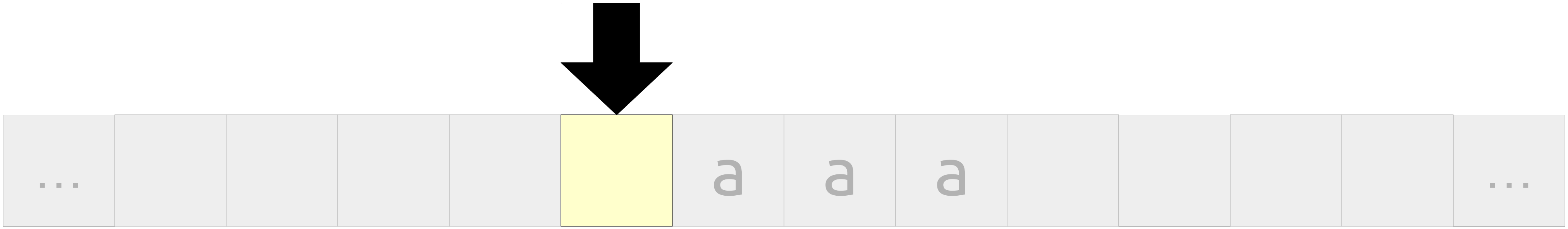

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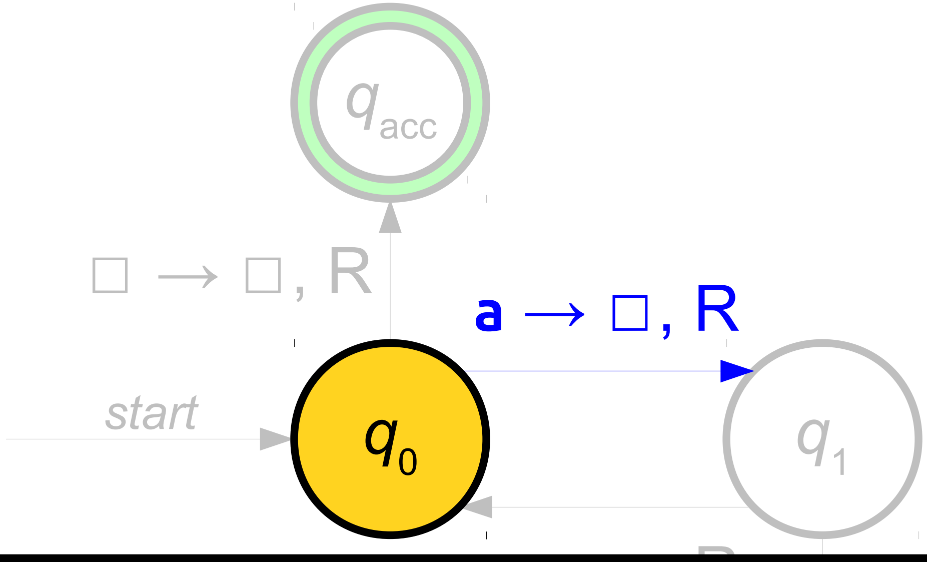

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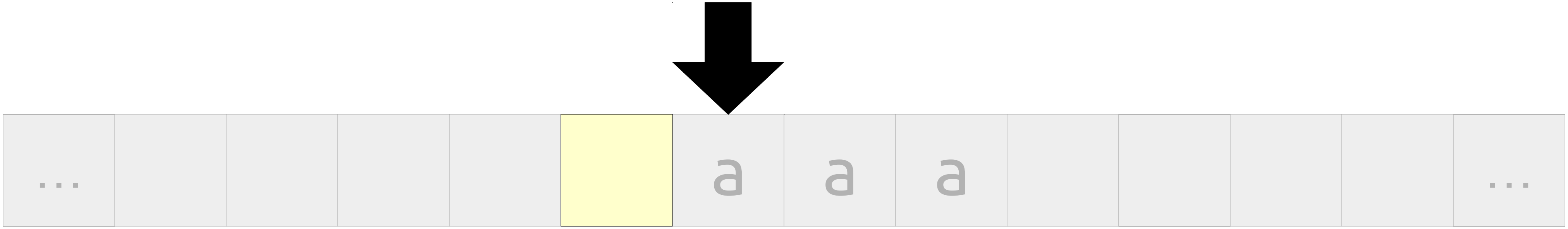

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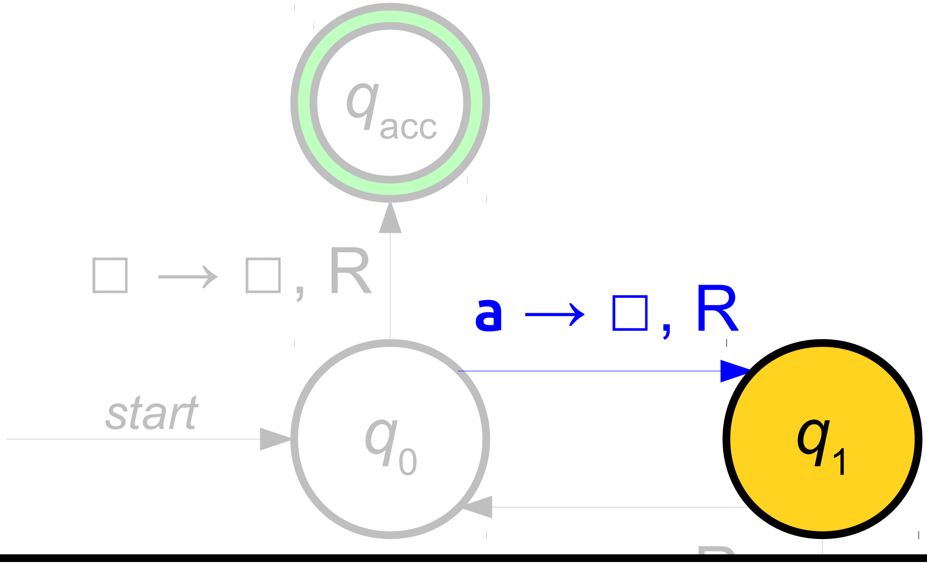

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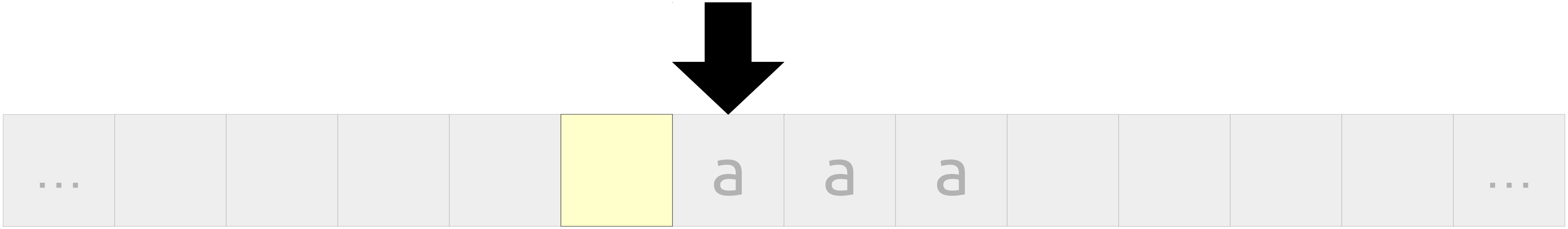

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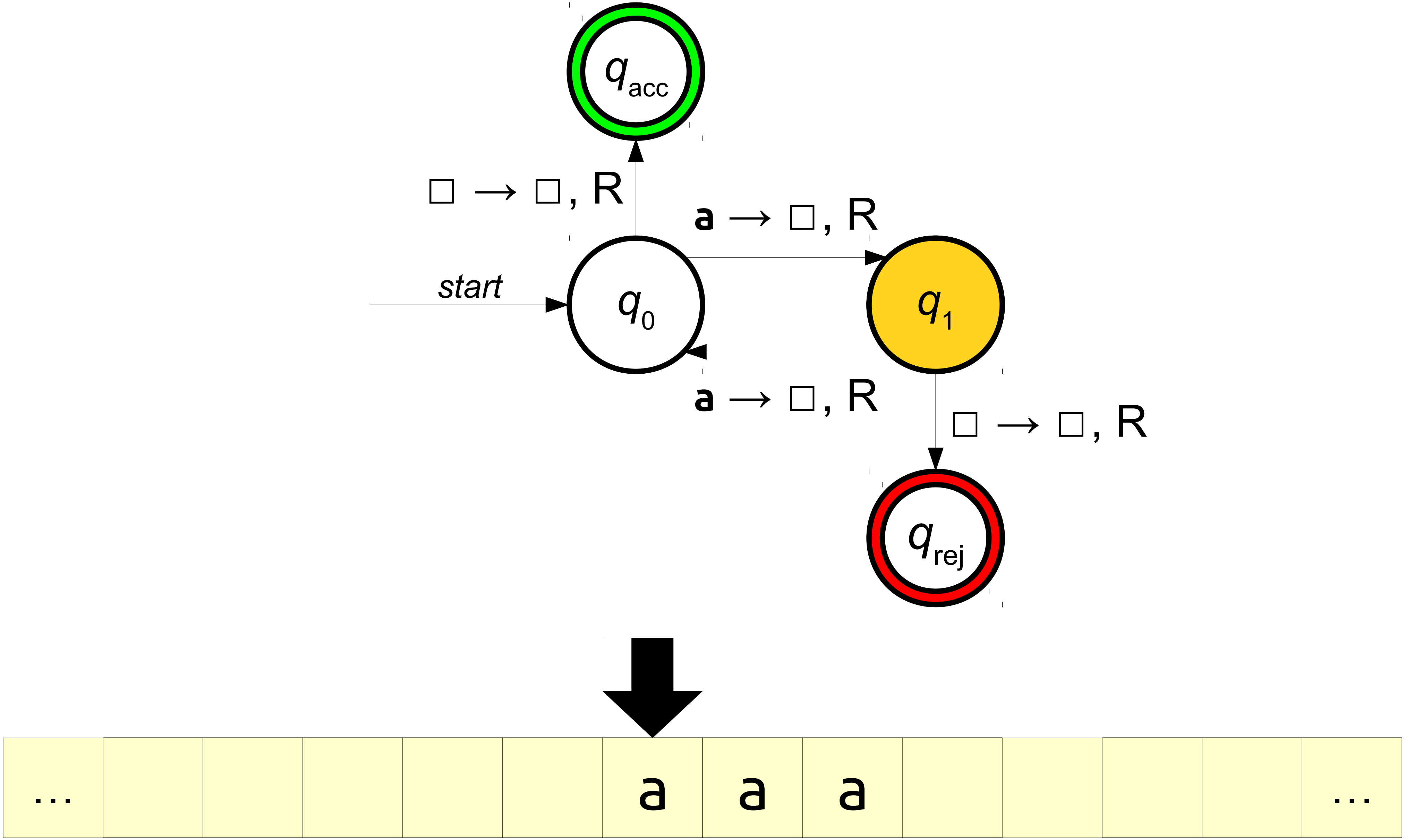

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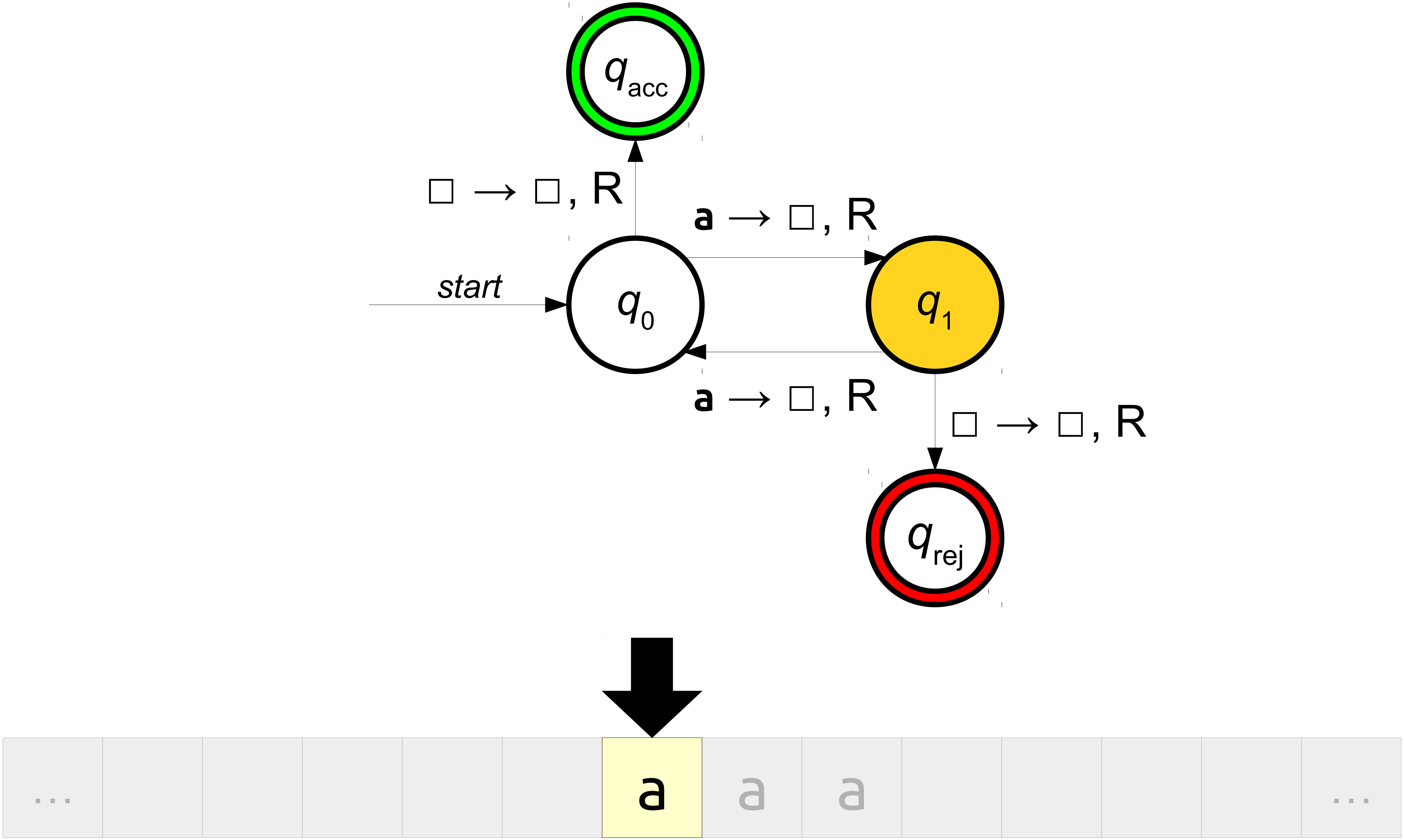

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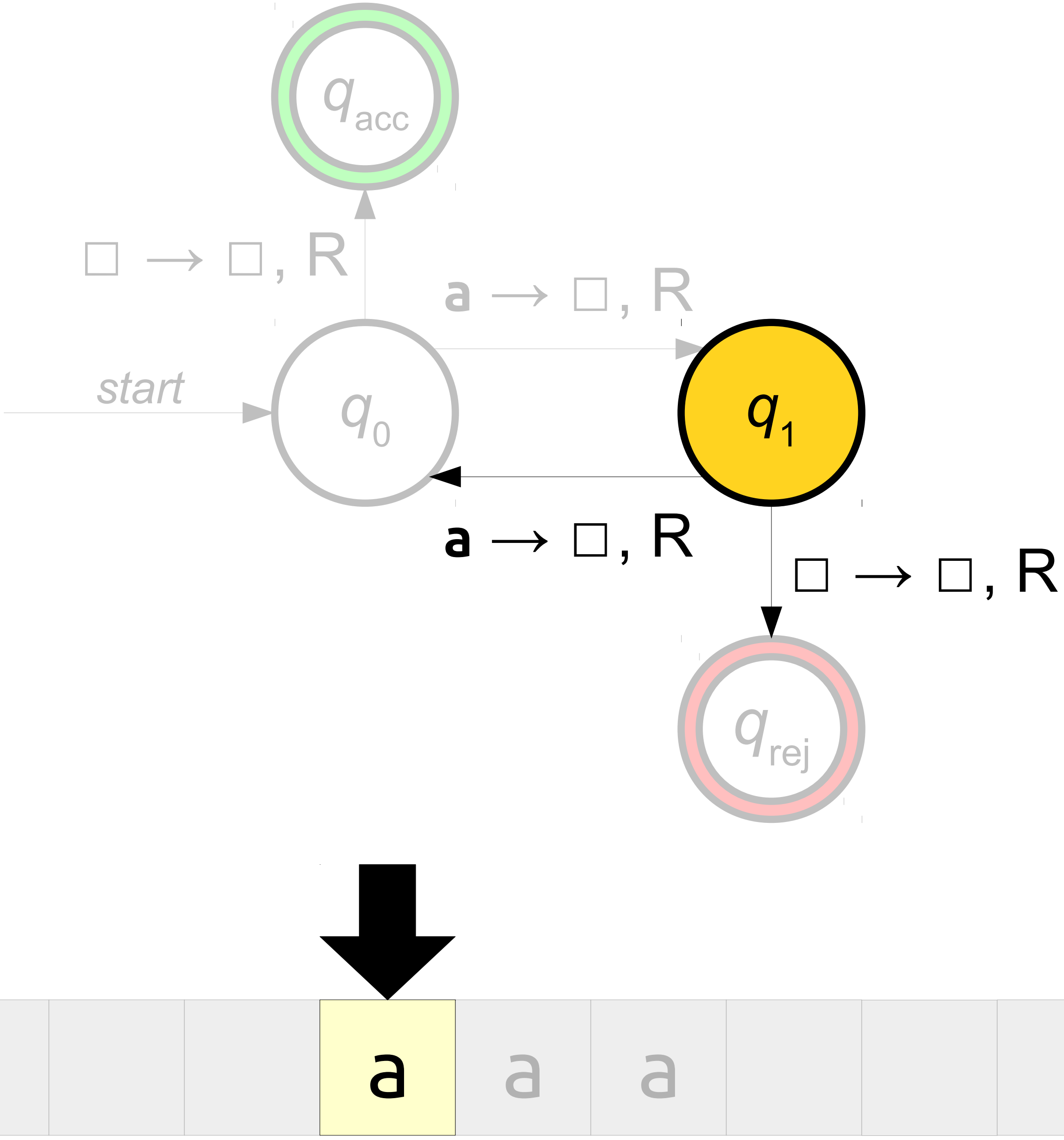

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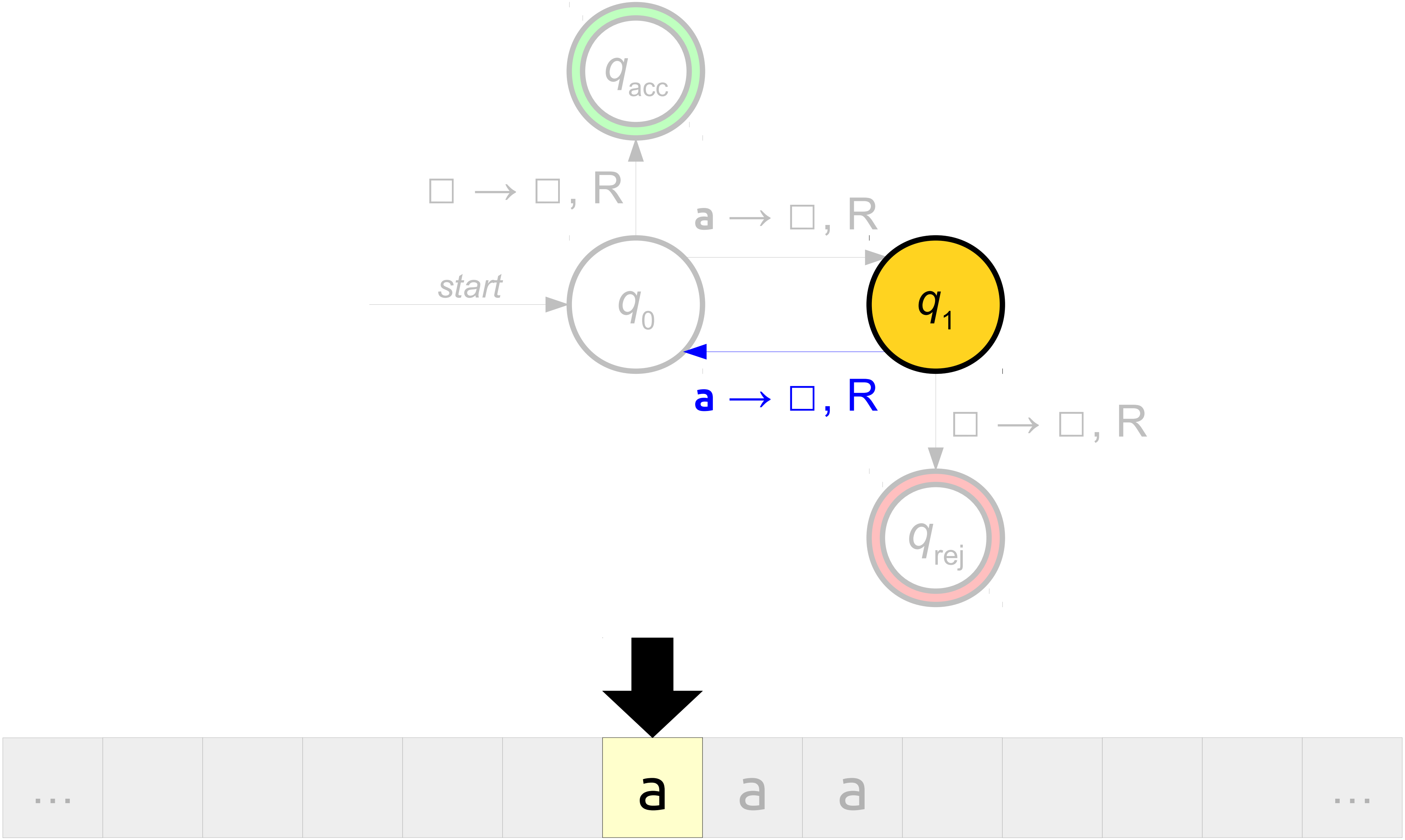

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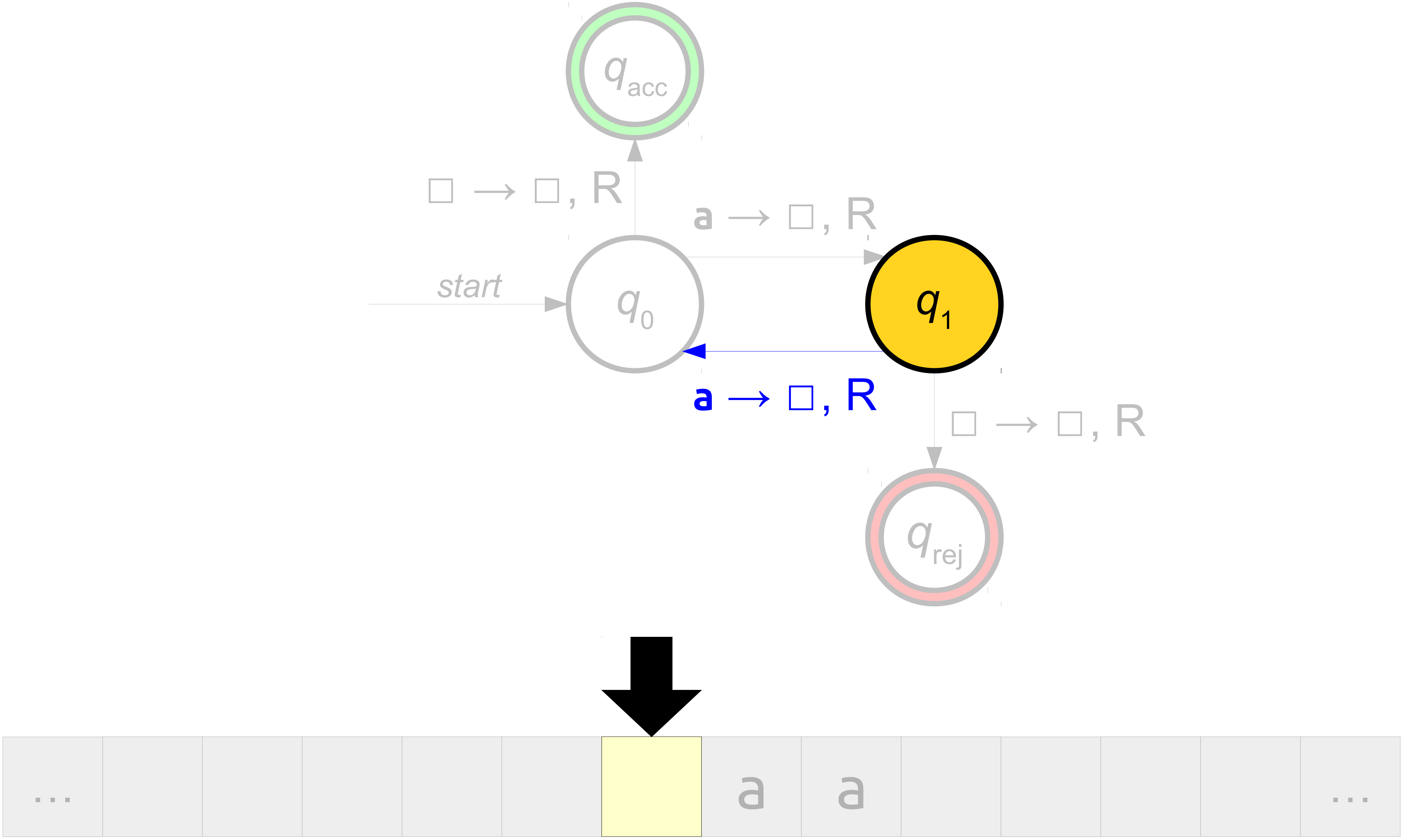

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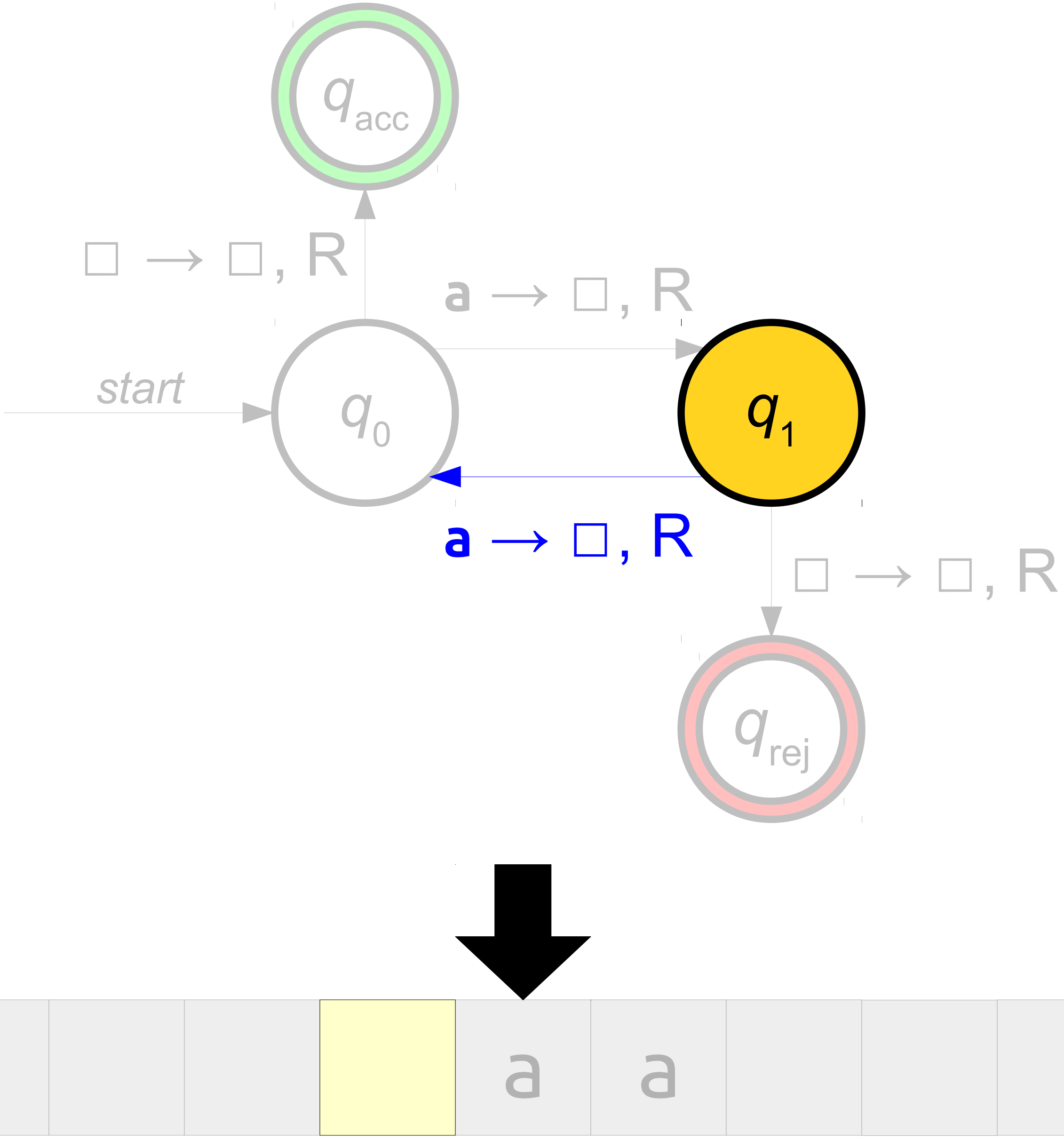

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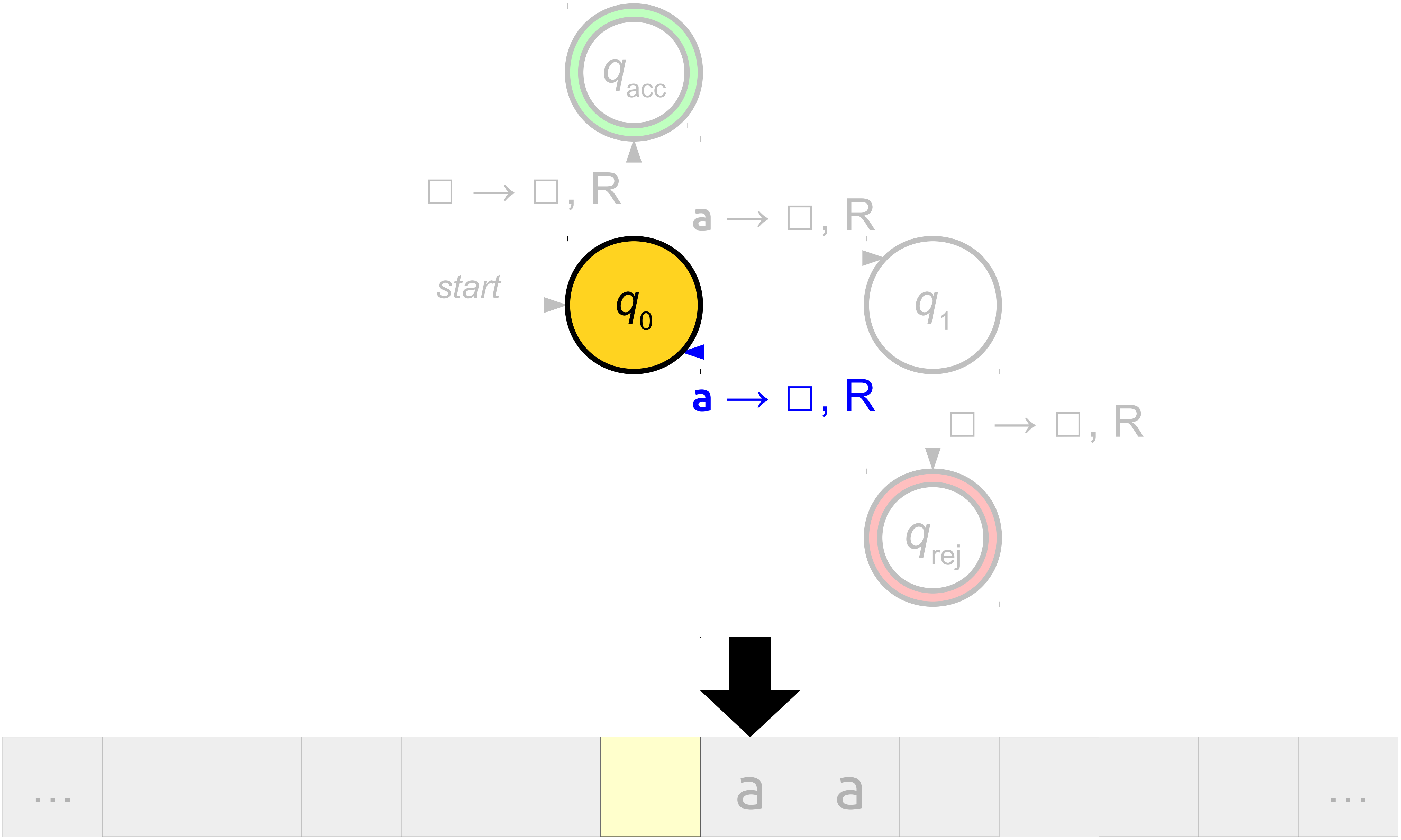

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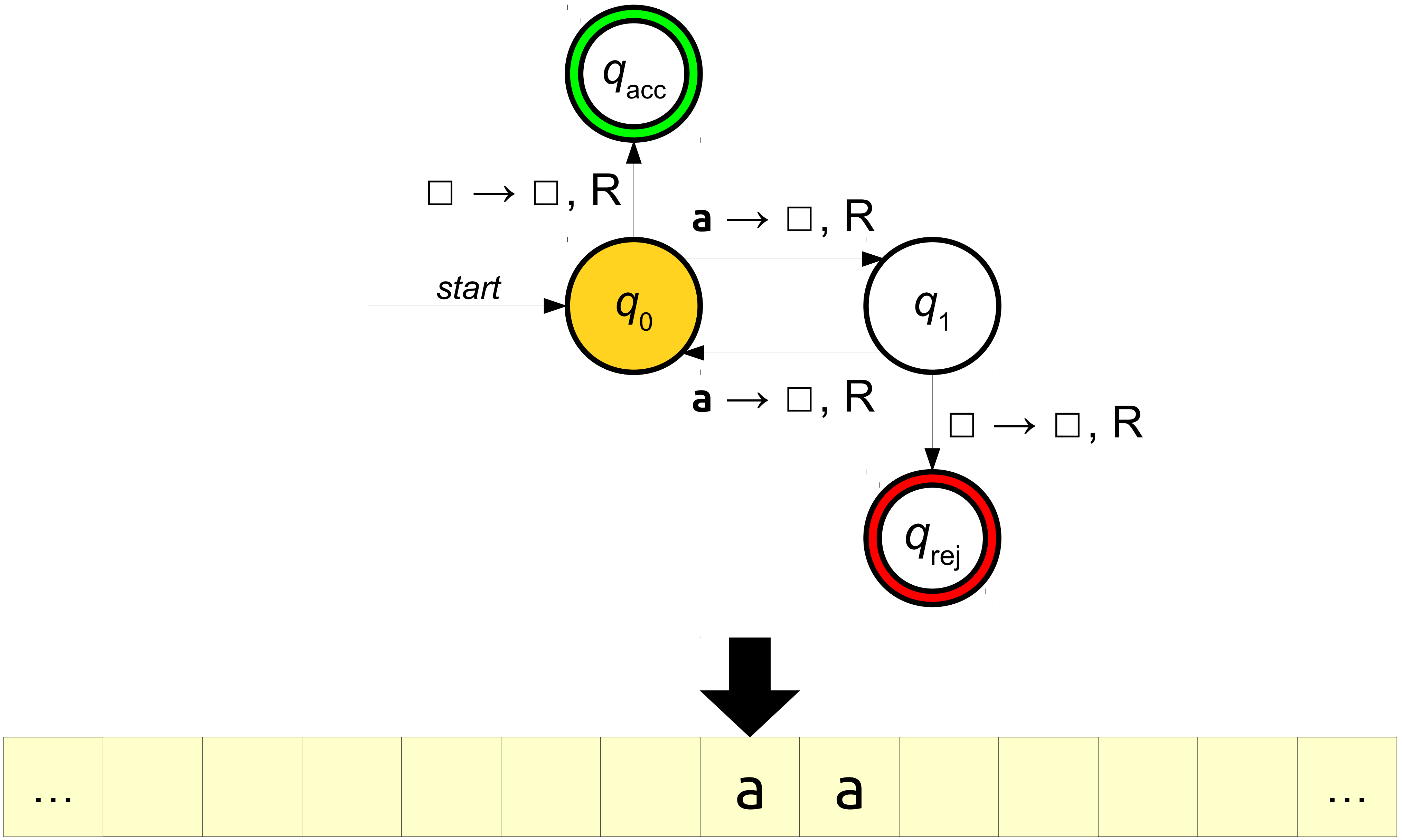

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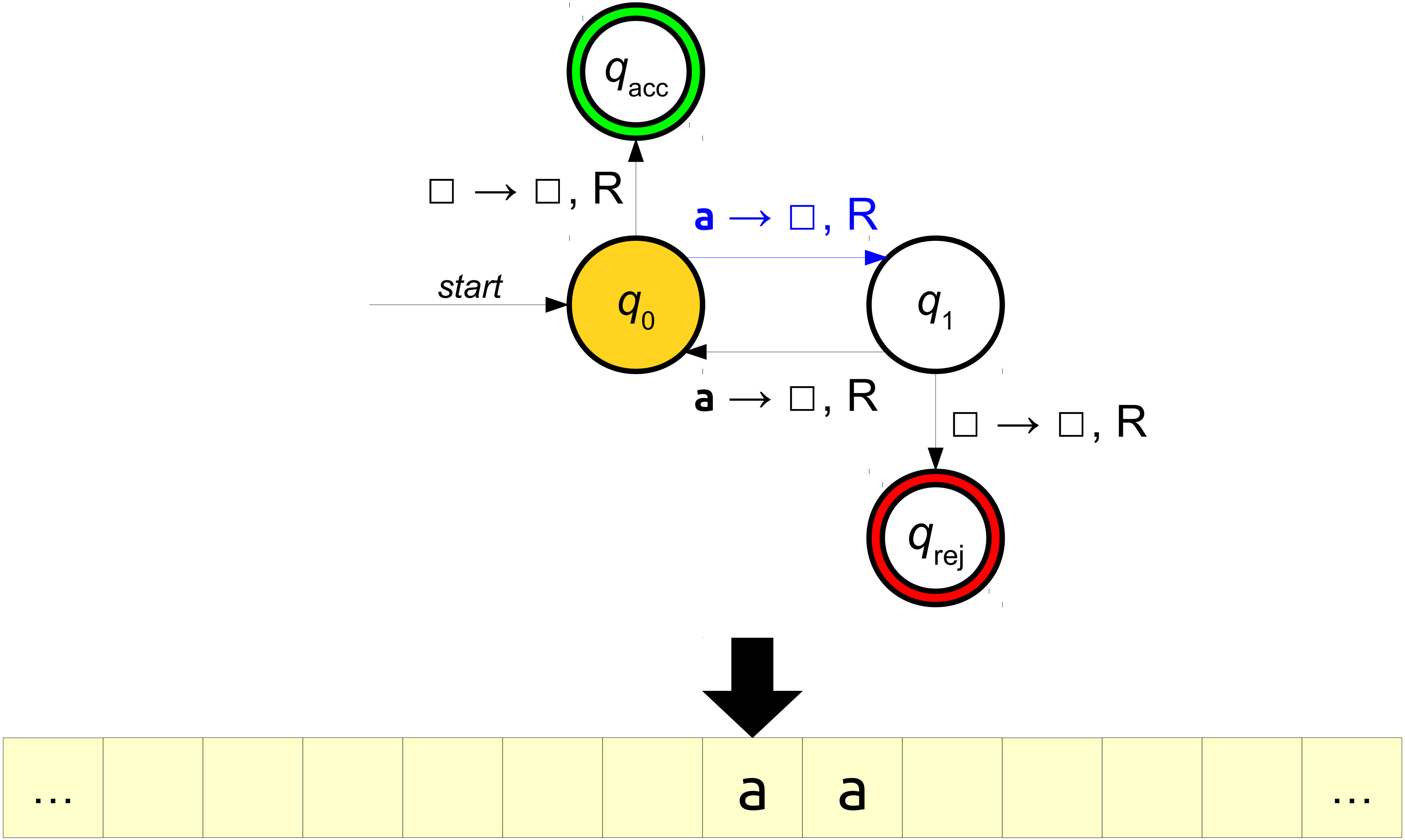

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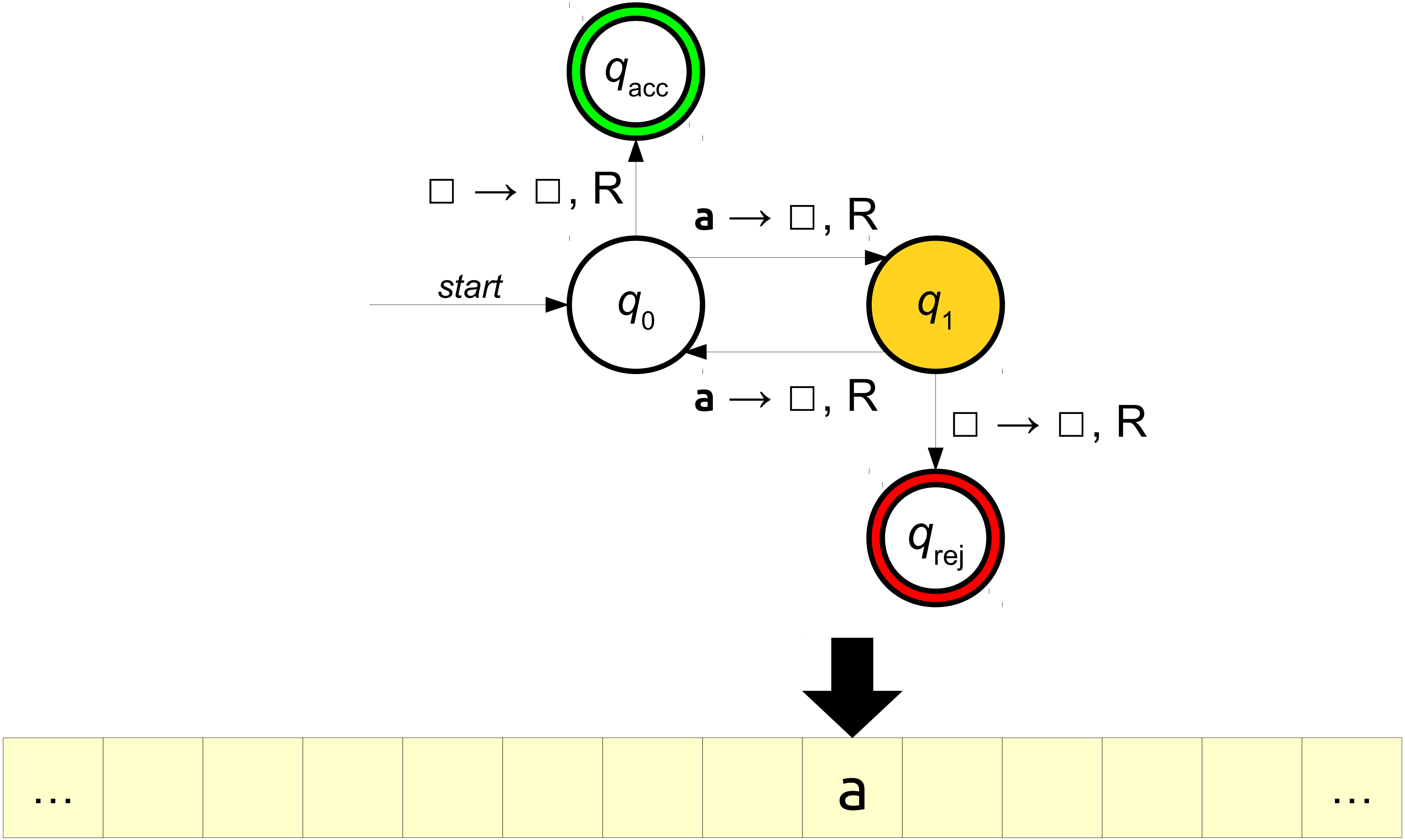

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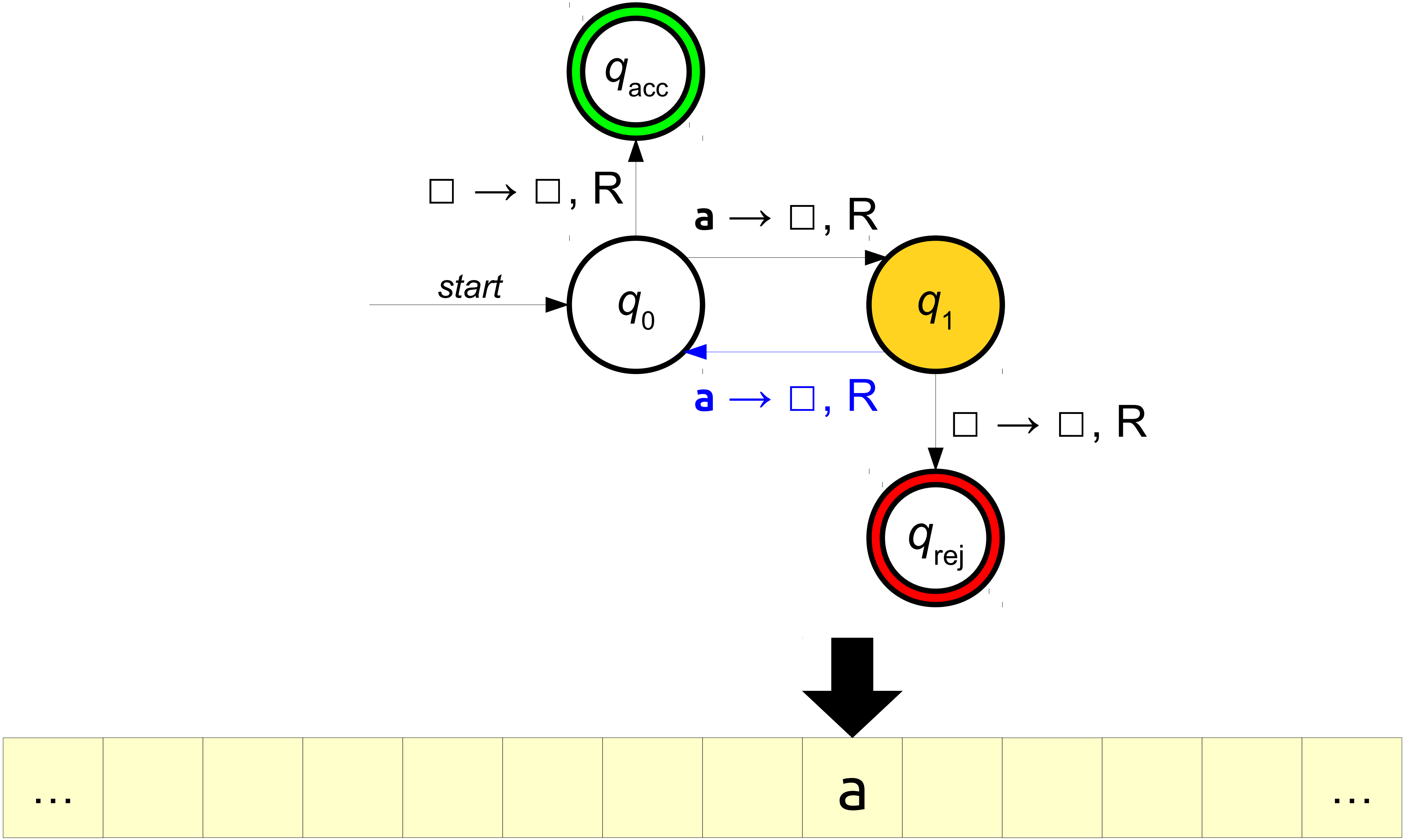

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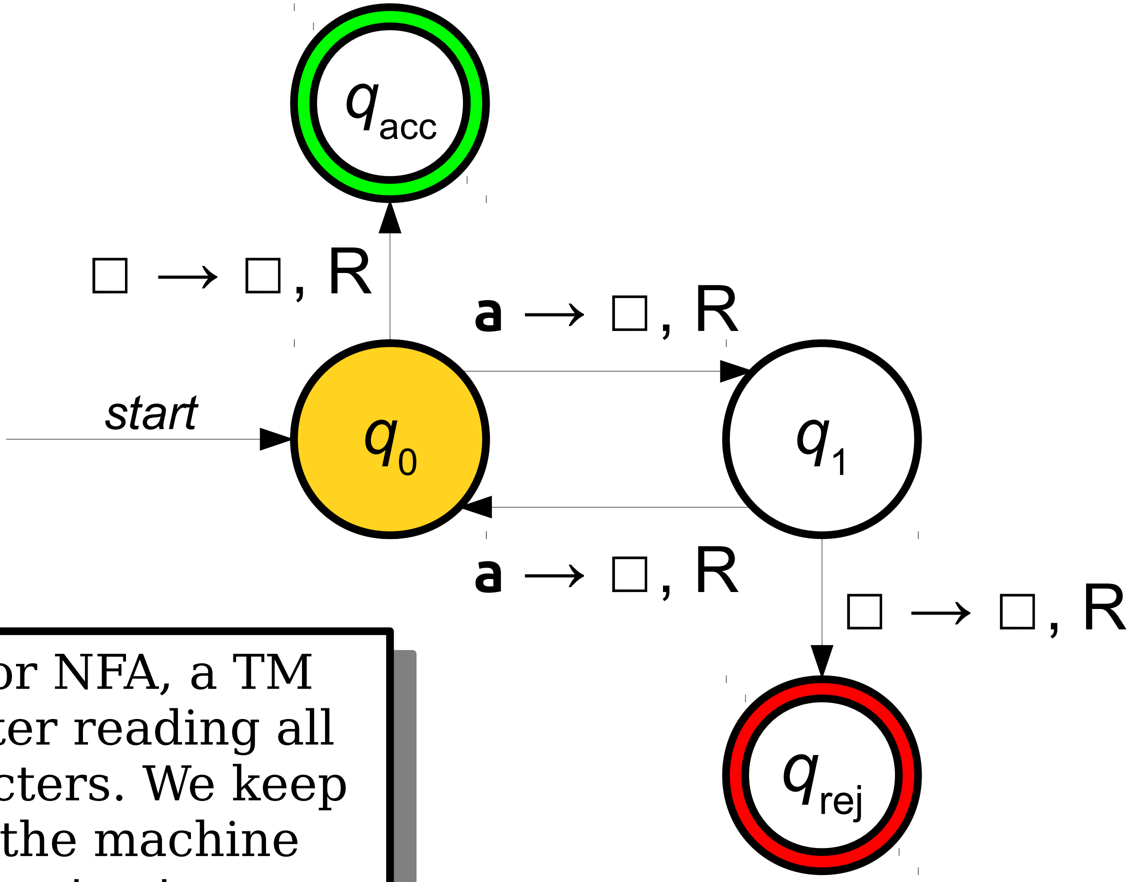
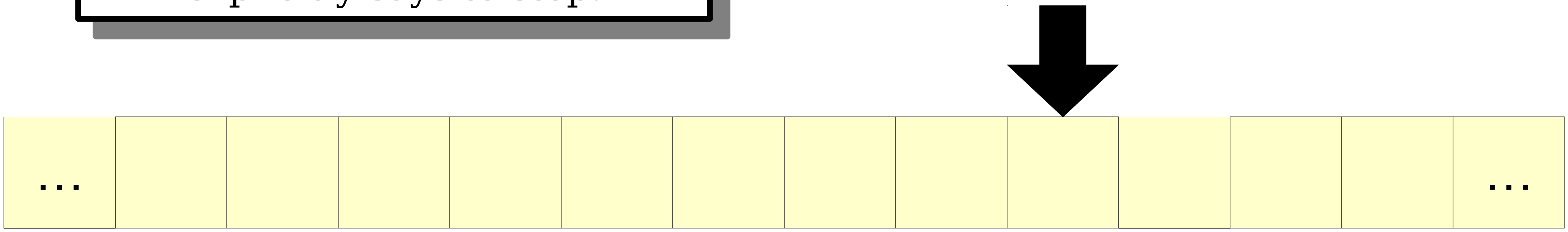


Unlike a DFA or NFA, a TM doesn't stop after reading all the input characters. We keep running until the machine explicitly says to stop.



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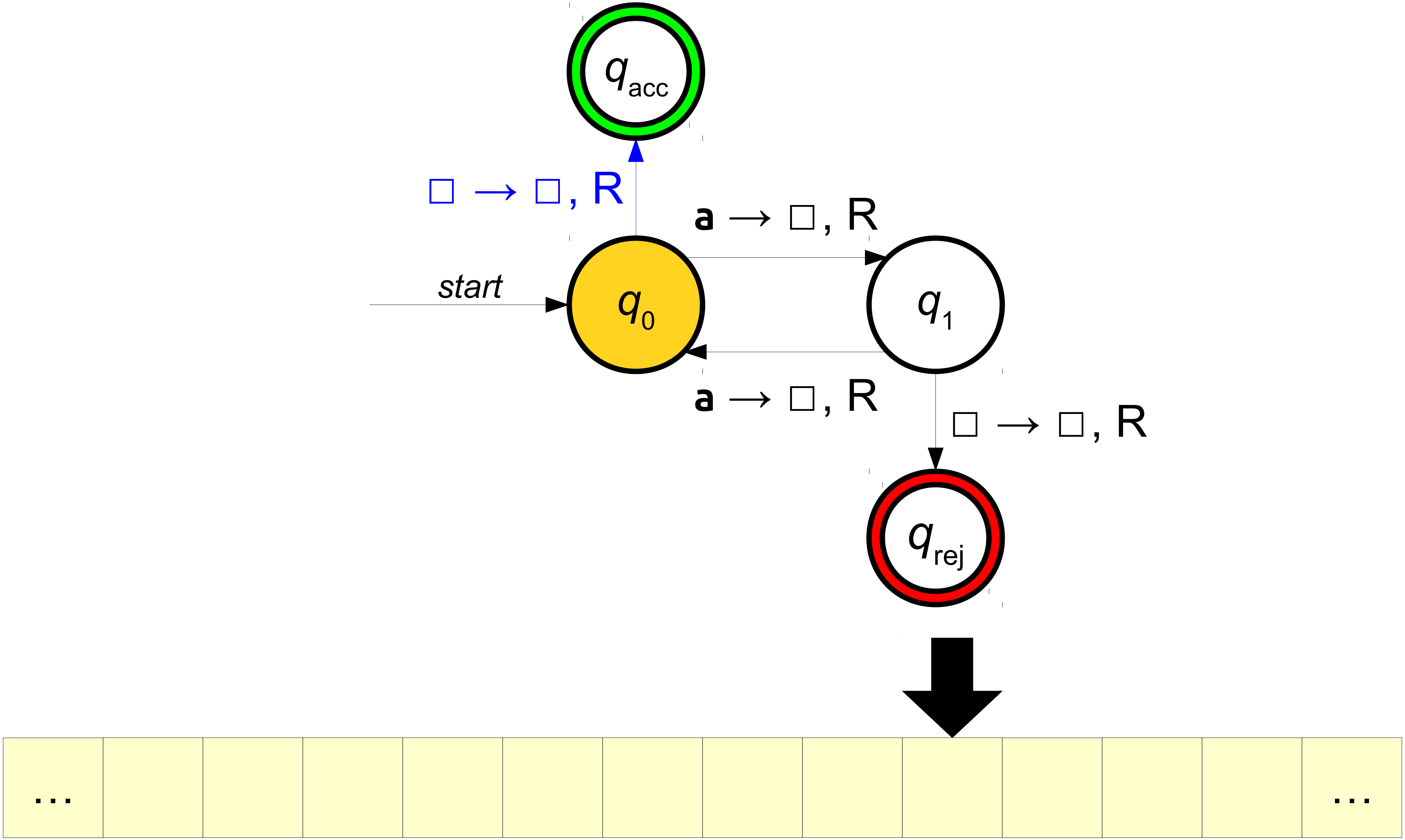

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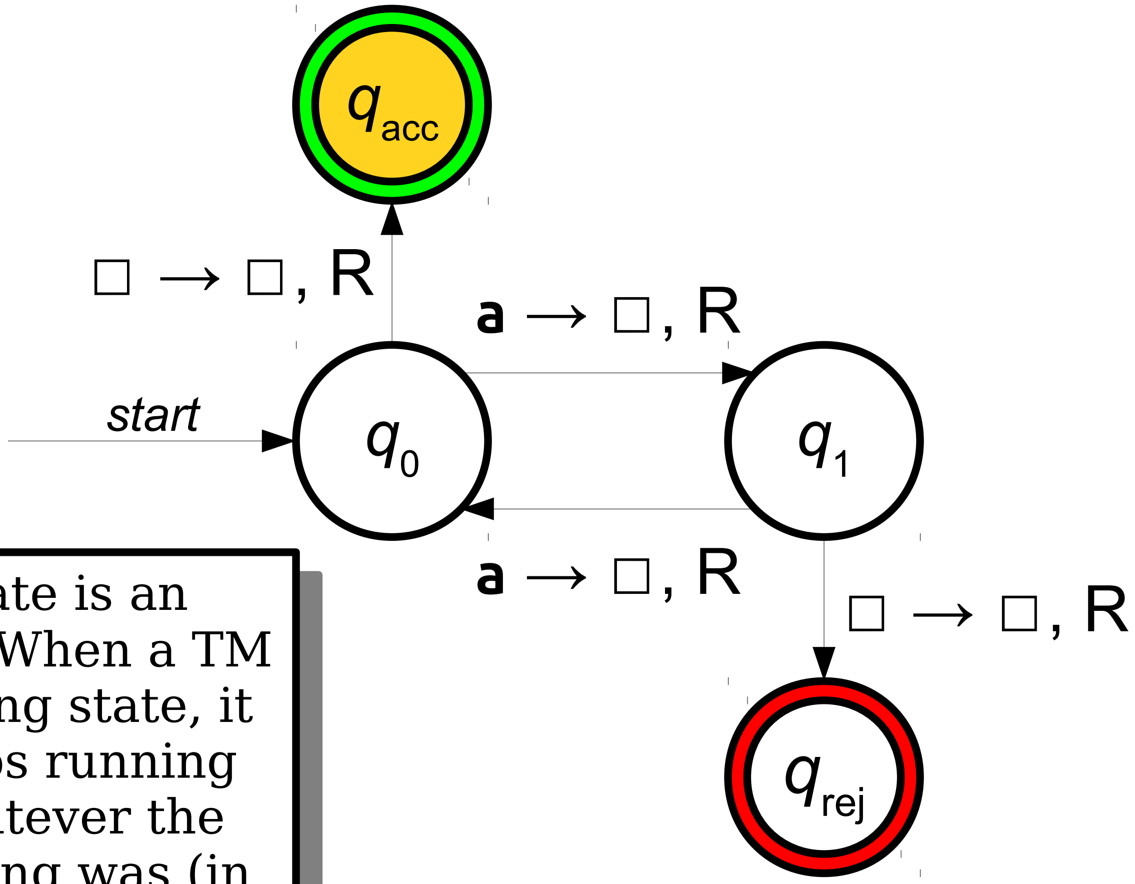
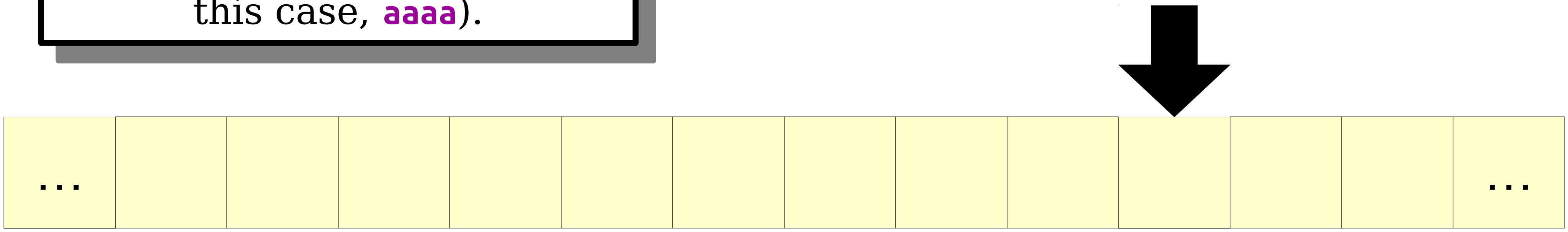


This special state is an ***accepting state***. When a TM enters an accepting state, it *immediately* stops running and accepts whatever the original input string was (in this case, **aaaa**).



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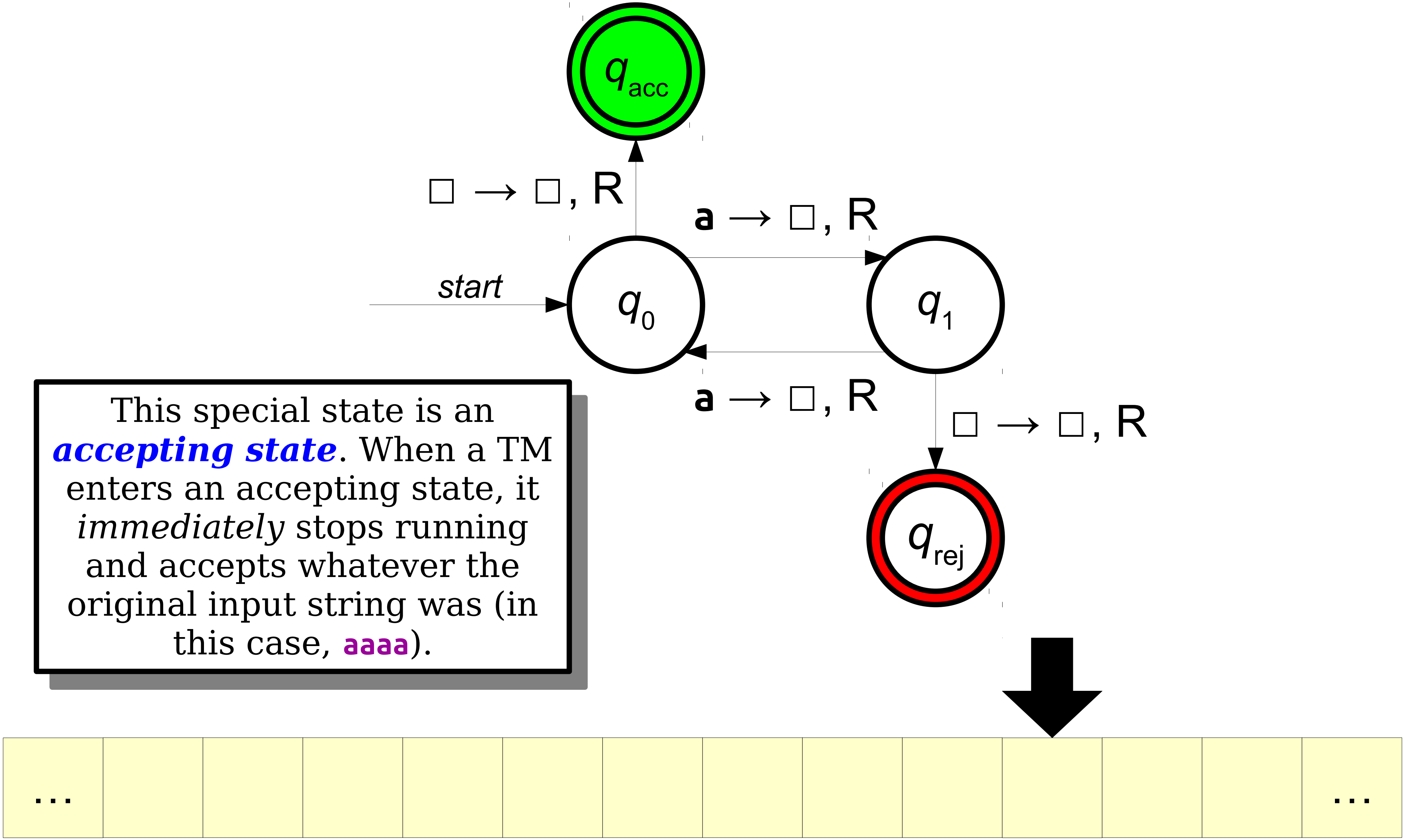

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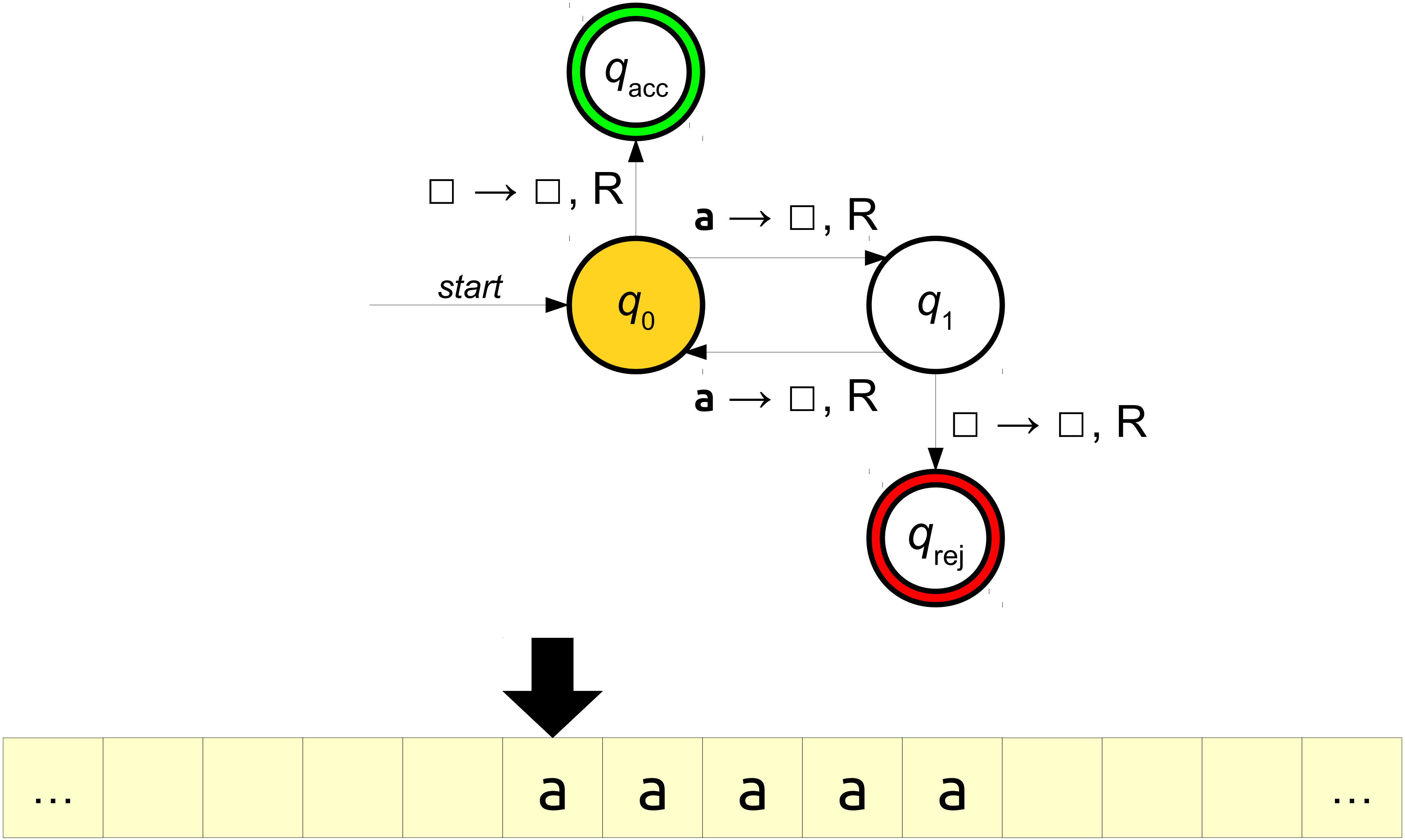

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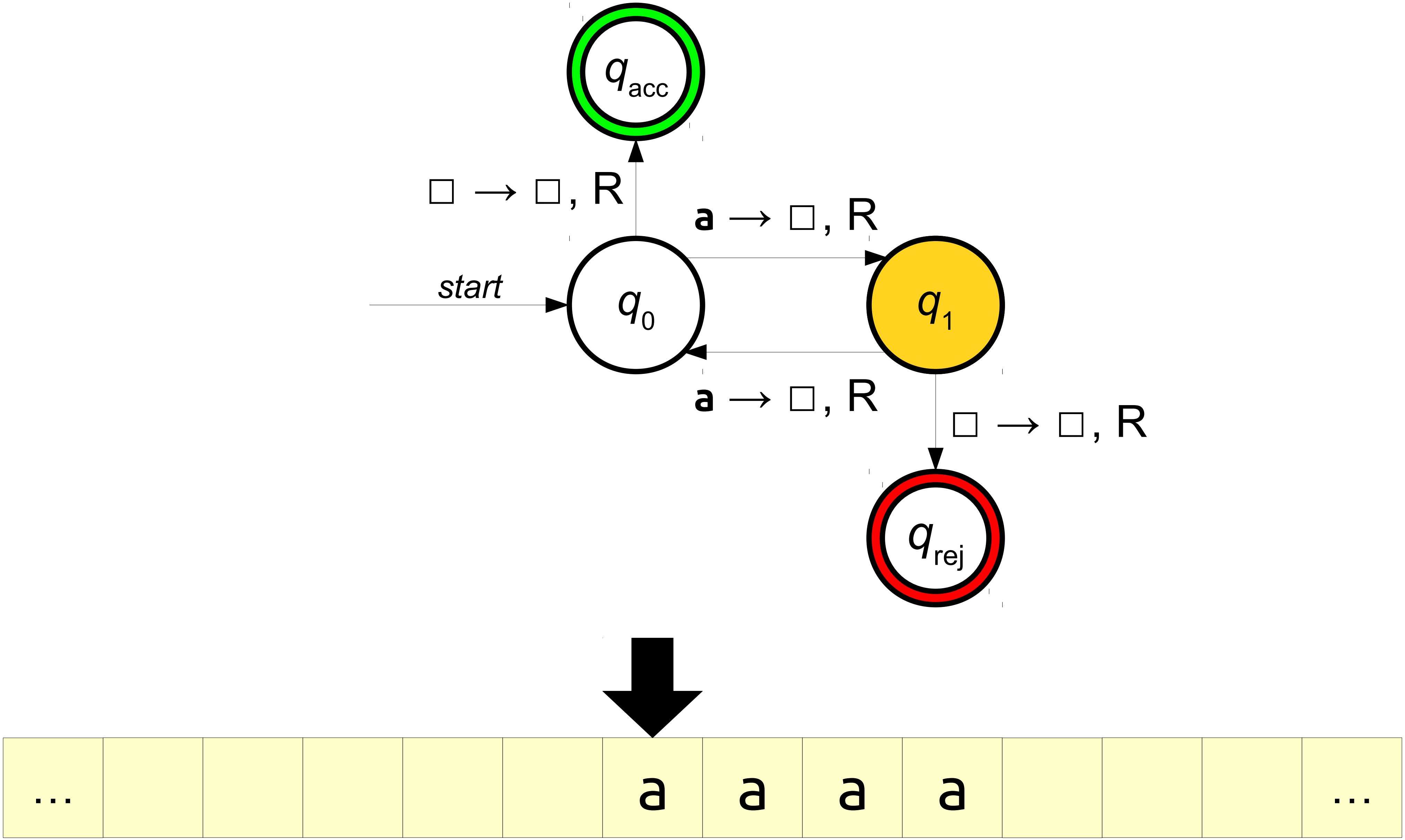

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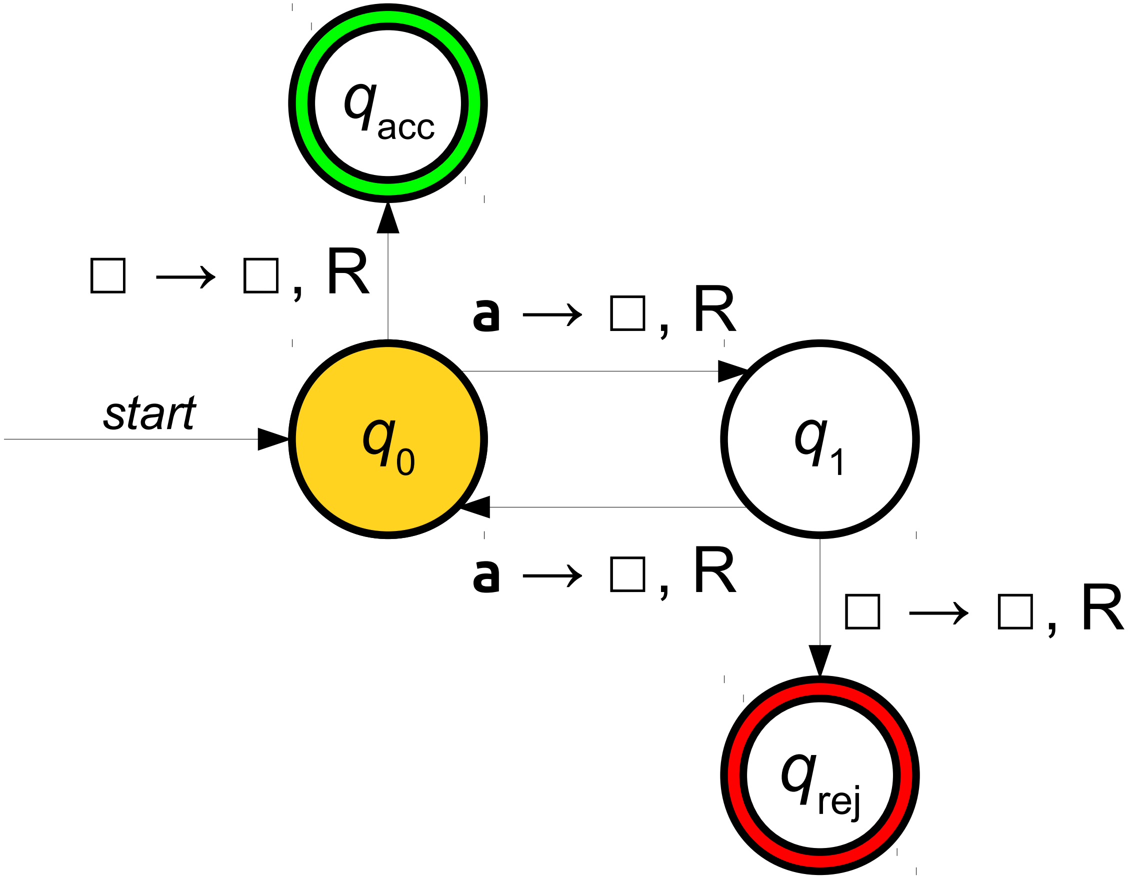

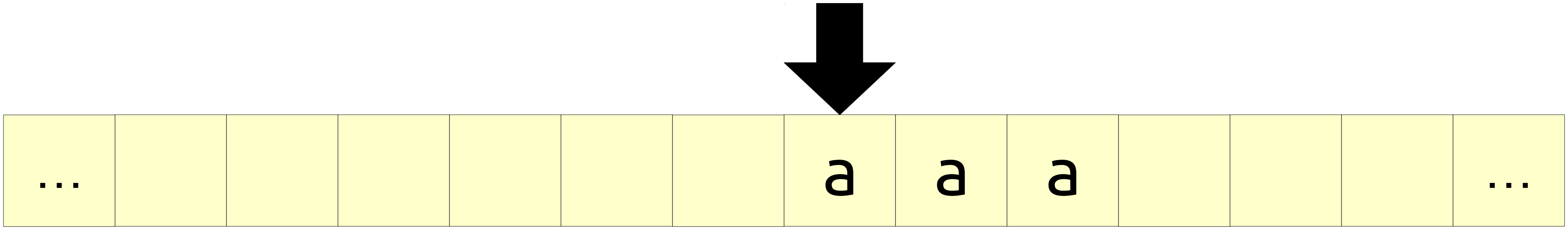

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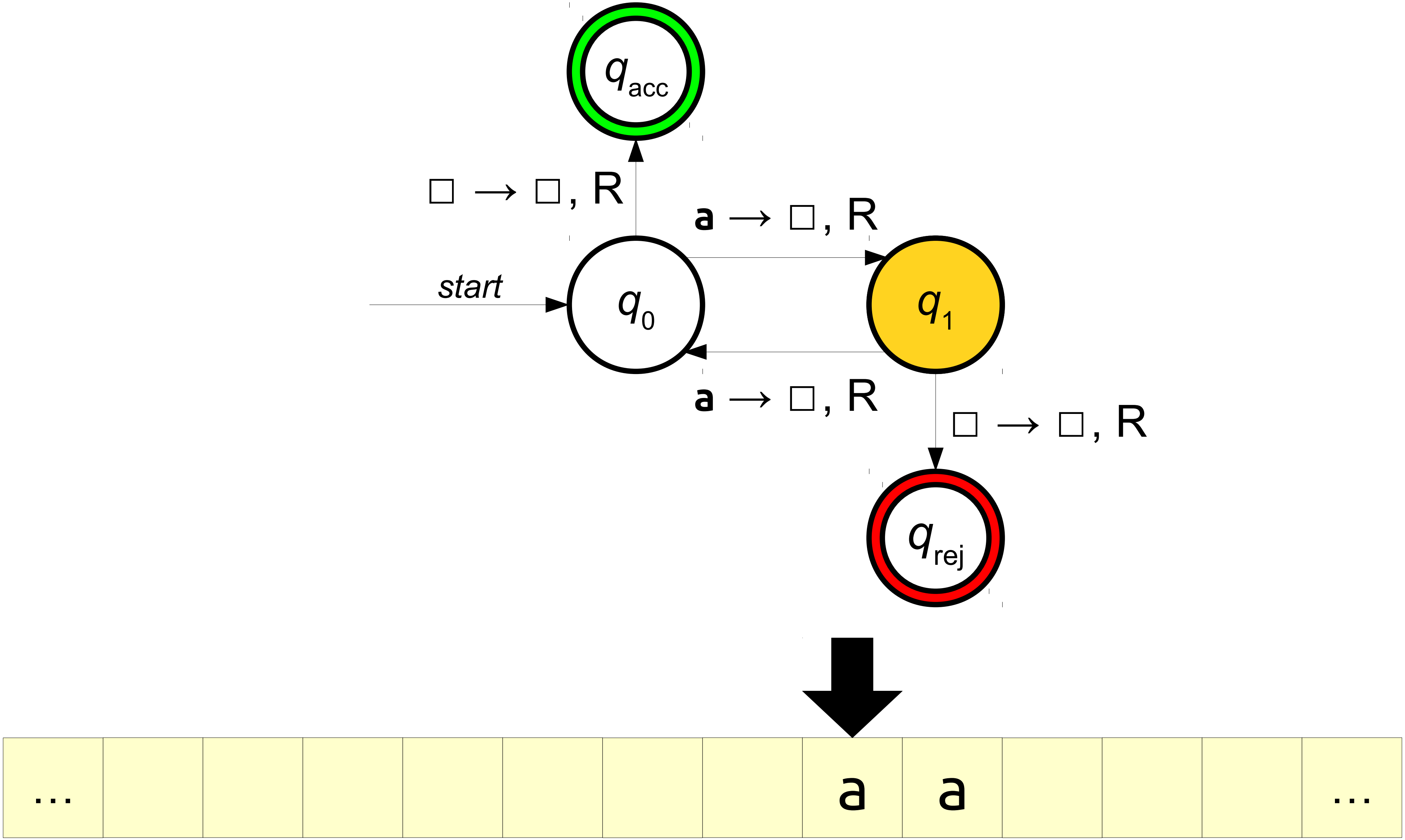

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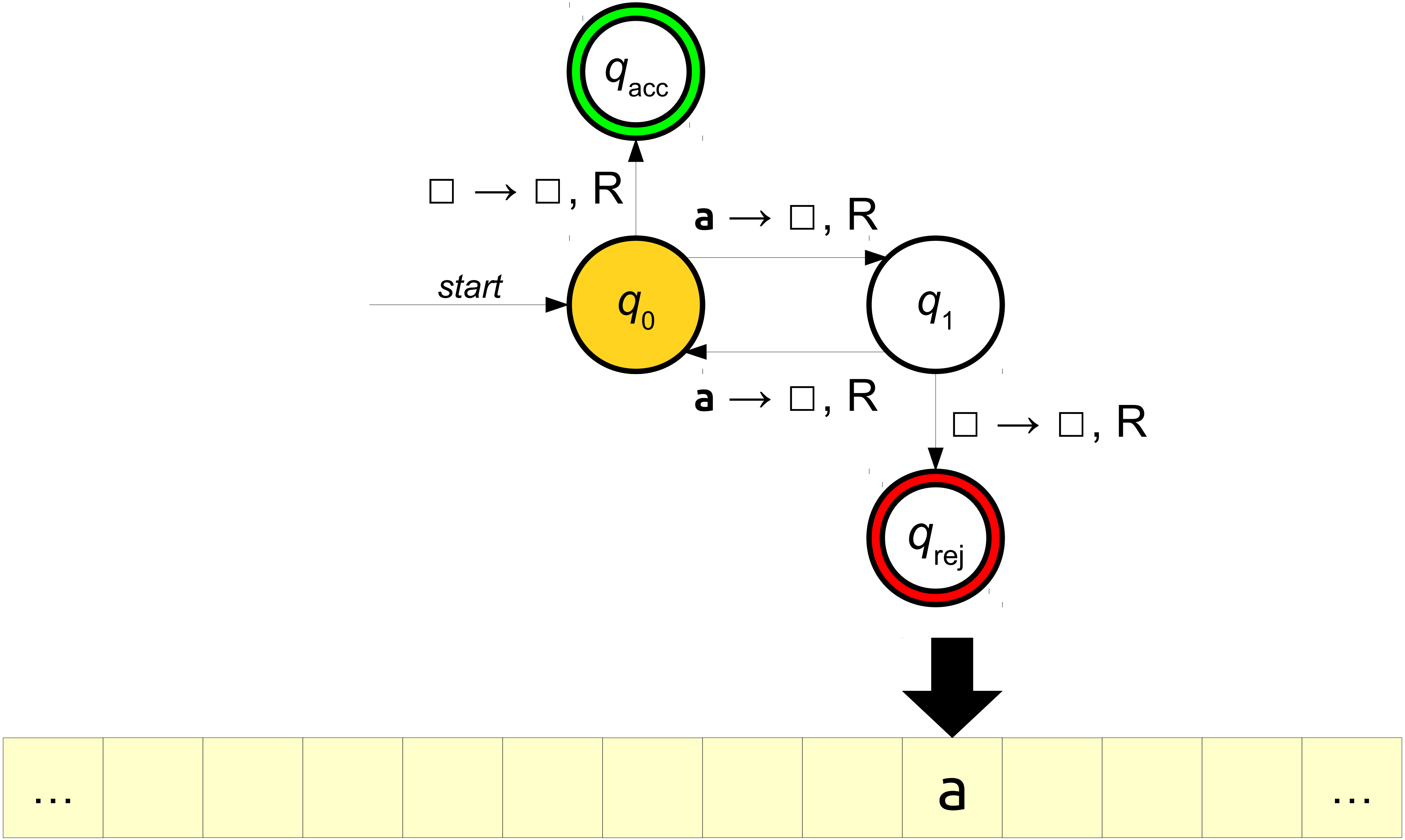

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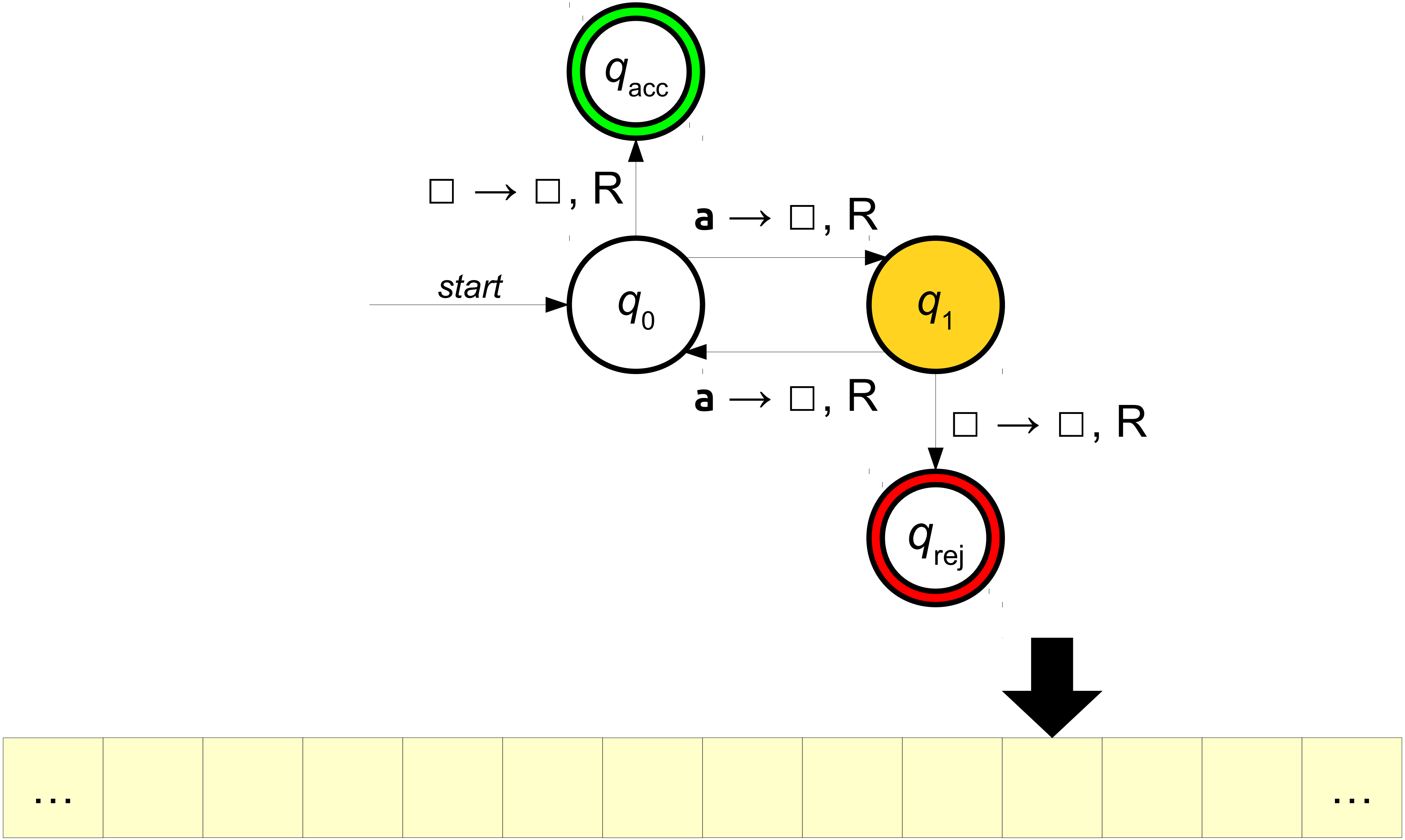

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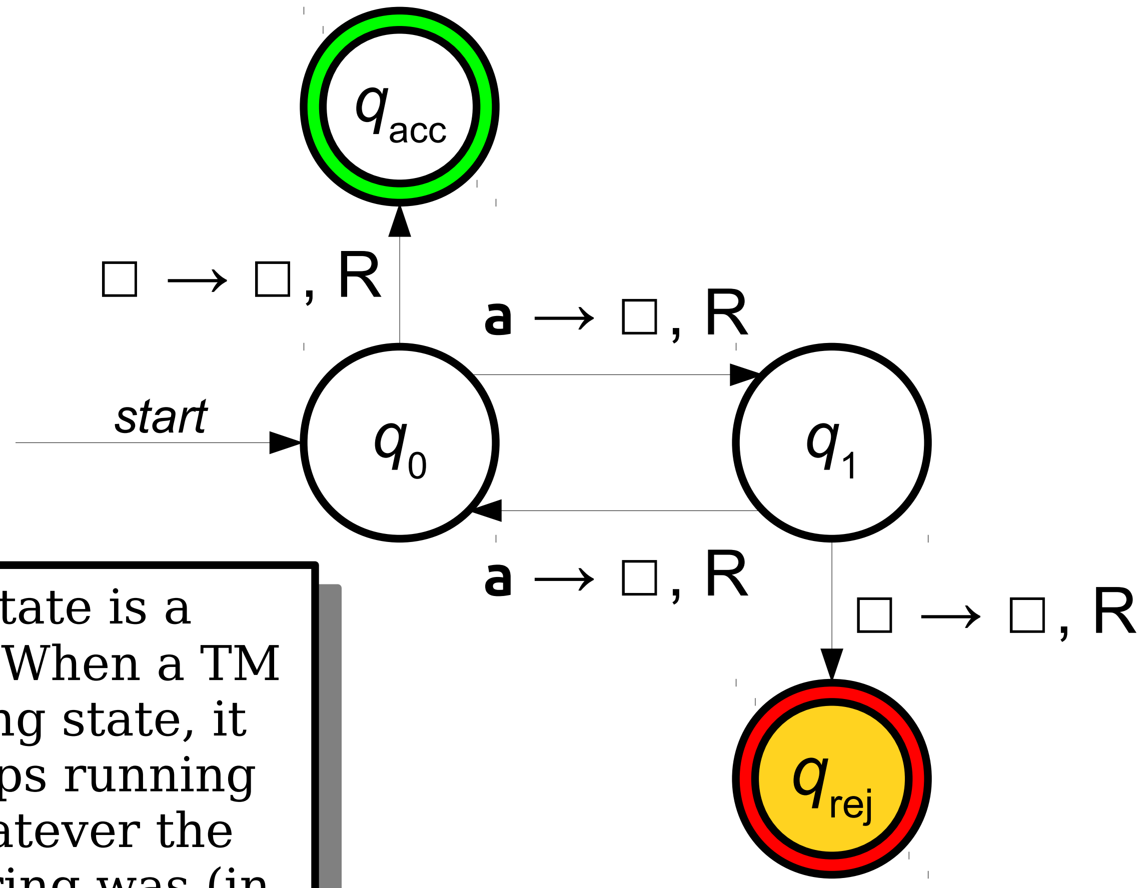
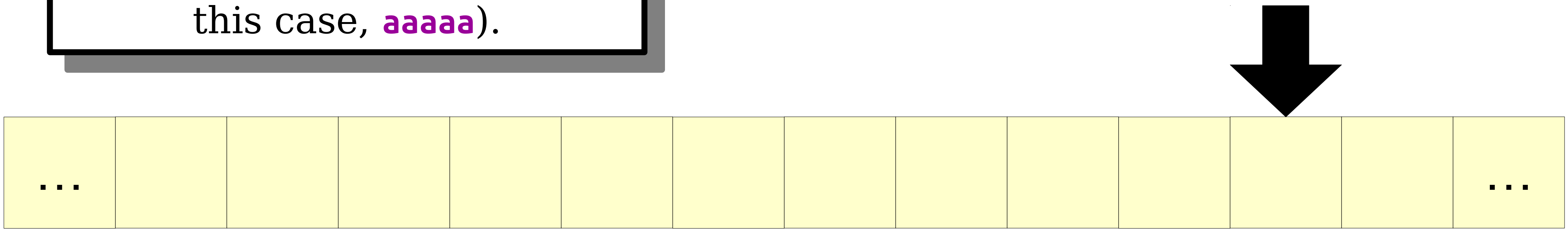


This special state is a ***rejecting state***. When a TM enters a rejecting state, it *immediately* stops running and rejects whatever the original input string was (in this case, **aaaaa**).



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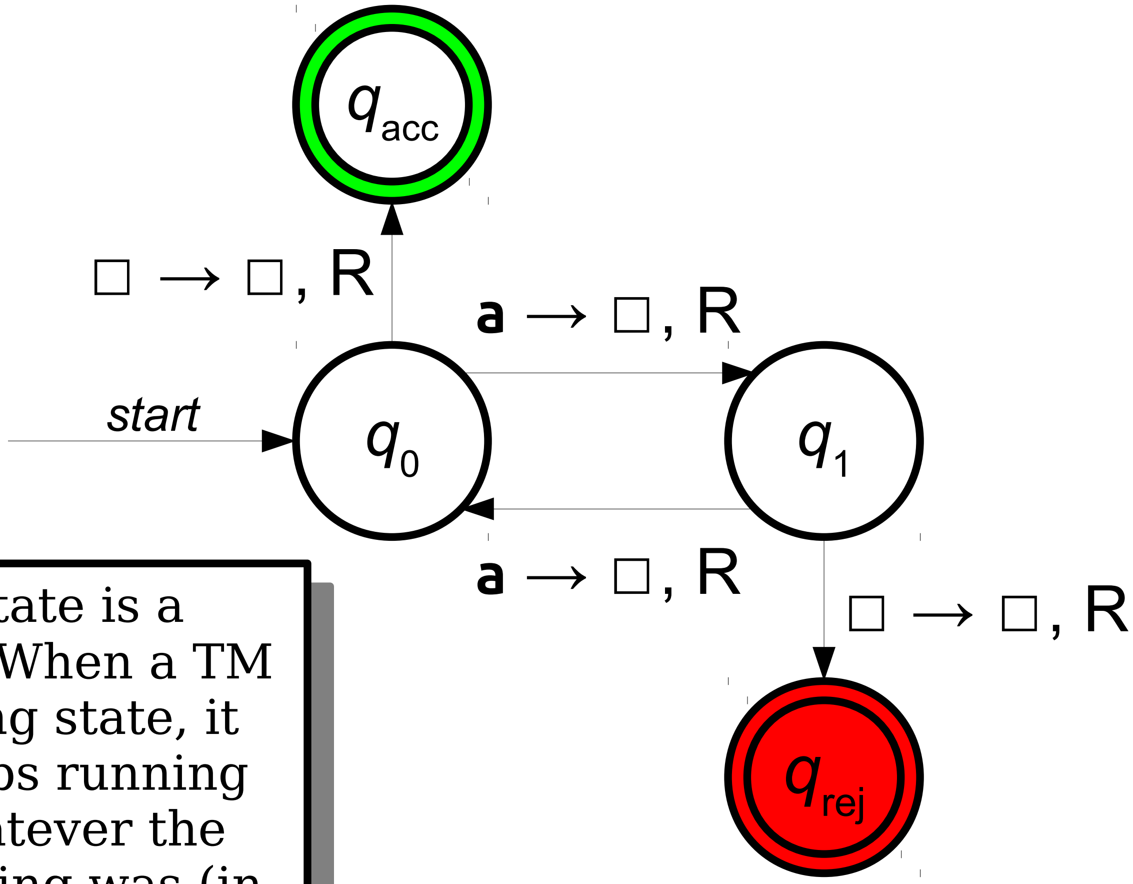

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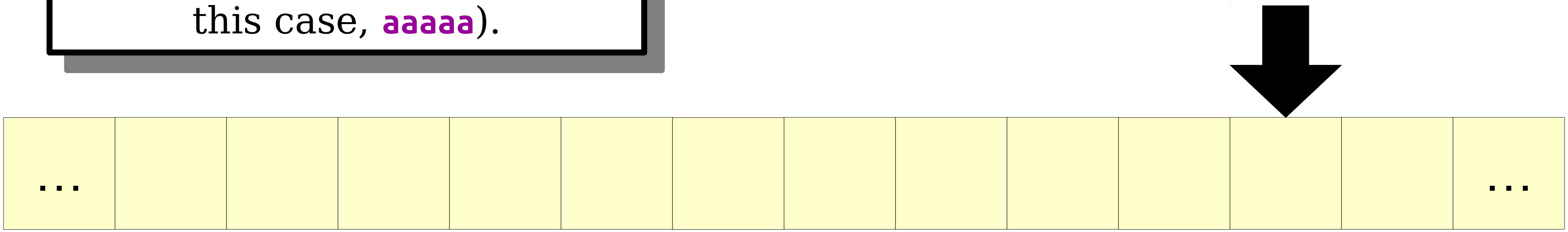

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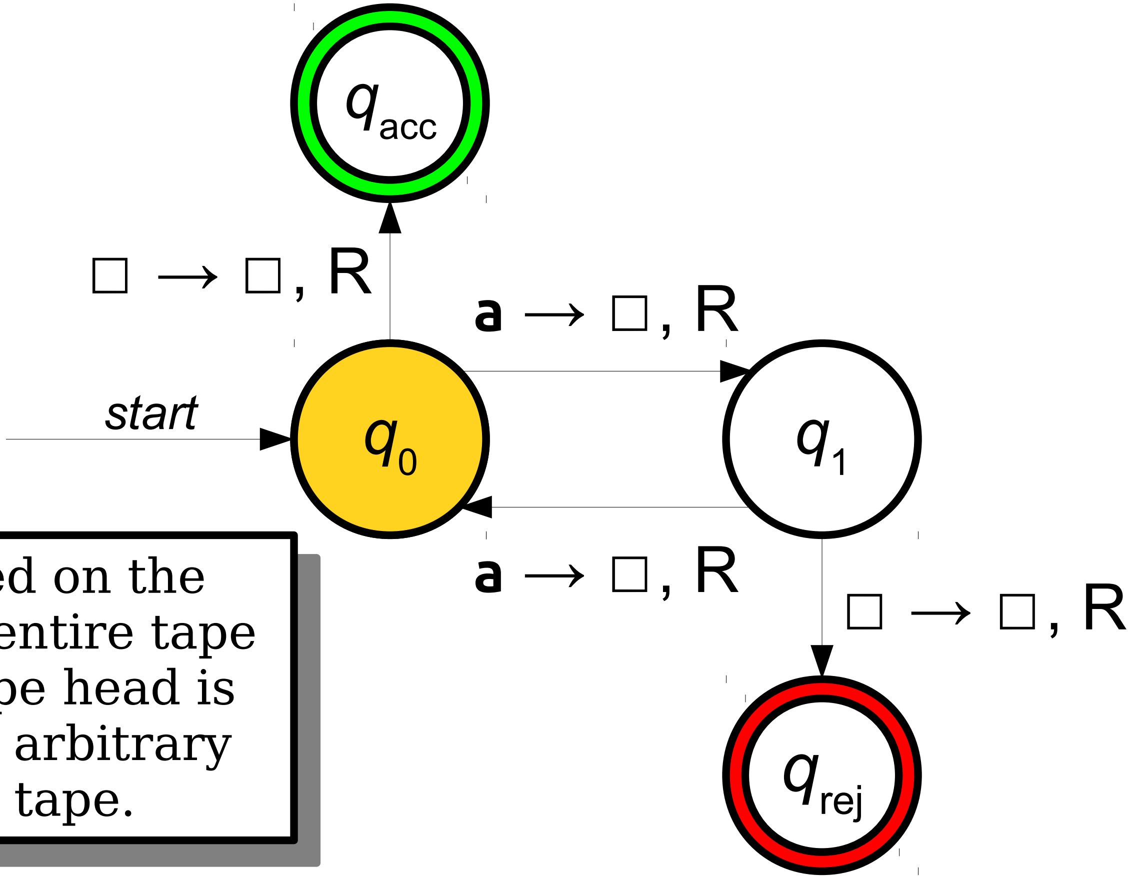
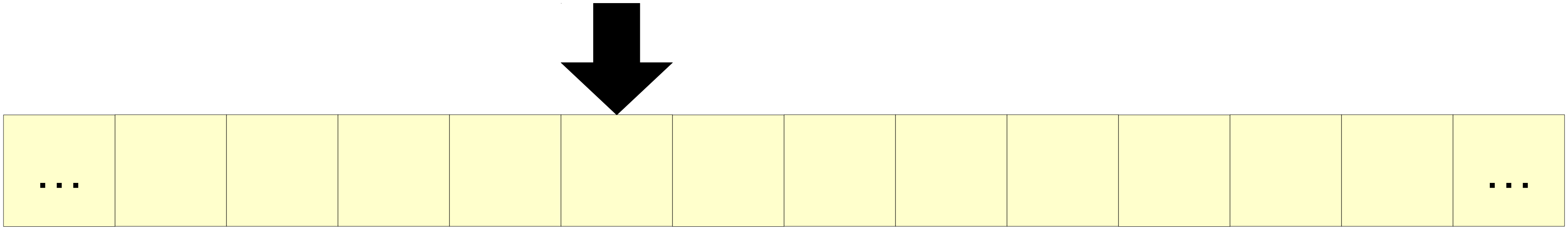


If the TM is started on the empty string ε, the entire tape is blank and the tape head is positioned at some arbitrary location on the tape.



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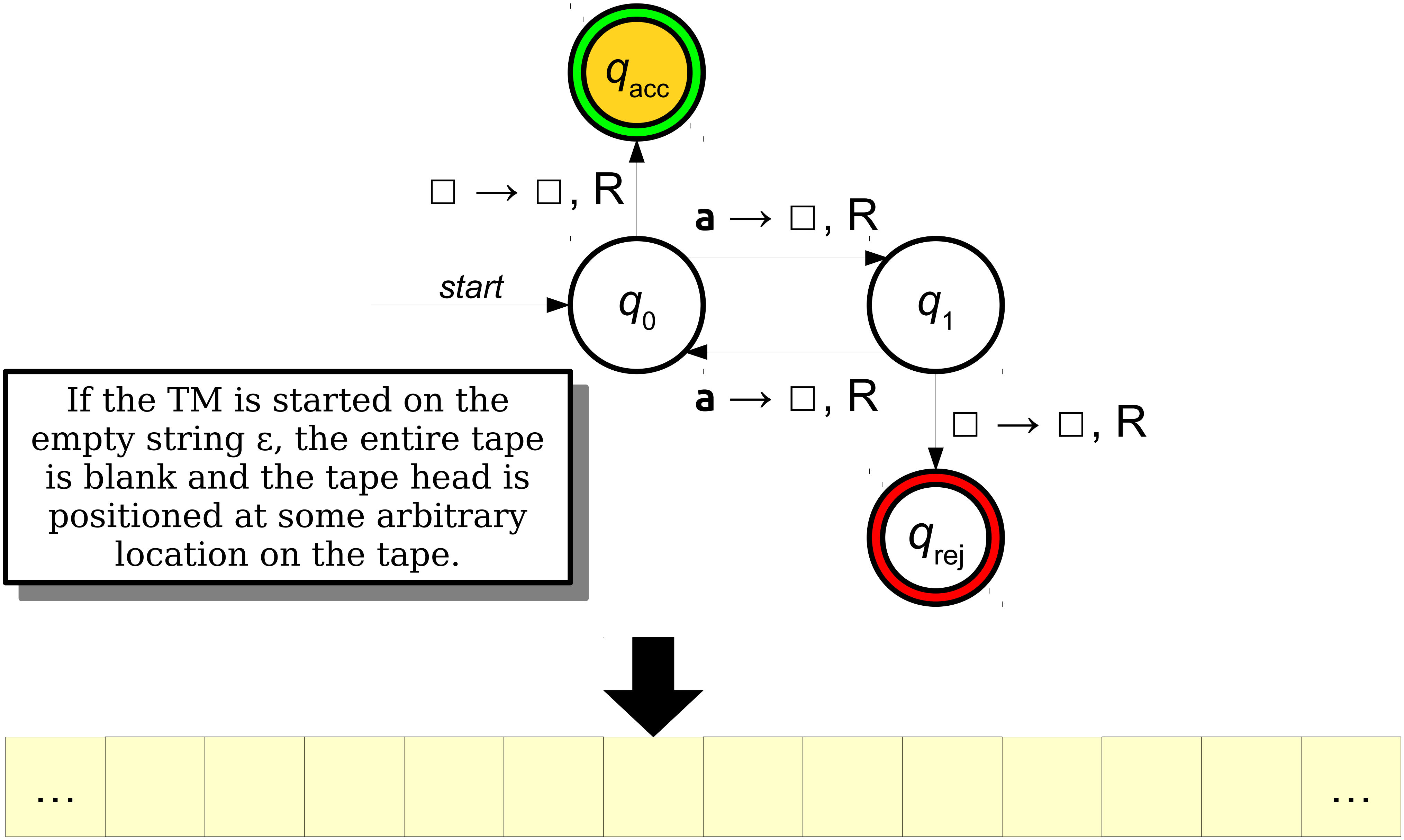

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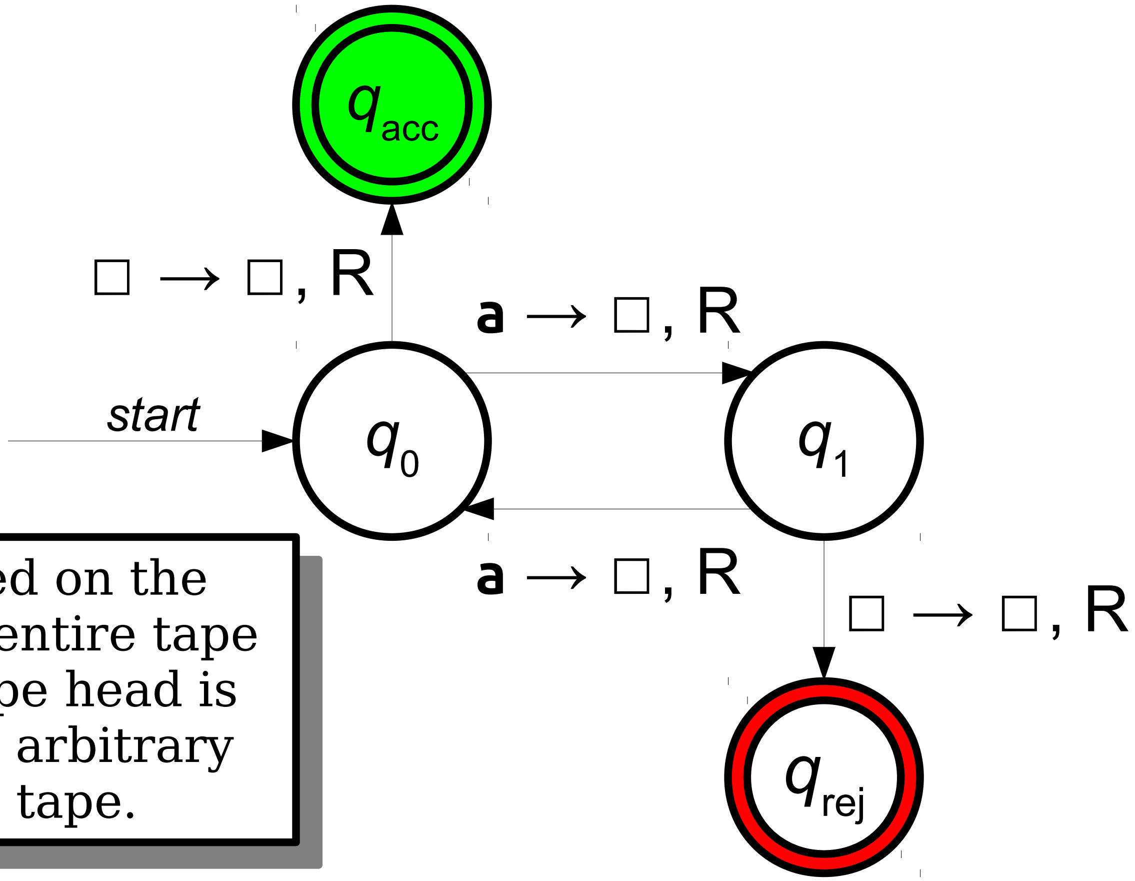

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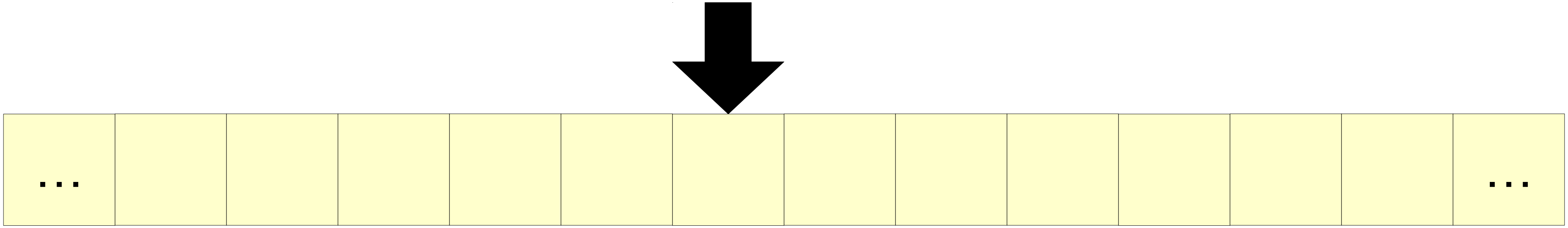


The Turing Machine

- A Turing machine consists of three parts:
  - A ***finite-state control*** that issues commands,
  - an ***infinite tape*** for input and scratch space, and
  - a ***tape head*** that can read and write a single tape cell.
- At each step, the Turing machine
  - writes a symbol to the tape cell under the tape head,
  - changes state, and
  - moves the tape head to the left or to the right.

Input and Tape Alphabets

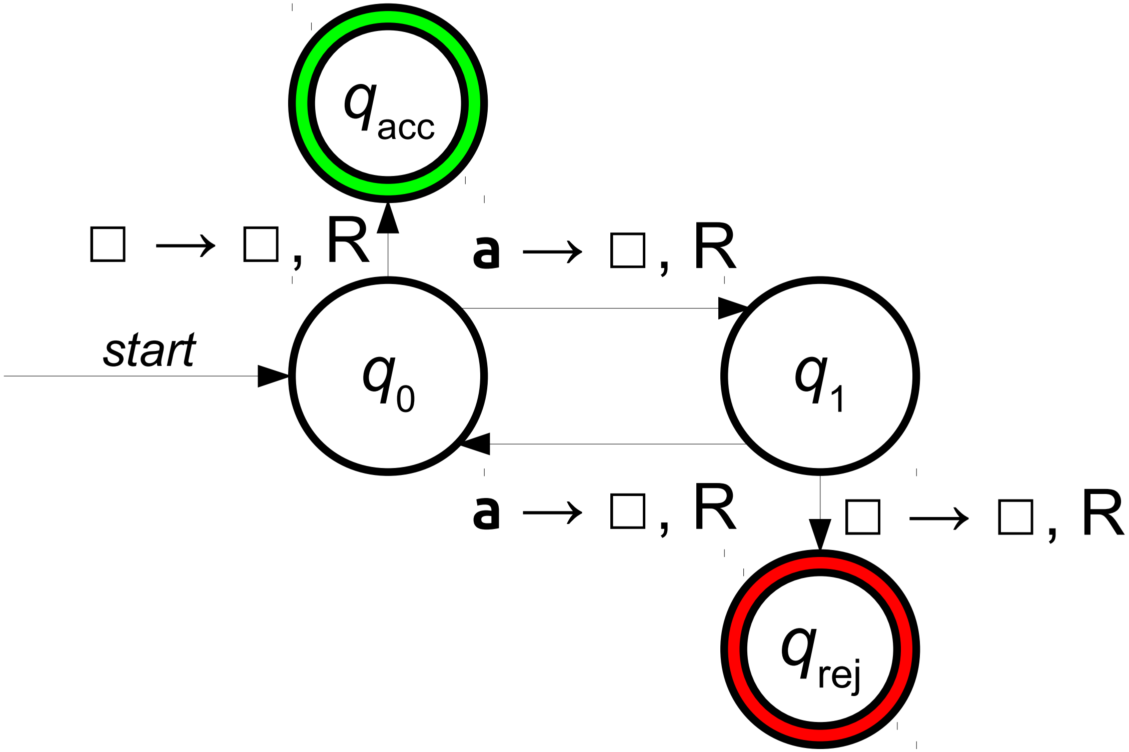
- A Turing machine has two alphabets:
  - An ***input alphabet*** Σ. All input strings are written in the input alphabet.
  - A ***tape alphabet*** Γ, where $\Sigma \subsetneq \Gamma$. The tape alphabet contains all symbols that can be written onto the tape.
- The tape alphabet Γ can contain any number of symbols, but always contains at least one ***blank symbol***, denoted $\Box$. You are guaranteed $\Box \notin \Sigma$.
- At startup, the Turing machine begins with an infinite tape of $\Box$ symbols with the input written at some location. The tape head is positioned at the start of the input.

Accepting and Rejecting States

- Unlike DFAs, Turing machines do not stop processing the input when they finish reading it.
- Turing machines decide when (and if!) they will accept or reject their input.
- Turing machines can enter infinite loops and never accept or reject; more on that later…

Determinism

- Turing machines are ***deterministic***: for every combination of a (non-accepting, non-rejecting) state q and a tape symbol $a \in \Gamma$, there must be exactly one transition defined for that combination of q and a.
- Any transitions that are missing implicitly go straight to a rejecting state. We'll use this later to simplify our designs.



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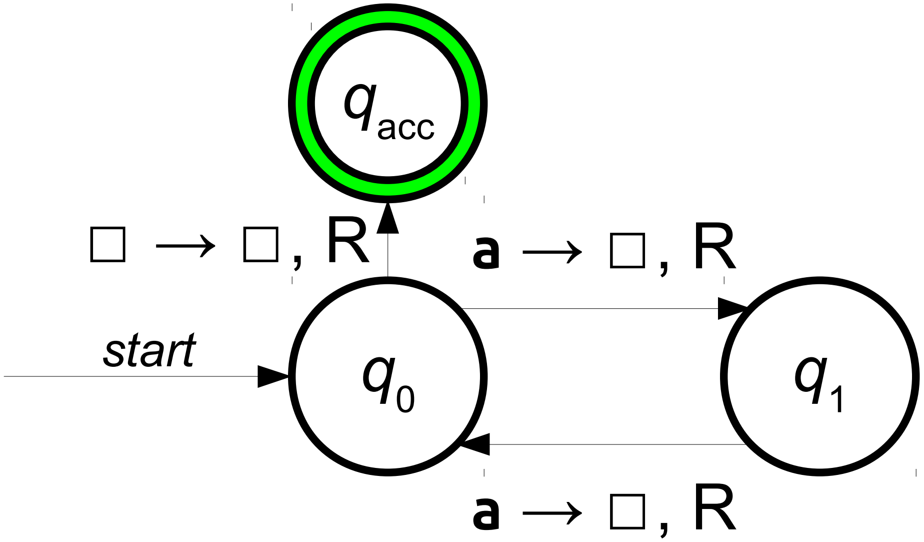
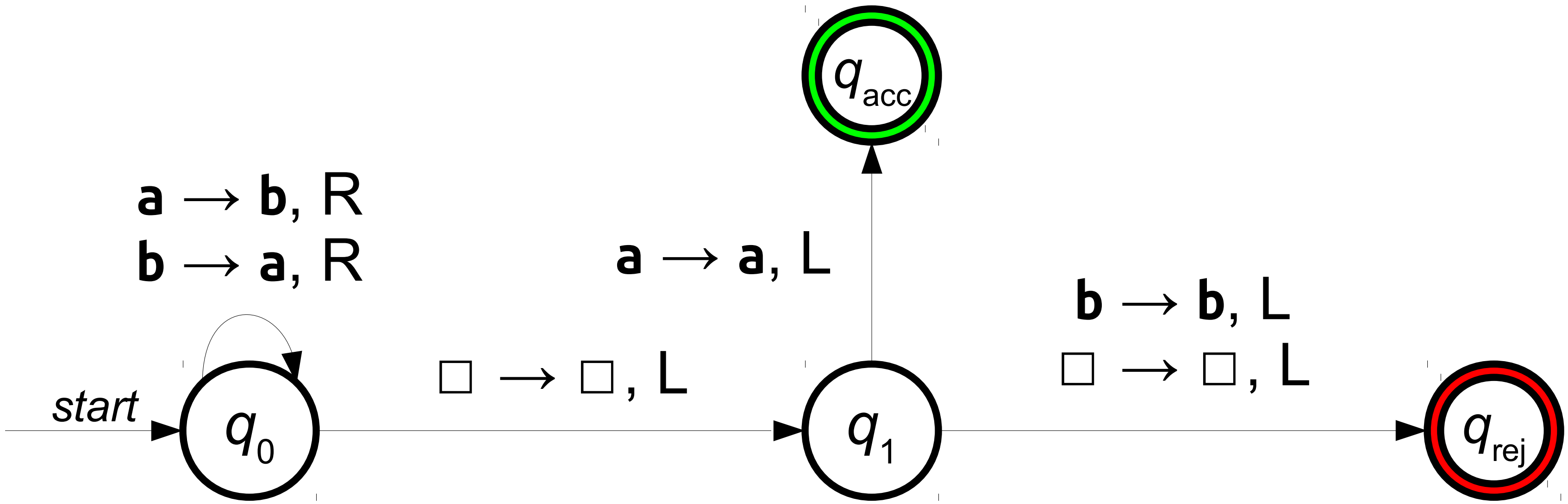


This machine is exactly the same as the previous one.



Run the TM shown above on the input string **bba**.
What will the tape look like when the TM finishes running?

***A*.**

					↓	
…		b	b	a		…

***B*.**

			↓			
…		a	a	b		…

***C*.**

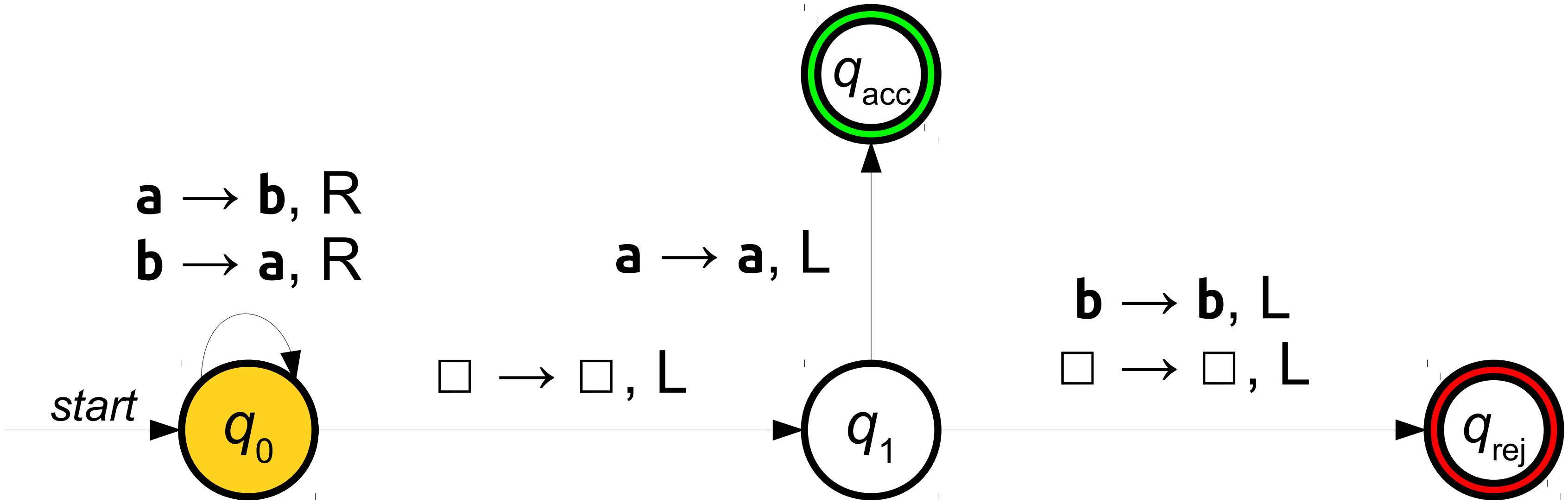
			↓			
…		b	b	a		…

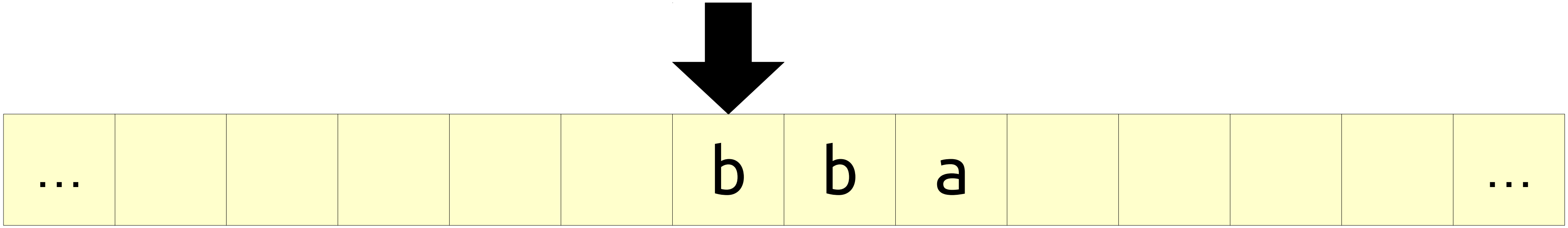
***D*.**

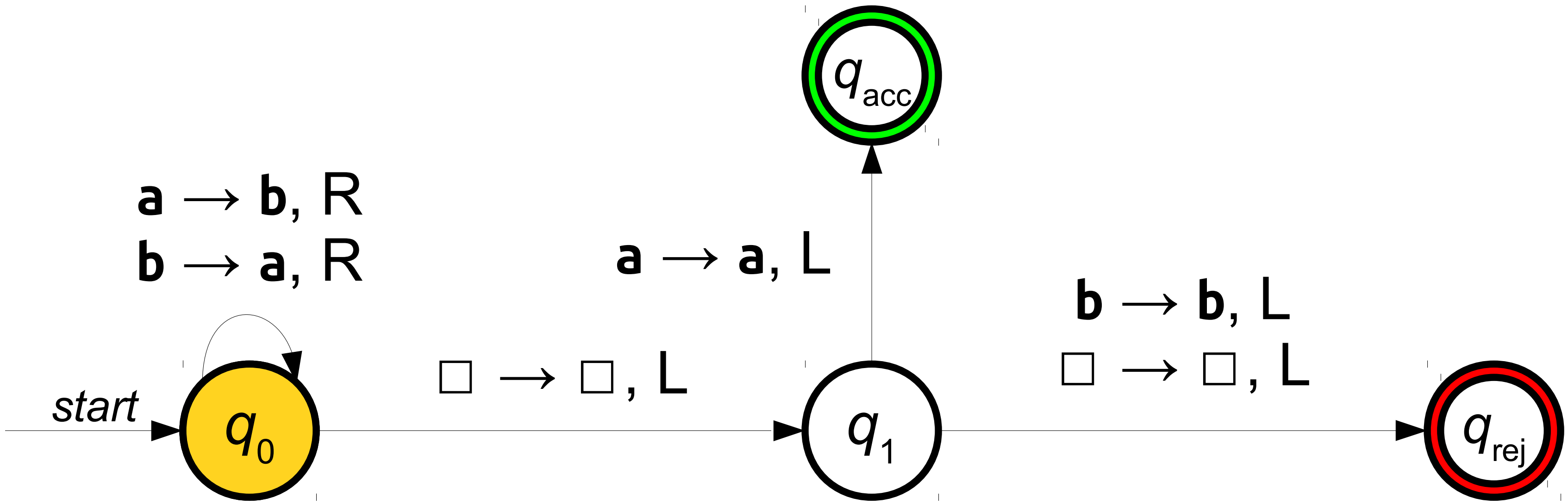
					↓	
…		a	a	b		…

***E*.** None of these, or two or more of these.

Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then ***A***, ***B***, ***C***, ***D***, or ***E***.


a → b, R
b → a, R
start
q_0
□ → □, L
q_1
a → a, L
q_{acc}
b → b, L
□ → □, L
q_{rej}


…
b
b
a
…


a → b, R
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start
q_0
□ → □, L
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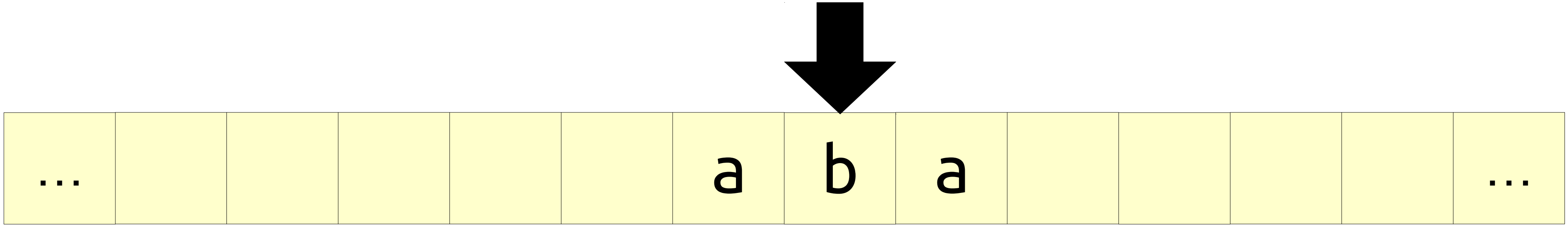
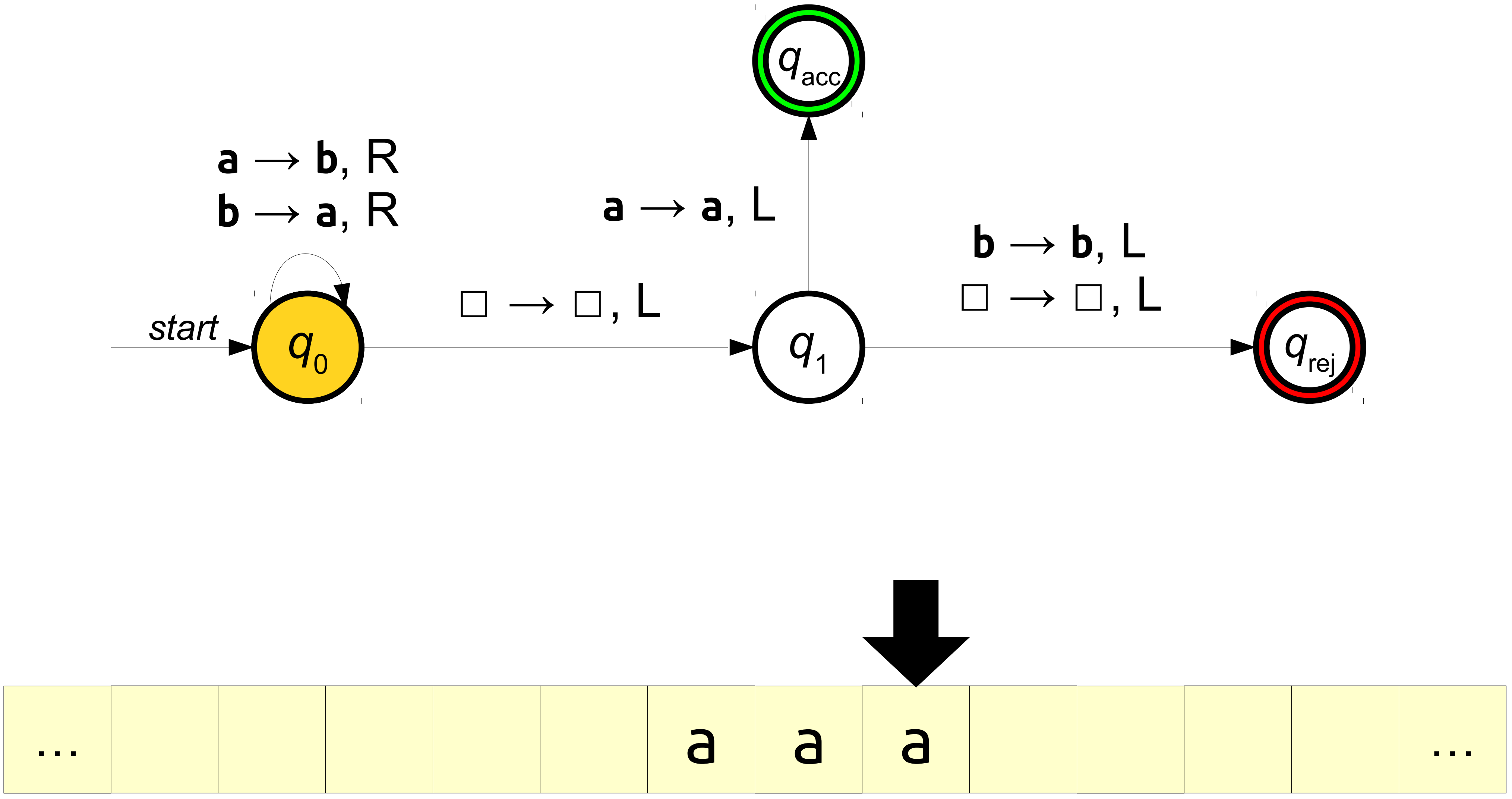
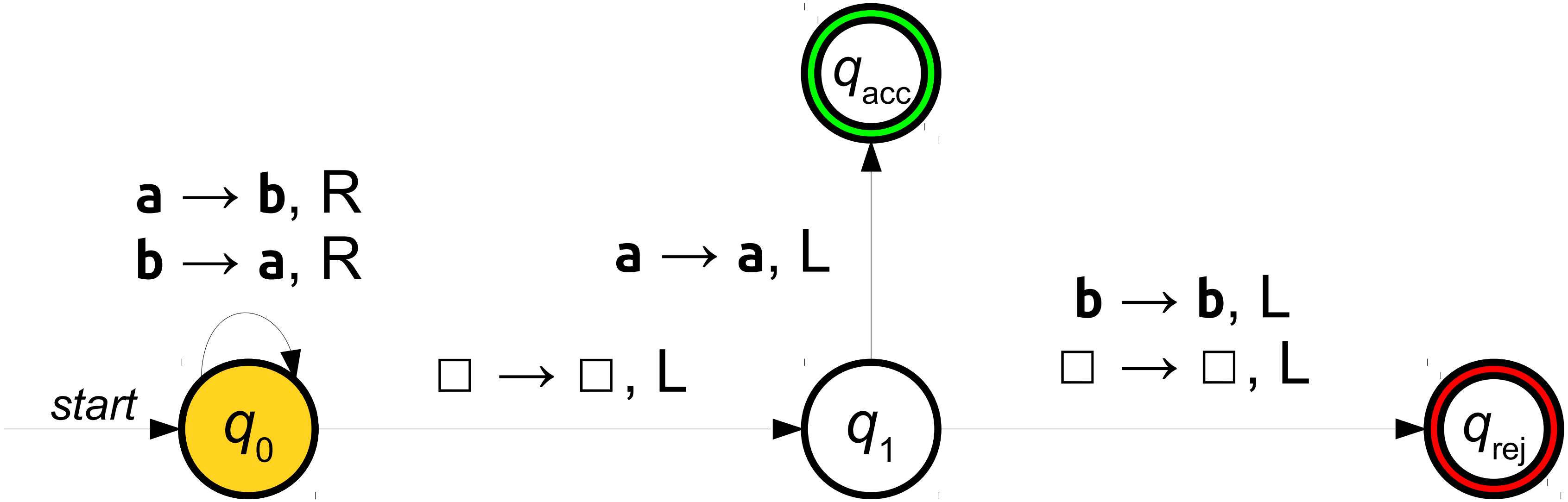
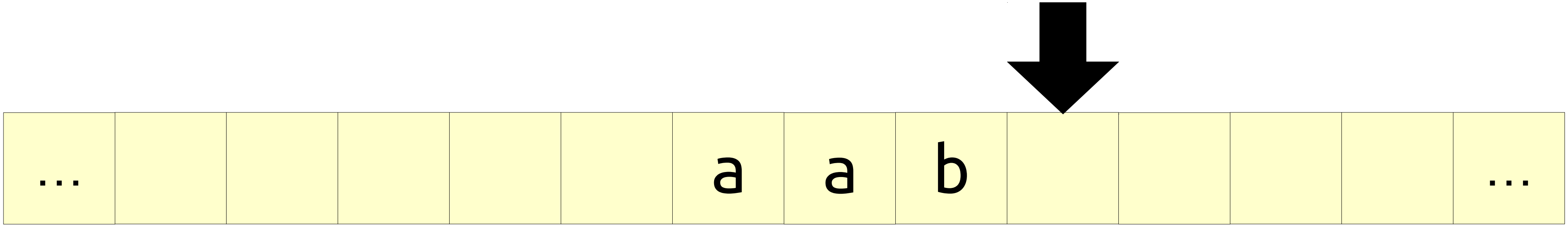
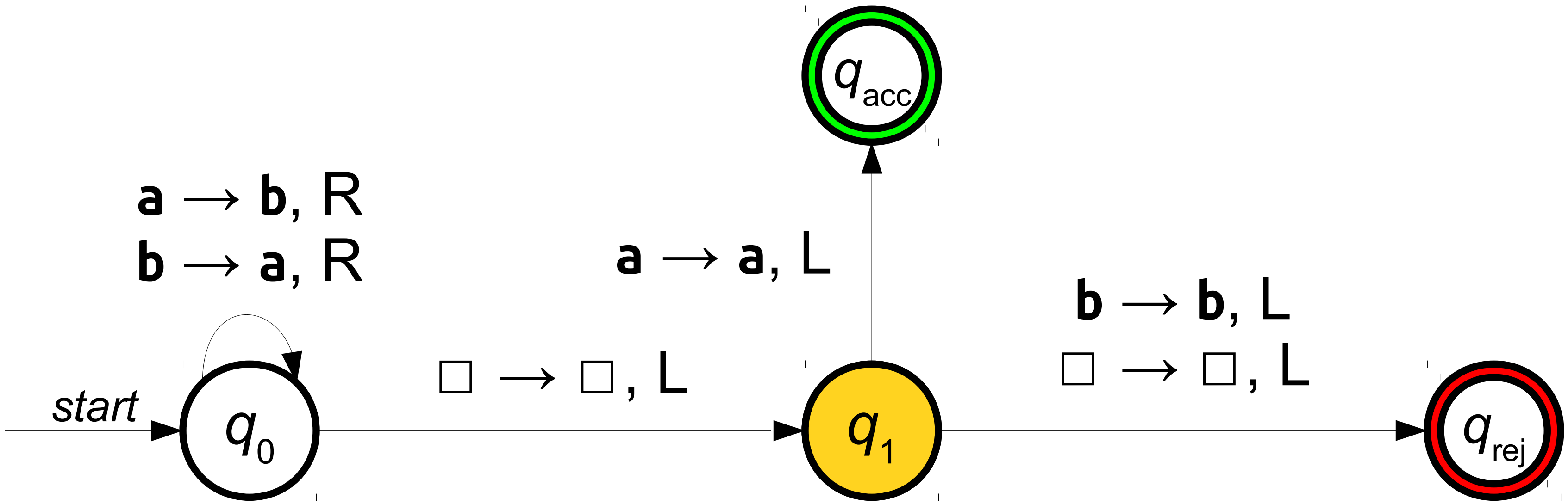
a → a, L
q_{acc}
b → b, L
□ → □, L
q_{rej}


…
a
b
a
…


a → b, R
b → a, R
start
q_0
□ → □, L
q_1
a → a, L
q_{acc}
b → b, L
□ → □, L
q_{rej}
…
a
a
a
…


a → b, R
b → a, R
start
q_0
□ → □, L
a → a, L
q_{acc}
q_1
b → b, L
□ → □, L
q_{rej}


…
a
a
b
…



a → b, R
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start
q_0
□ → □, L
q_1
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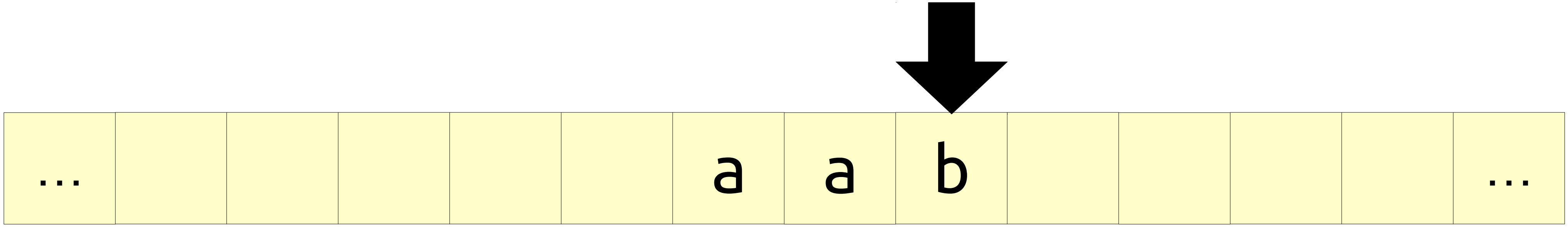
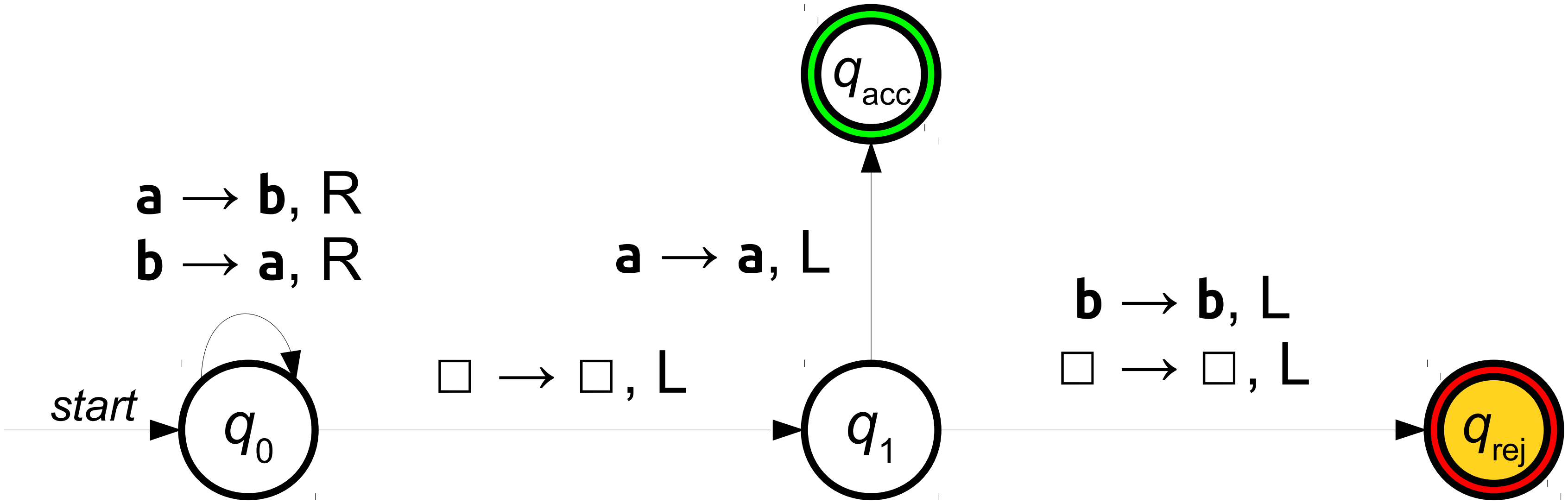
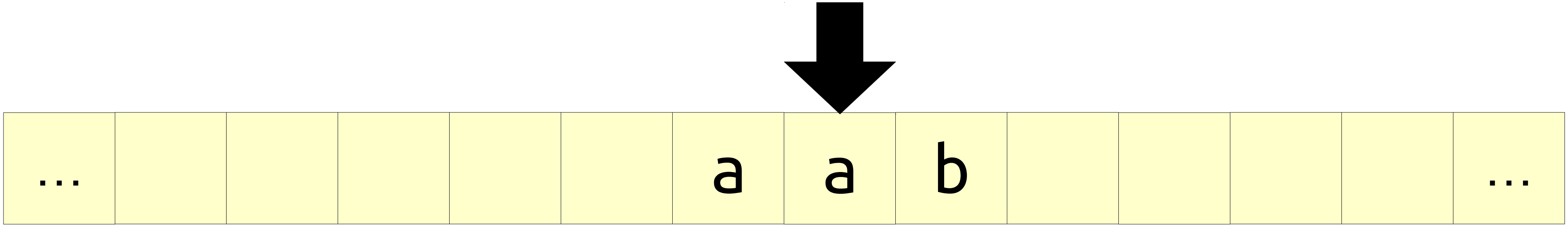
q_{acc}
b → b, L
□ → □, L
q_{rej}



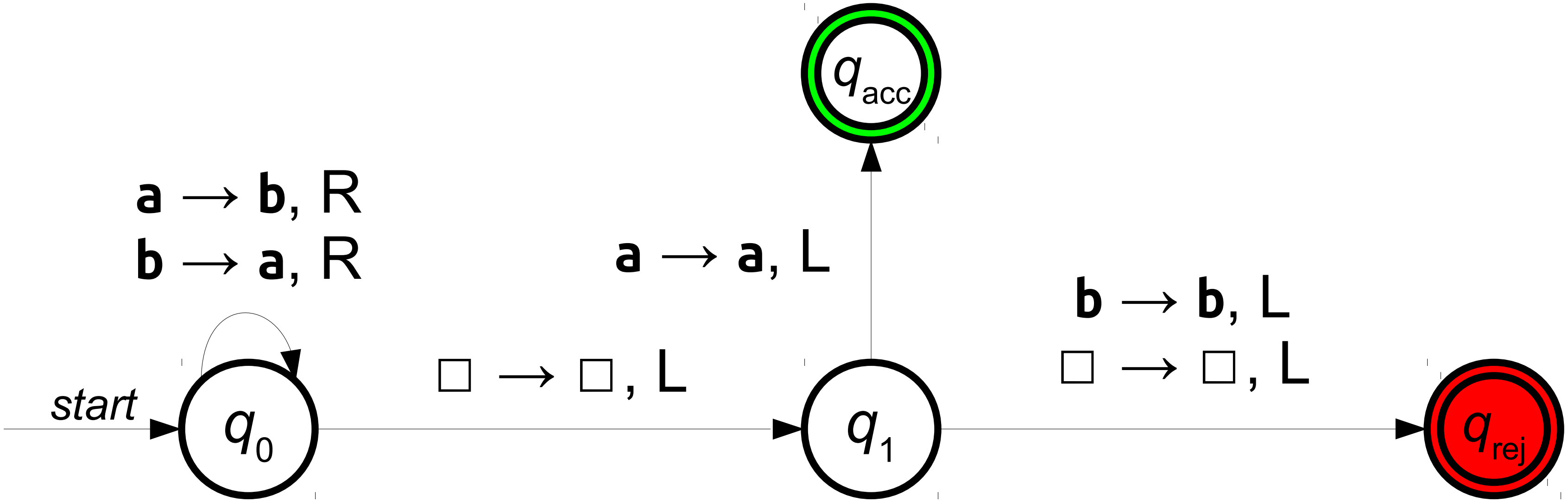
…
a
a
b
…

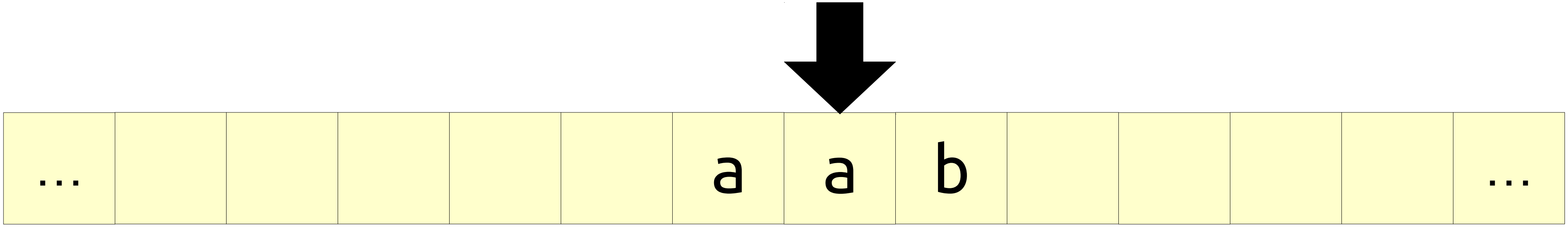


a → b, R
b → a, R
start
q_0
□ → □, L
a → a, L
q_{acc}
q_1
b → b, L
□ → □, L
q_{rej}

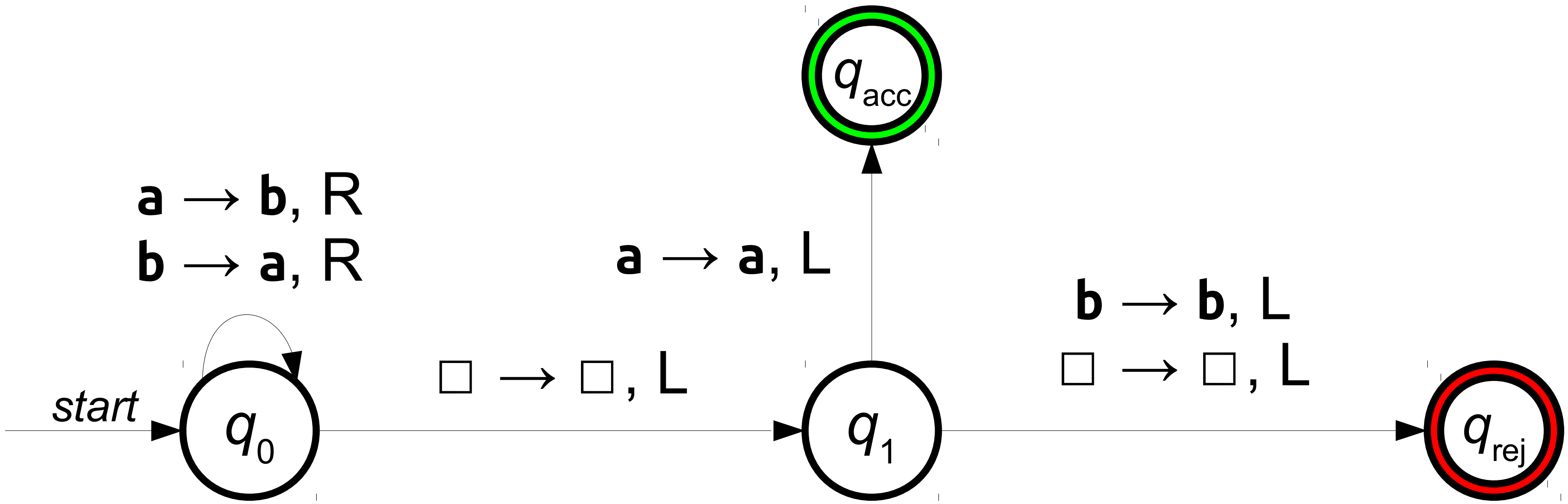


…
a
a
b
…



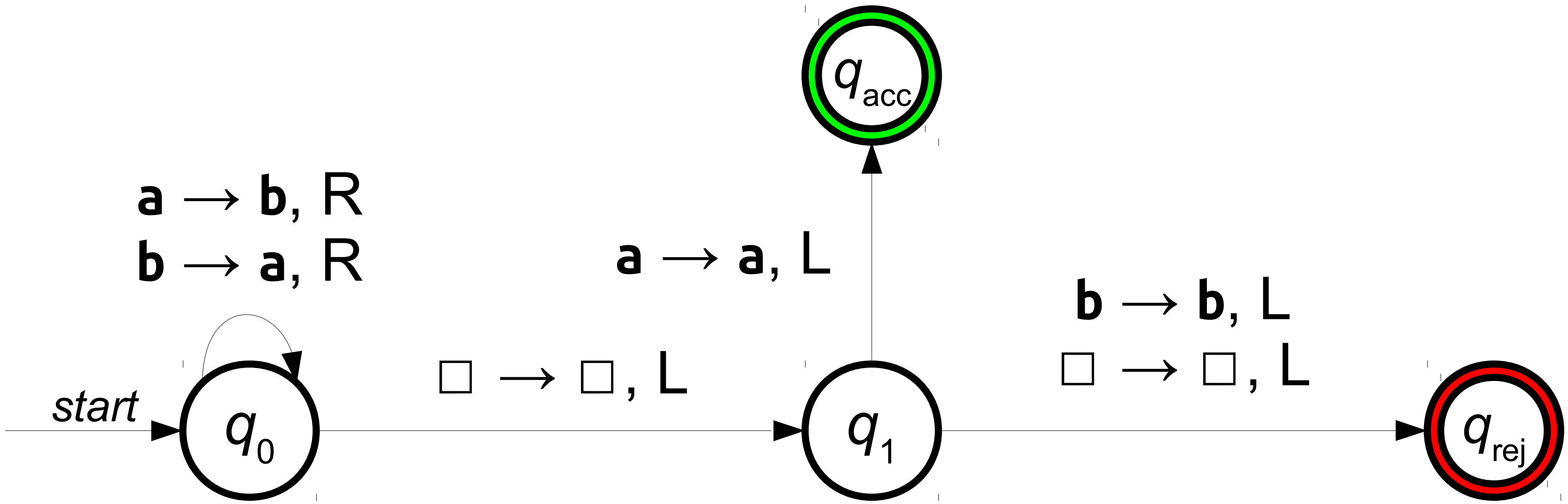


…						a	a	b					…



If M is a Turing machine with input alphabet Σ, then the ***language of M***, denoted $\mathcal{L}(M)$, is the set

$$\mathcal{L}(M) = \{ w \in \Sigma^* \mid M \text{ accepts } w \}$$



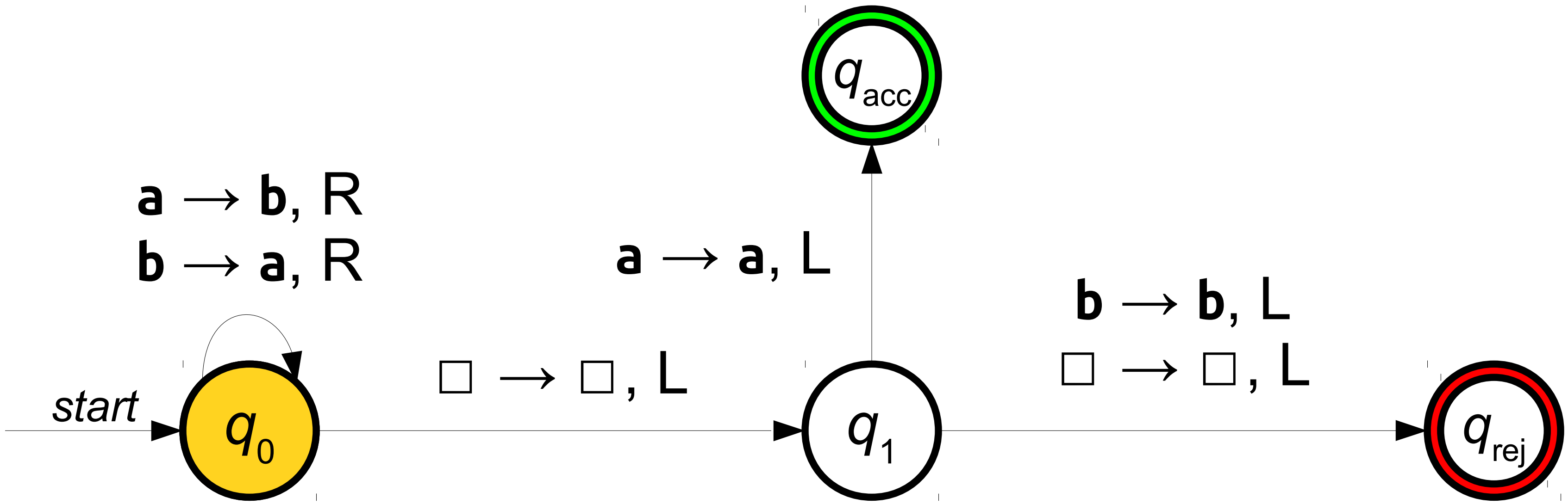
If M is a Turing machine with input alphabet Σ, then the ***language of M***, denoted $\mathscr{L}(M)$, is the set

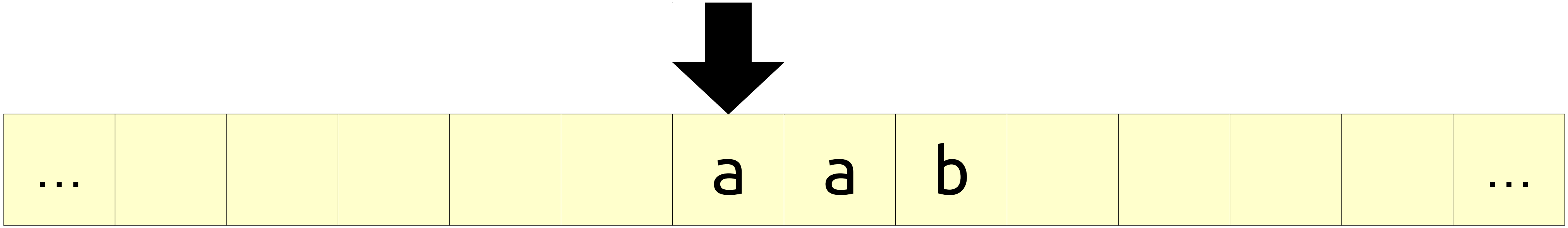
$$\mathscr{L}(M) = \{\ w \in \Sigma^* \mid M \text{ accepts } w\ \}$$

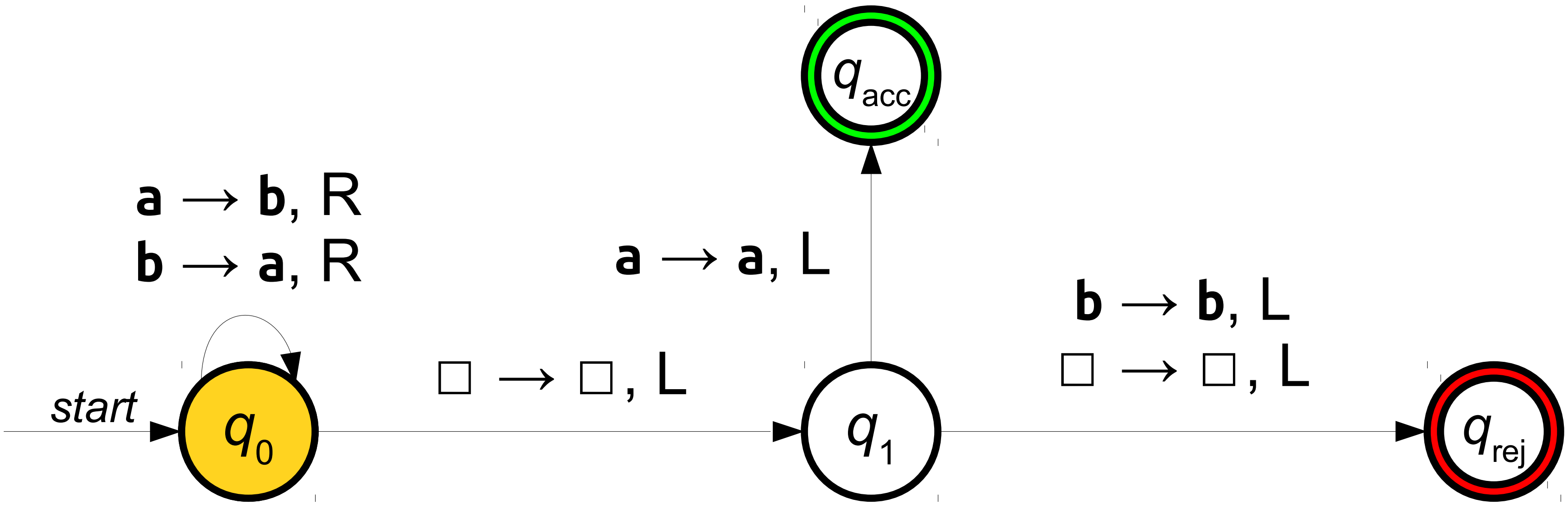
Let M be the above TM, and assume its input alphabet is $\{$a, b$\}$. What is $\mathscr{L}(M)$?

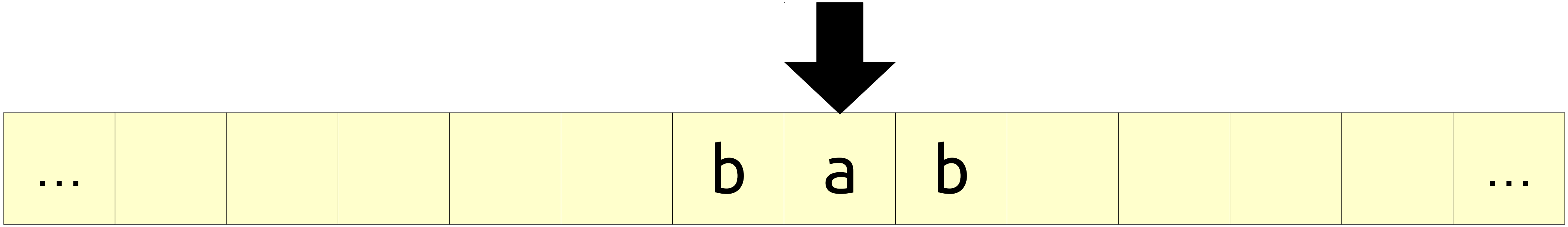
A. $\{\ w \in \{$a, b$\}^* \mid w$ ends in a $\}$
B. $\{\ w \in \{$a, b$\}^* \mid w$ ends in b $\}$
C. Ø
D. None of these, or two or more of these.

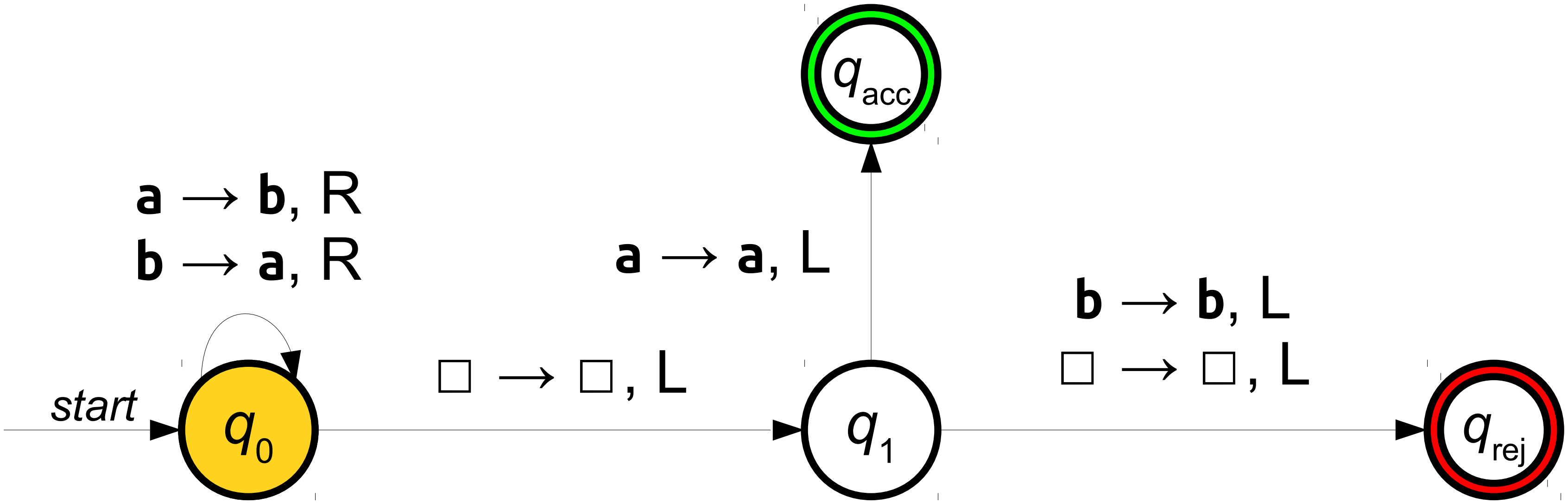
Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then ***A***, ***B***, ***C***, or ***D***.

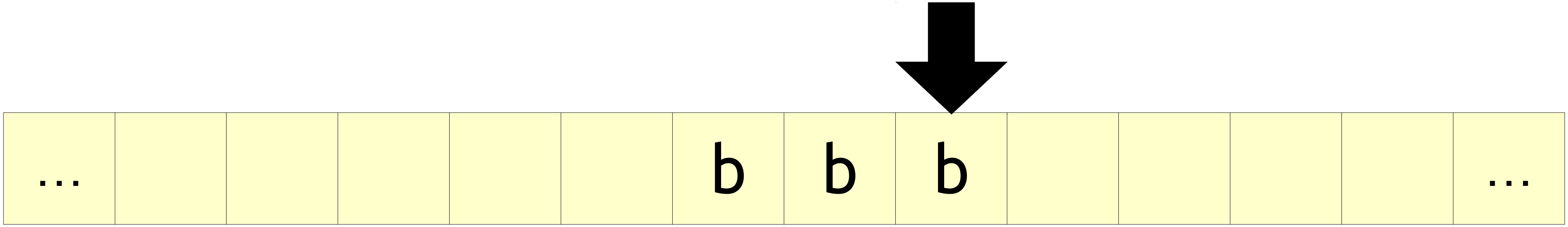

a → b, R
b → a, R
start
q0
□ → □, L
q1
a → a, L
qacc
b → b, L
□ → □, L
qrej

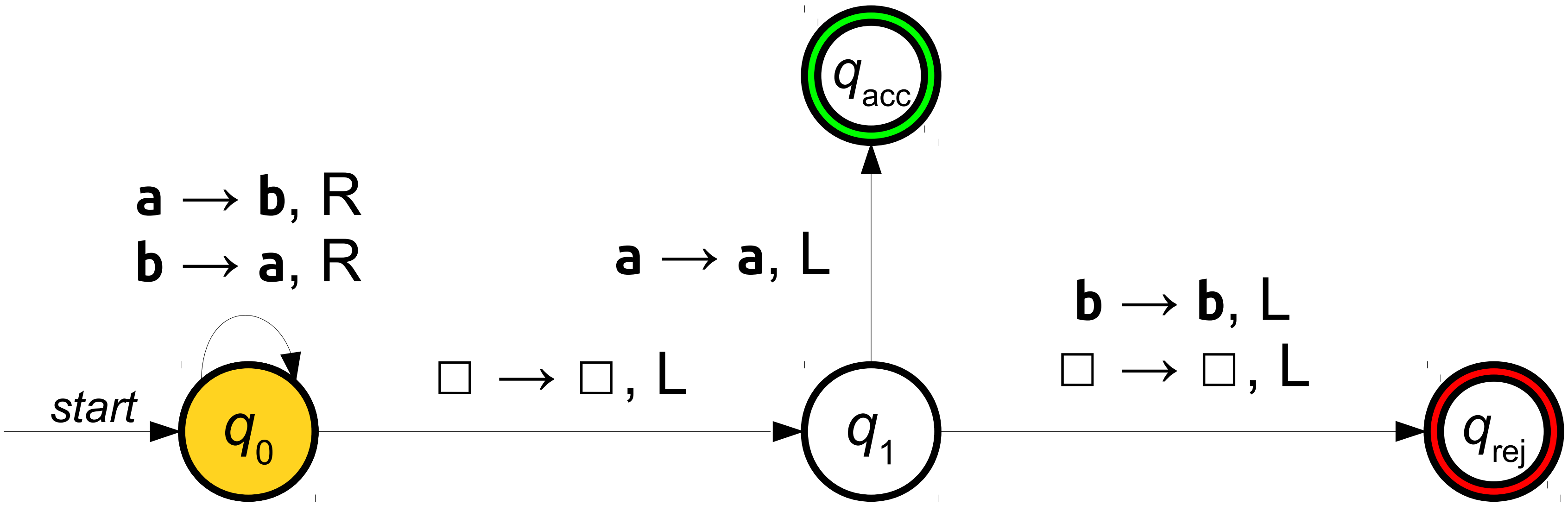

…
a
a
b
…

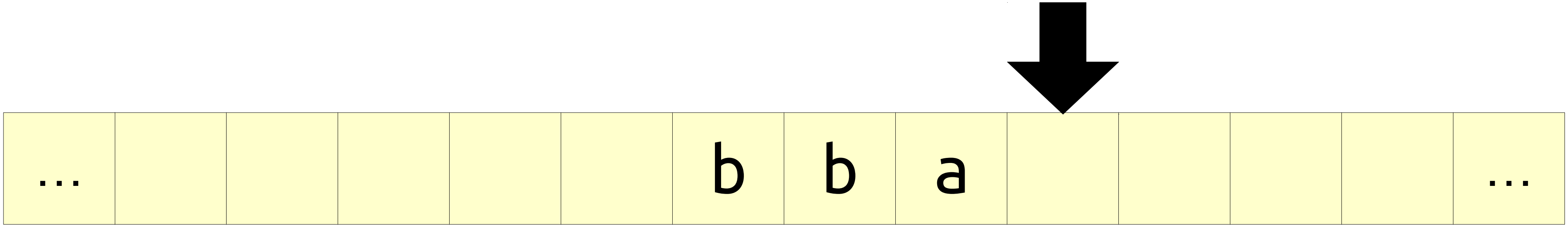

a → b, R
b → a, R
start
q_0
□ → □, L
a → a, L
q_{acc}
q_1
b → b, L
□ → □, L
q_{rej}

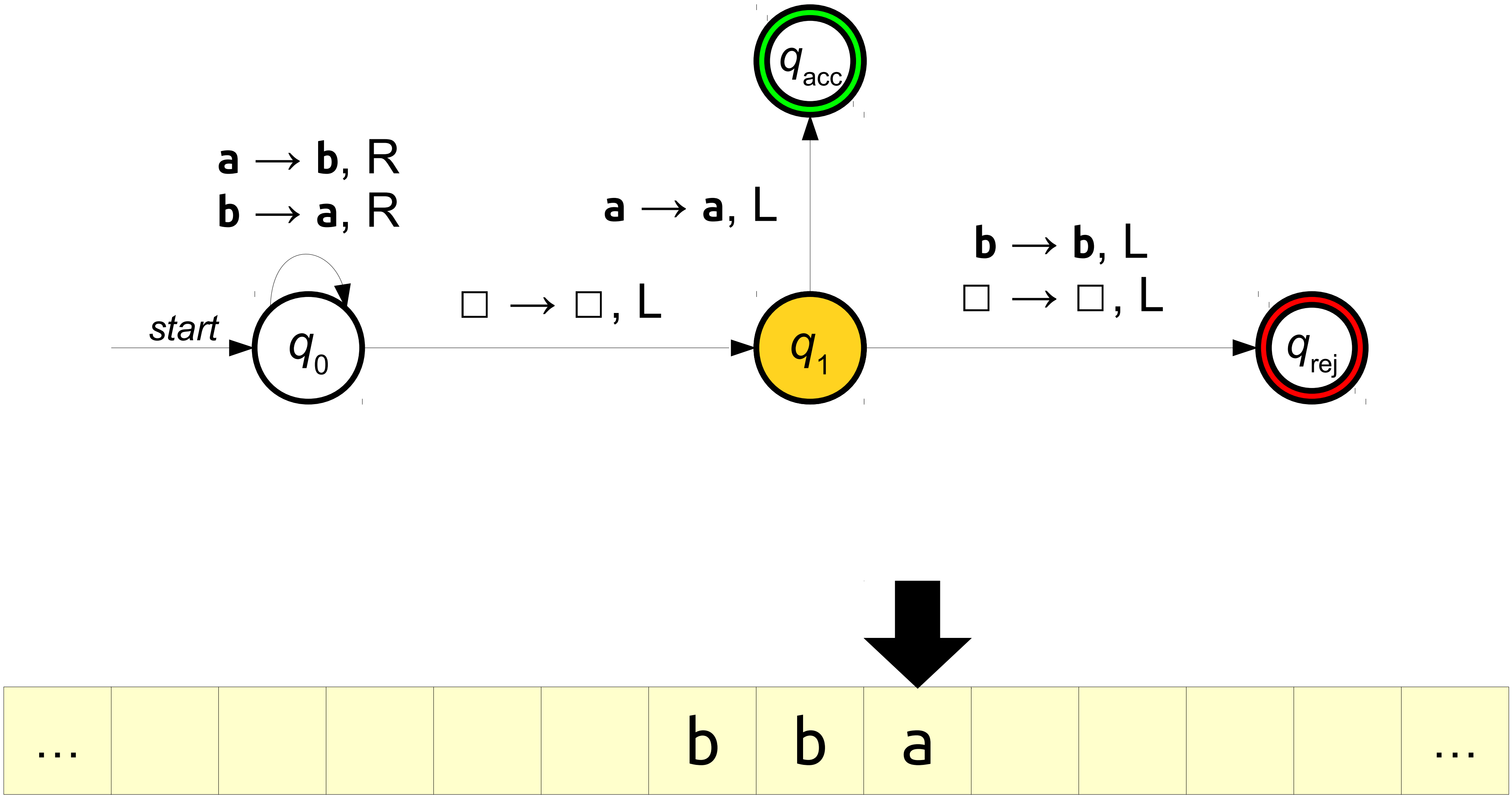

…
b
a
b
…

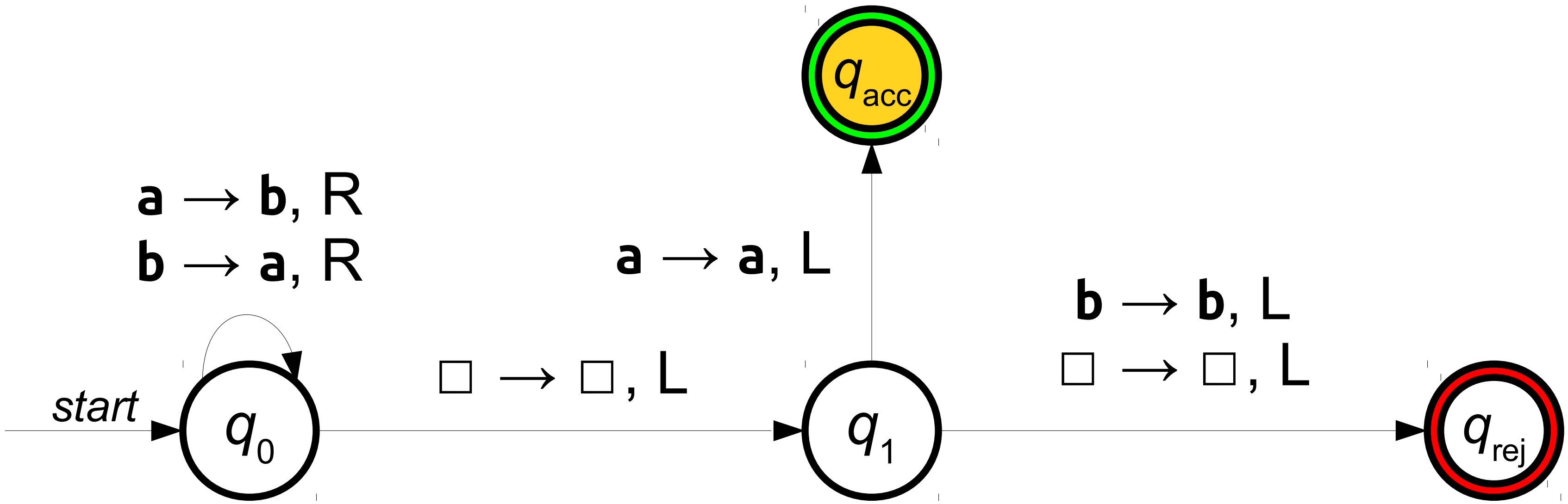

start
q_0
a → b, R
b → a, R
□ → □, L
q_1
a → a, L
q_{acc}
b → b, L
□ → □, L
q_{rej}


…
b
b
b
…


a → b, R
b → a, R
start
q0
□ → □, L
q1
a → a, L
qacc
b → b, L
□ → □, L
qrej


…
b
b
a
…


a → b, R
b → a, R
start
q_0
□ → □, L
a → a, L
q_{acc}
q_1
b → b, L
□ → □, L
q_{rej}
…
b
b
a
…


a → b, R
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q_0
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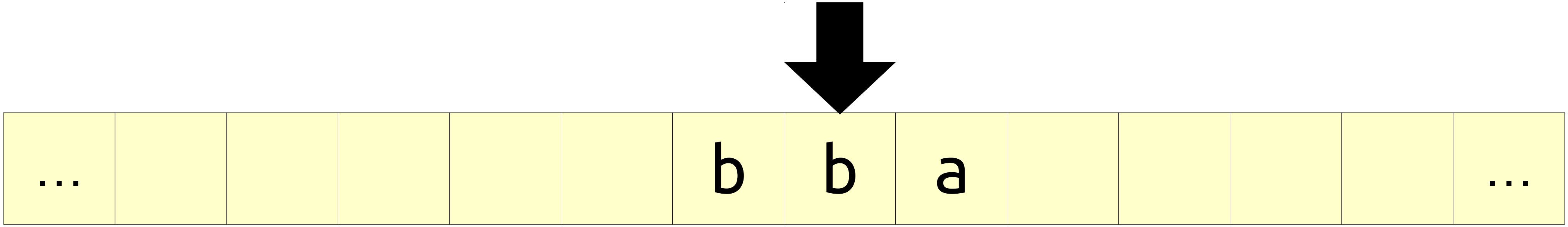
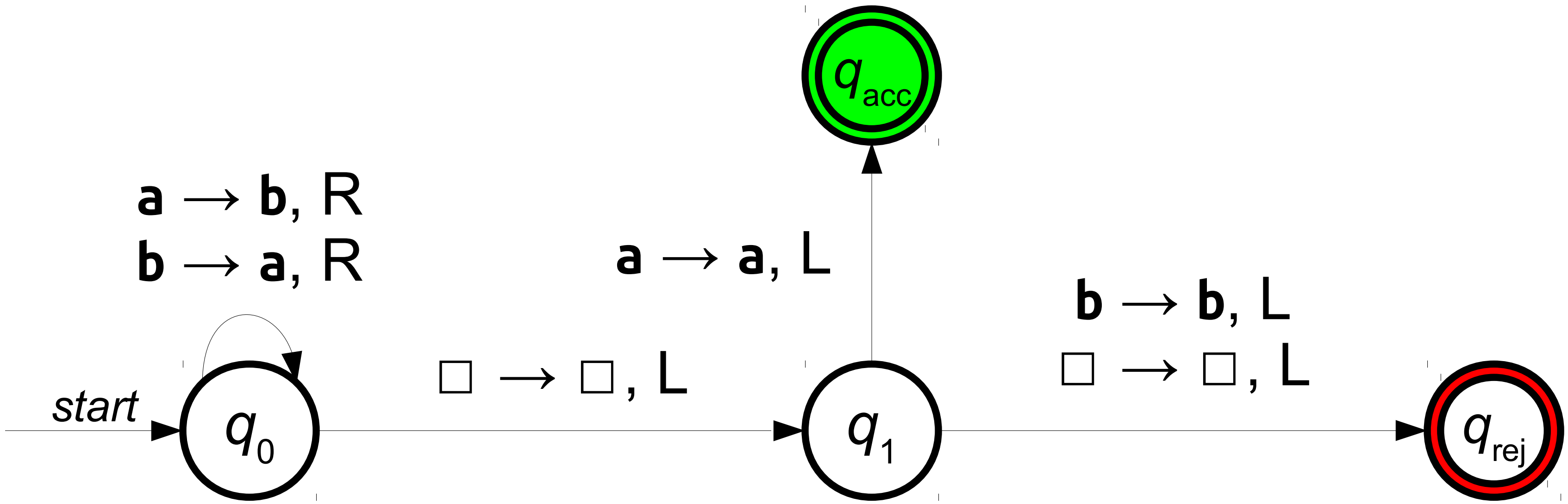
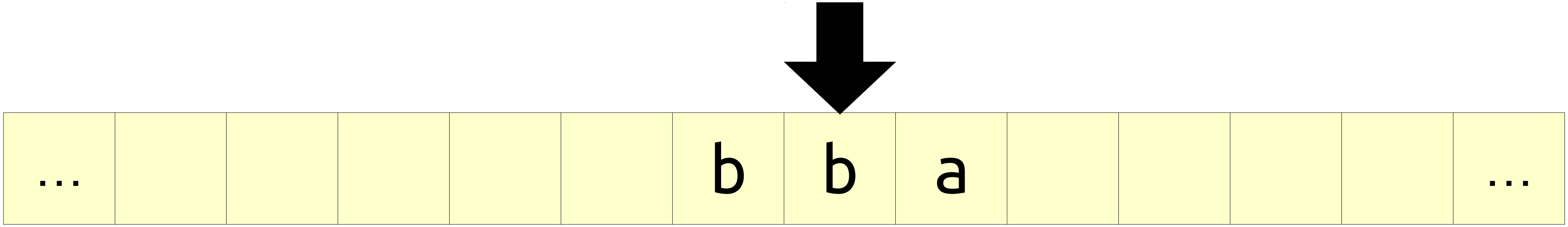
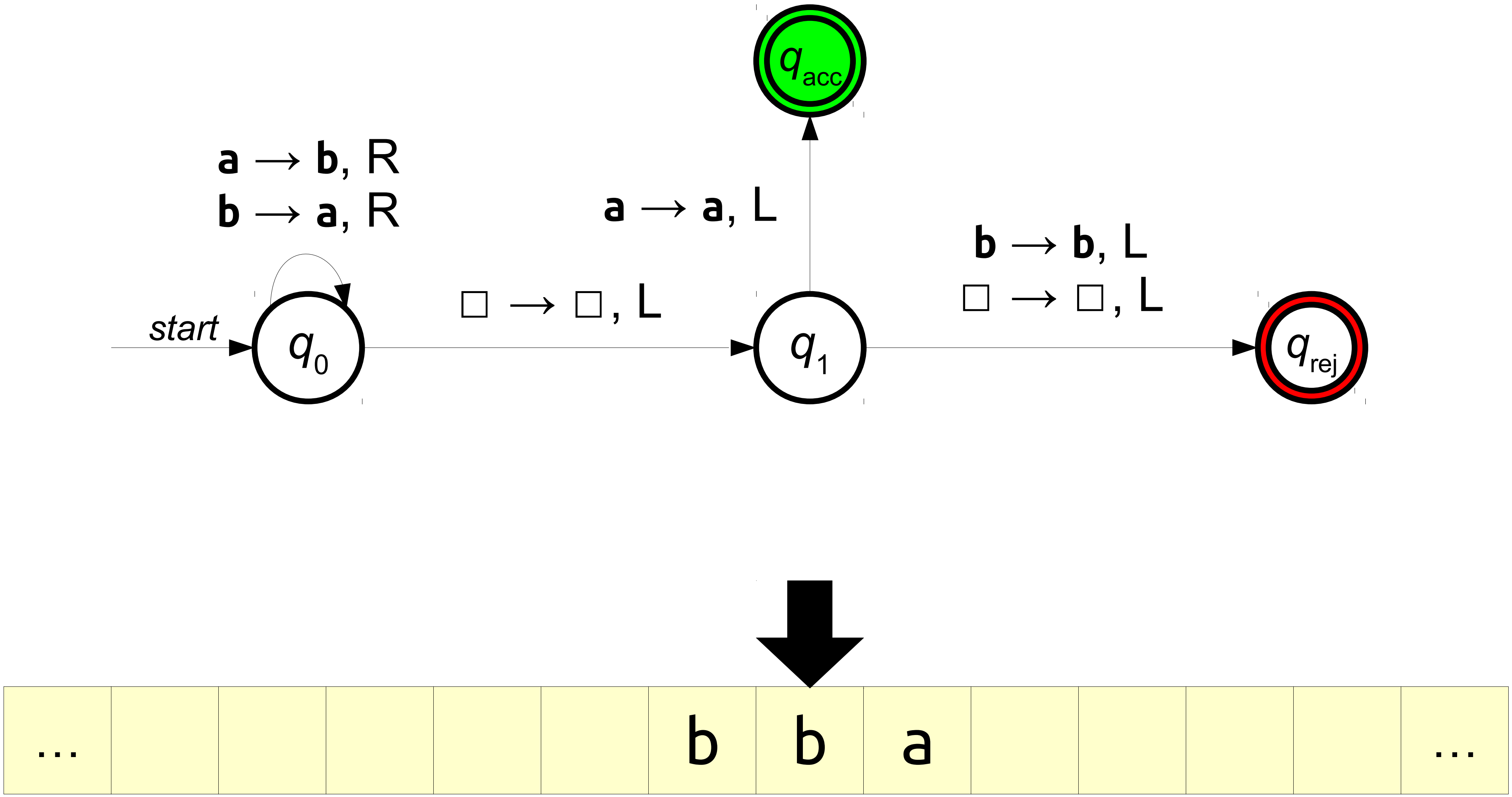
a → a, L
q_{acc}
q_1
b → b, L
□ → □, L
q_{rej}


…
b
b
a
…


a → b, R
b → a, R
start
q_0
□ → □, L
q_1
a → a, L
q_{acc}
b → b, L
□ → □, L
q_{rej}


…
b
b
a
…



Although the tape ends with **bba** written on it, the original input string was **aab**. This shows that the TM accepts **aab**, not **bba**.

So $\mathscr{L}(M) = \{\, w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ ends in } \mathtt{b} \,\}$

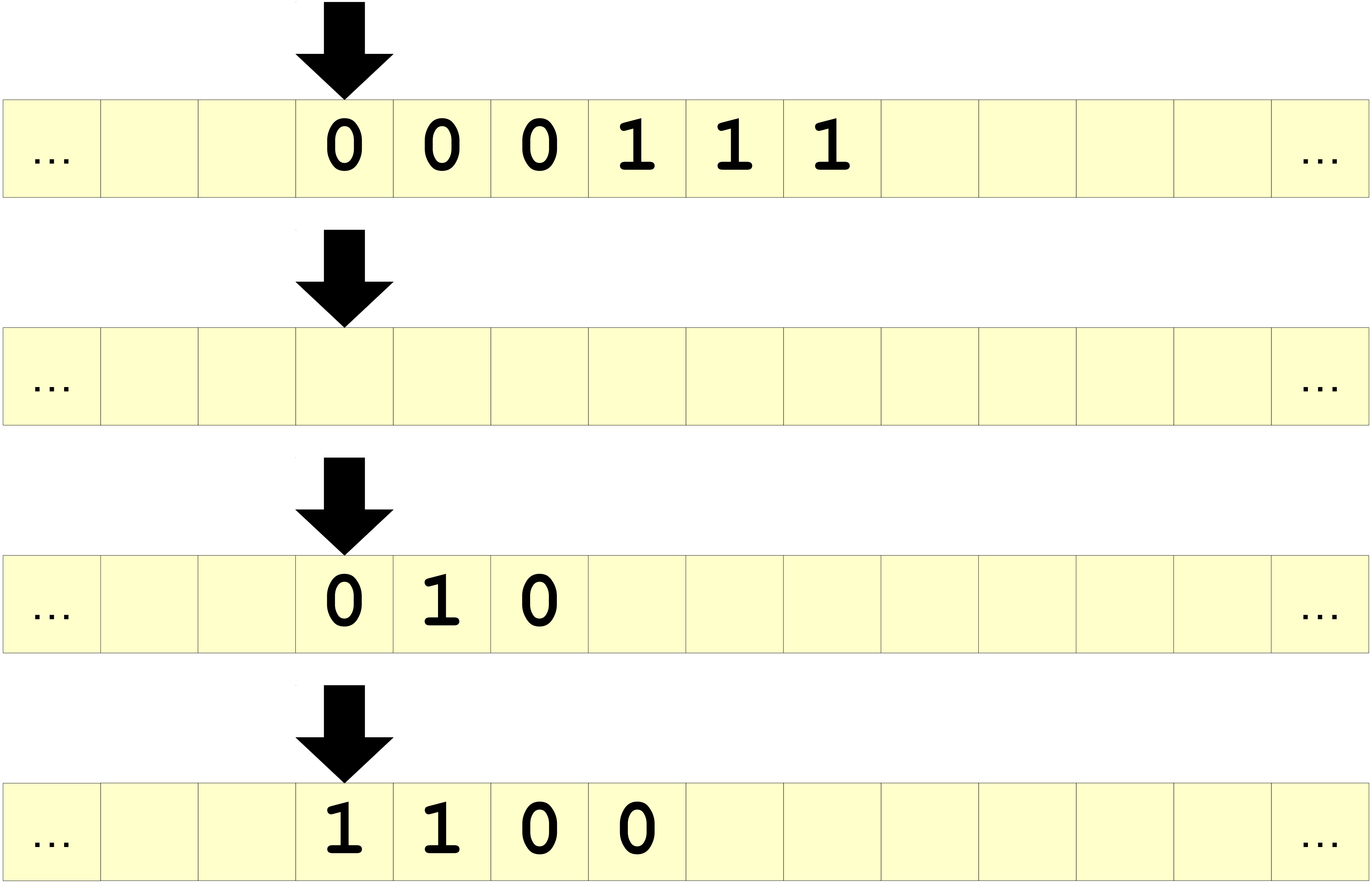
Designing Turing Machines

- Despite their simplicity, Turing machines are very powerful computing devices.
- Today's lecture explores how to design Turing machines for various languages.

Designing Turing Machines

- Let $\Sigma = \{\texttt{0}, \texttt{1}\}$ and consider the language $L = \{\texttt{0}^n\texttt{1}^n \mid n \in \mathbb{N}\}$.
- We know that L is context-free.
- How might we build a Turing machine for it?

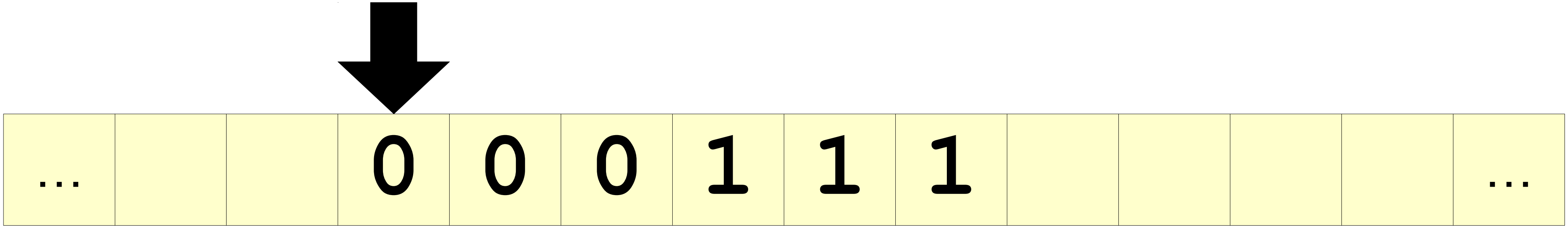
$$L = \{ \mathtt{0}^n \mathtt{1}^n \mid n \in \mathbb{N} \}$$



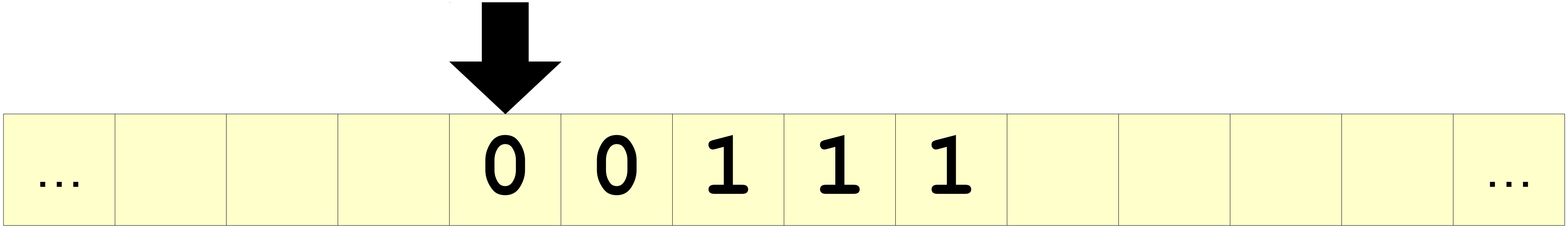
A Recursive Approach

- The string ε is in L.
- The string `0`w`1` is in L iff w is in L.
- Any string starting with `1` is not in L.
- Any string ending with `0` is not in L.

A Sketch of the TM



A Sketch of the TM



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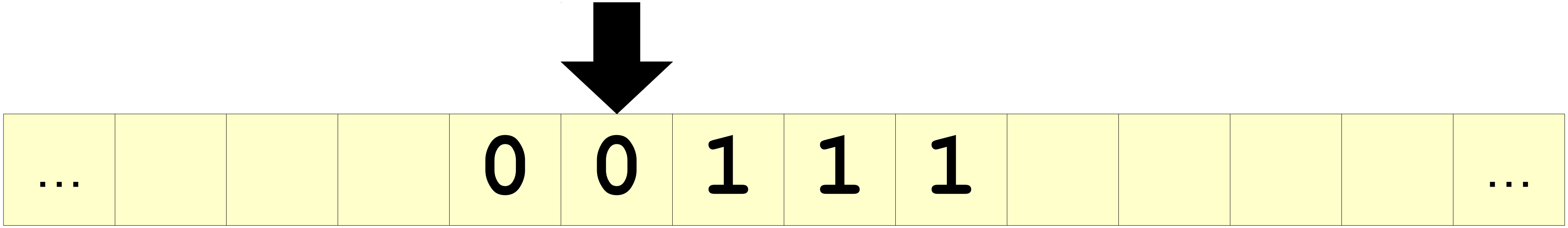

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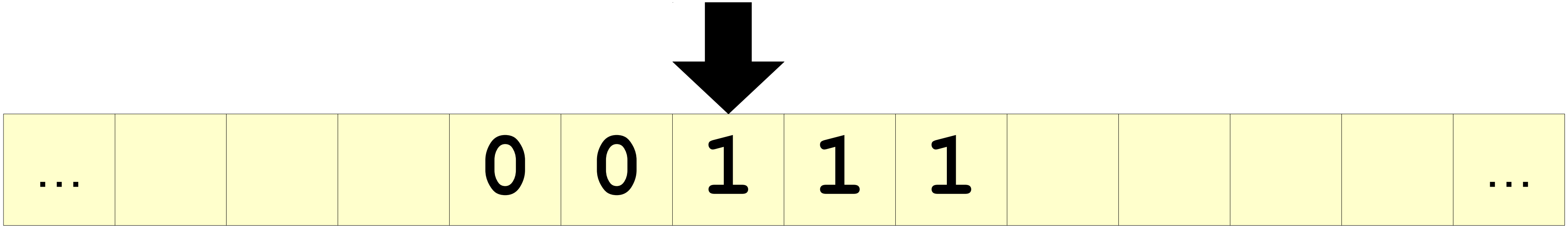

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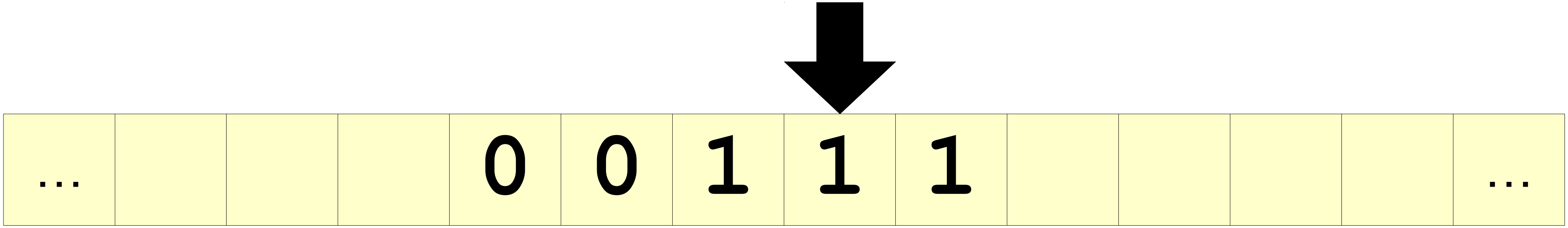

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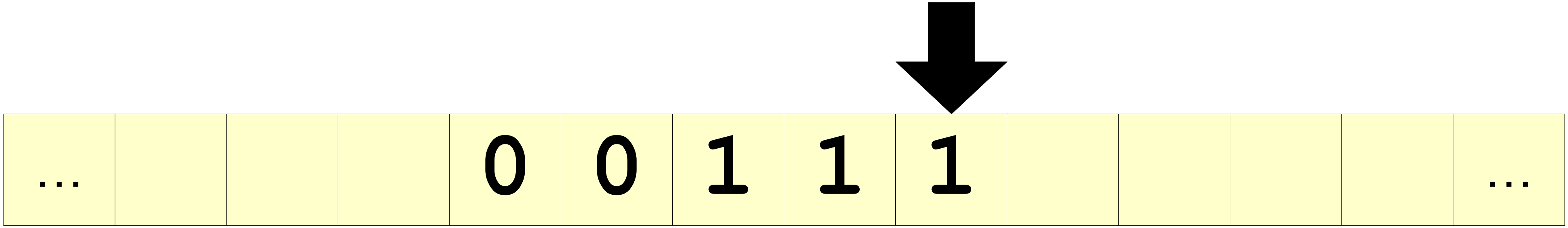
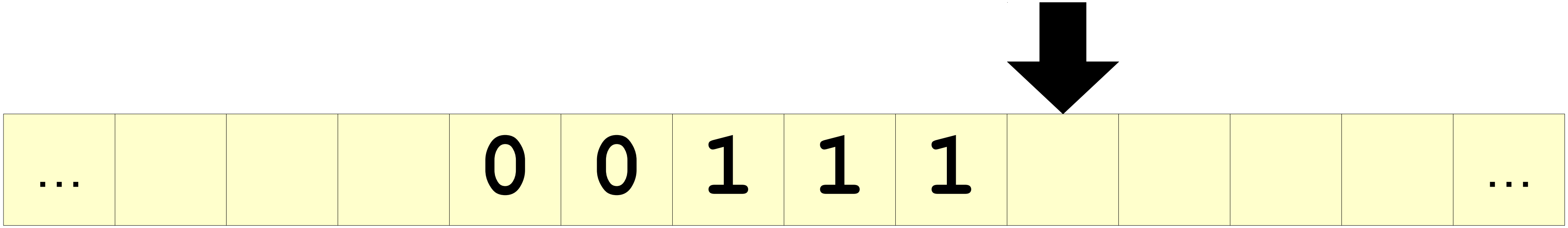
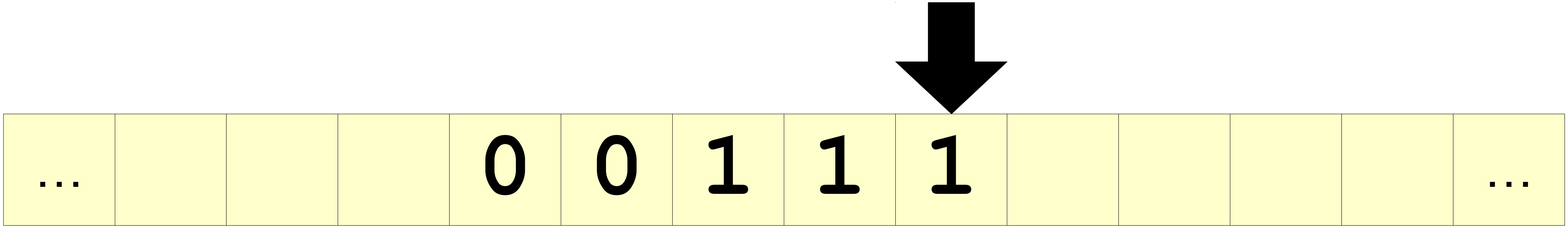


A Sketch of the TM

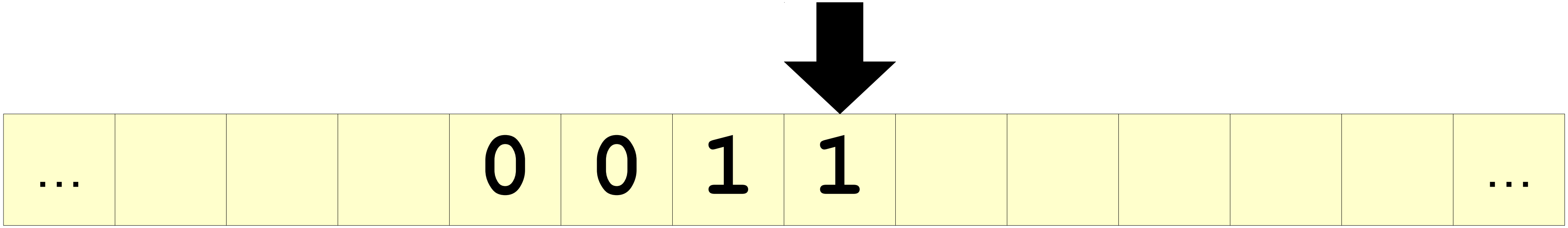


… 0 0 1 1 1 …

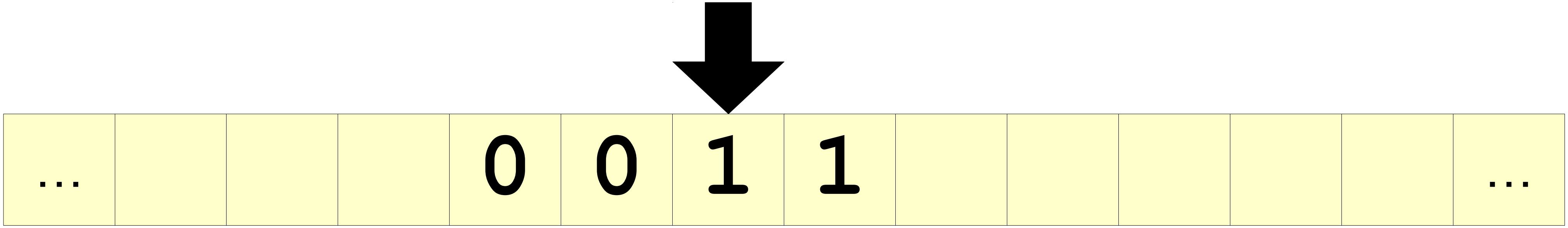
A Sketch of the TM



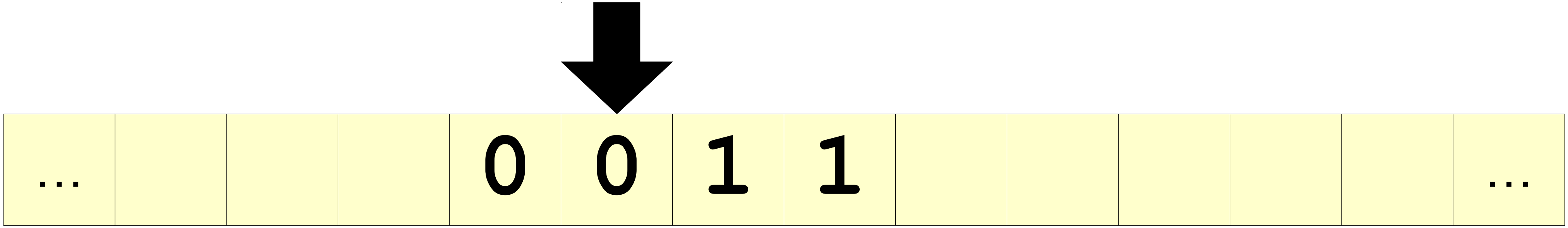
A Sketch of the TM



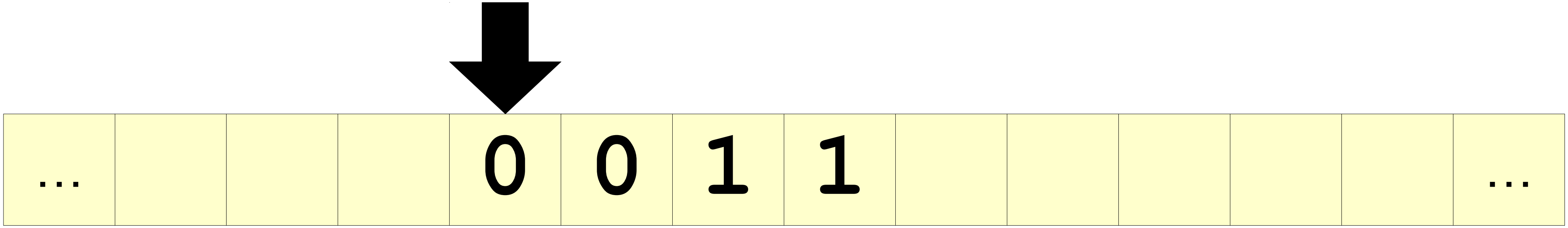
A Sketch of the TM



A Sketch of the TM



A Sketch of the TM



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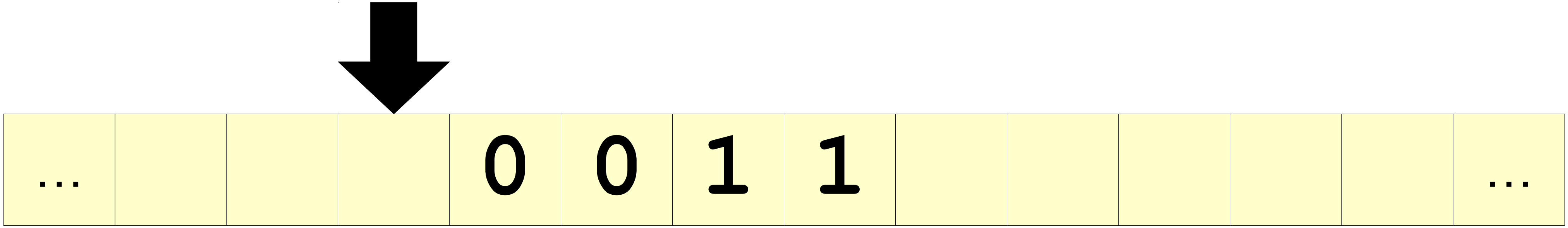

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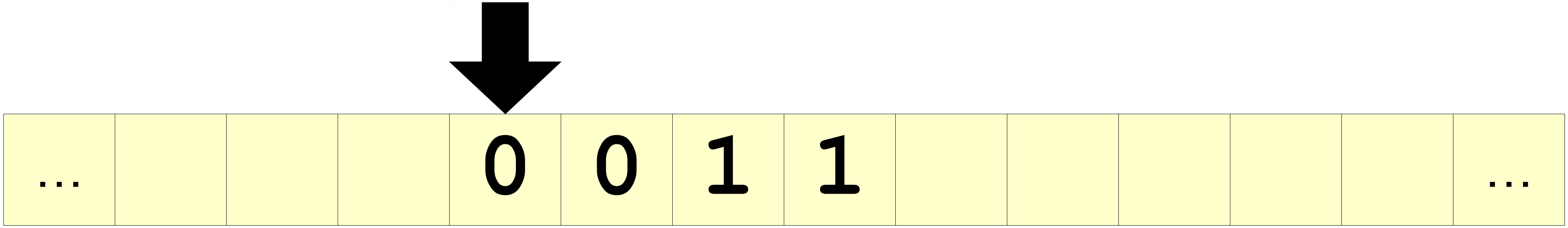

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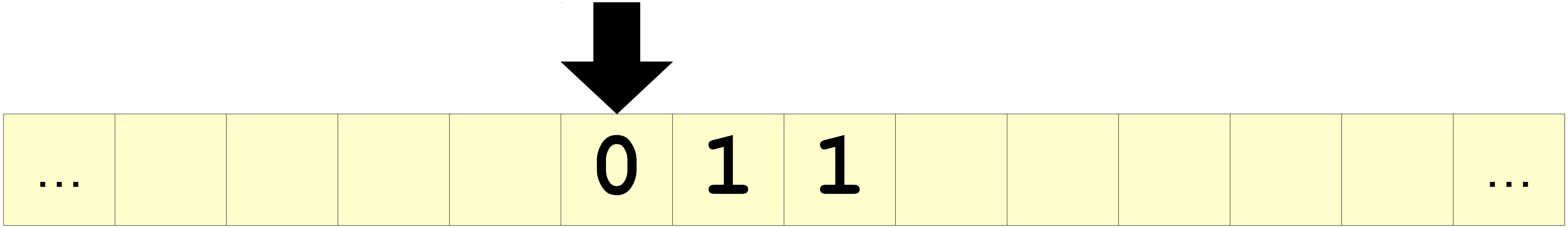

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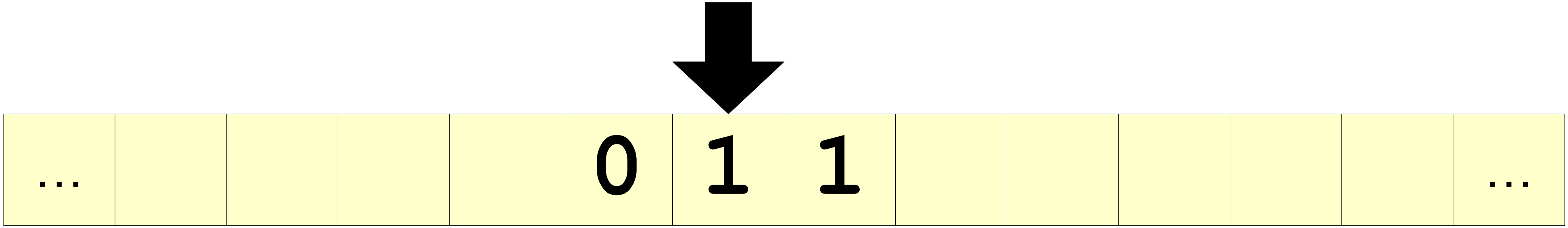

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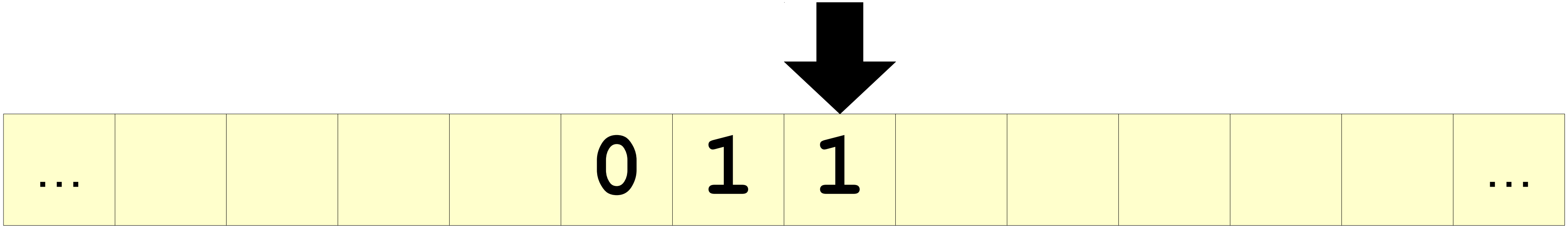
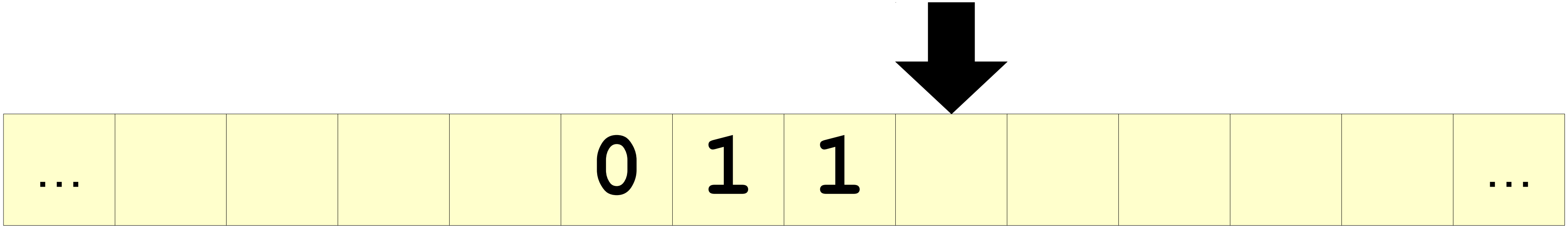


…					0	1	1						…

A Sketch of the TM



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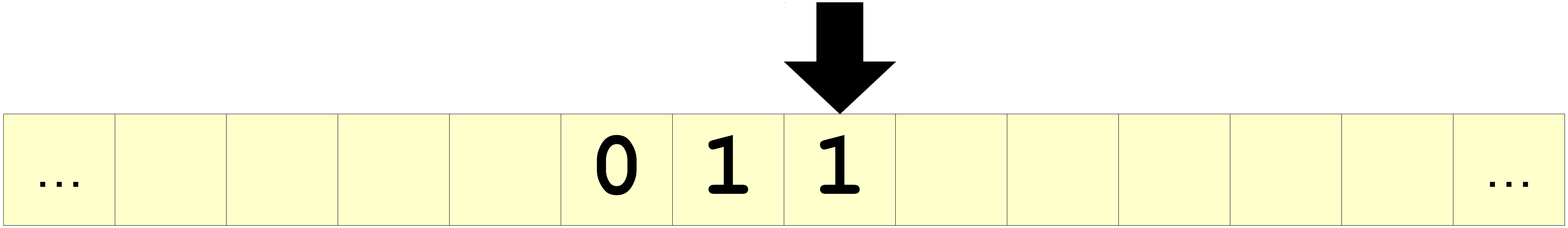

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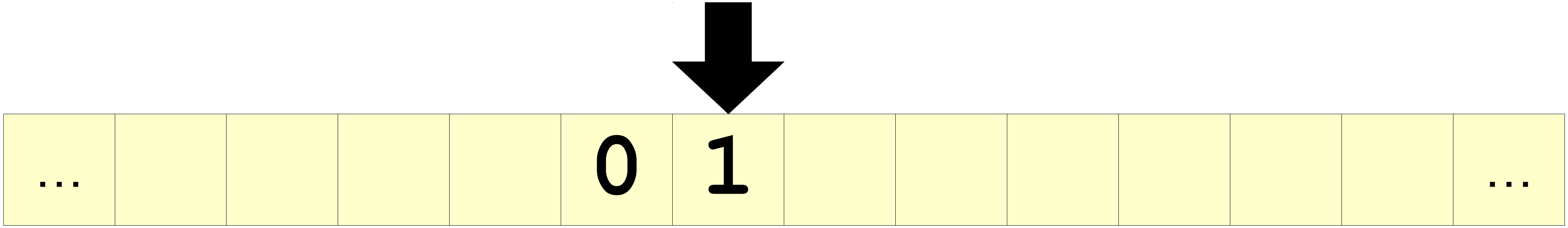

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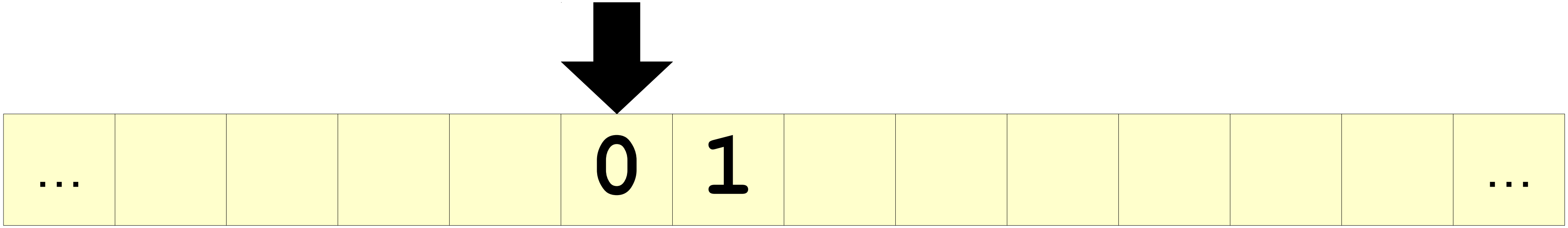

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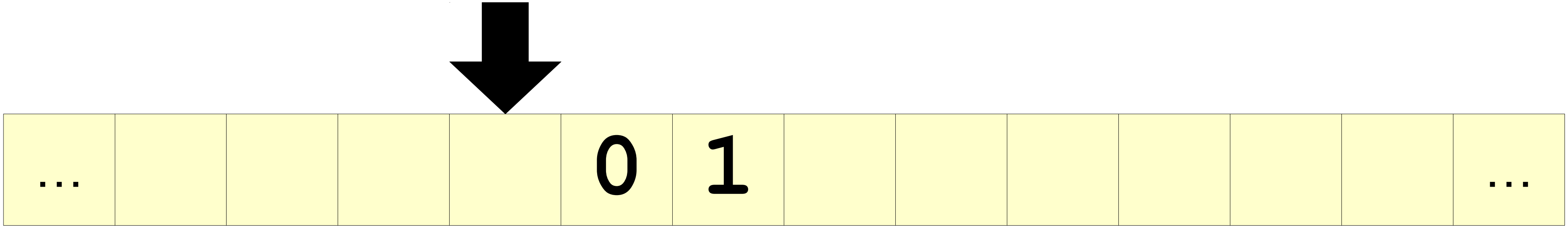
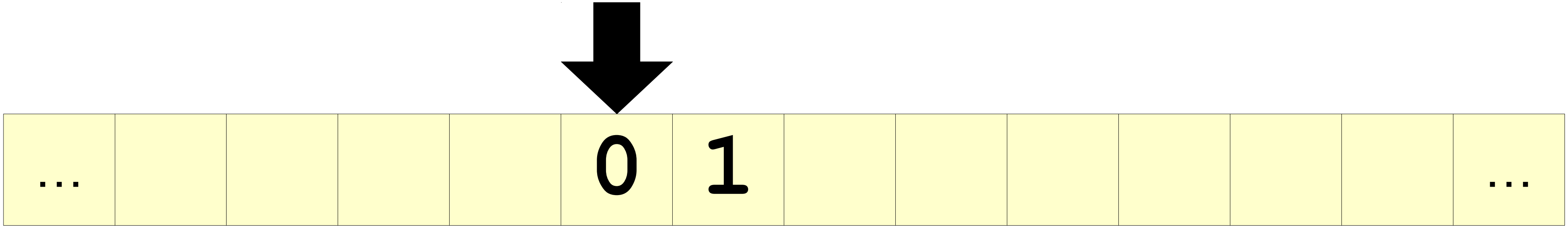
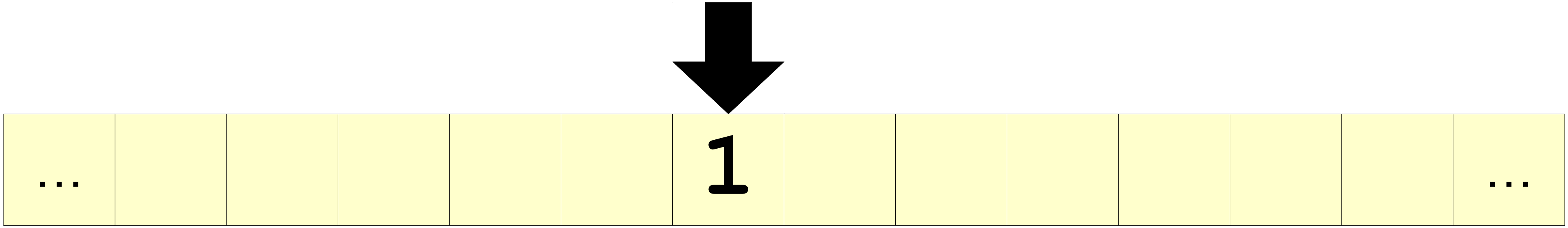


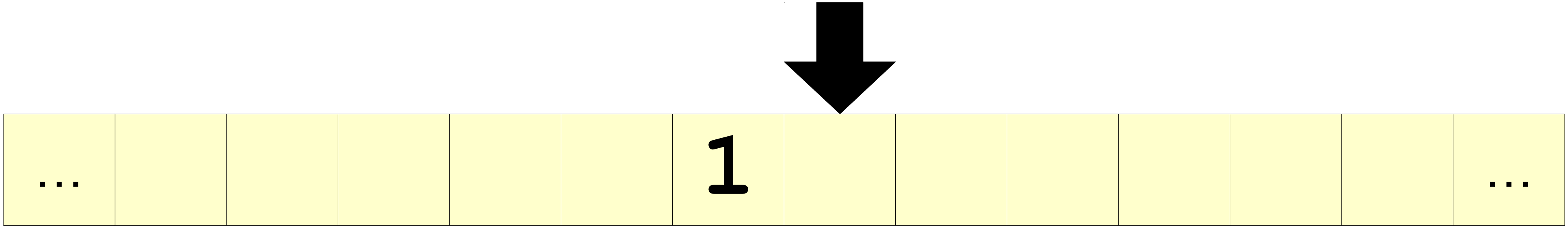
A Sketch of the TM



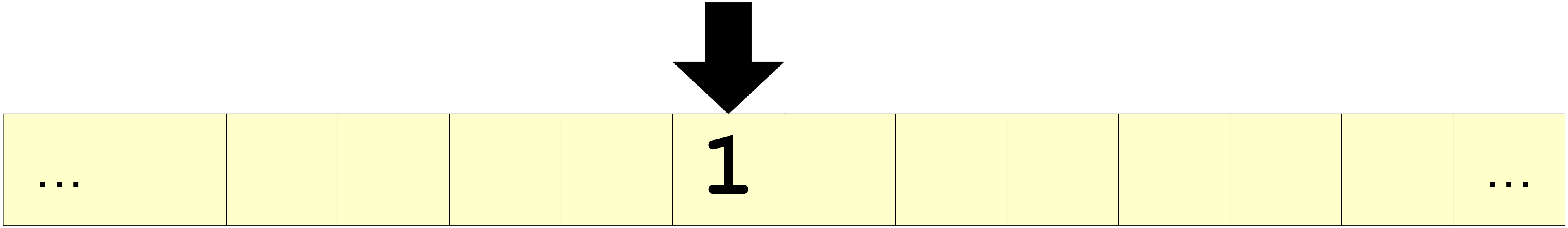
A Sketch of the TM



A Sketch of the TM

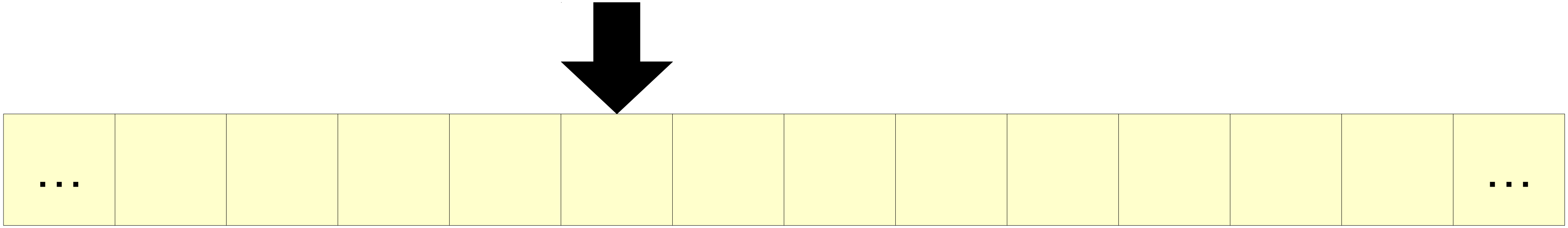


A Sketch of the TM

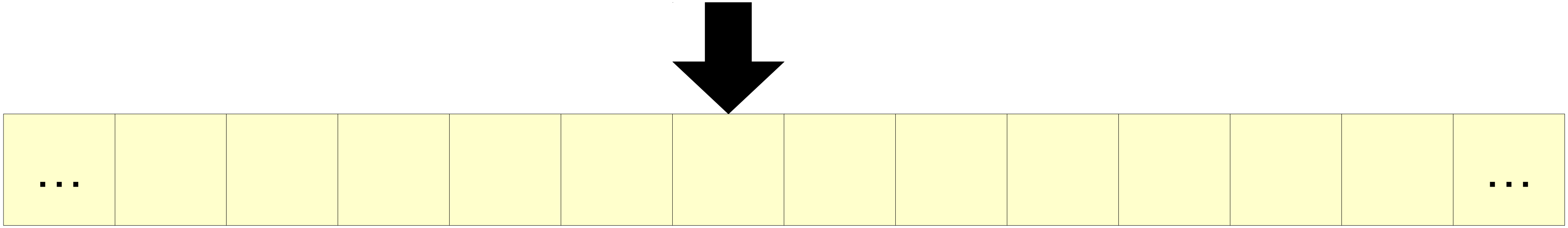


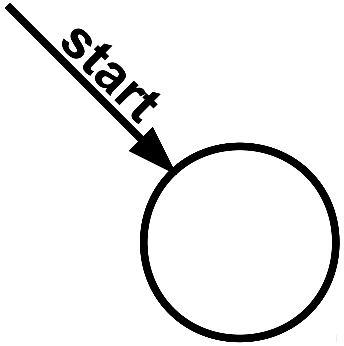
… 1 …

A Sketch of the TM



A Sketch of the TM

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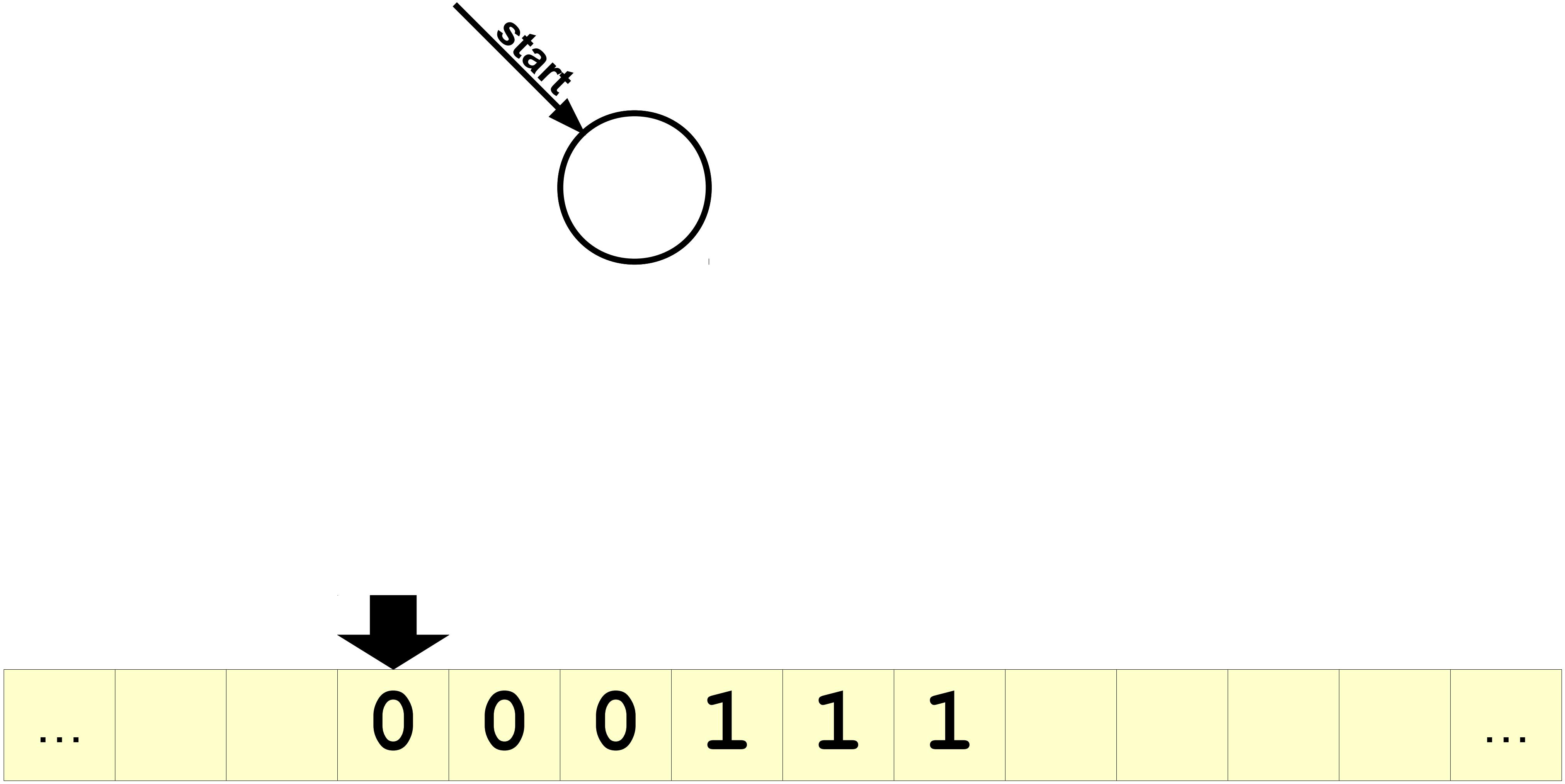
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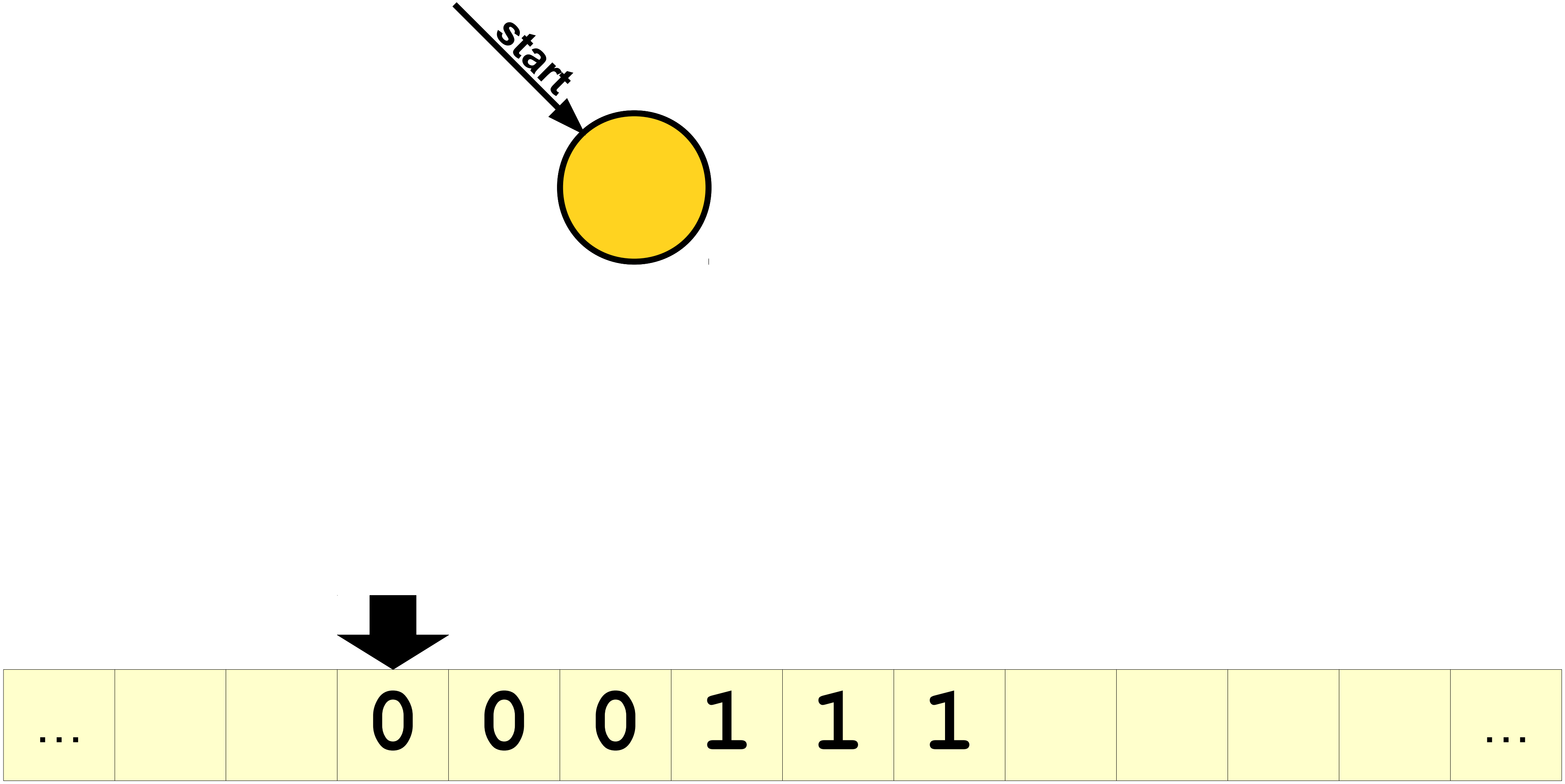
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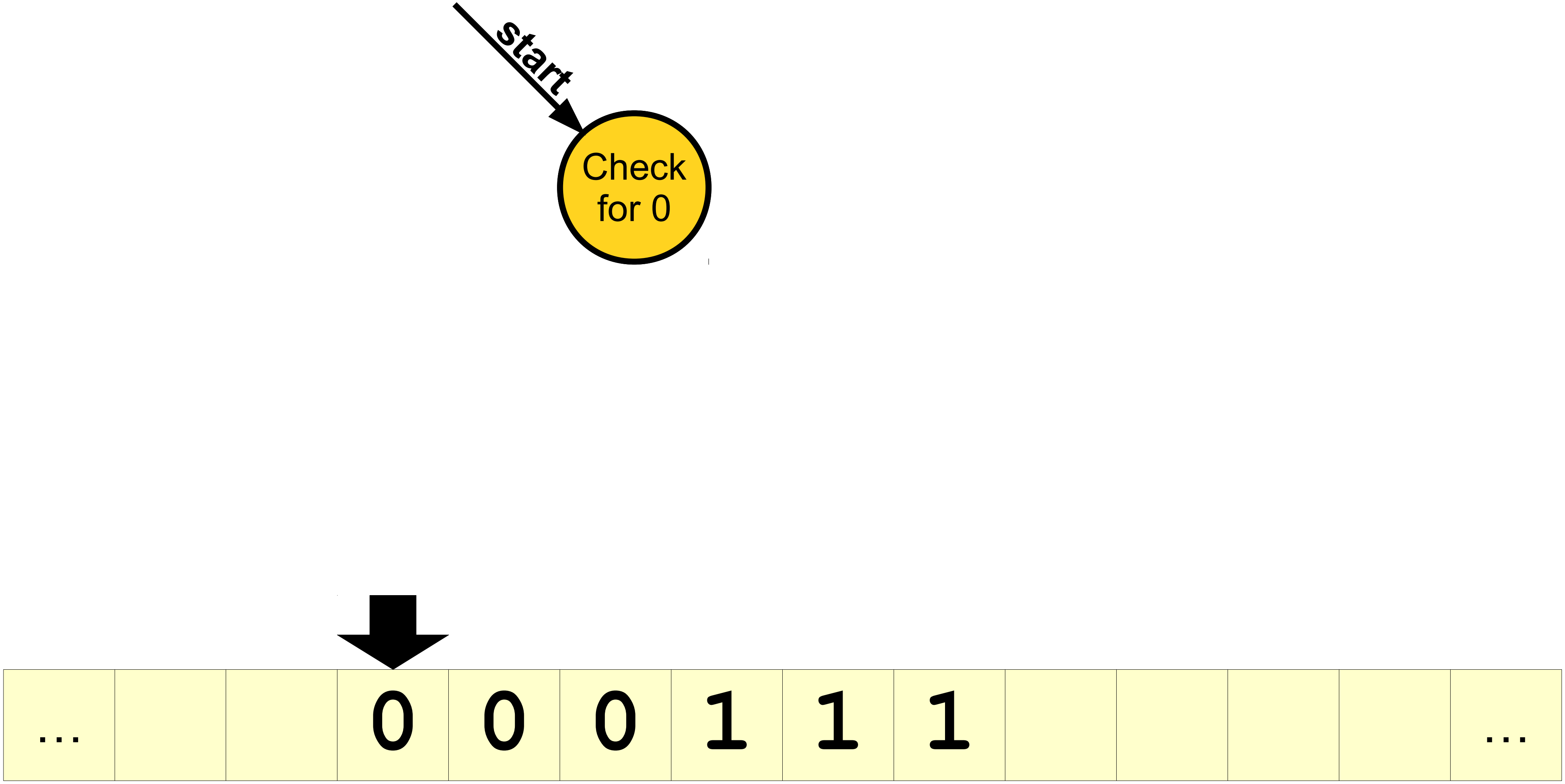
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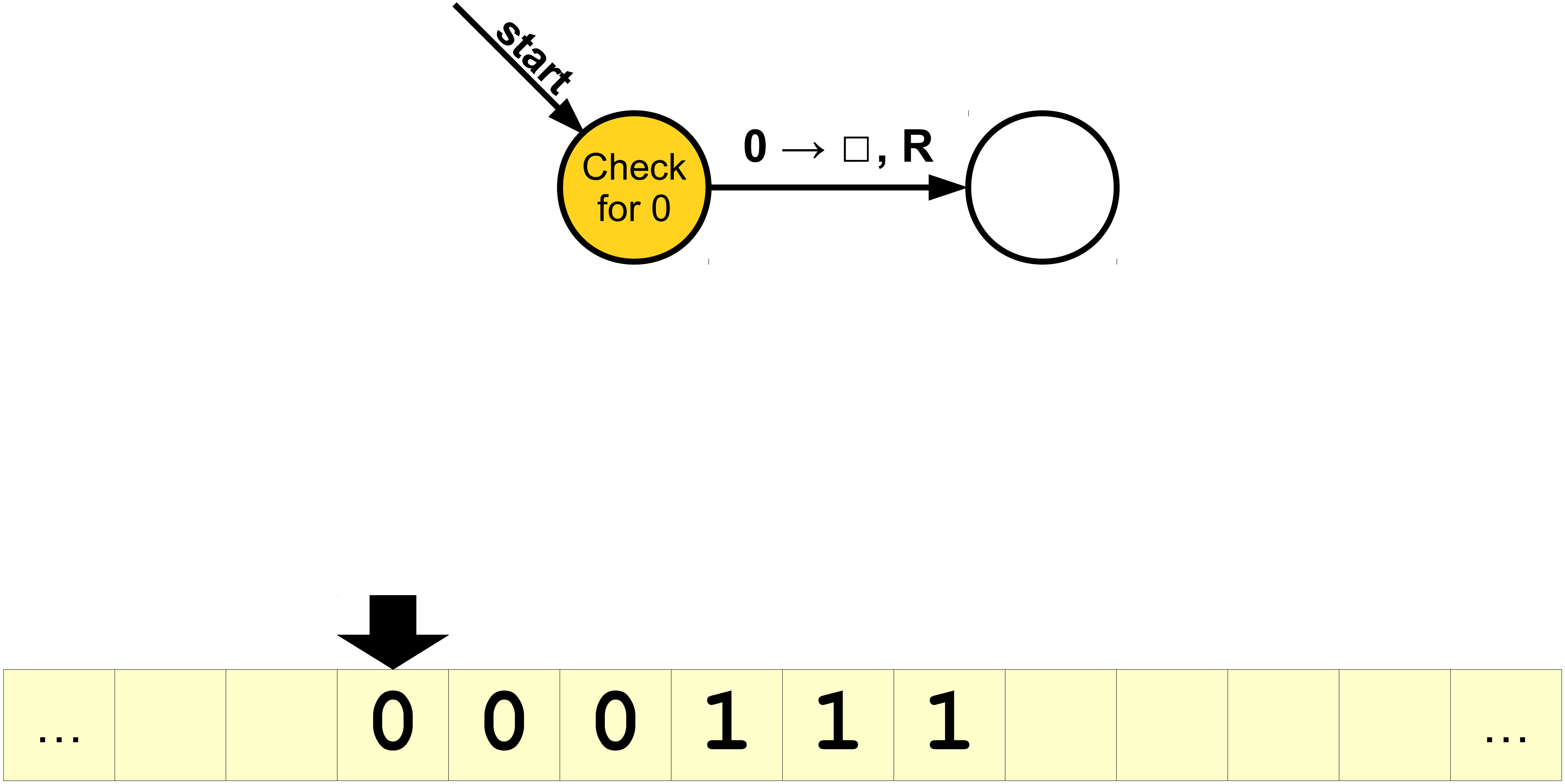
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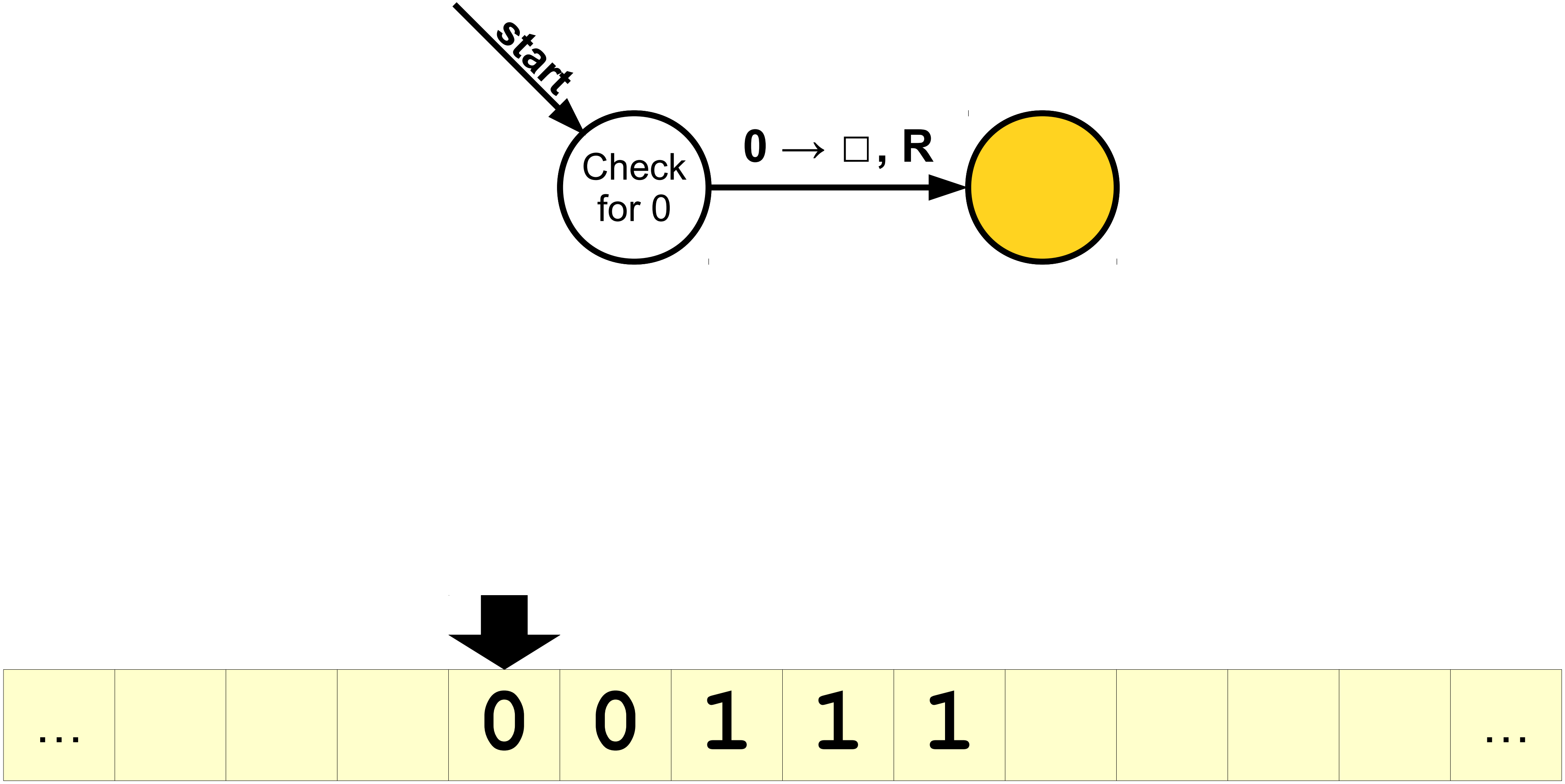
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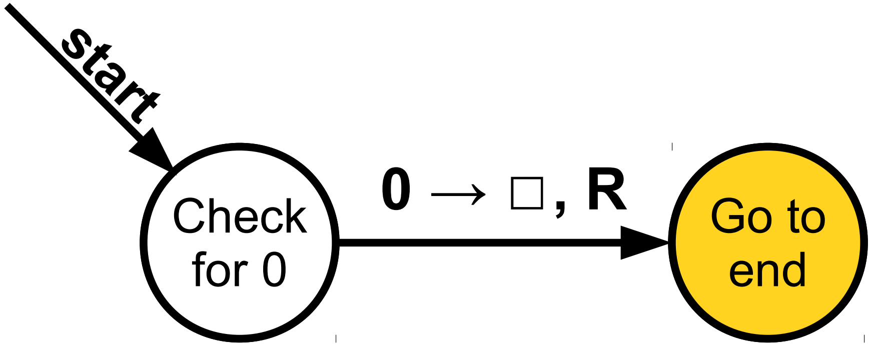

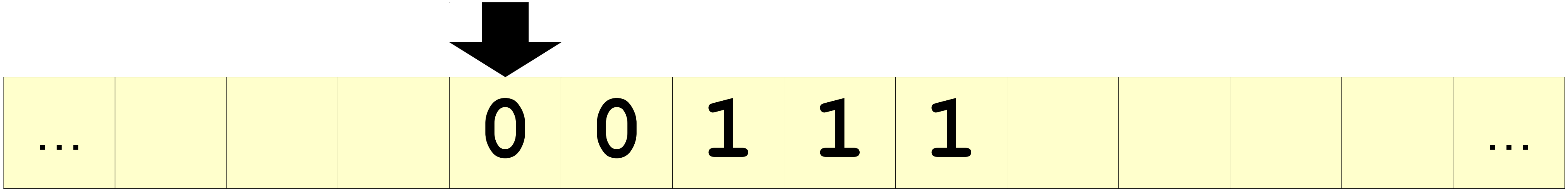

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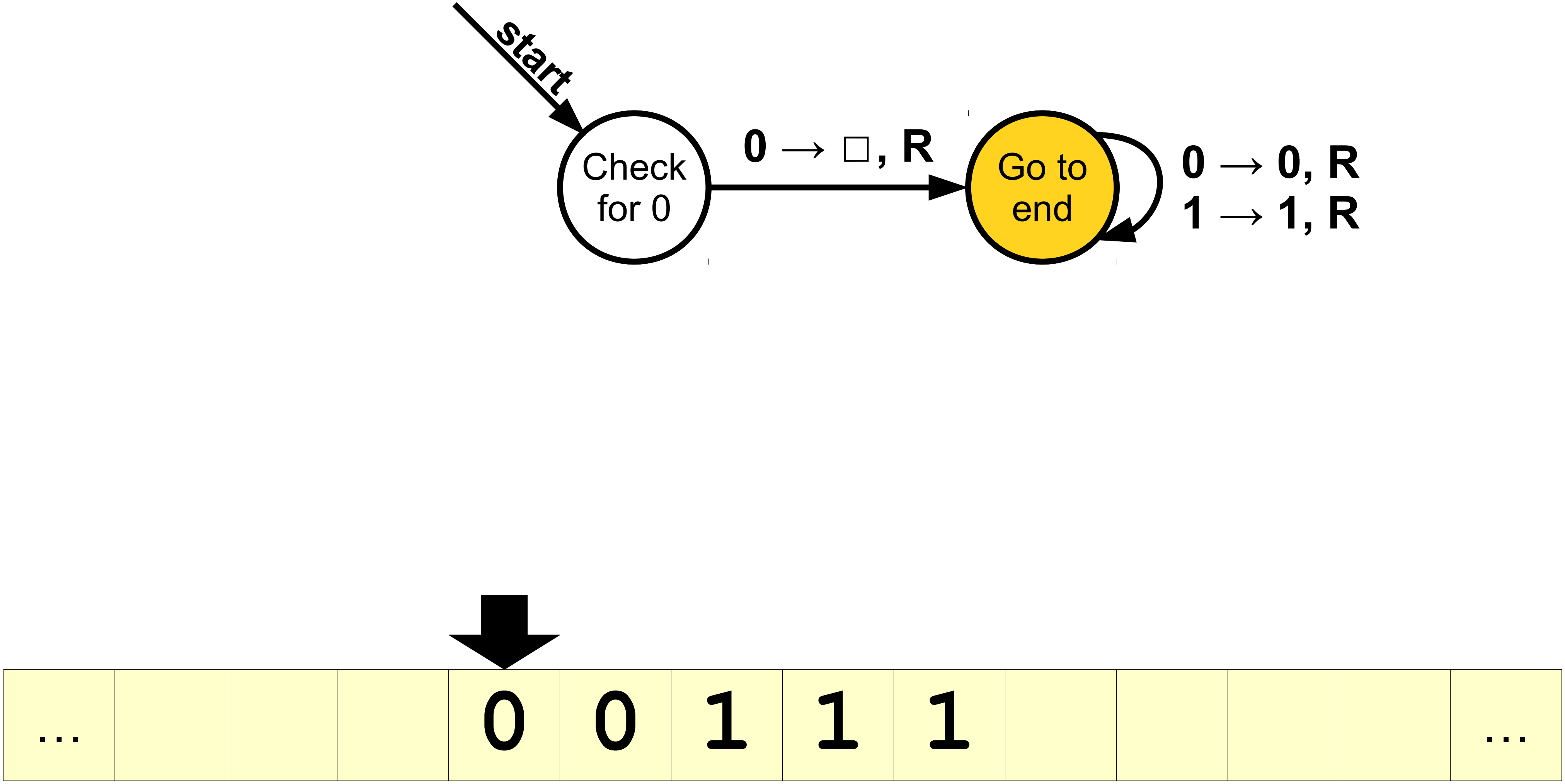
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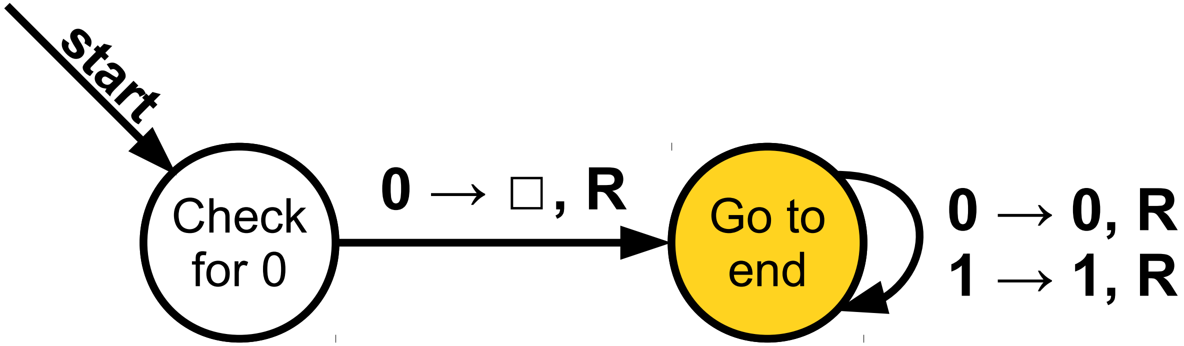
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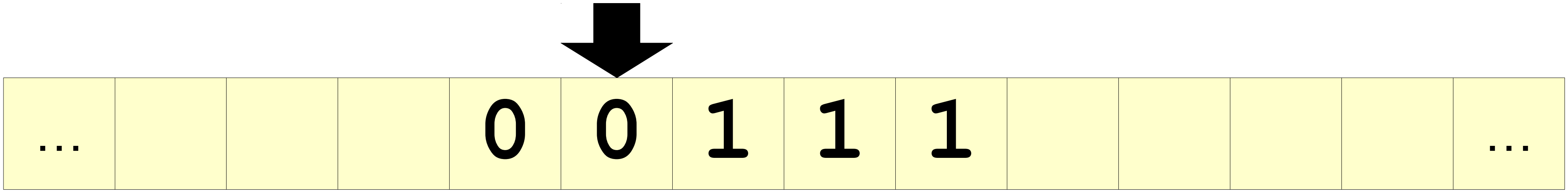
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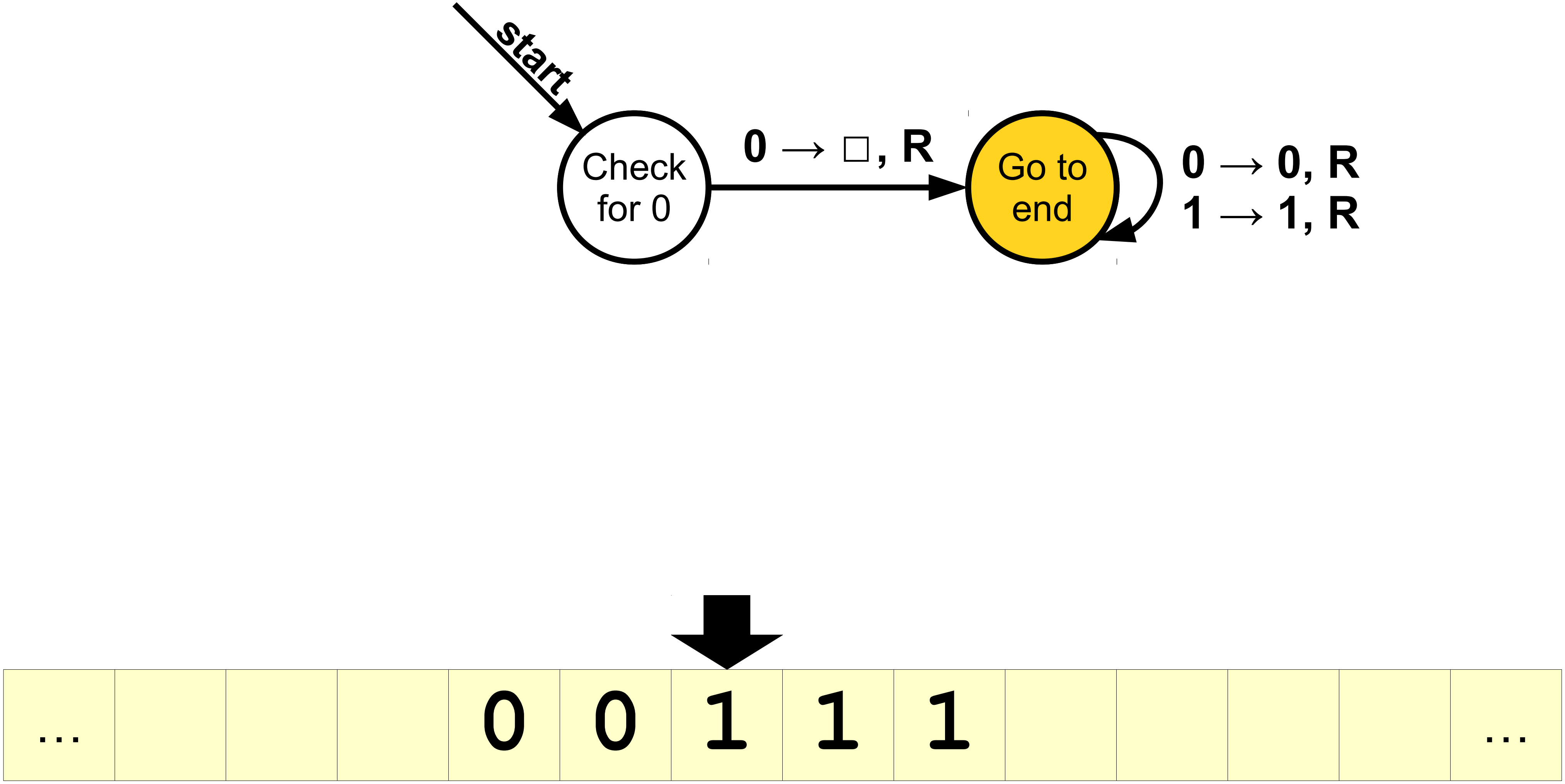
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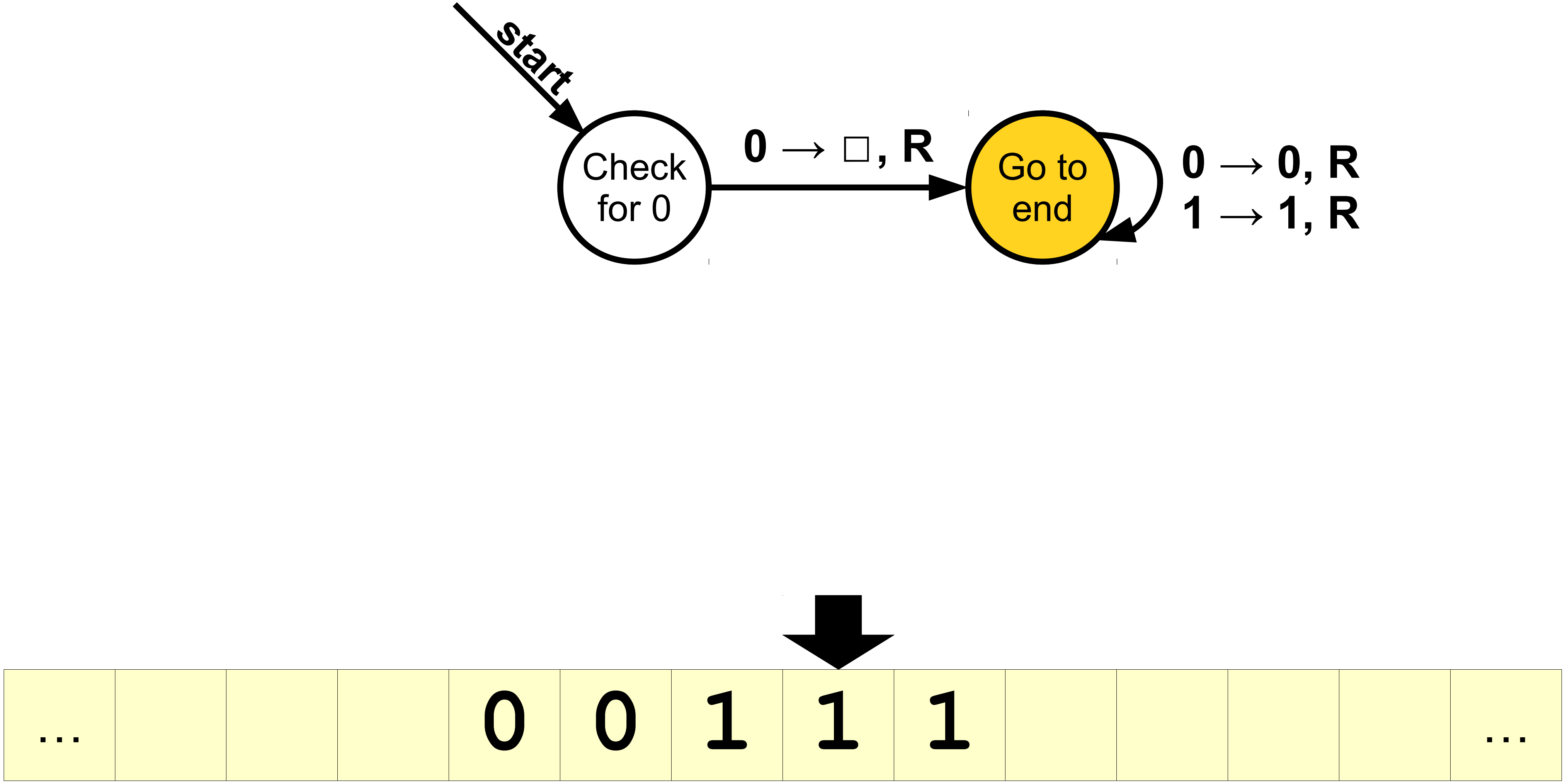
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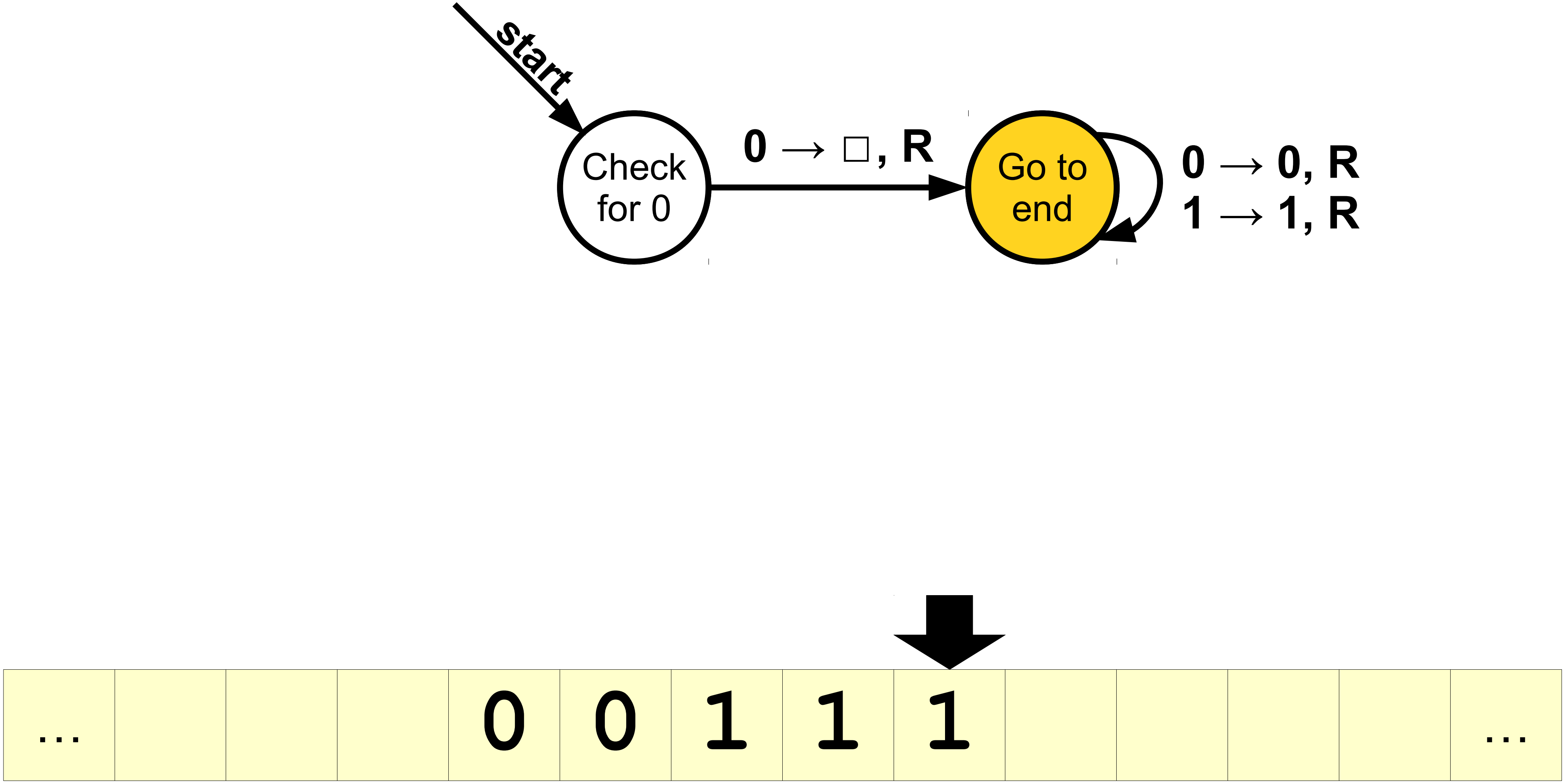
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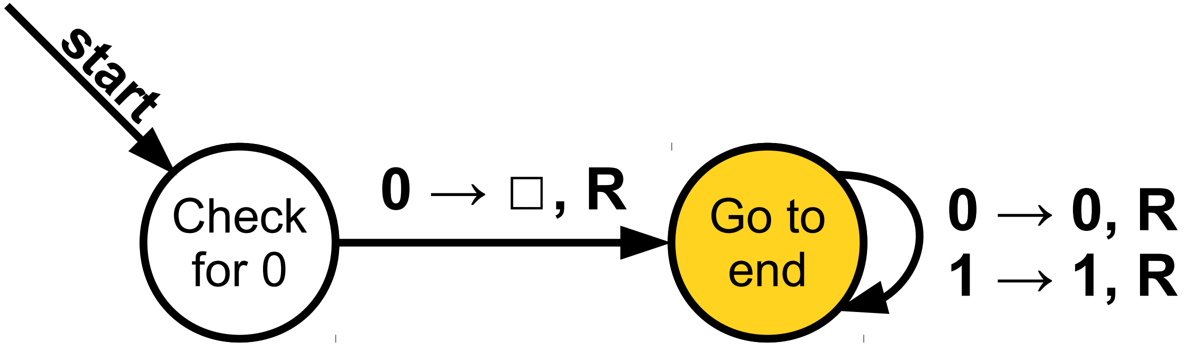

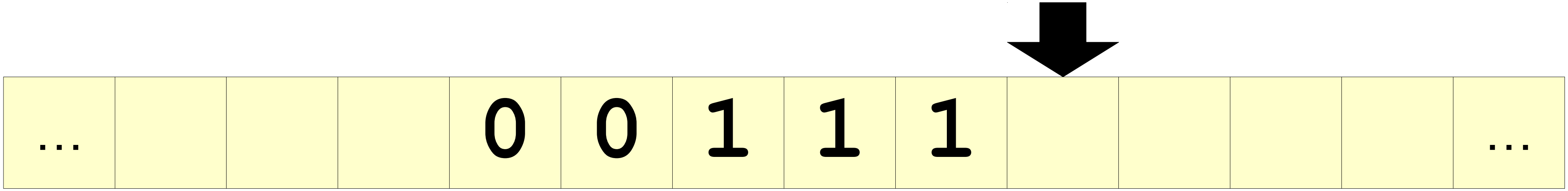

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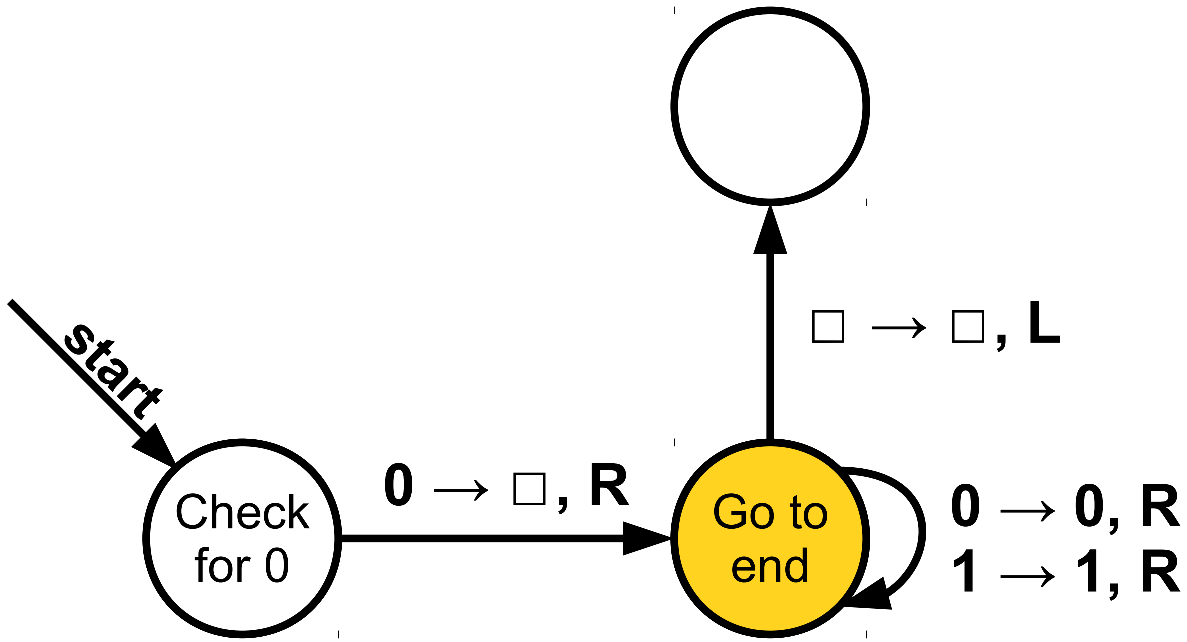
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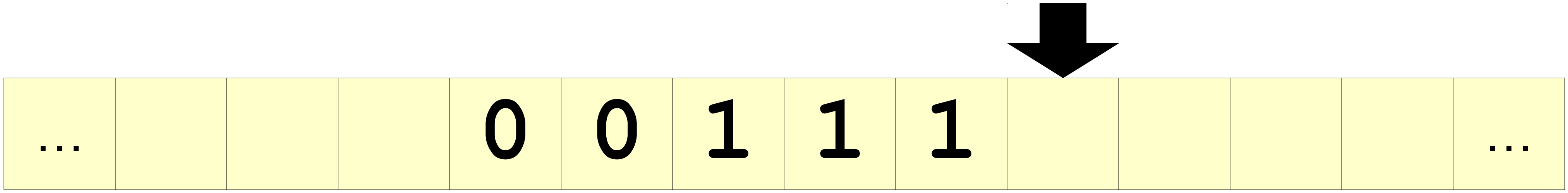
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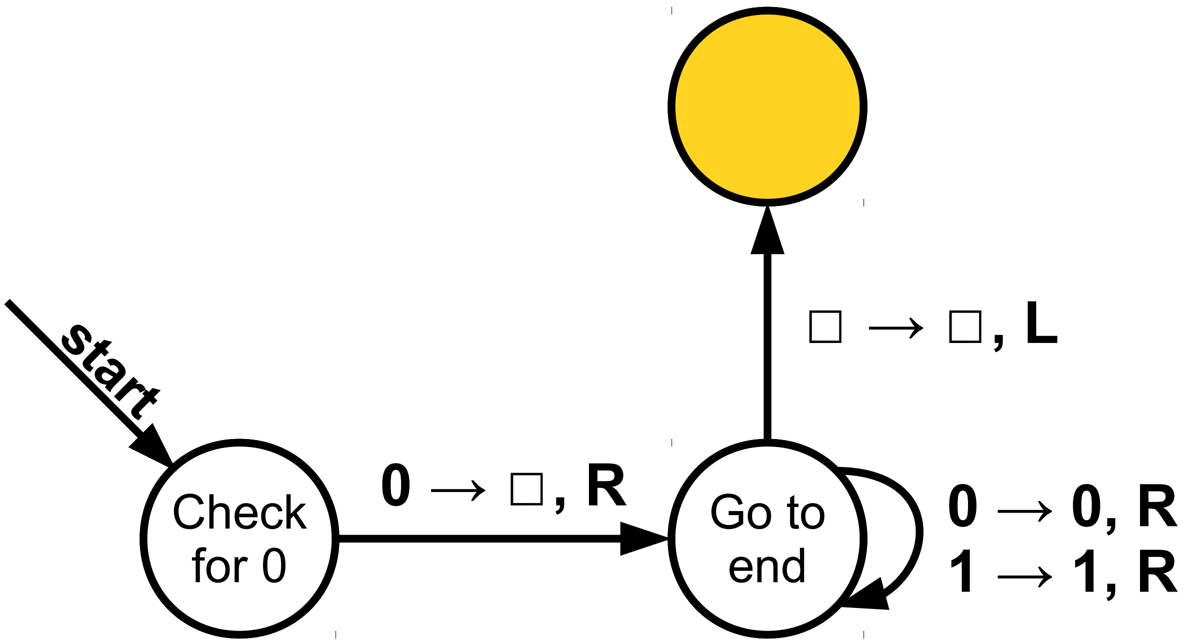

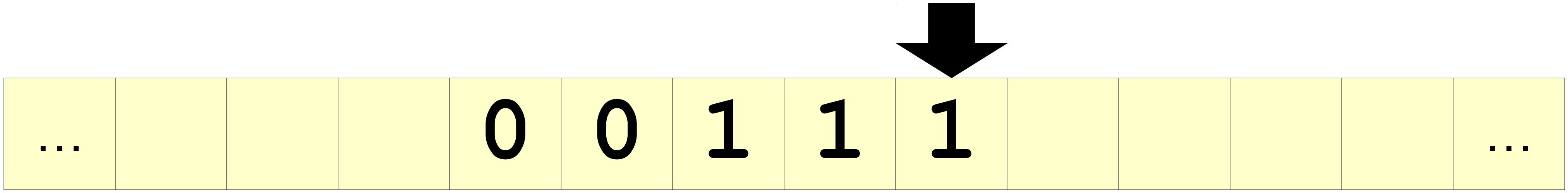

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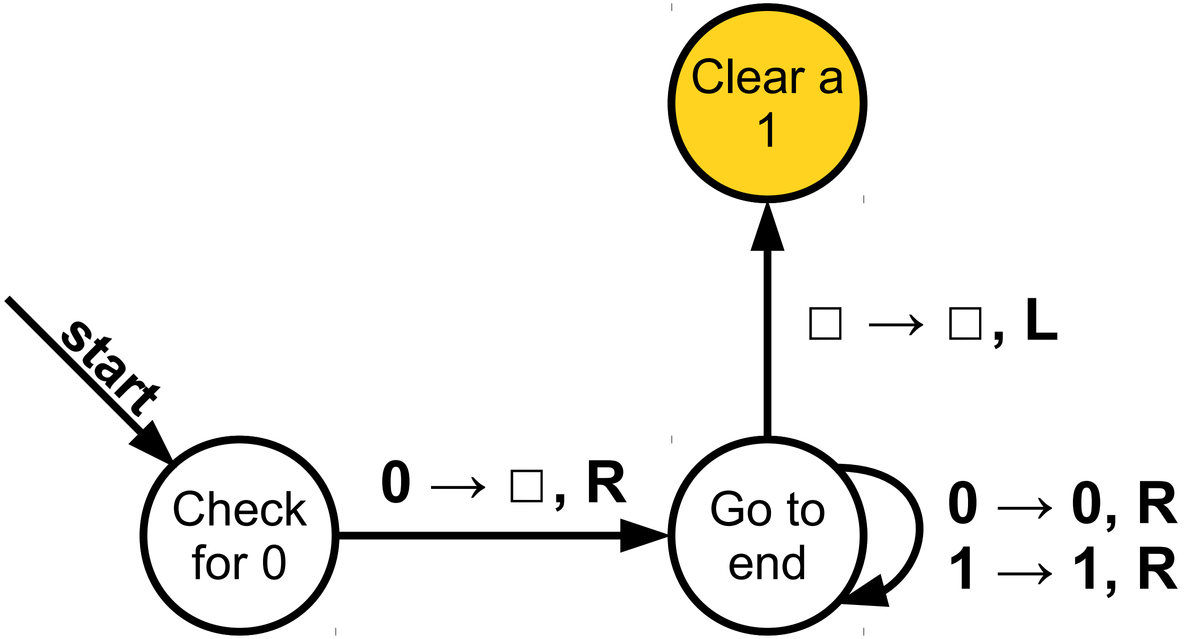

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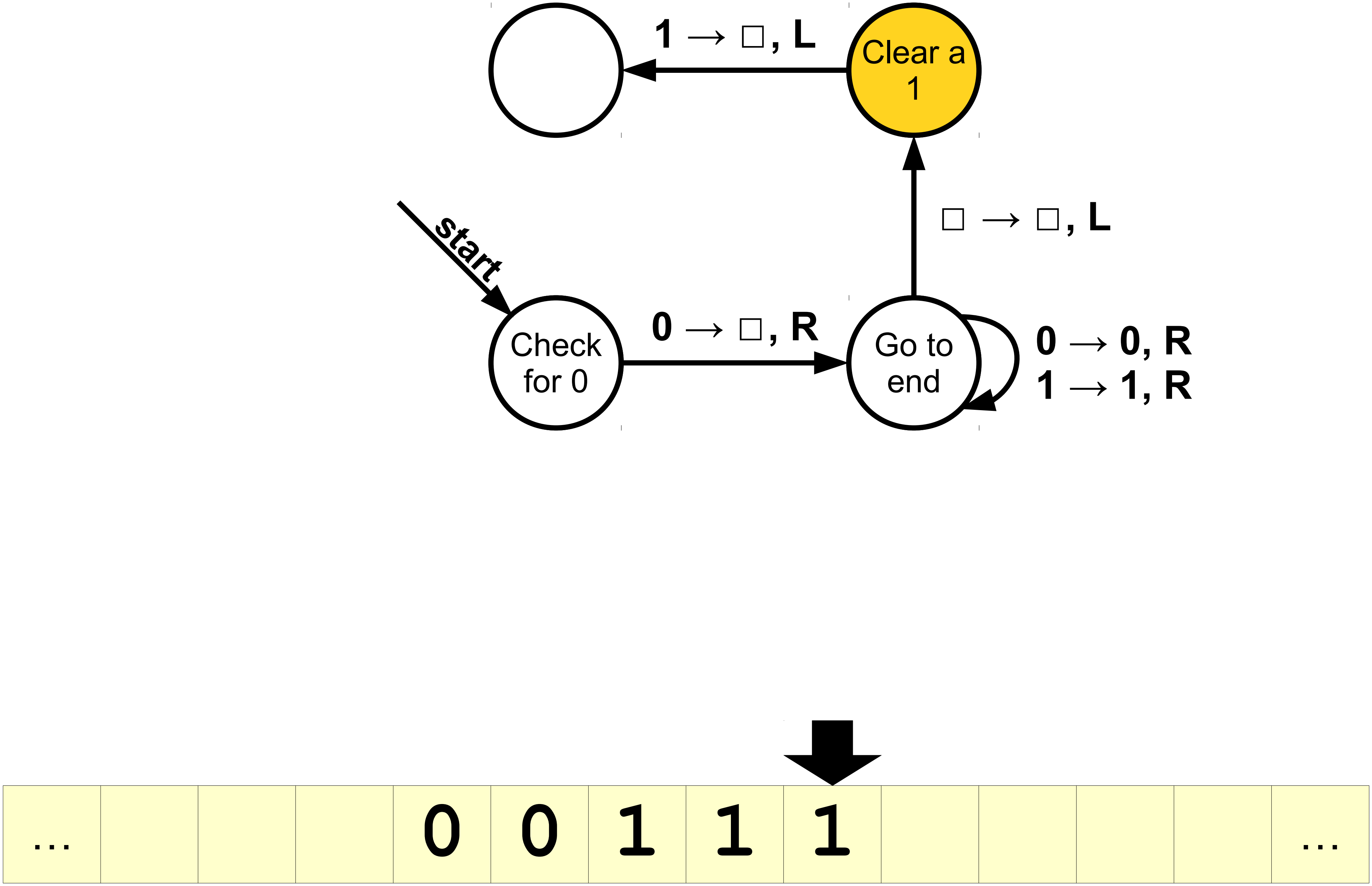
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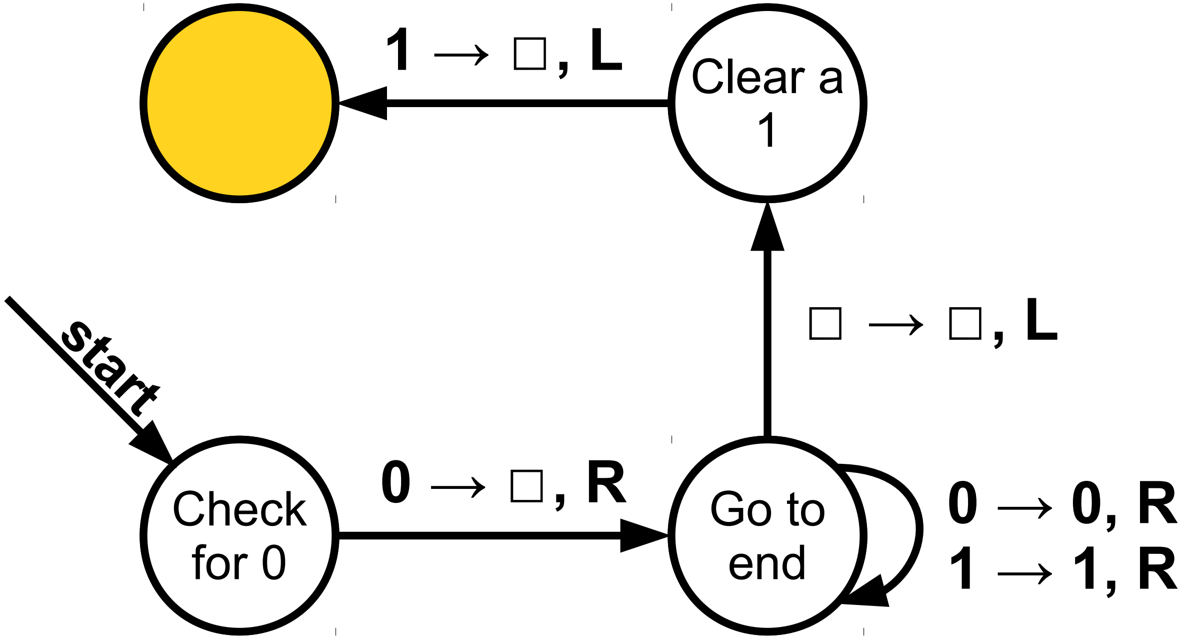

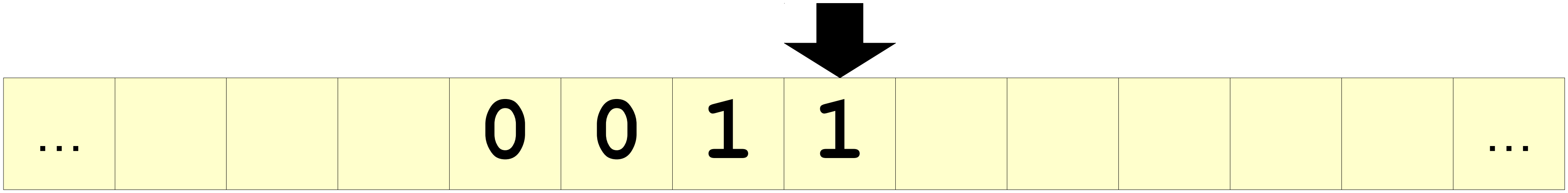

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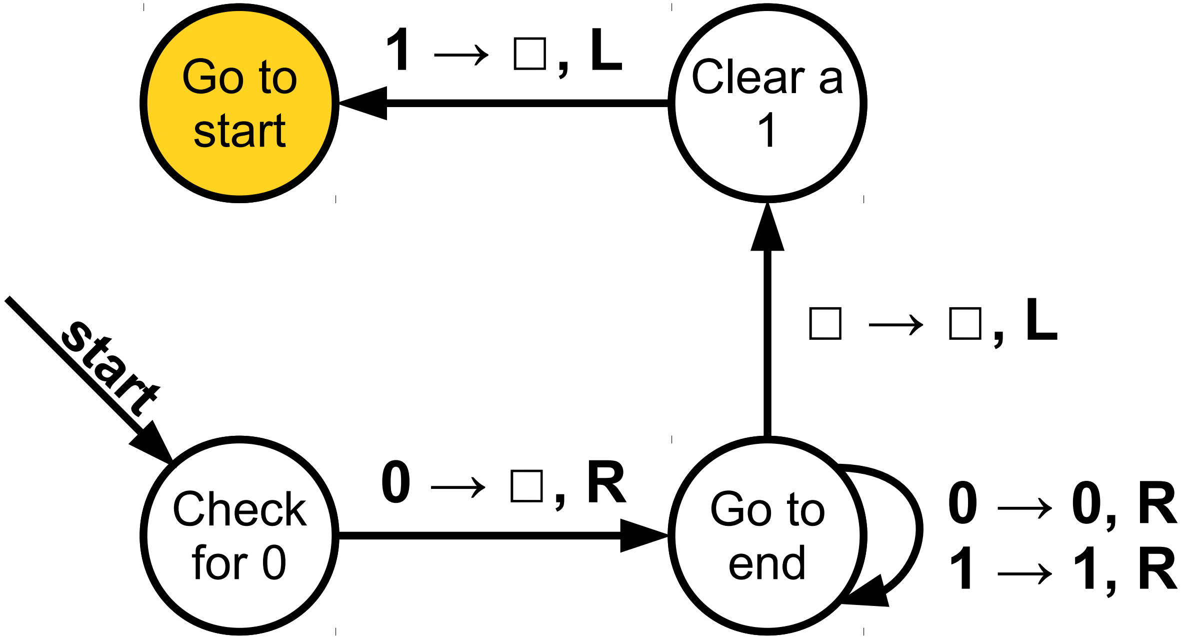

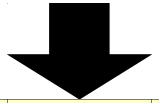

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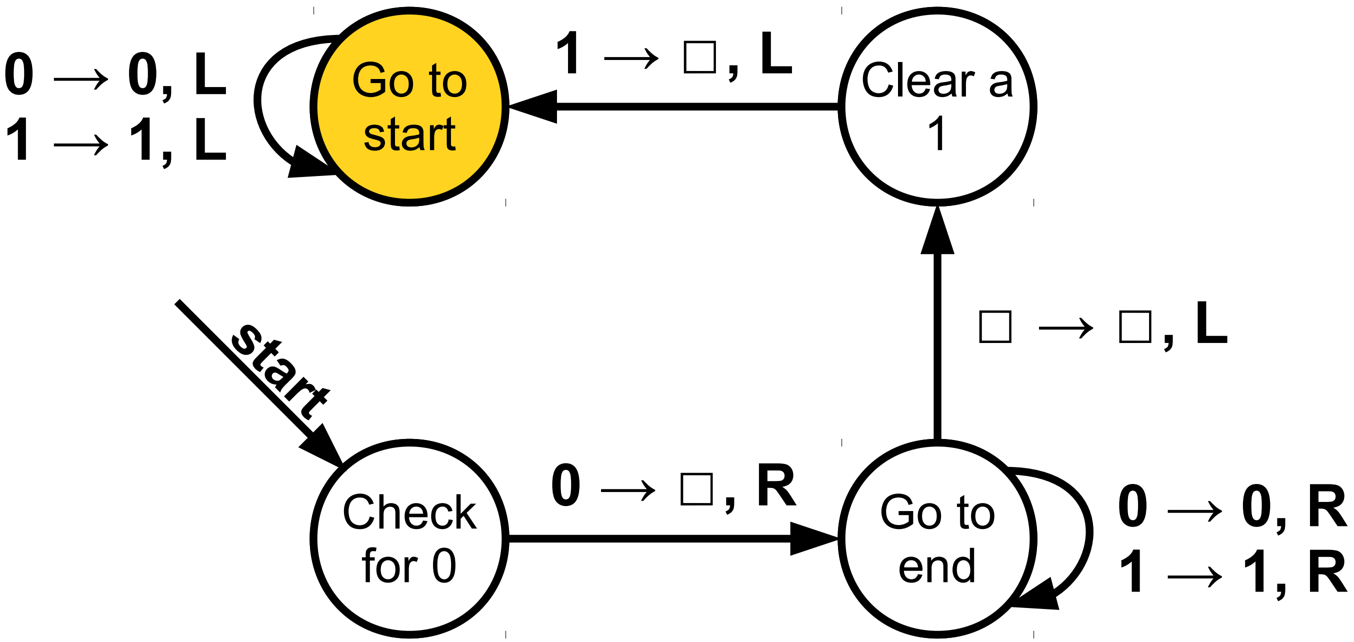
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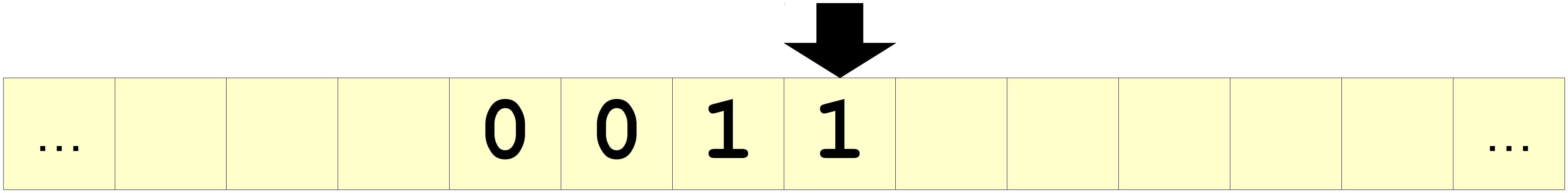

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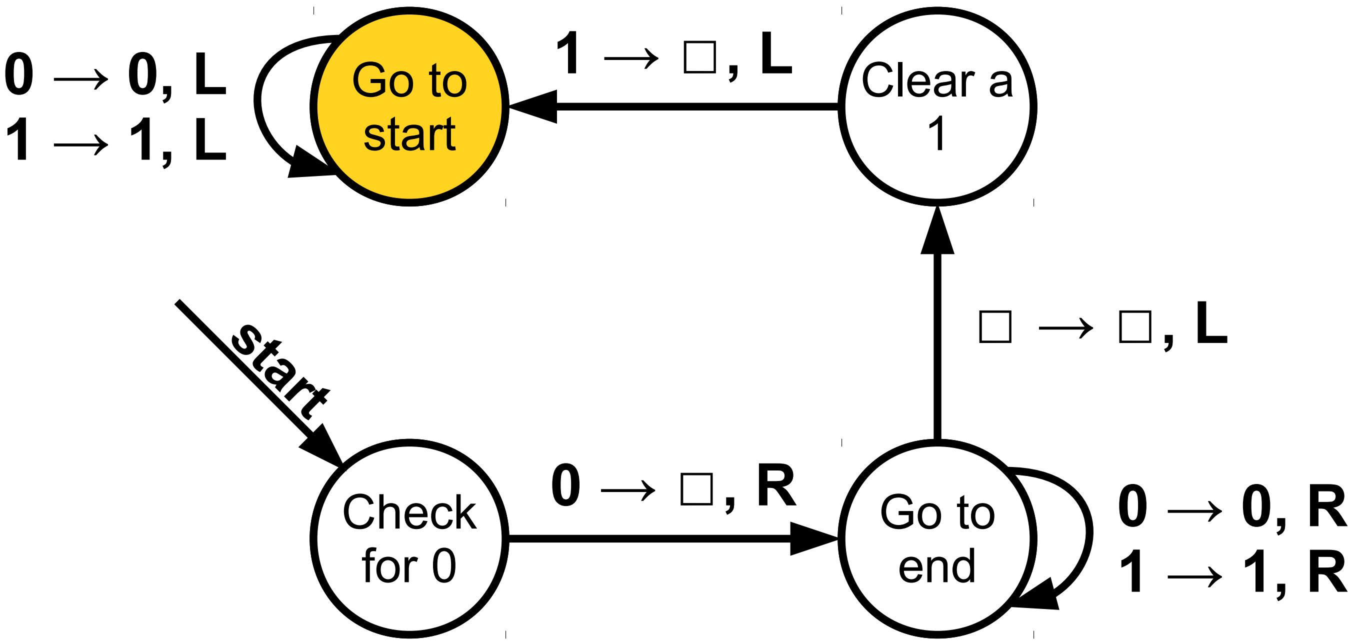
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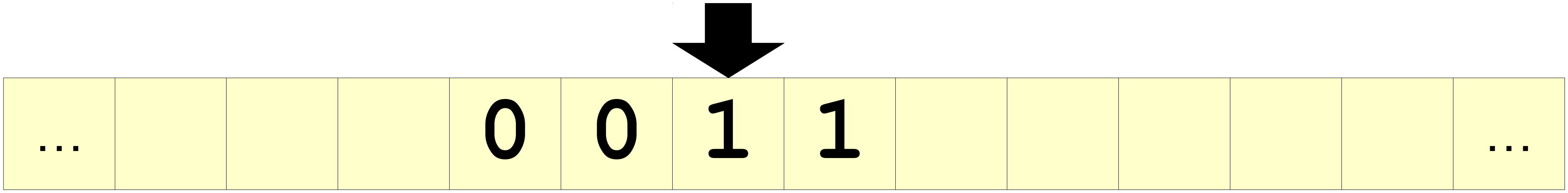
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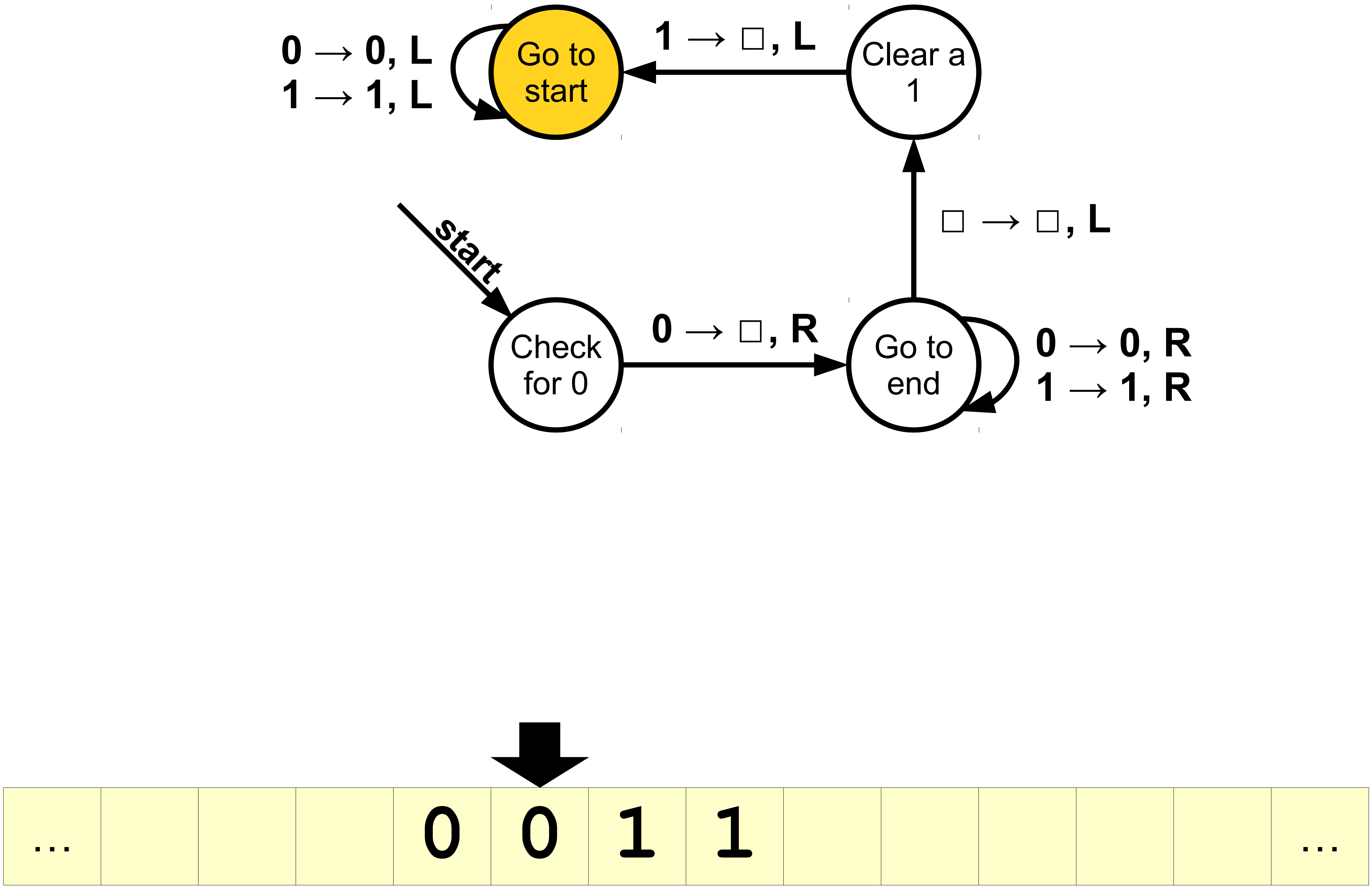

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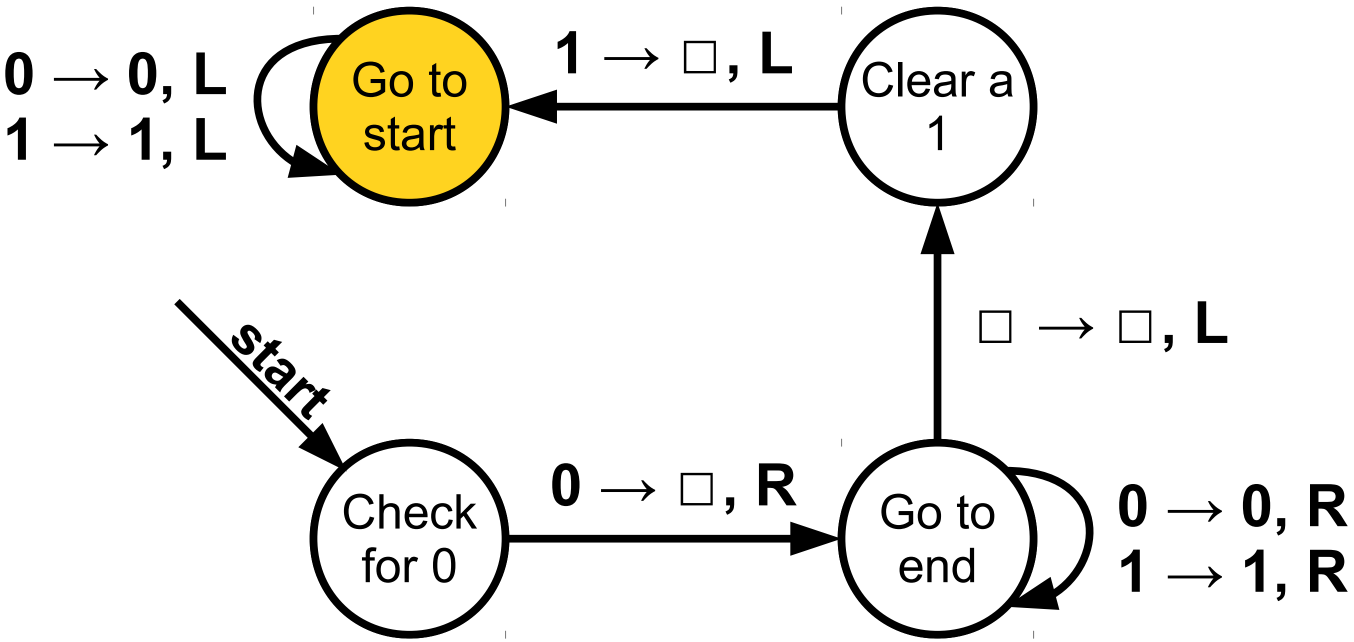
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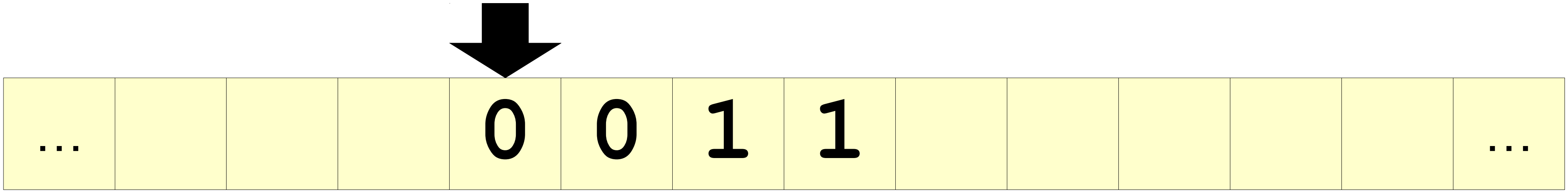
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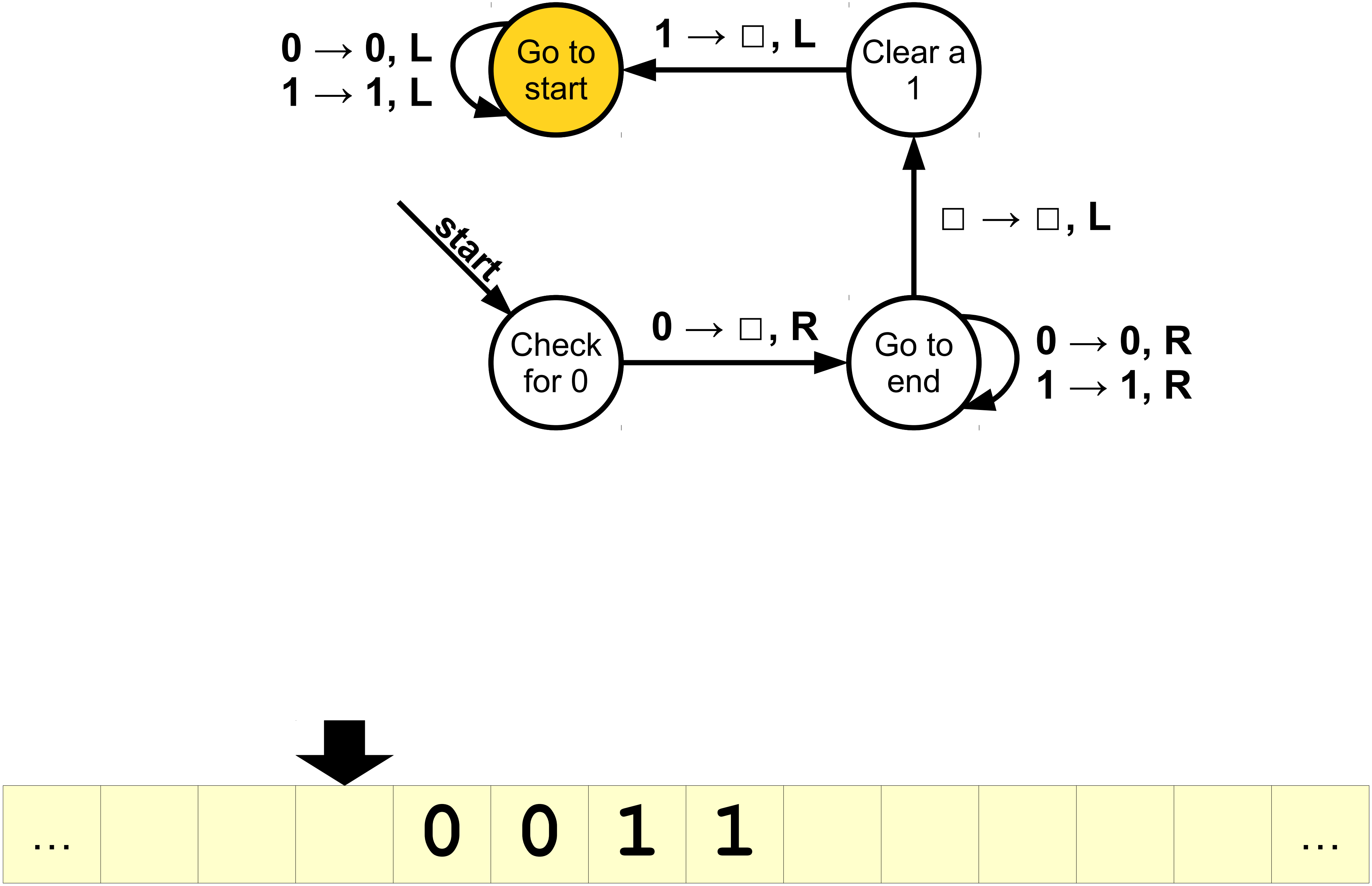
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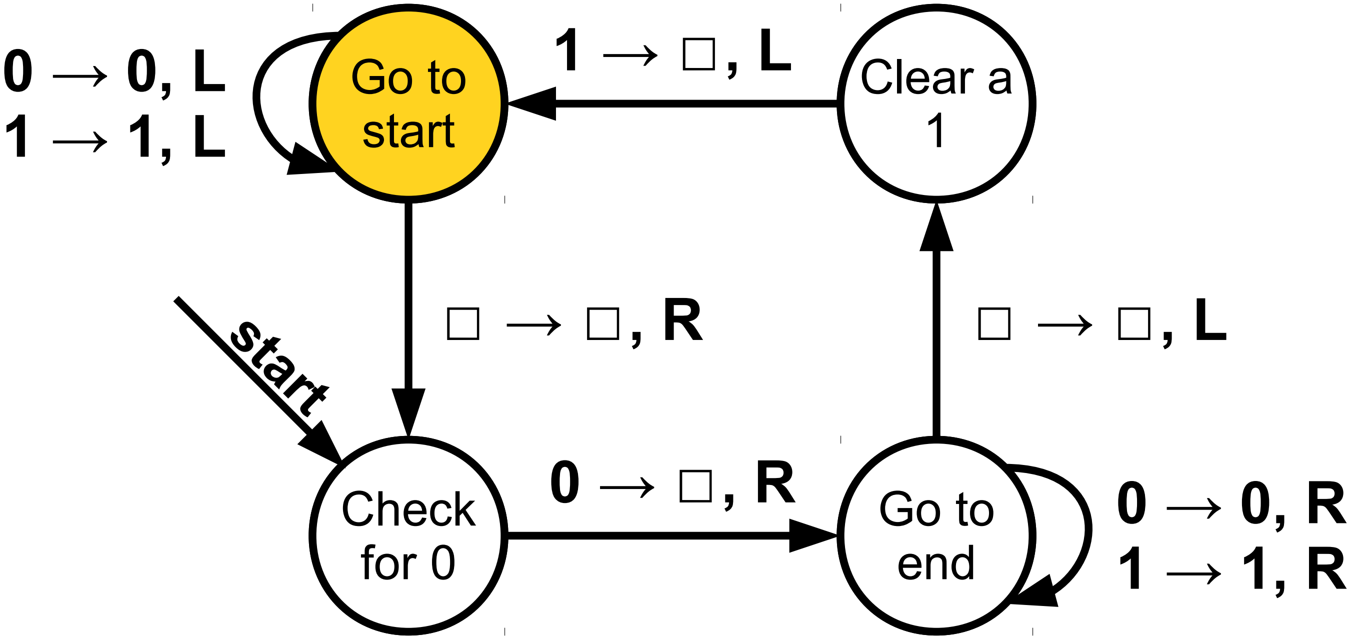
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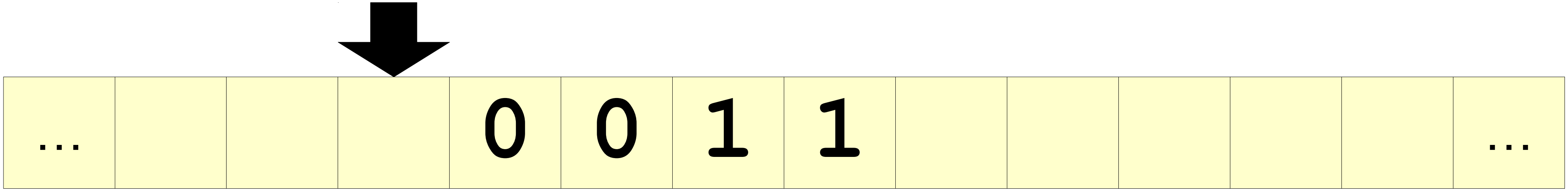

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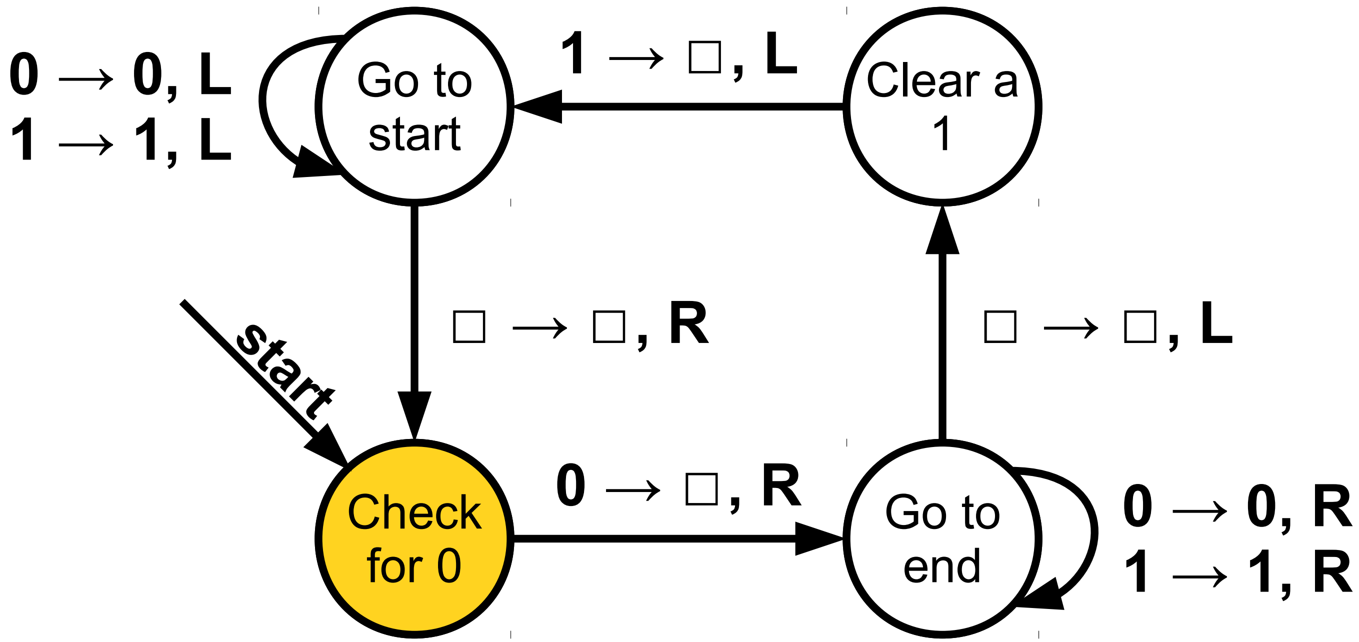
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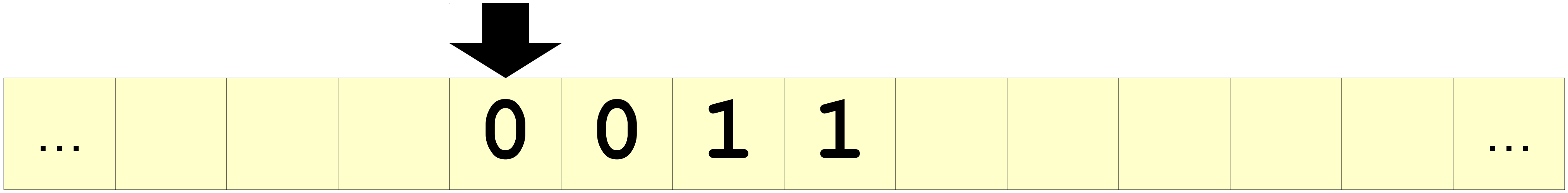

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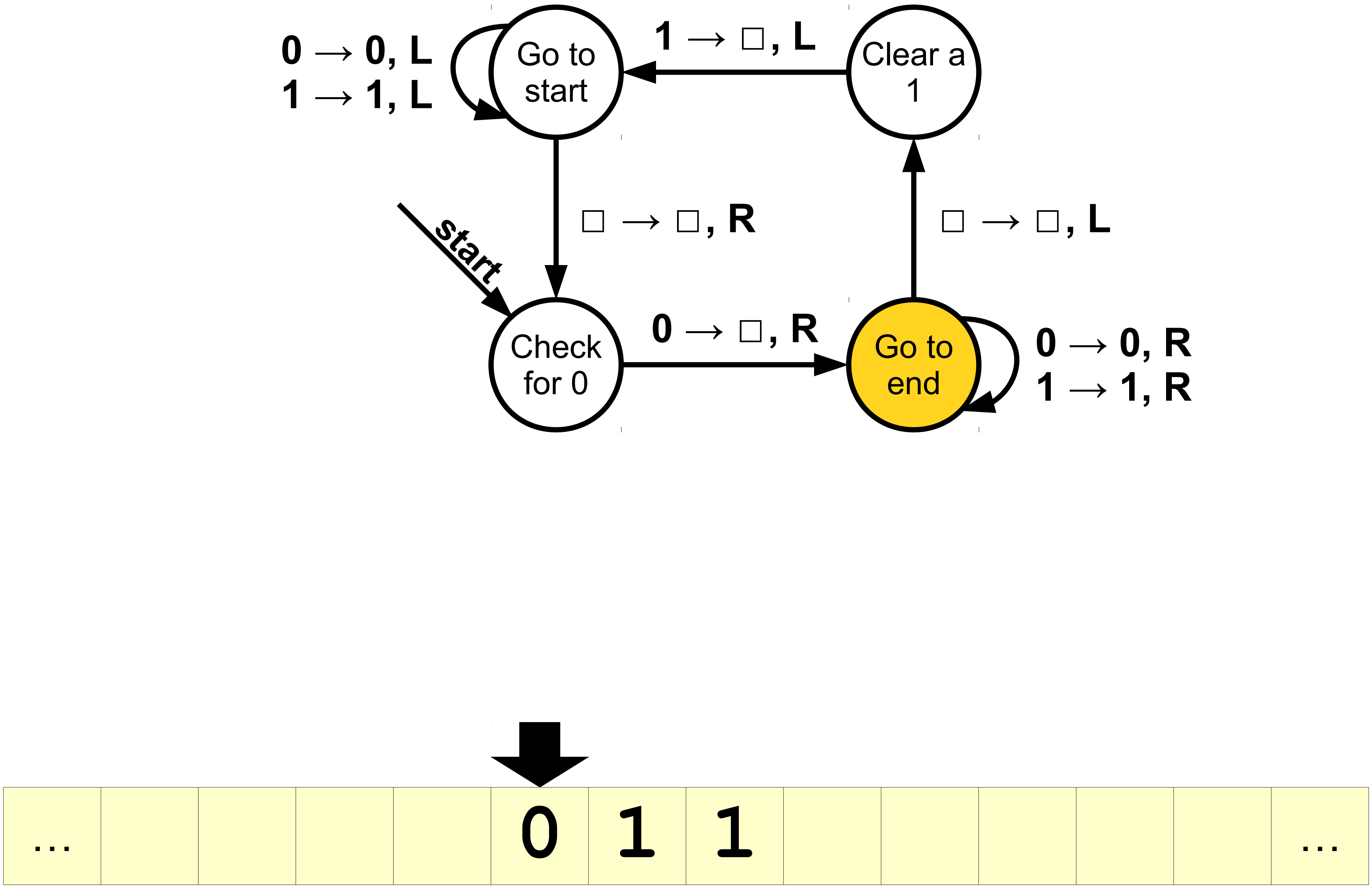
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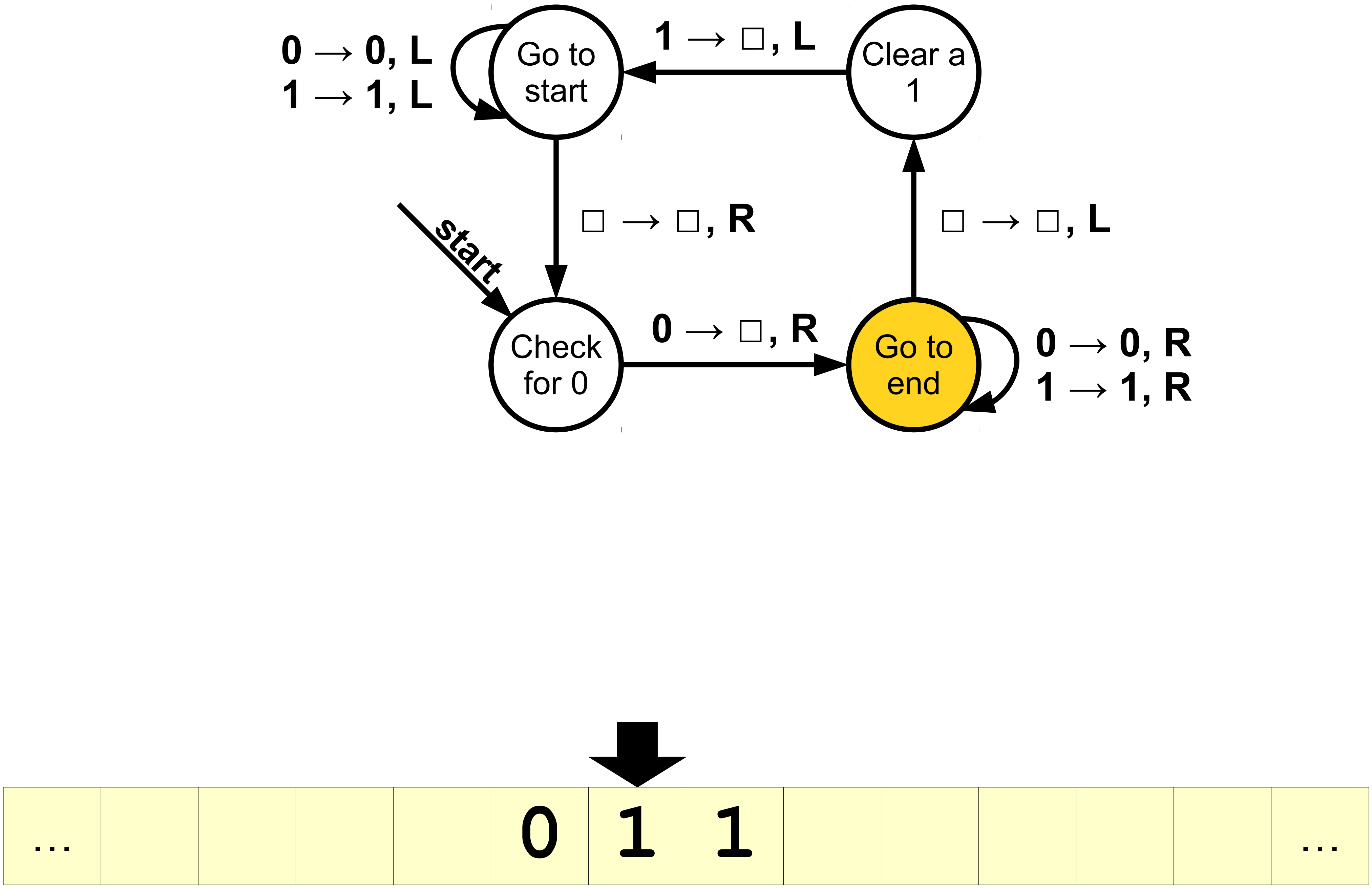
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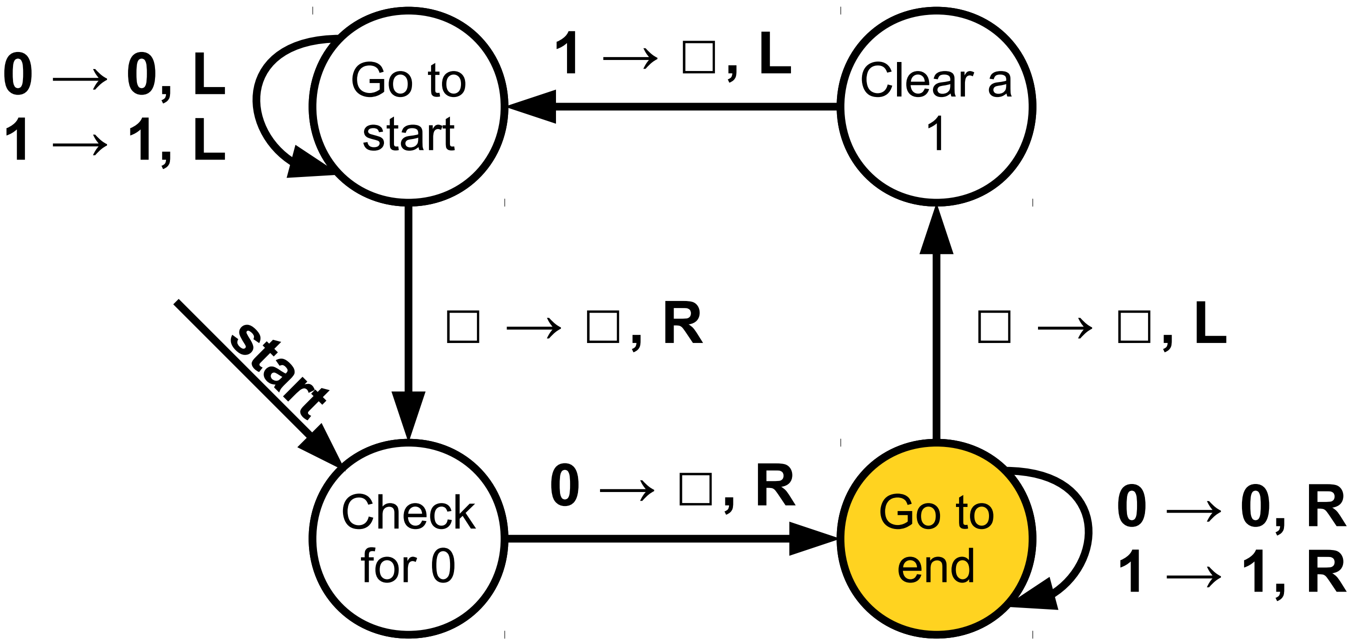
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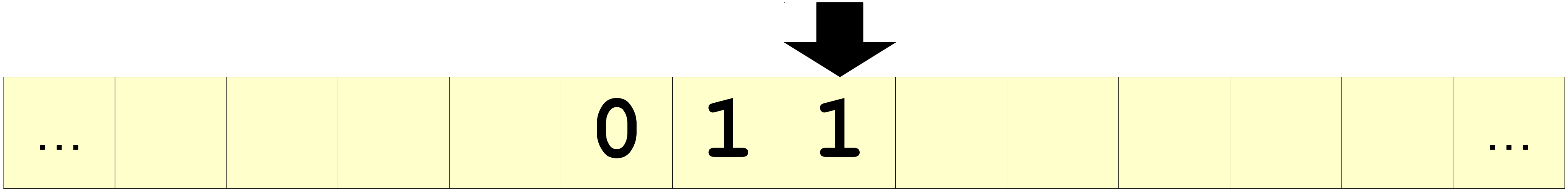
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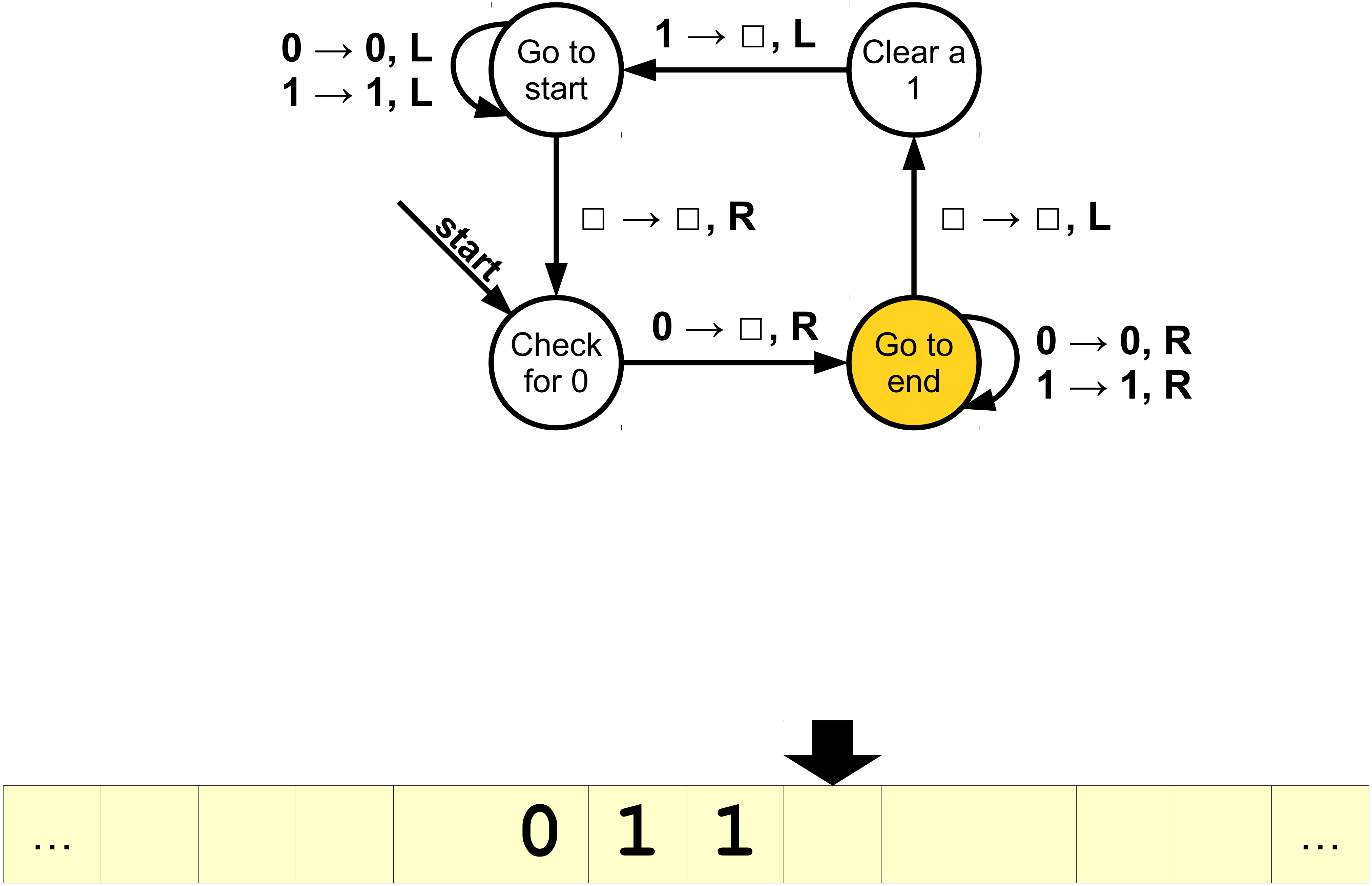
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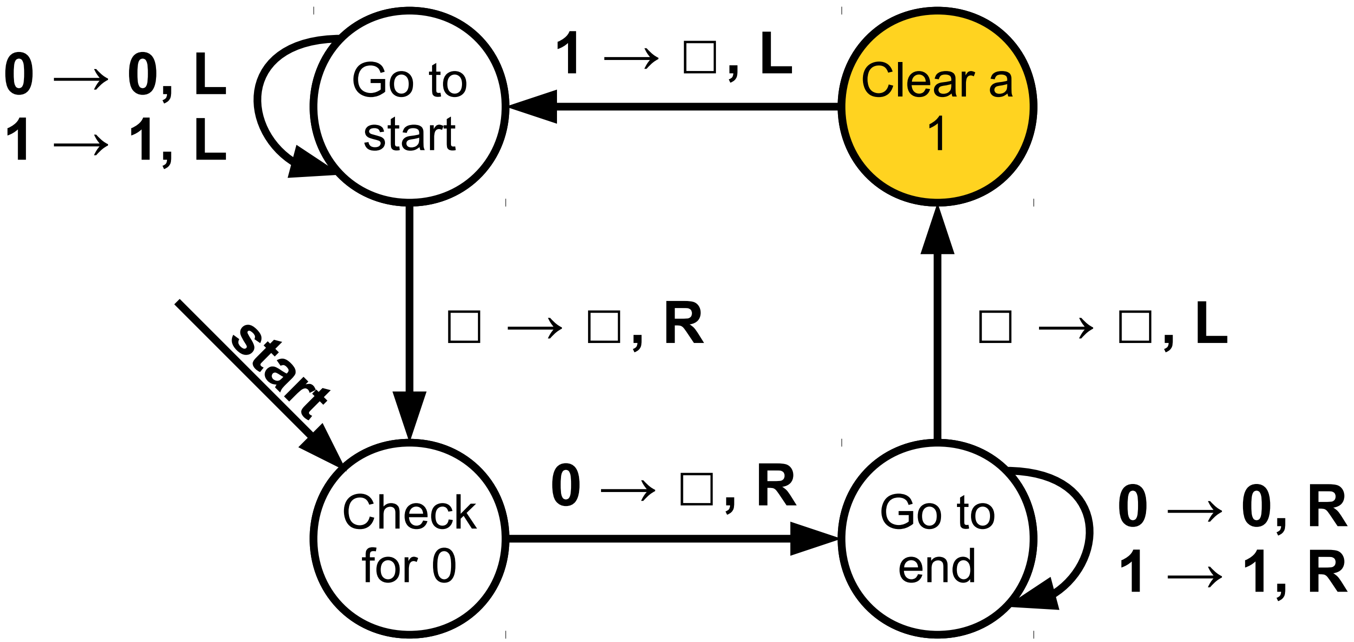
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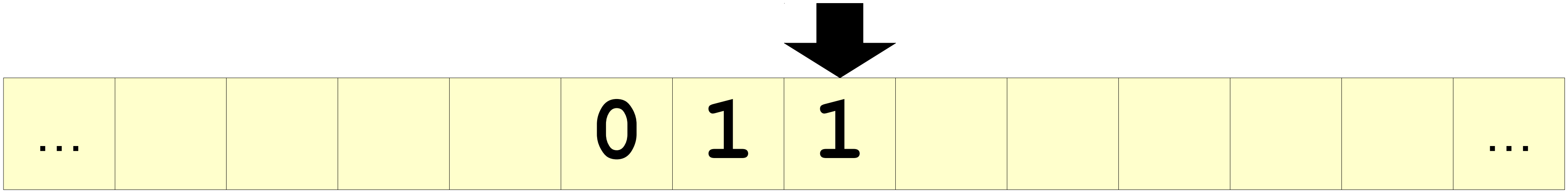
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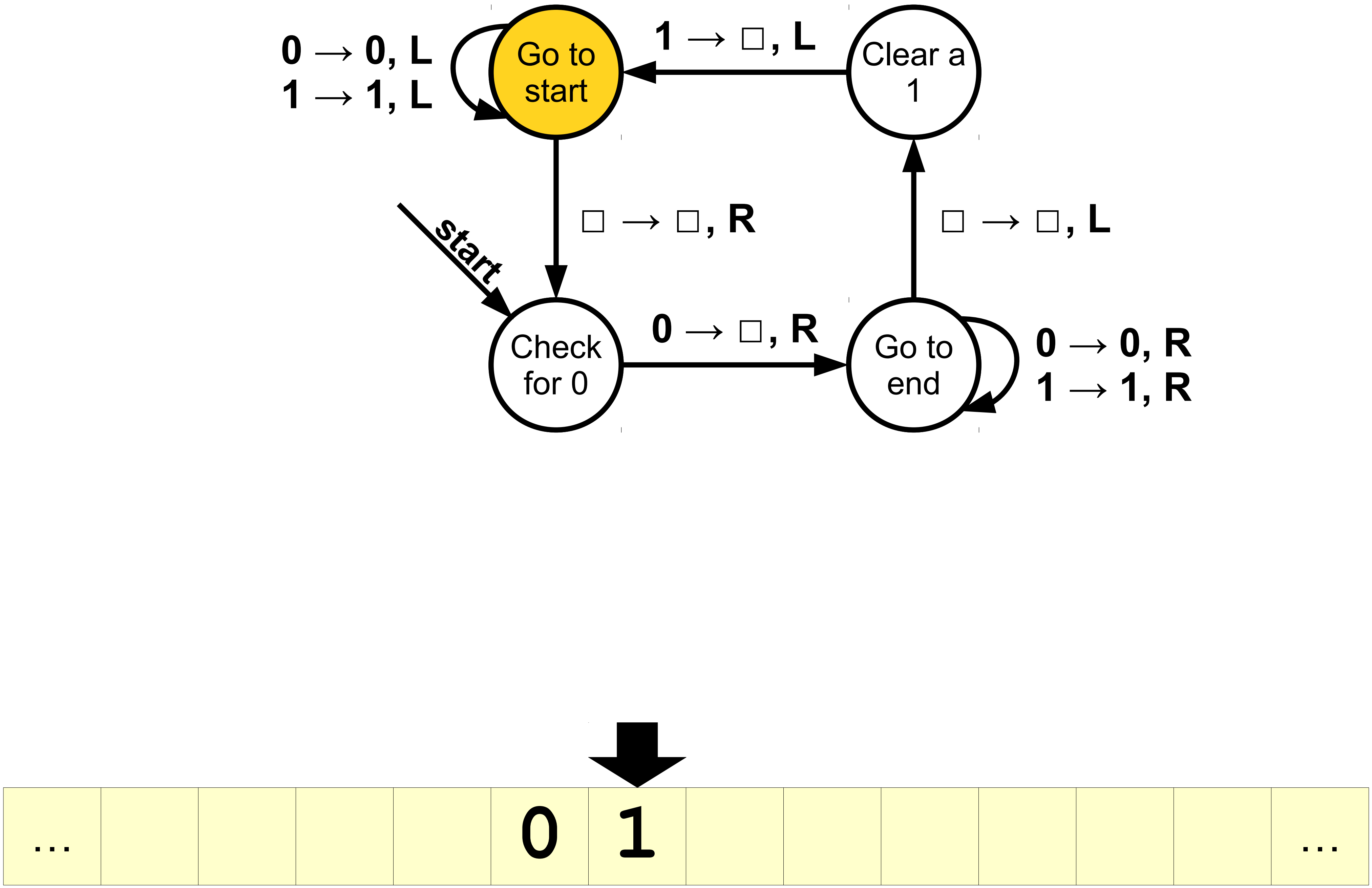
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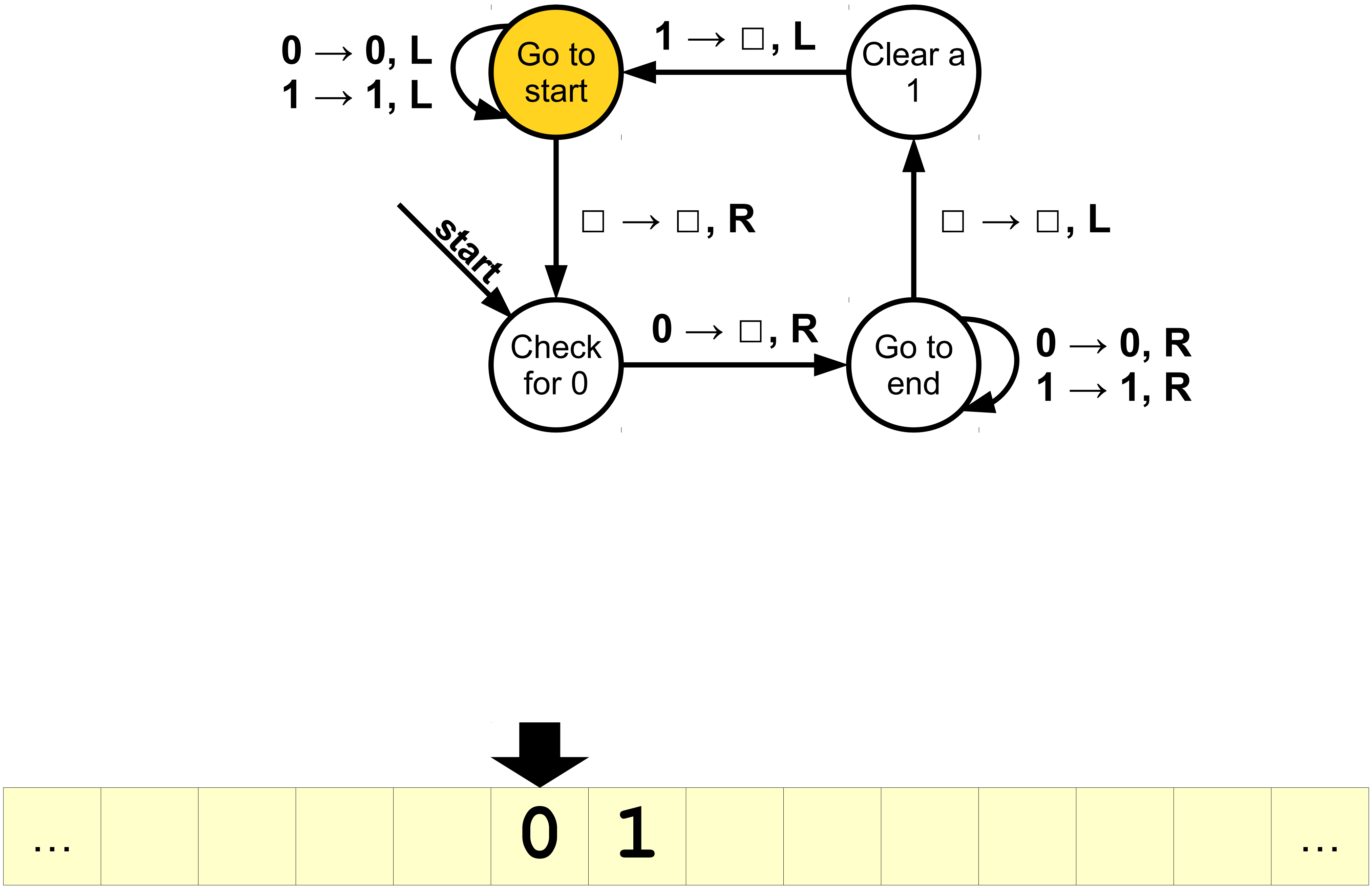
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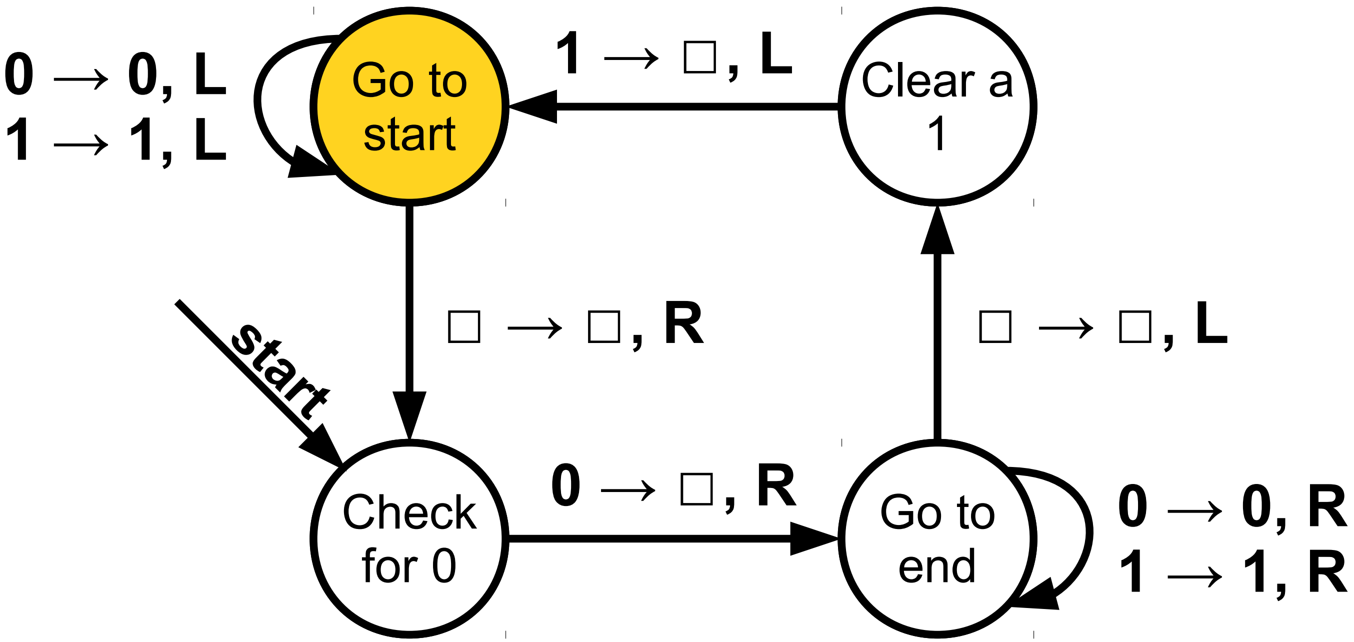
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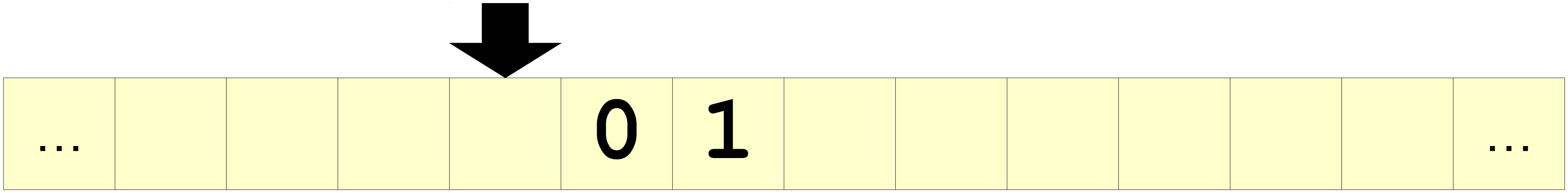
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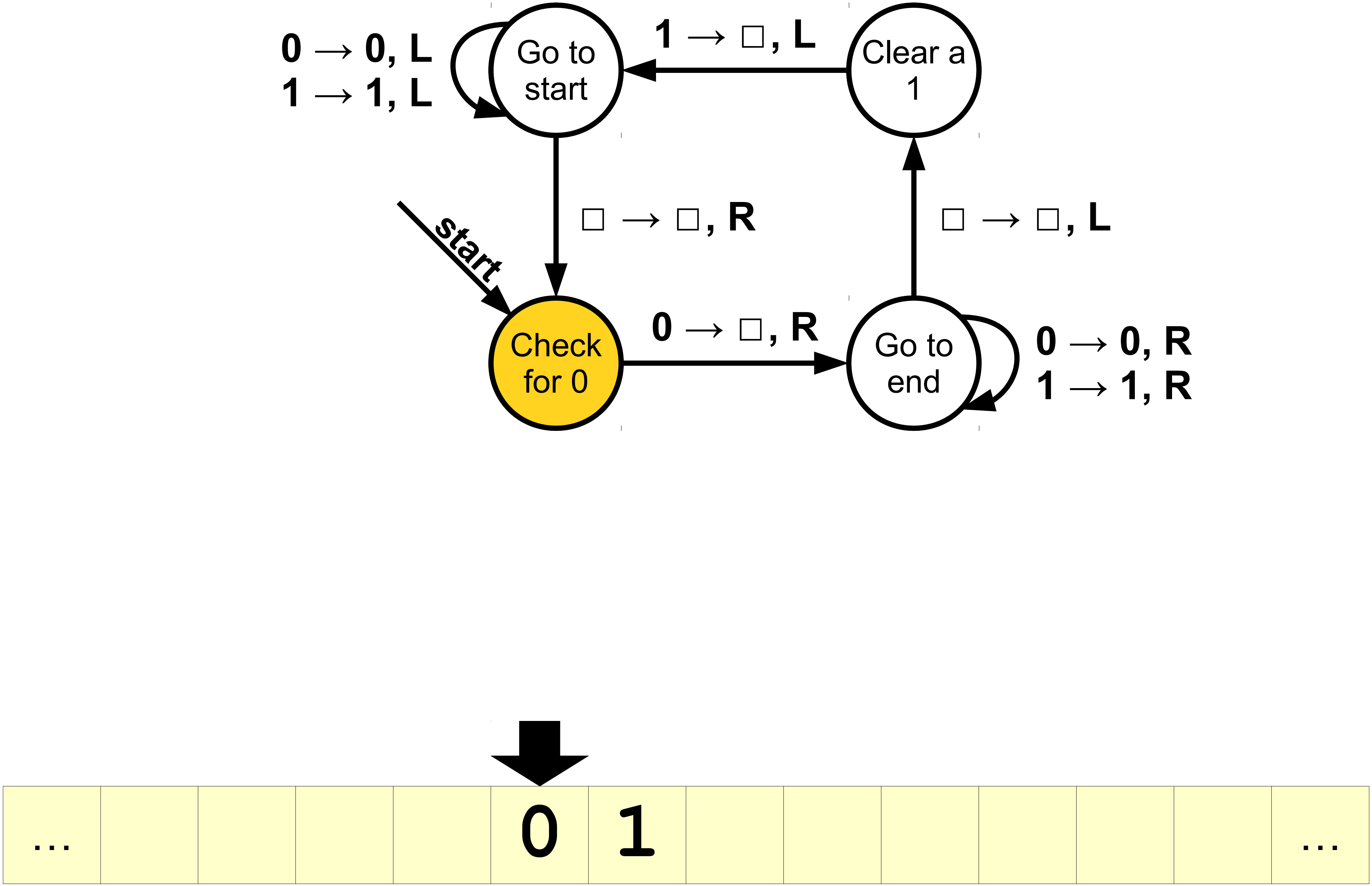
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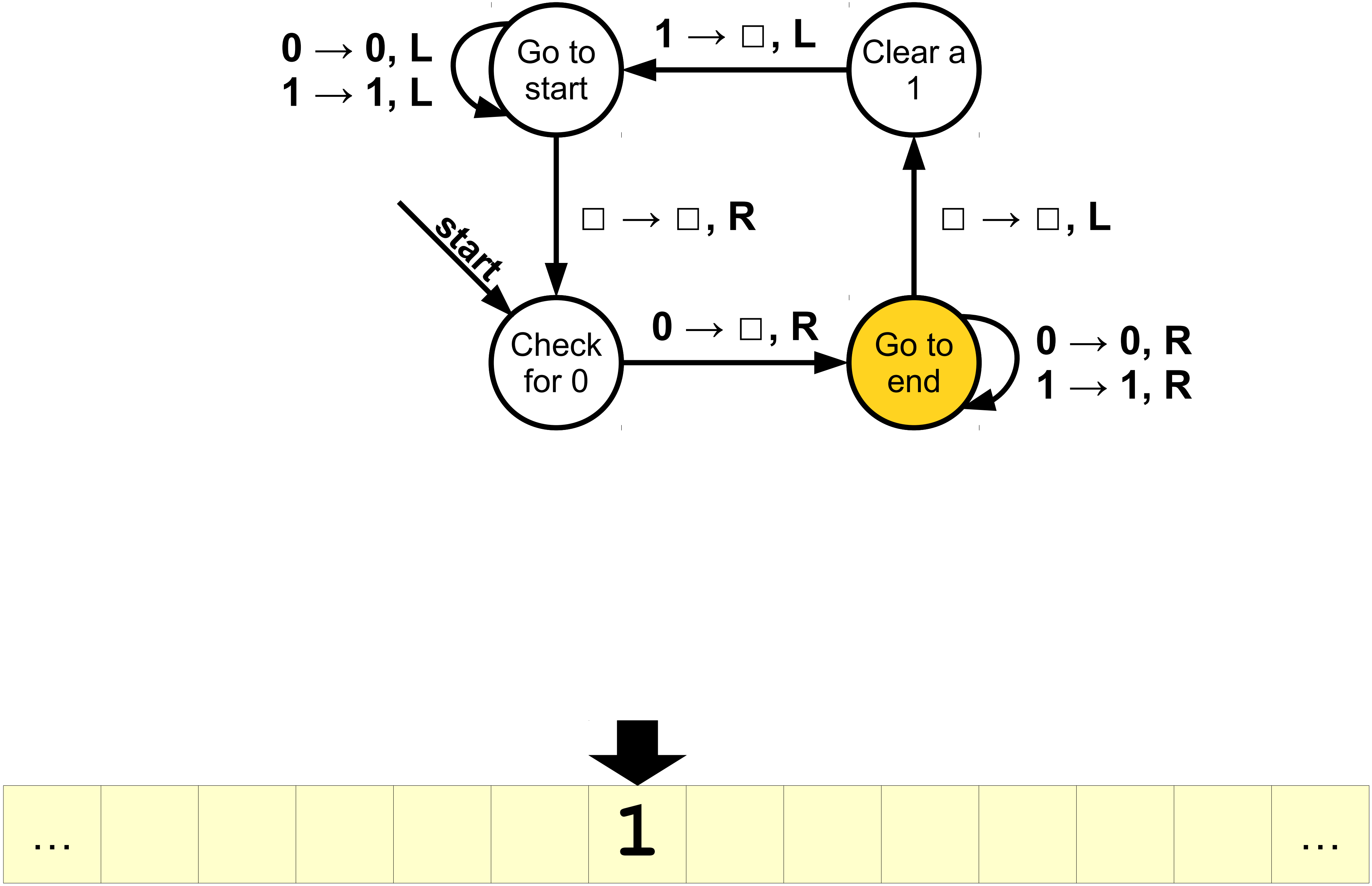
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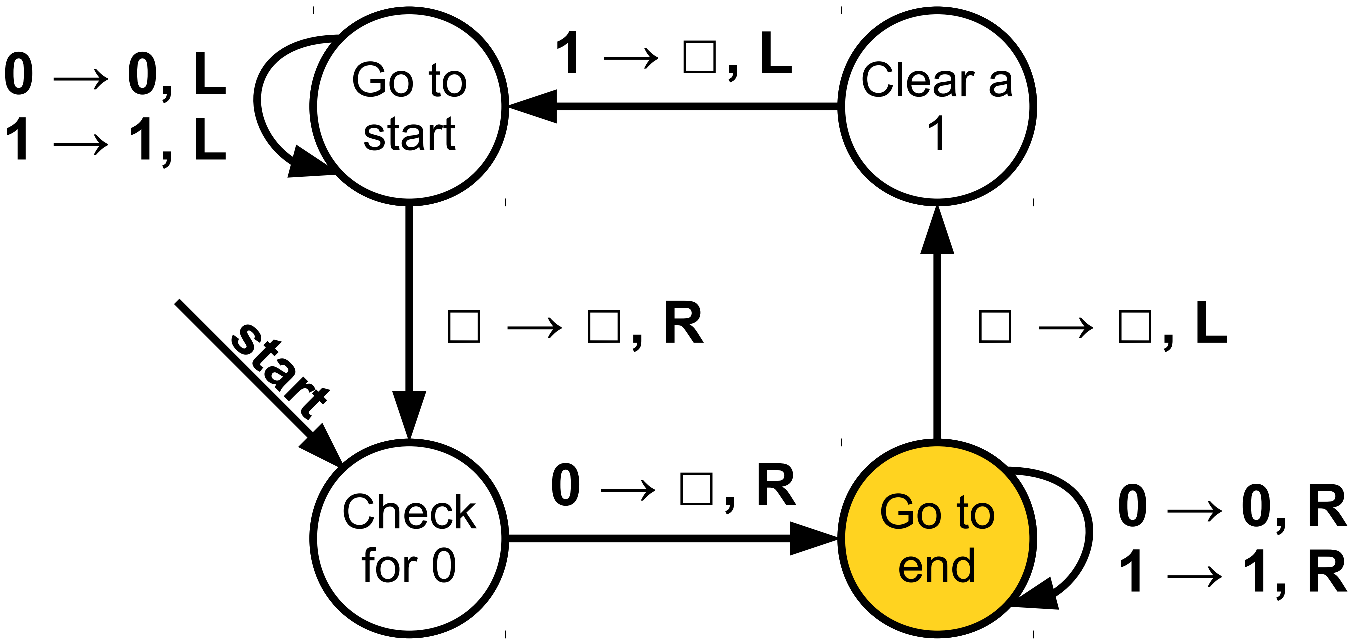
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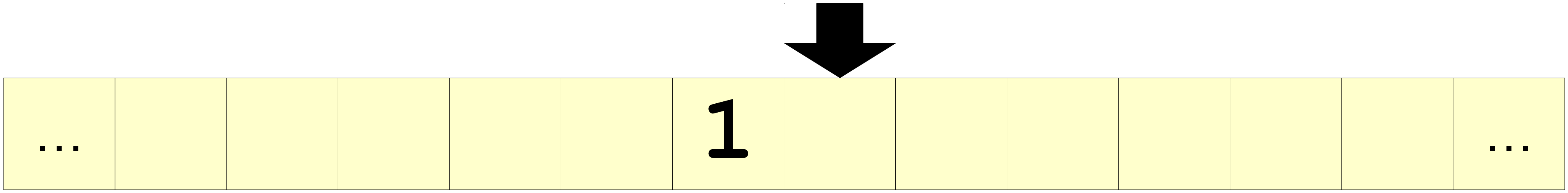
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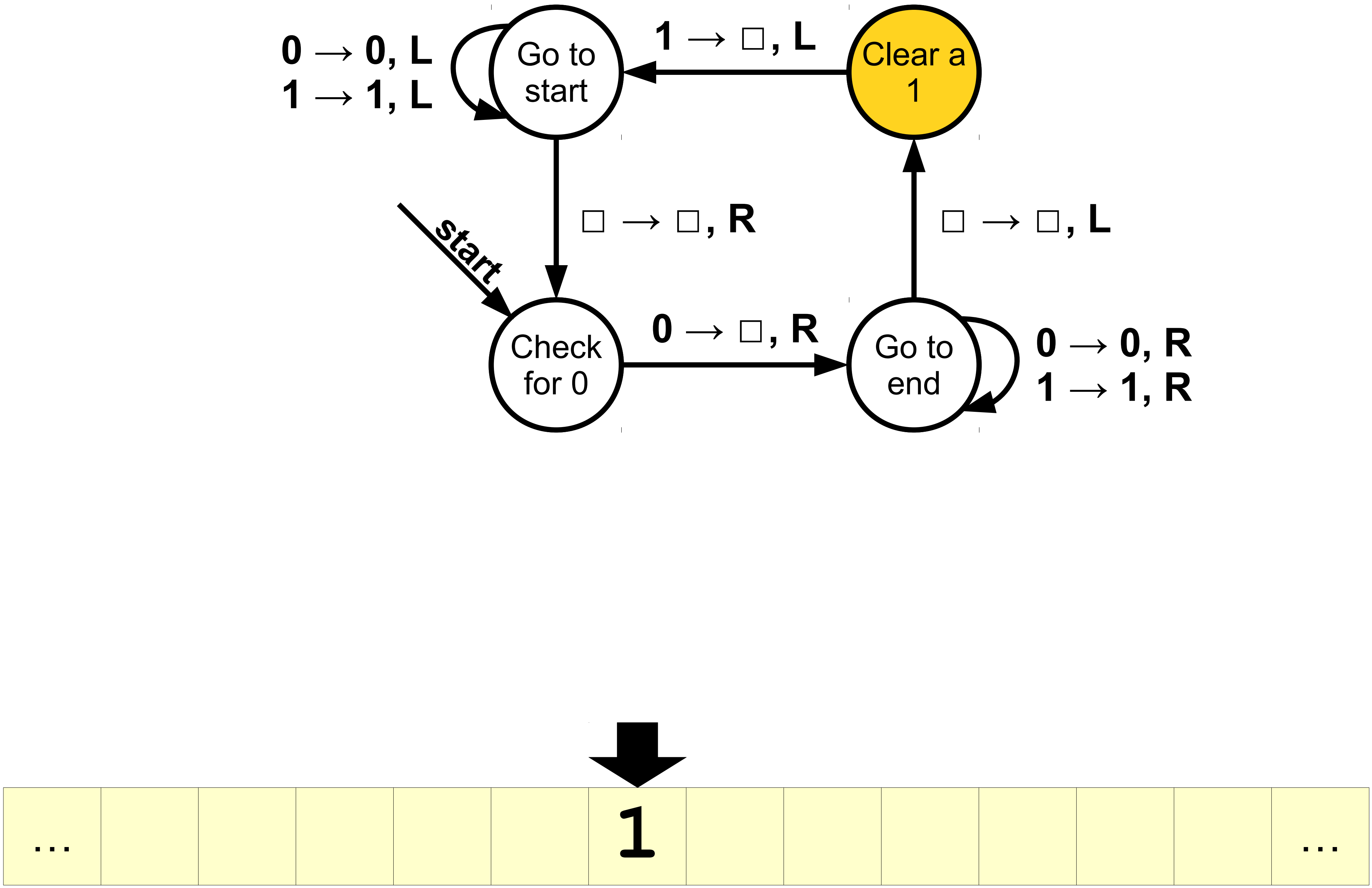
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1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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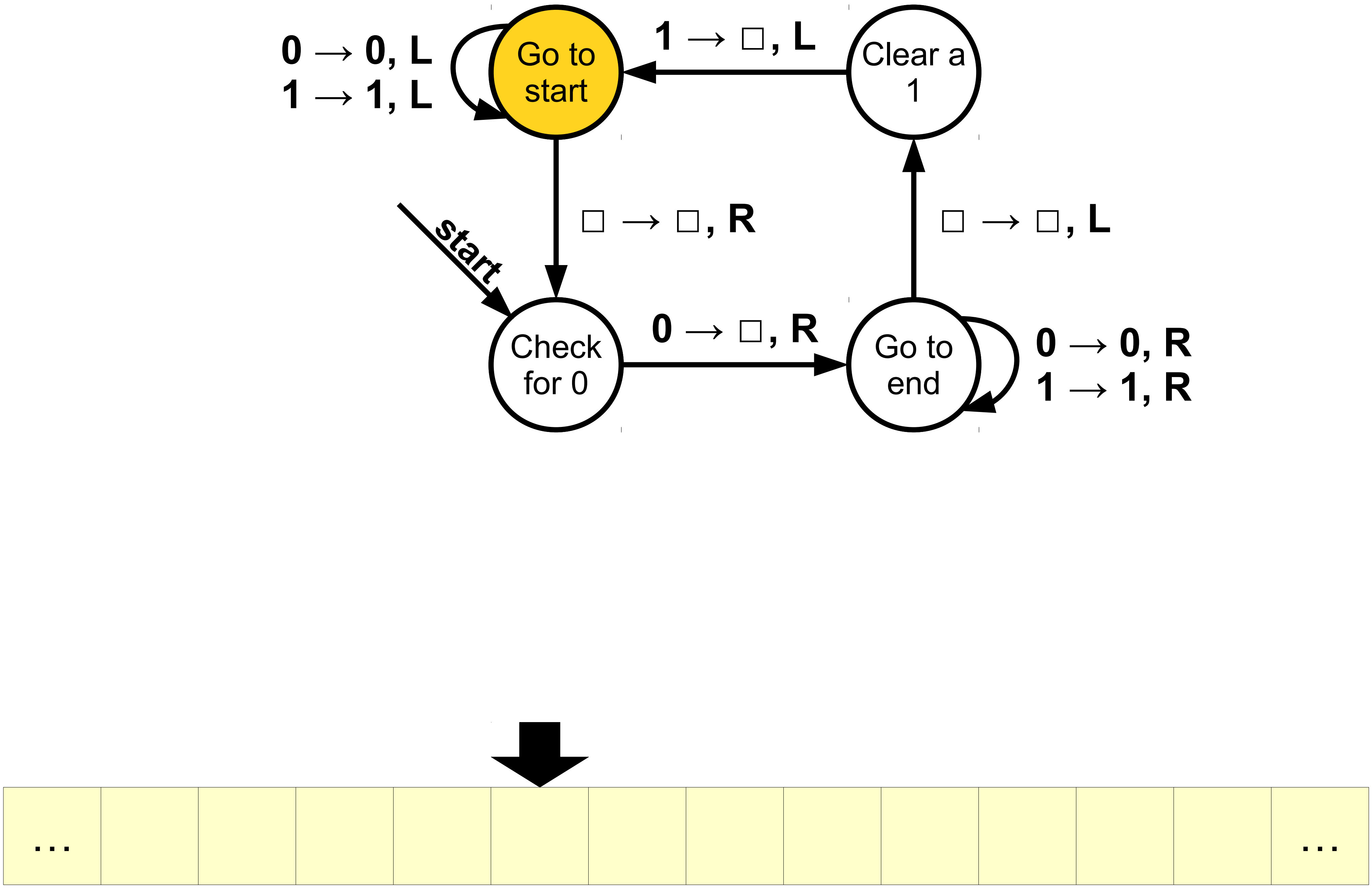
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1 → 1, L
Go to start
1 → □, L
Clear a 1
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Check for 0
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Go to end
0 → 0, R
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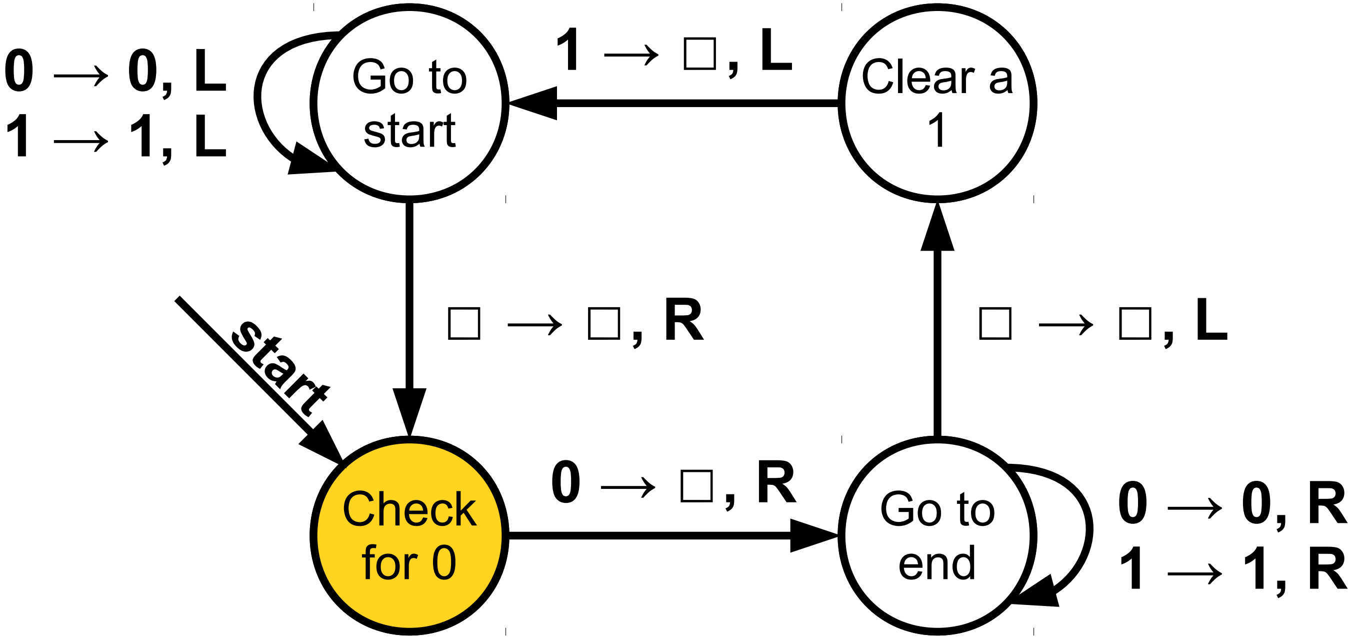
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Go to start
1 → □, L
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Go to end
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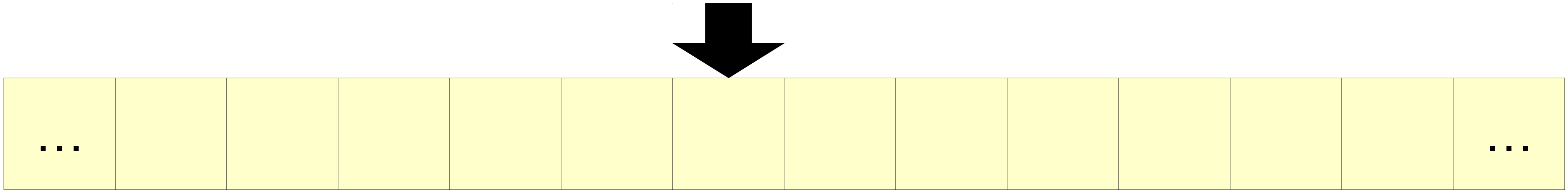
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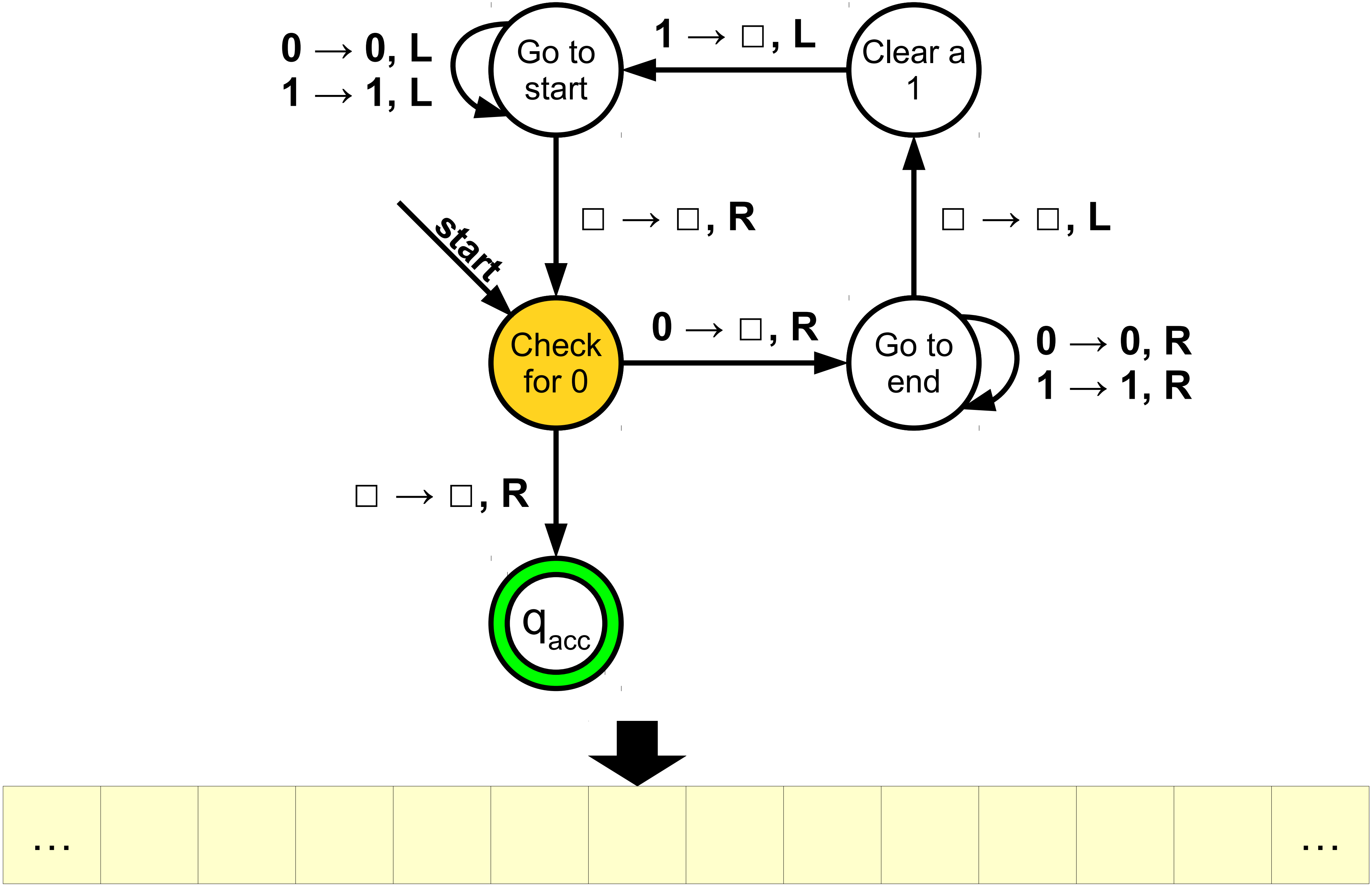
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Go to start
1 → □, L
Clear a 1
□ → □, R
start
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0 → □, R
Go to end
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1 → 1, R
□ → □, R
q_{acc}
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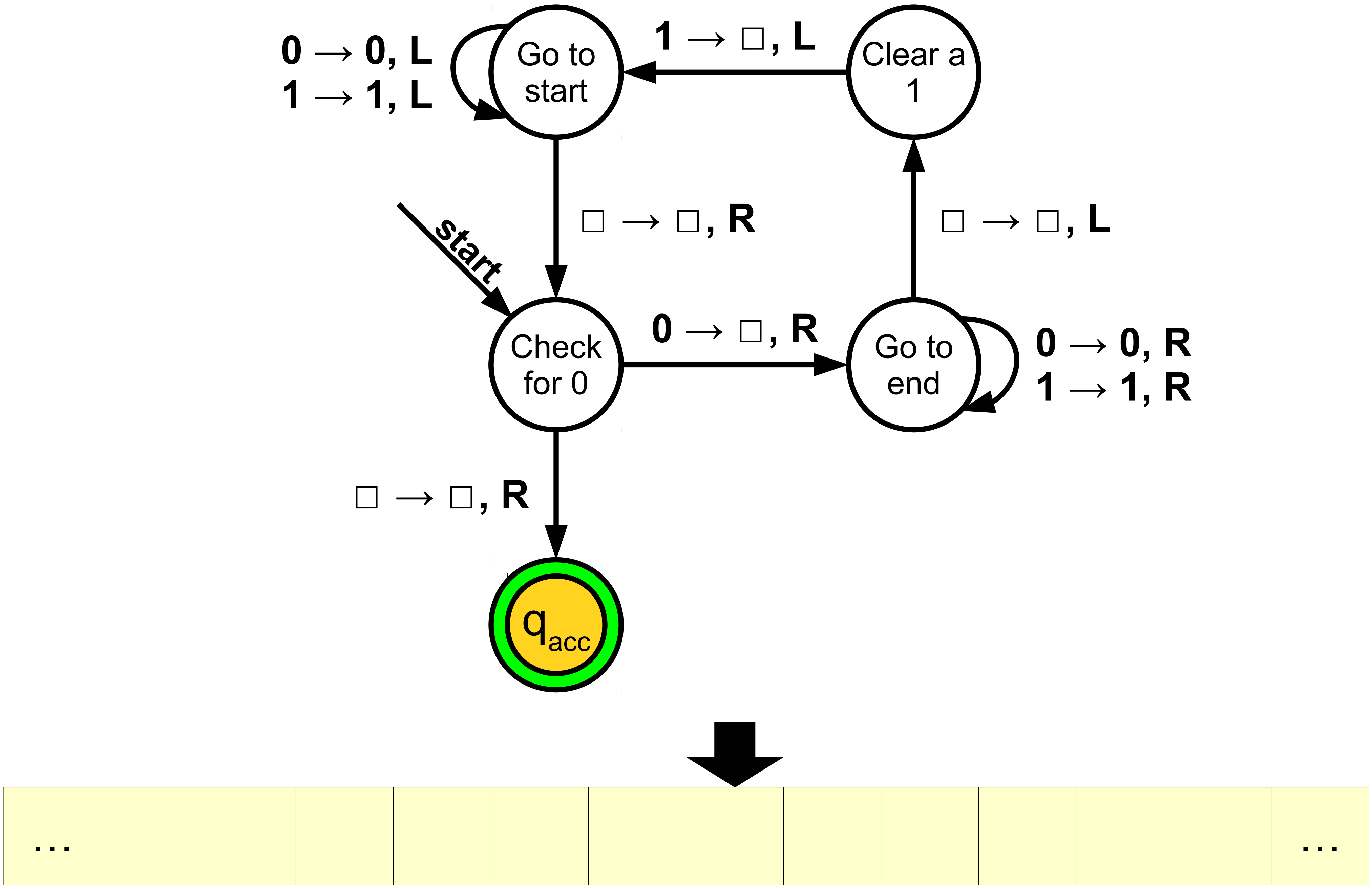

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Clear a 1
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Go to end
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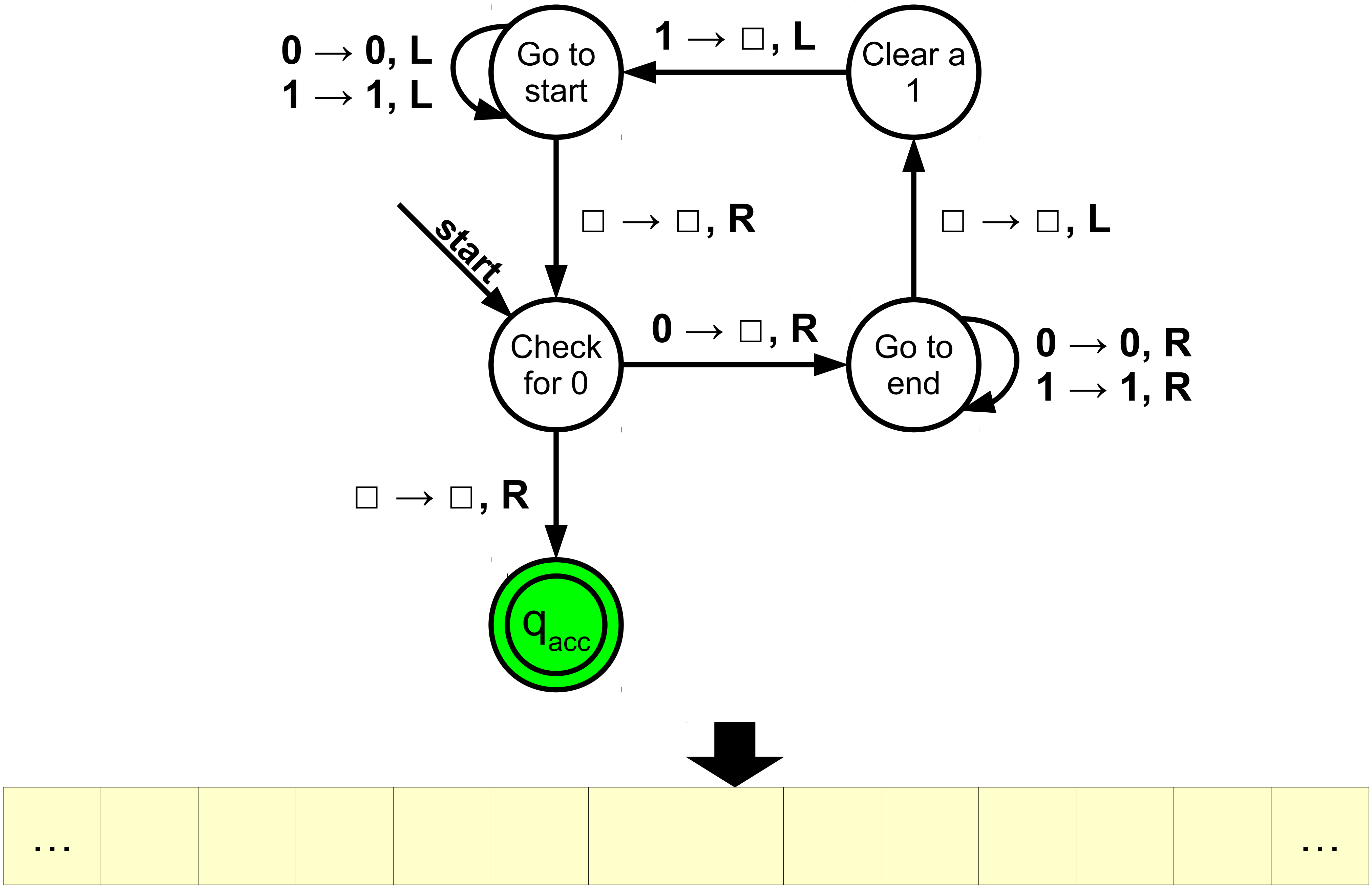
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□ → □, R
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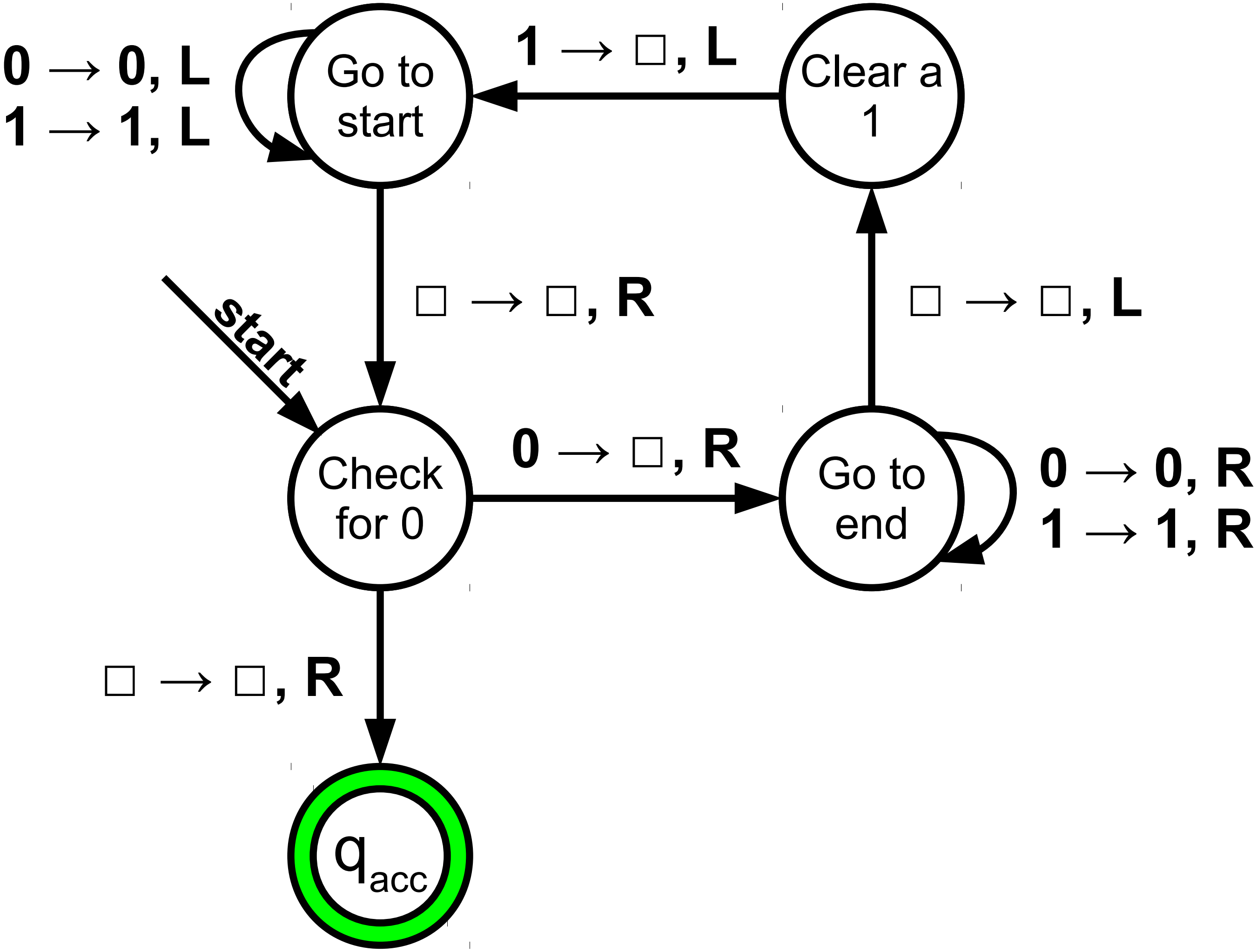
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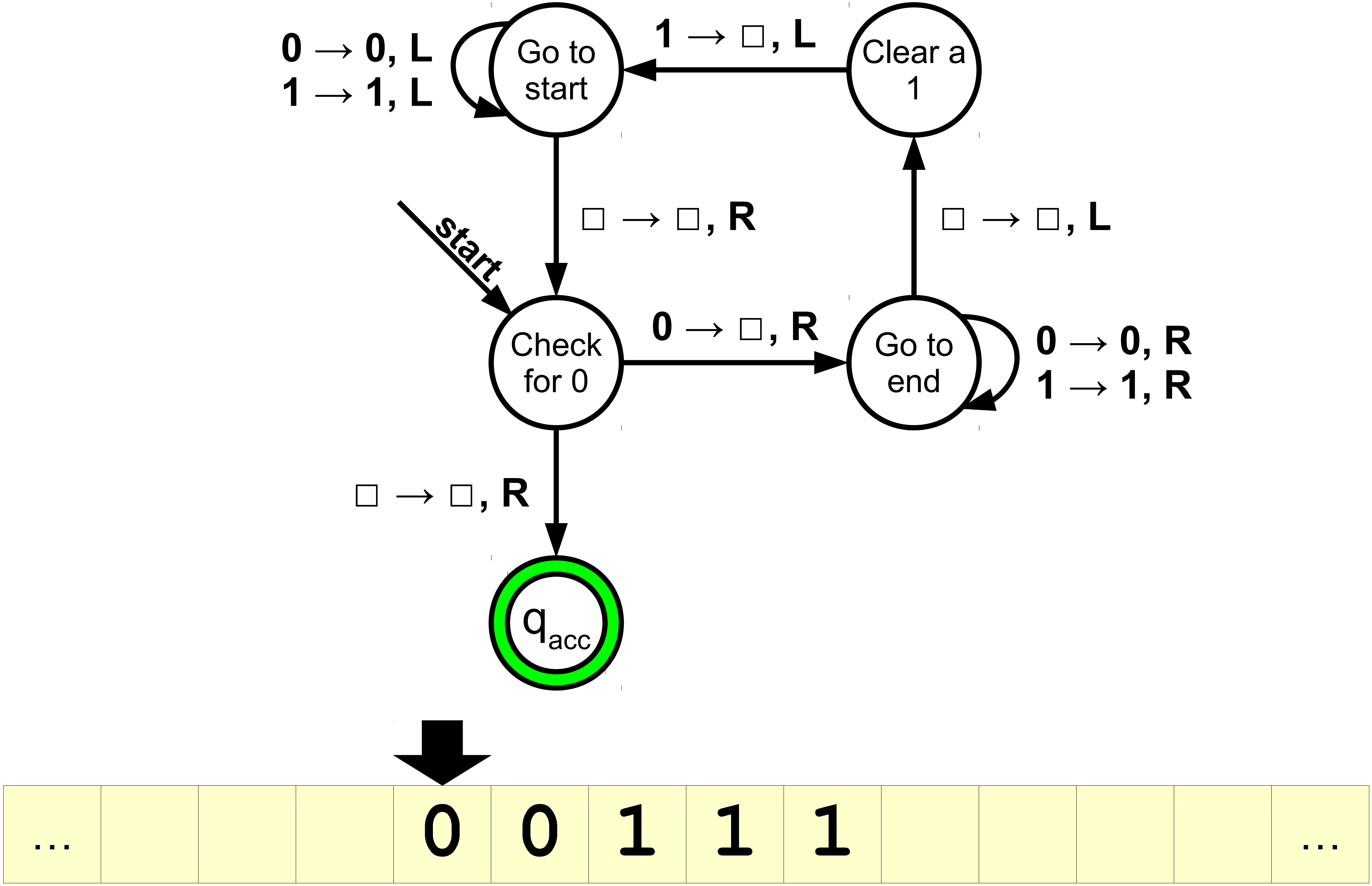
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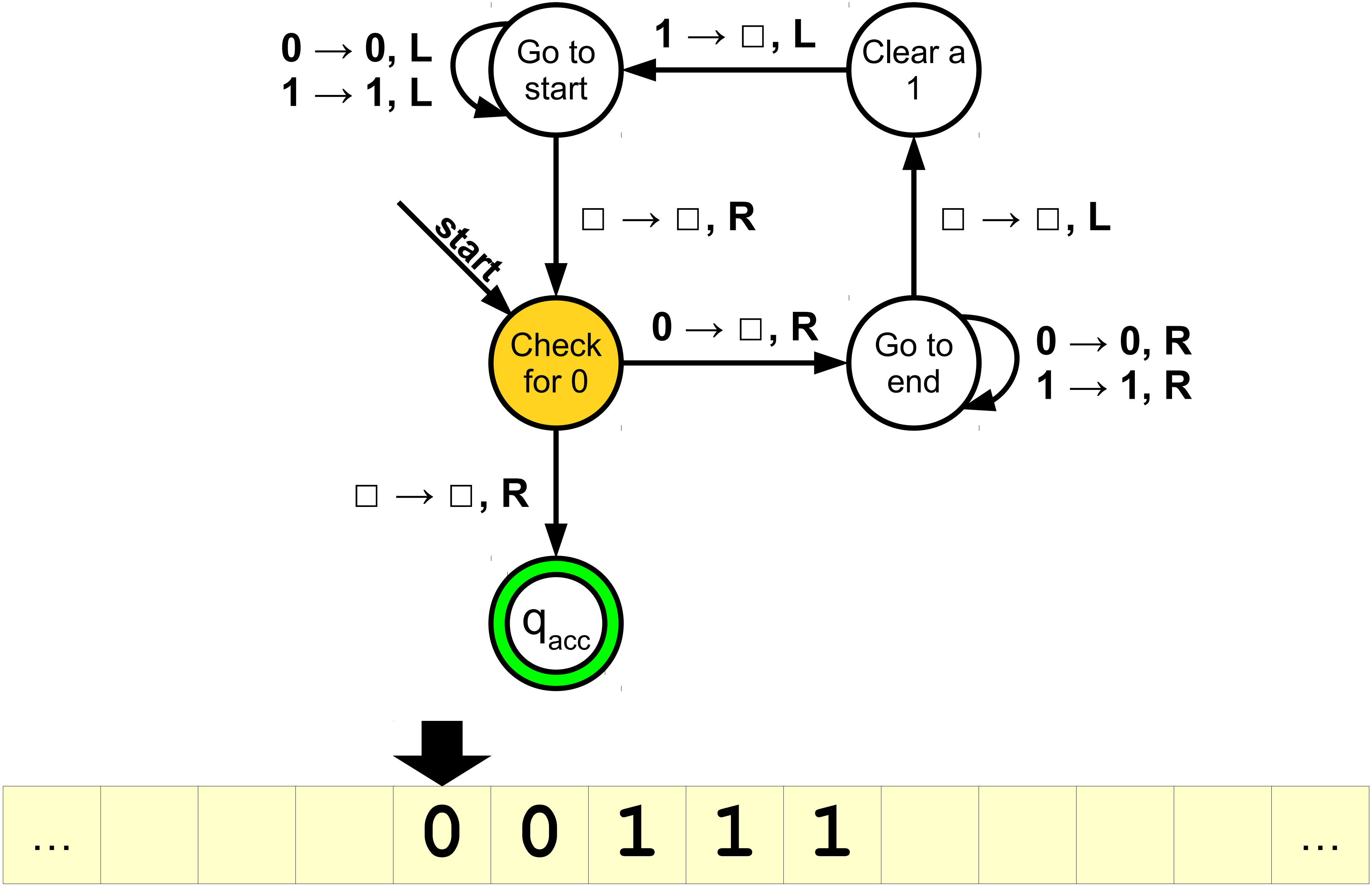

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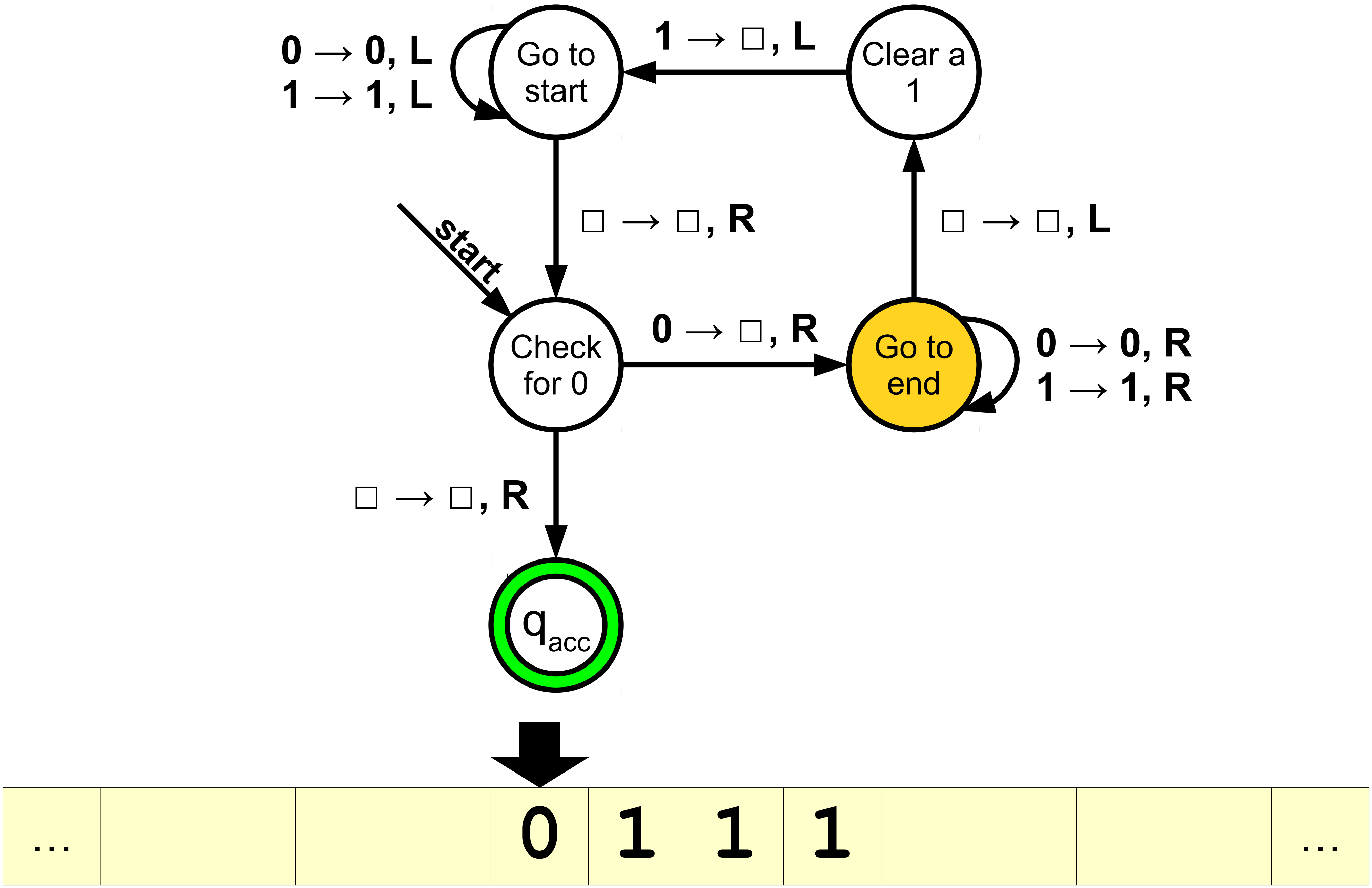
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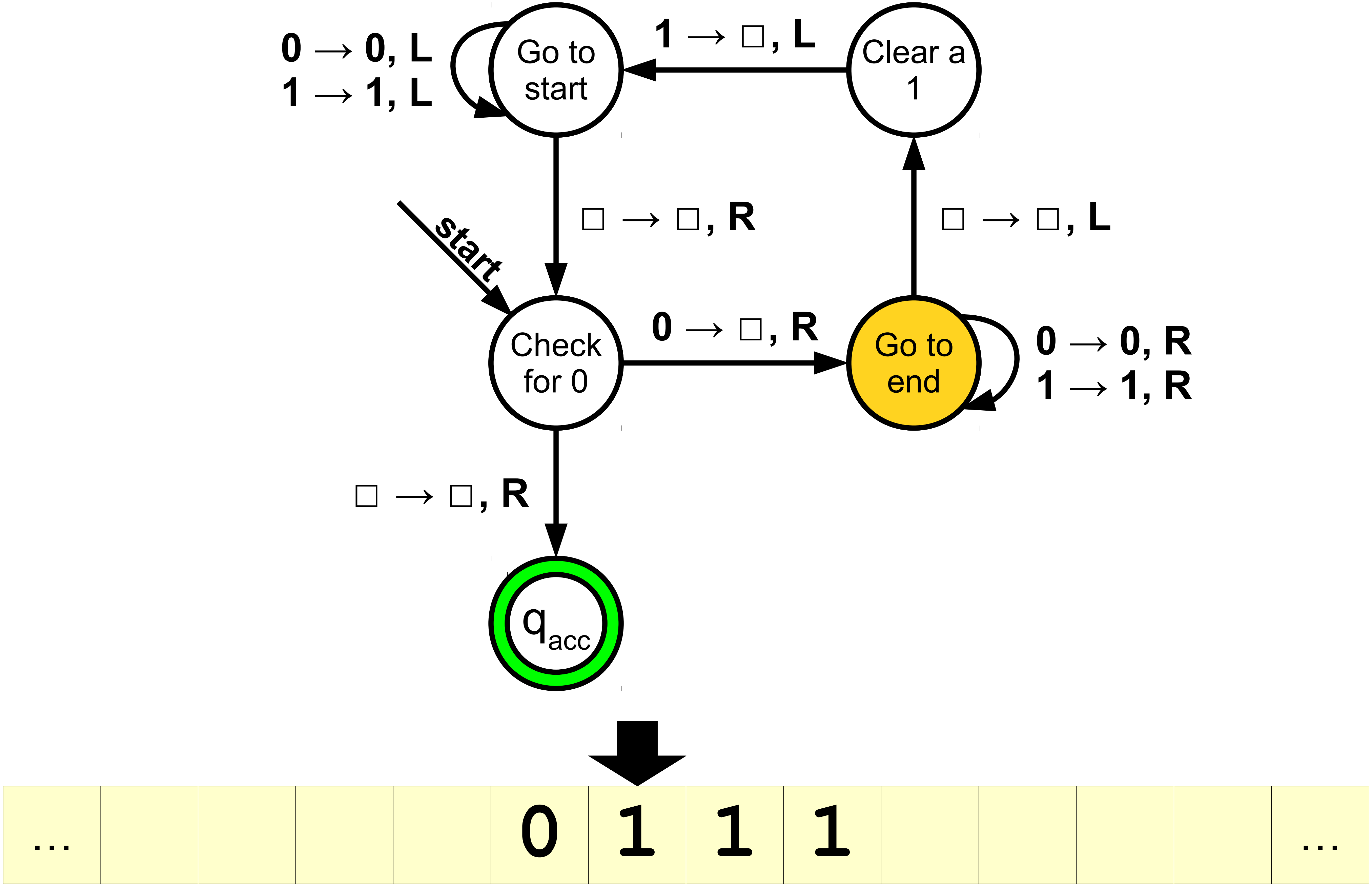
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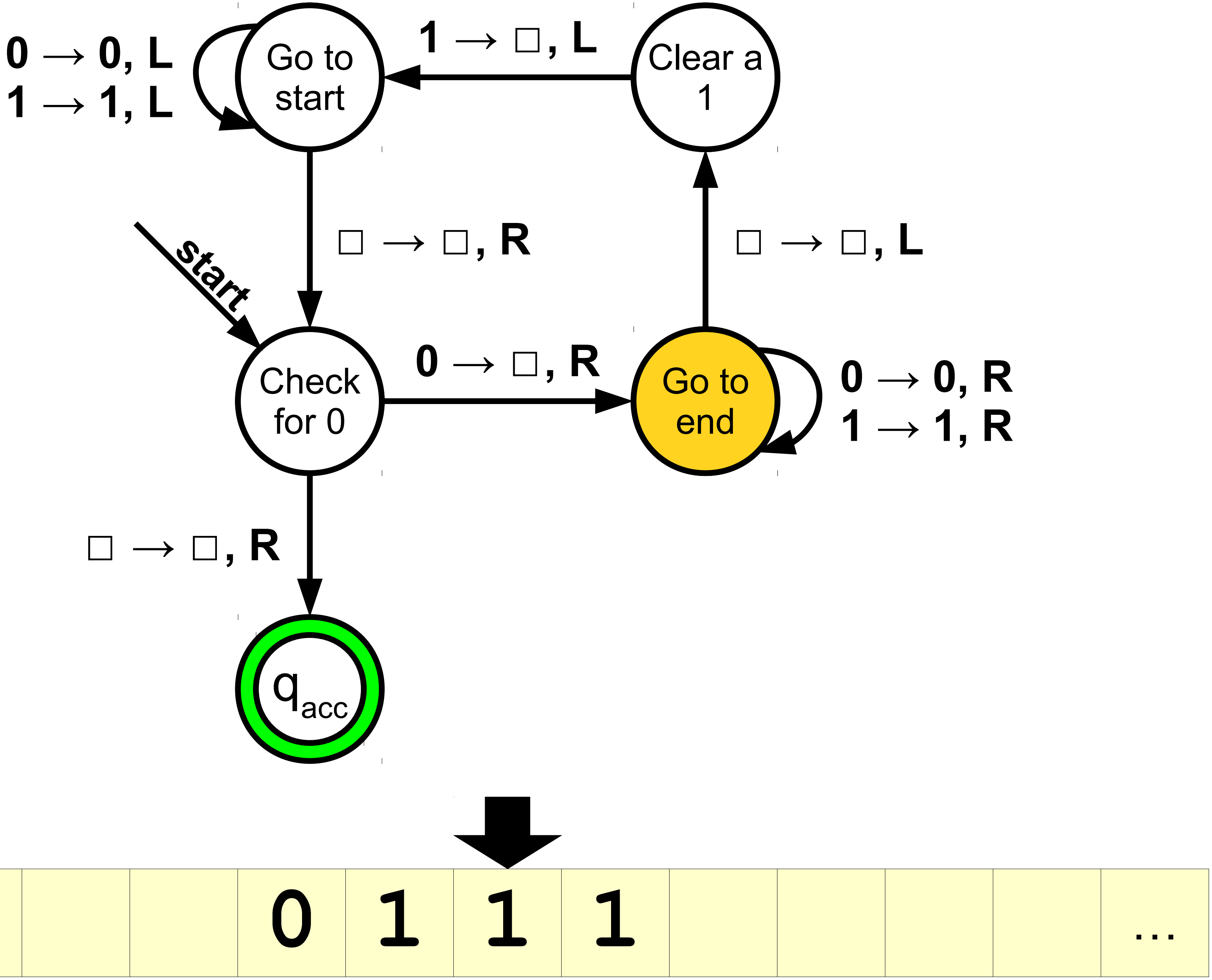
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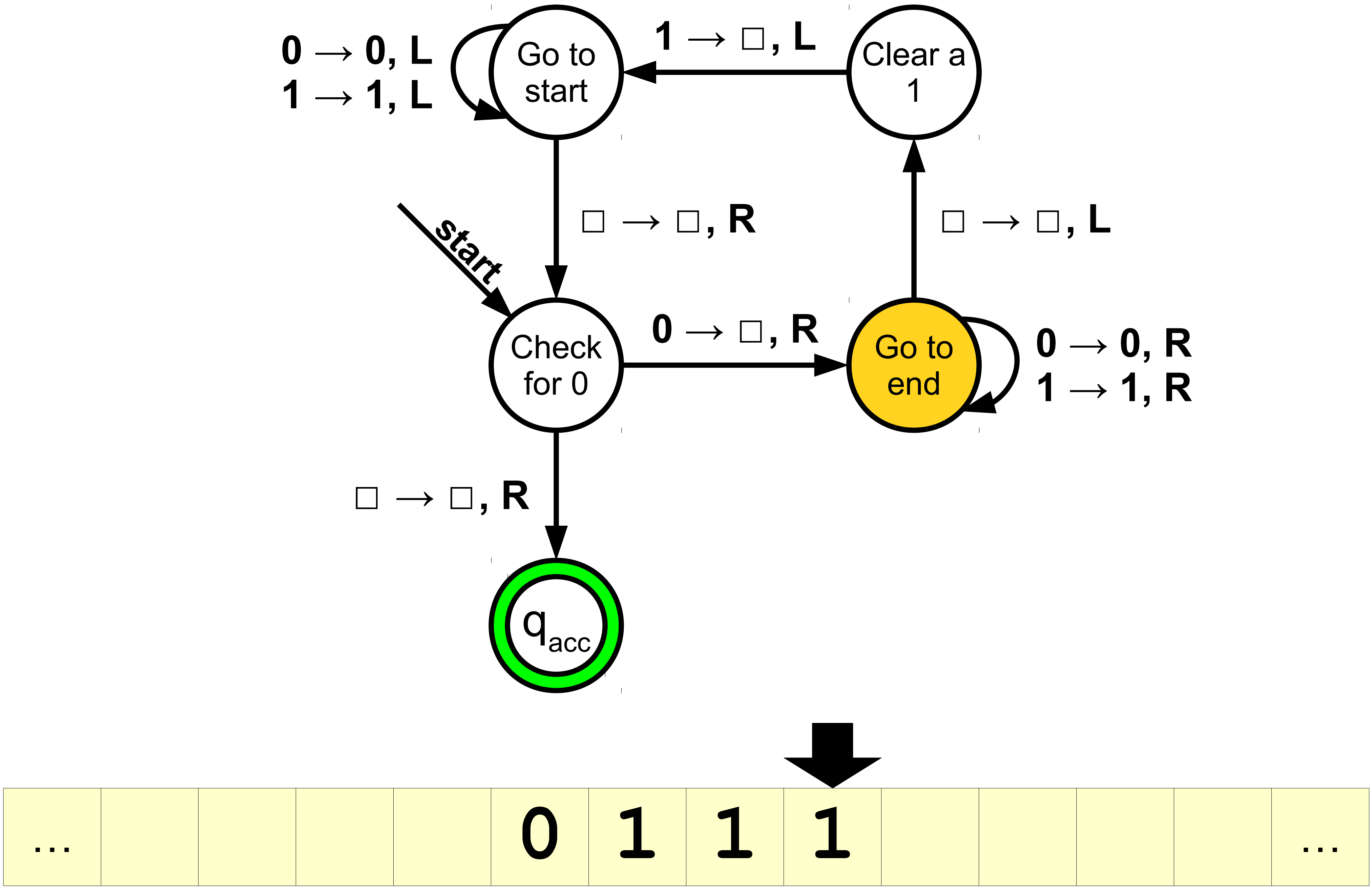
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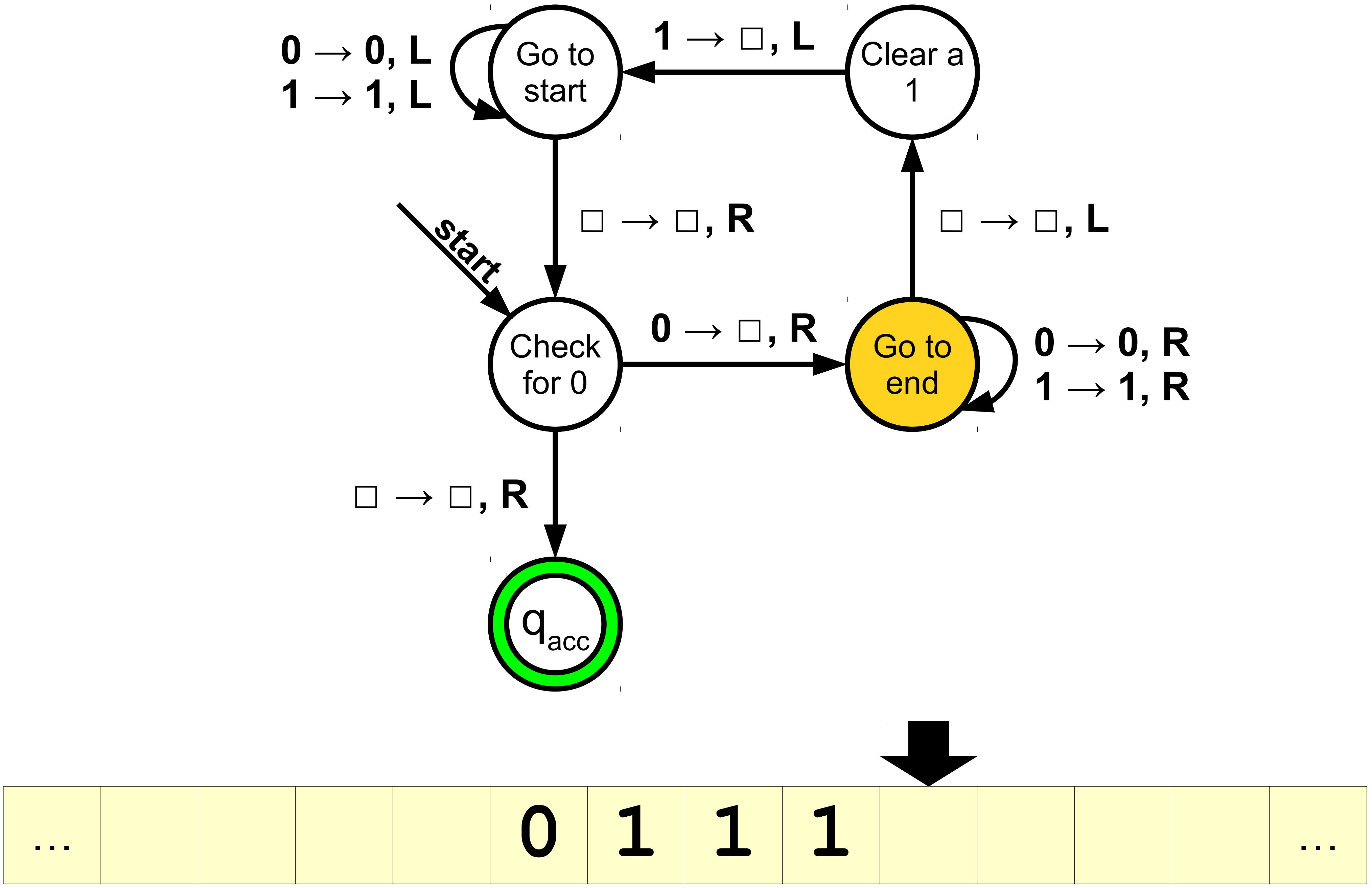
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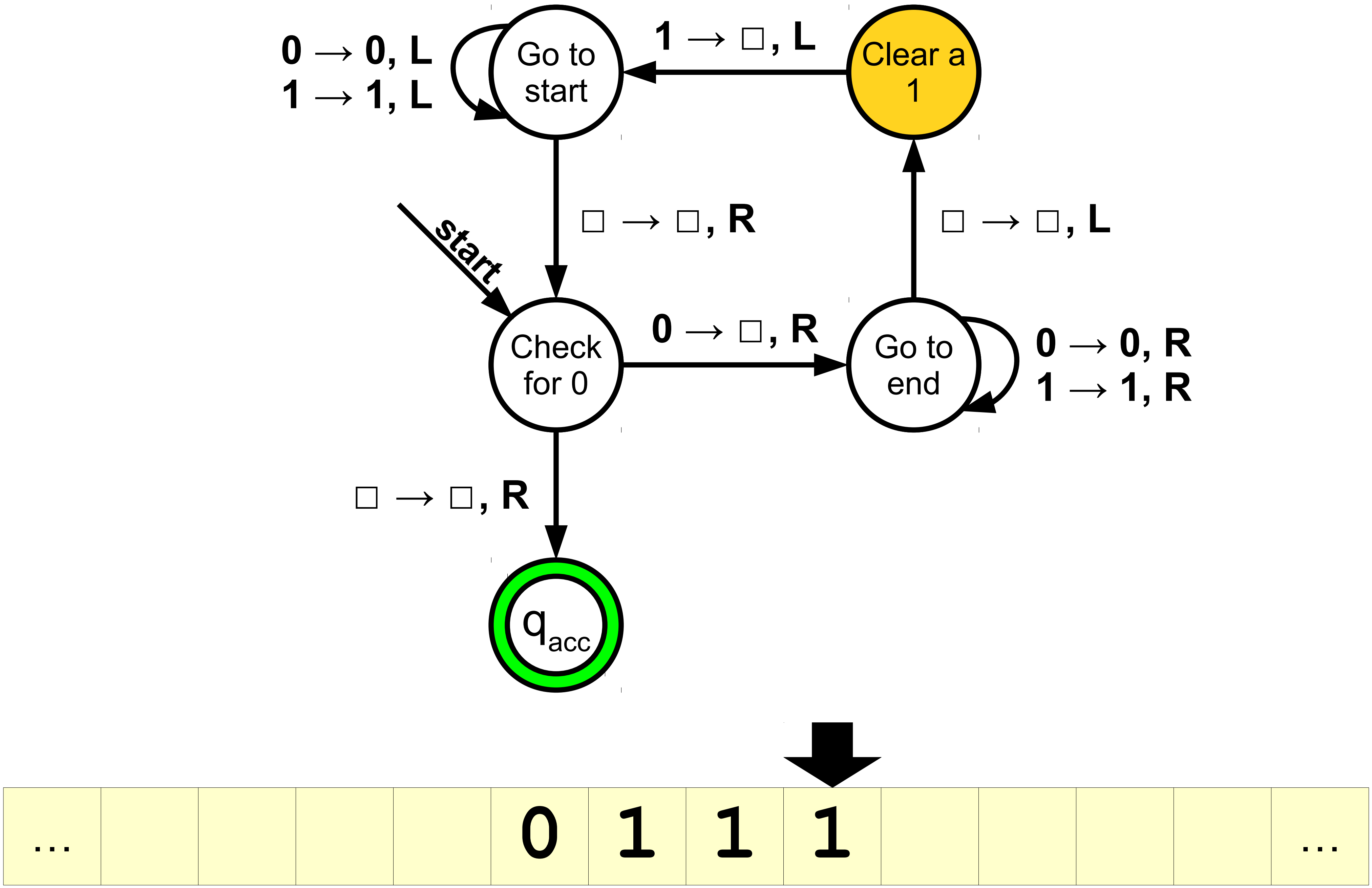
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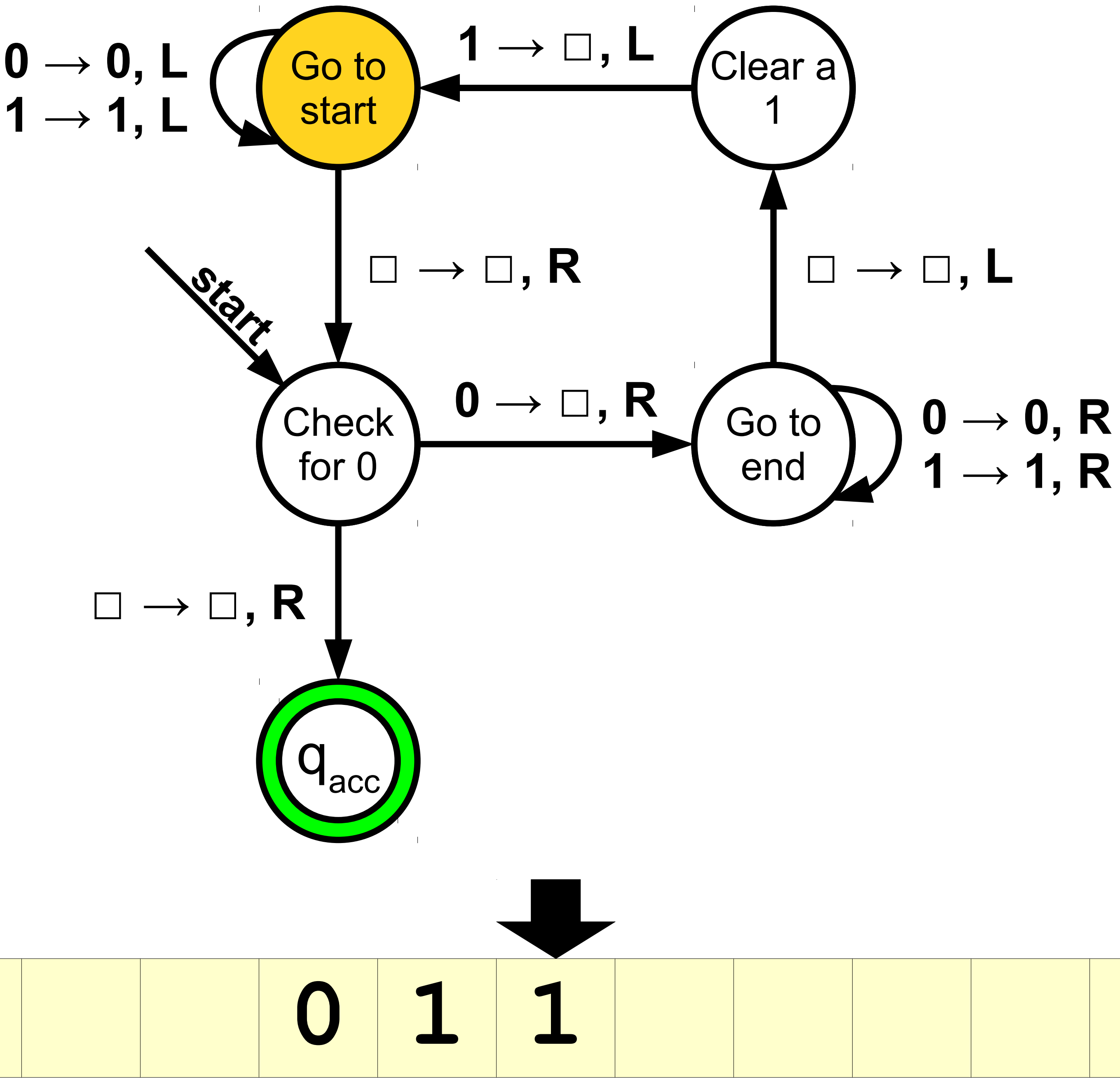
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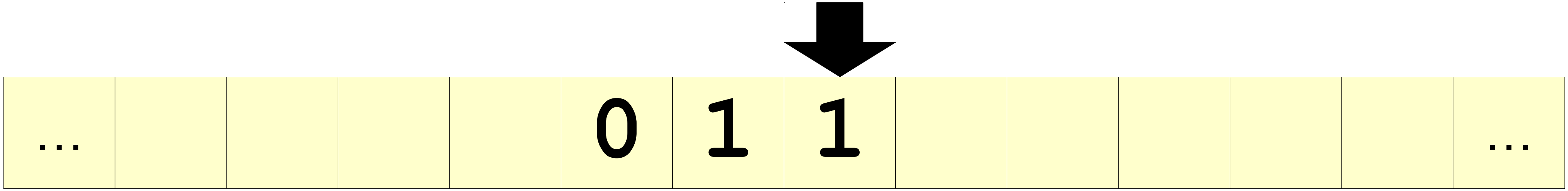
q_{acc}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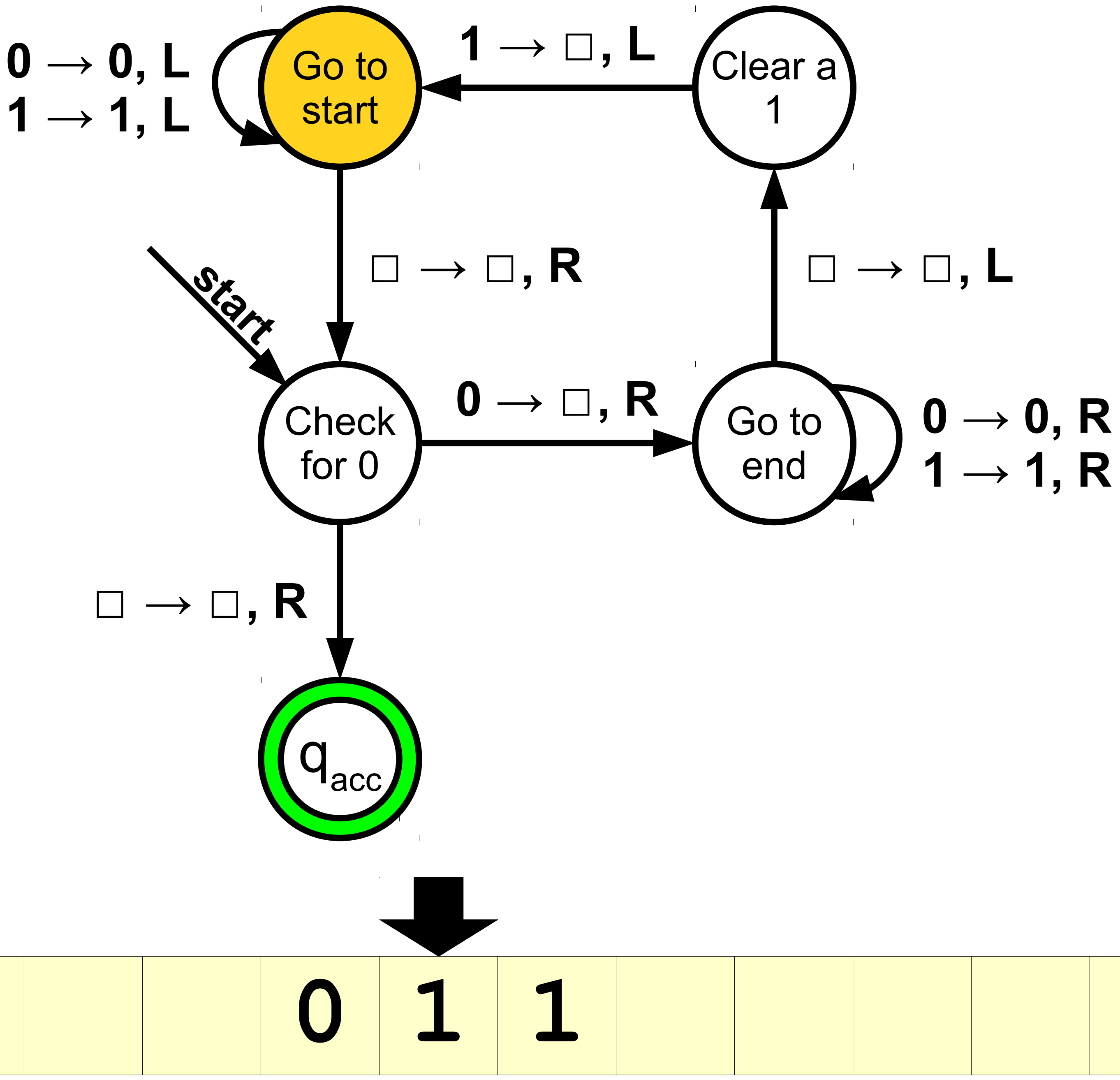
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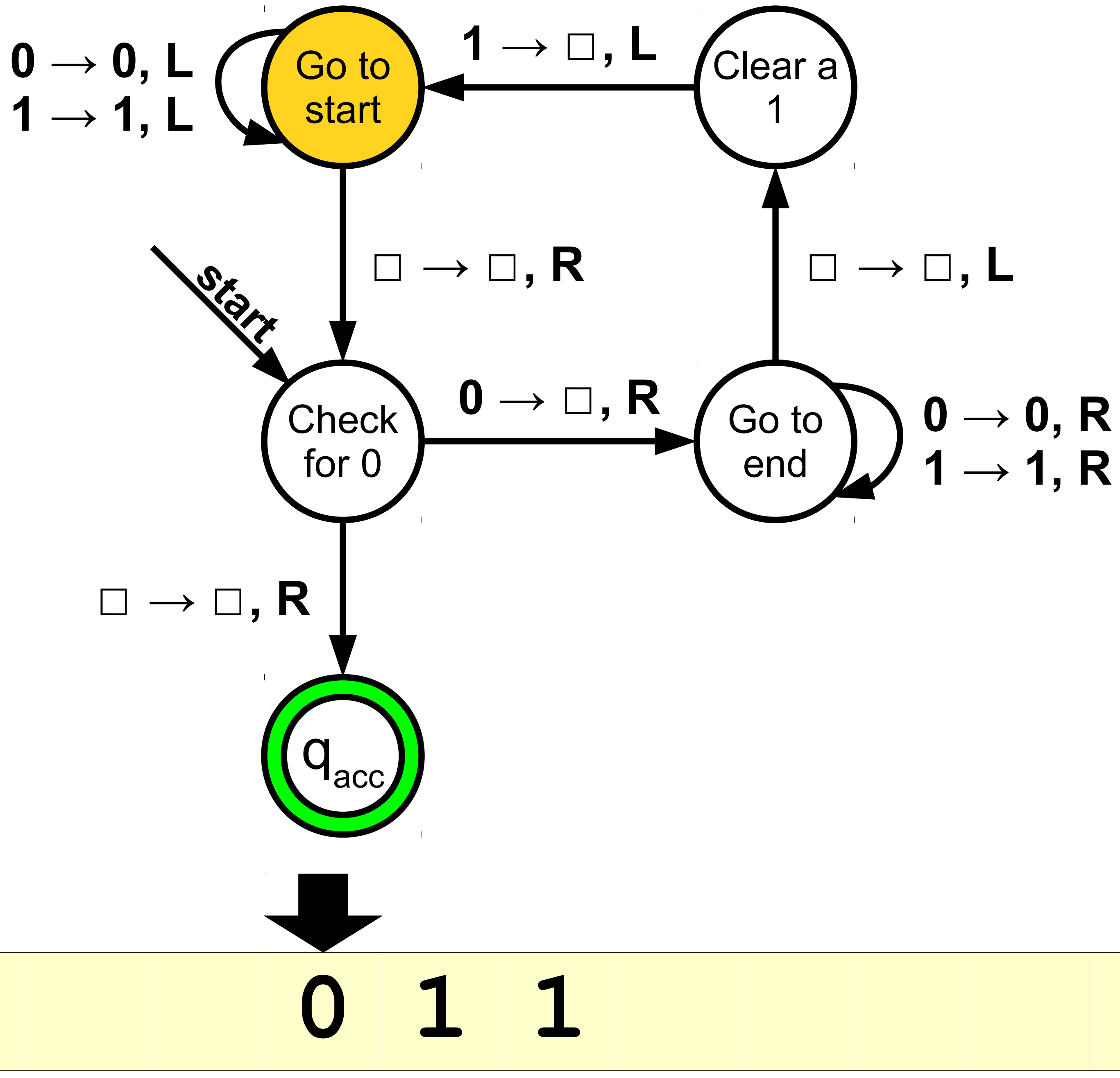
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Go to end
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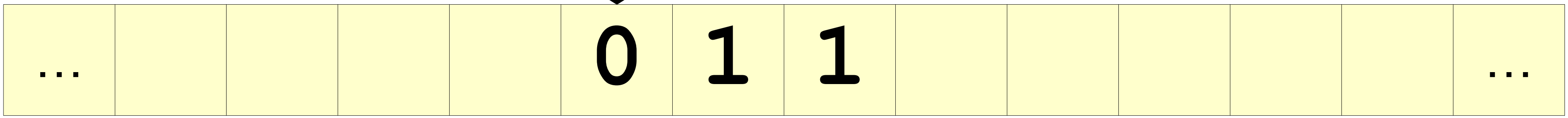
q_acc



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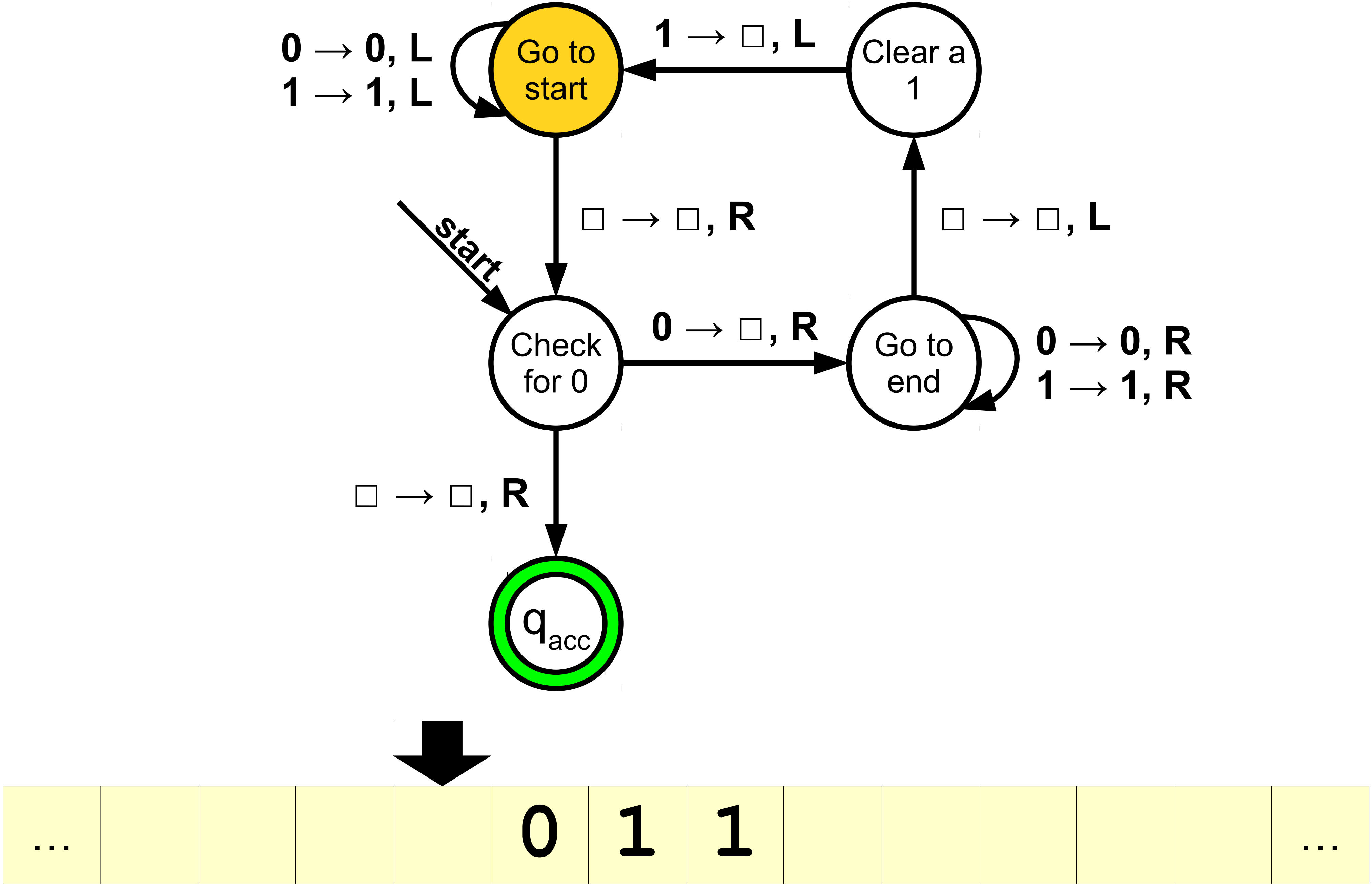
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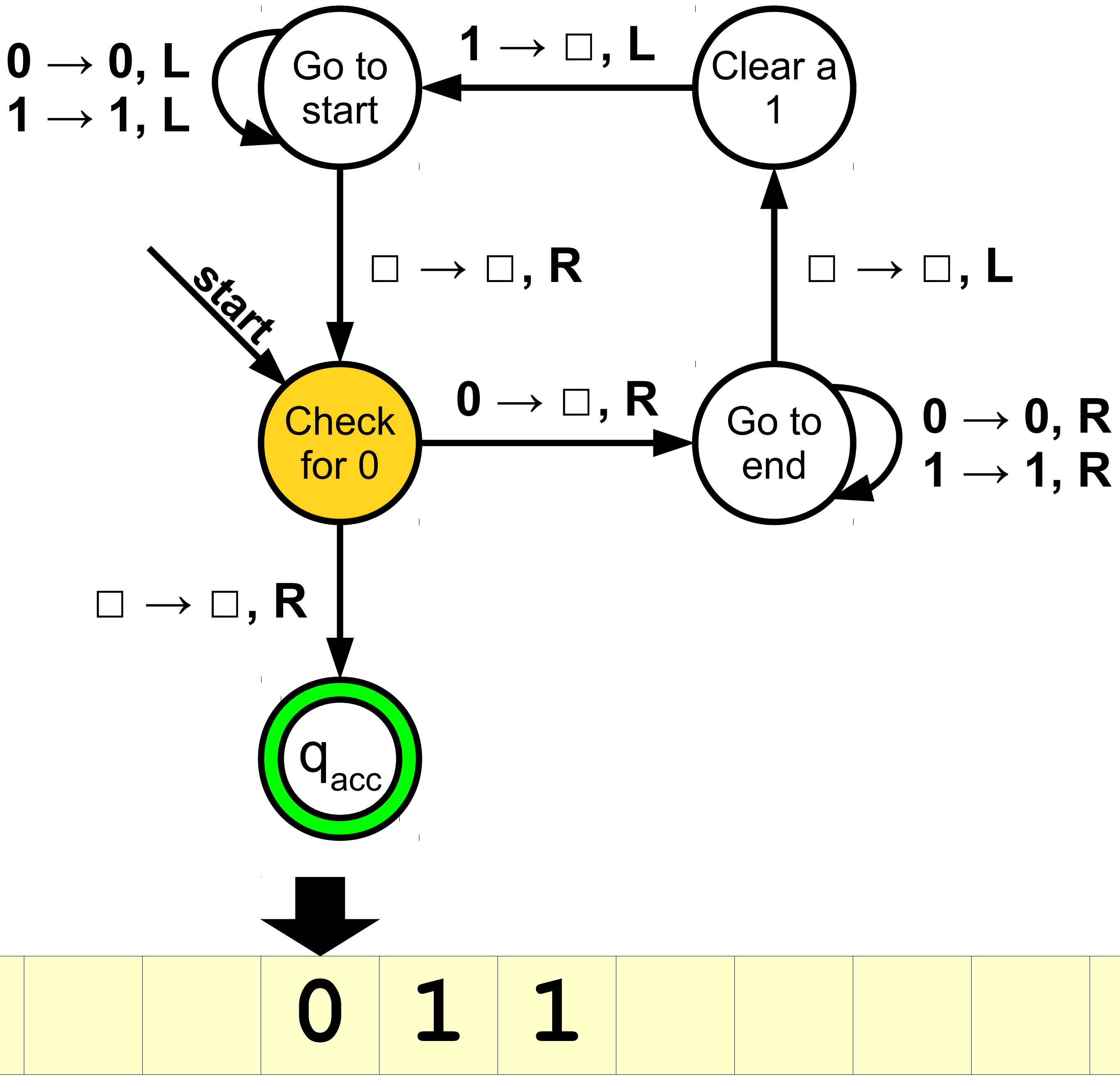
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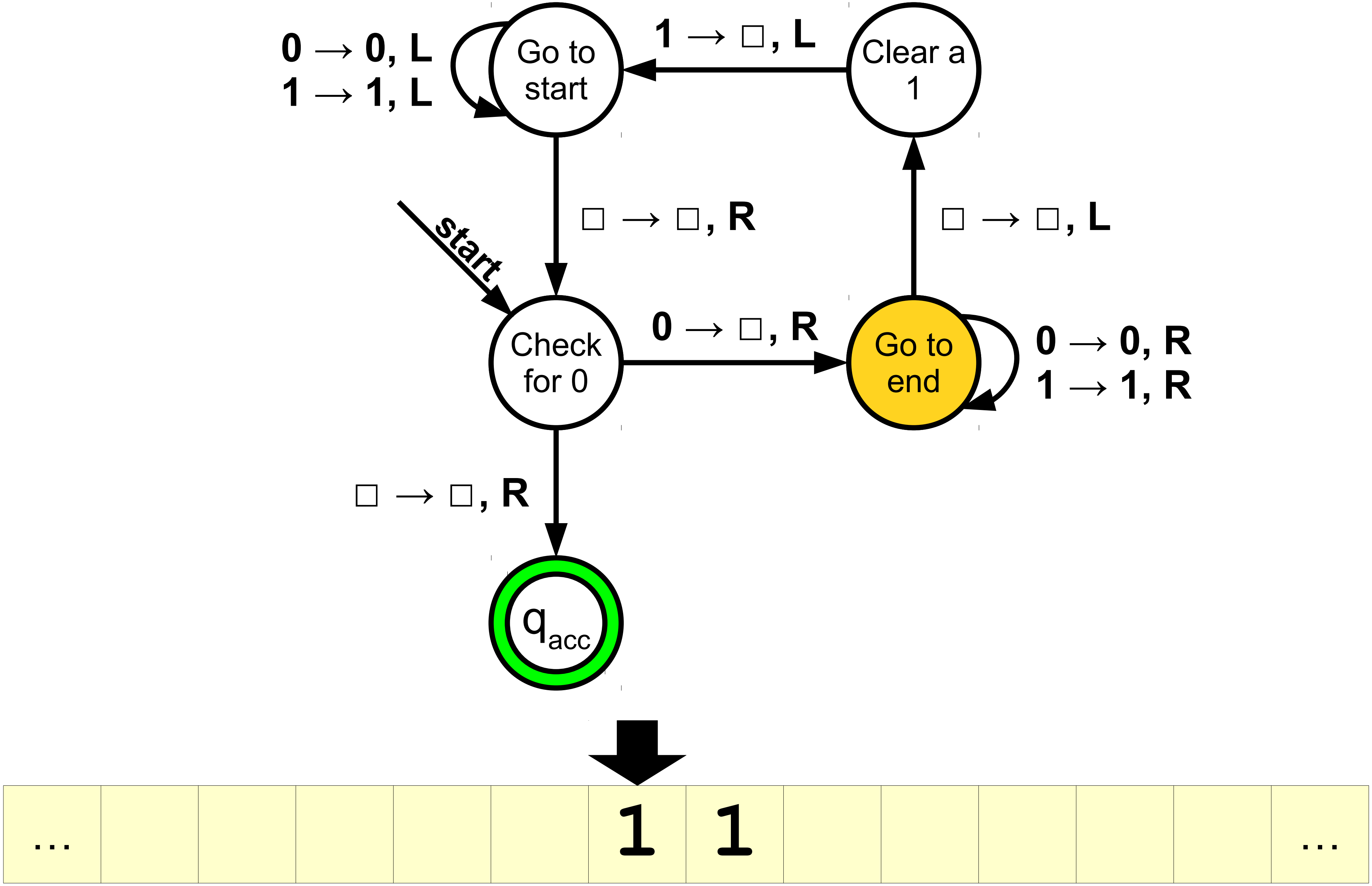
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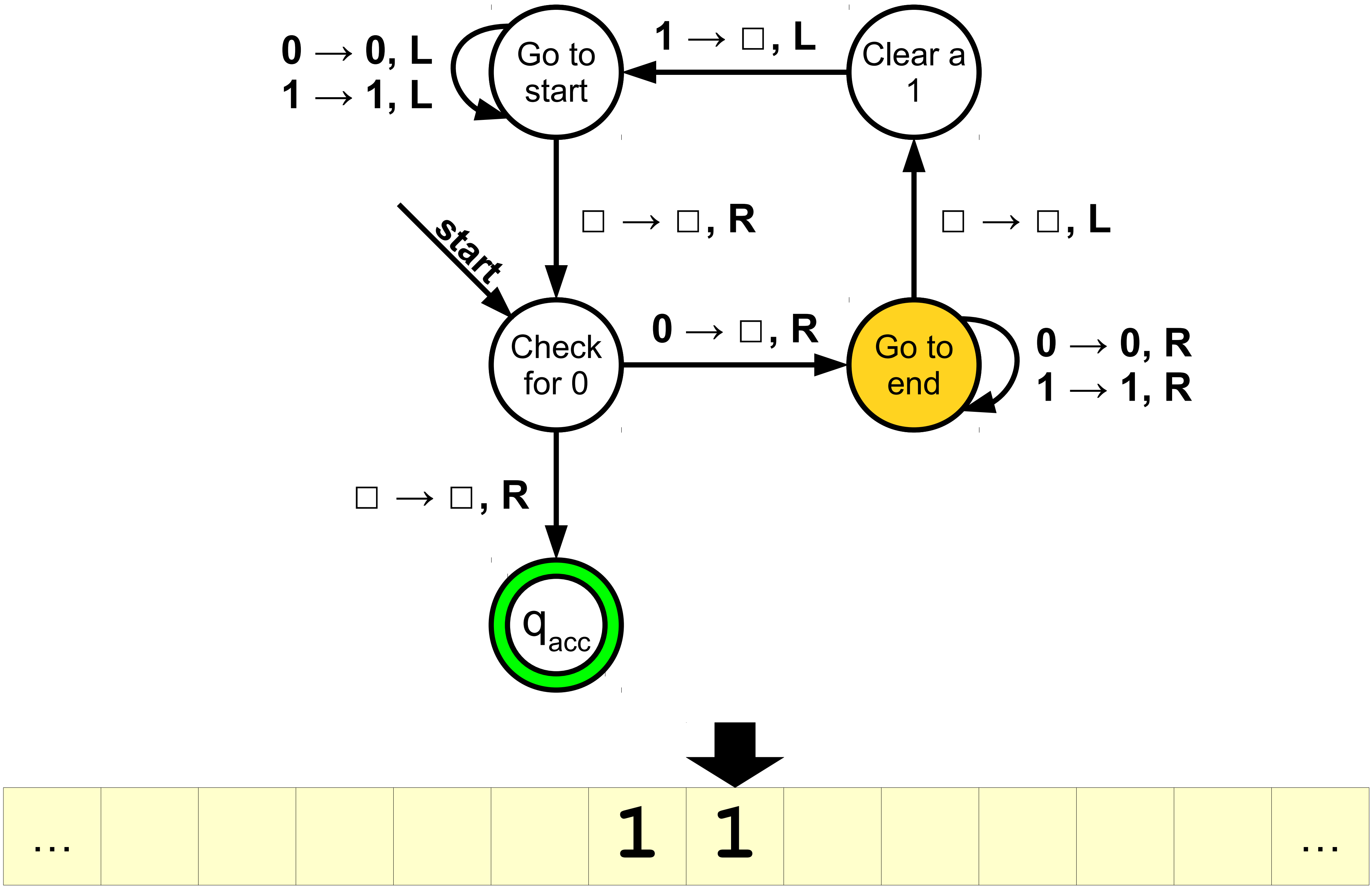
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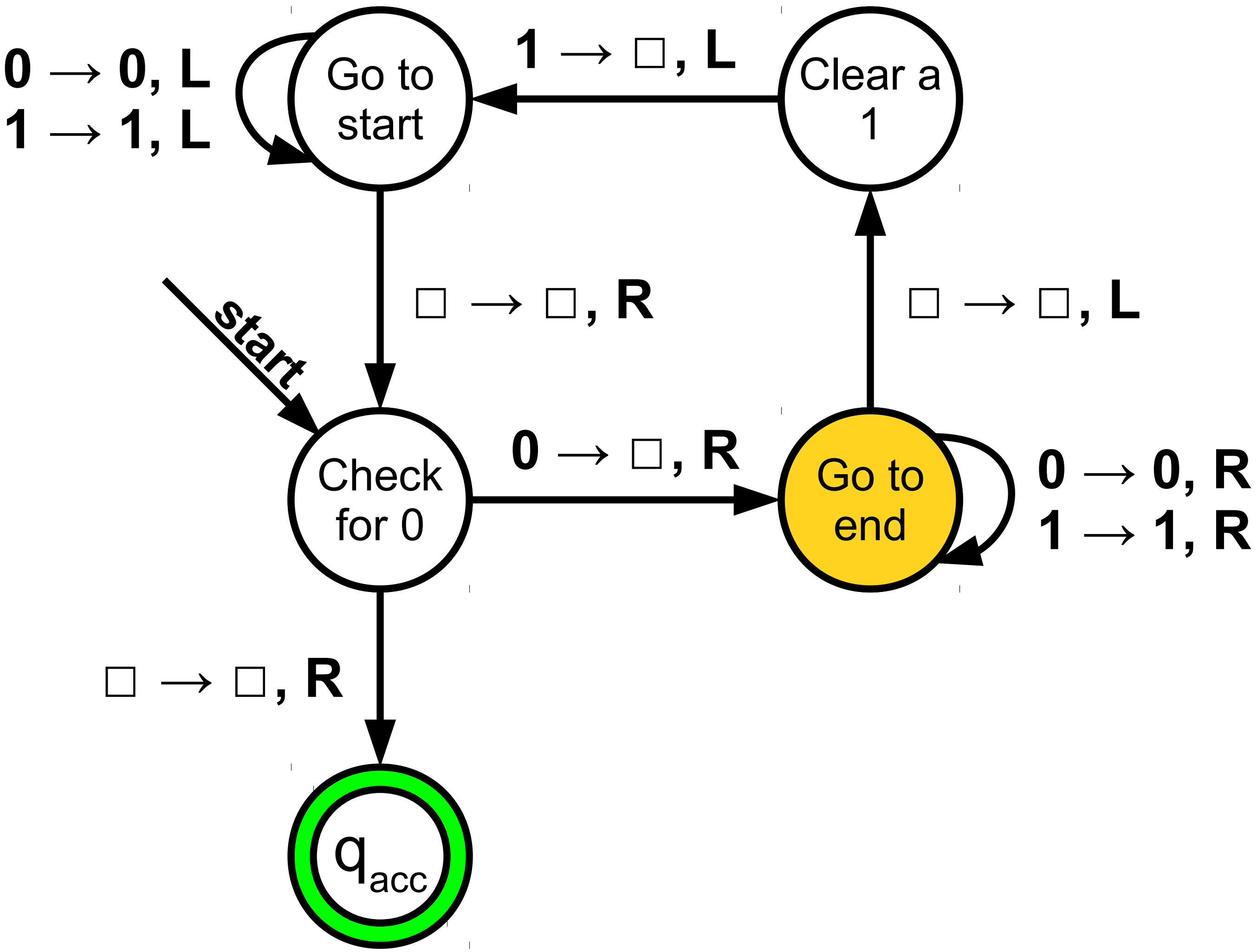
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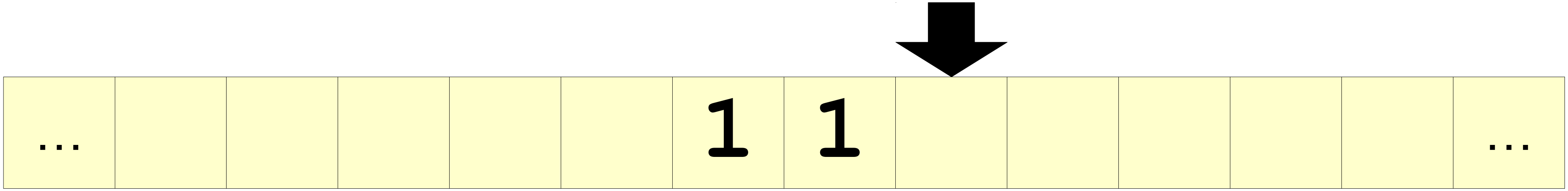
q_acc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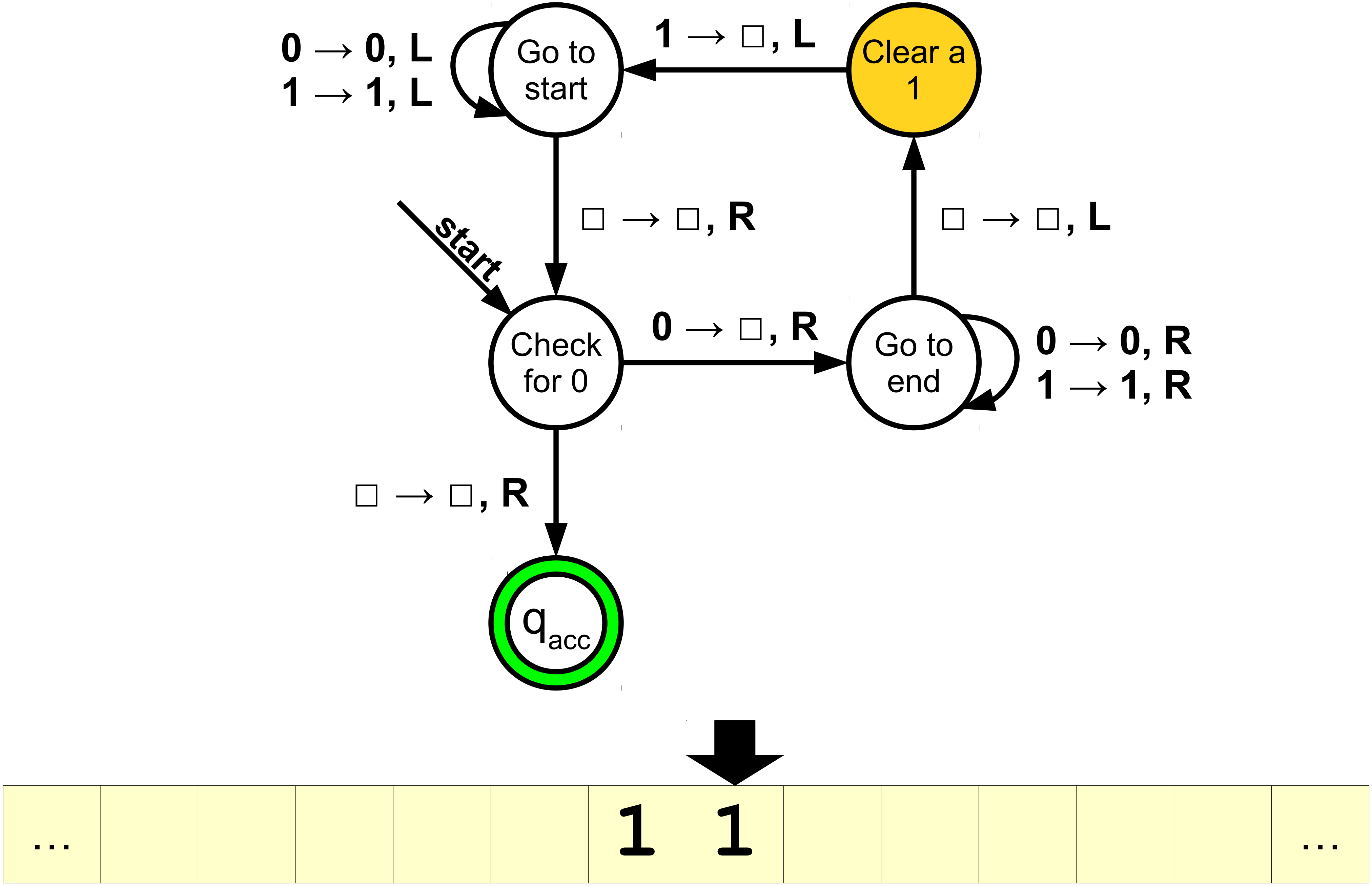
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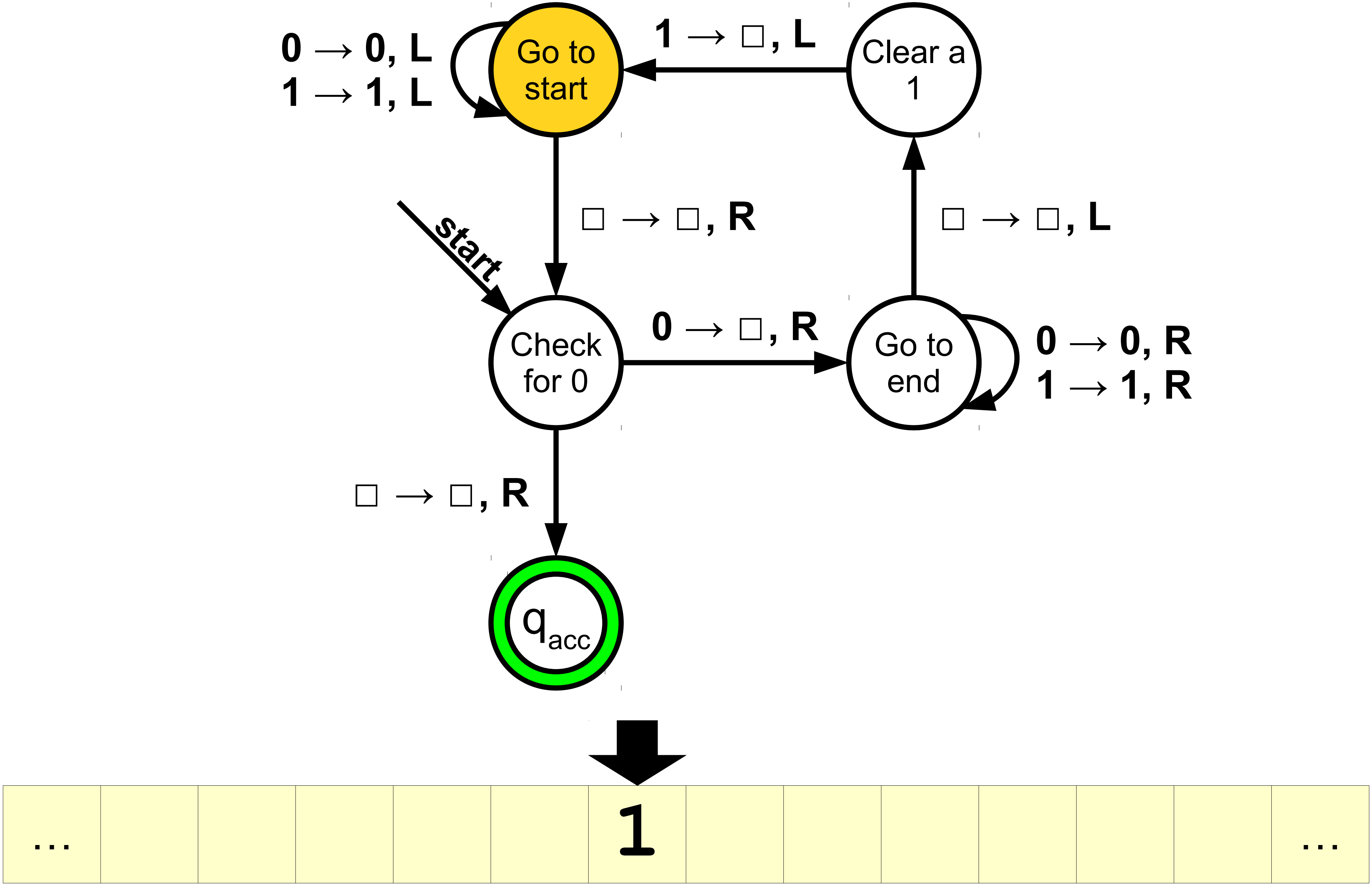
0 → 0, L
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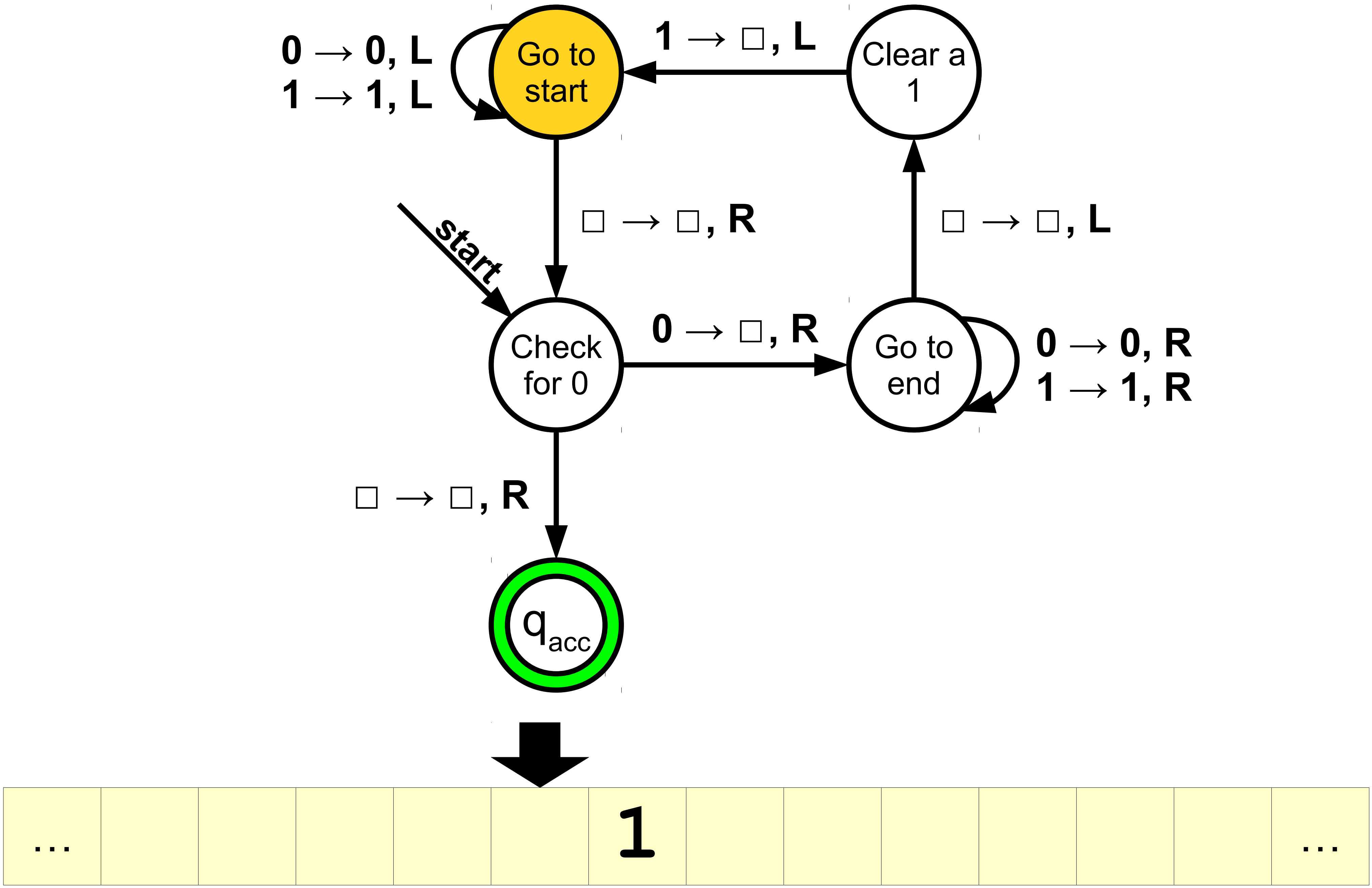
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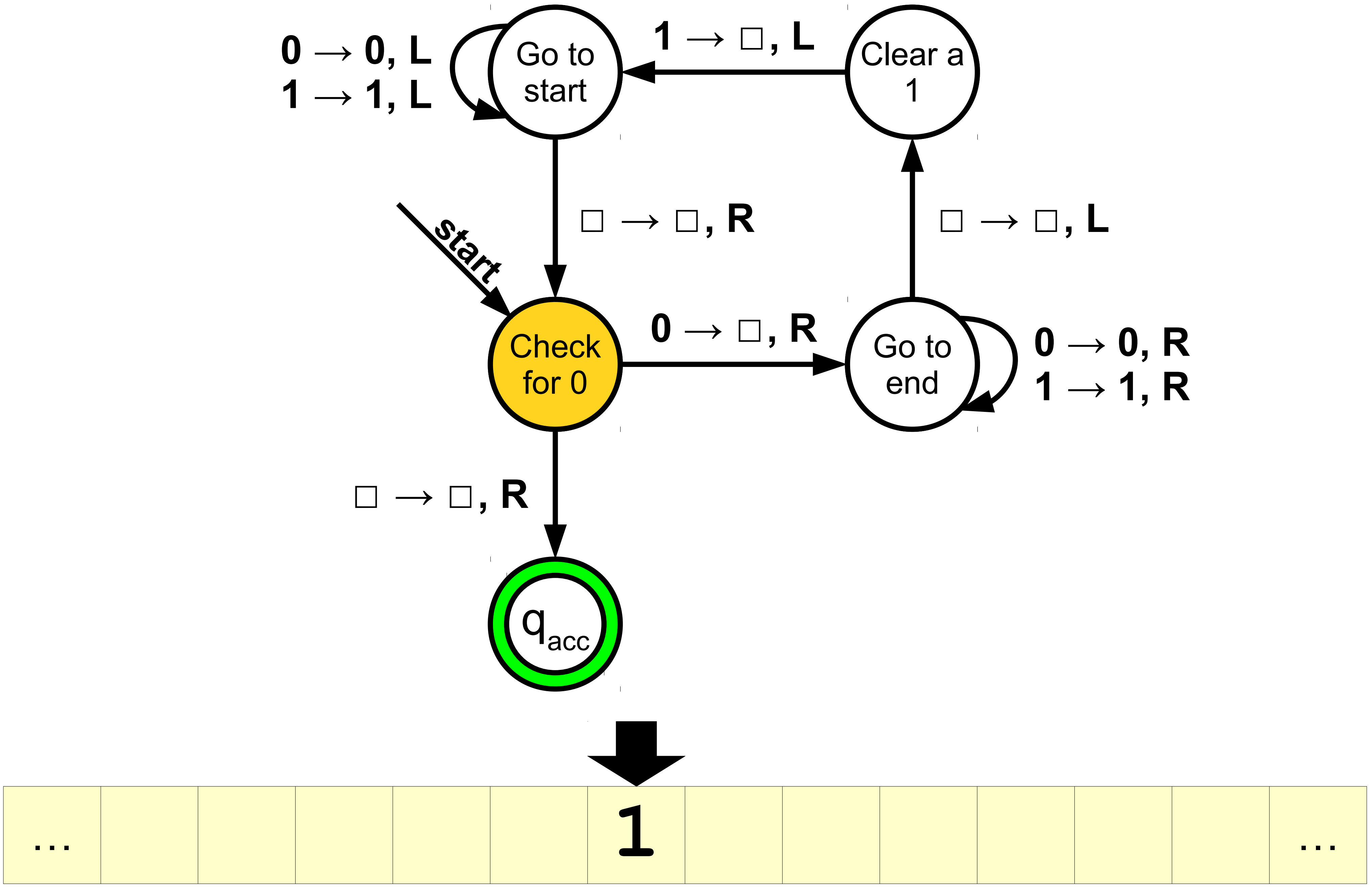
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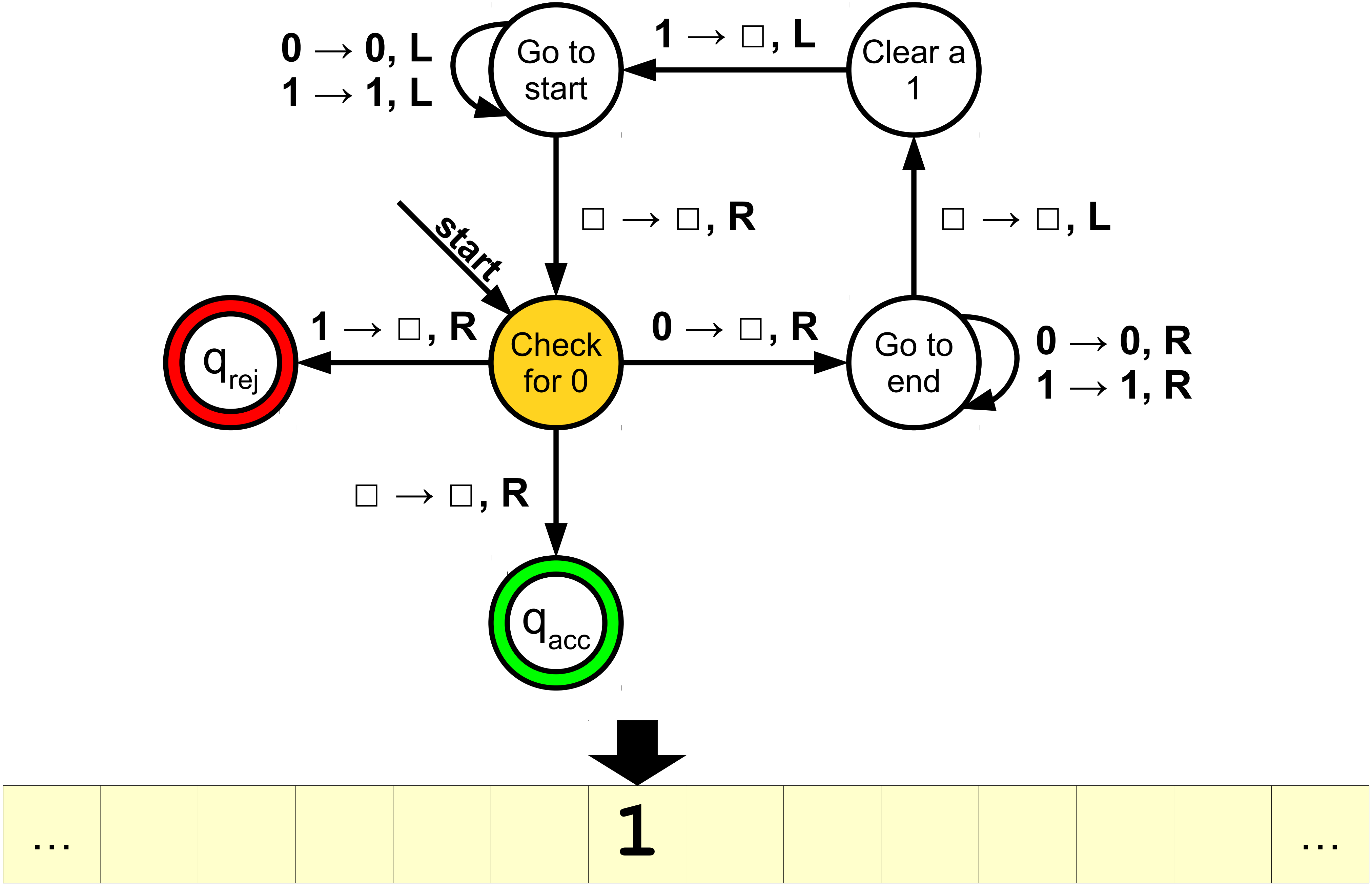
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start
q_rej
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Go to end
0 → 0, R
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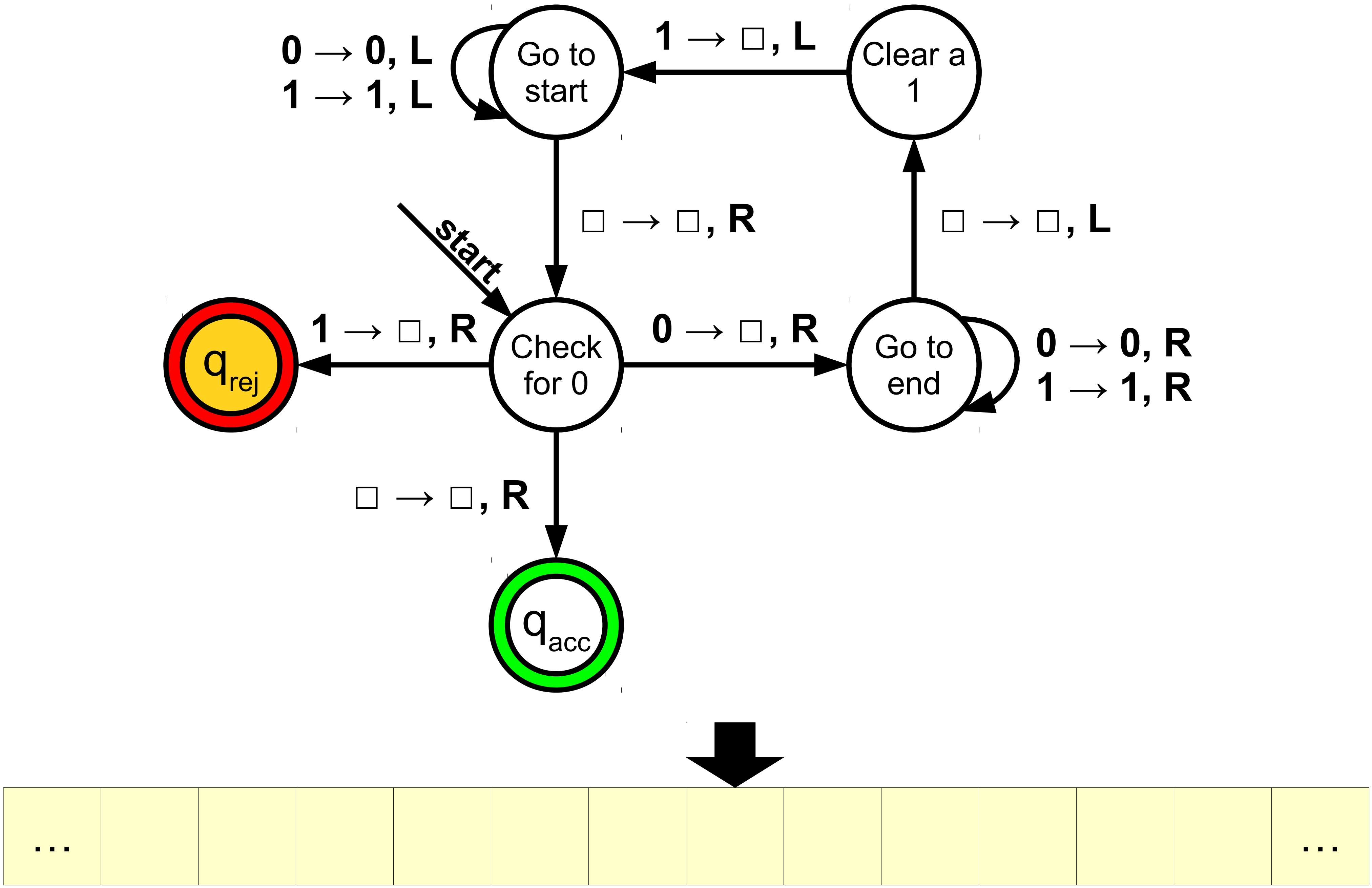
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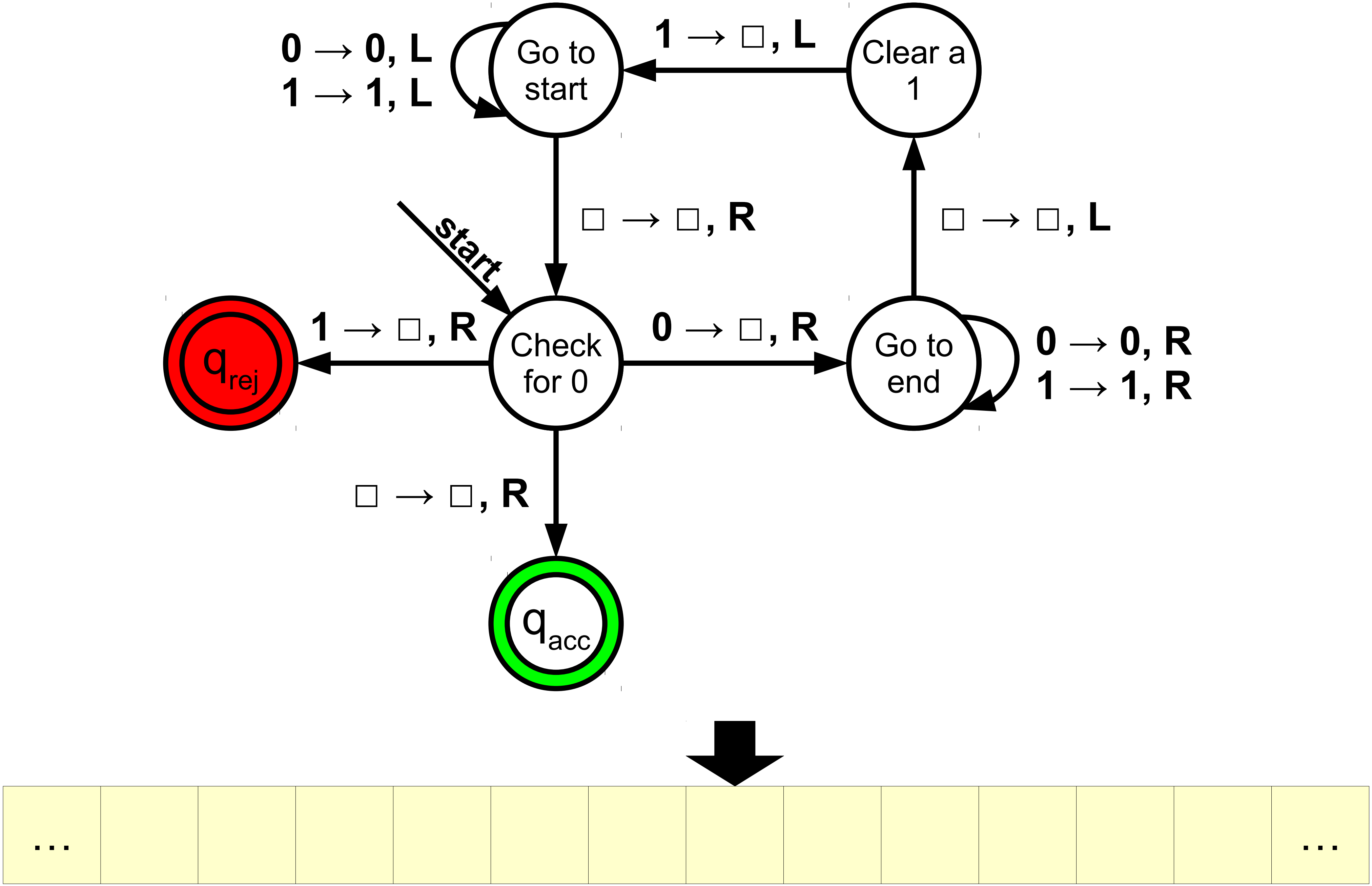
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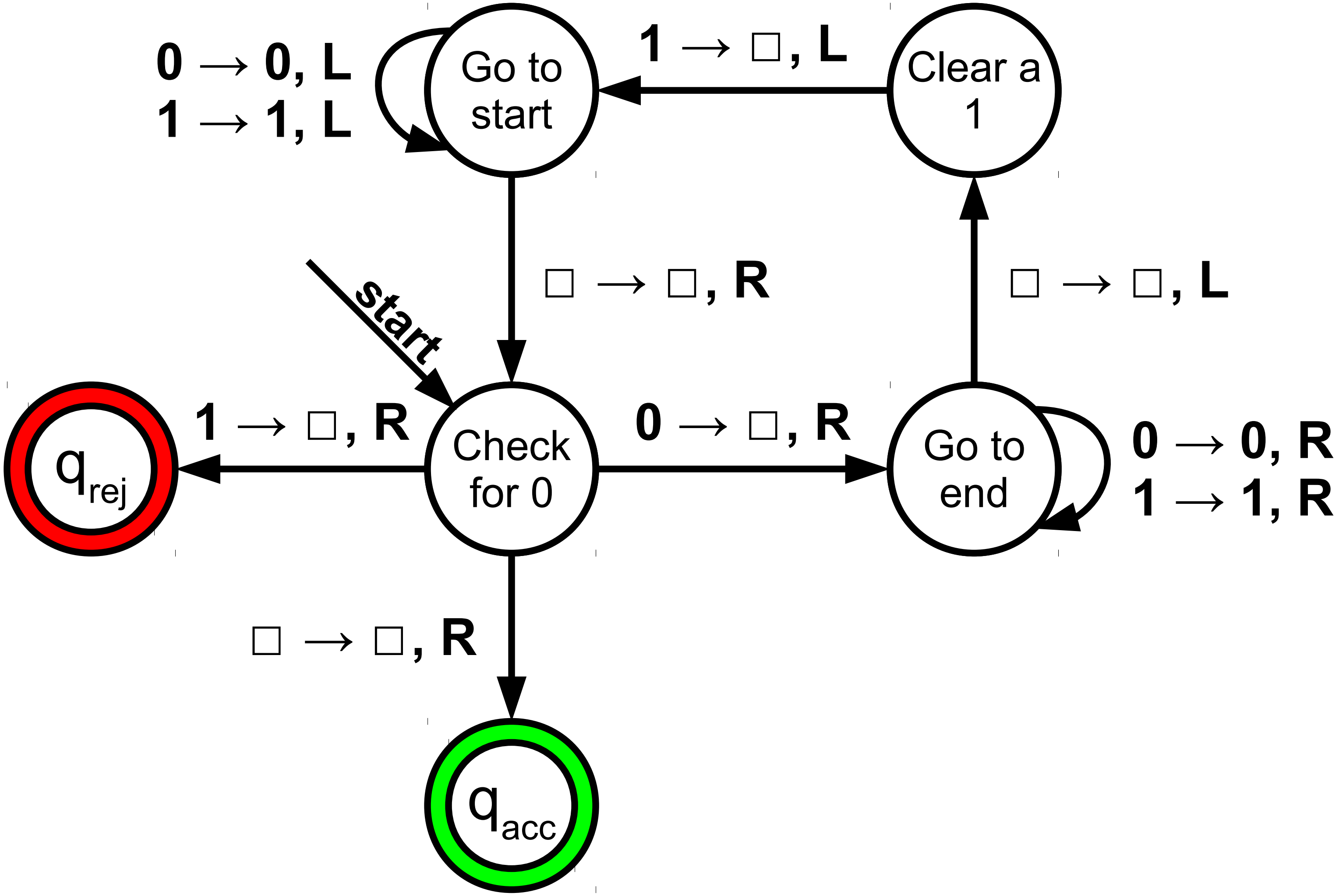
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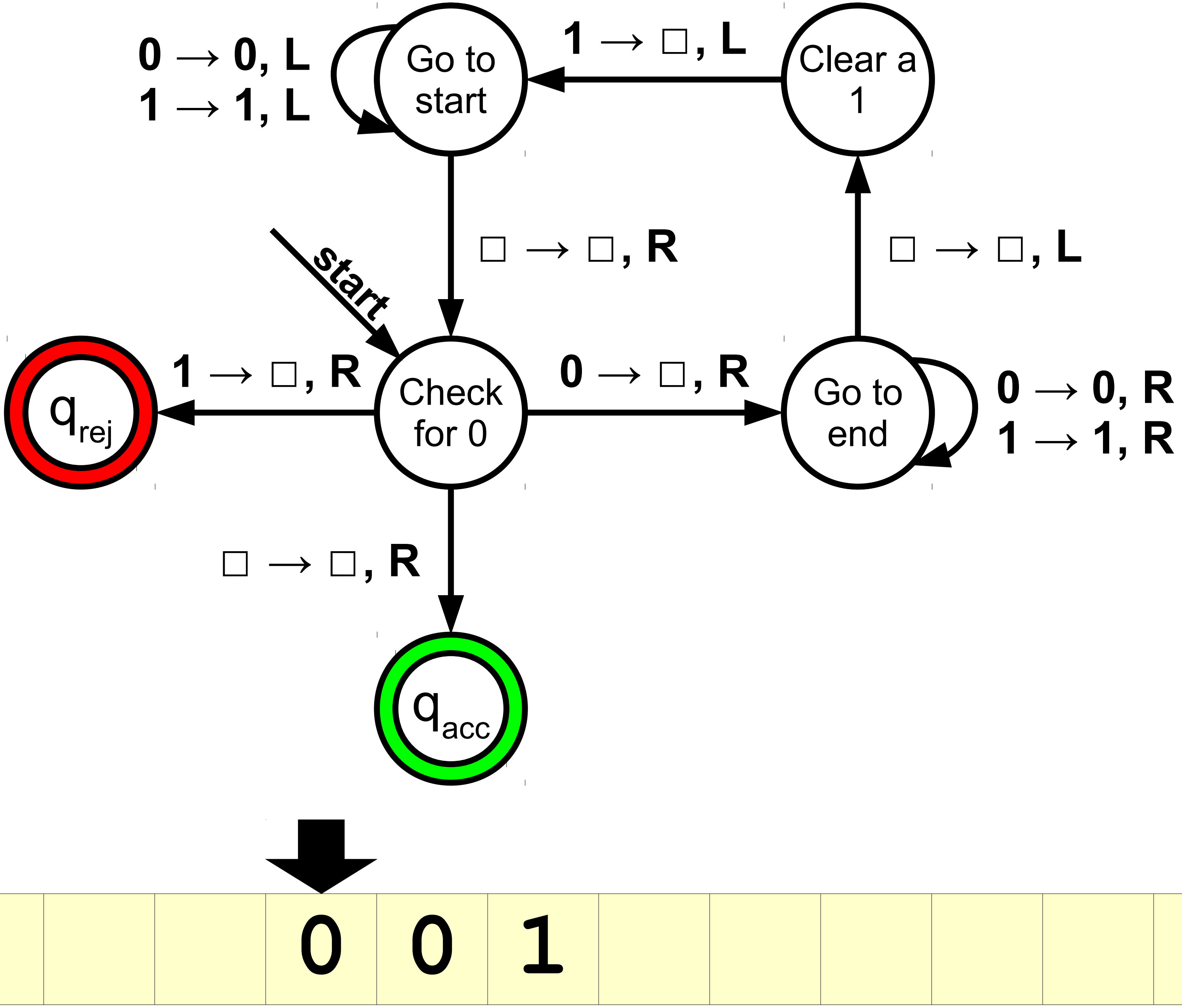
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Check for 0
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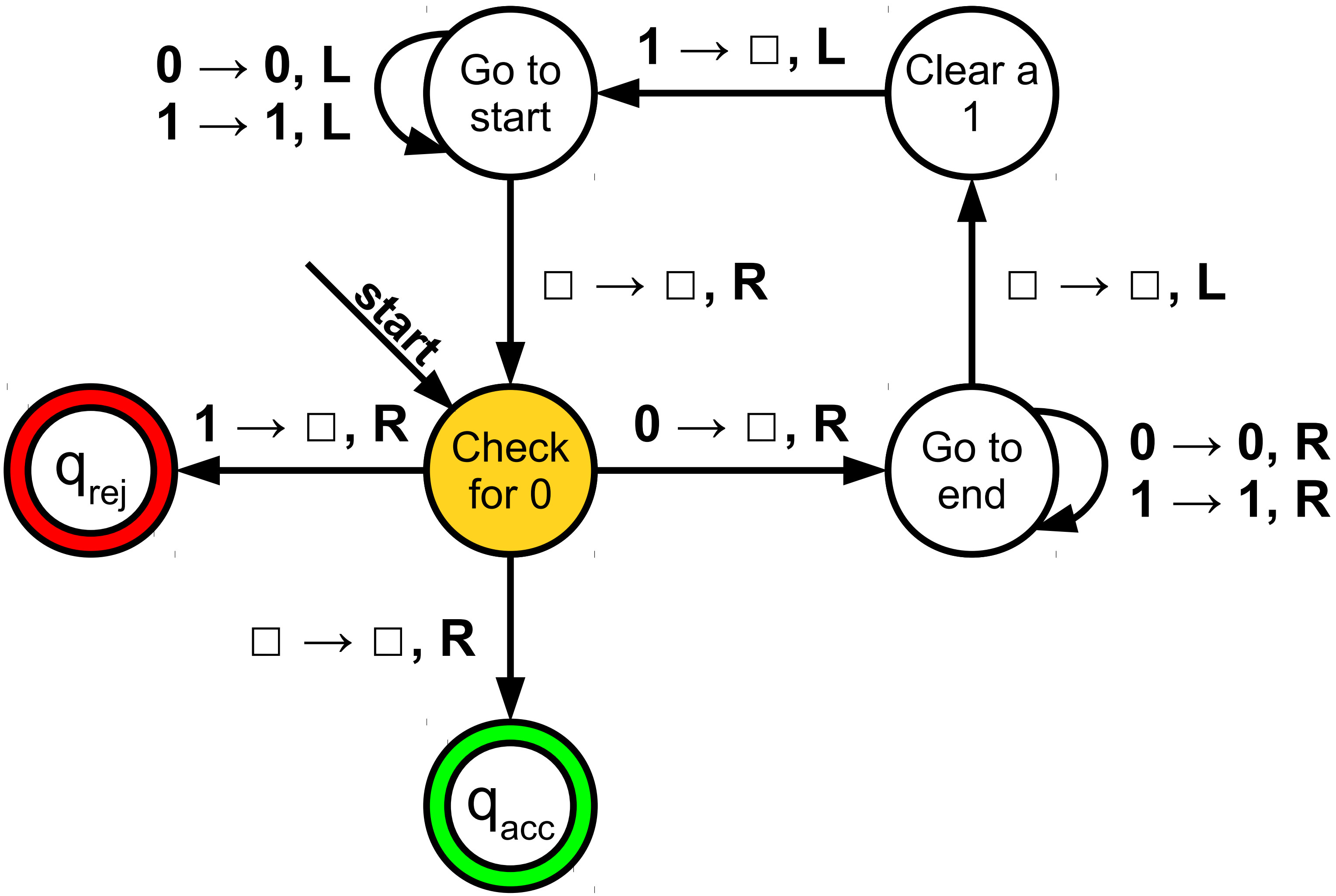
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Go to start
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Check for 0
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Go to end
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1 → 1, R
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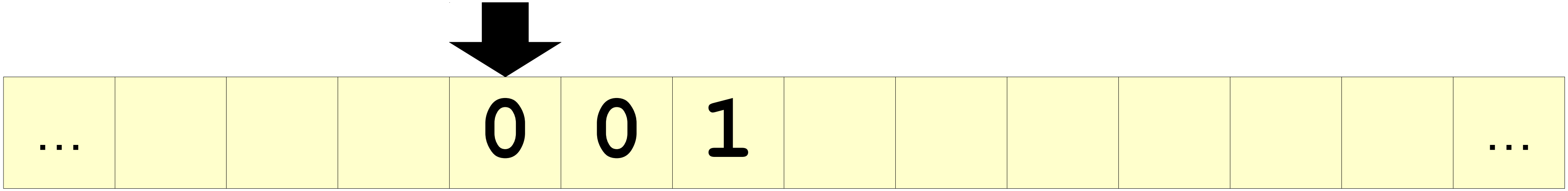
q_acc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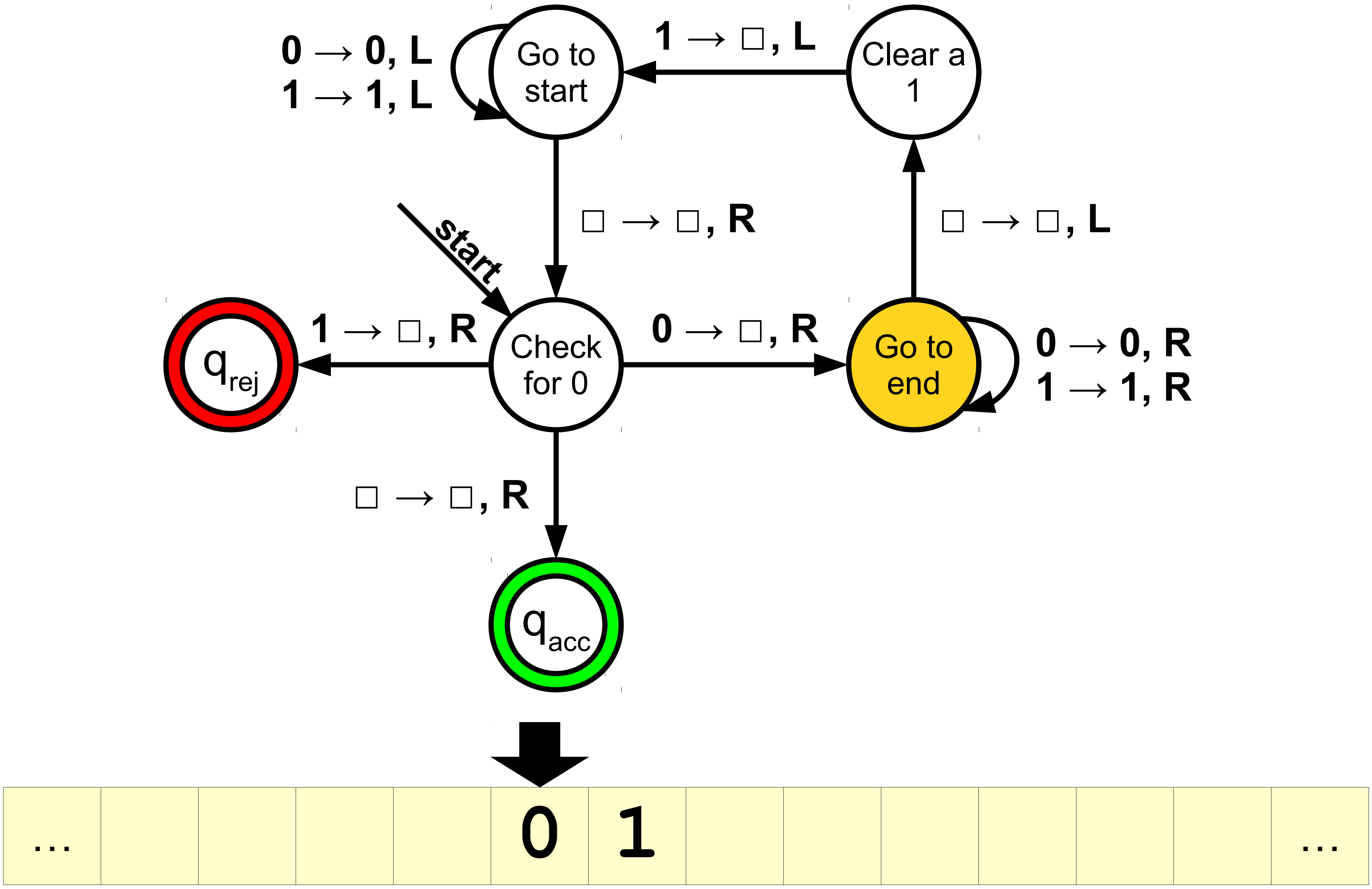
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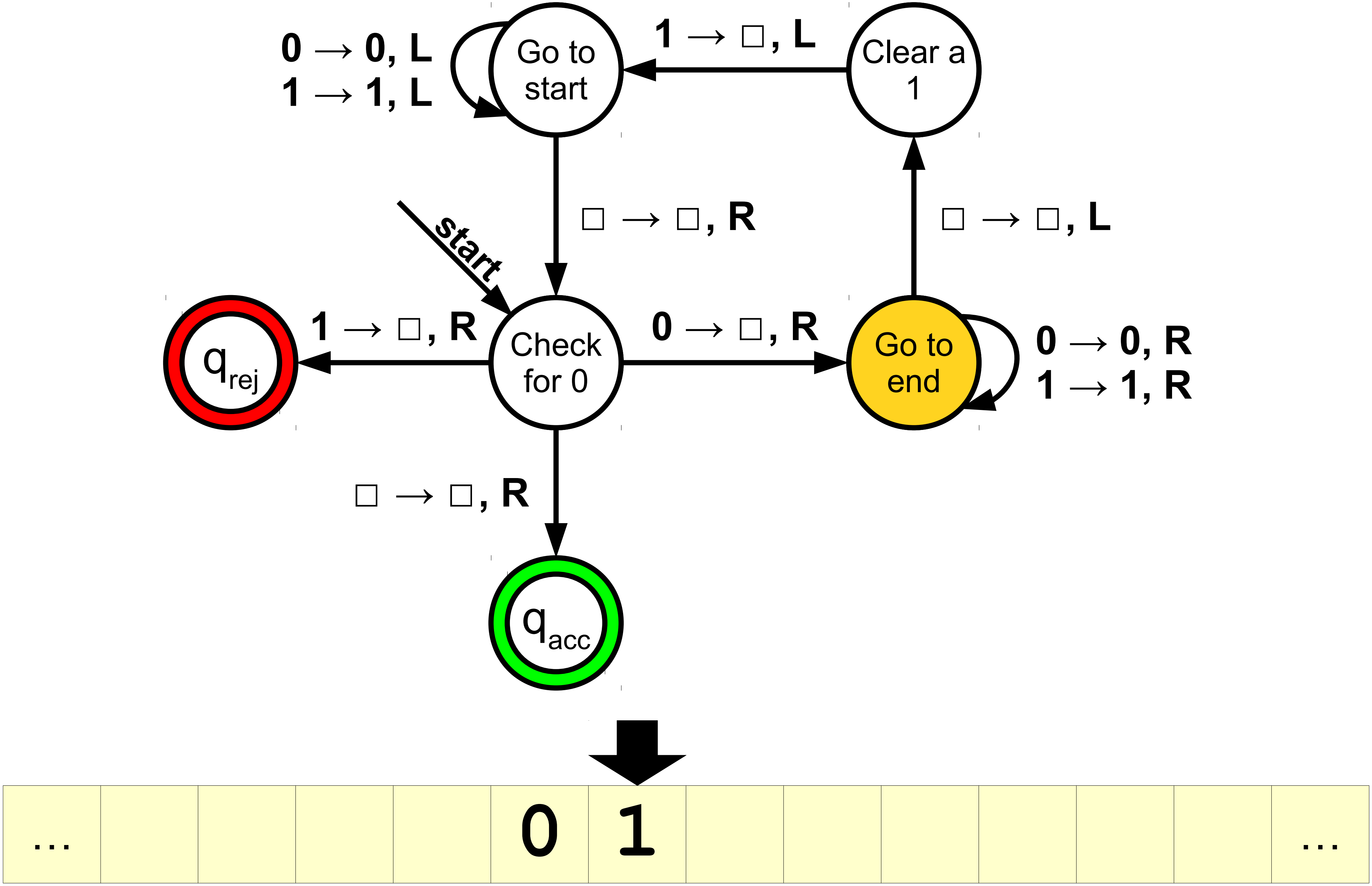
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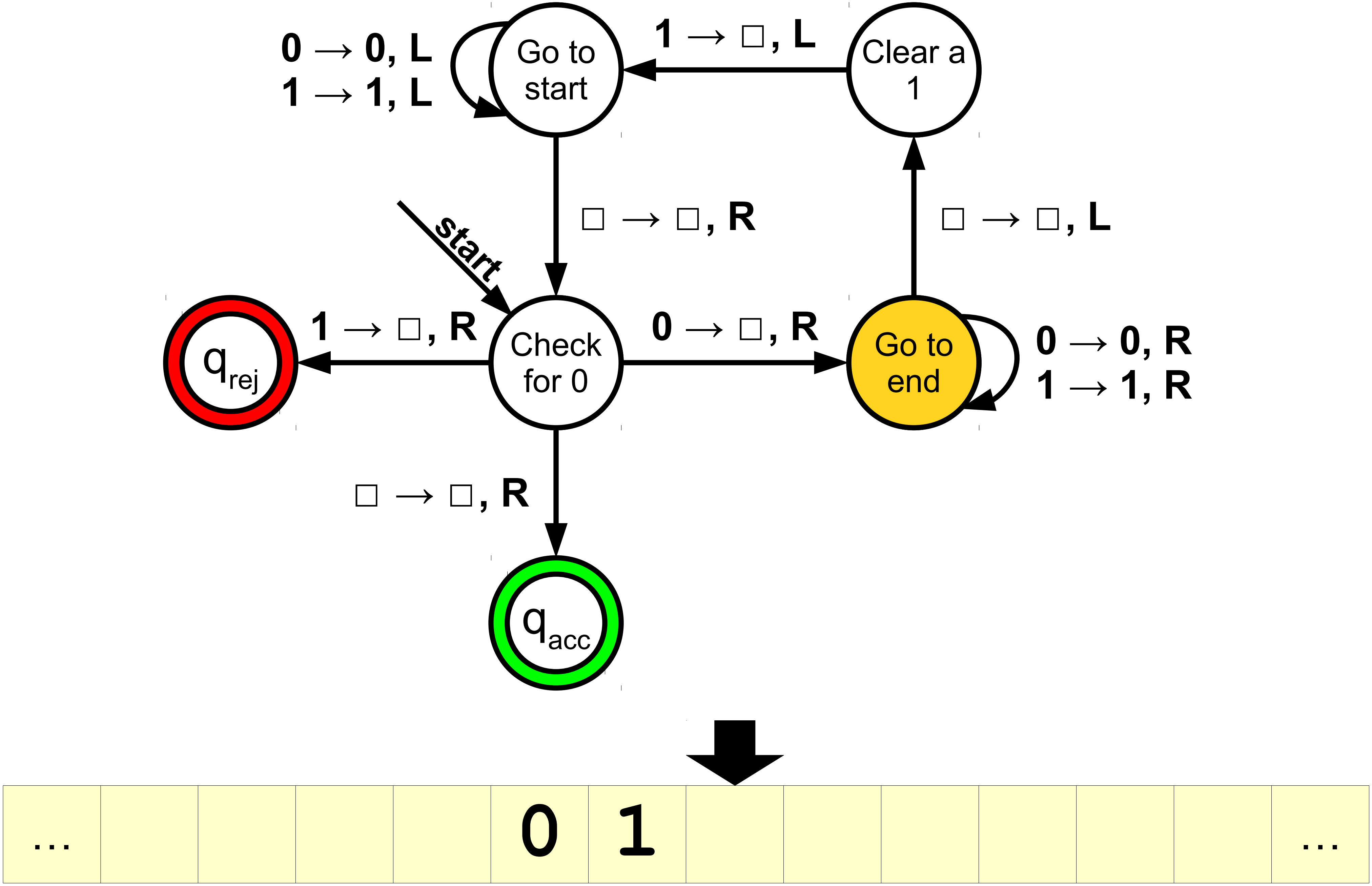
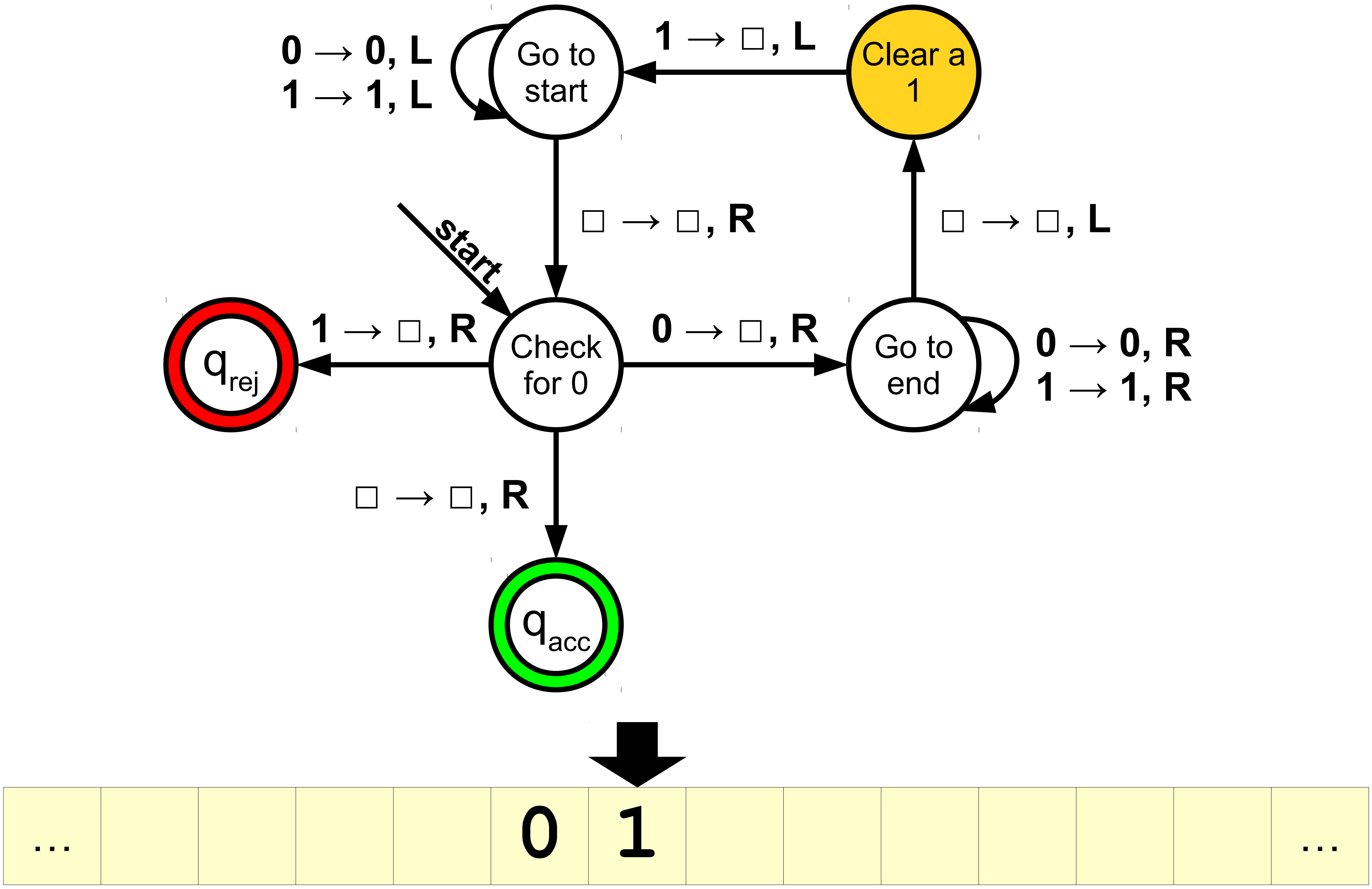
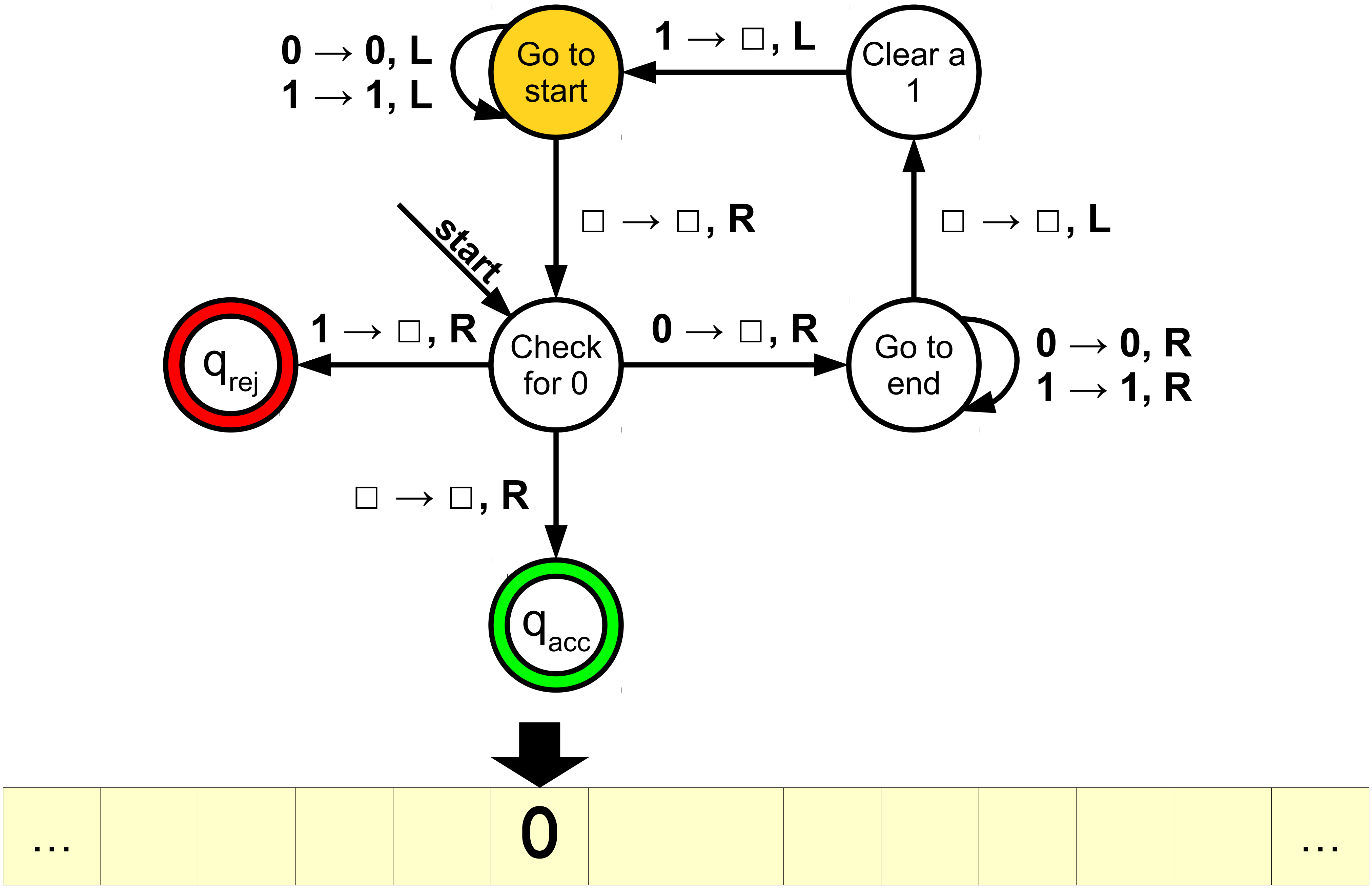
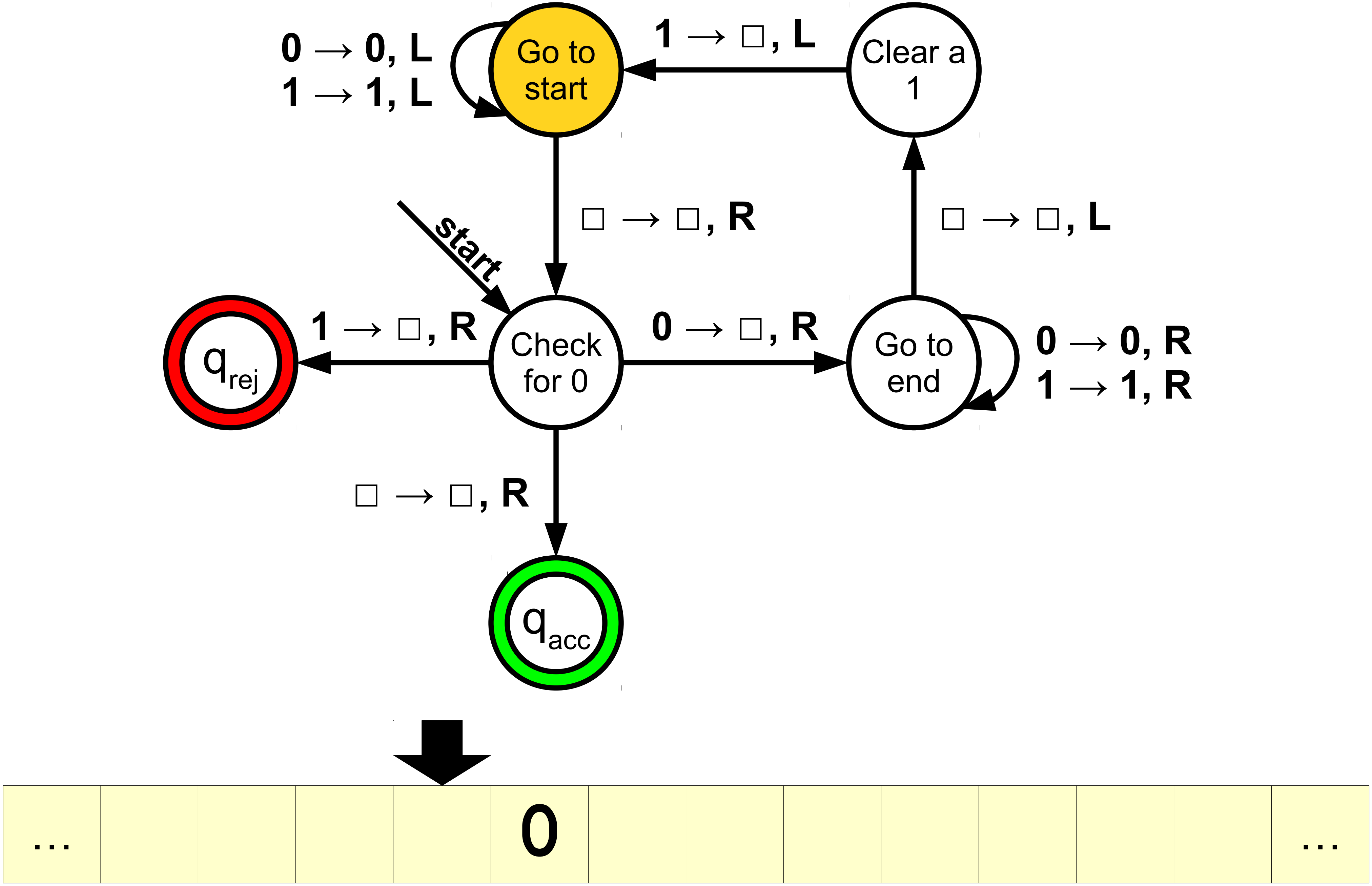
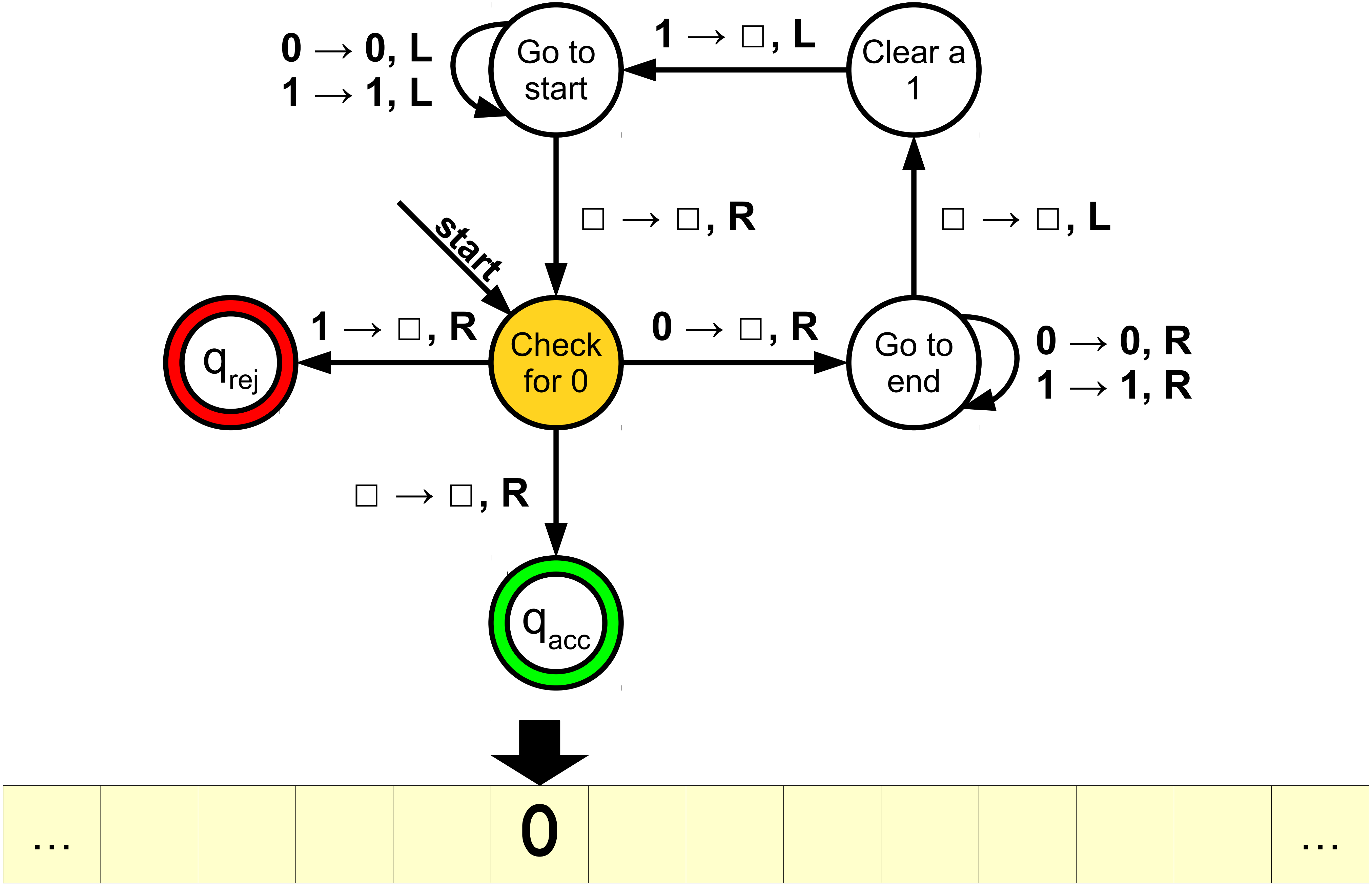
1
…

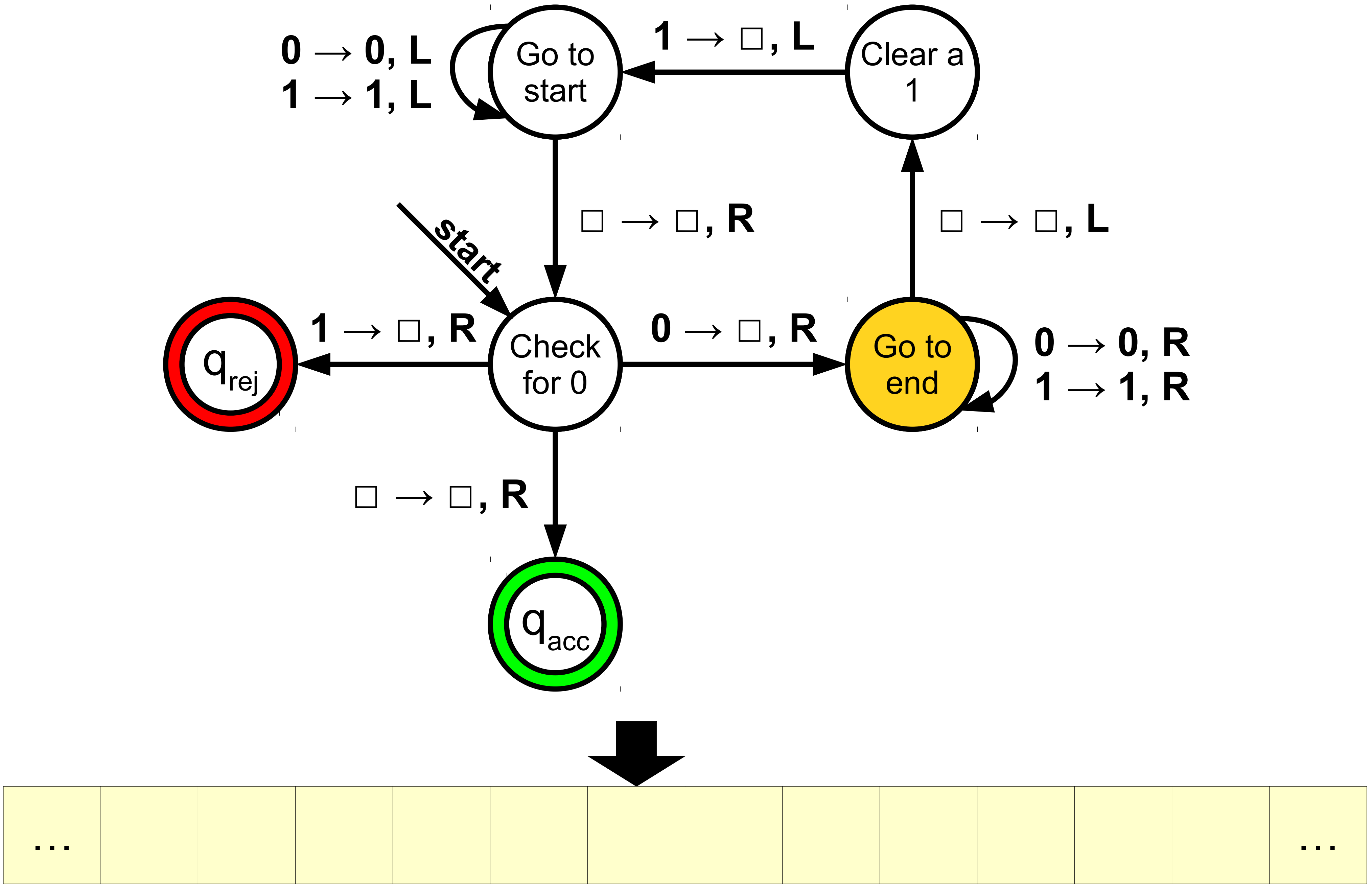

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_{rej}
□ → □, R
q_{acc}
…
0
1
…

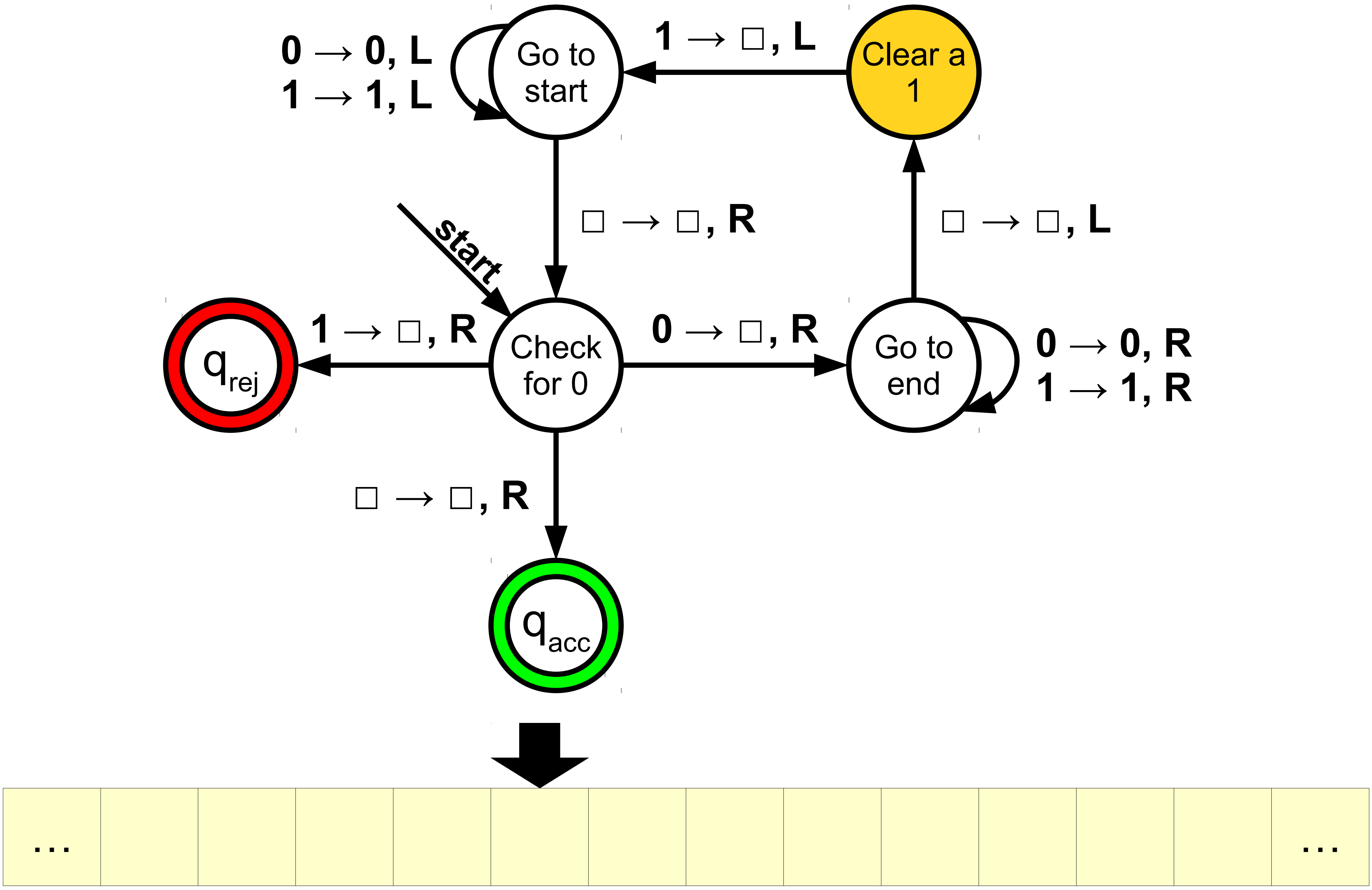

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
q_rej
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_{rej}
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
…
0
…

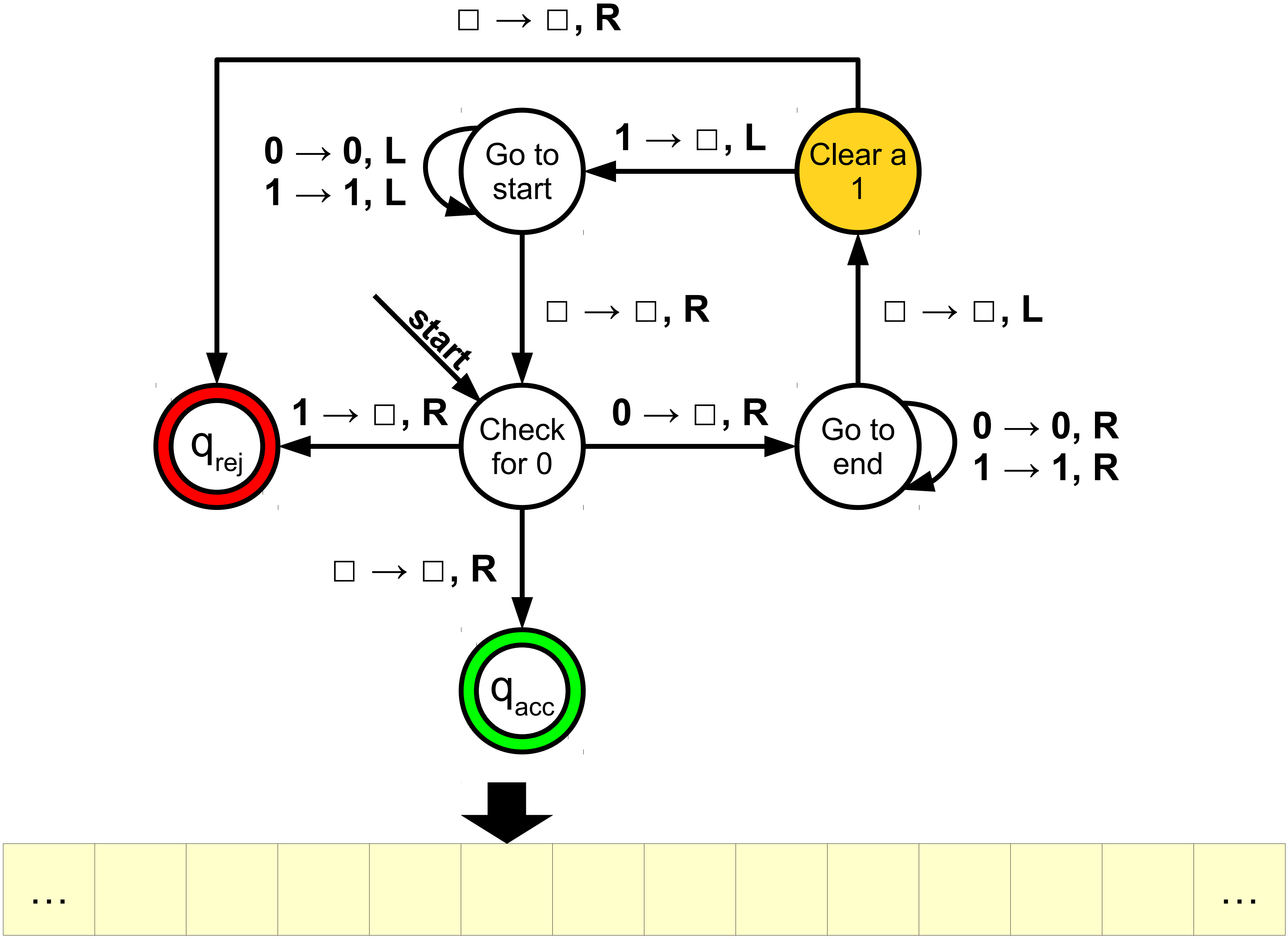

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
…

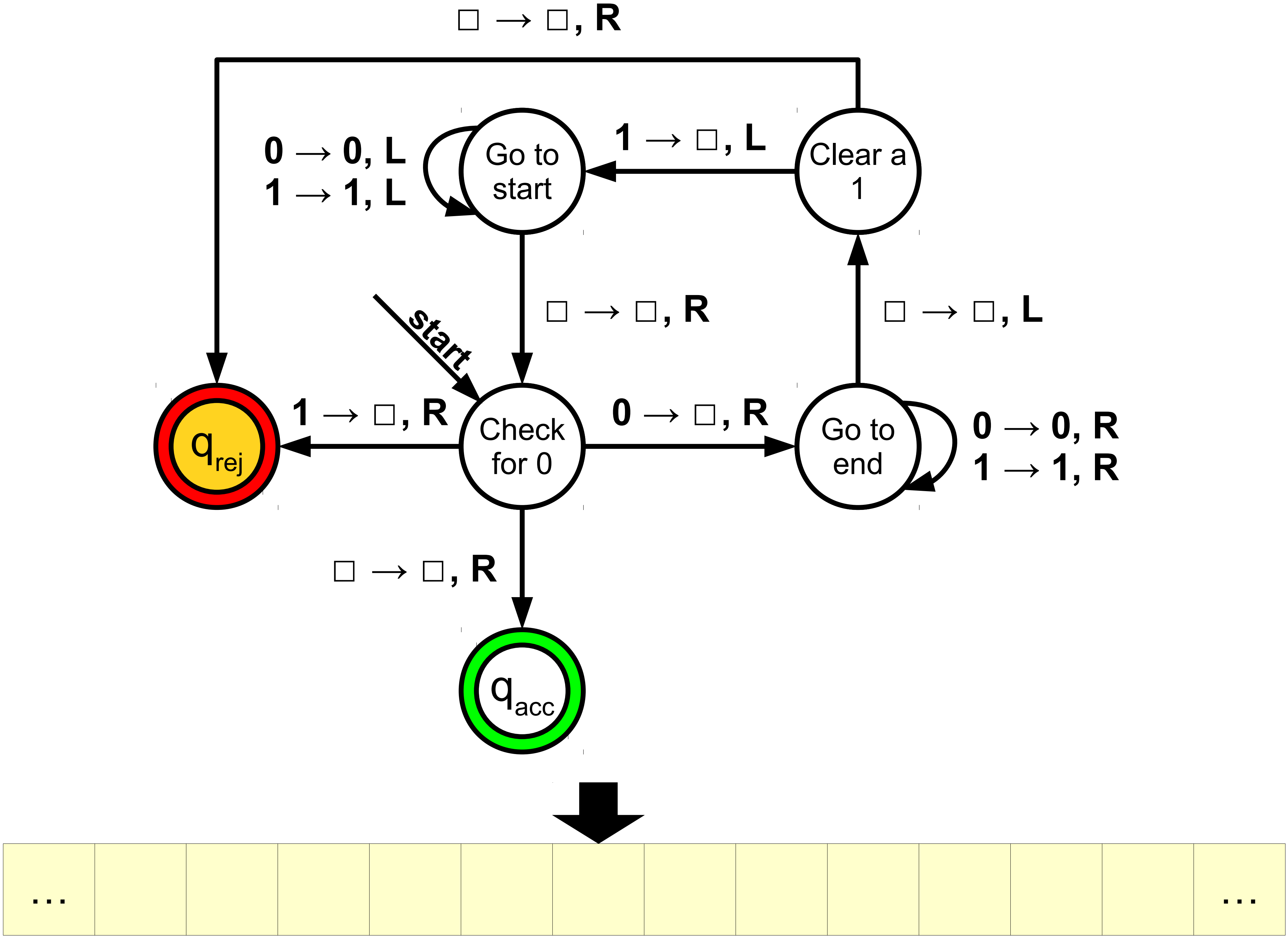

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
…

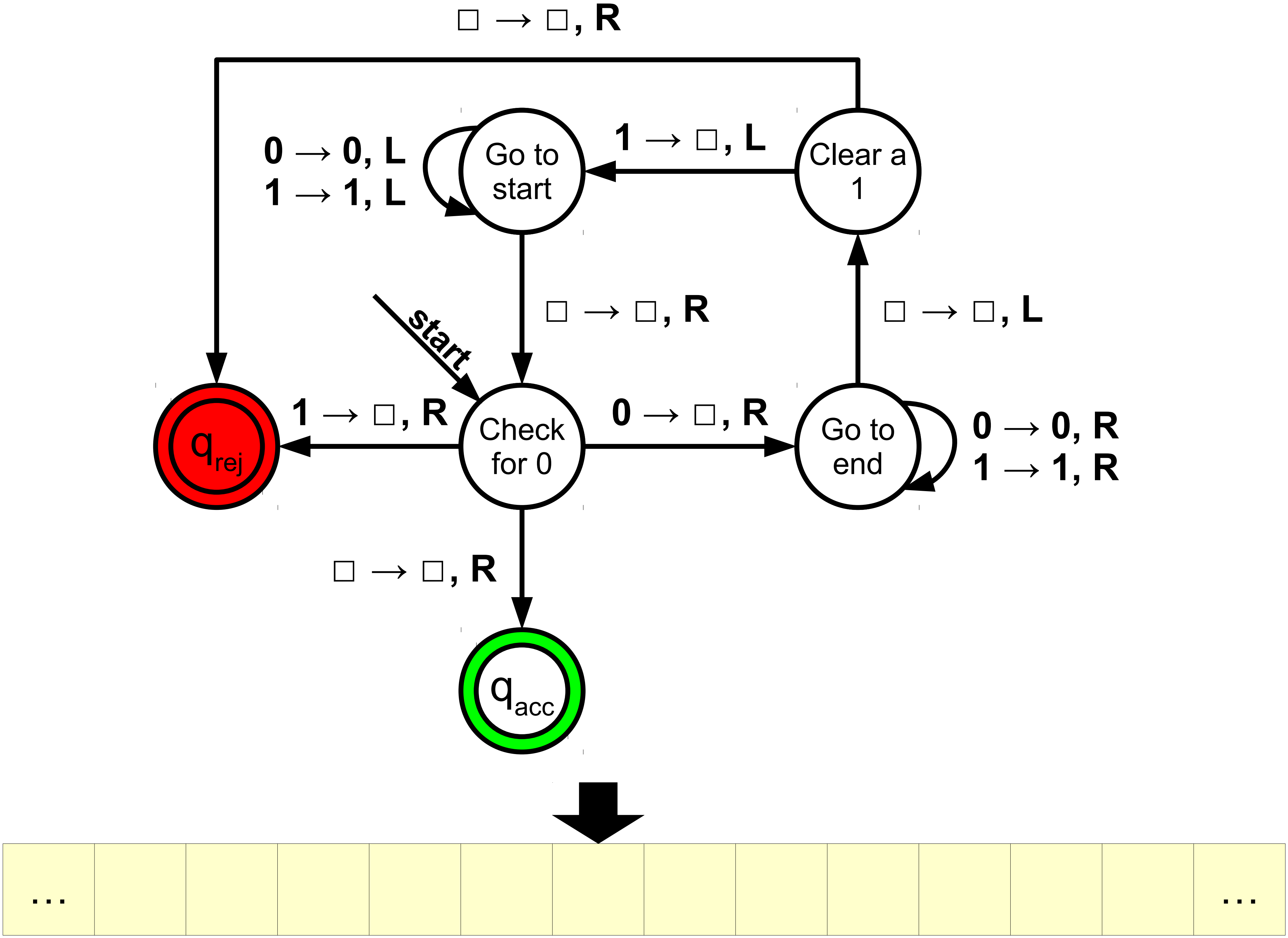

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
q$_{rej}$
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q$_{acc}$
…
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
q_rej
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
…


□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
…


□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_rej
□ → □, R
q_acc
…
…


□ → □, R
0 → 0, L
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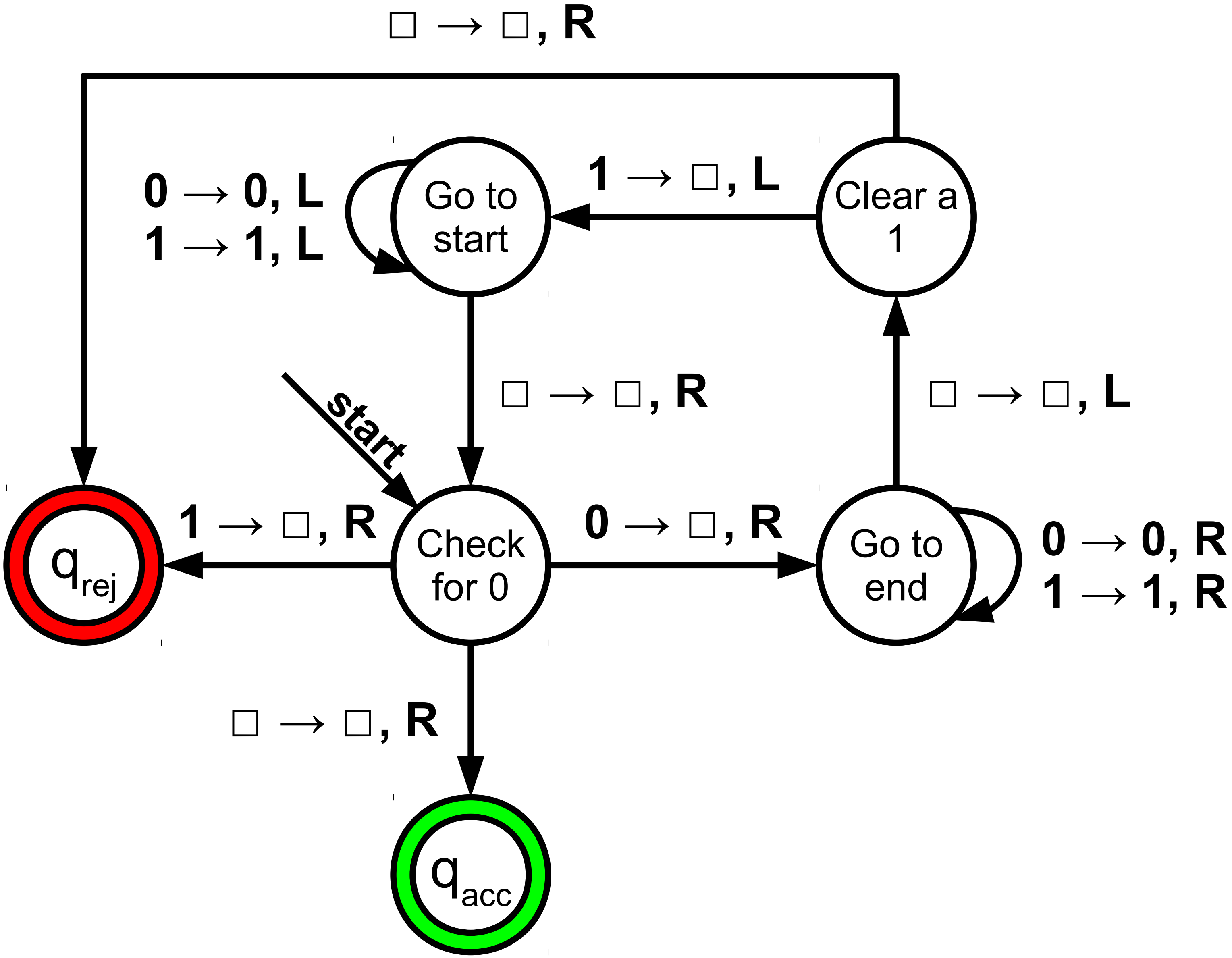
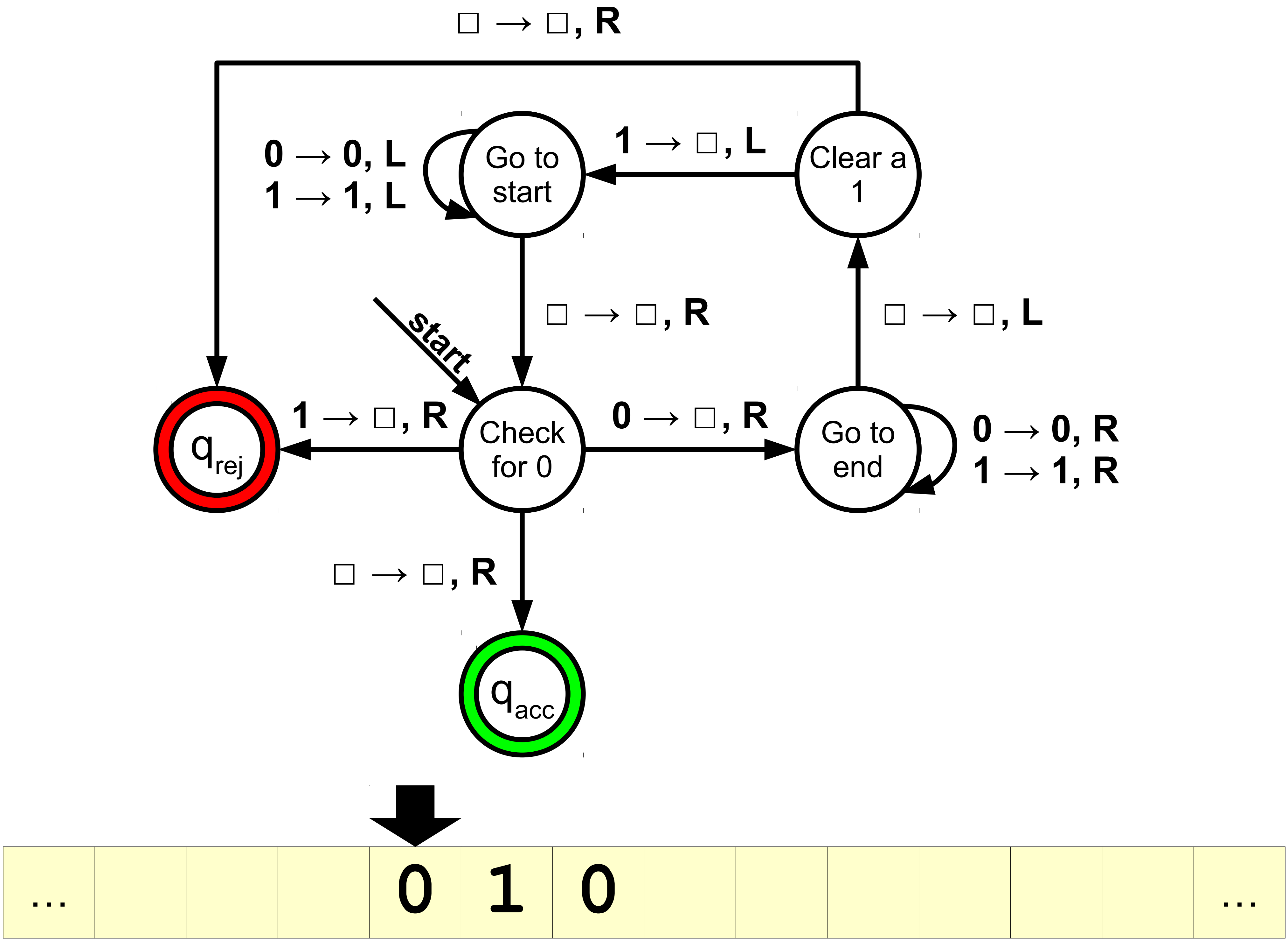
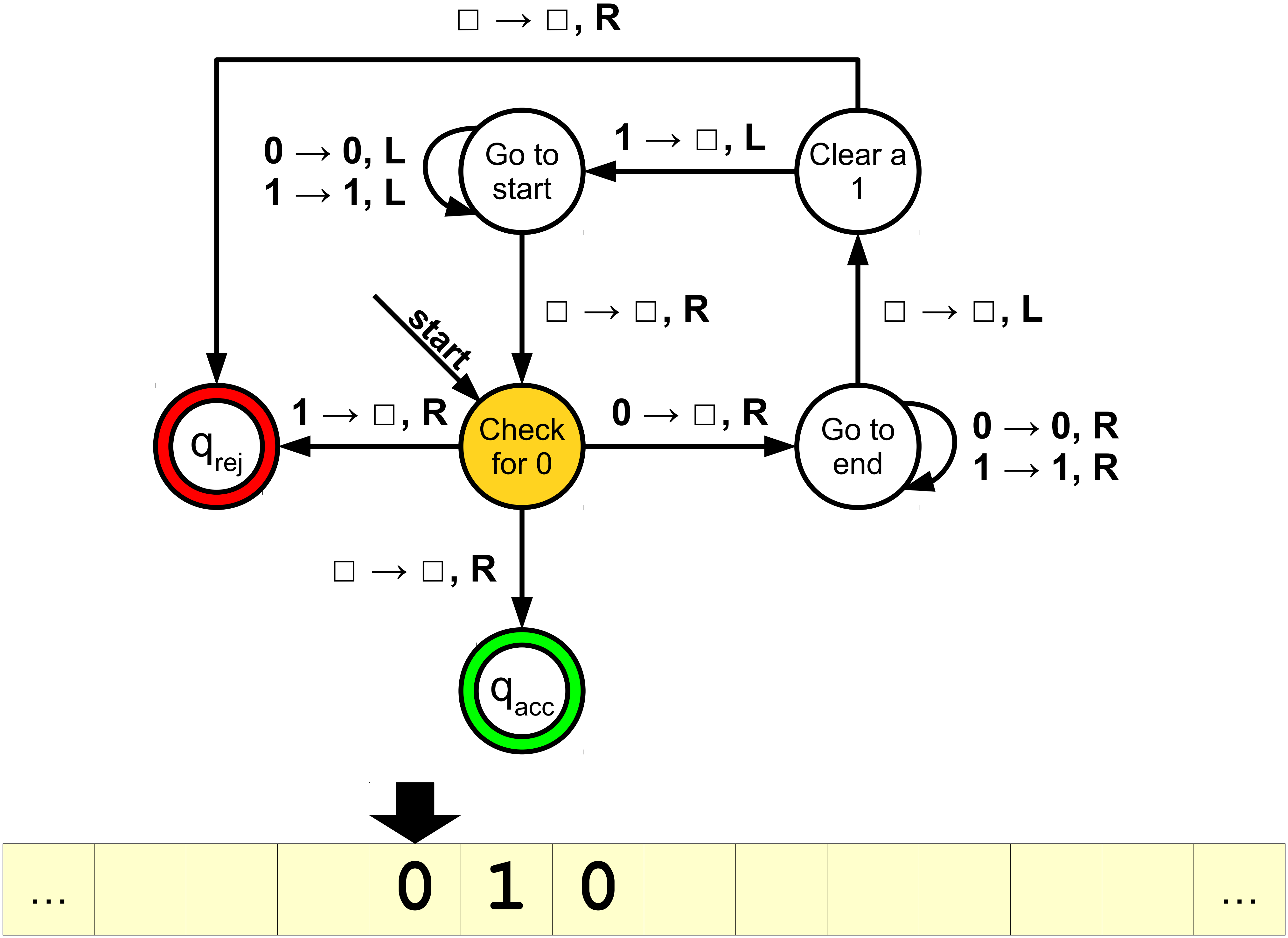
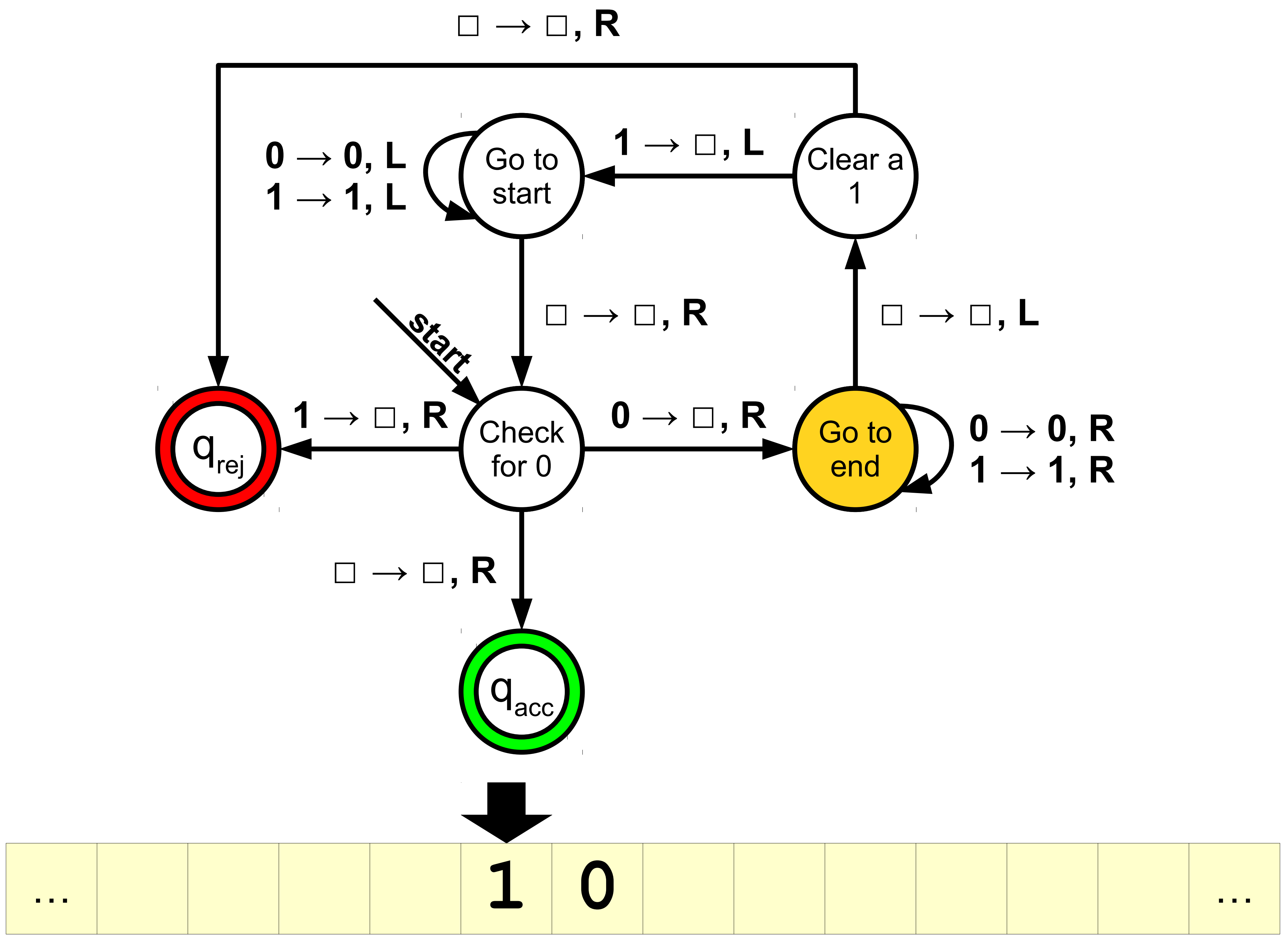
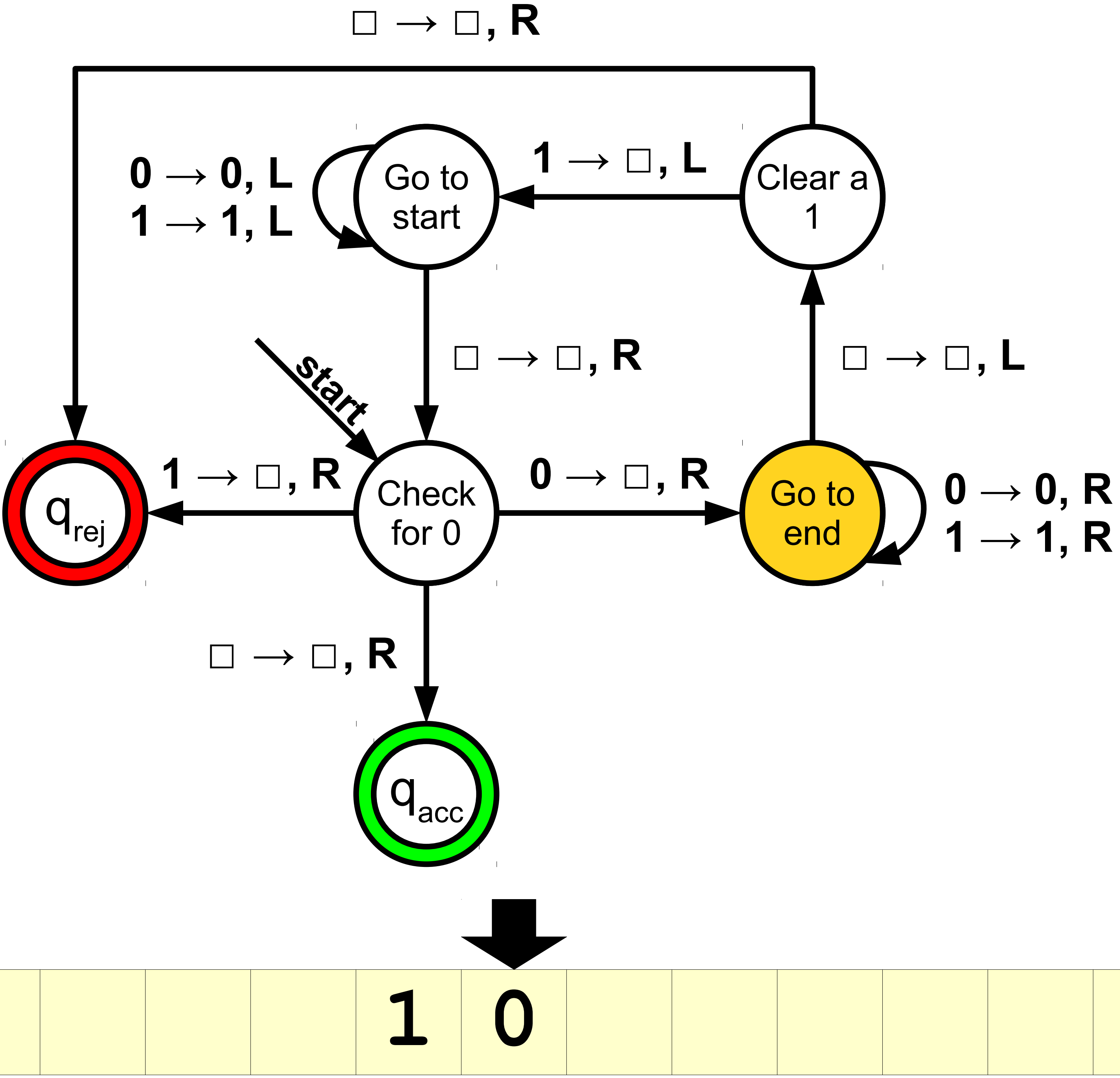
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
…

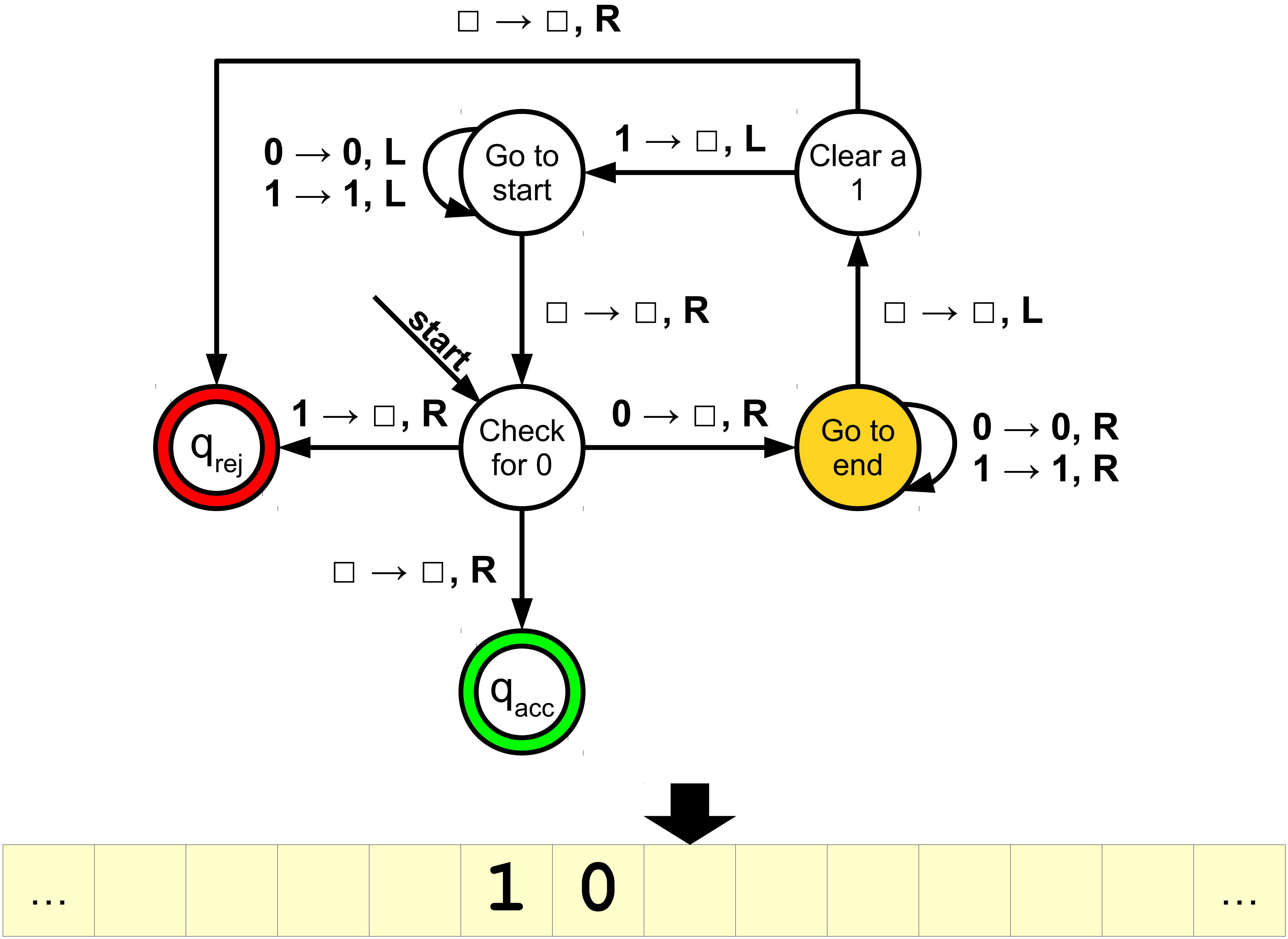

□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc

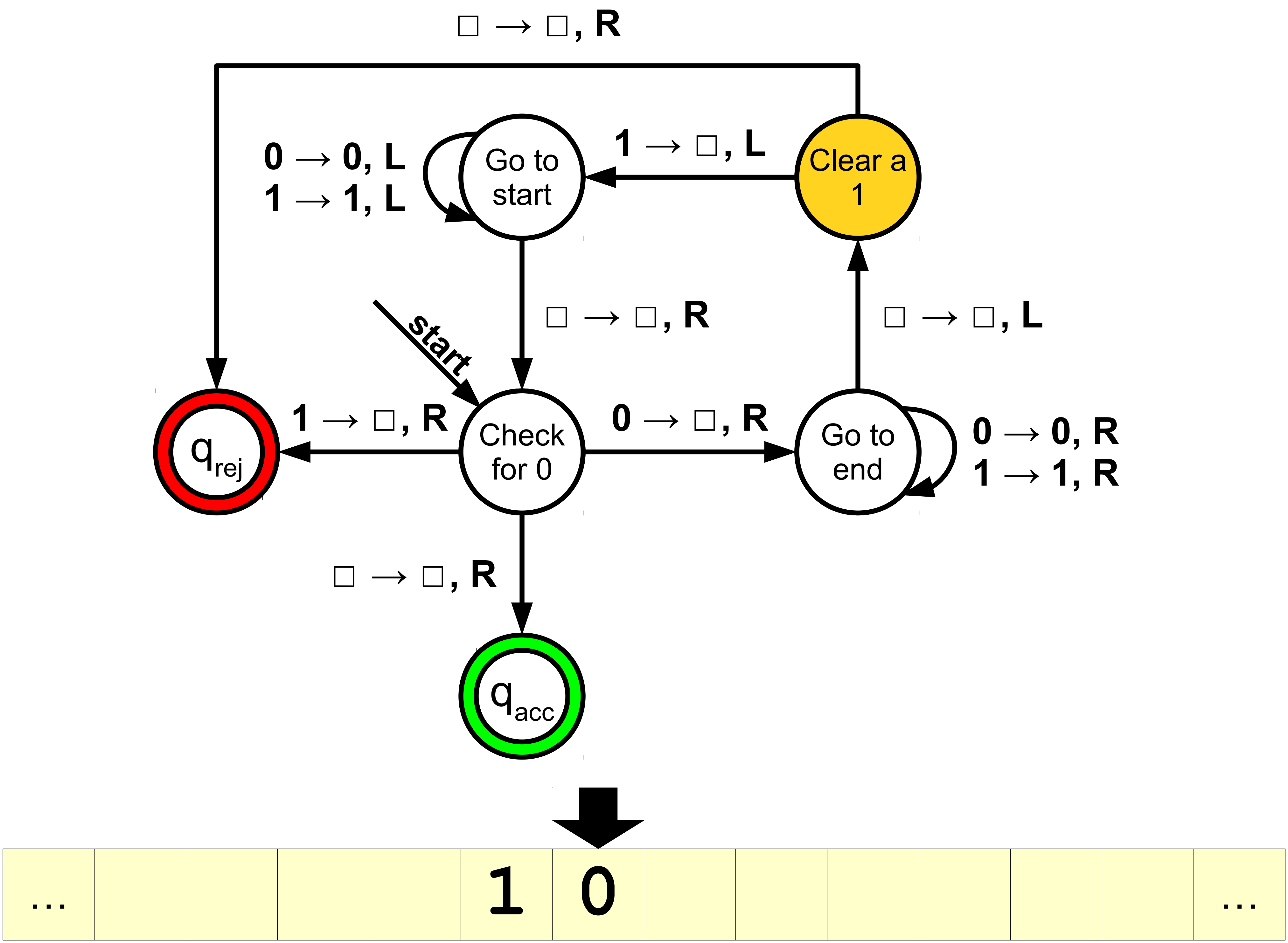

□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
1
0
…


□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
1
0
…

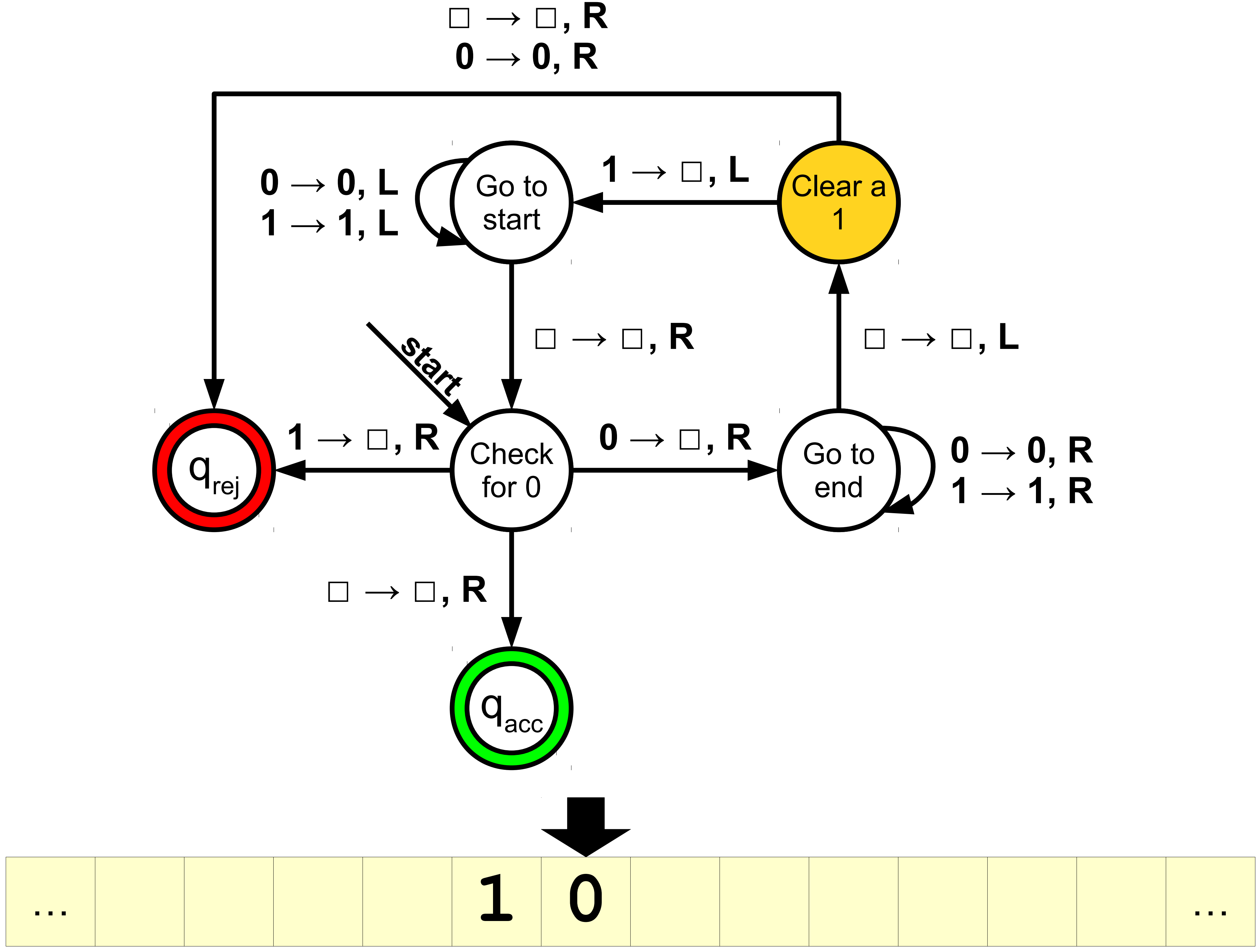

□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
0
…

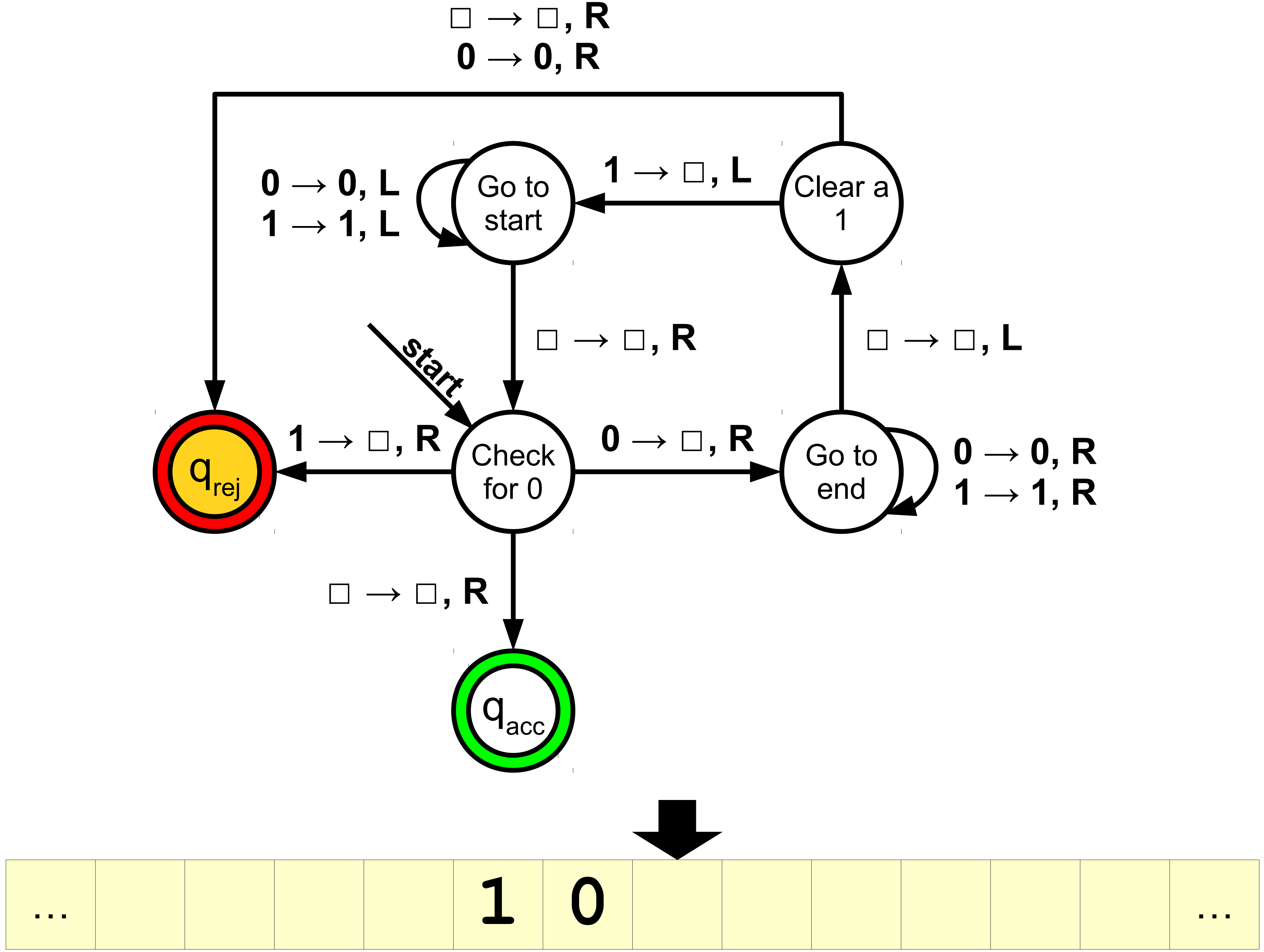

□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
0
…

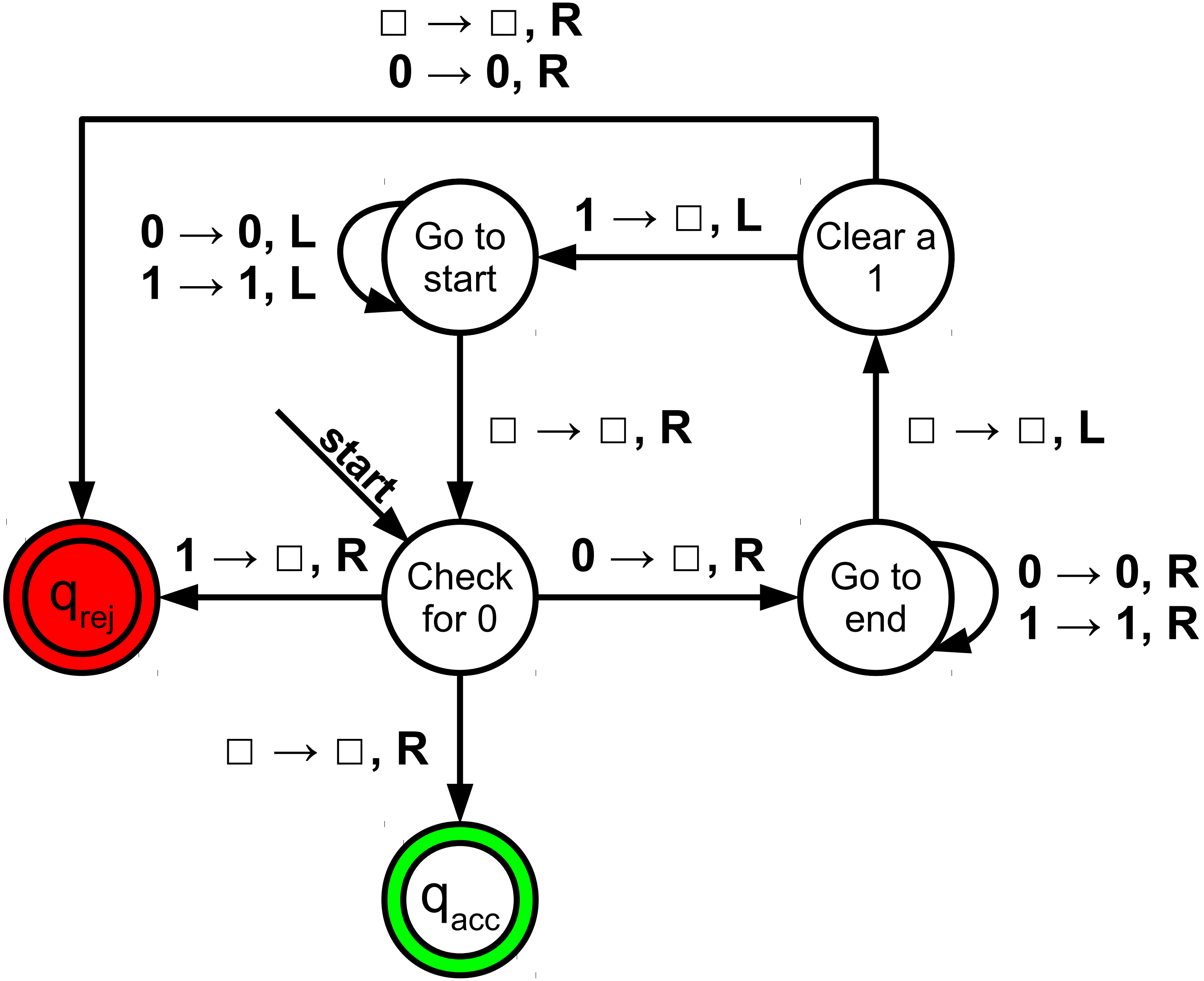

□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
1 → □, R
q_rej
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
0
…

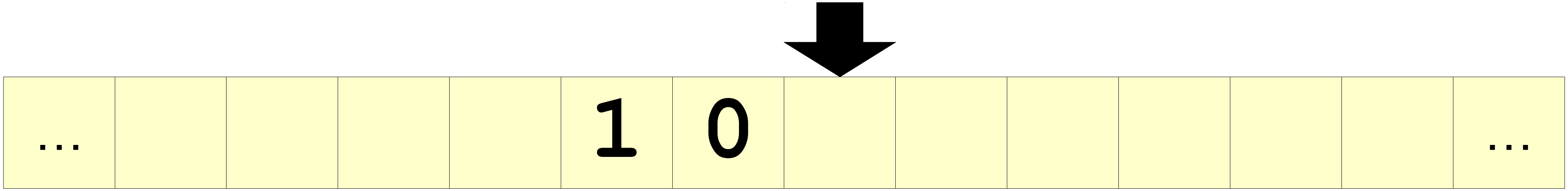


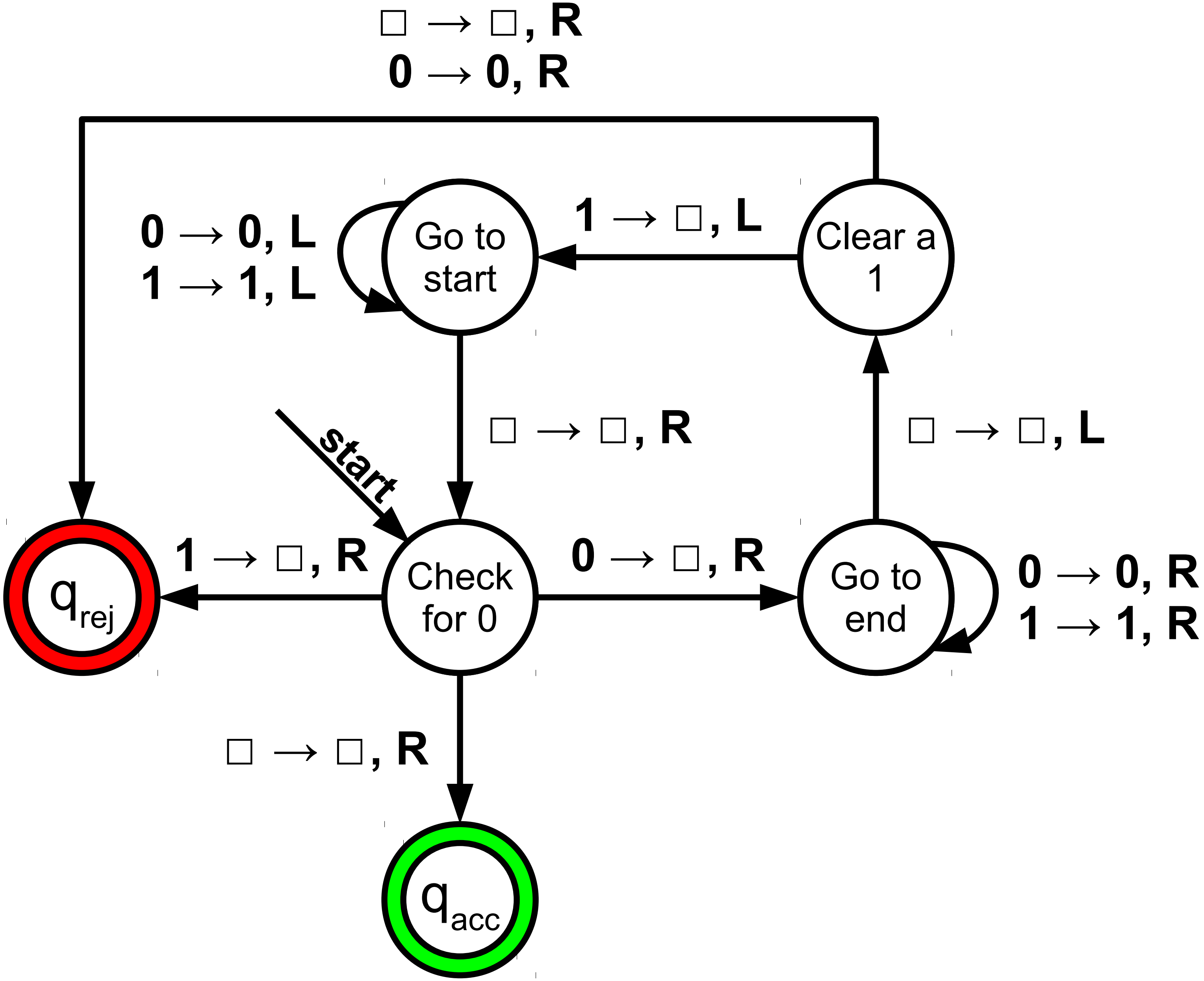
□ → □, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
0
…

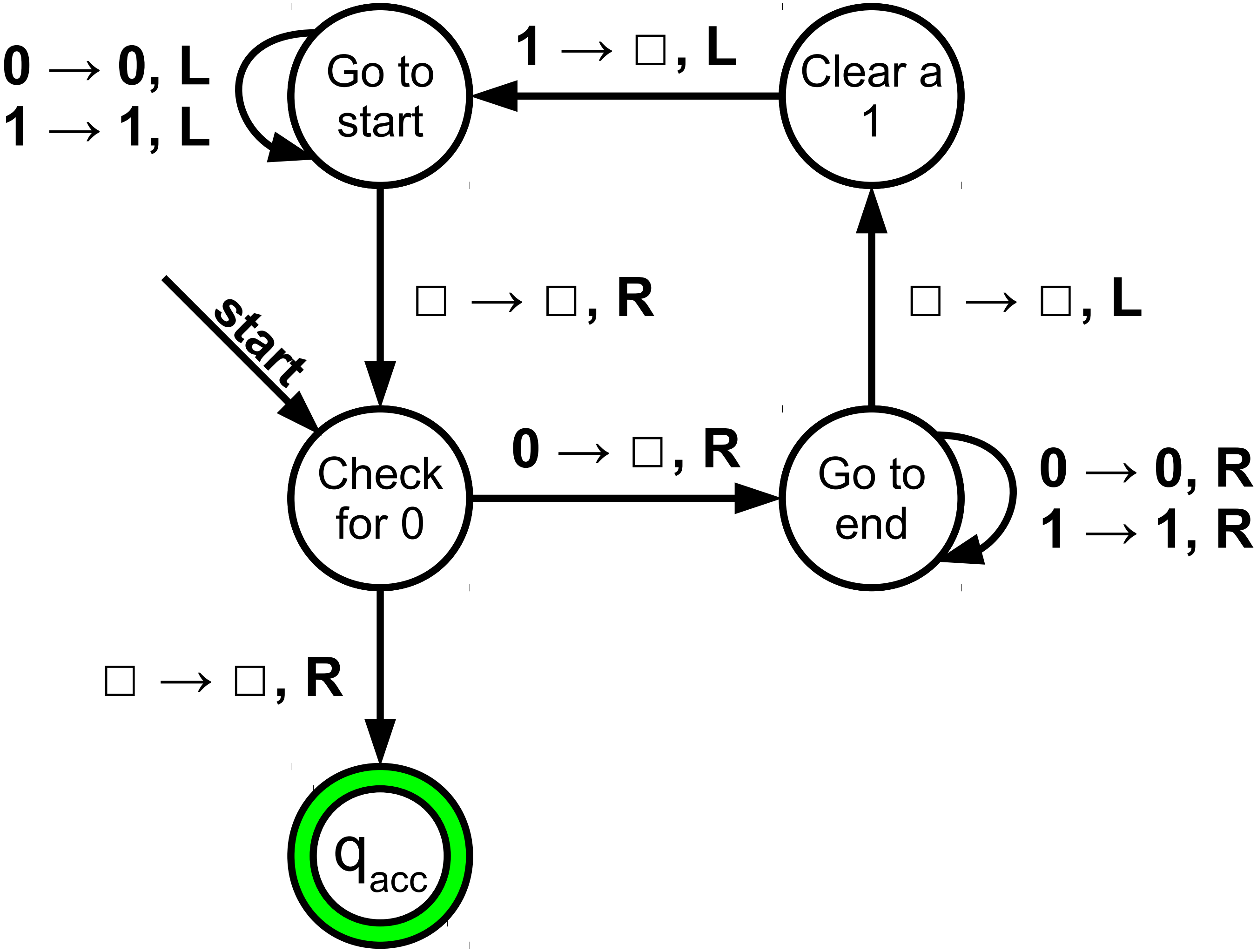

□ → □, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
0
…


□ → □, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
0
…


□ → □, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_rej
□ → □, R
q_acc


…
1
0
…


□ → □, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
q_{rej}
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}

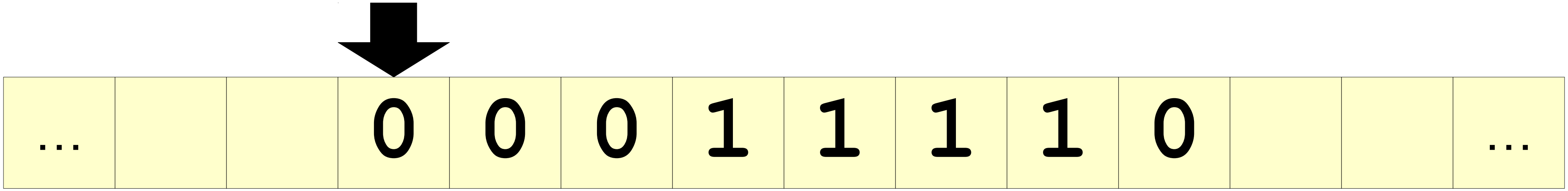
Another TM Design

- We've designed a TM for $\{\mathtt{0}^n\mathtt{1}^n \mid n \in \mathbb{N}\}$.
- Consider this language over $\Sigma = \{\mathtt{0}, \mathtt{1}\}$:

$$L = \{ w \in \Sigma^* \mid w \text{ has the same number of } \mathtt{0}\text{s and } \mathtt{1}\text{s} \}$$

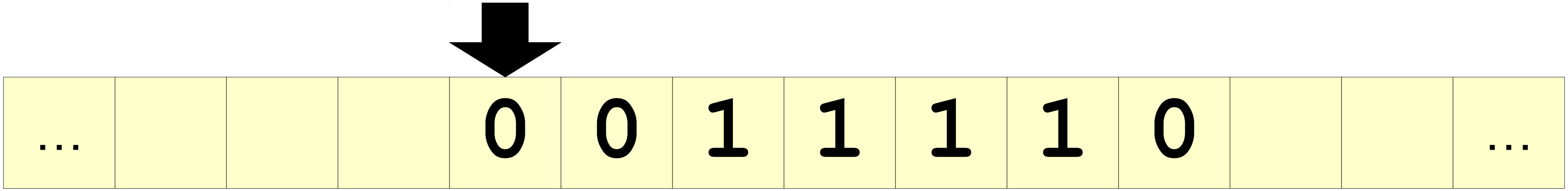
- This language is also not regular, but it is context-free.
- How might we design a TM for it?

A Caveat

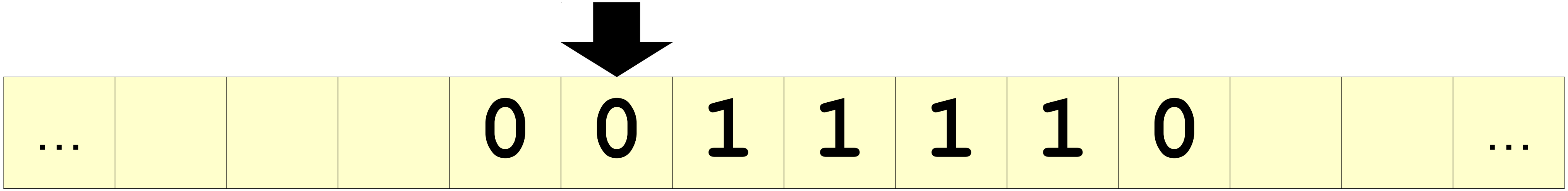


…			0	0	0	1	1	1	1	0			…

A Caveat

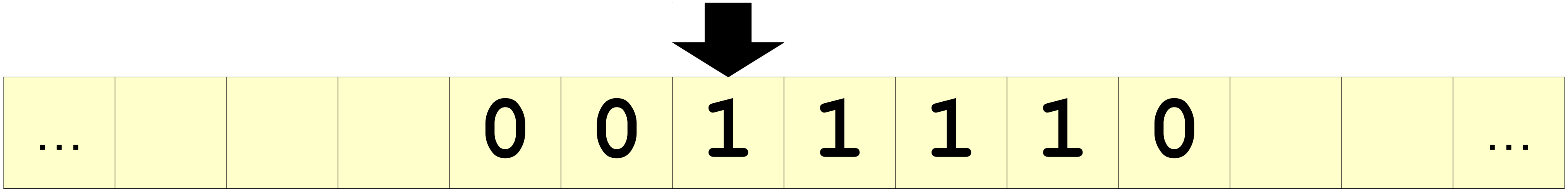


A Caveat

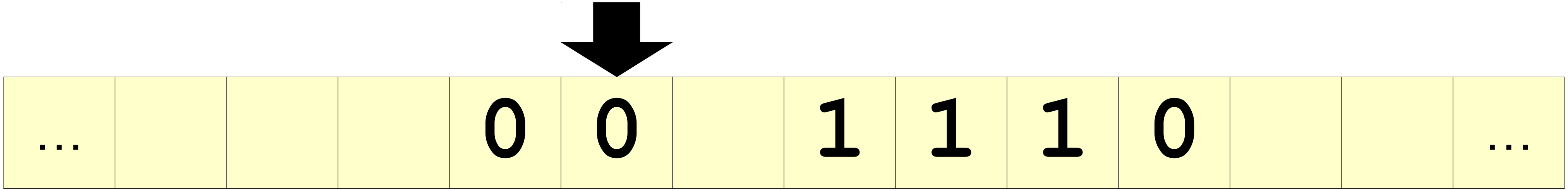


…				0	0	1	1	1	1	0			…

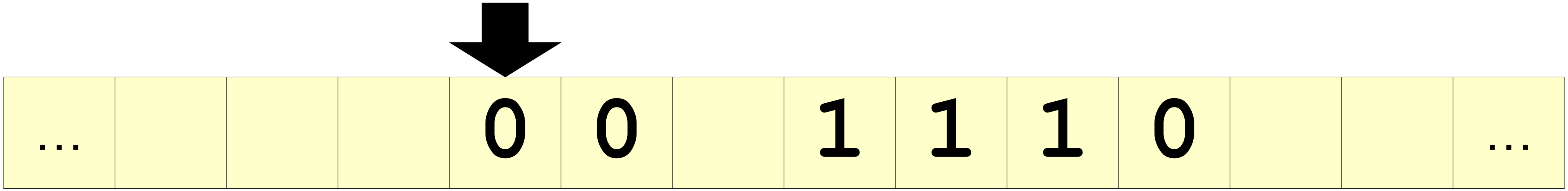
A Caveat



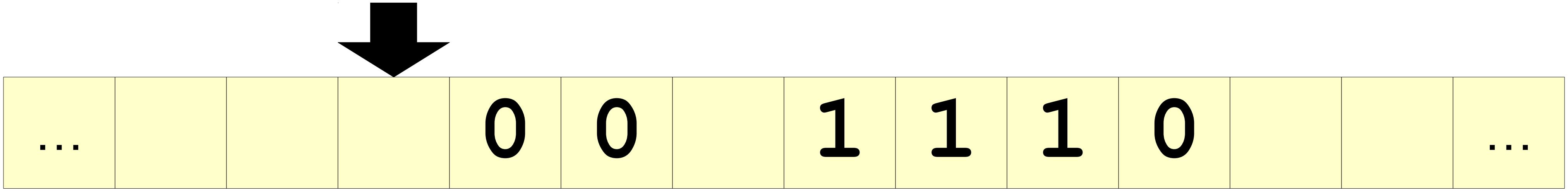
A Caveat



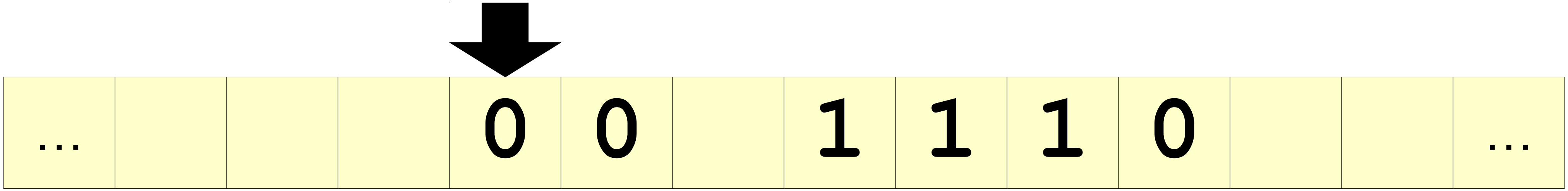
A Caveat



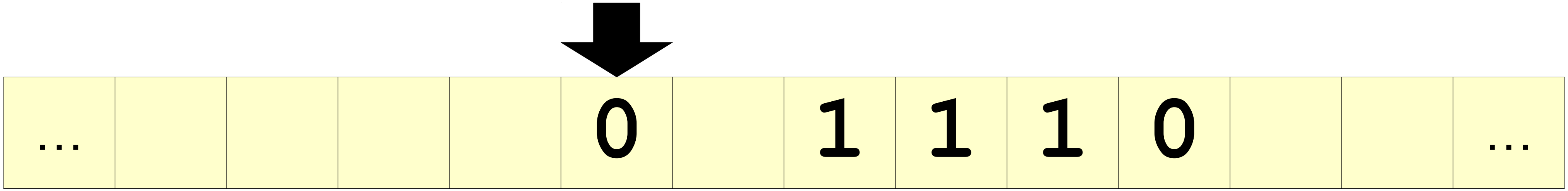
A Caveat



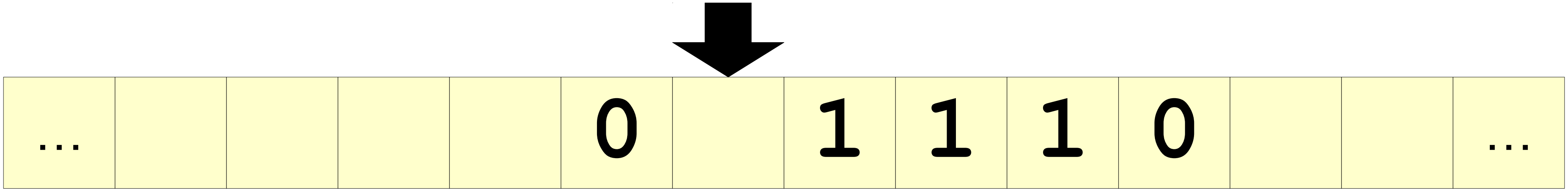
A Caveat



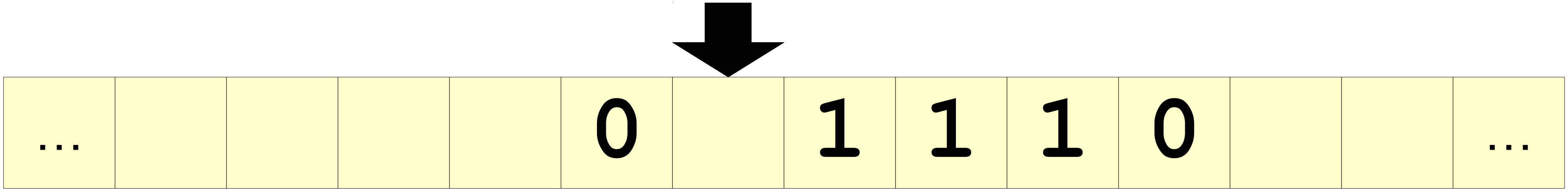
A Caveat



A Caveat

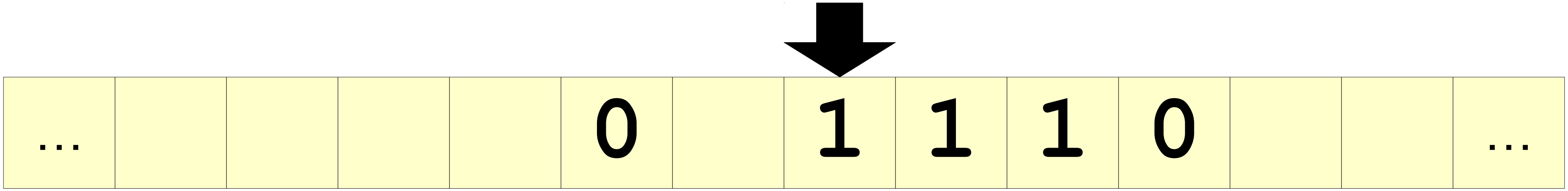


A Caveat

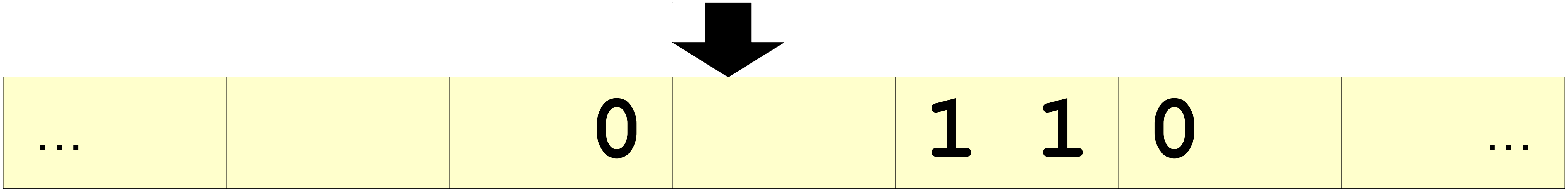


How do we know that this blank isn't one of the infinitely many blanks after our input string?

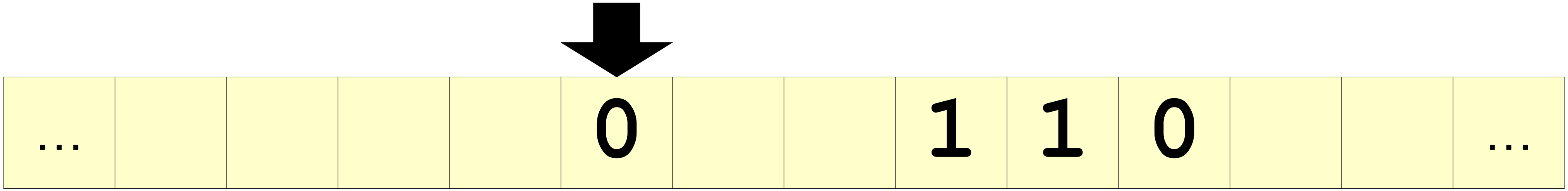
A Caveat



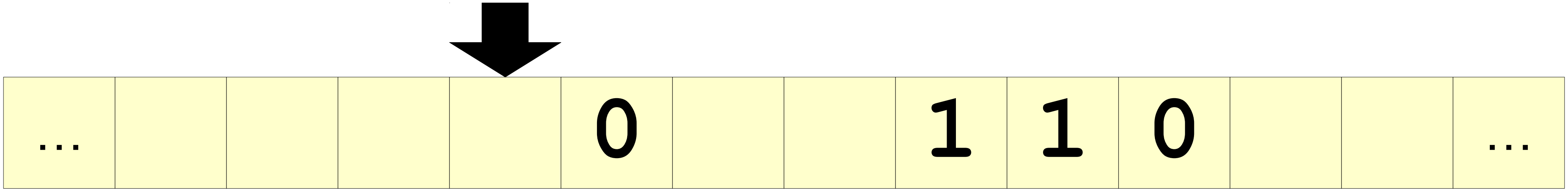
A Caveat



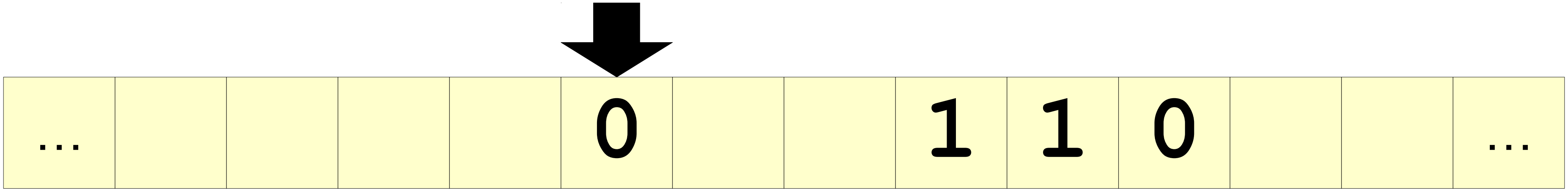
A Caveat



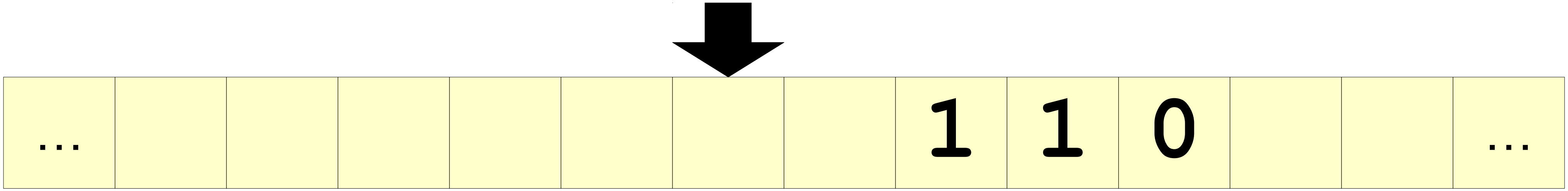
A Caveat



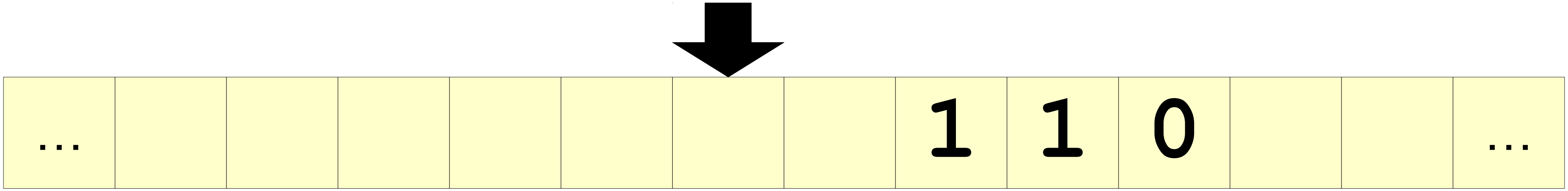
A Caveat



A Caveat

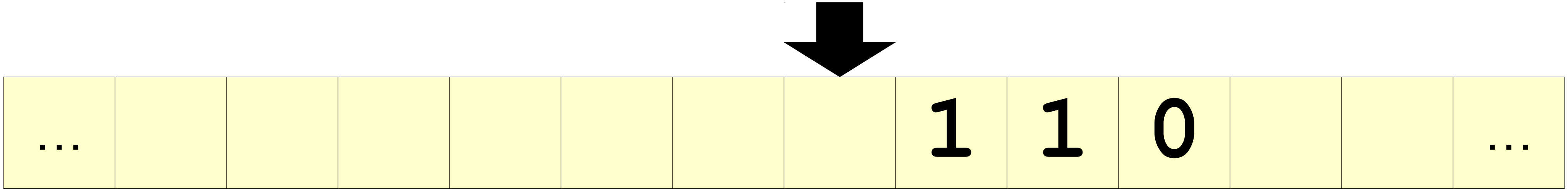


A Caveat

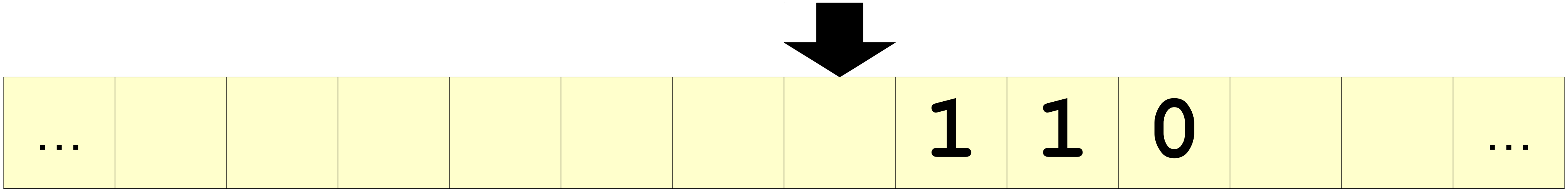


How do we know that this blank isn't one of the infinitely many blanks after our input string?

A Caveat

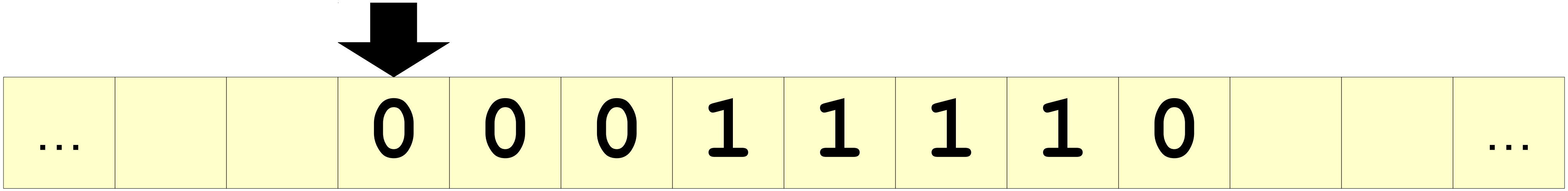


A Caveat



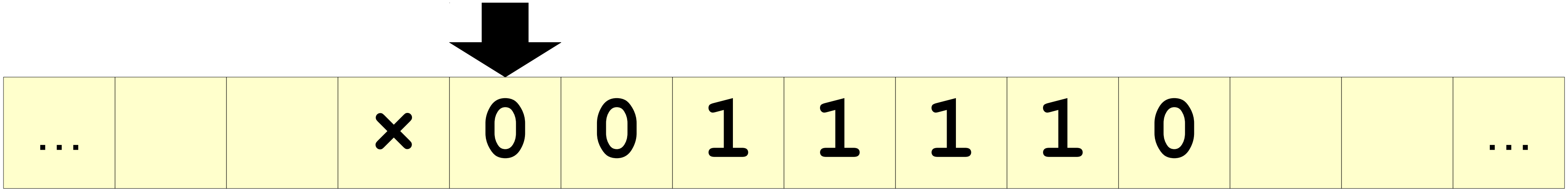
How do we know that this blank isn't one of the infinitely many blanks after our input string?

One Solution

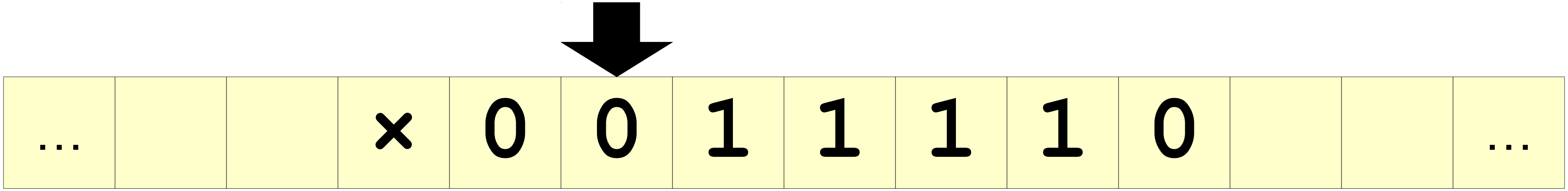


… 0 0 0 1 1 1 1 0 …

One Solution

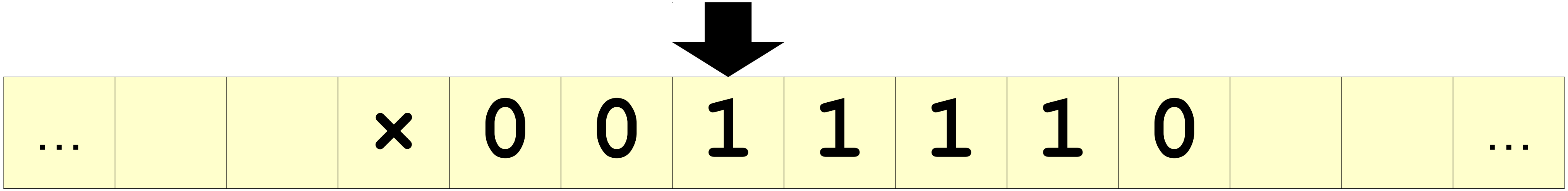


One Solution

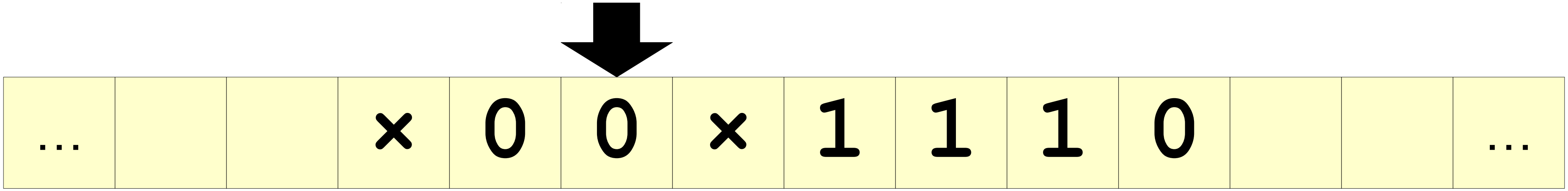


…			×	0	0	1	1	1	1	0			…

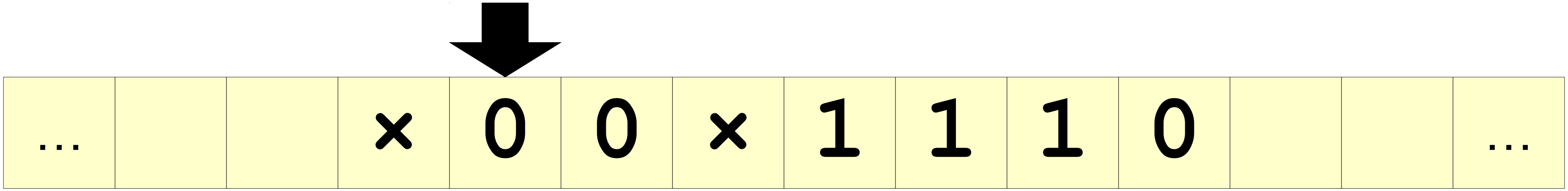
One Solution



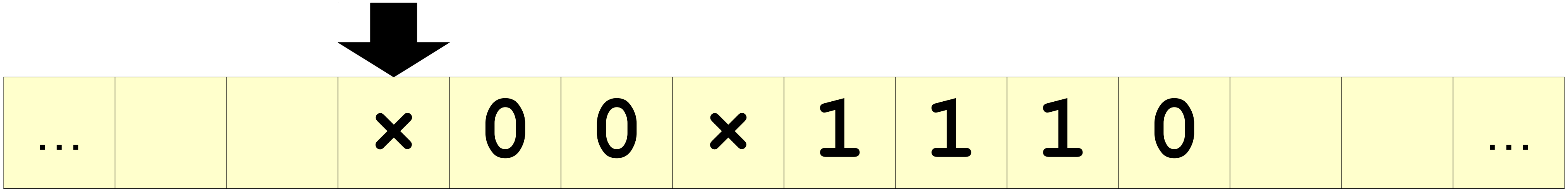
One Solution



One Solution

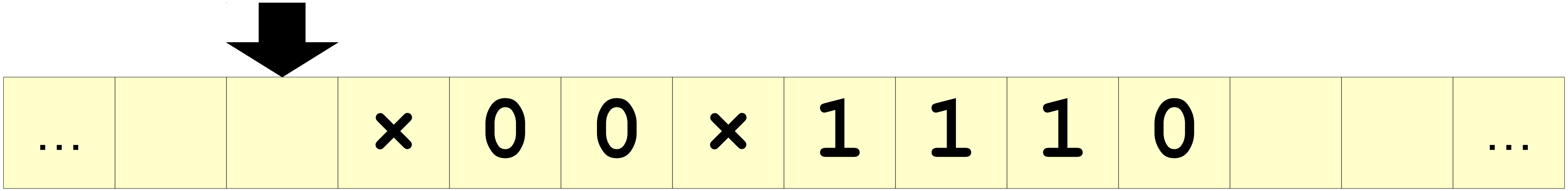


One Solution

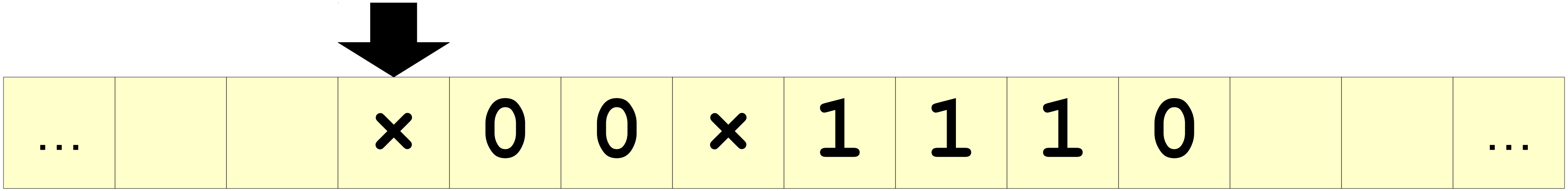


…			×	0	0	×	1	1	1	0			…

One Solution

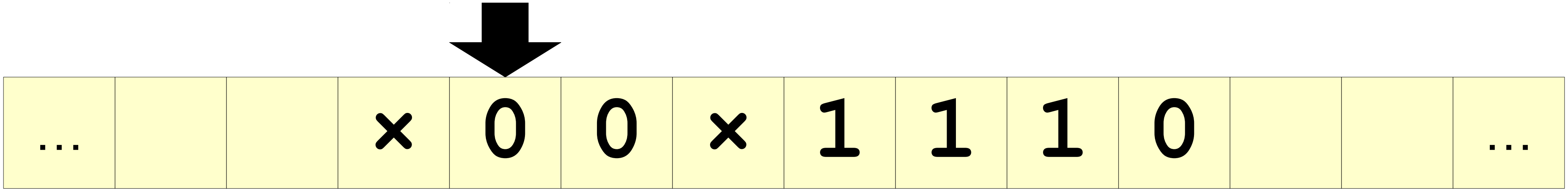


One Solution

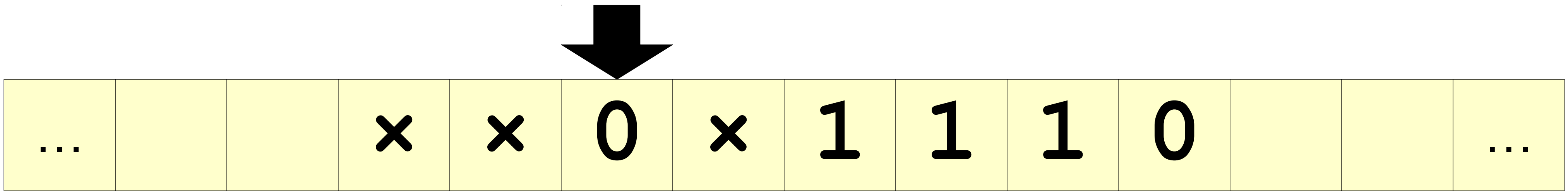


…			×	0	0	×	1	1	1	0			…

One Solution

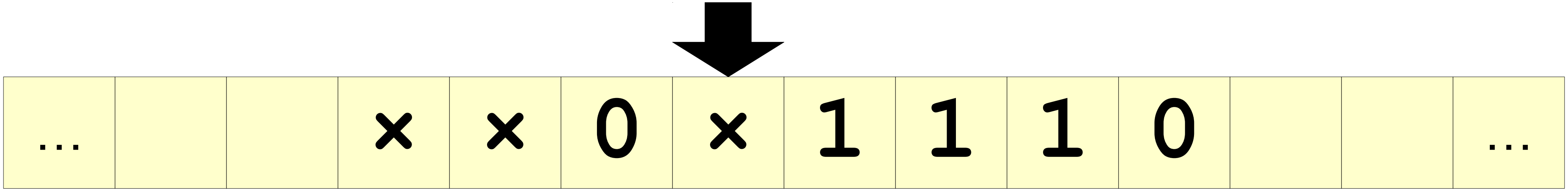


One Solution



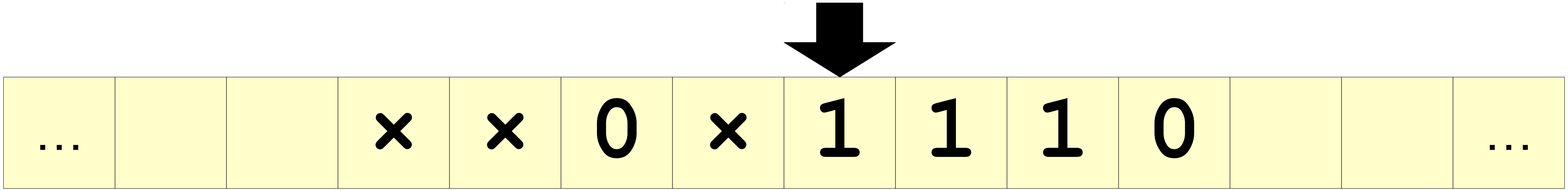
…			×	×	0	×	1	1	1	0			…

One Solution



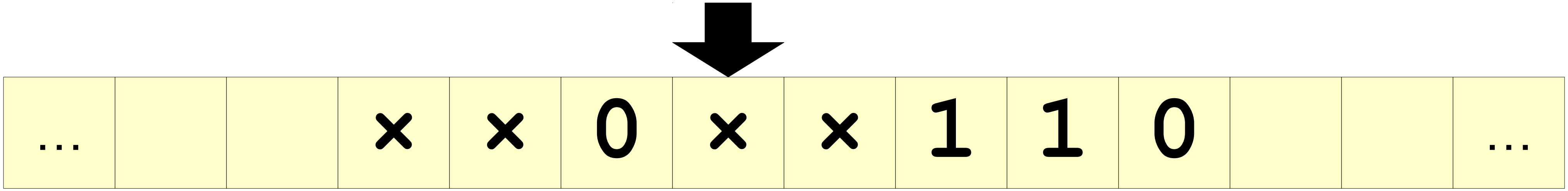
…			×	×	0	×	1	1	1	0			…

One Solution



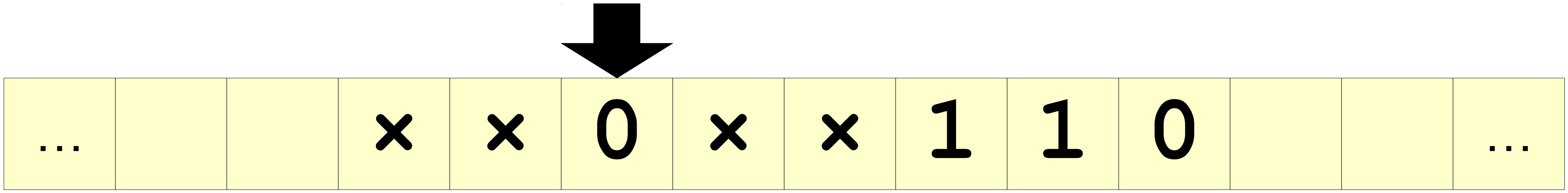
…			×	×	0	×	1	1	1	0			…

One Solution



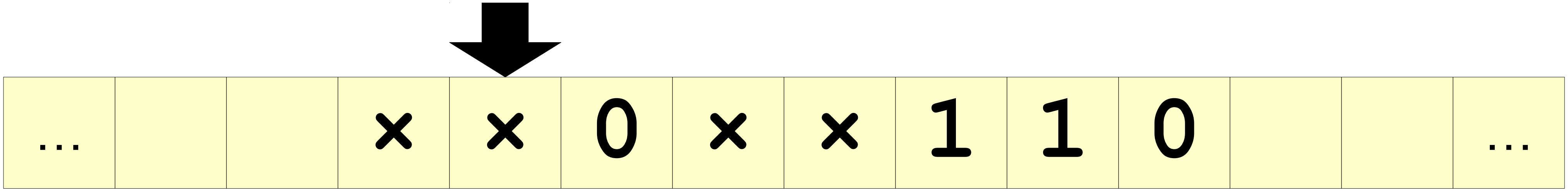
…			×	×	0	×	×	1	1	0			…

One Solution

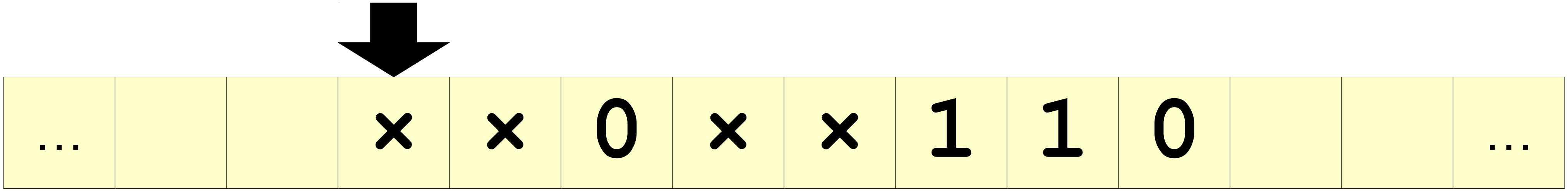


…			×	×	0	×	×	1	1	0			…

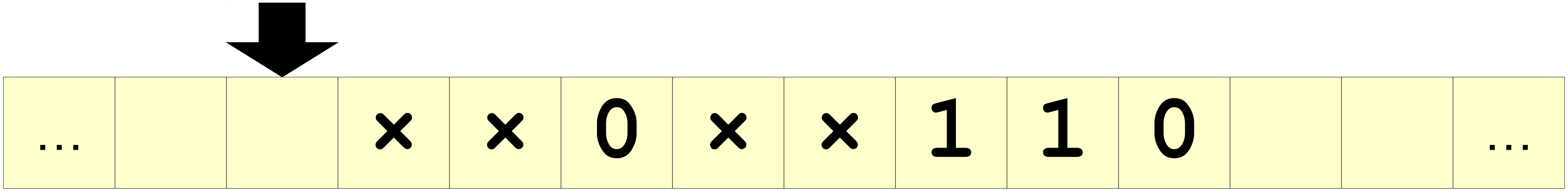
One Solution



One Solution

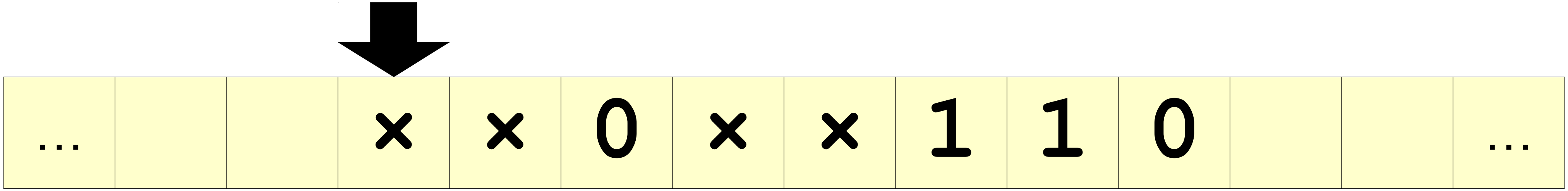


One Solution



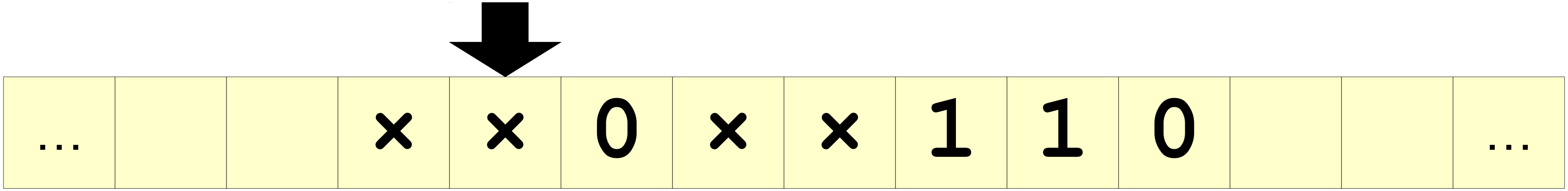
…			×	×	0	×	×	1	1	0			…

One Solution



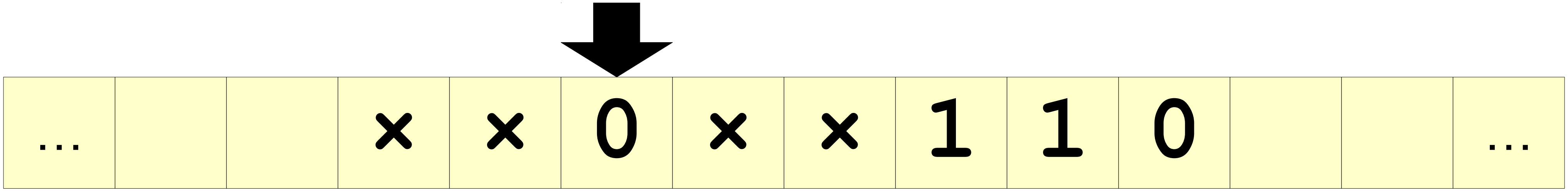
…			×	×	0	×	×	1	1	0			…

One Solution



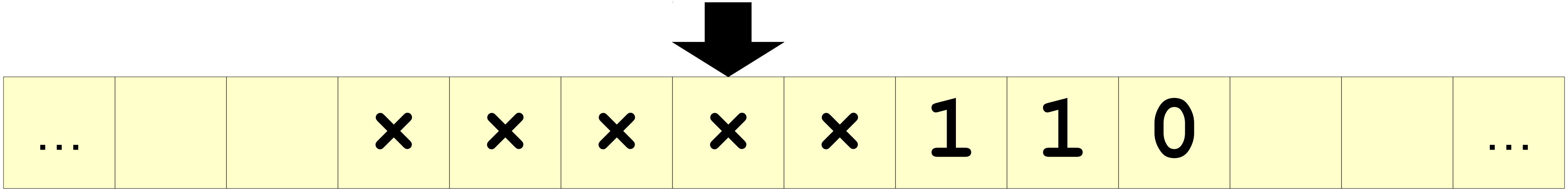
…			×	×	0	×	×	1	1	0			…

One Solution

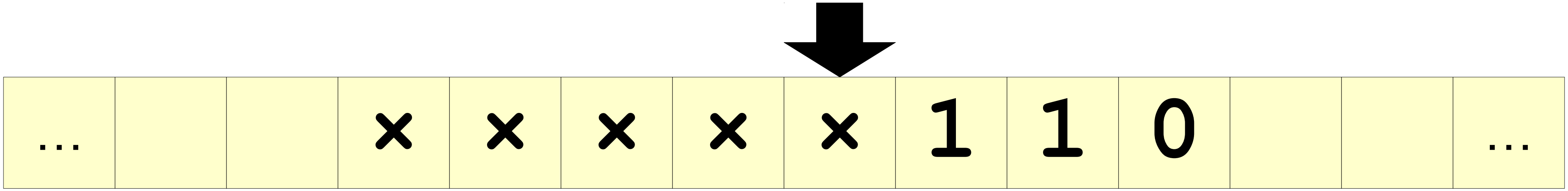


…			×	×	0	×	×	1	1	0			…

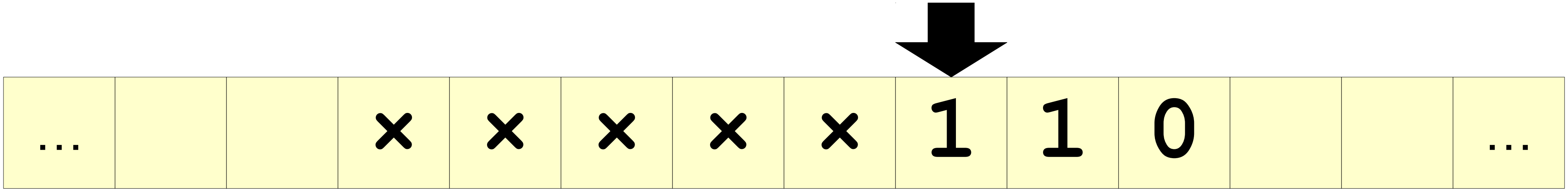
One Solution



One Solution

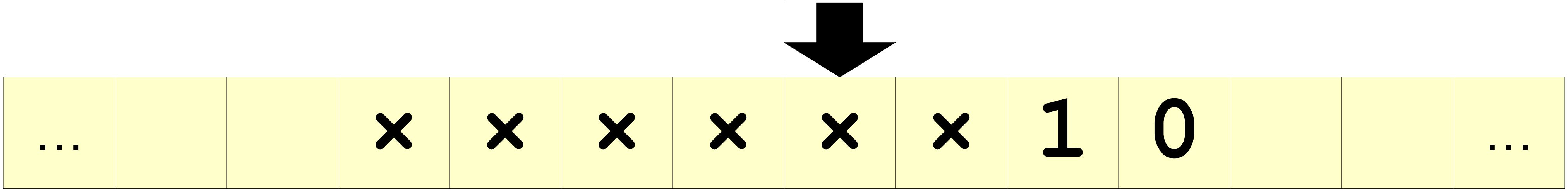


One Solution

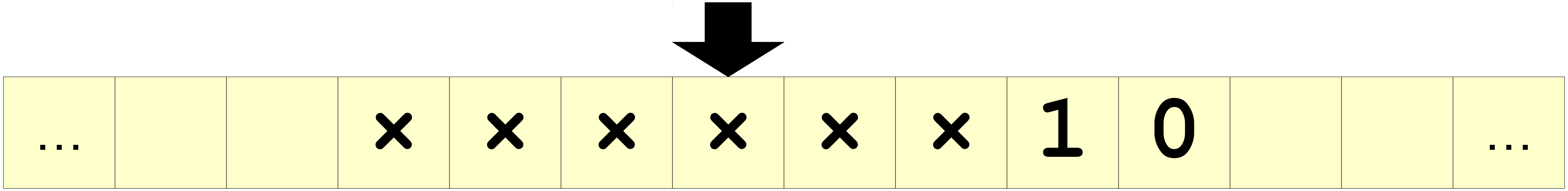


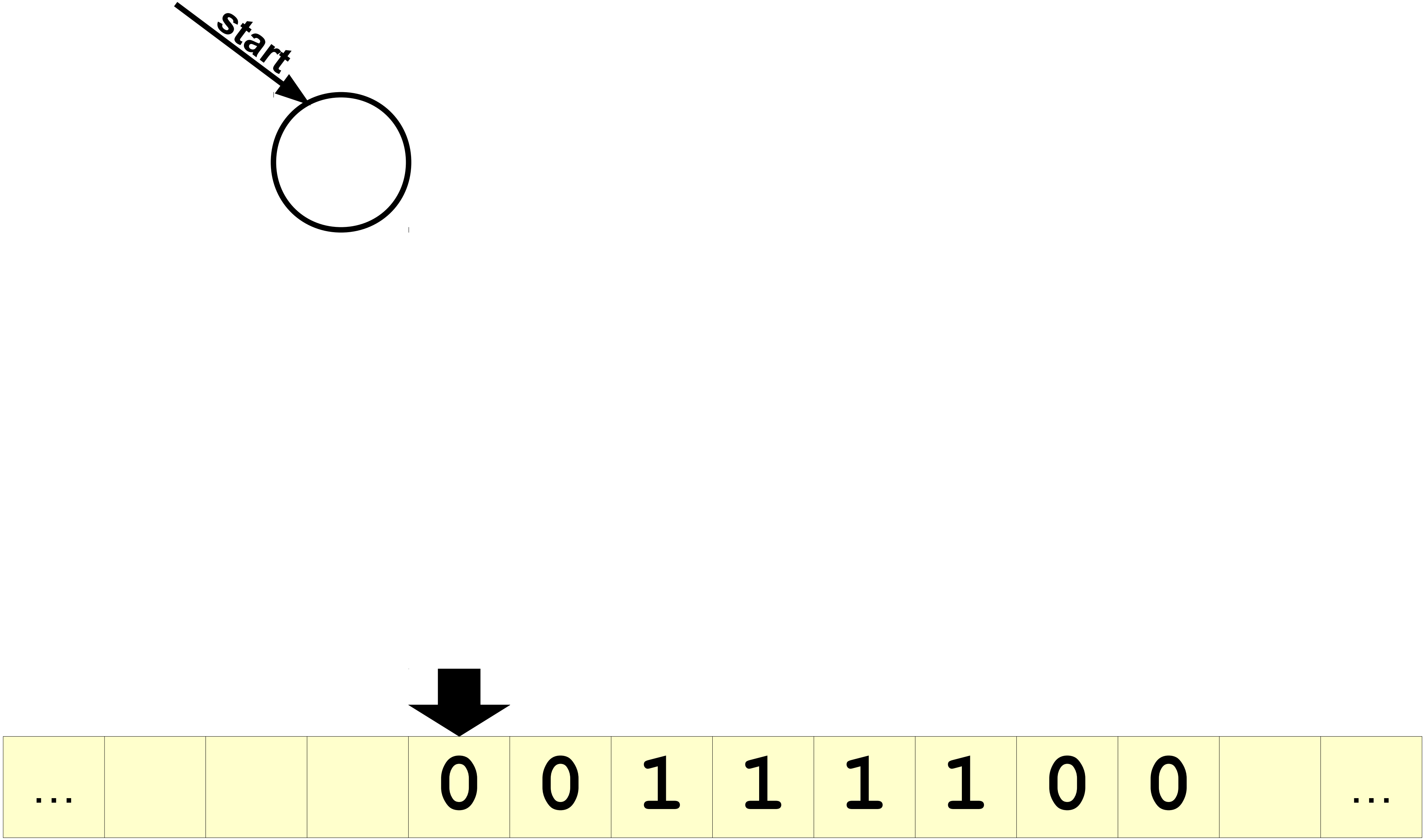
…			×	×	×	×	×	1	1	0			…

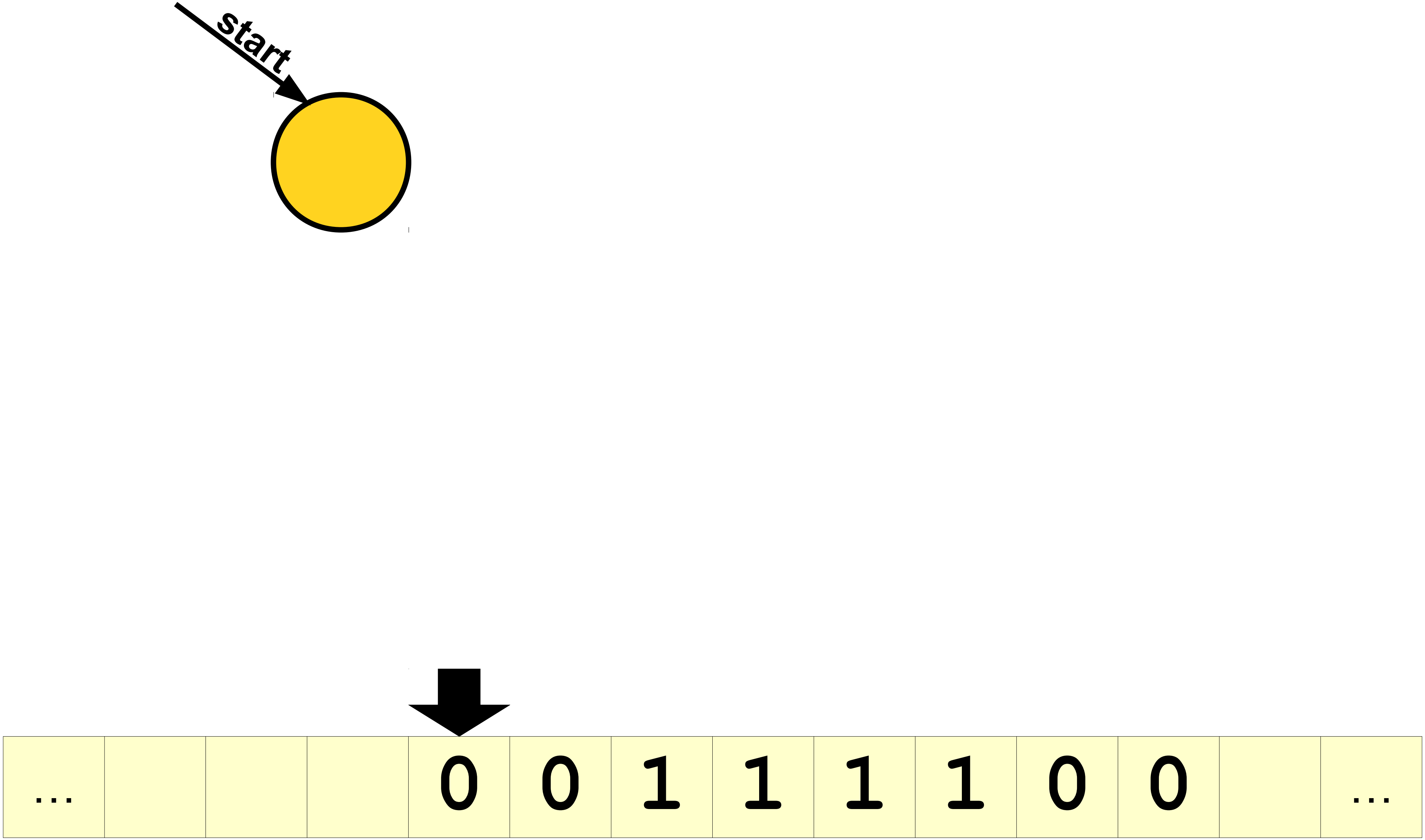
One Solution

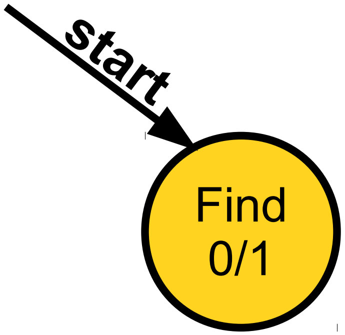


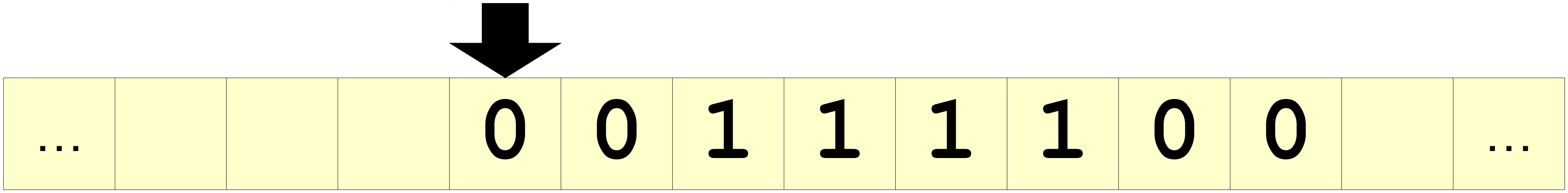
One Solution



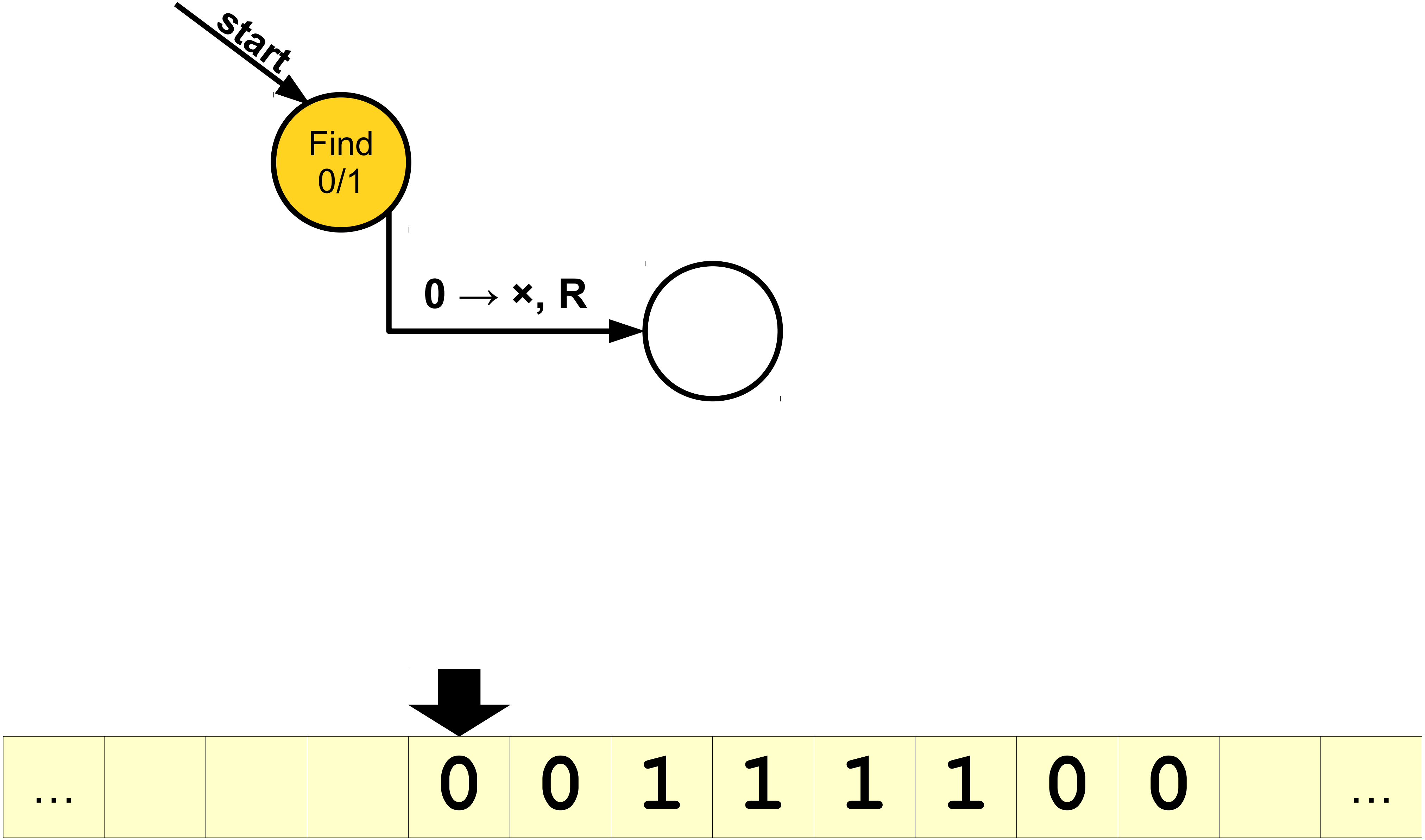

start
...
0
0
1
1
1
1
0
0
...

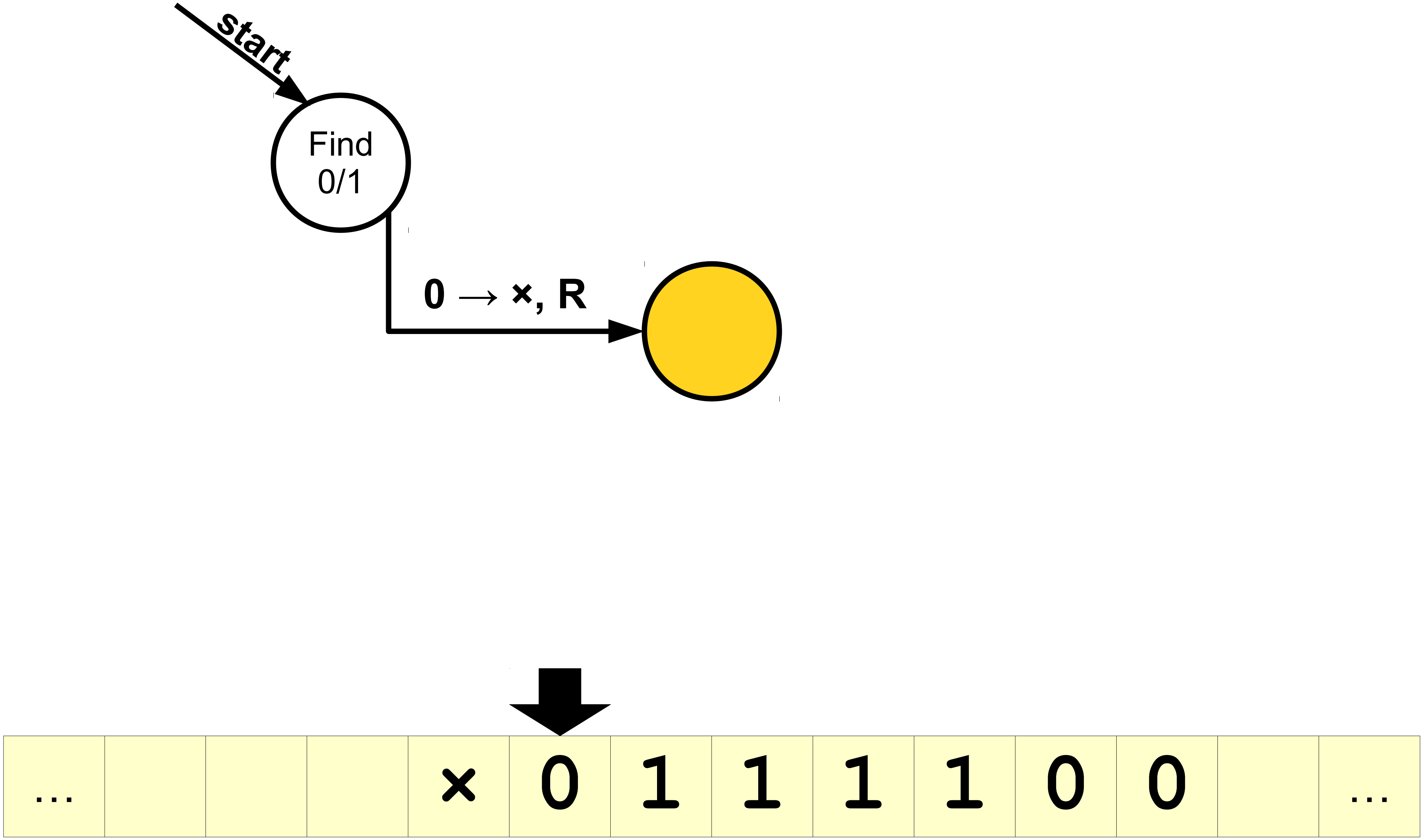

start
…
0
0
1
1
1
1
0
0
…

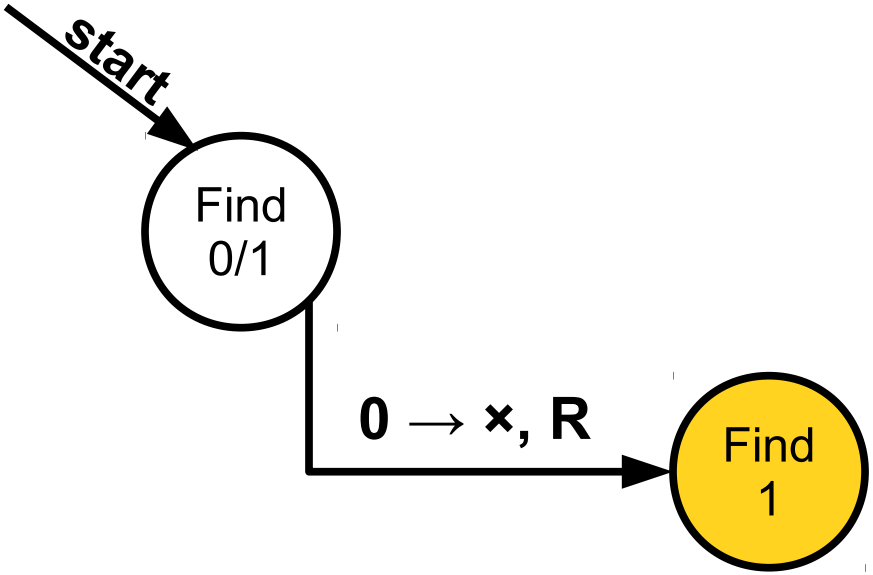


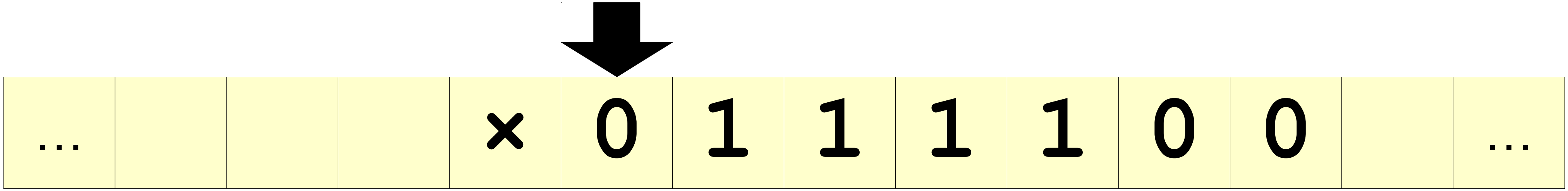


...				0	0	1	1	1	1	0	0		...

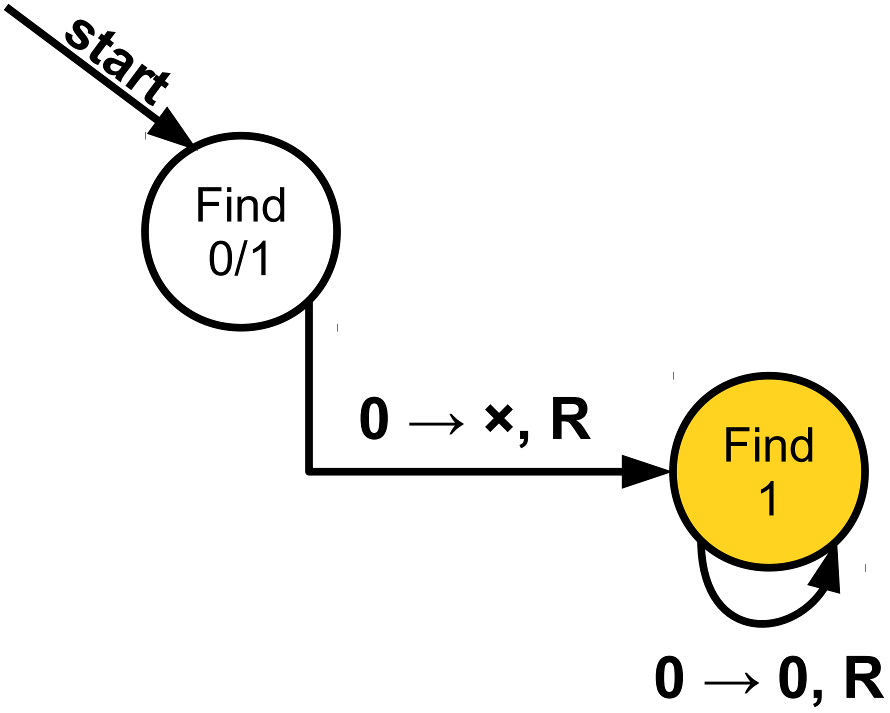

start
Find 0/1
0 → ×, R
…
0
0
1
1
1
1
0
0
…

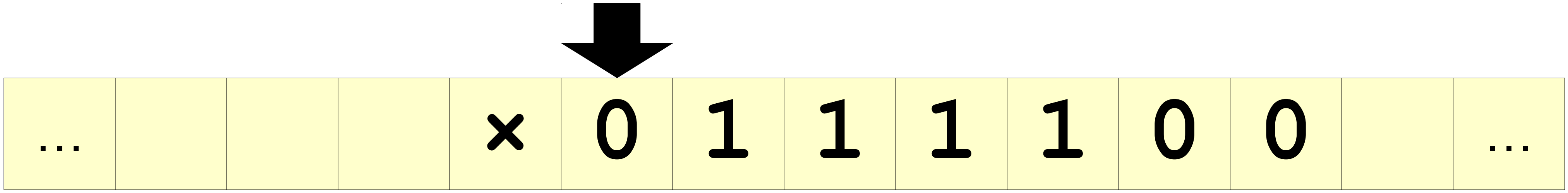

start
Find
0/1
0 → ×, R
…
×
0
1
1
1
1
0
0
…



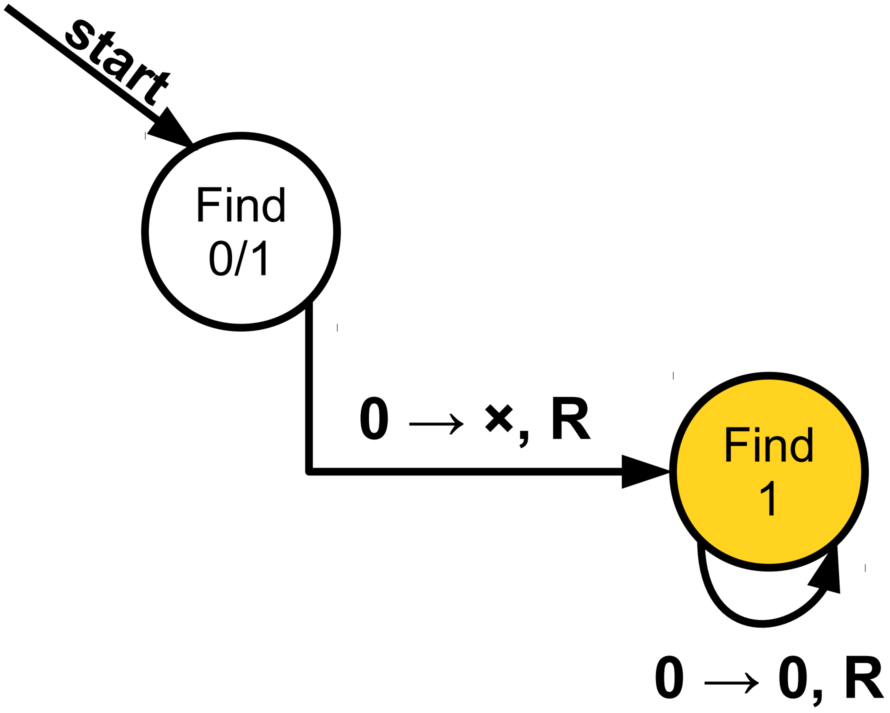


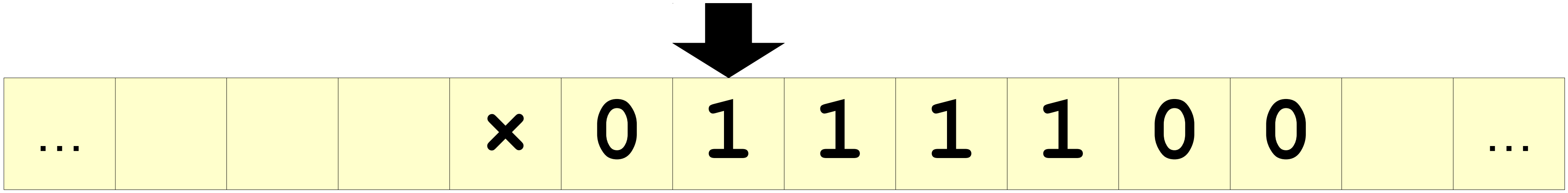
…				×	0	1	1	1	1	0	0		…

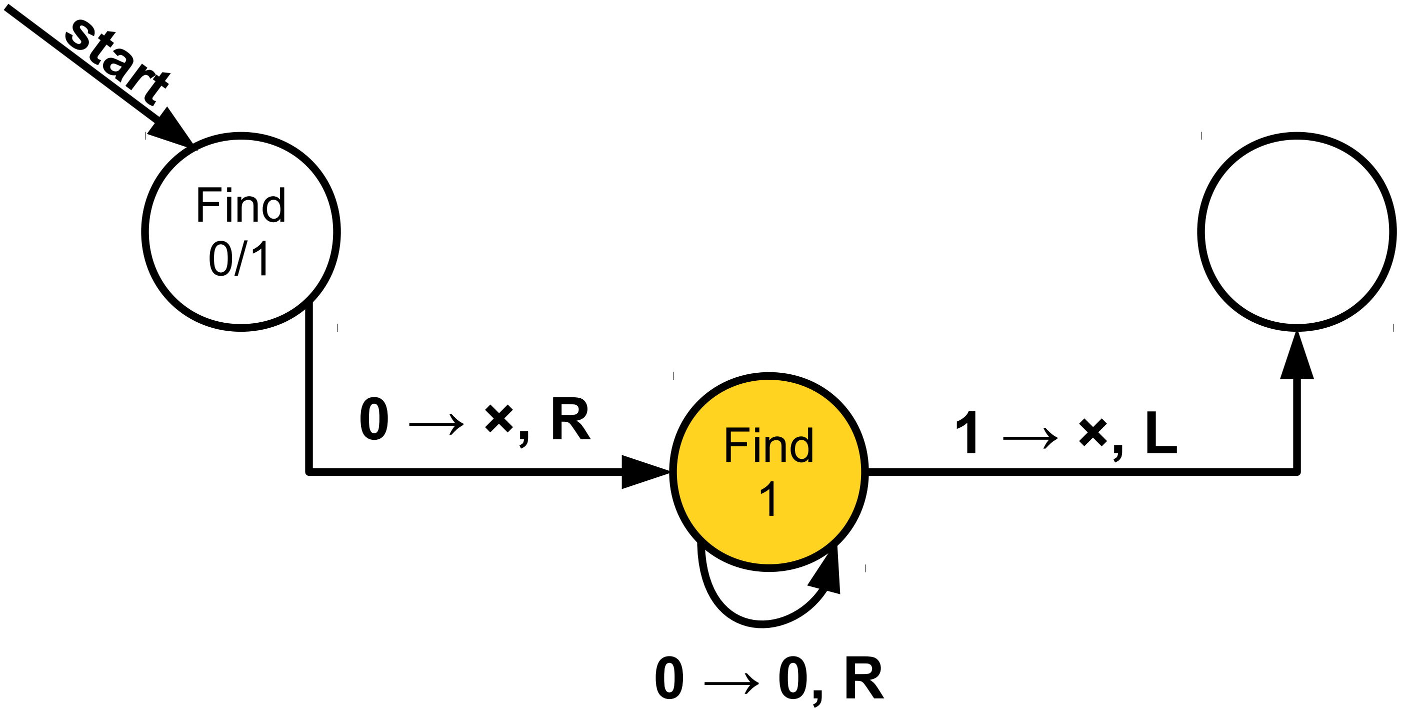


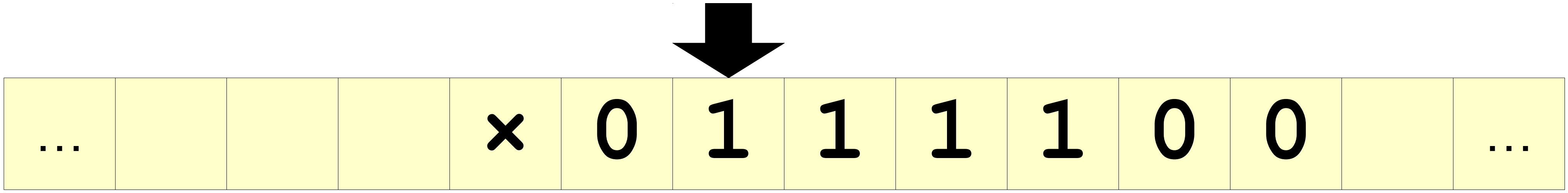


…				×	0	1	1	1	1	0	0		…

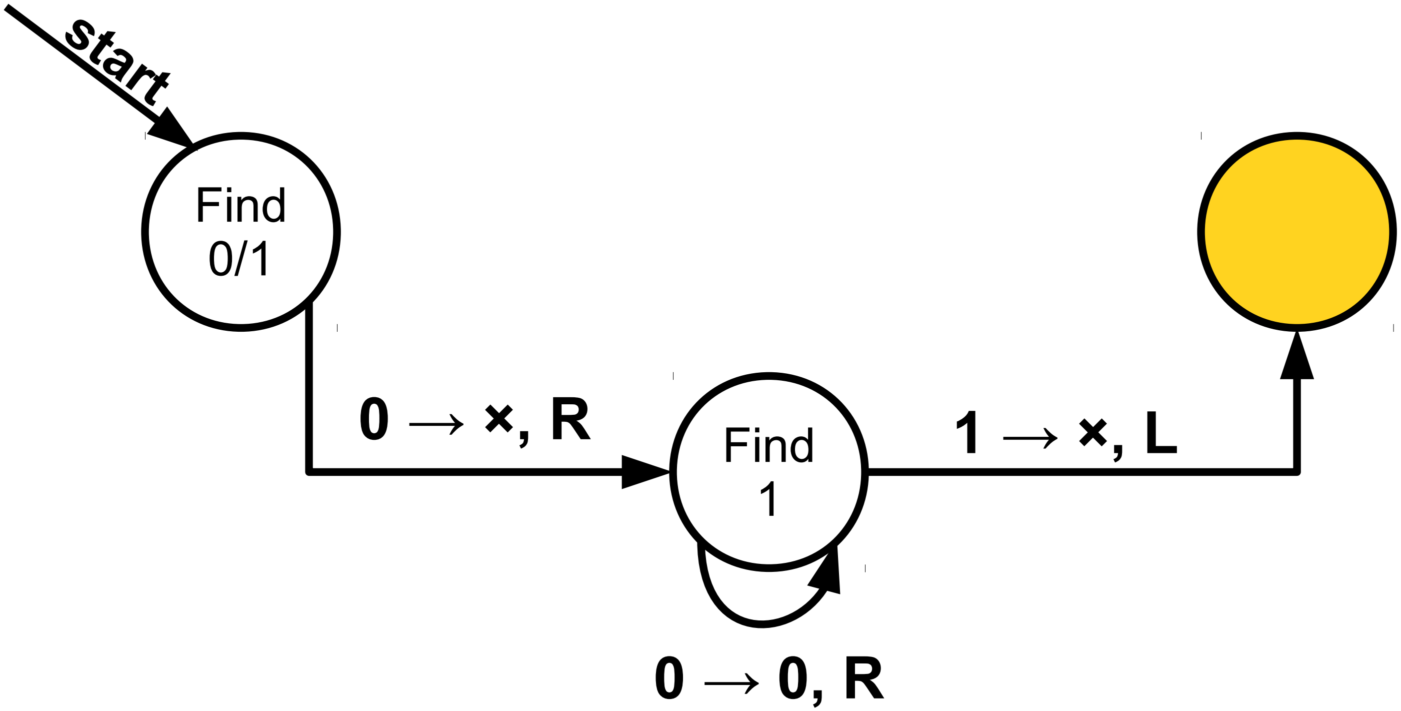

start
Find 0/1
0 → ×, R
Find 1
0 → 0, R


…
×
0
1
1
1
1
0
0
…





…				×	0	1	1	1	1	0	0		…



start

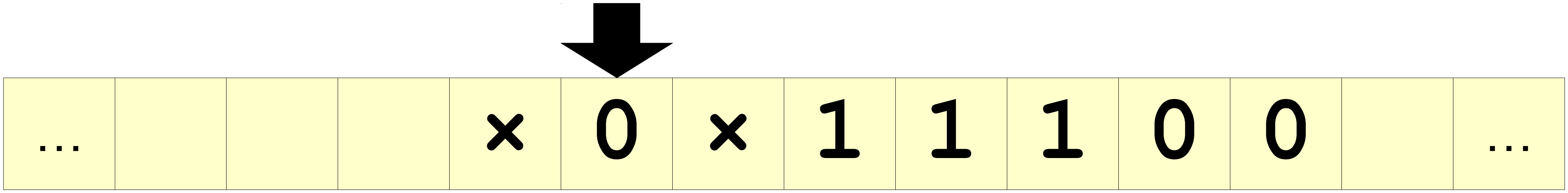
Find 0/1

0 → ×, R

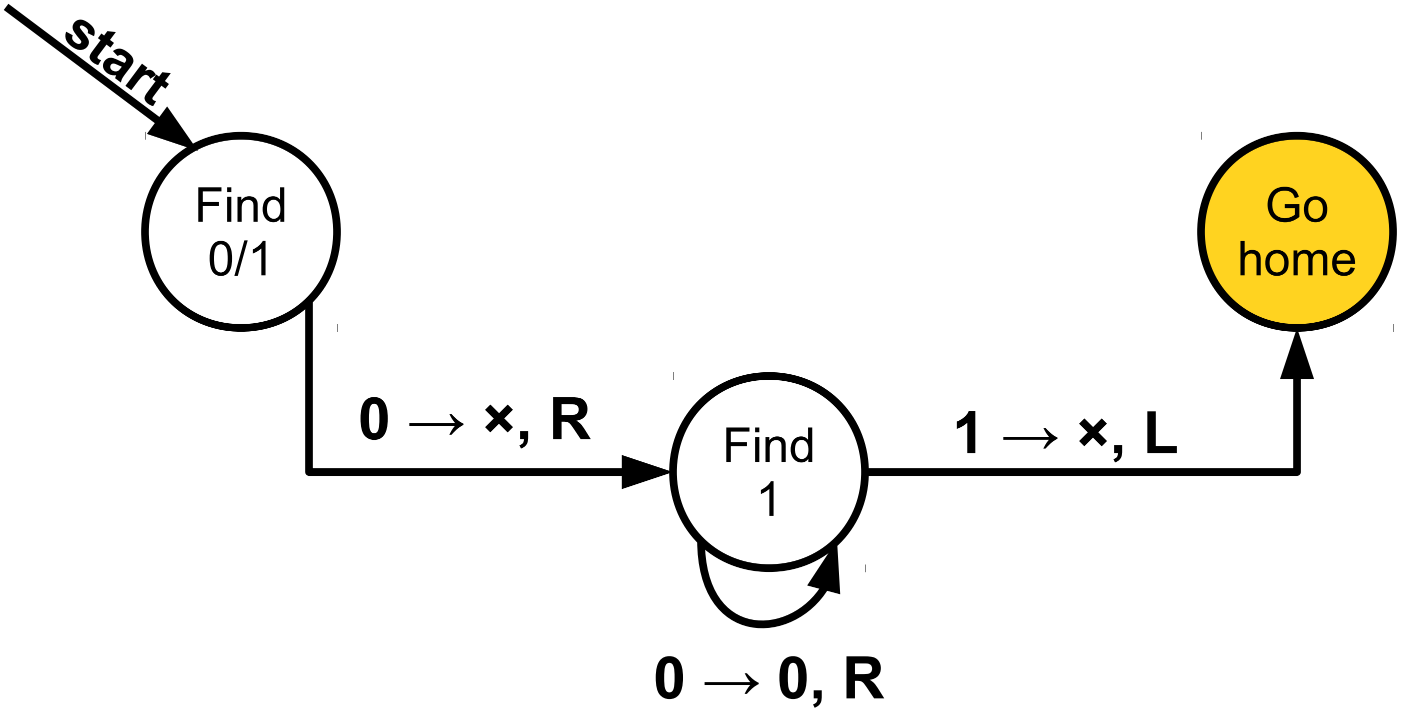
Find 1

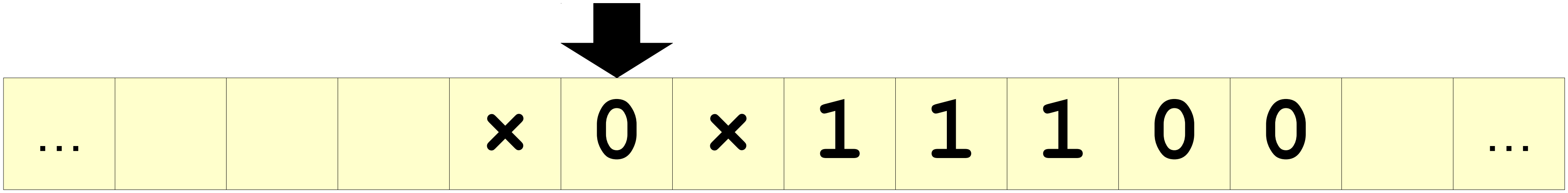
0 → 0, R

1 → ×, L

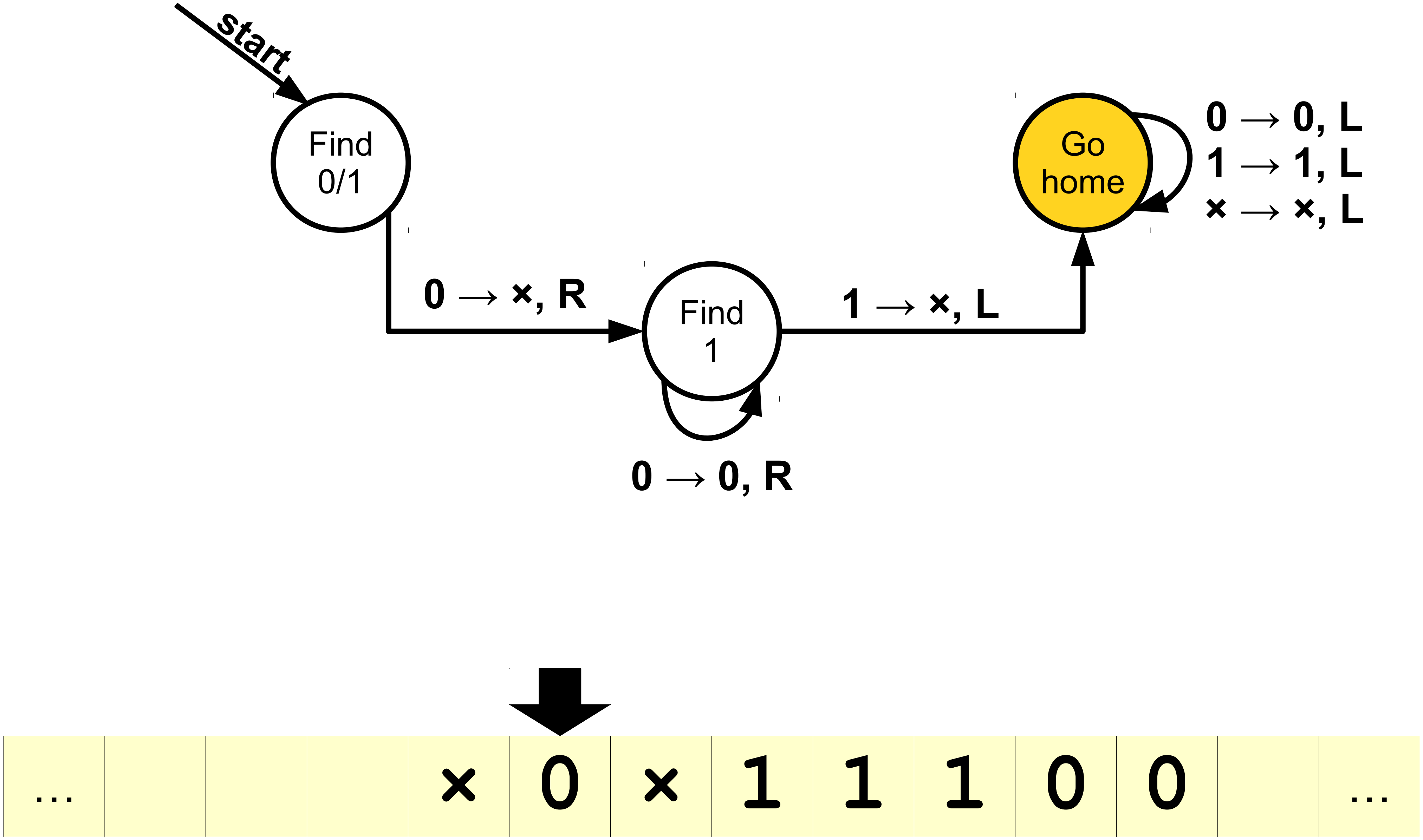


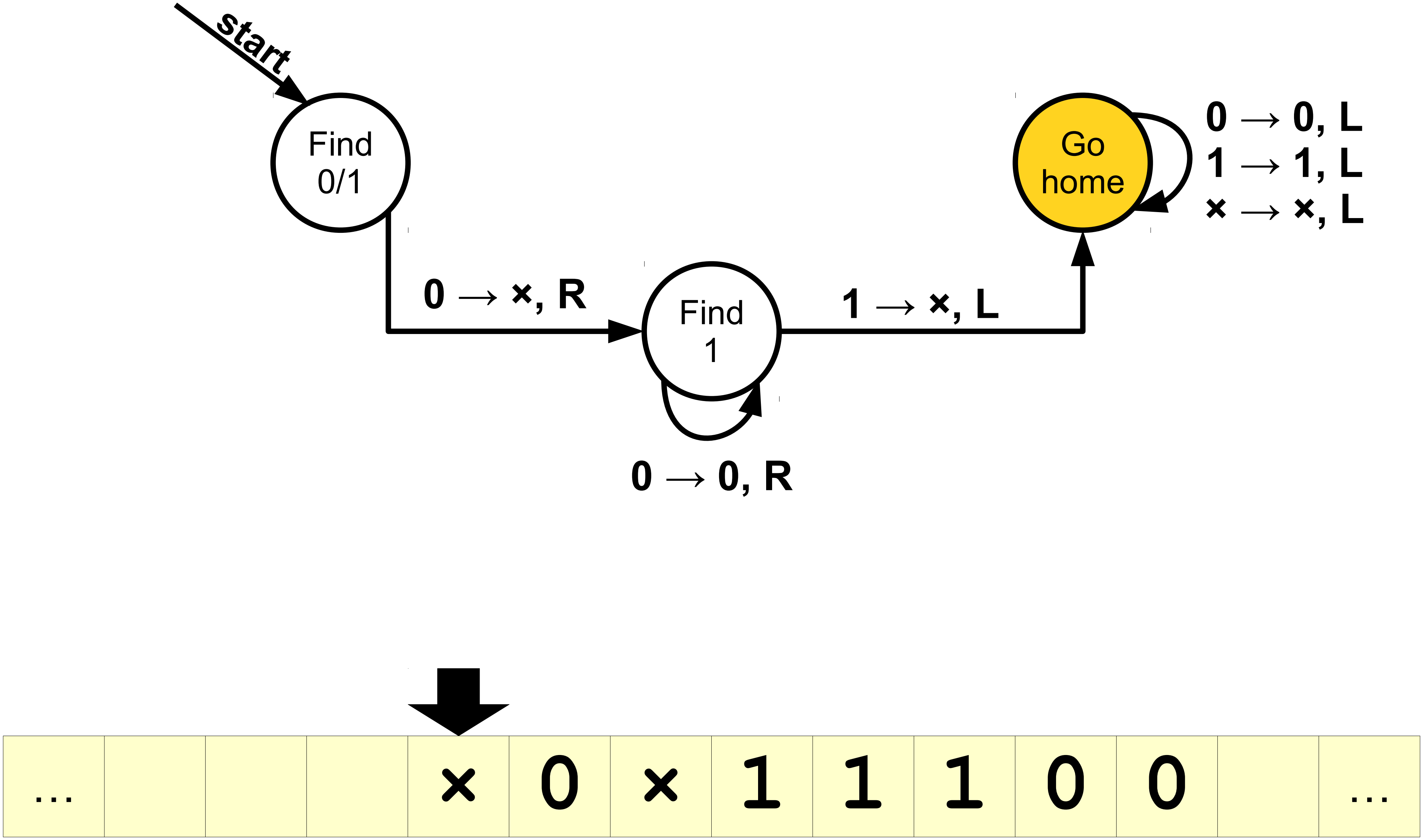
…				×	0	×	1	1	1	0	0		…

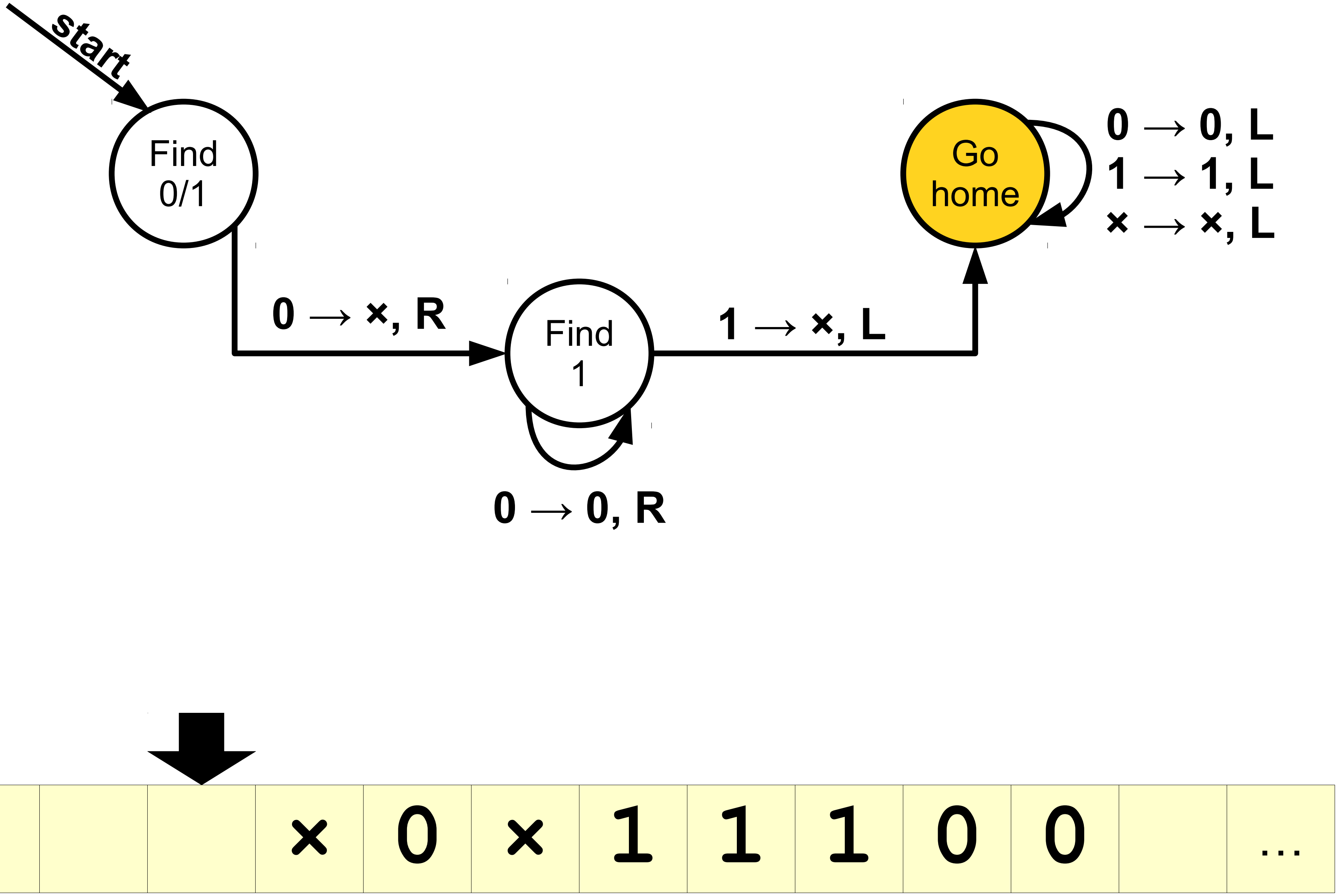




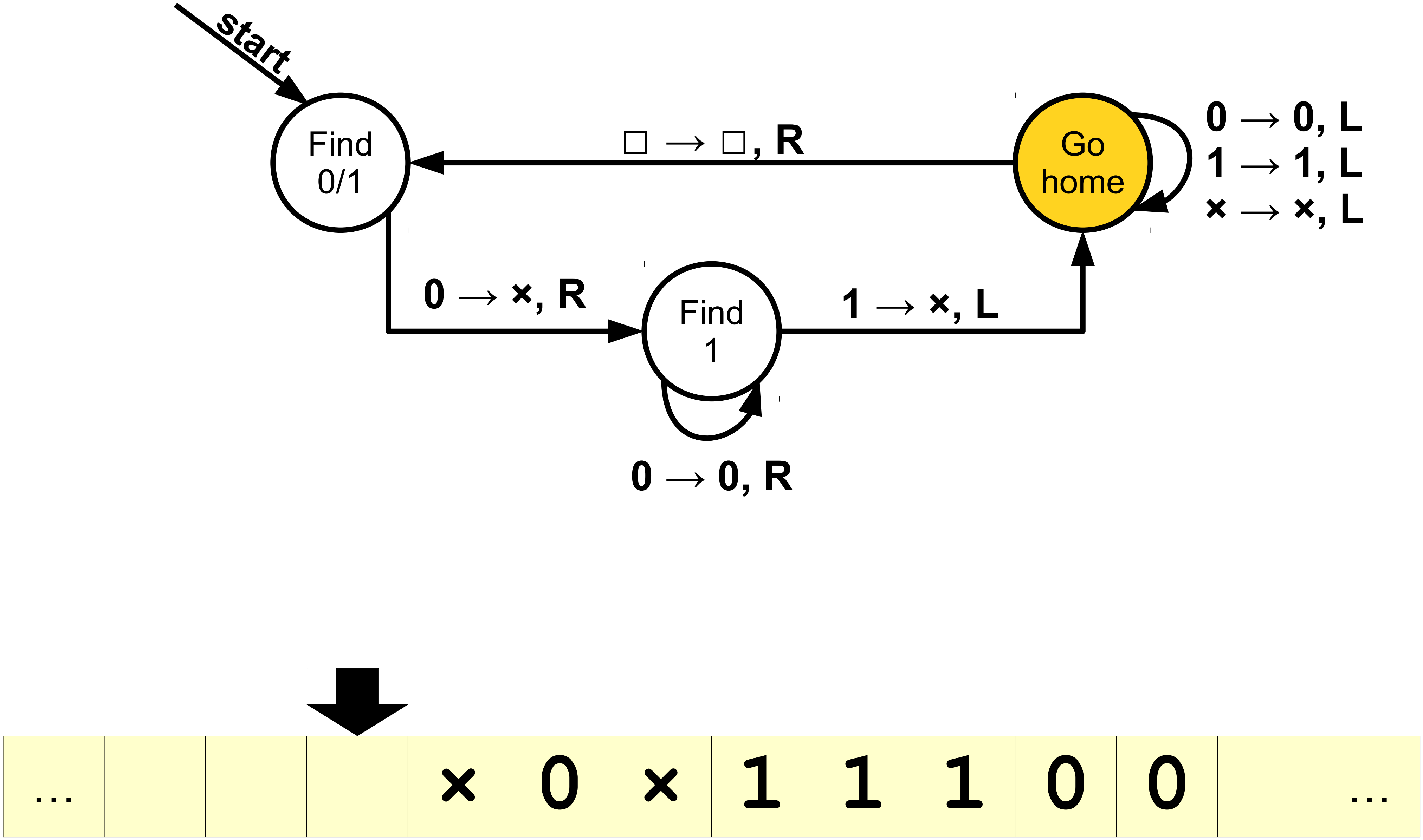
…				×	0	×	1	1	1	0	0		…

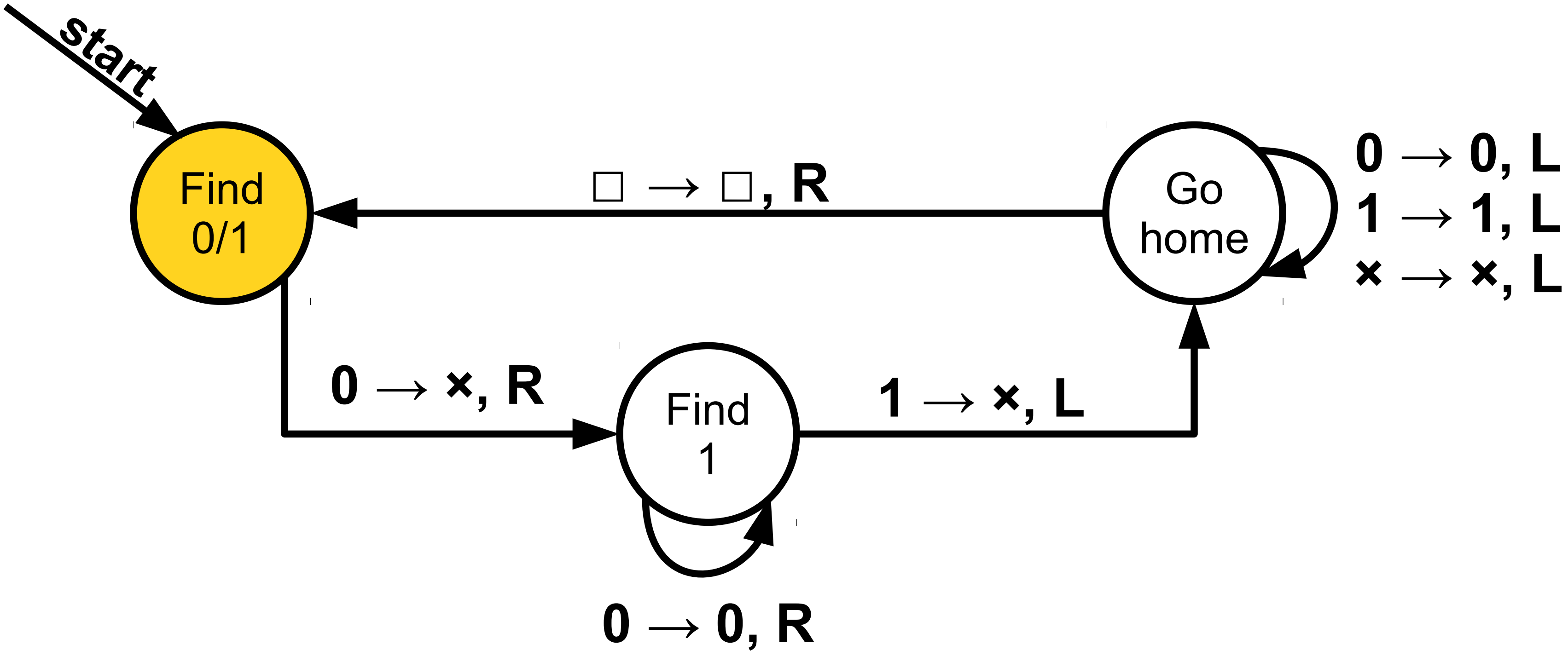

start
Find 0/1
0 → ×, R
Find 1
0 → 0, R
1 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
…
×
0
×
1
1
1
0
0
…


start
Find 0/1
0 → ×, R
Find 1
0 → 0, R
1 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
…
×
0
×
1
1
1
0
0
…

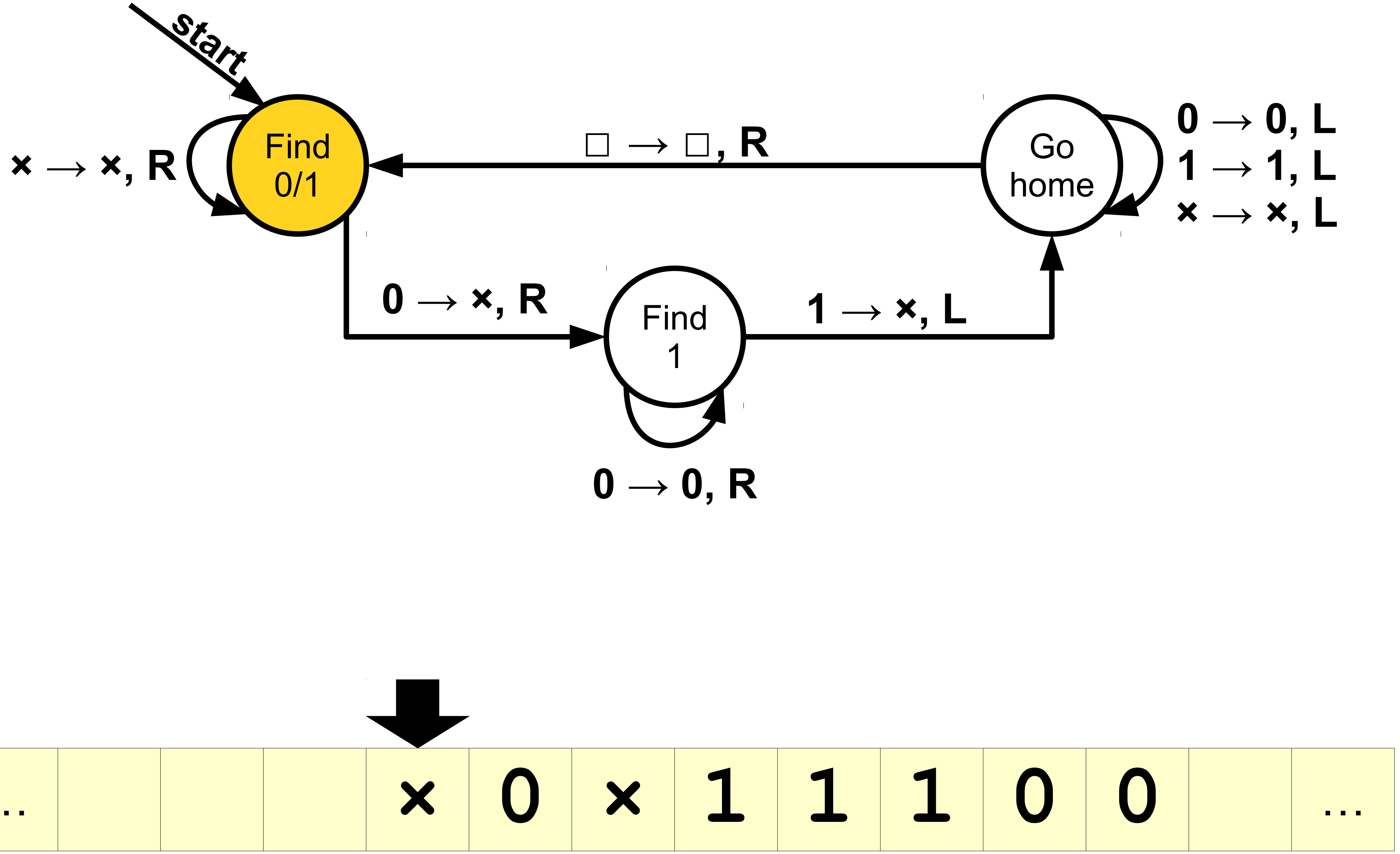


…				×	0	×	1	1	1	0	0		…

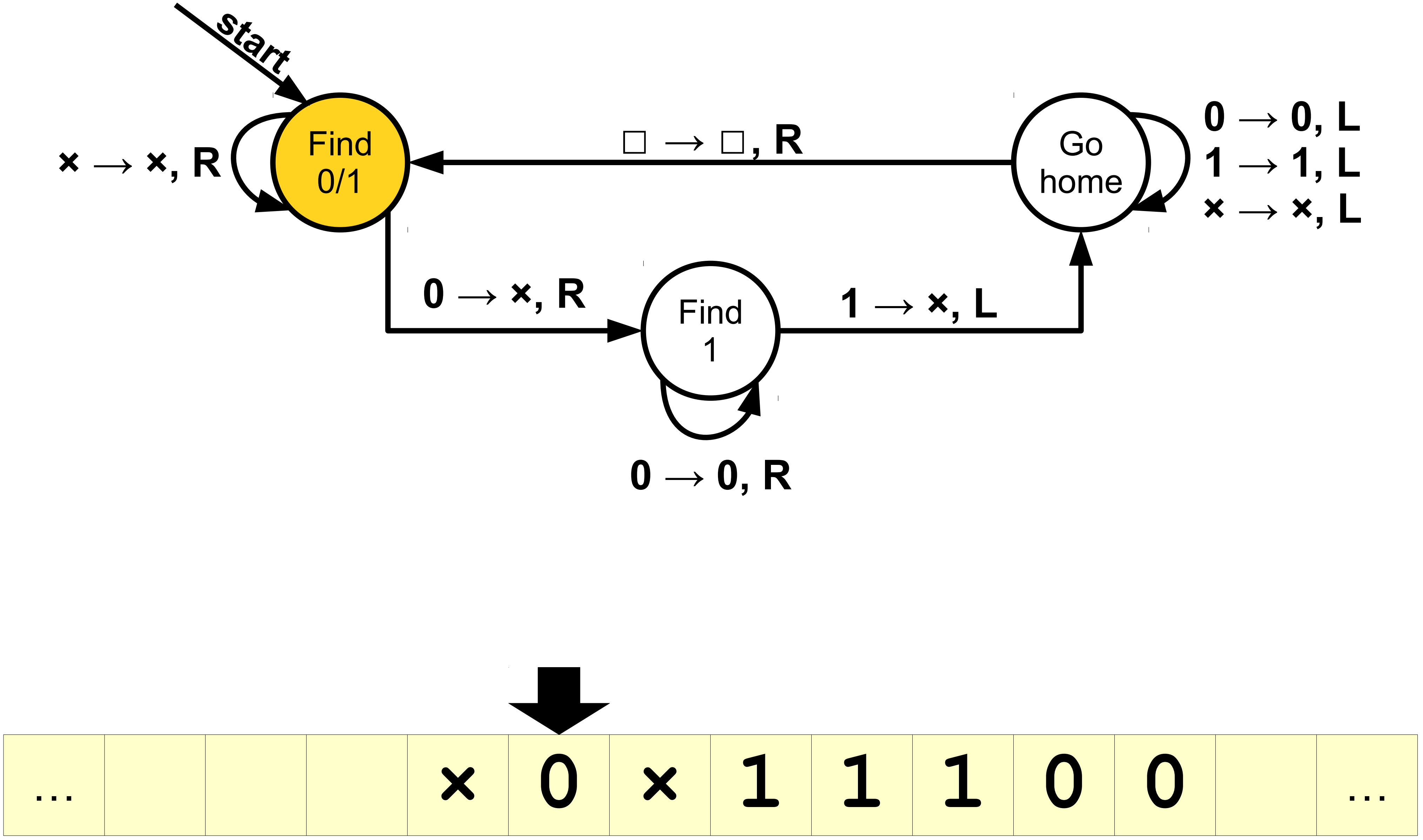

start
Find 0/1
Find 1
Go home
□ → □, R
0 → ×, R
1 → ×, L
0 → 0, R
0 → 0, L
1 → 1, L
× → ×, L
…
×
0
×
1
1
1
0
0
…

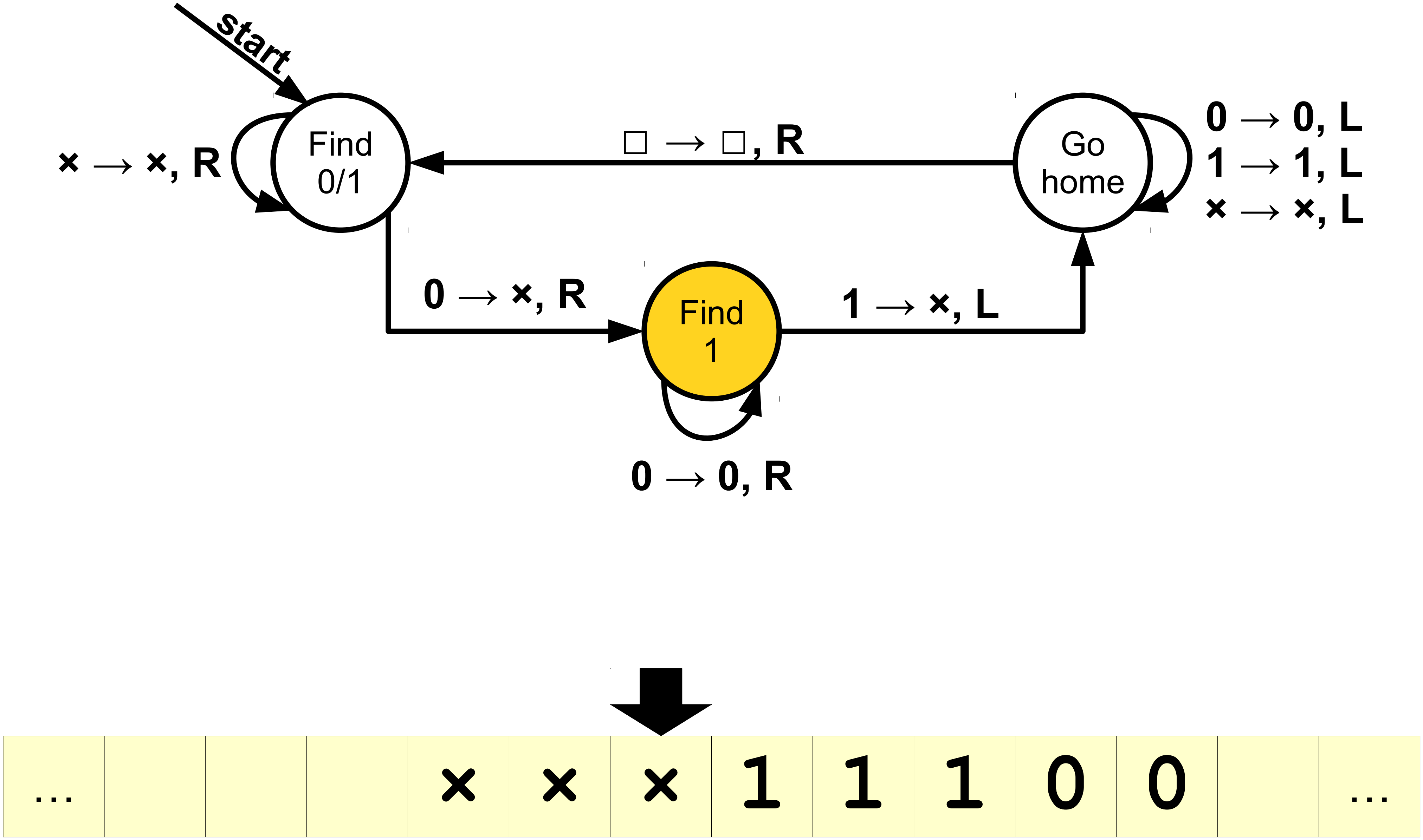


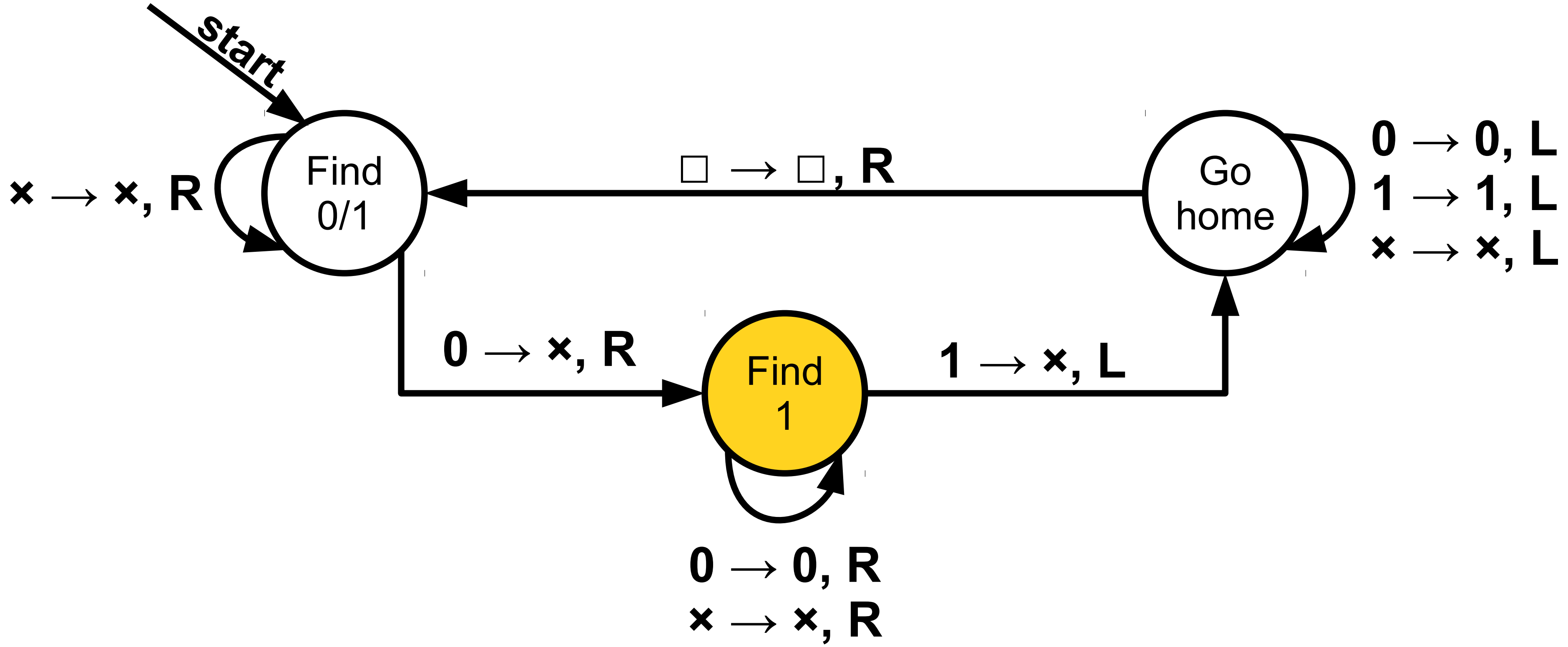
…				×	0	×	1	1	1	0	0		…

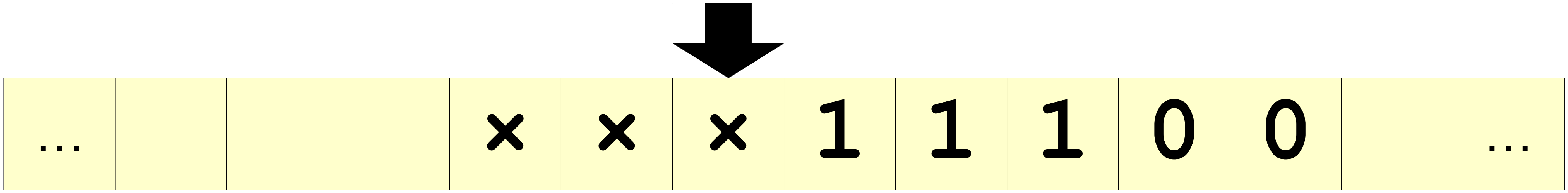


…				×	0	×	1	1	1	0	0		…

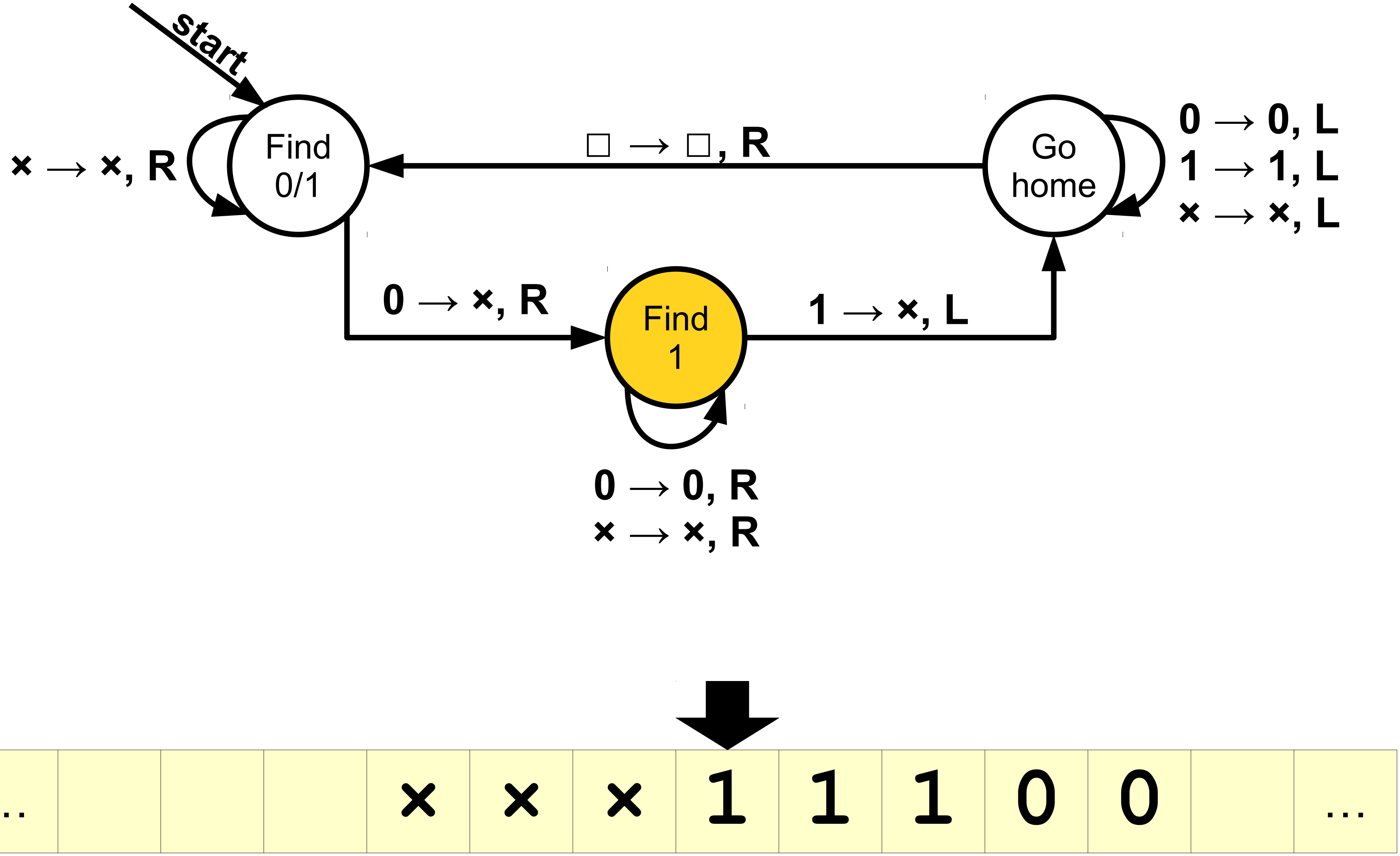

start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
… × 0 × 1 1 1 0 0 …

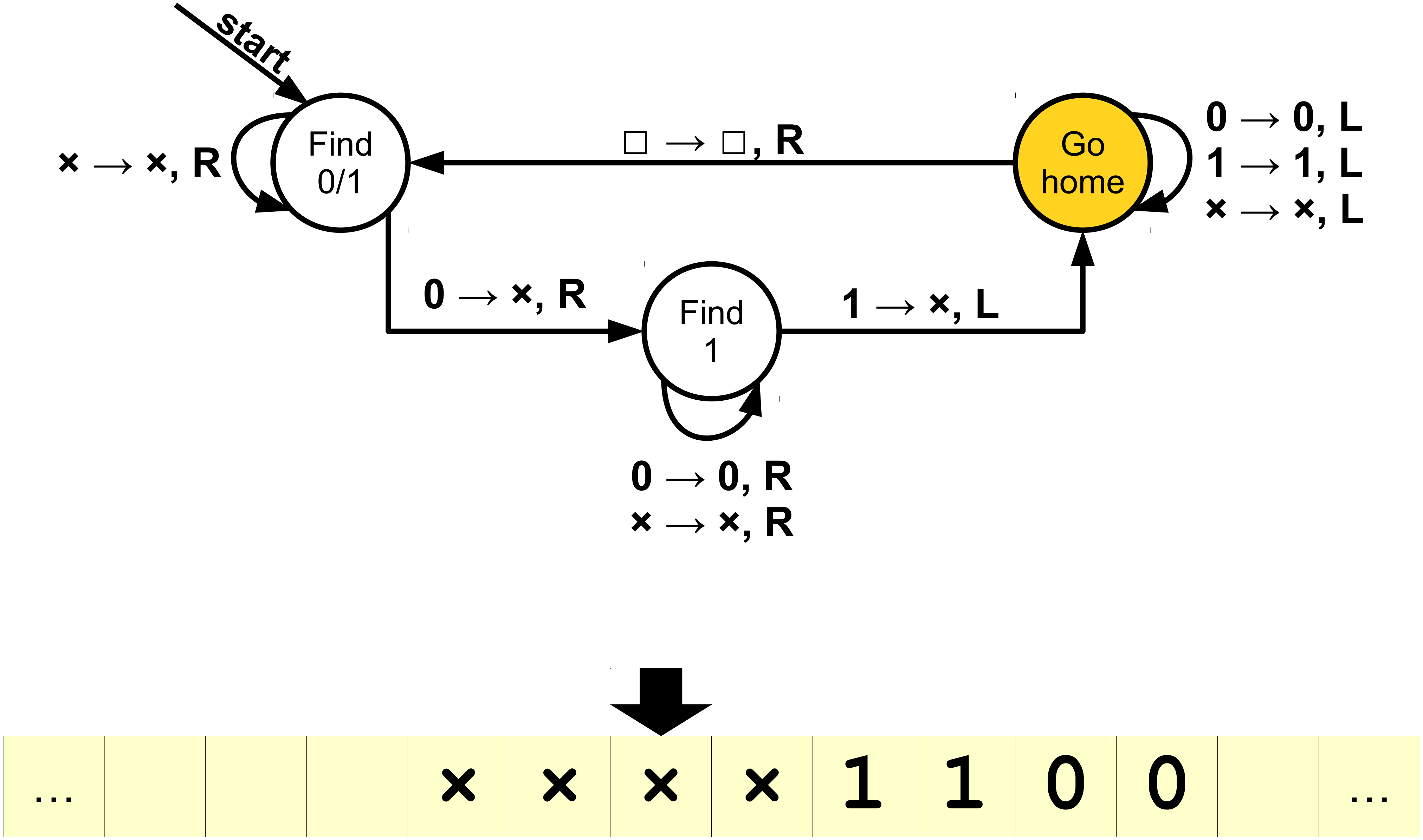

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
…
×
×
×
1
1
1
0
0
…

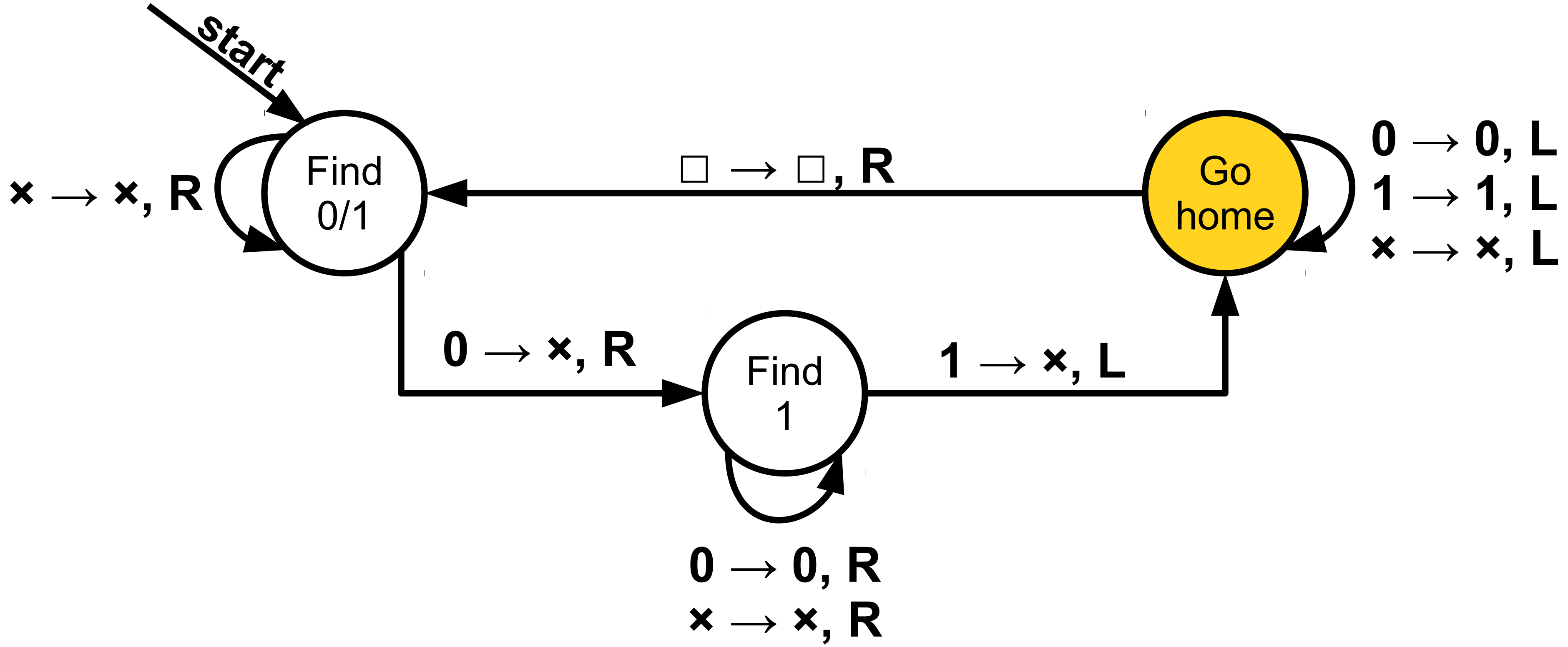


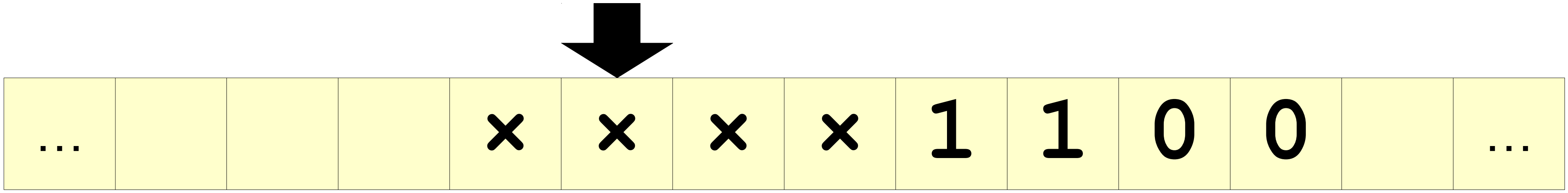


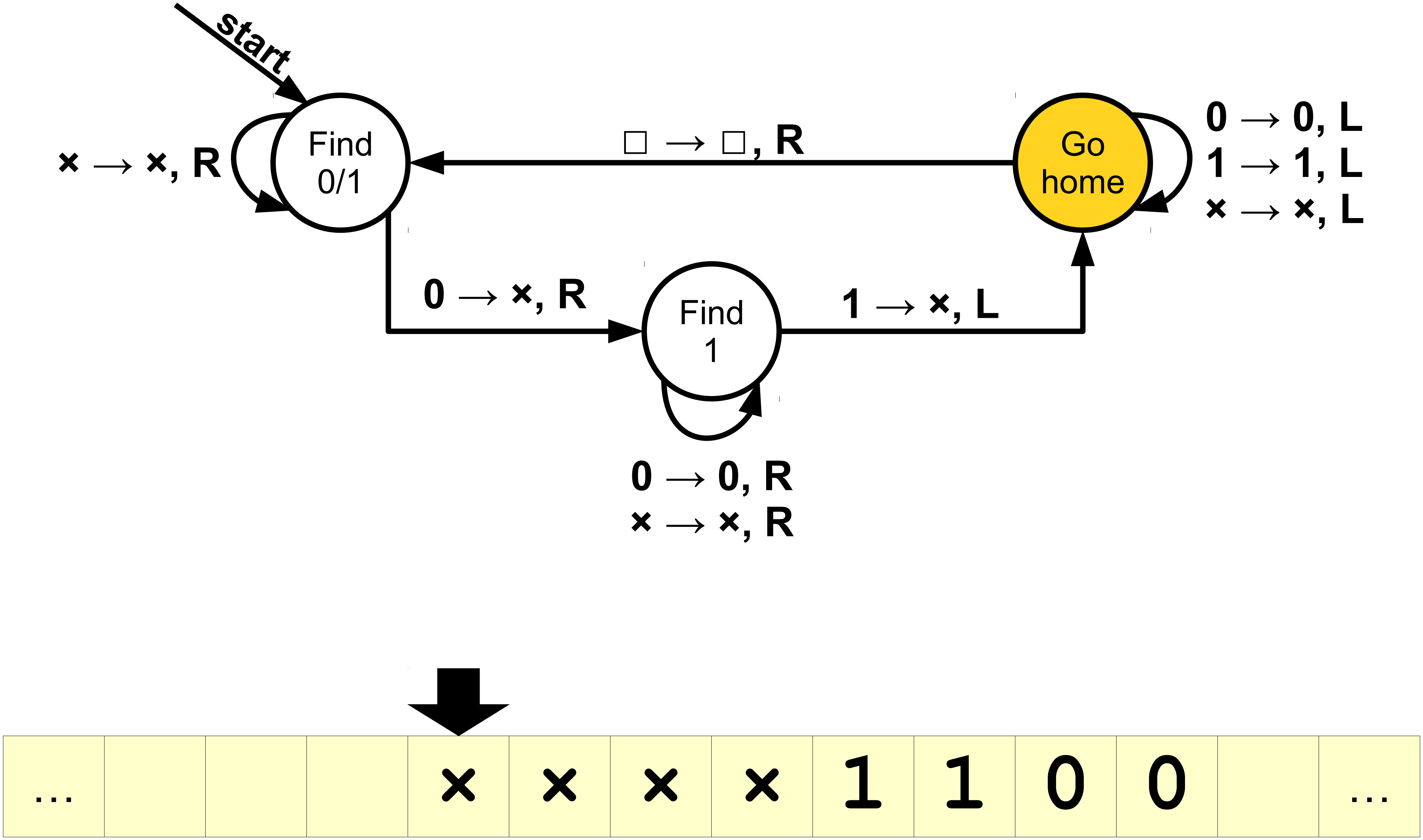
…				×	×	×	1	1	1	0	0		…


start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
… × × × 1 1 1 0 0 …

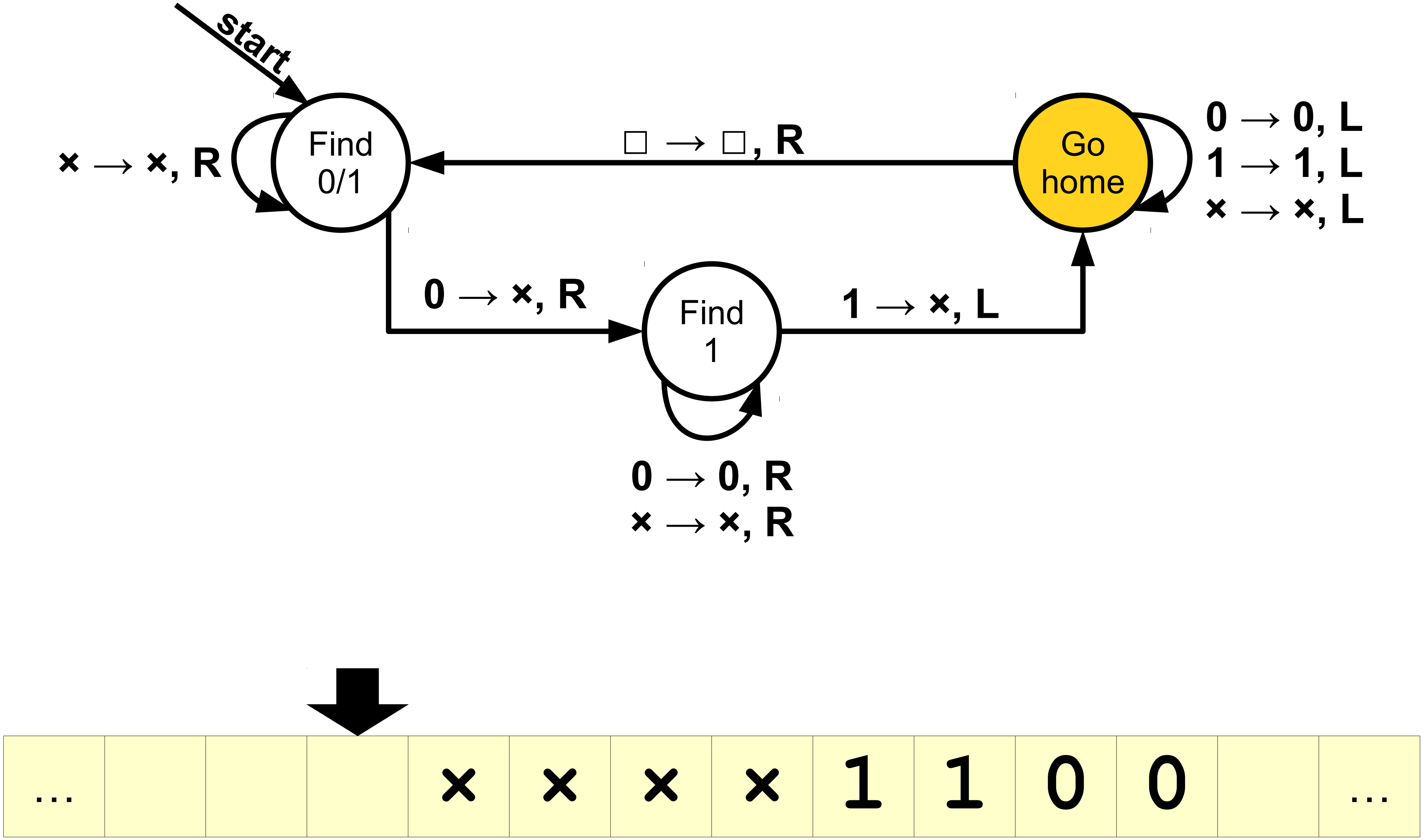

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
… × × × × 1 1 0 0 …

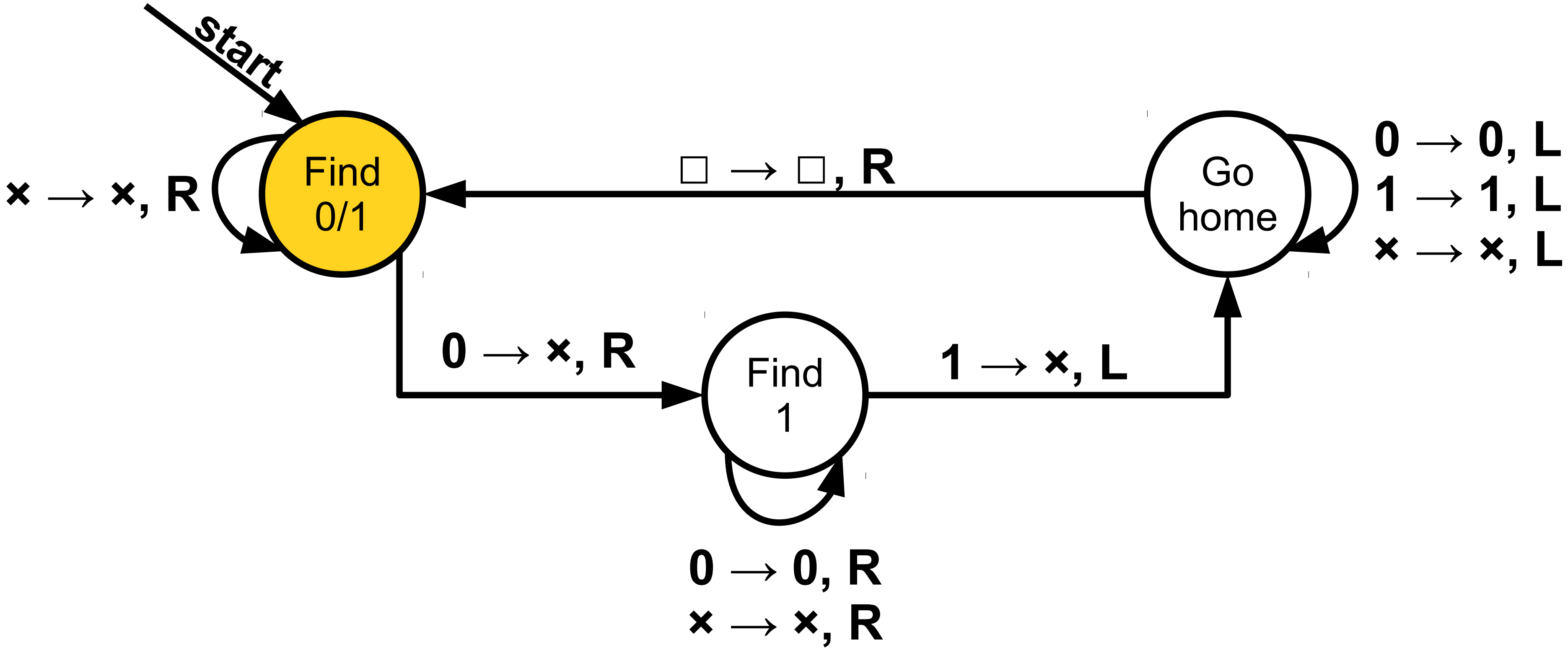

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

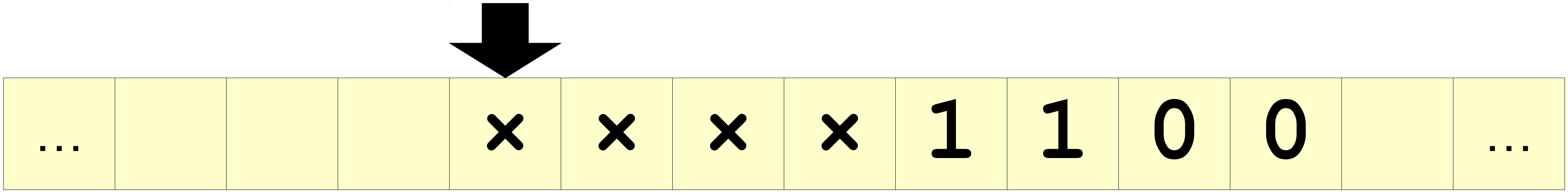

…
×
×
×
×
1
1
0
0
…



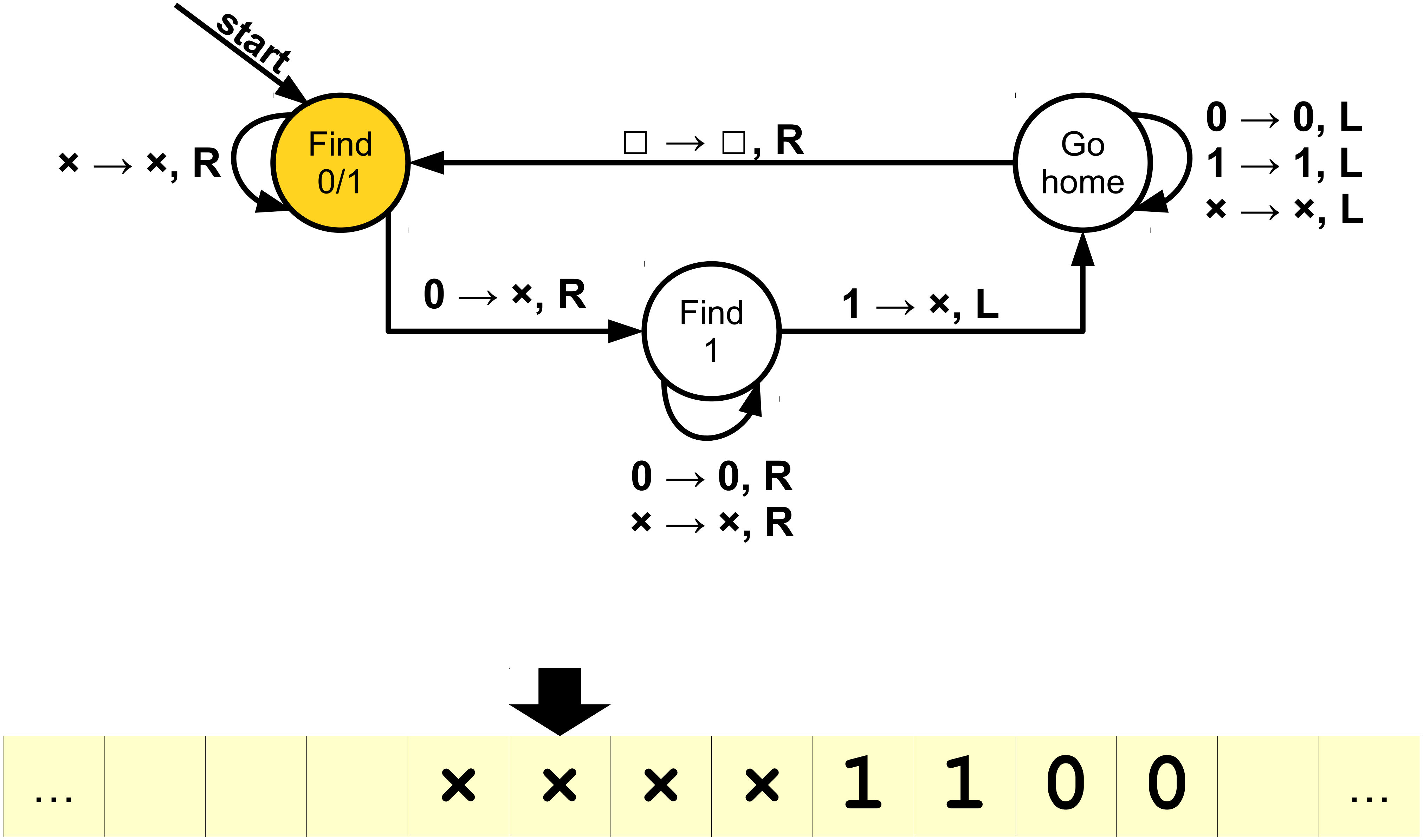
…				×	×	×	×	1	1	0	0		…

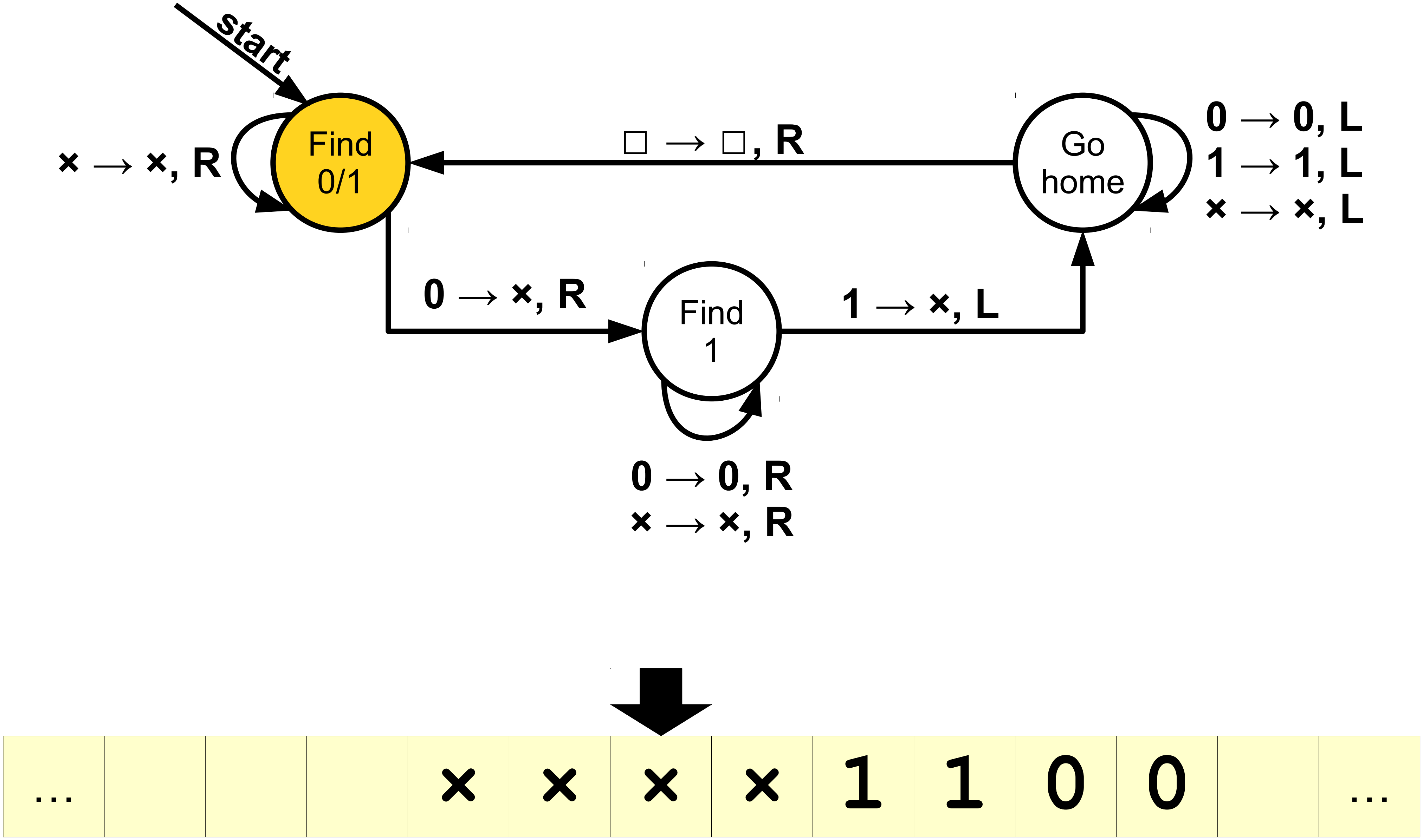

start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
1
1
0
0
…

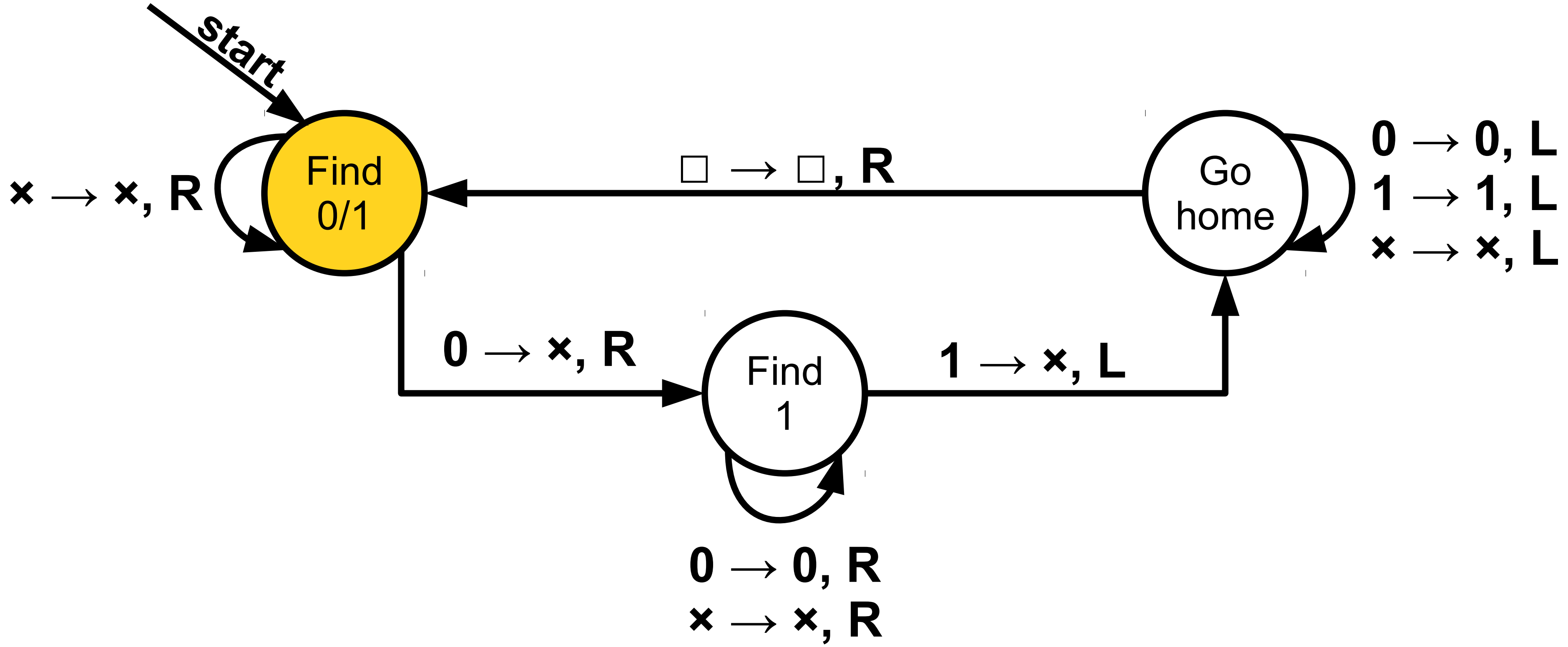


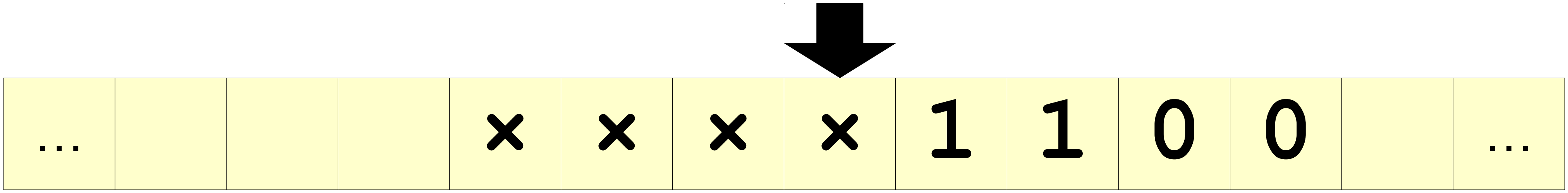


…				×	×	×	×	1	1	0	0		…

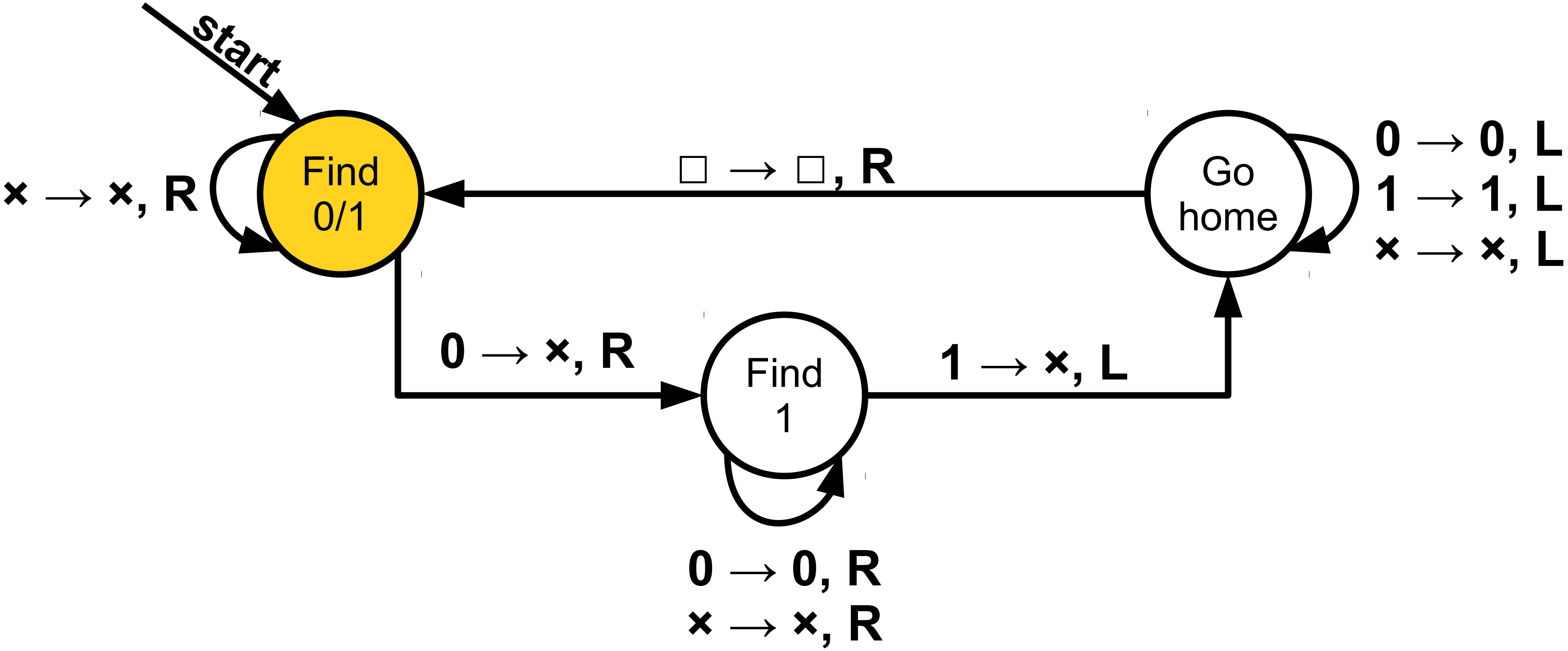

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
1
1
0
0
…

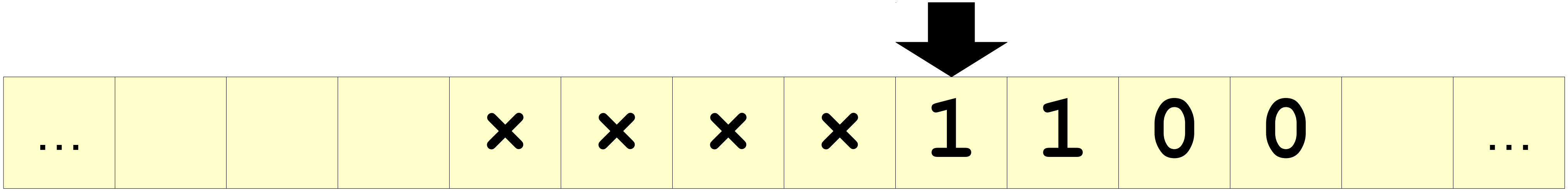

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
1
1
0
0
…

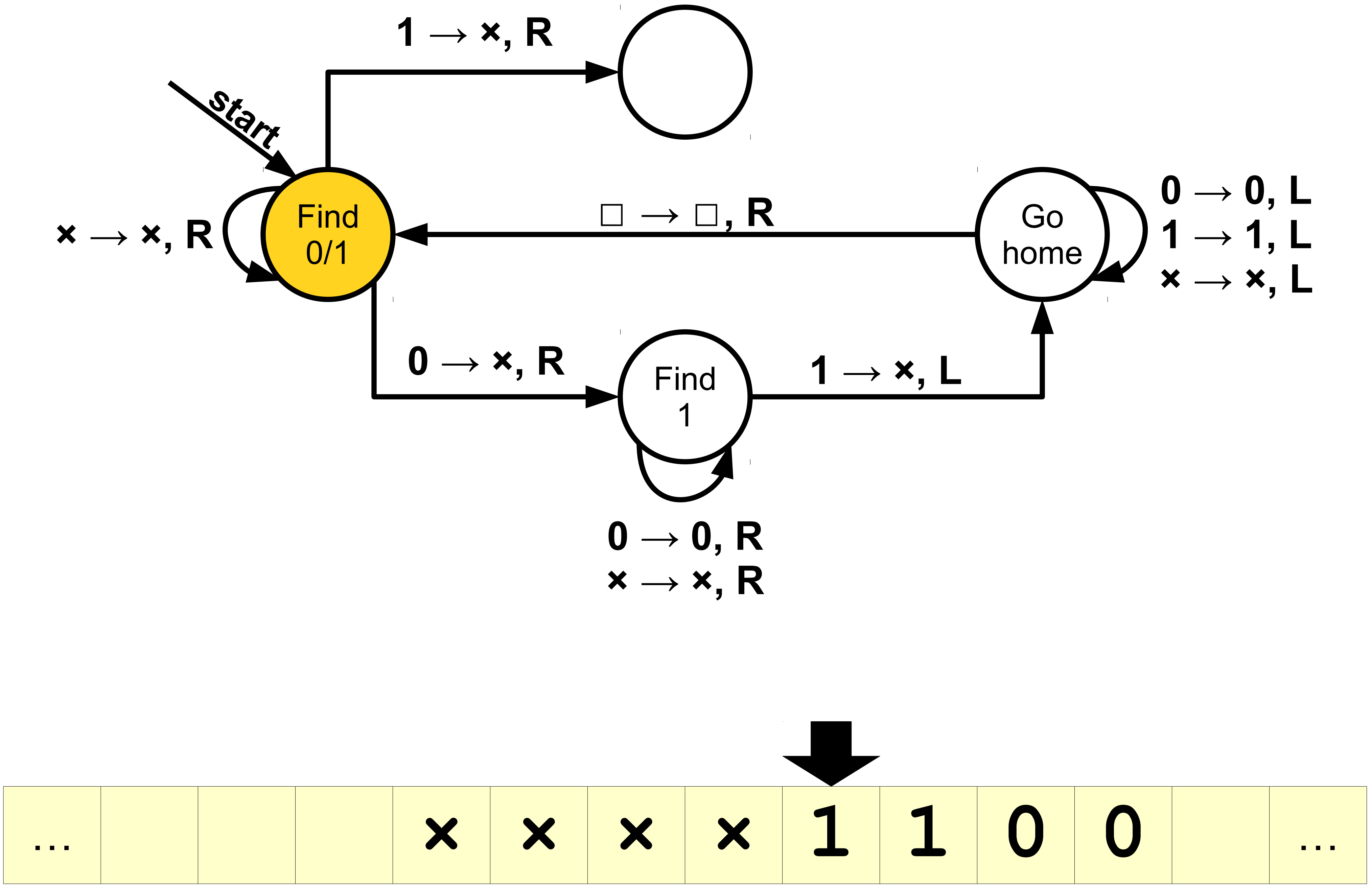


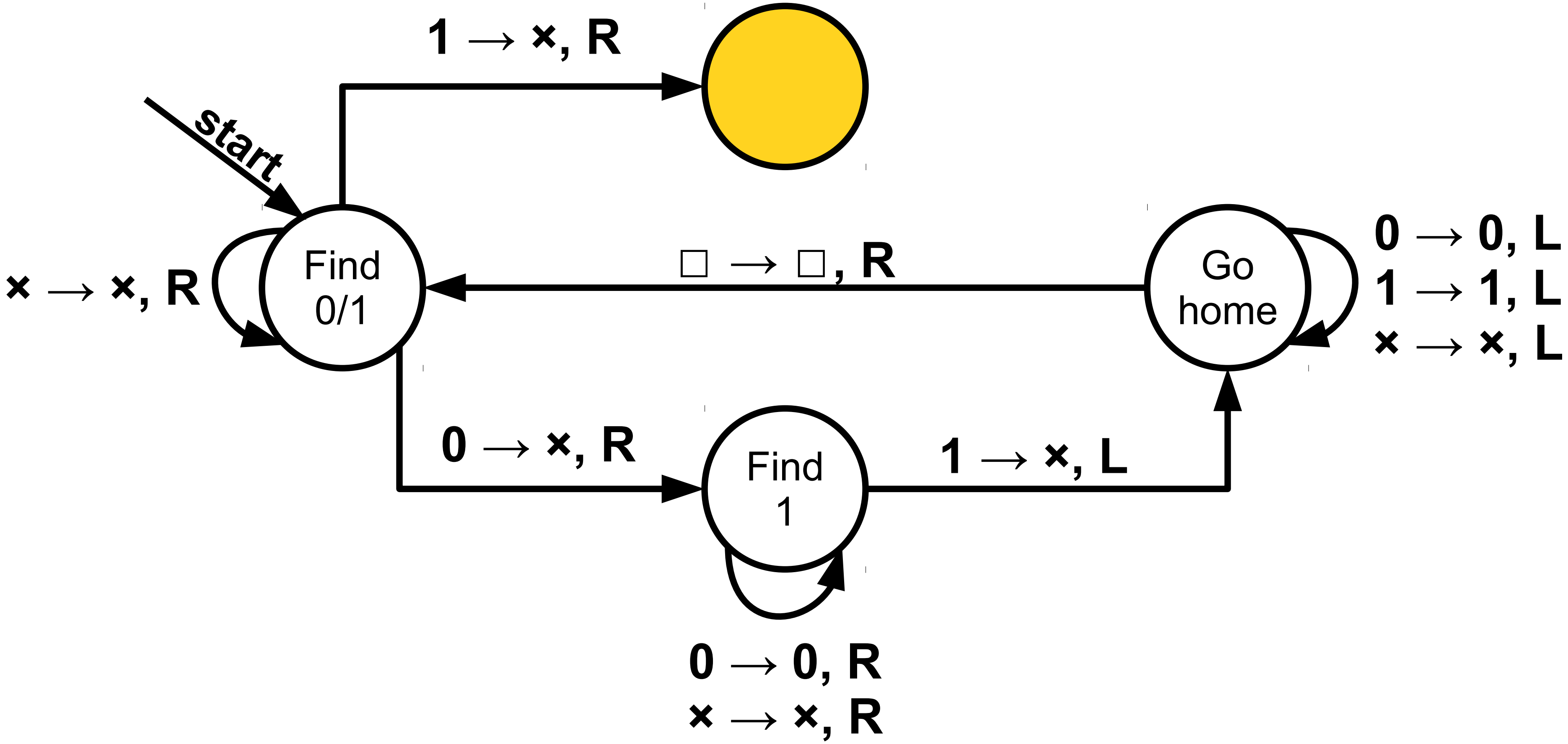


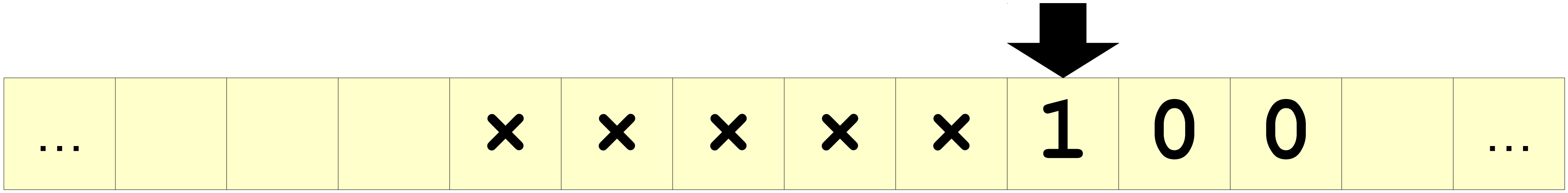
…				×	×	×	×	1	1	0	0		…


start
× → ×, R
Find
0/1
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R

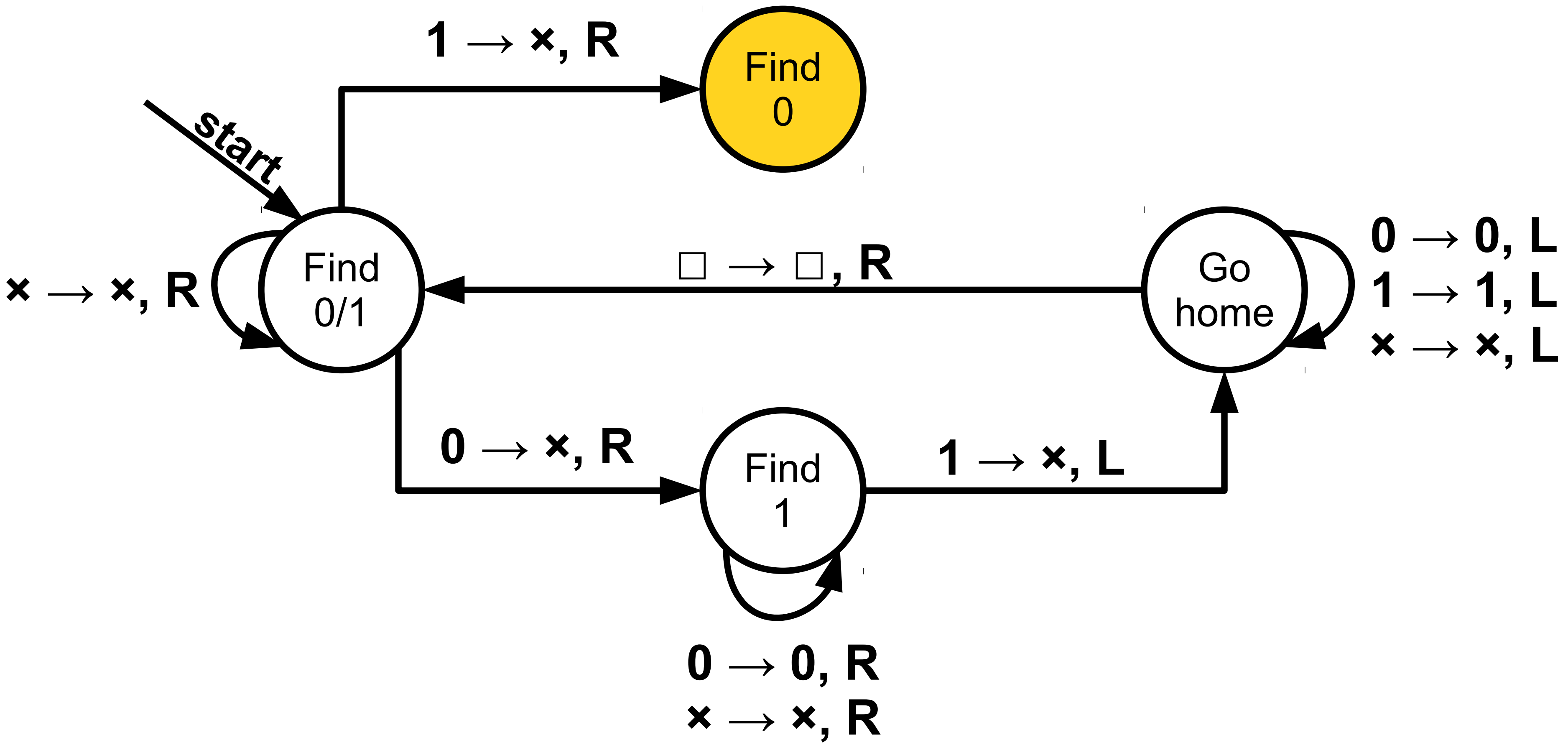

…
×
×
×
×
1
1
0
0
…

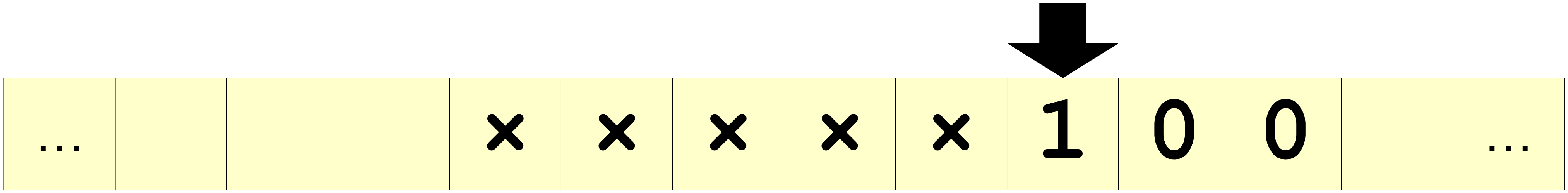

start
Find 0/1
× → ×, R
1 → ×, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
…
×
×
×
×
1
1
0
0
…

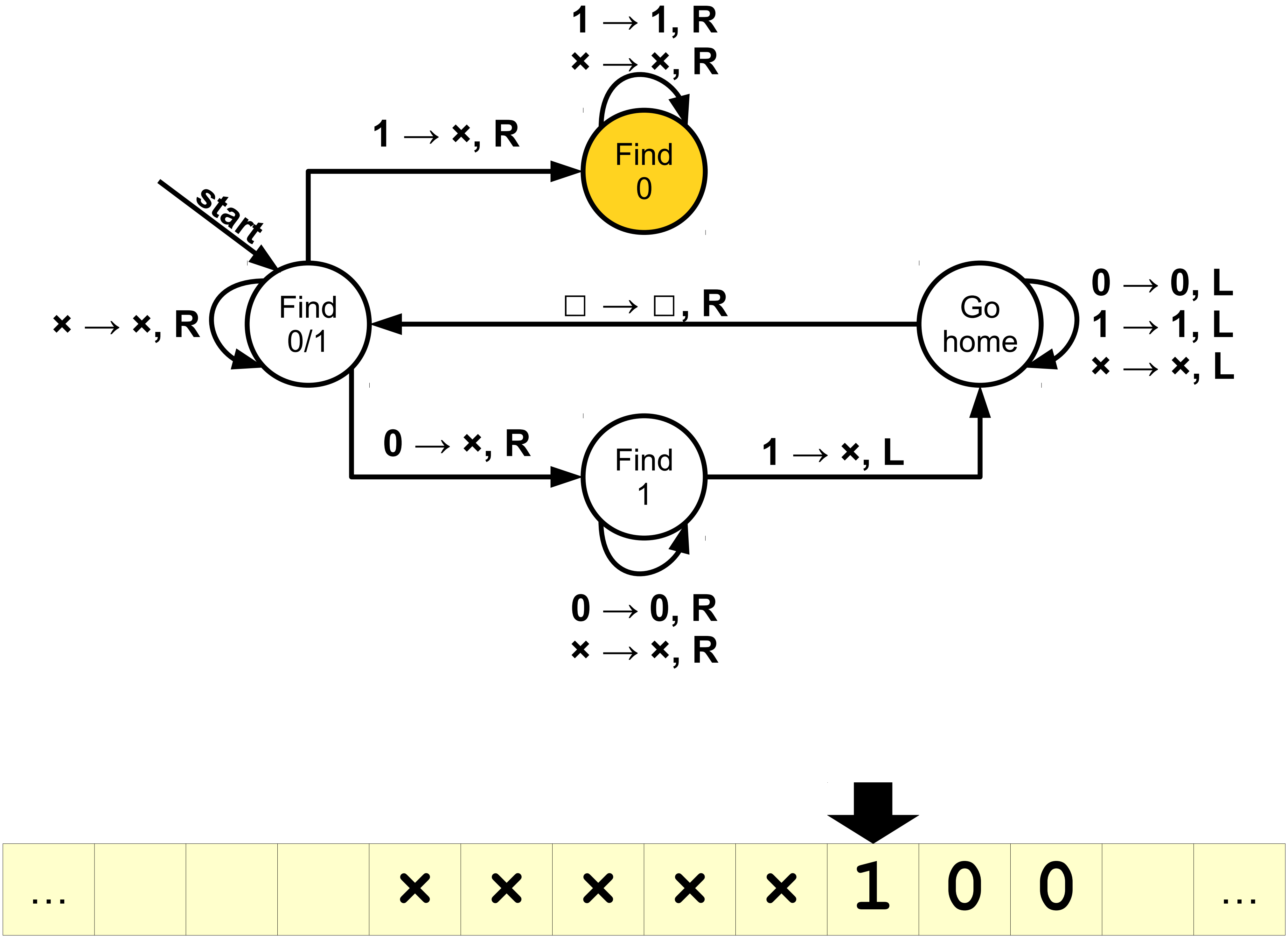


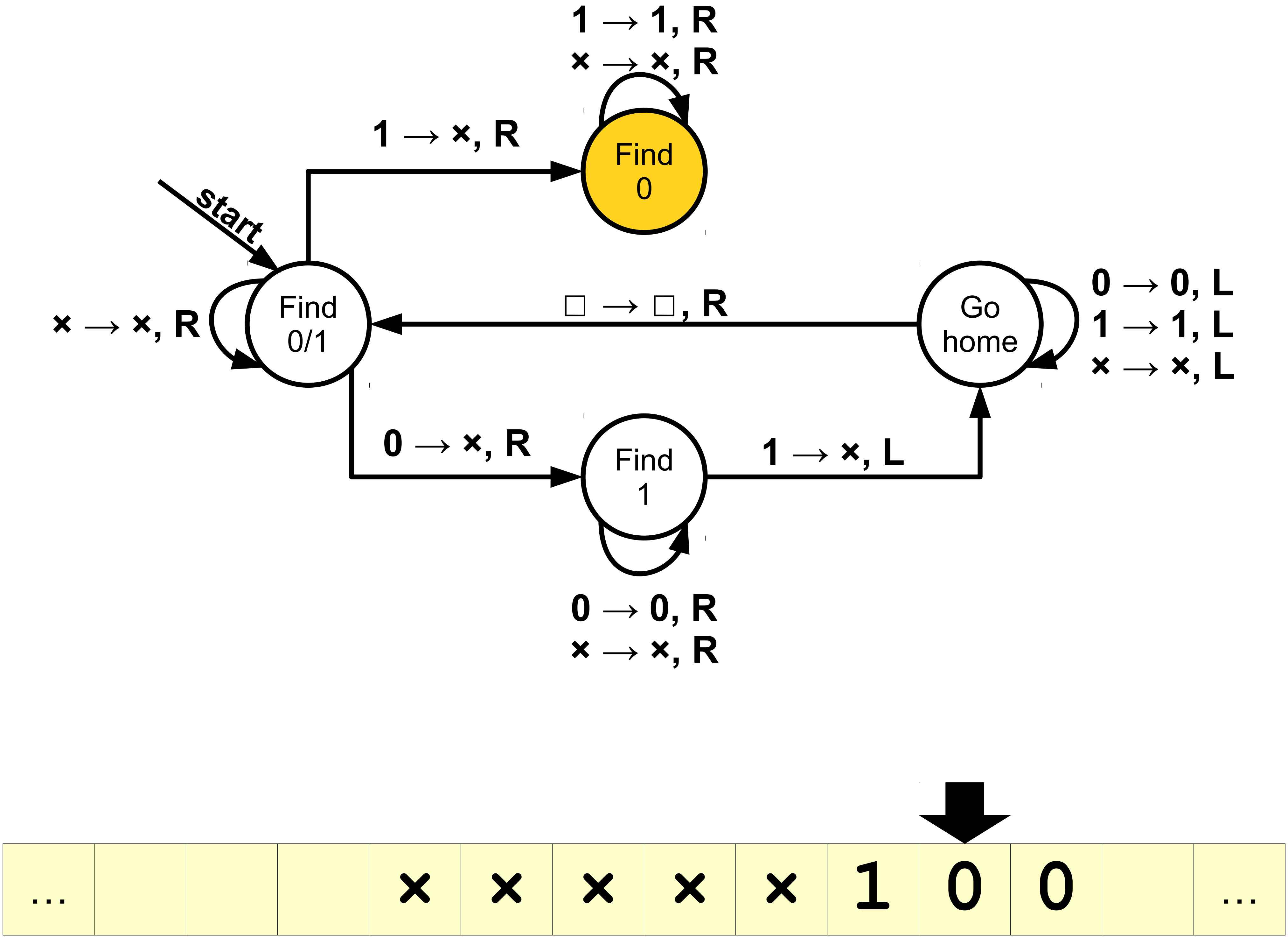


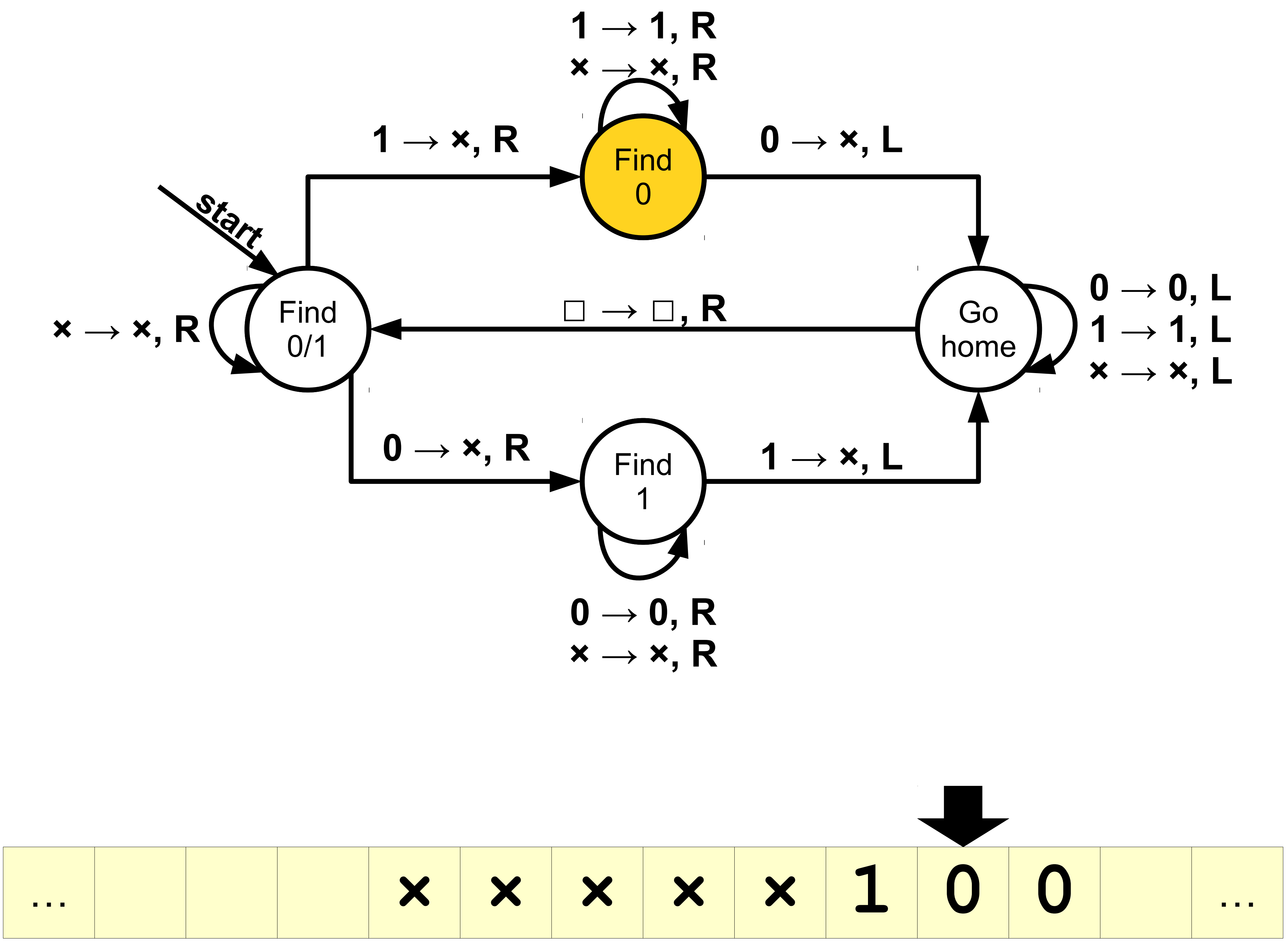
…				×	×	×	×	×	1	0	0		…

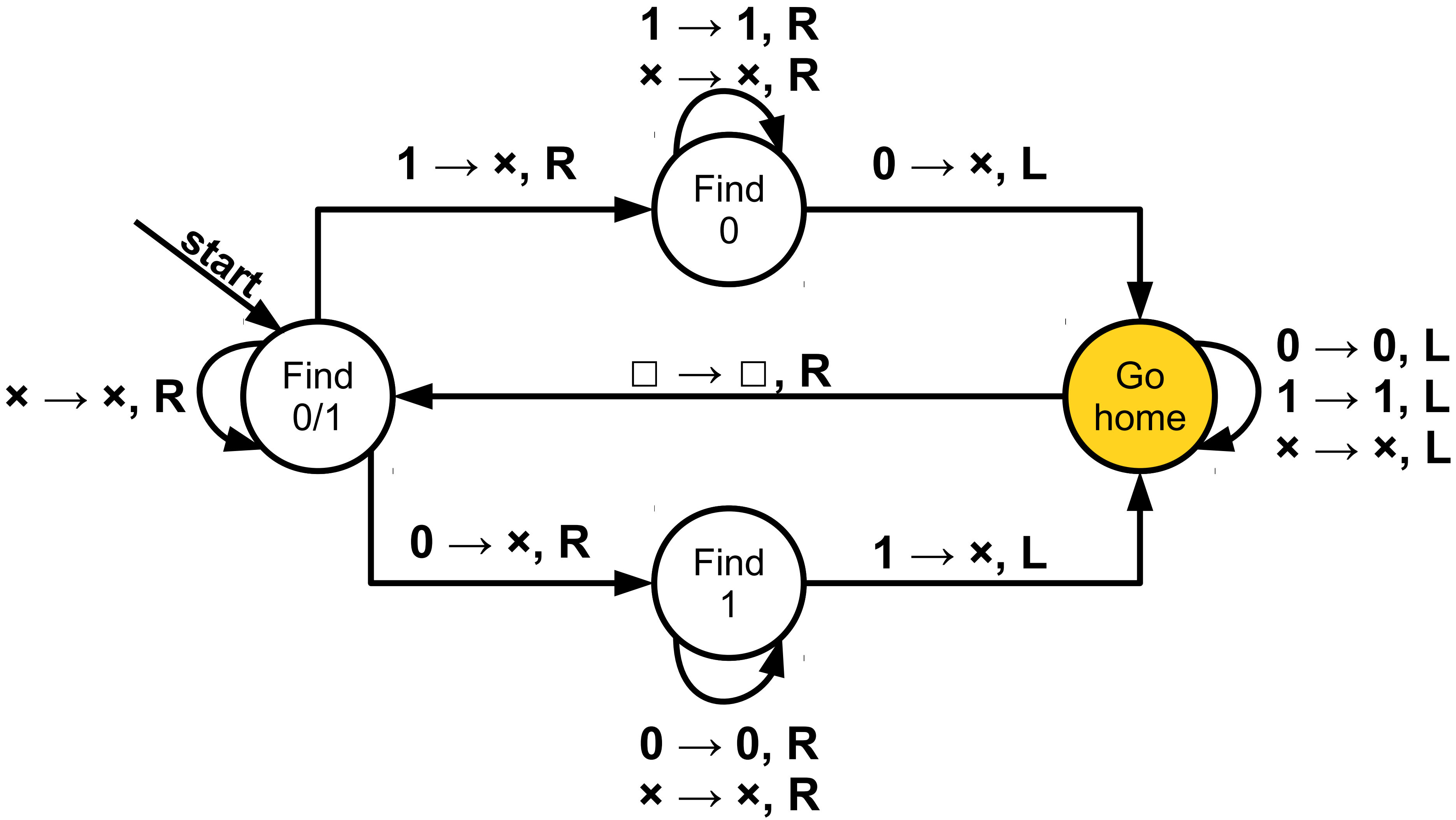

start
Find 0/1
Find 0
Find 1
Go home
1 → ×, R
× → ×, R
□ → □, R
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
1 → ×, L
0 → 0, R
× → ×, R


…
×
×
×
×
×
1
0
0
…

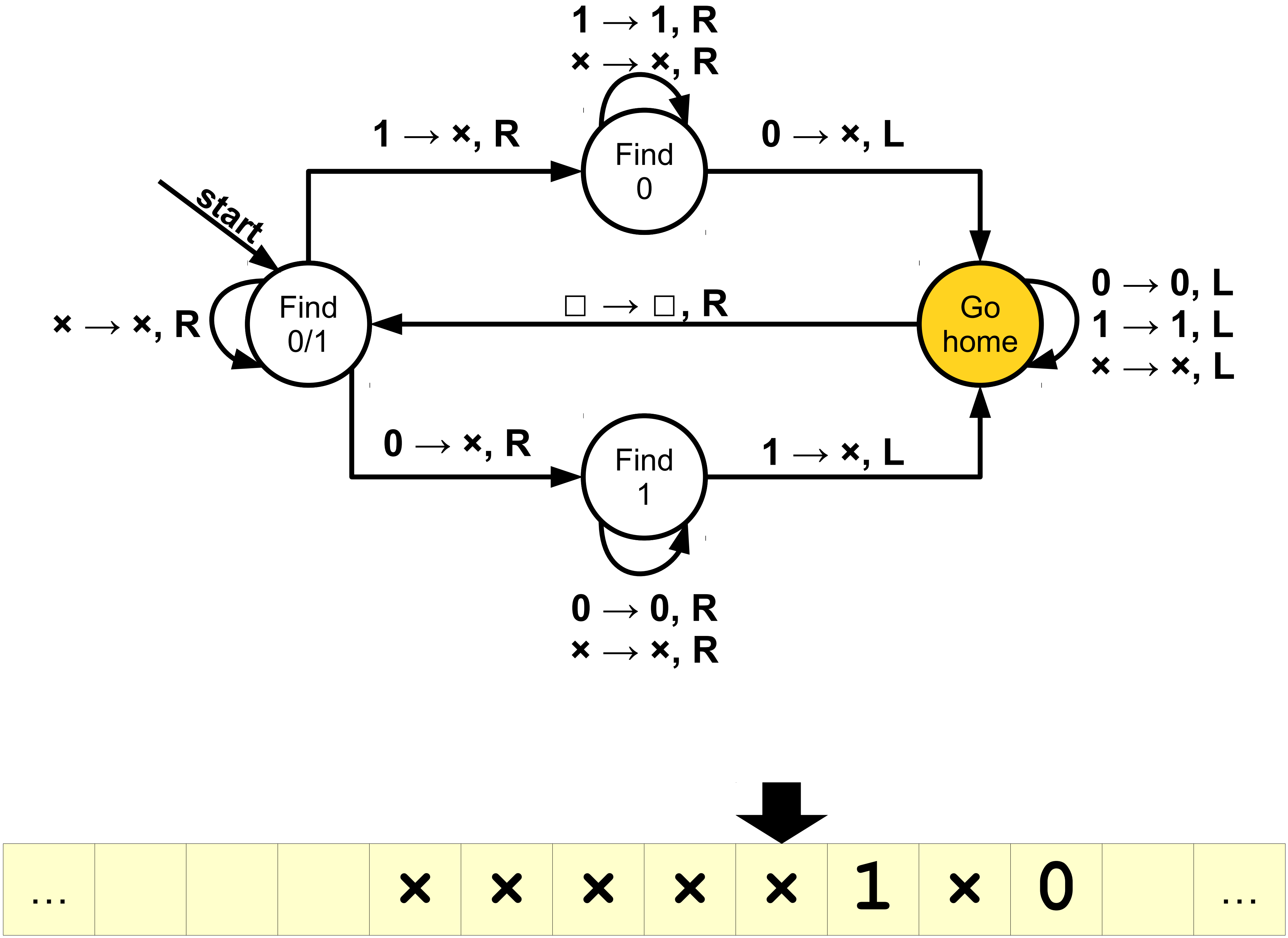

1 → 1, R
× → ×, R
1 → ×, R
Find 0
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
0
0
…

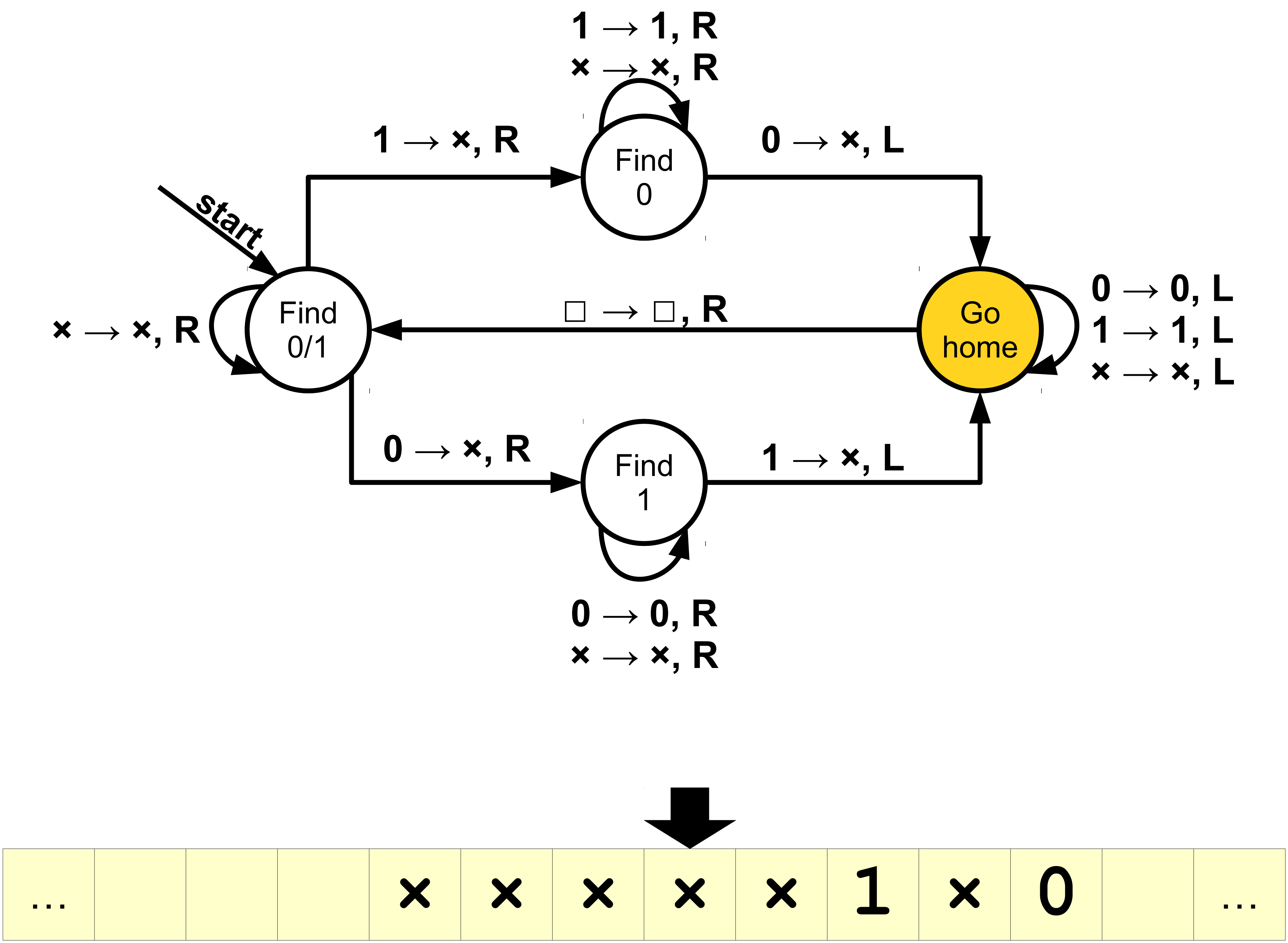

1 → 1, R
× → ×, R
1 → ×, R
Find 0
start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
0
0
…

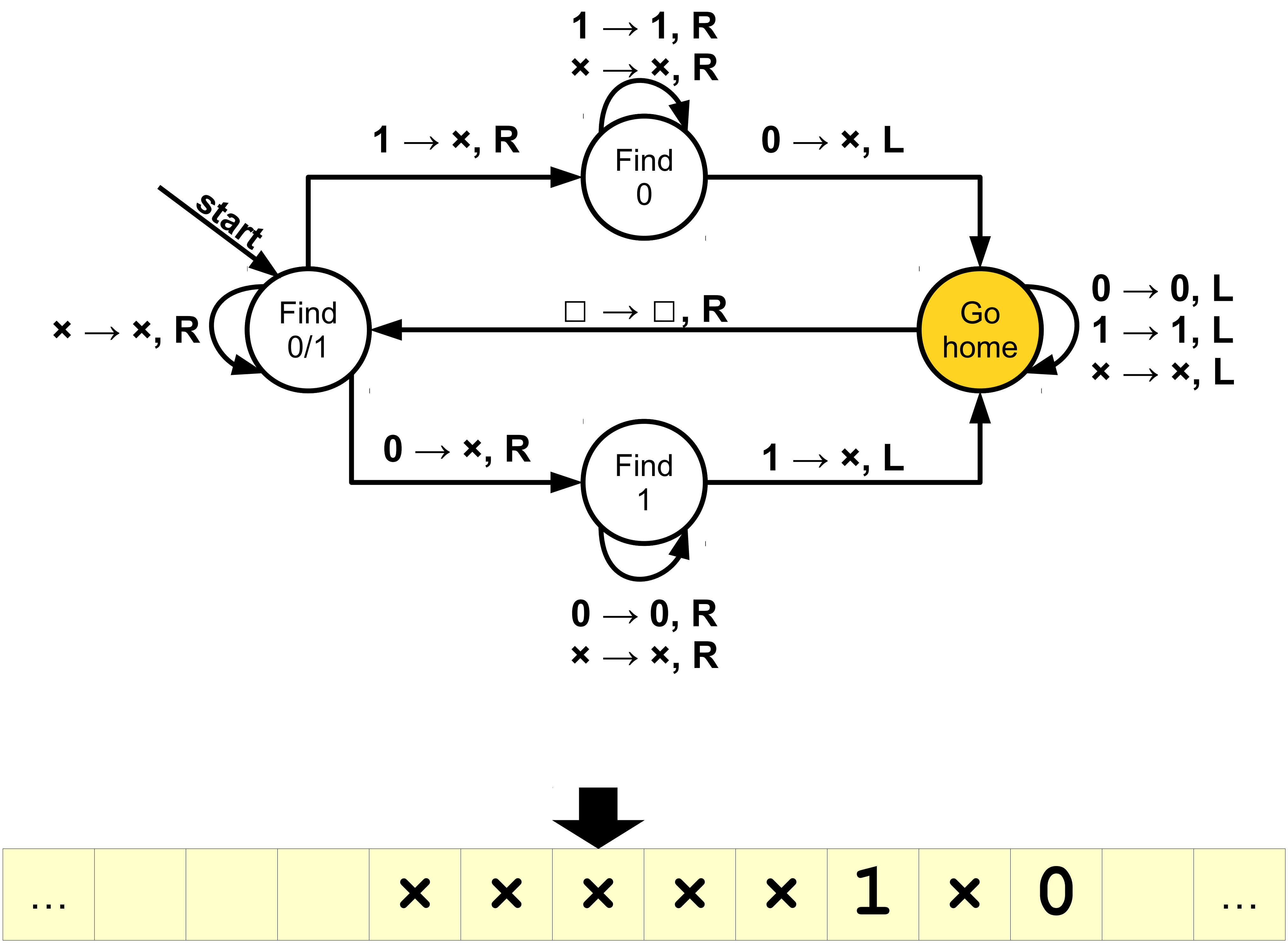

1 → 1, R
× → ×, R
1 → ×, R
Find
0
0 → ×, L
start
× → ×, R
Find
0/1
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
0
0
…

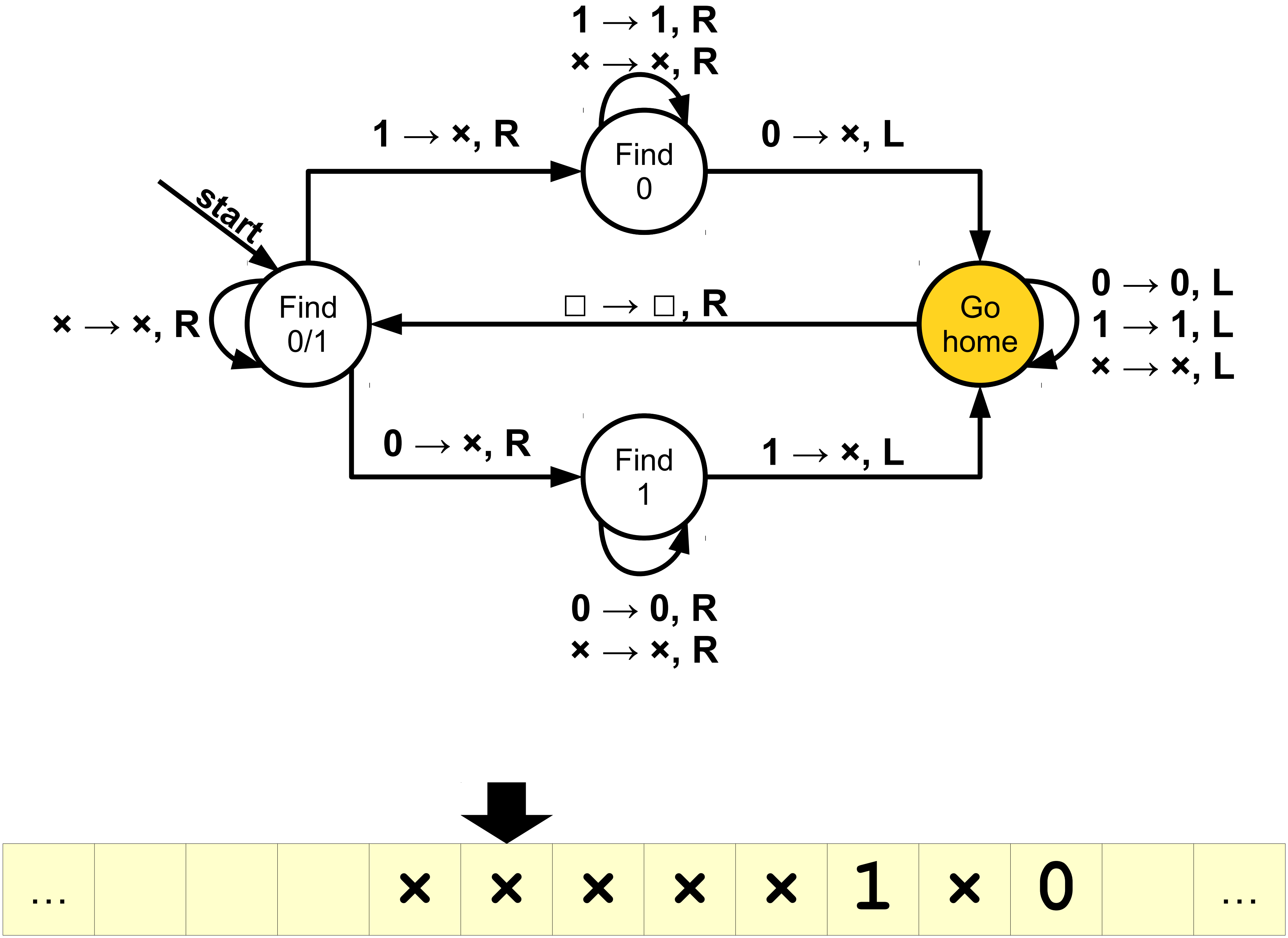

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
Find 0
start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

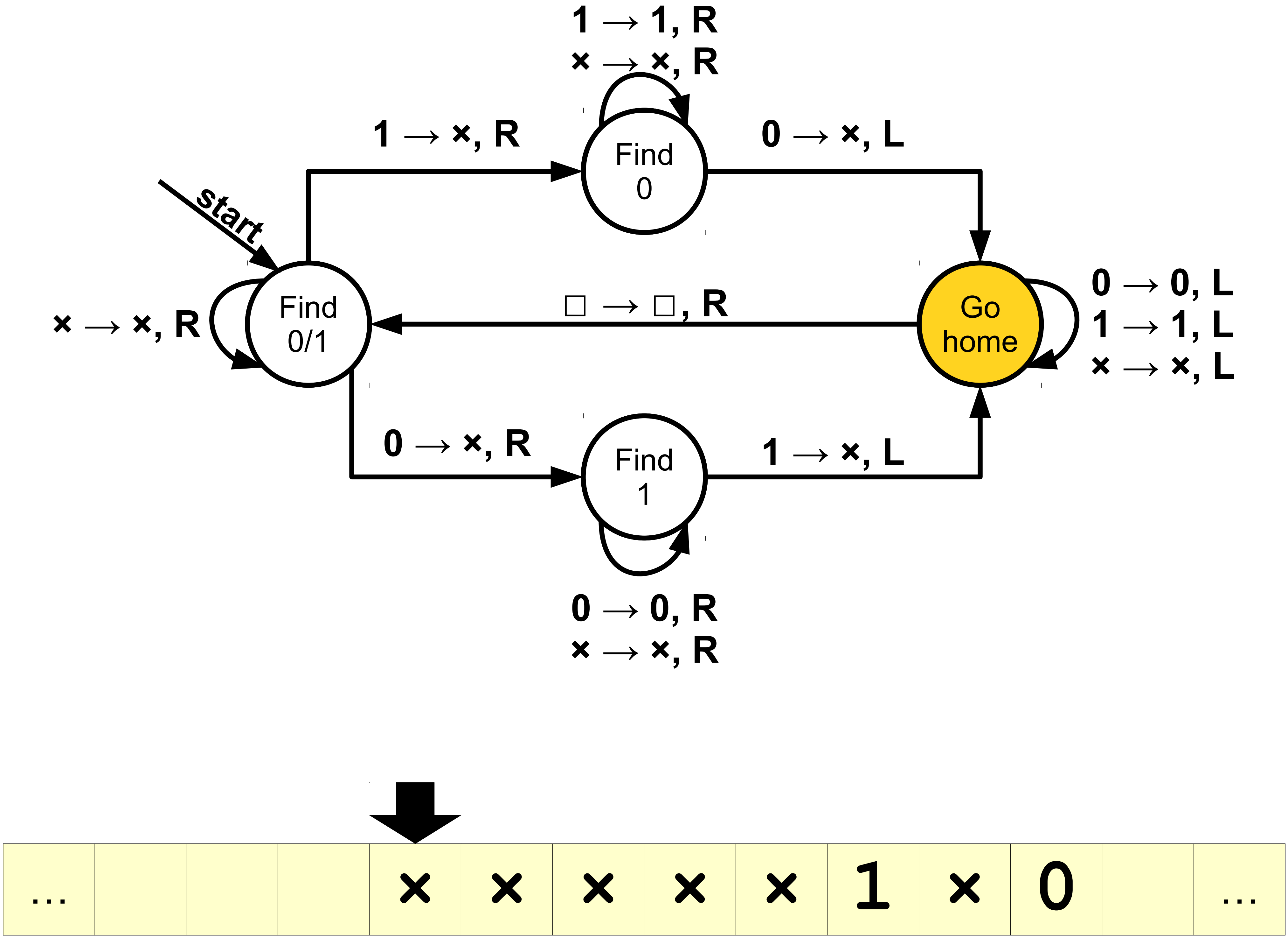
…				×	×	×	×	×	1	×	0		…

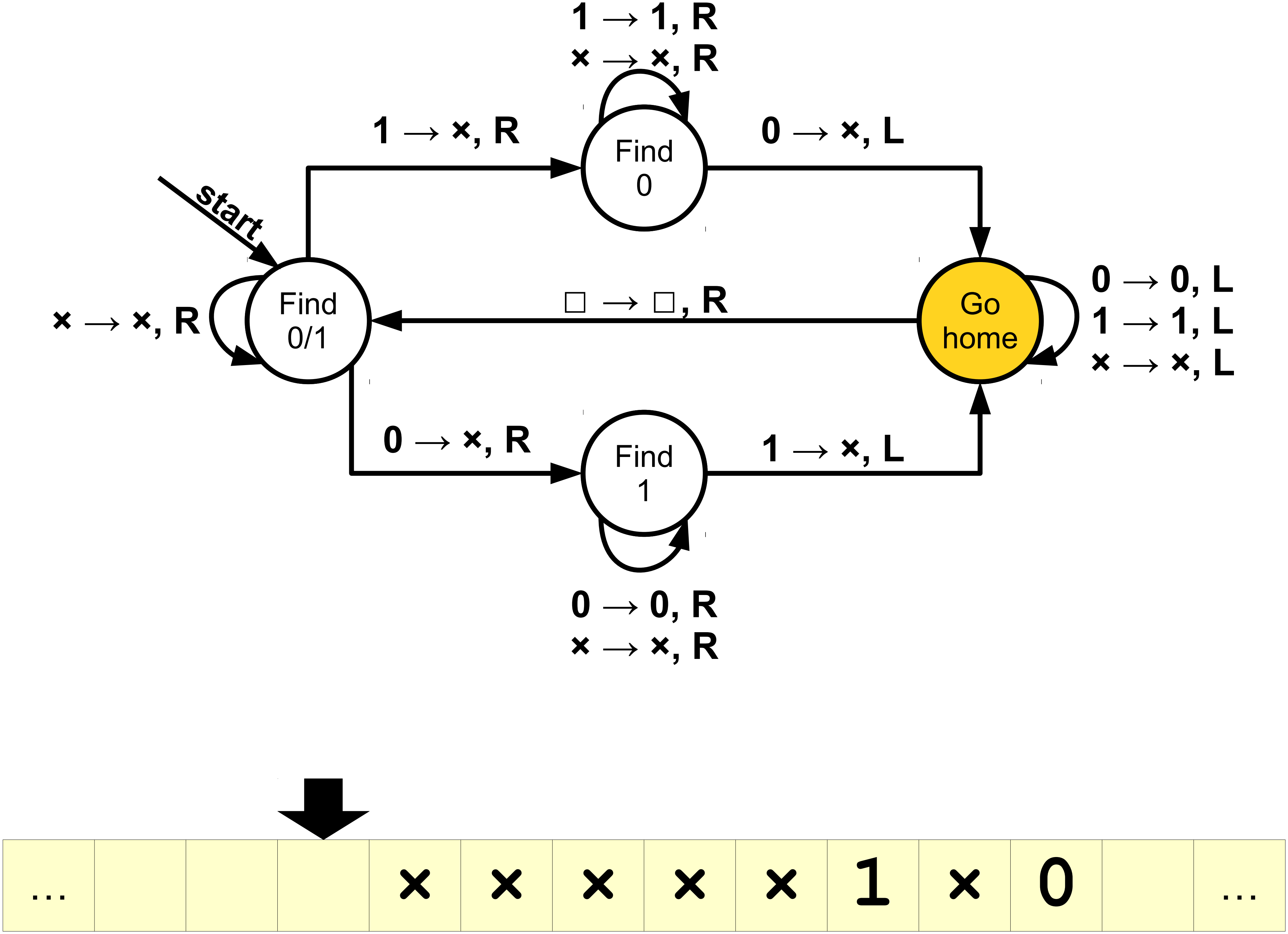

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

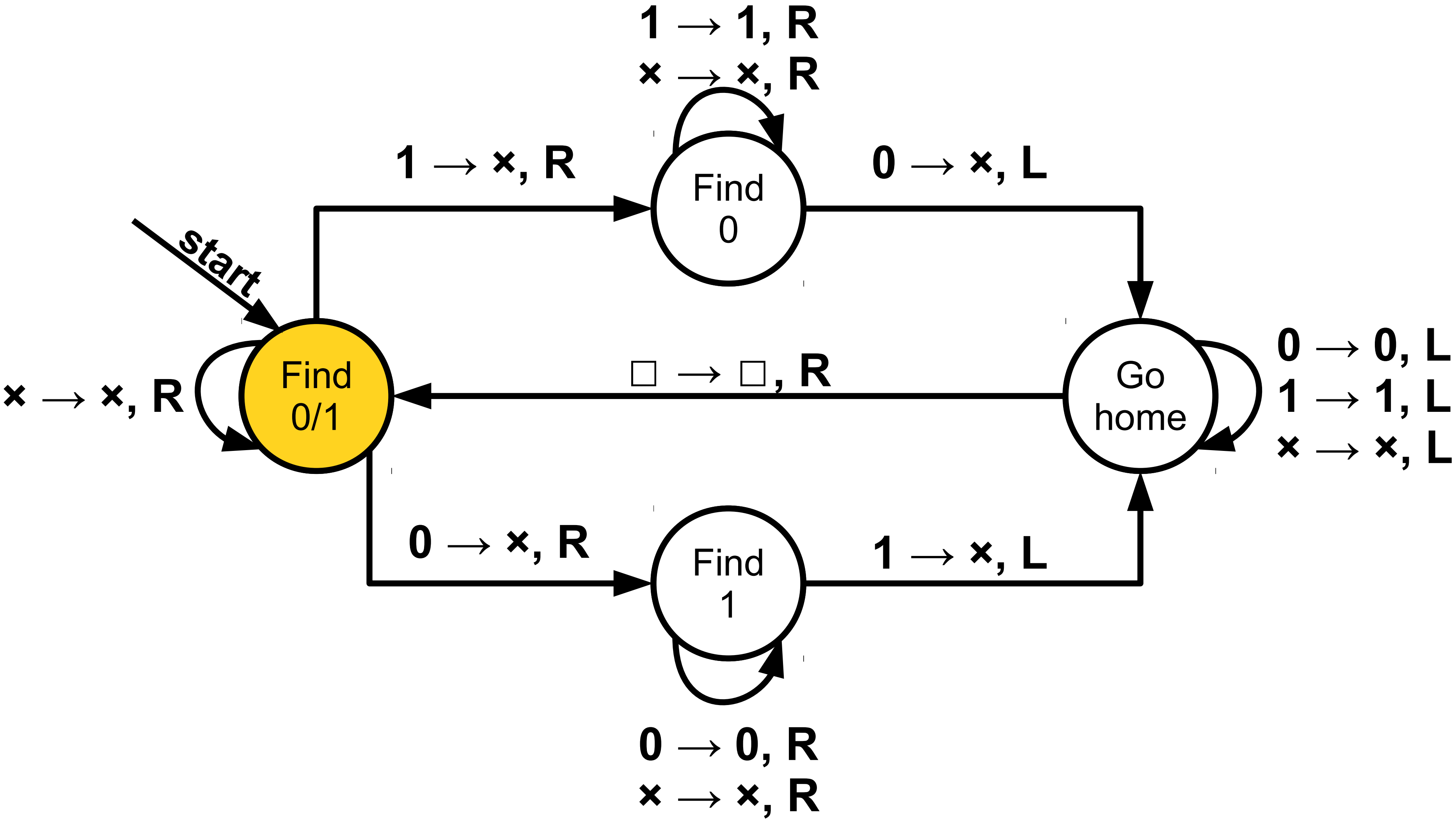

1 → 1, R
× → ×, R
1 → ×, R
Find
0
0 → ×, L
start
Find
0/1
× → ×, R
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
×
0
…


start
Find 0/1
Find 0
Find 1
Go home
× → ×, R
1 → ×, R
0 → ×, R
1 → 1, R
× → ×, R
0 → ×, L
0 → 0, R
× → ×, R
1 → ×, L
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
…
×
×
×
×
×
1
×
0
…


start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

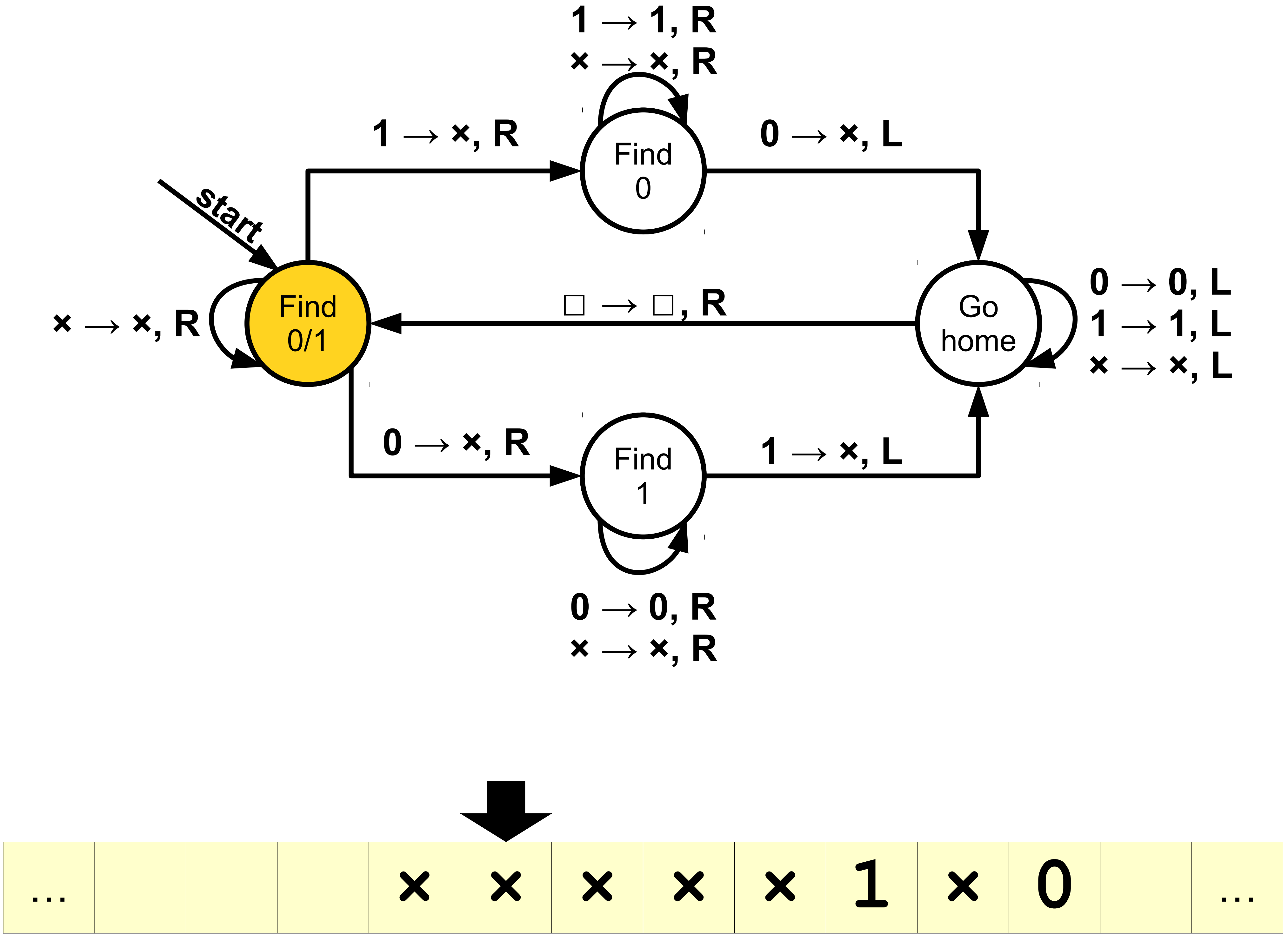

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

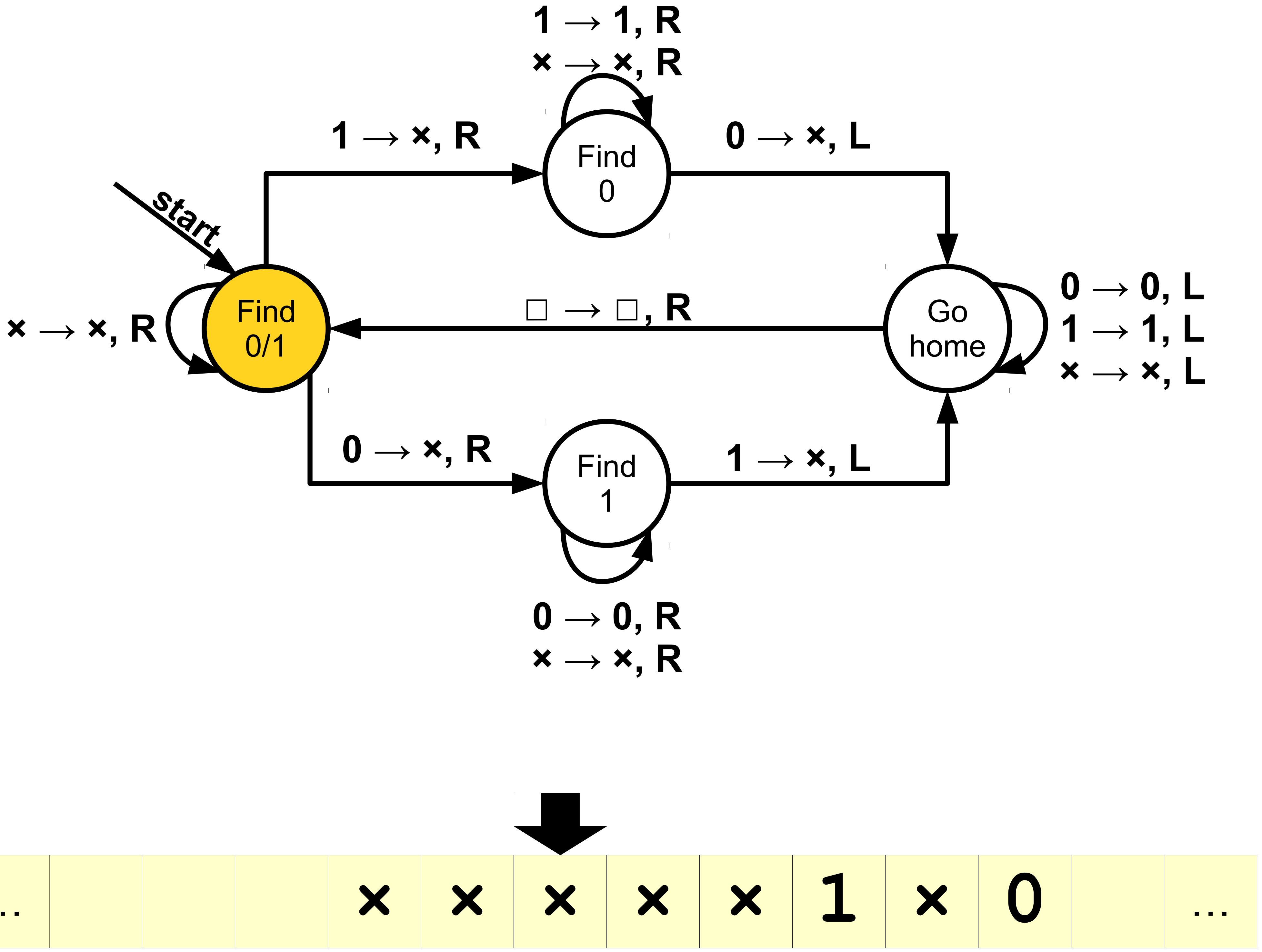

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

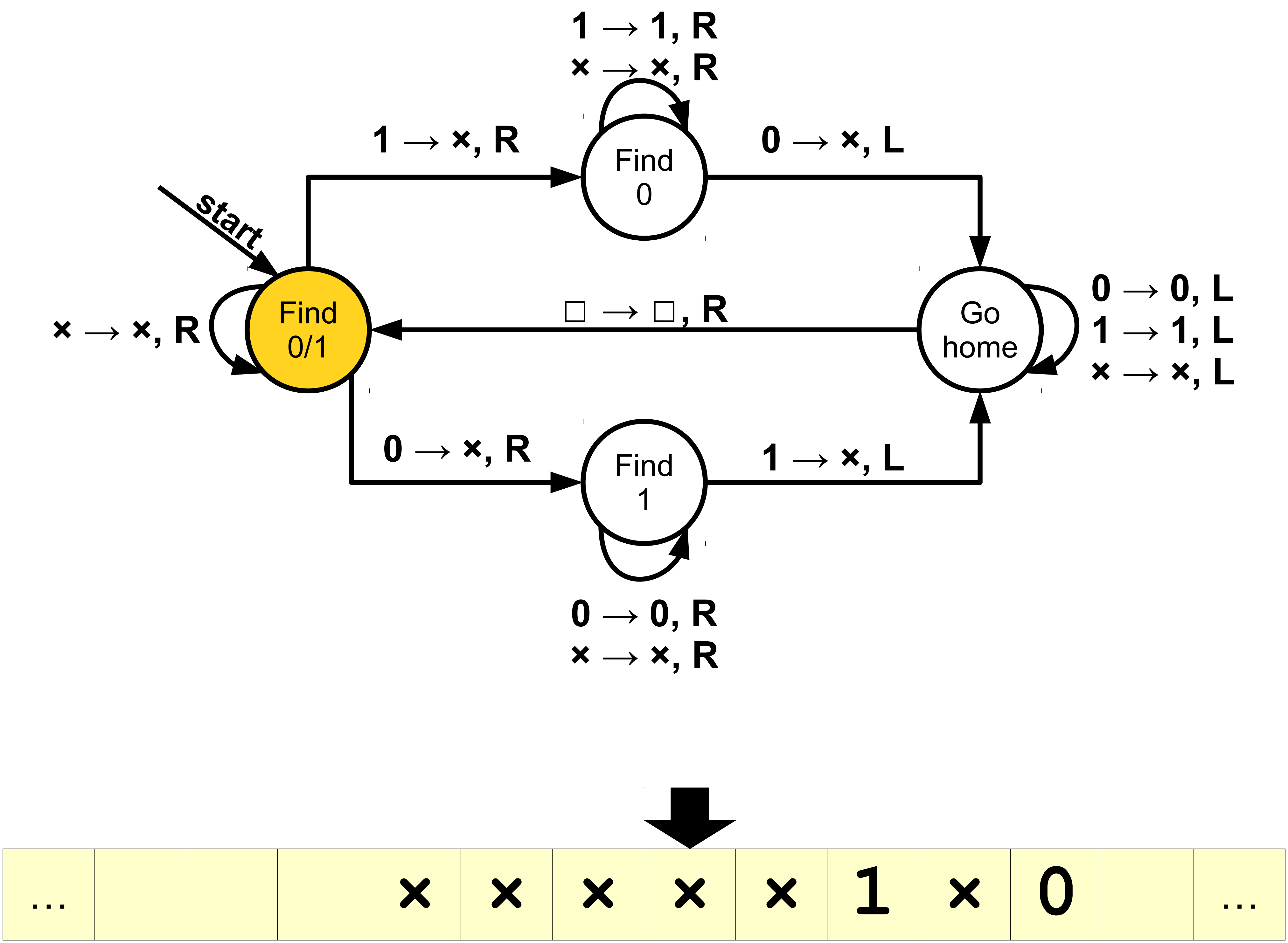


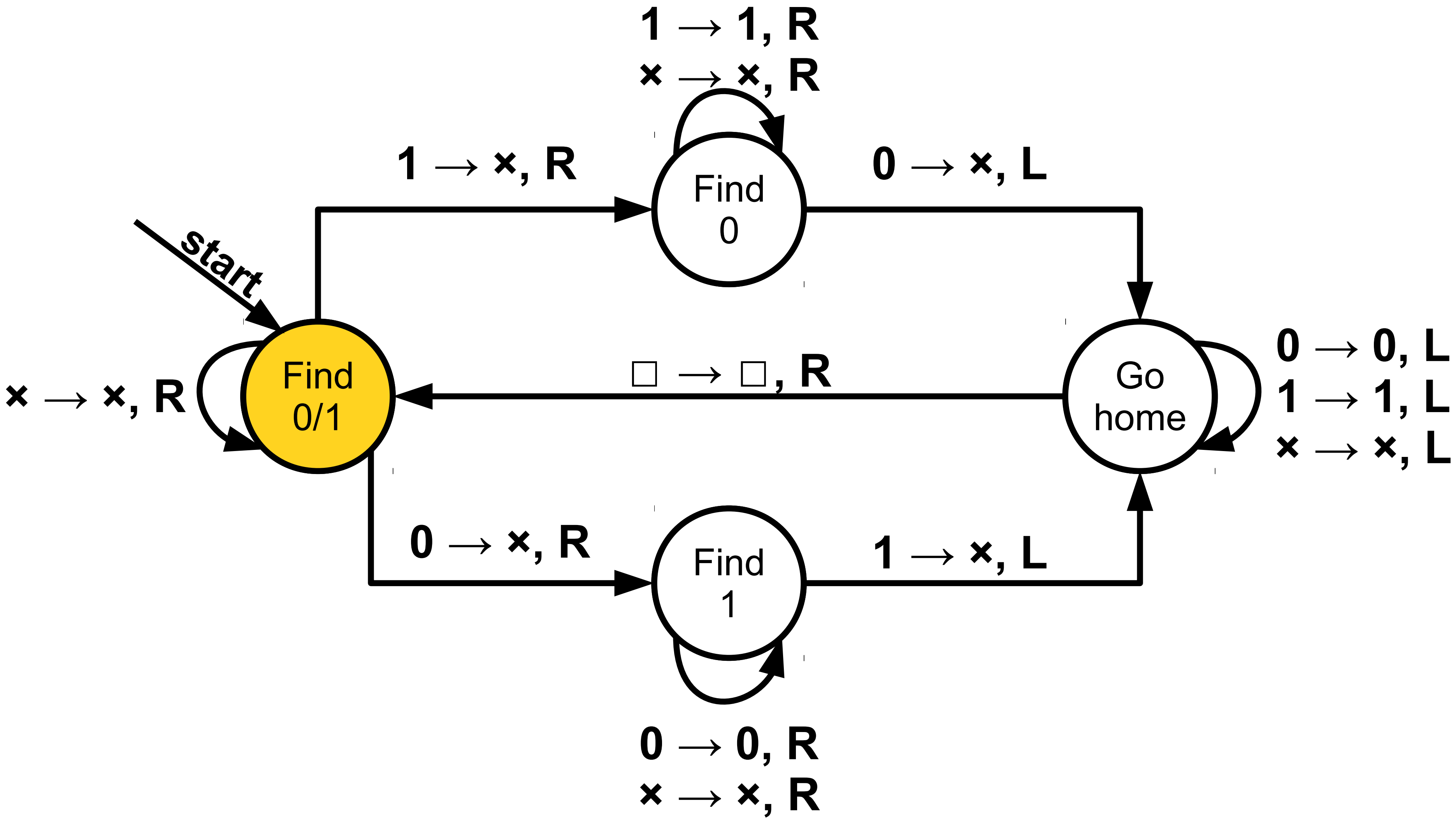
start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L

…				×	×	×	×	×	1	×	0		…

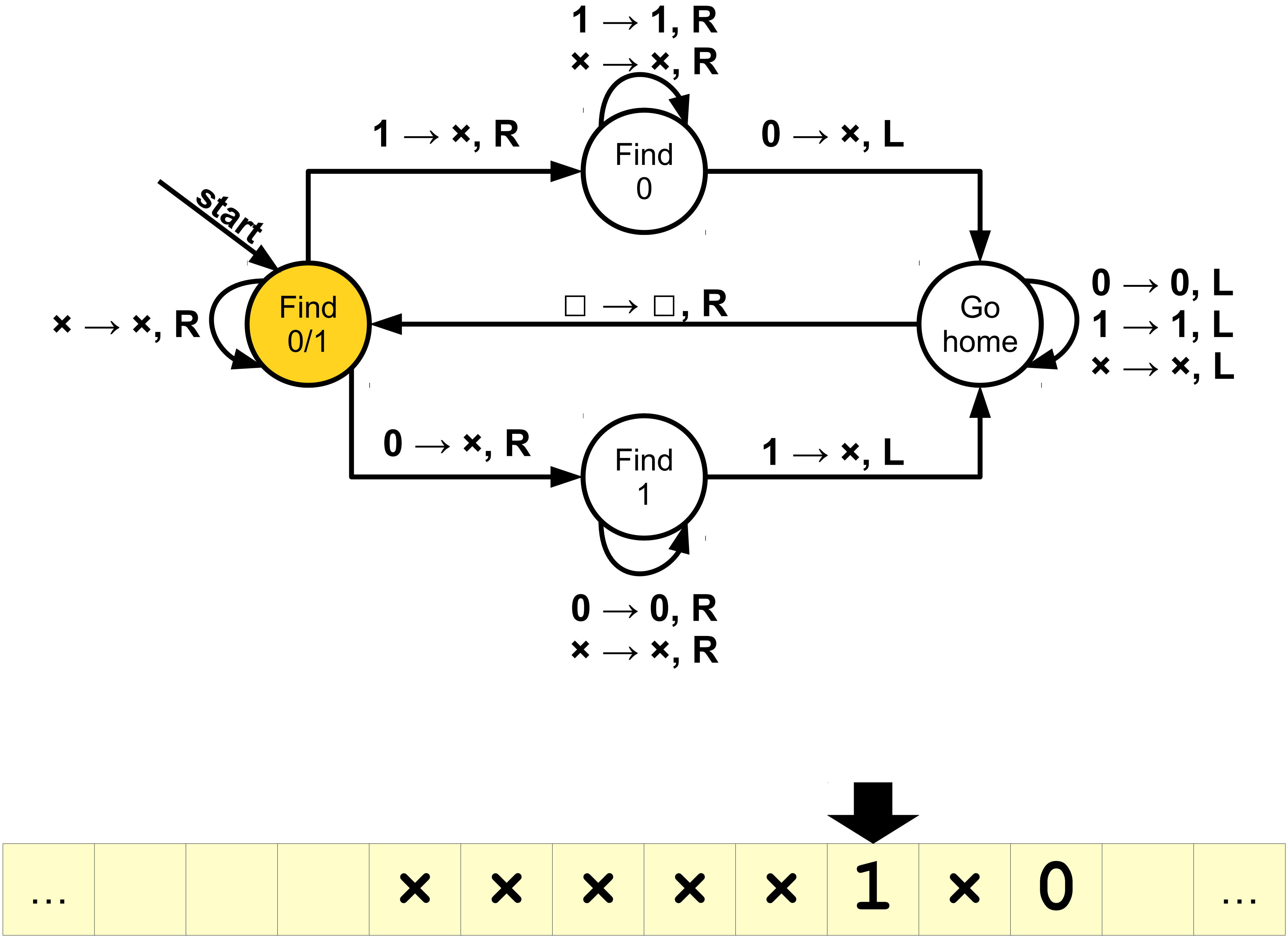

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

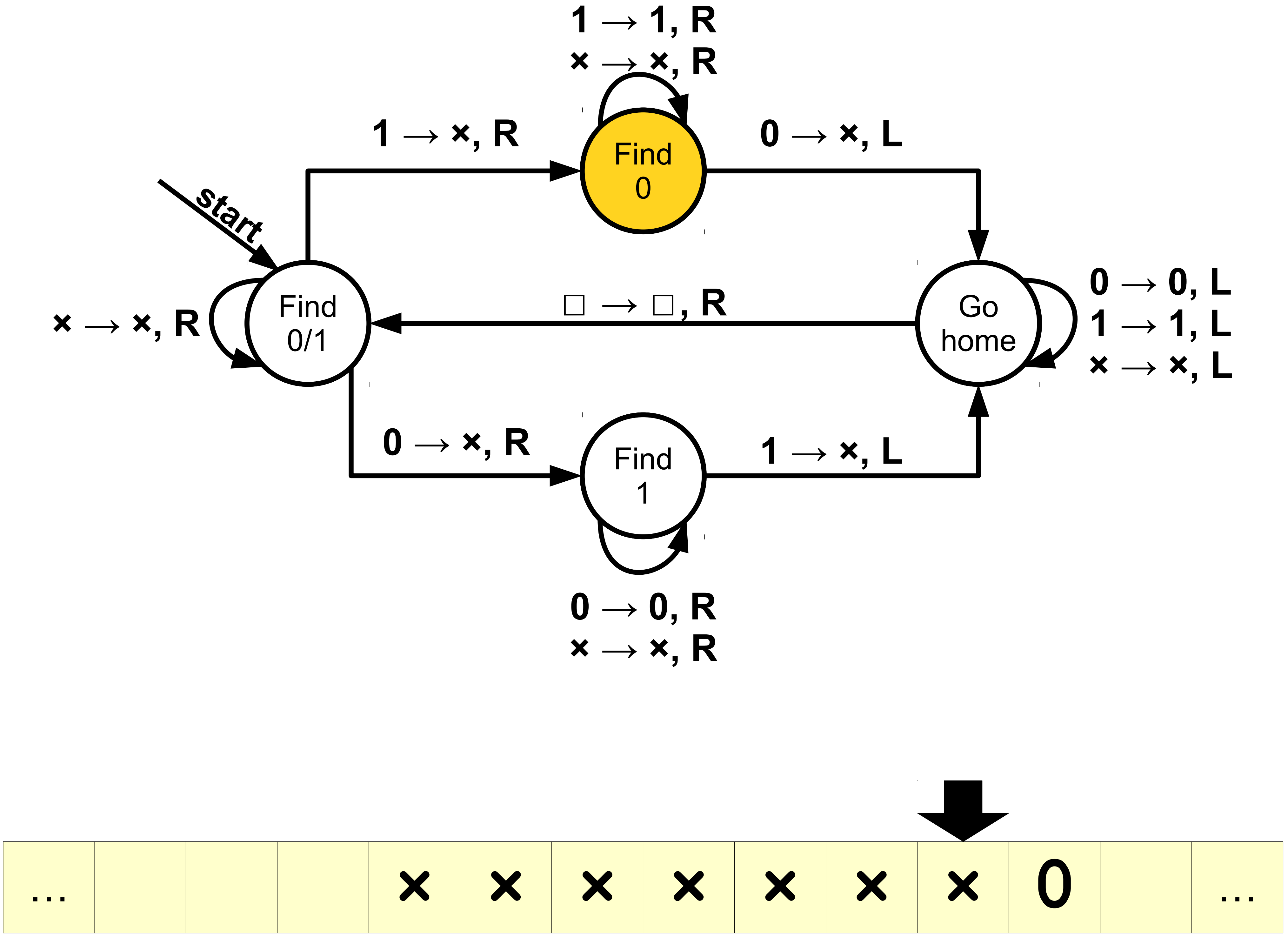

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

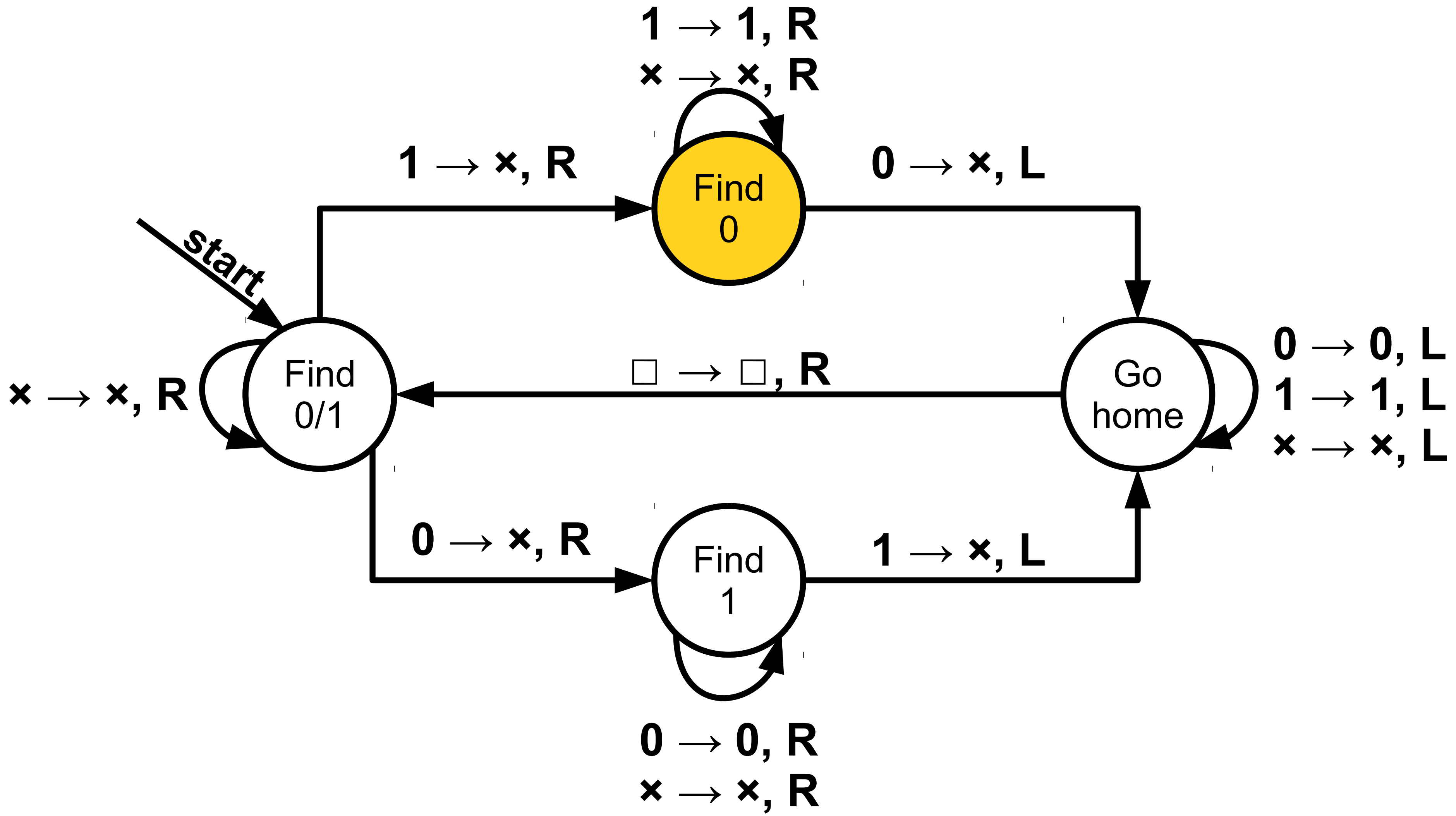

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

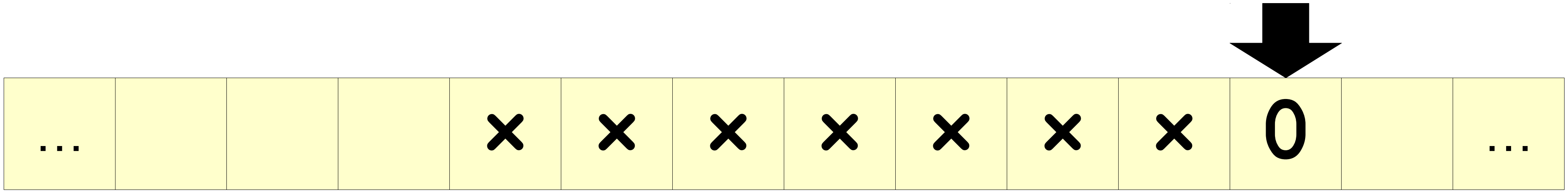

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

…				×	×	×	×	×	1	×	0		…

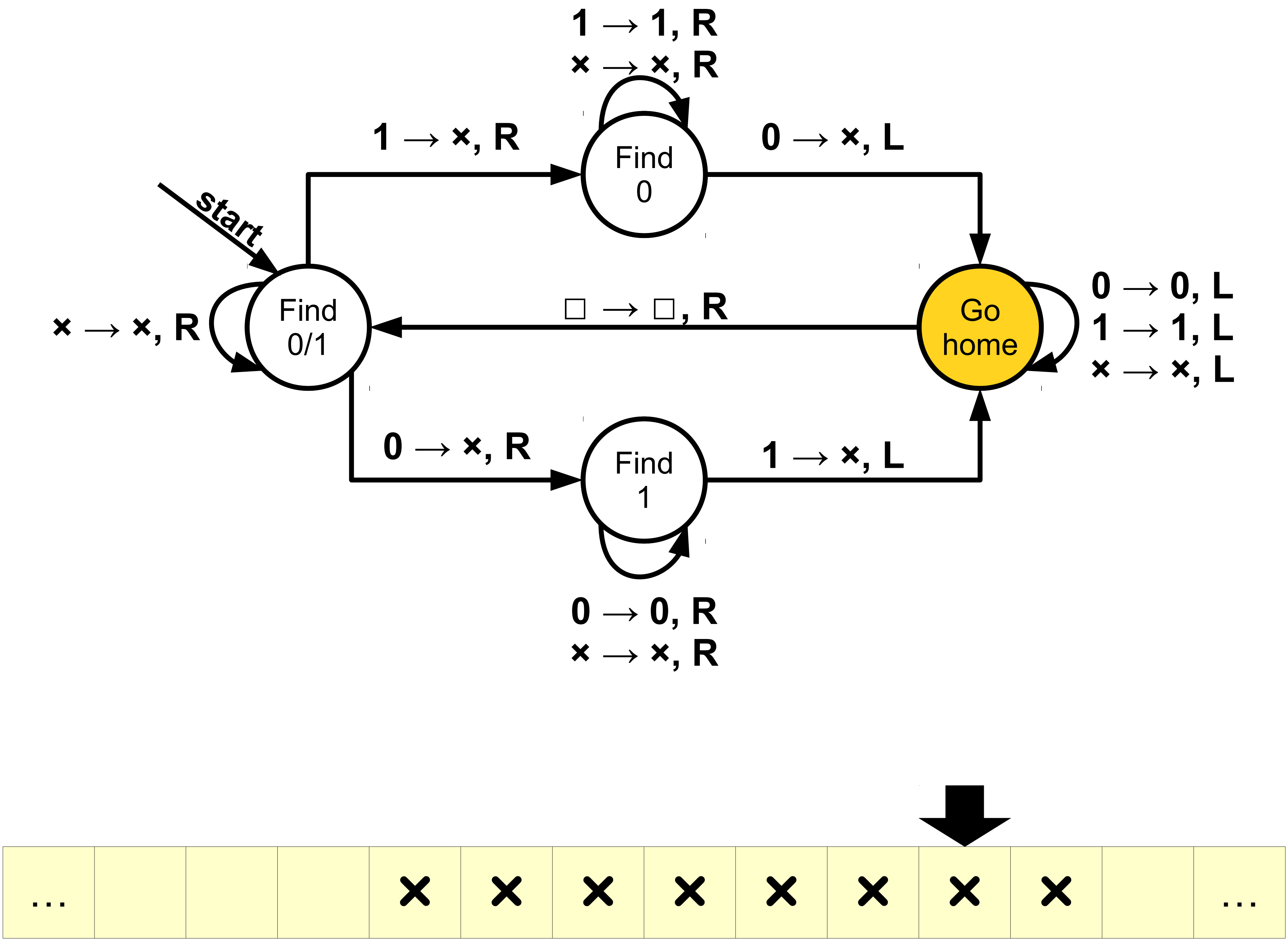

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

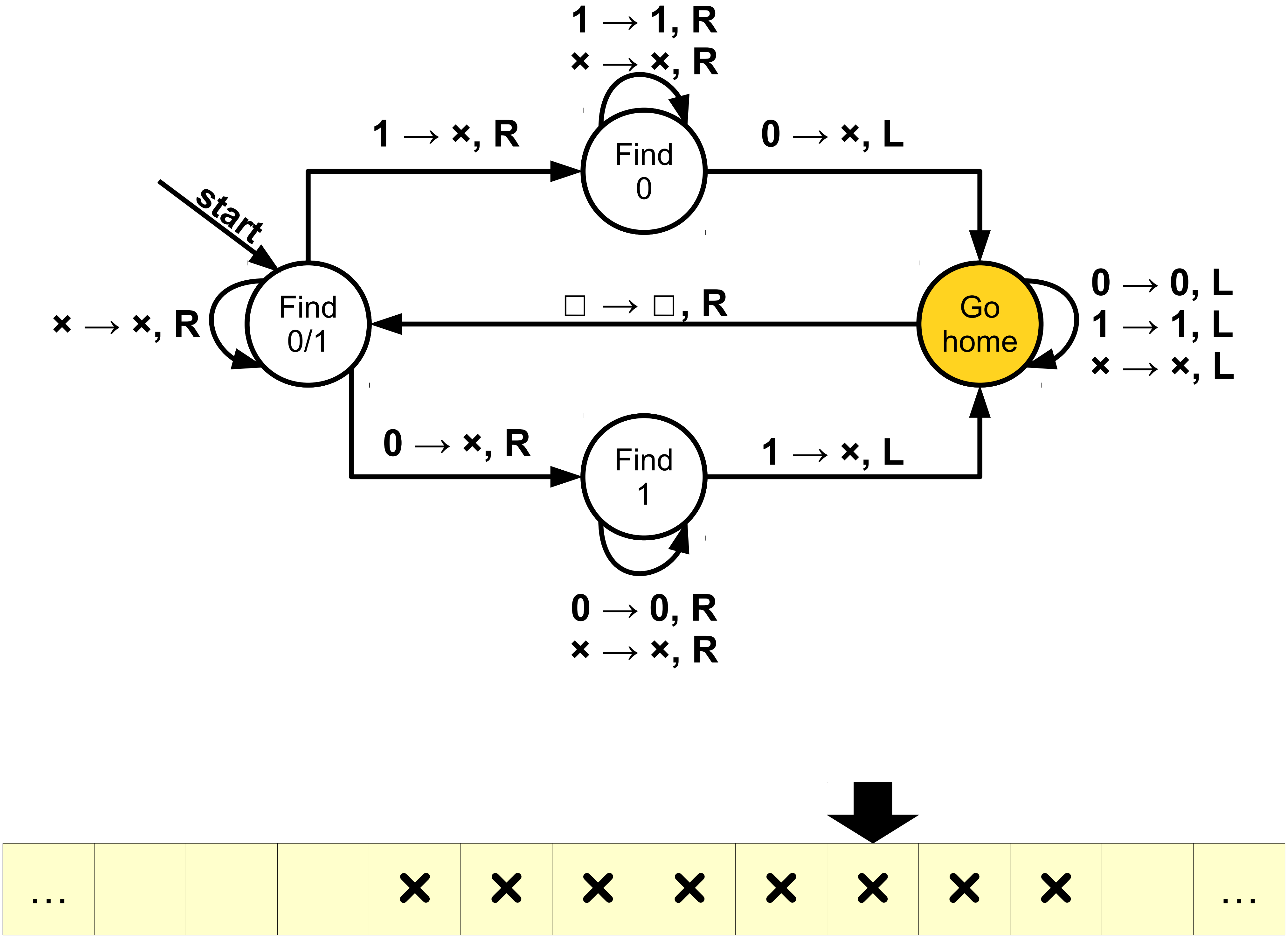

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
0
…

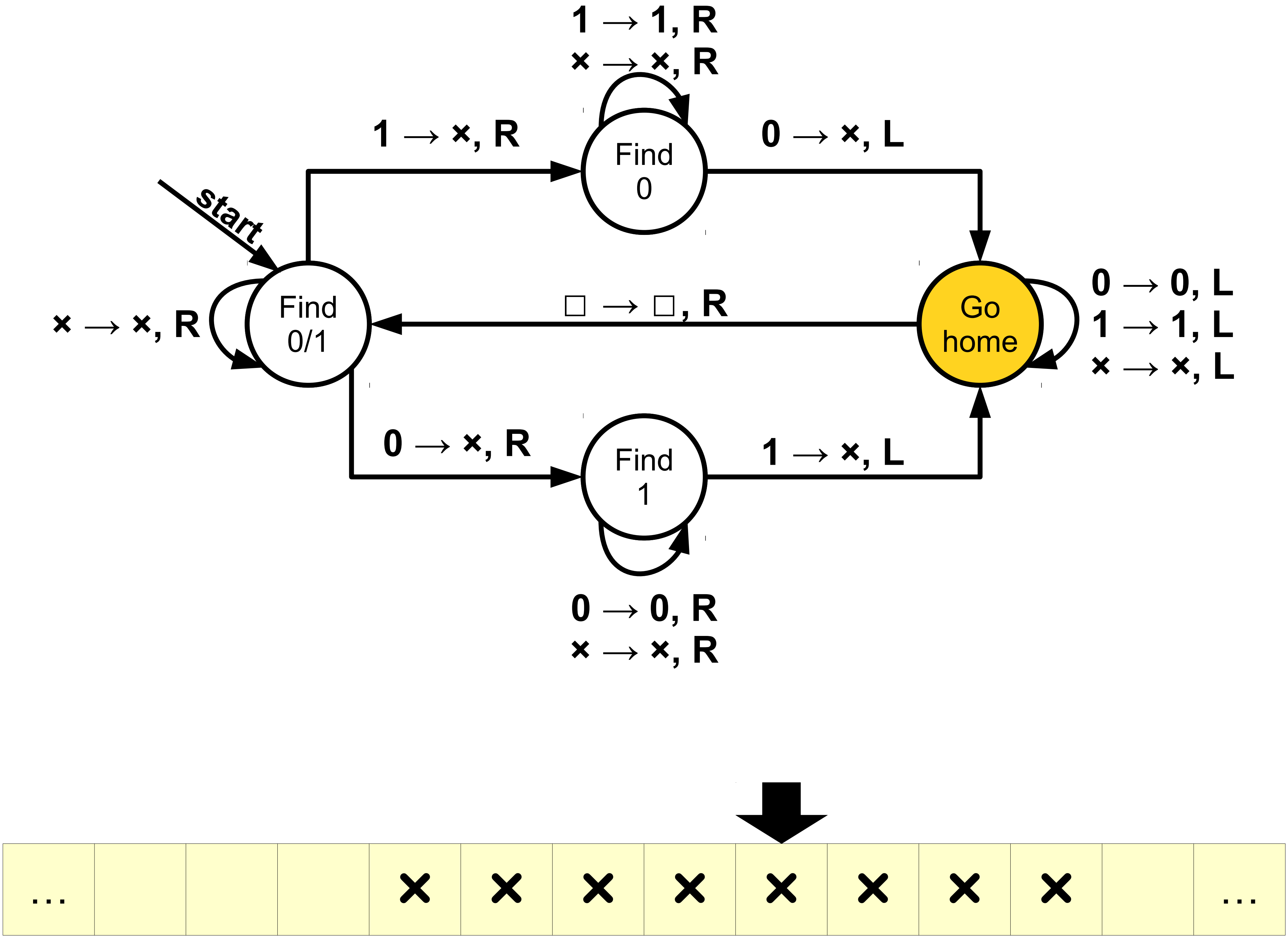

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

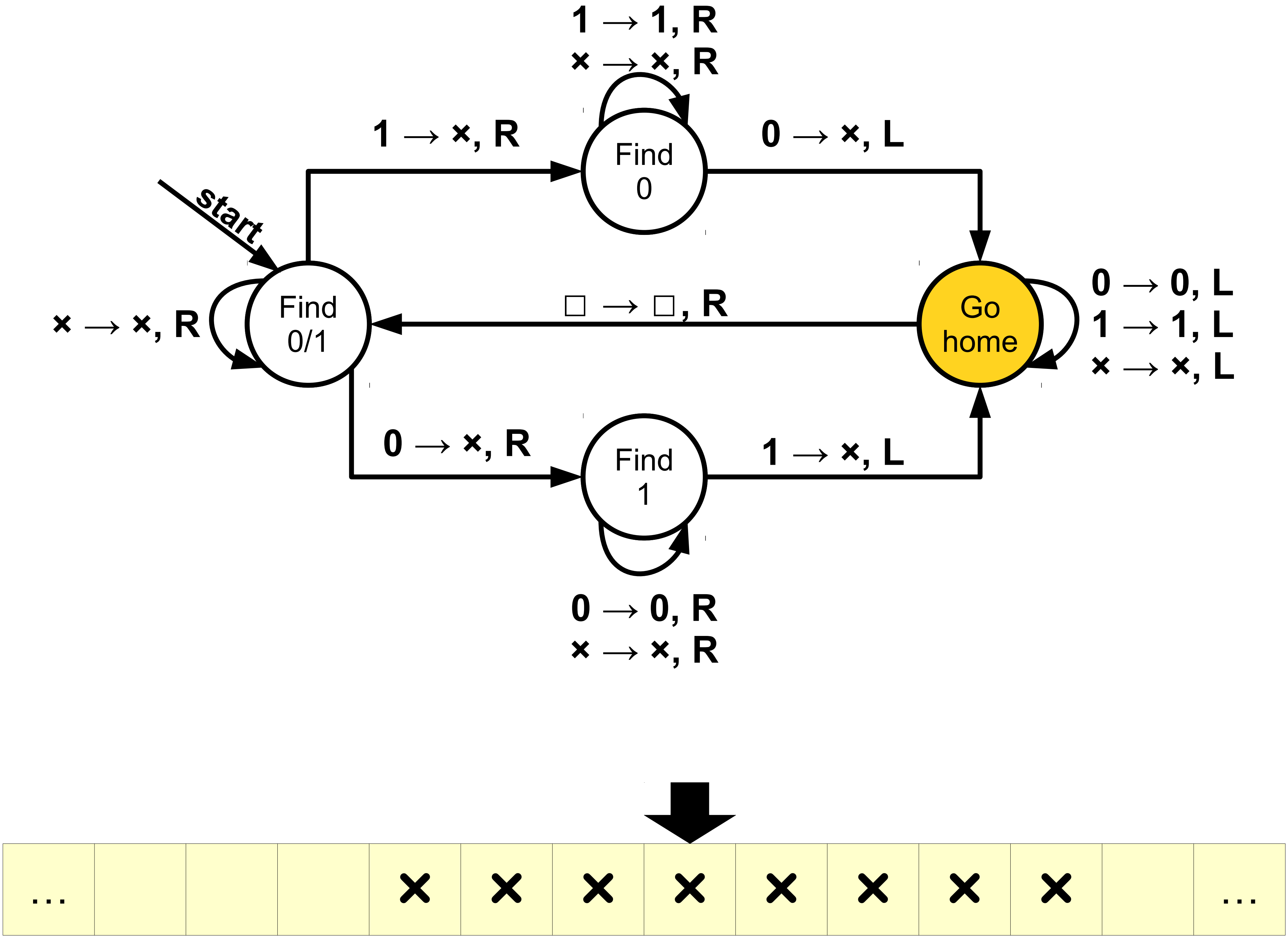


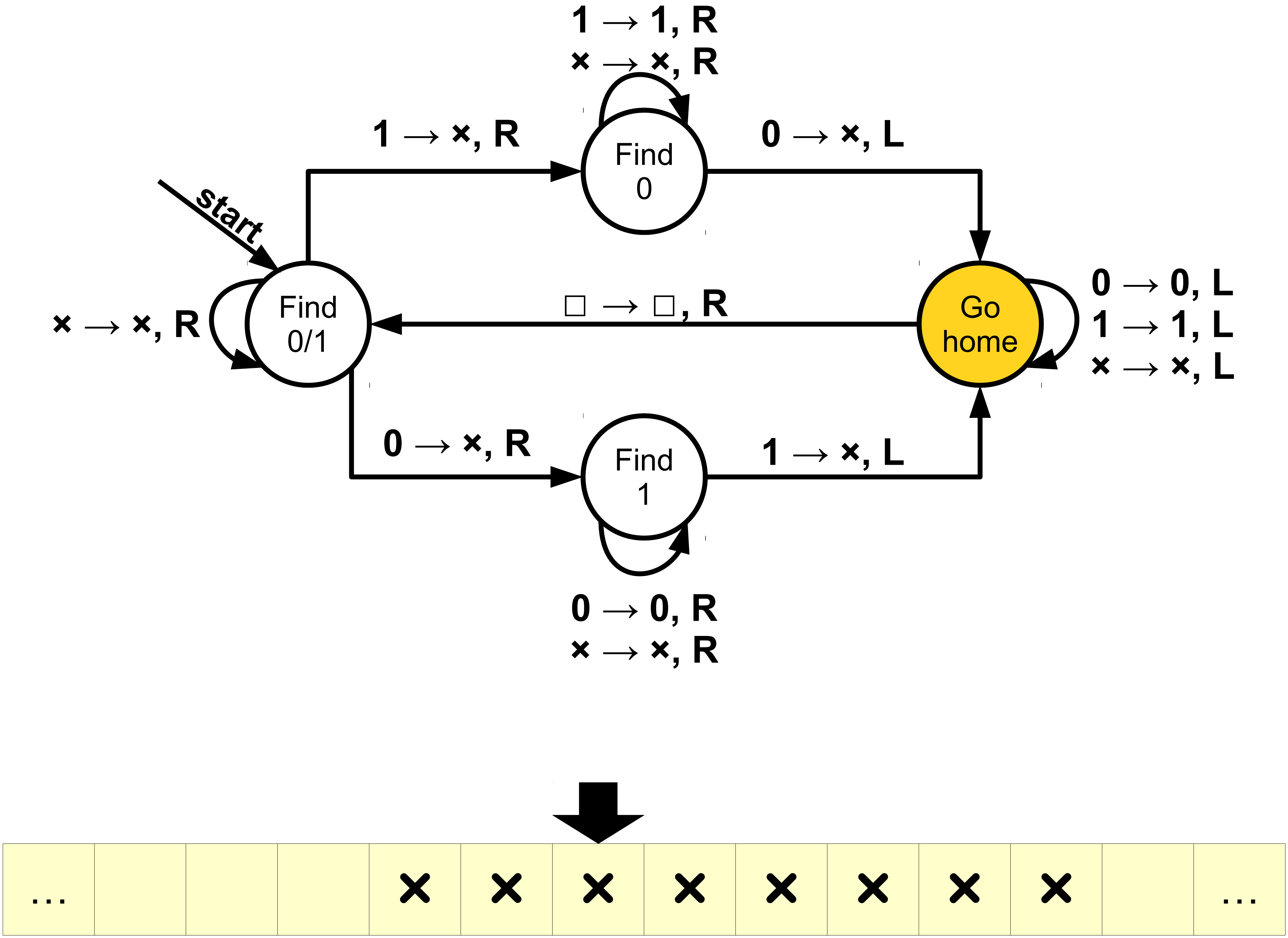
…				×	×	×	×	×	×	×	0		…

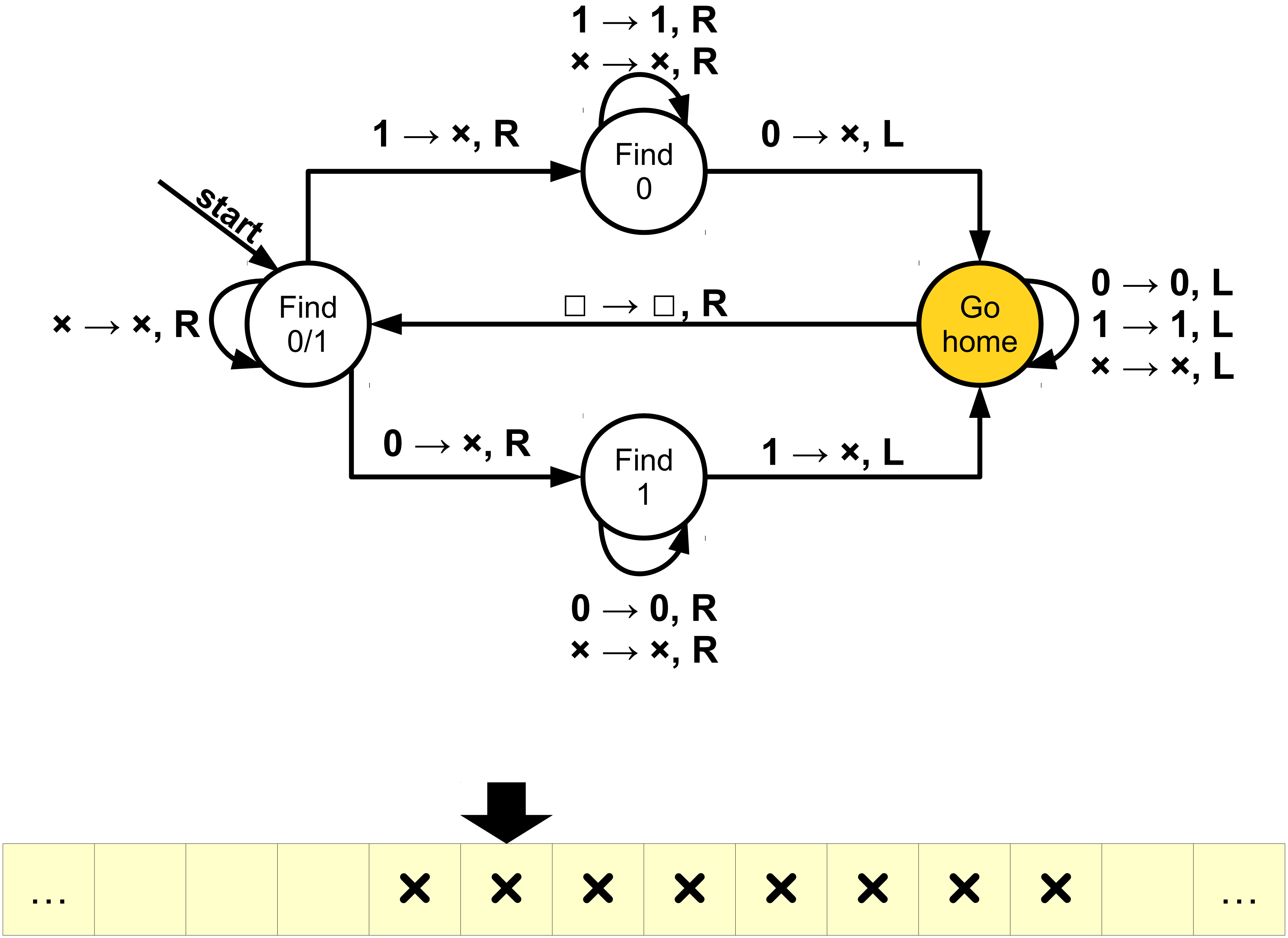

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

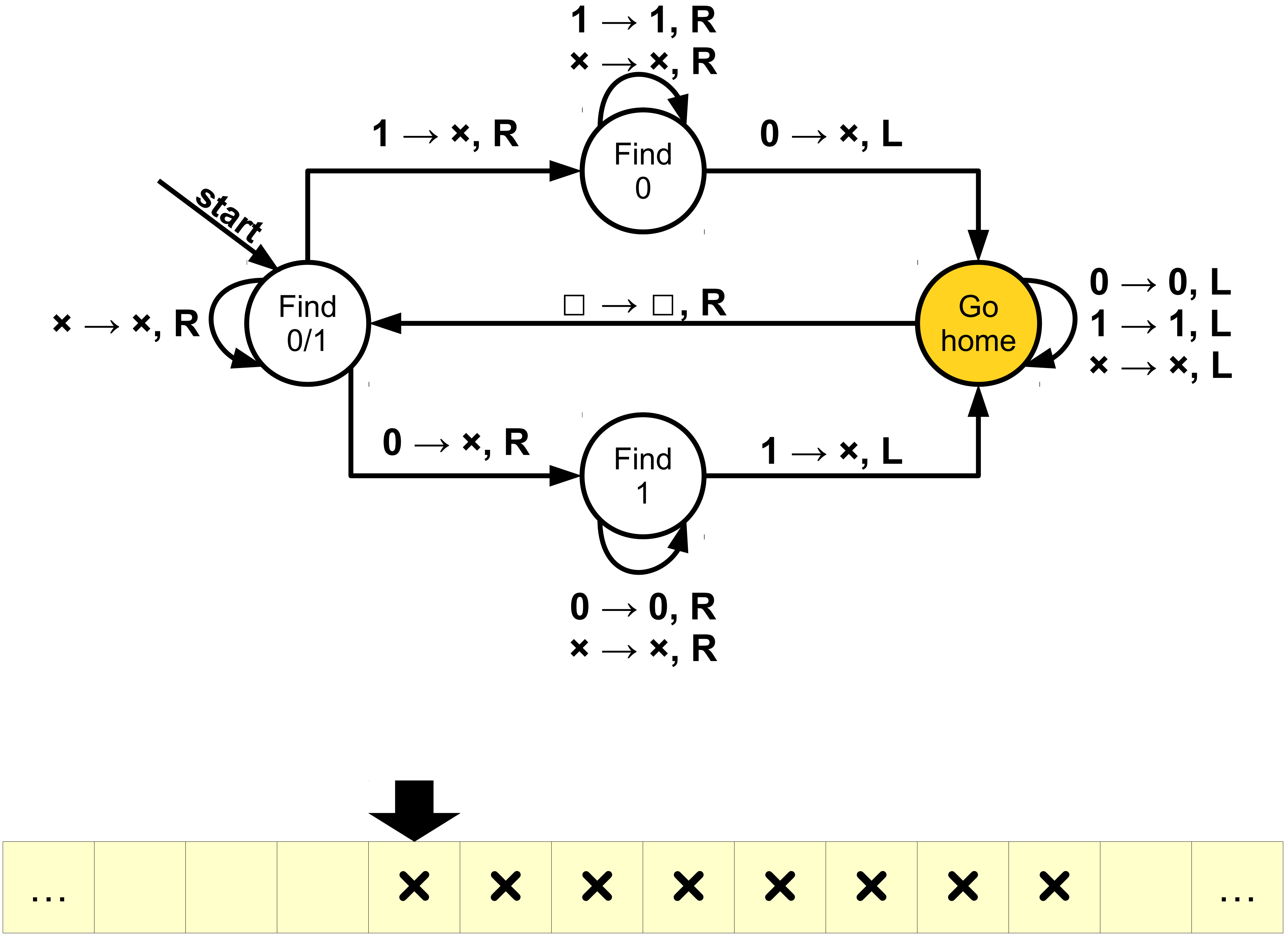

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…

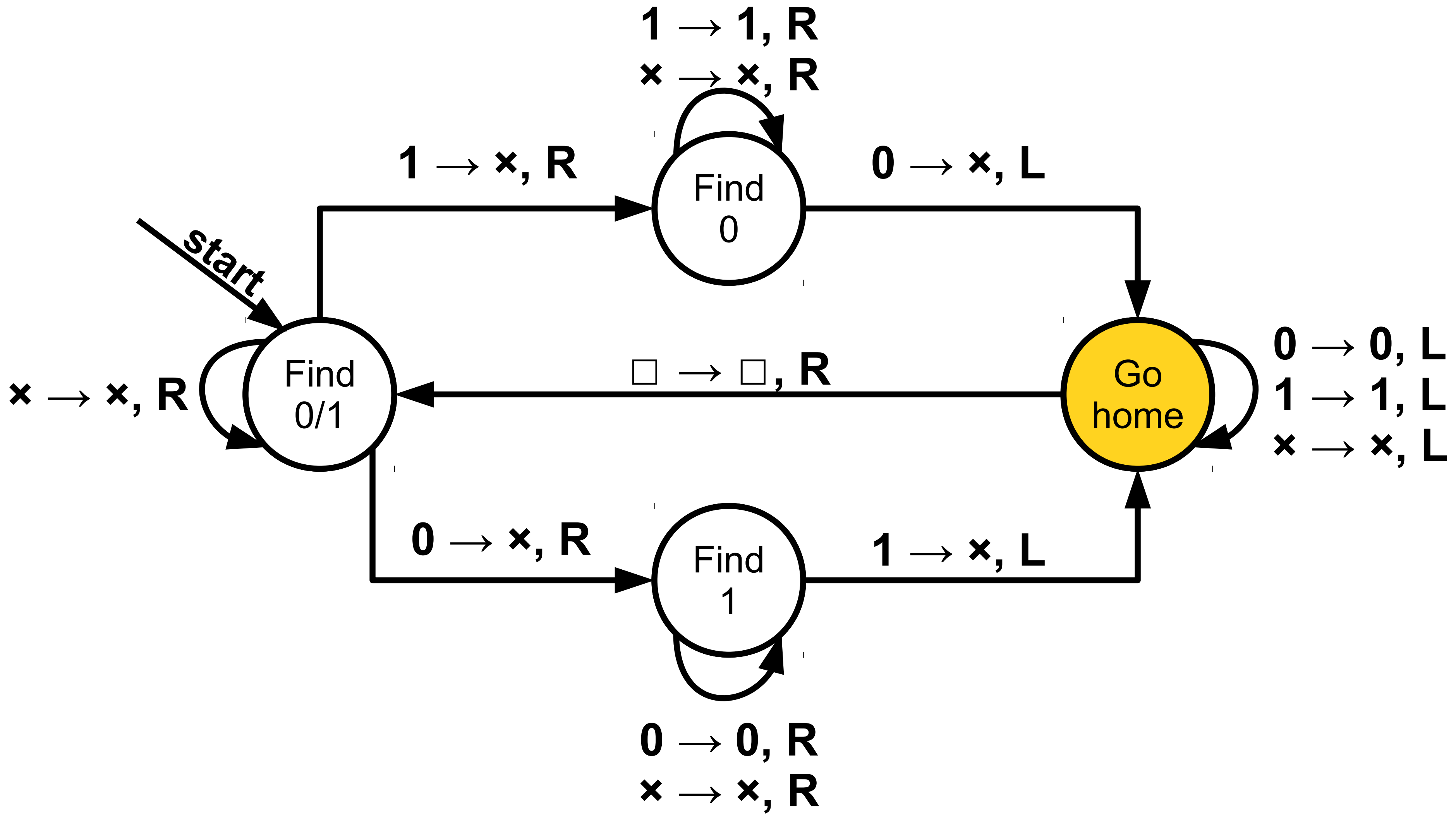

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

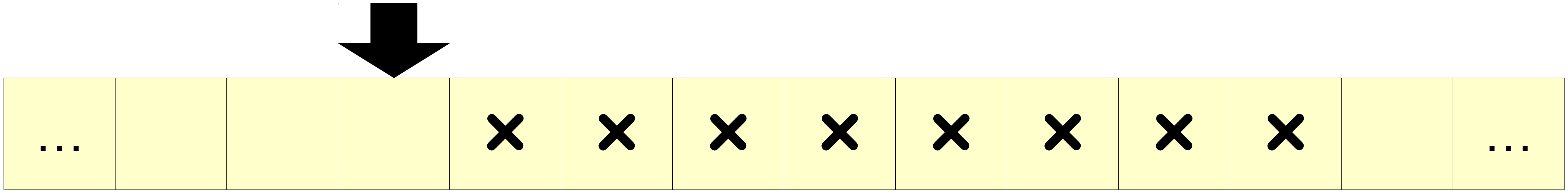

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…

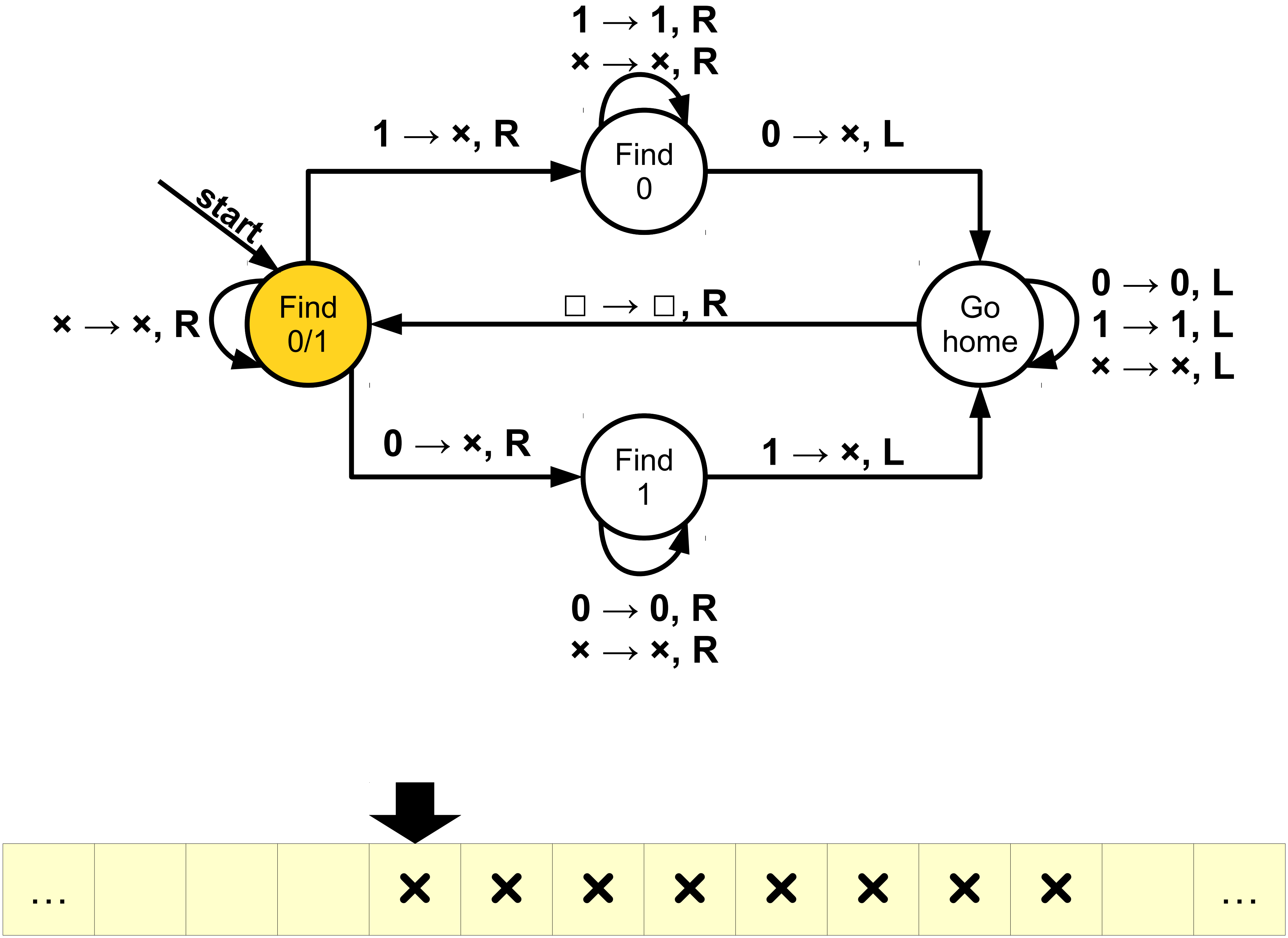

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

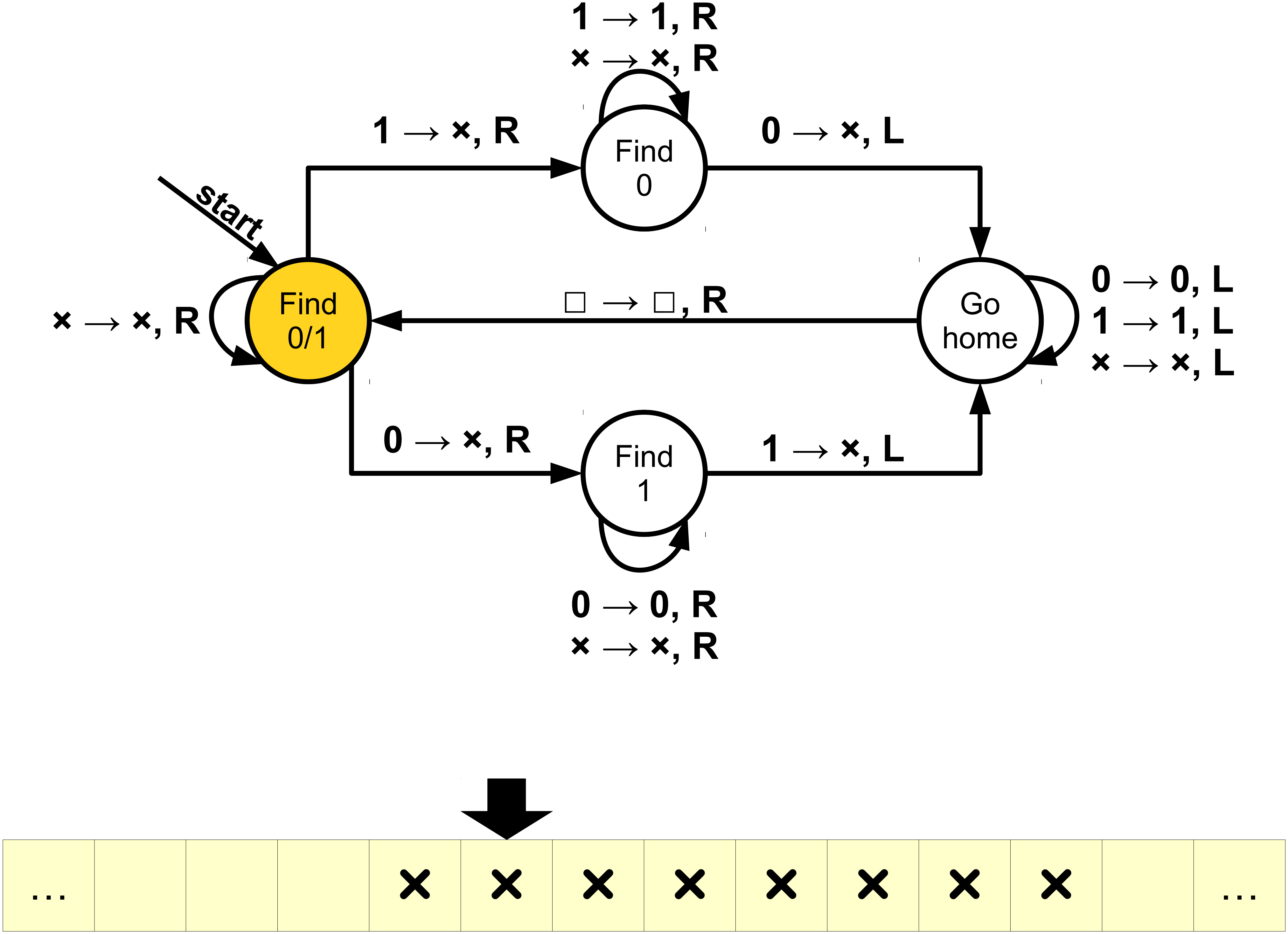

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0/1
× → ×, R
Find 0
Find 1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

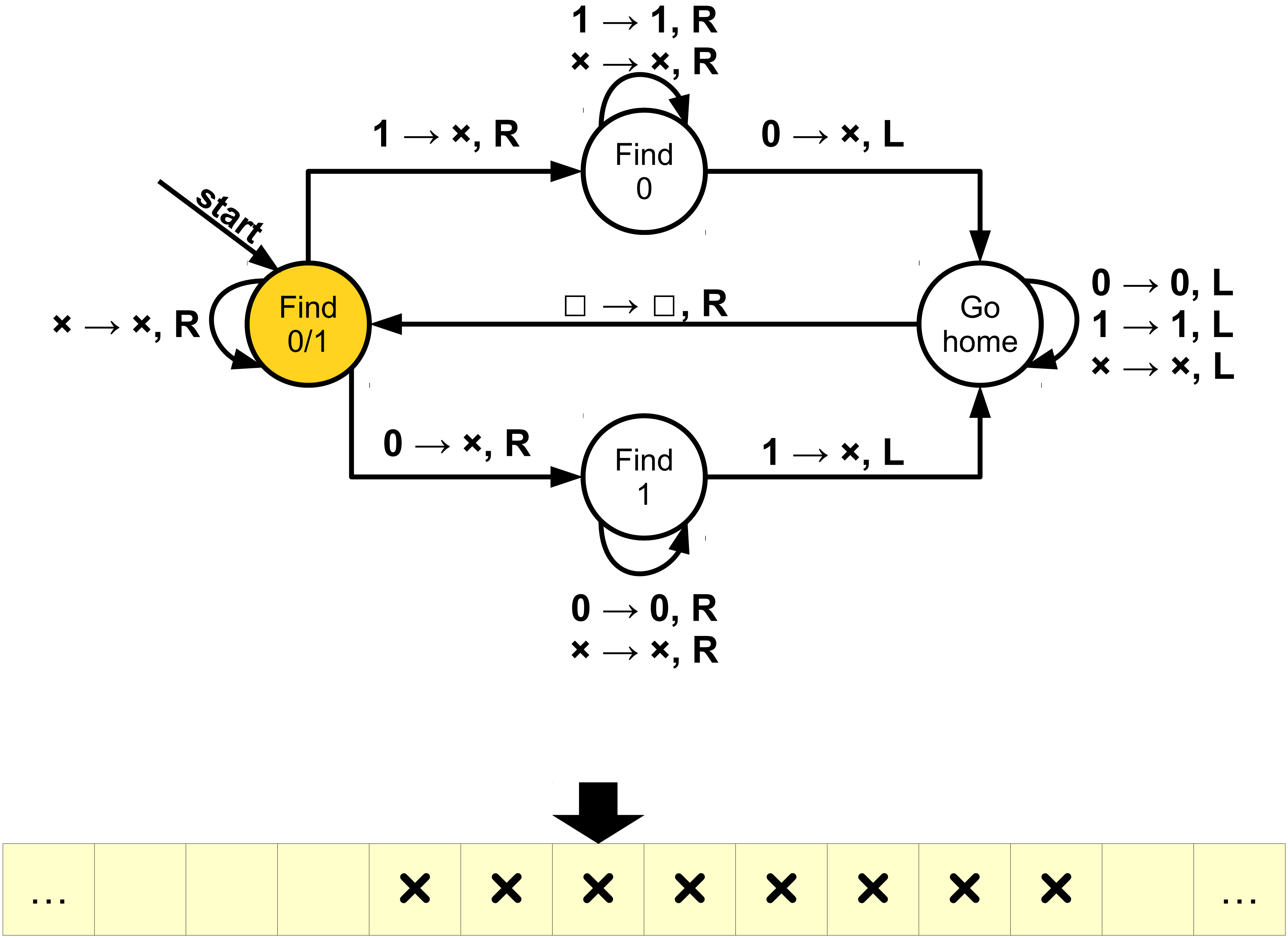

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

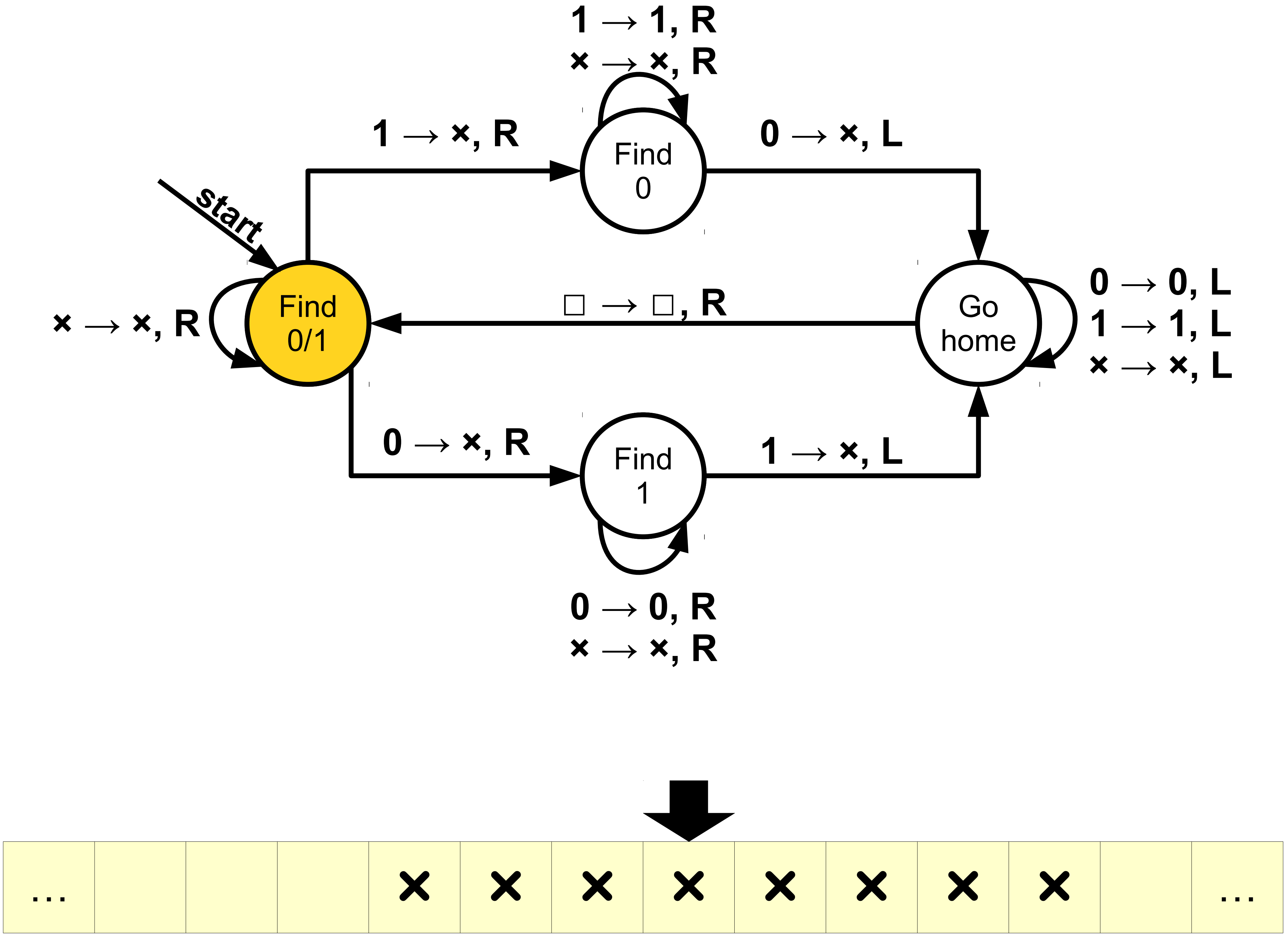

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

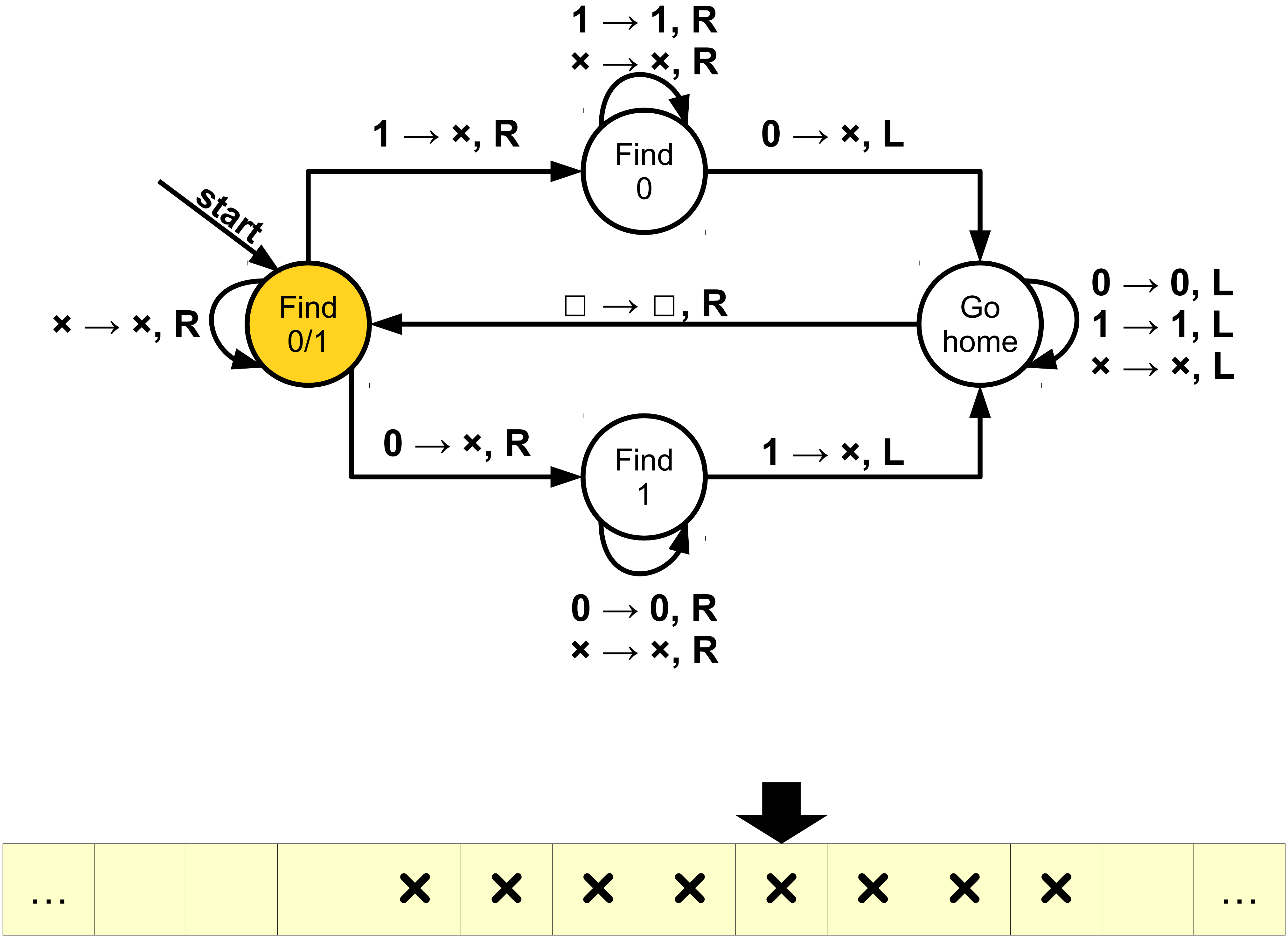

…
×
×
×
×
×
×
×
×
…

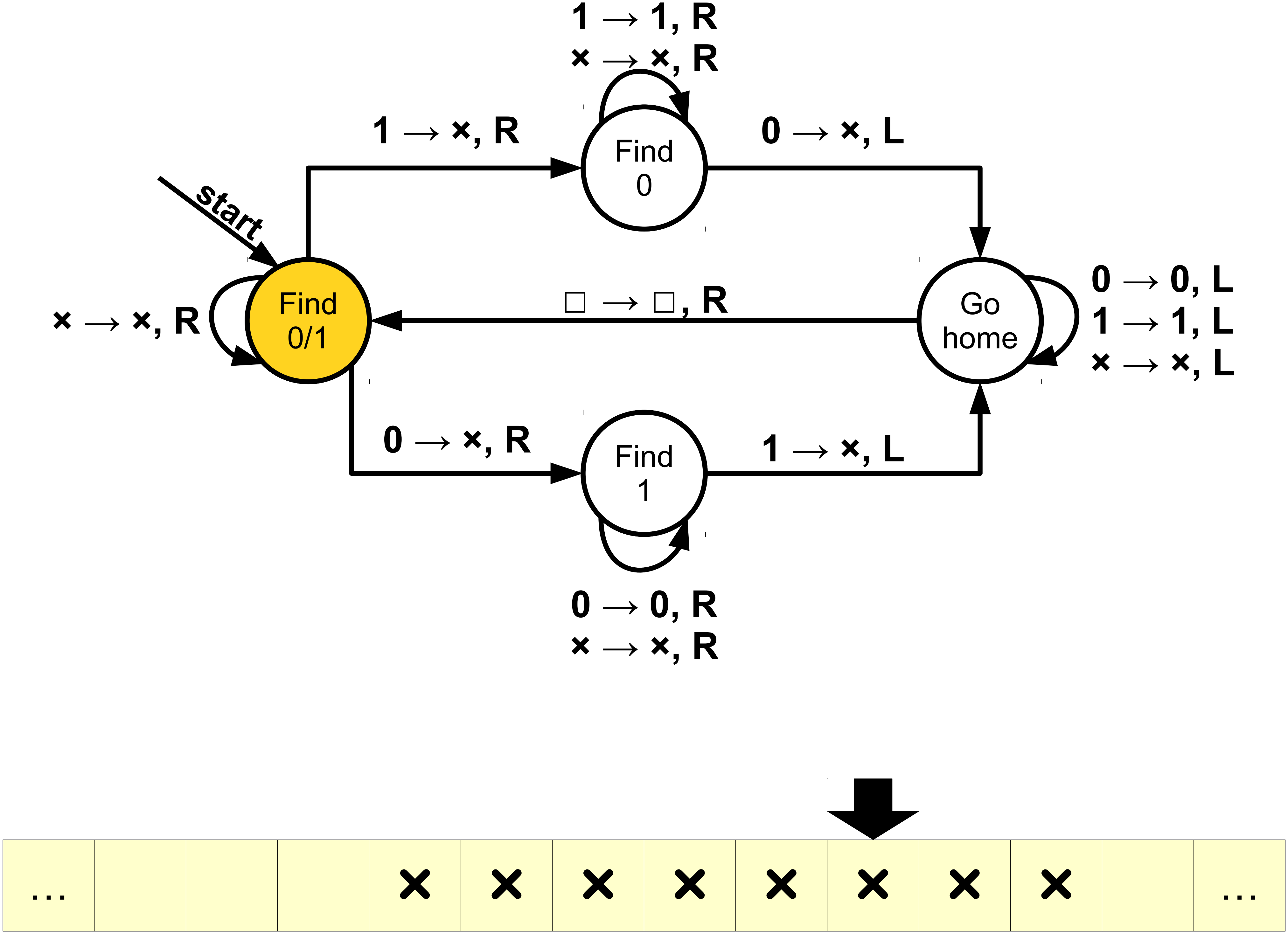

1 → 1, R
× → ×, R
1 → ×, R
Find
0
0 → ×, L
start
× → ×, R
Find
0/1
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

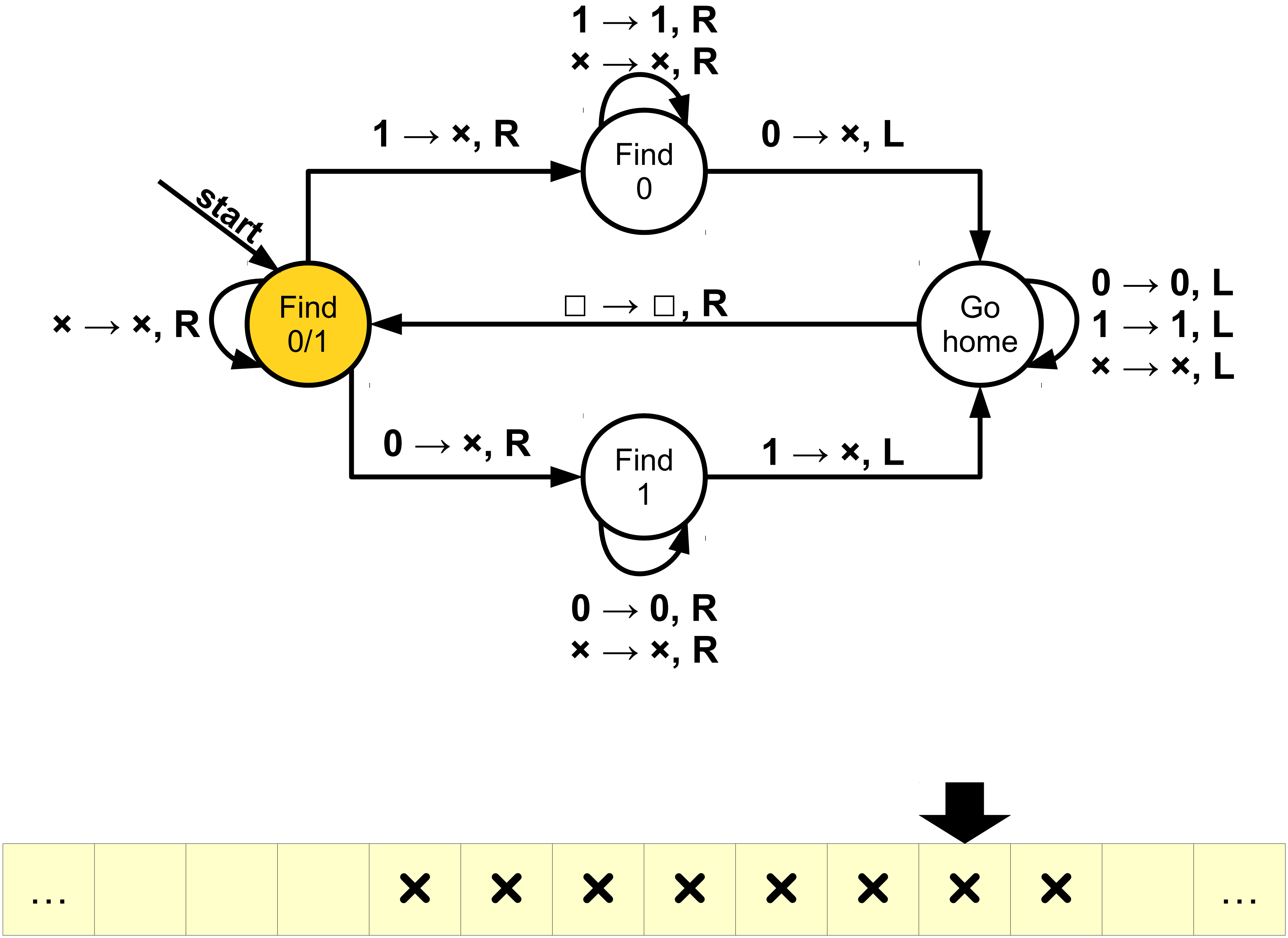

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…

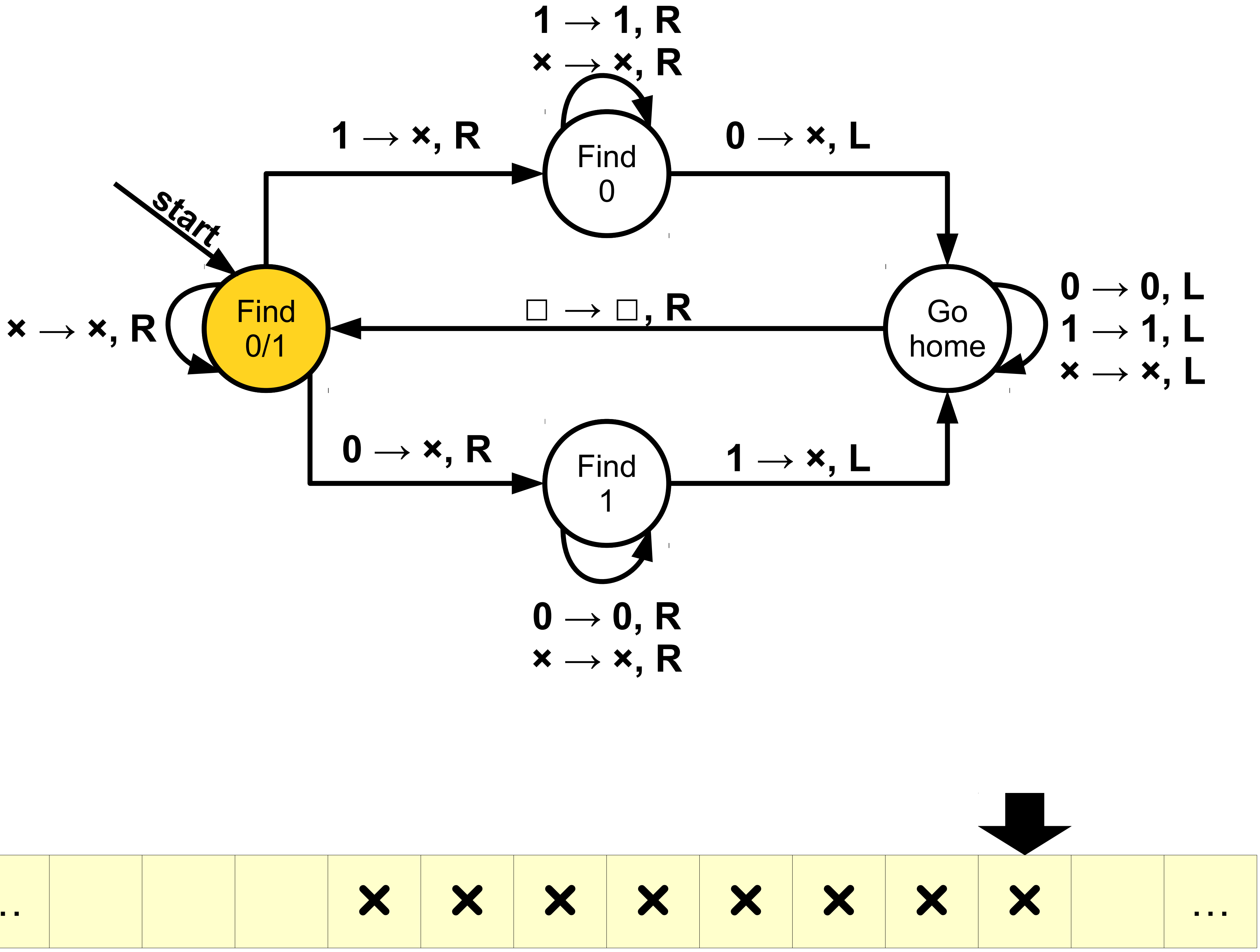

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

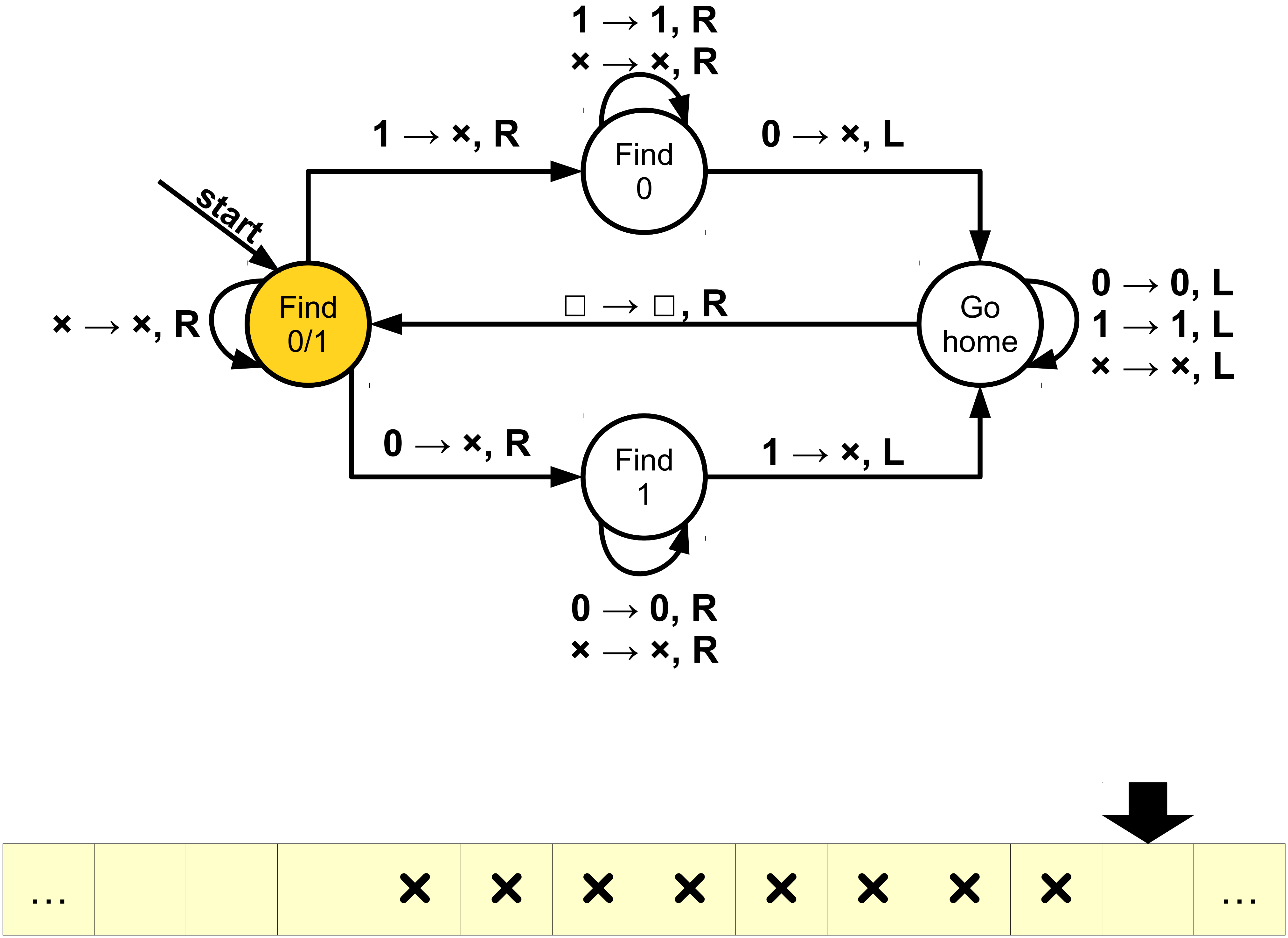

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…

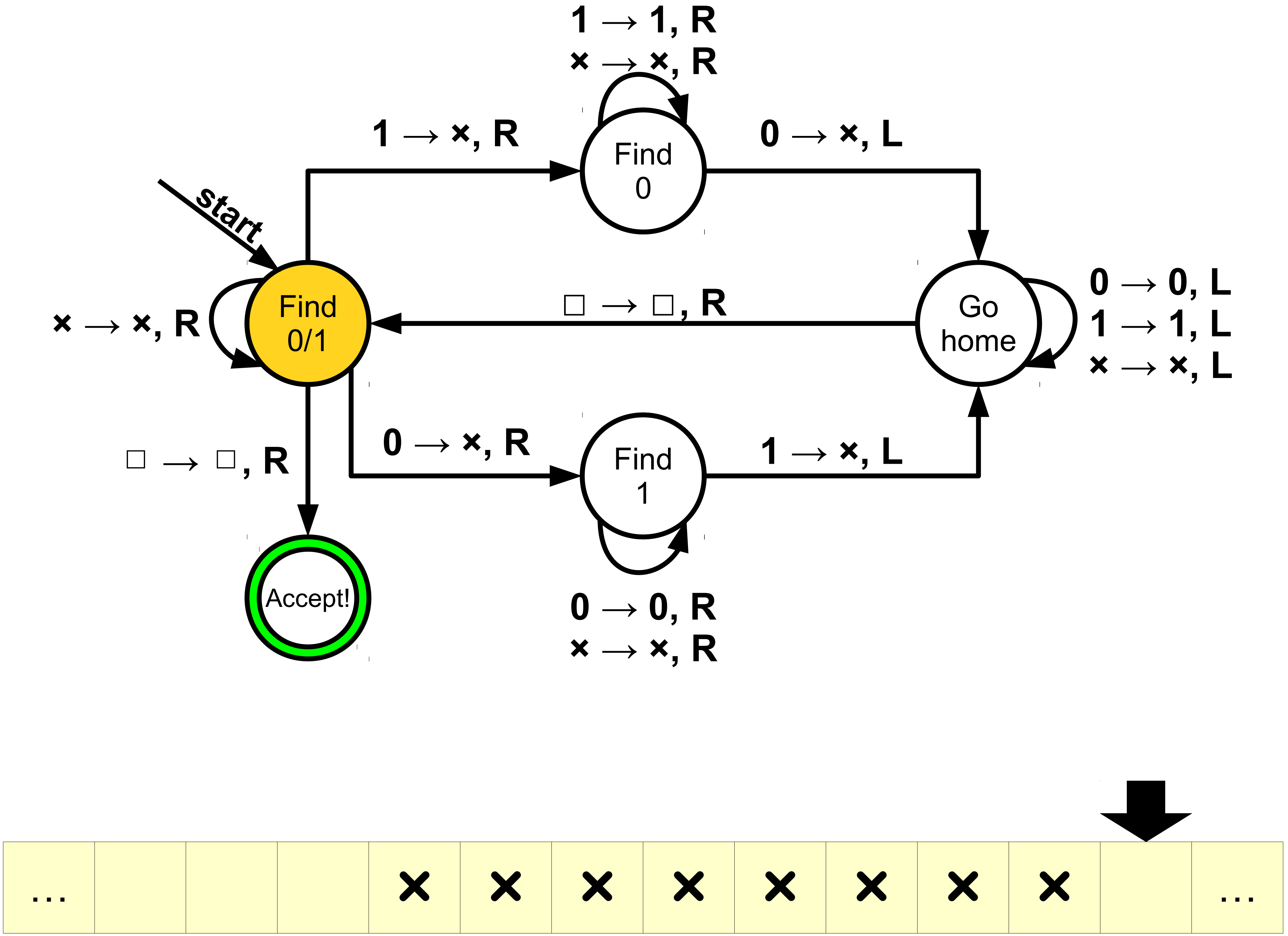

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…

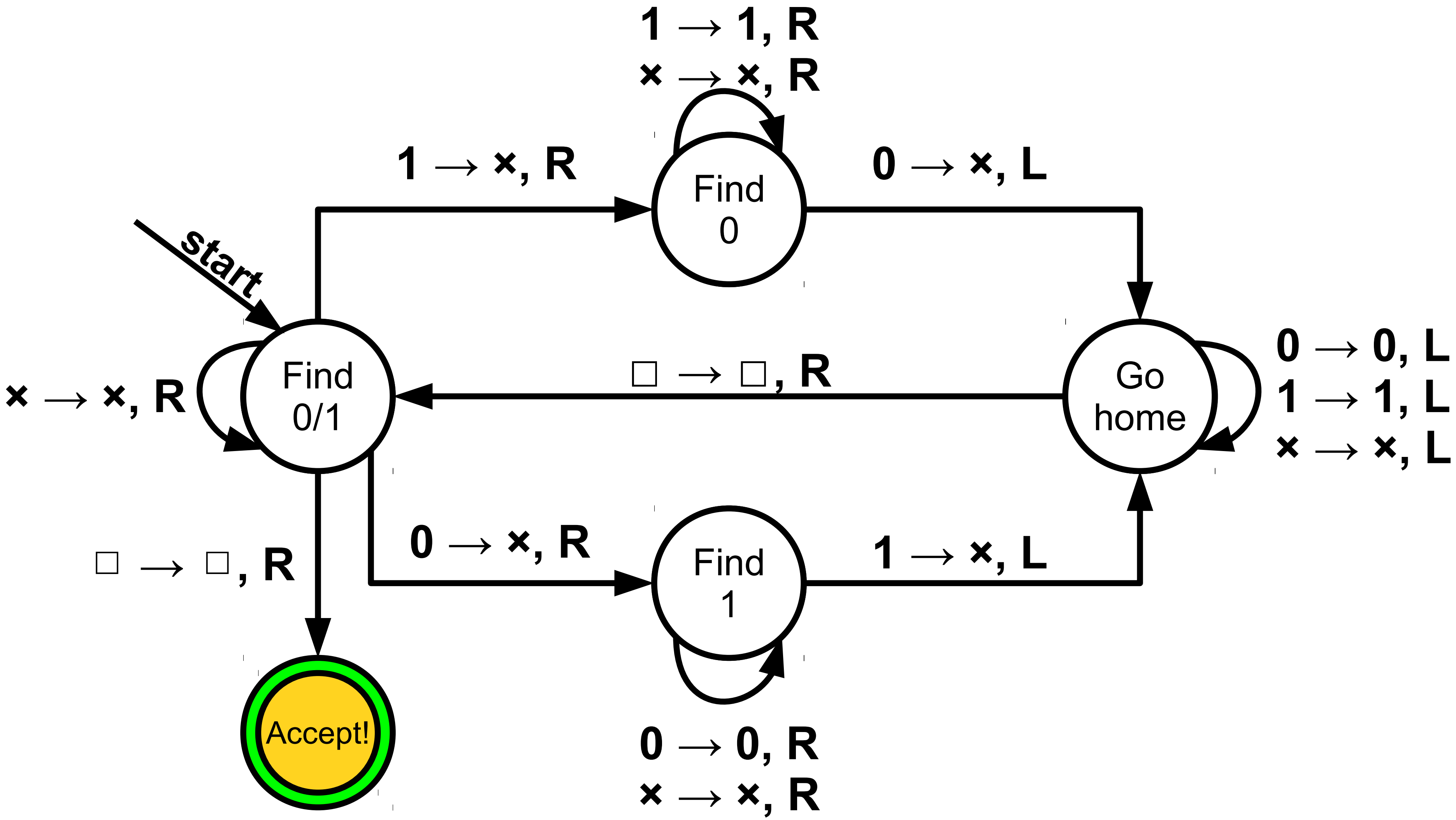

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…

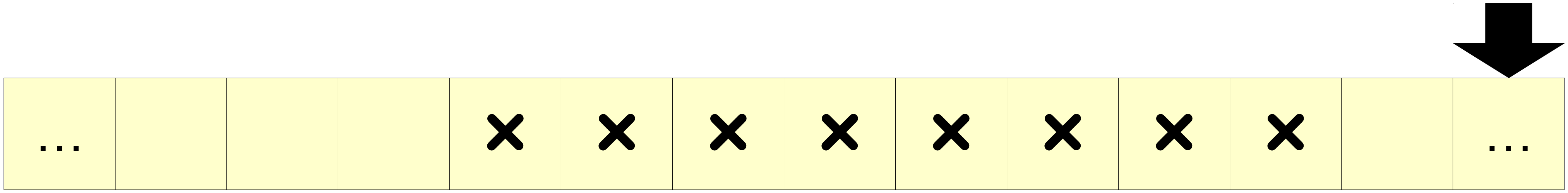

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
× × × × × × × ×
…

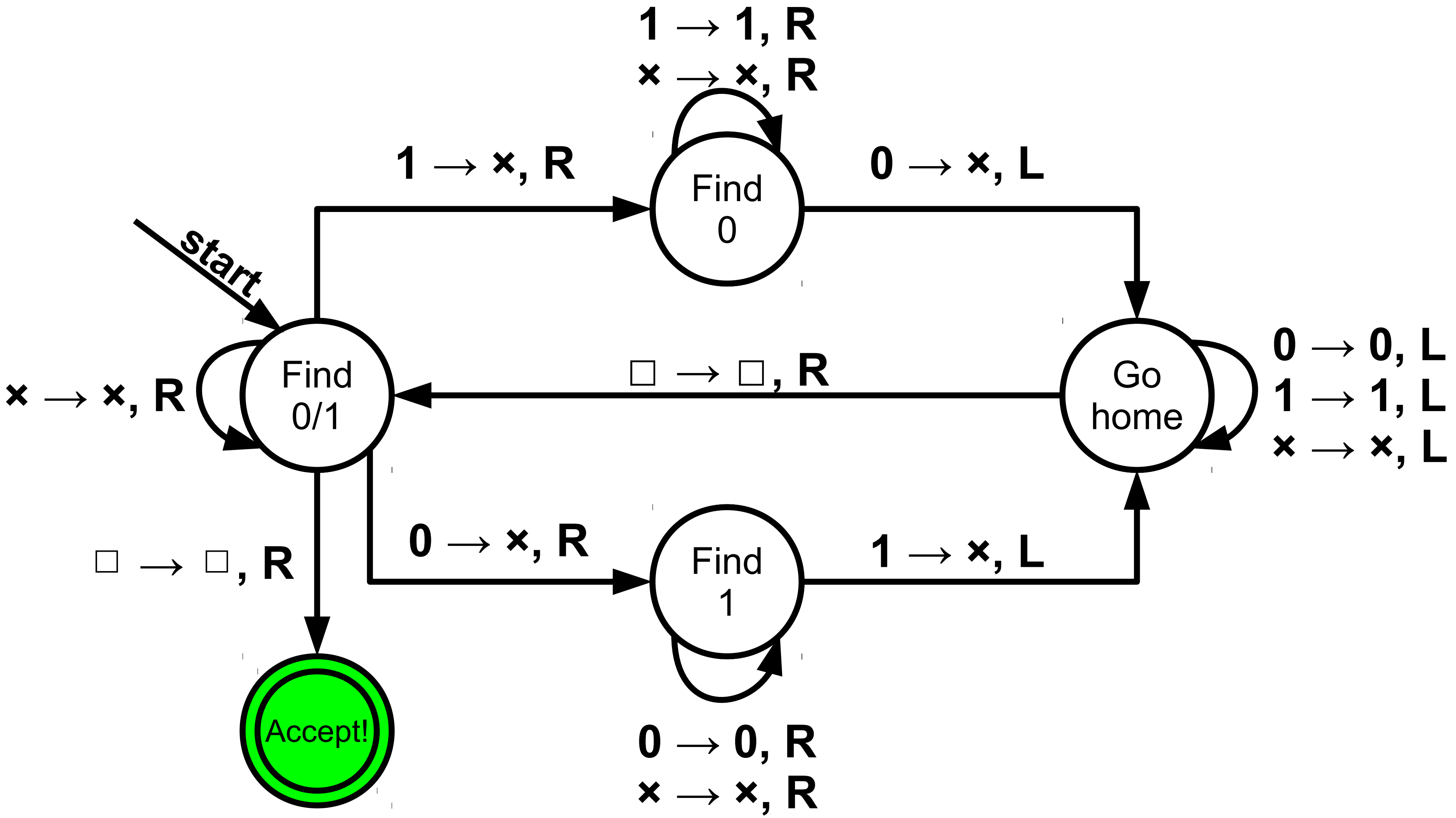

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…

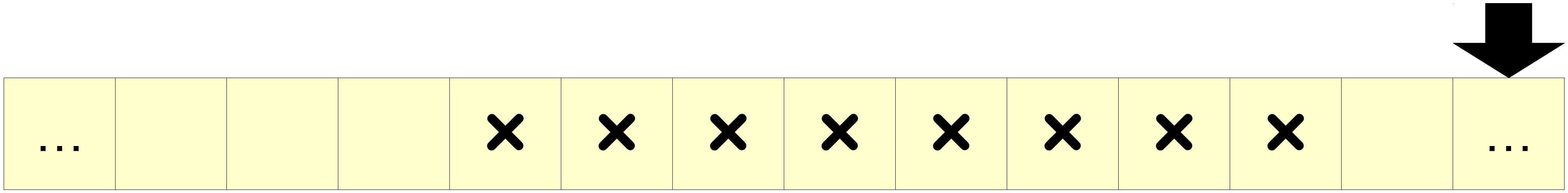

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…


start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
□ → □, R
Accept!
…
×
×
×
×
×
×
×
×
…

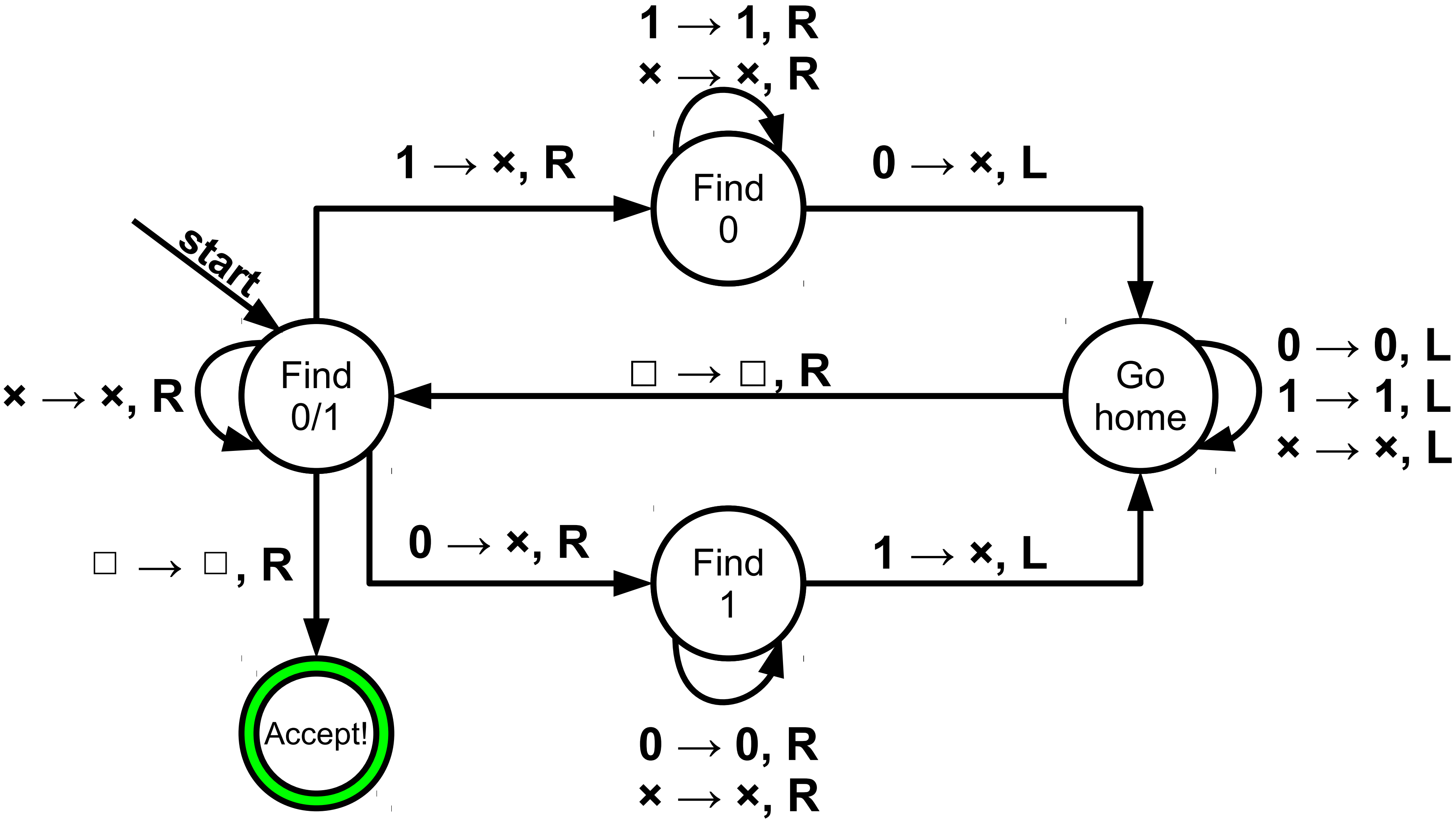

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
1 → ×, L
Find 1
□ → □, R
Accept!
0 → 0, R
× → ×, R

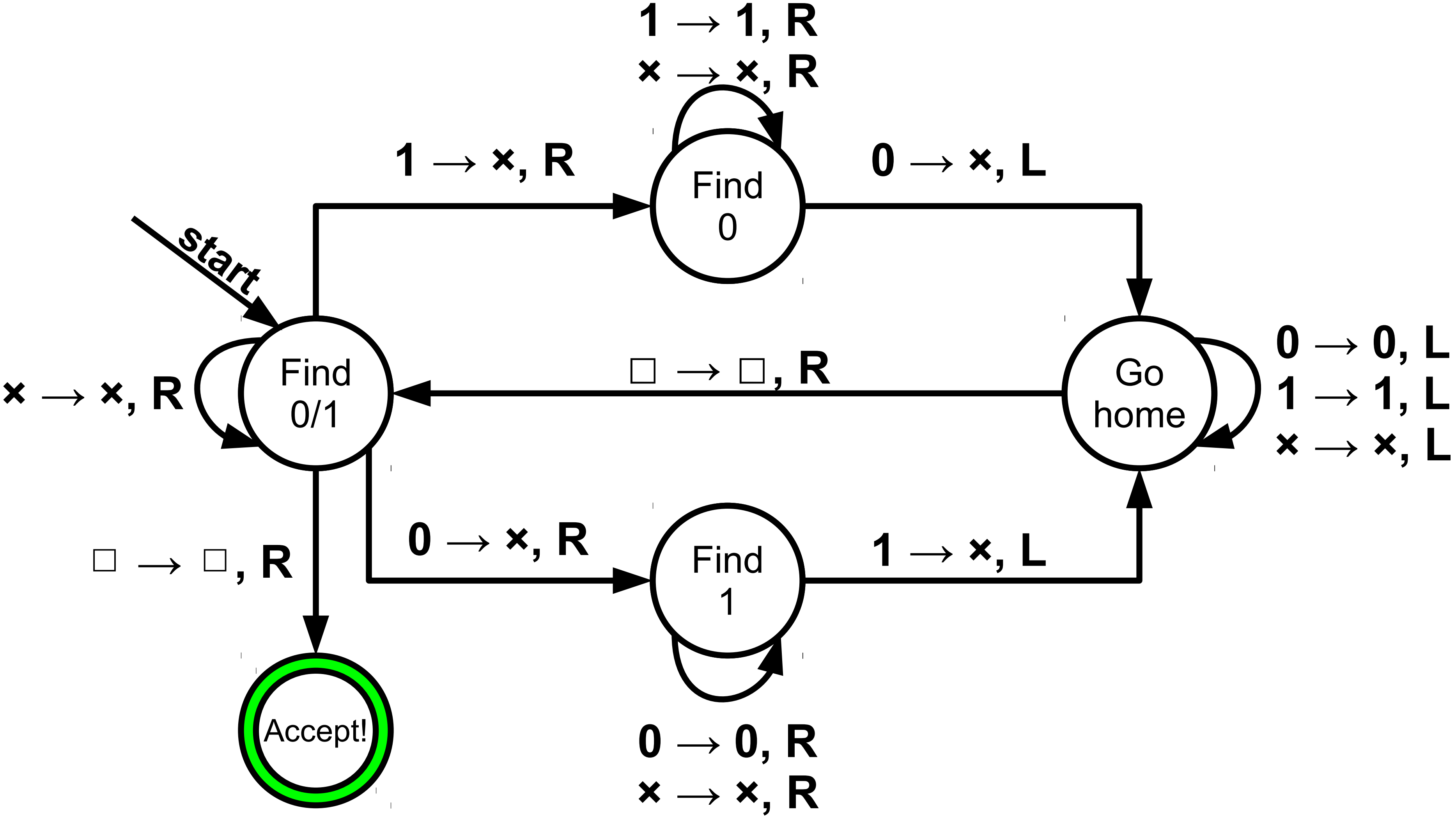

…
×
×
×
×
×
×
×
×
…


1 → 1, R
× → ×, R
1 → ×, R
Find
0
0 → ×, L
start
× → ×, R
Find
0/1
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find
1
1 → ×, L
Accept!
0 → 0, R
× → ×, R

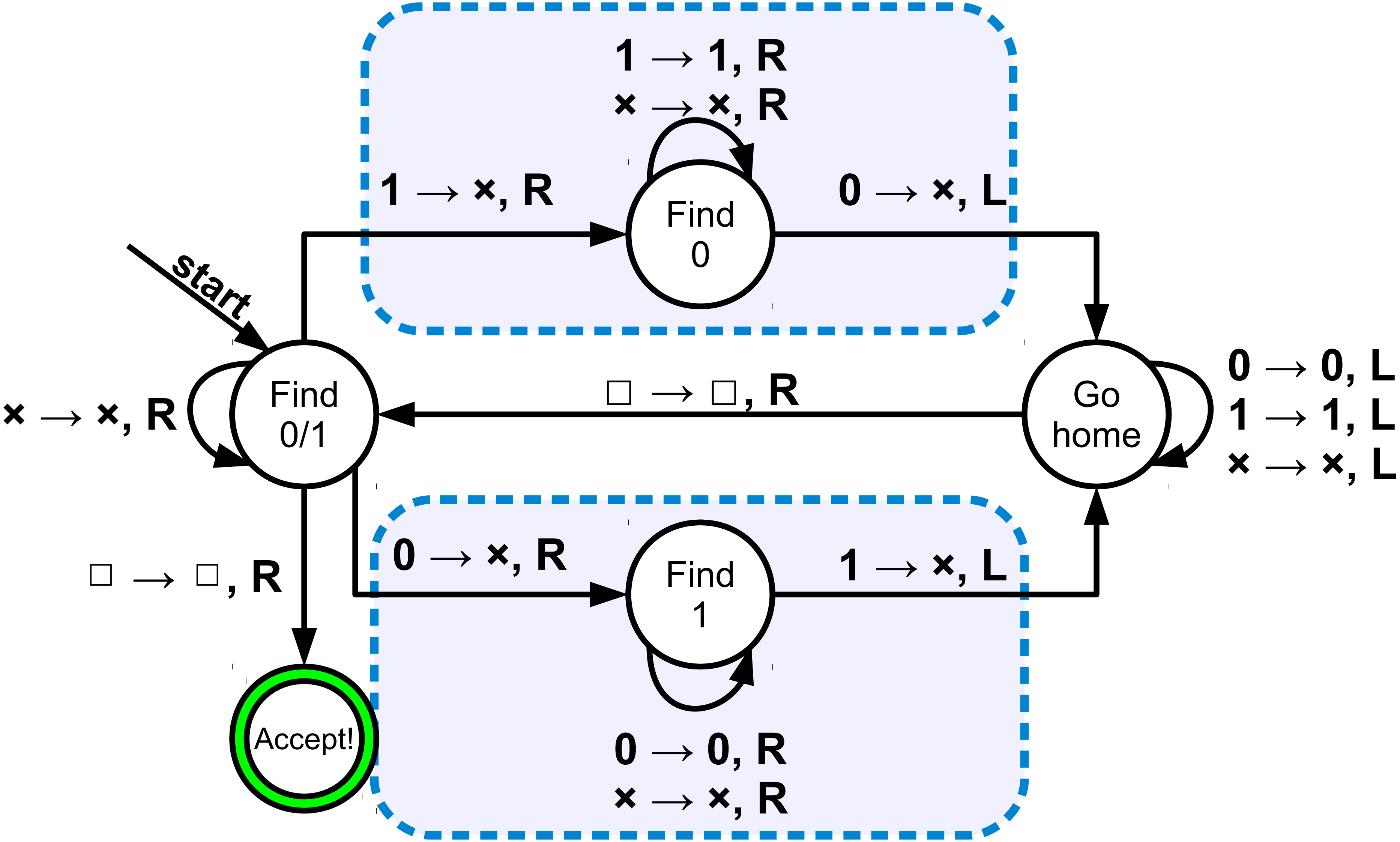


…				×	×	×	×	×	×	×	×		…


1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
1 → ×, L
Find 1
□ → □, R
Accept!
0 → 0, R
× → ×, R



Remember that all missing transitions implicitly reject.


1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
1 → ×, L
Accept!
0 → 0, R
× → ×, R

Constant Storage

- Sometimes, a TM needs to remember some additional information that can't be put on the tape.
- In this case, you can use similar techniques from DFAs and introduce extra states into the TM's finite-state control.
- The finite-state control can only remember one of finitely many things, but that might be all that you need!

Time-Out for Announcements!

Problem Set Seven

- Problem Set Seven is due this Friday at 2:30PM.
  - As always, if you have questions, feel free to stop by office hours or ask on Piazza!
- Problem Set Eight will go out on Friday.

Back to CS103!

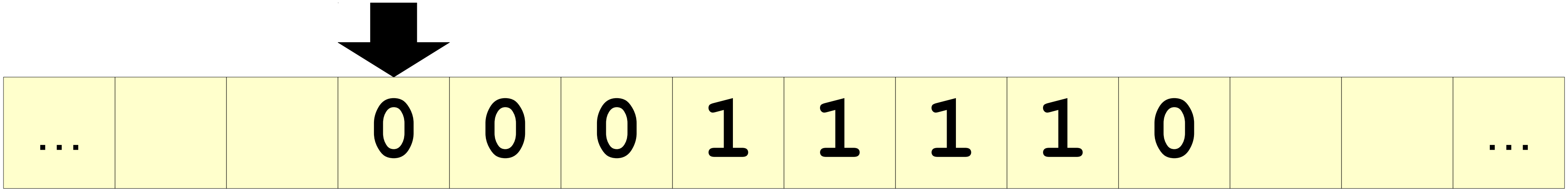
Another TM Design

- We just designed a TM for this language over $\Sigma = \{$**0**, **1**$\}$:

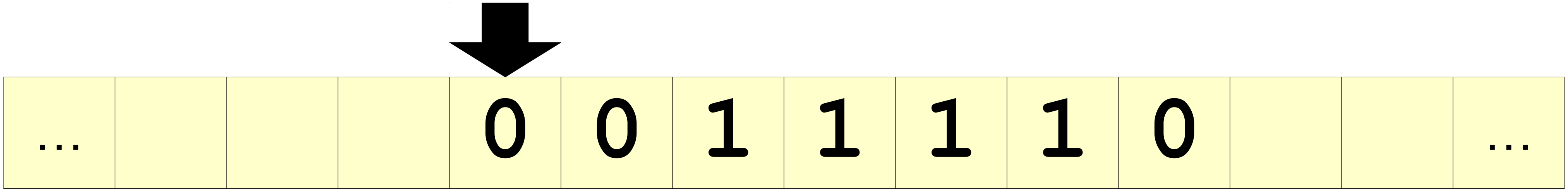
$$L = \{\ w \in \Sigma^* \mid w \text{ has the same number of } \mathtt{0}\text{s and } \mathtt{1}\text{s}\ \}$$

- Let's do a quick review of how it worked.

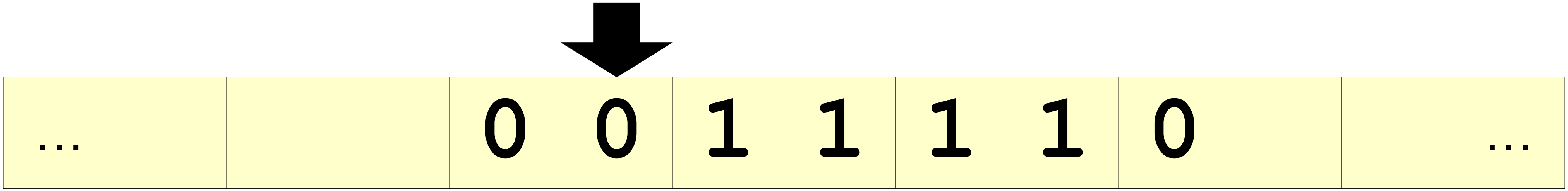
A Leap of Faith



A Leap of Faith

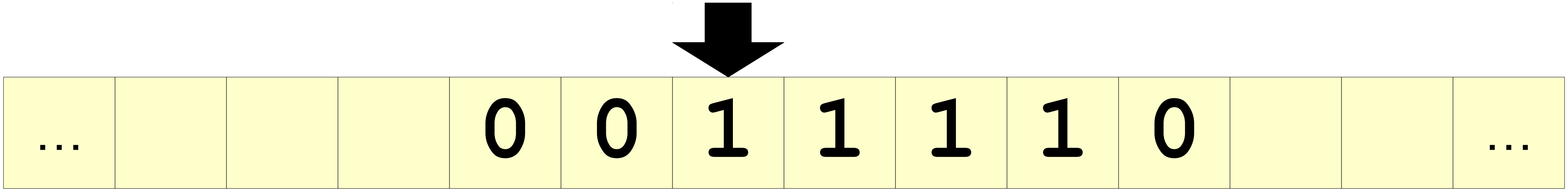


A Leap of Faith



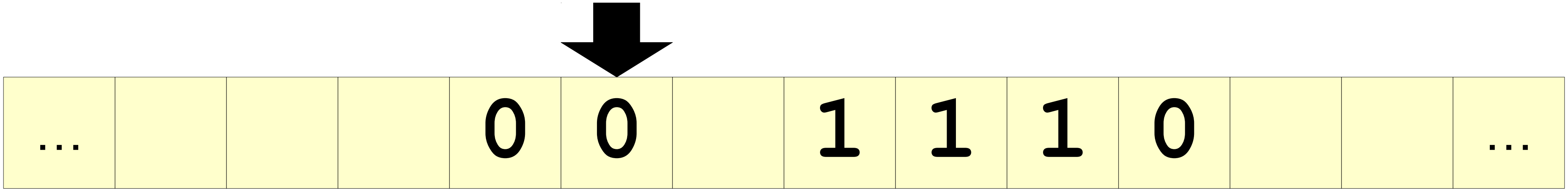
…				0	0	1	1	1	1	0			…

A Leap of Faith



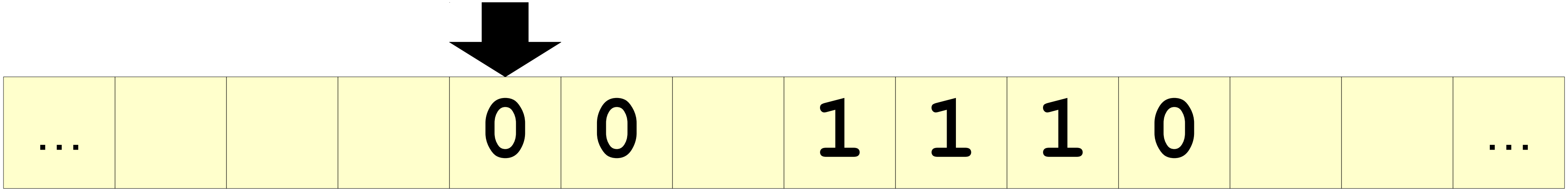
…				0	0	1	1	1	1	0			…

A Leap of Faith

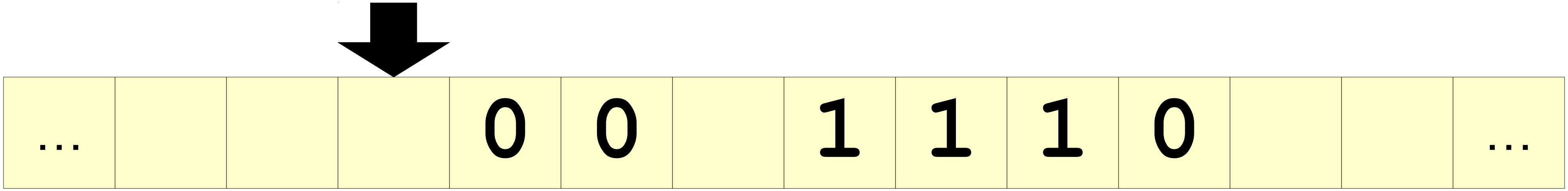


…				0	0		1	1	1	0			…

A Leap of Faith

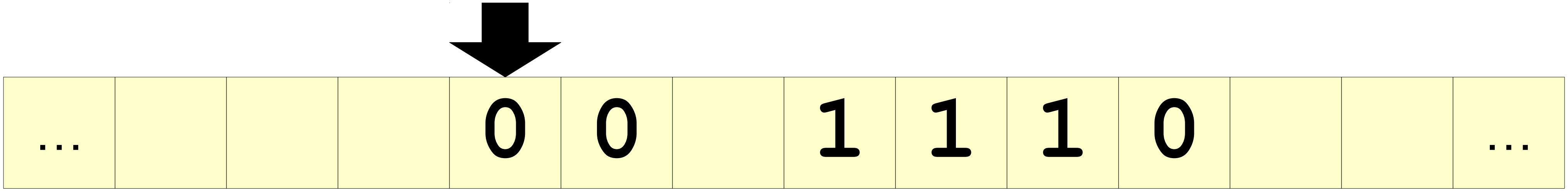


A Leap of Faith

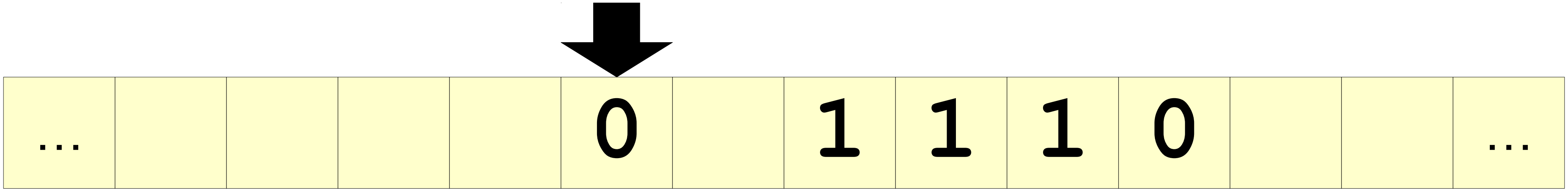


…				0	0		1	1	1	0			…

A Leap of Faith

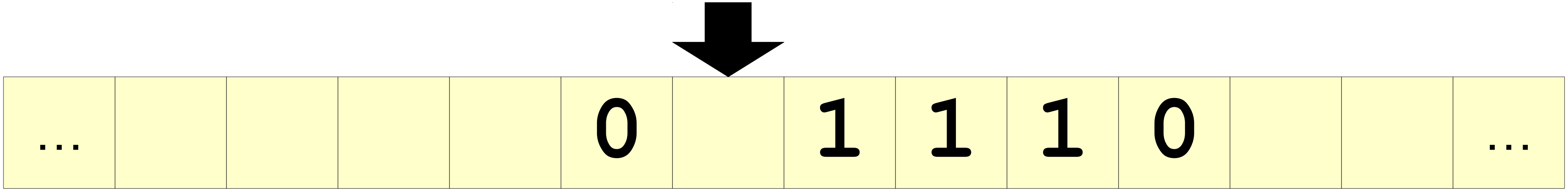


A Leap of Faith



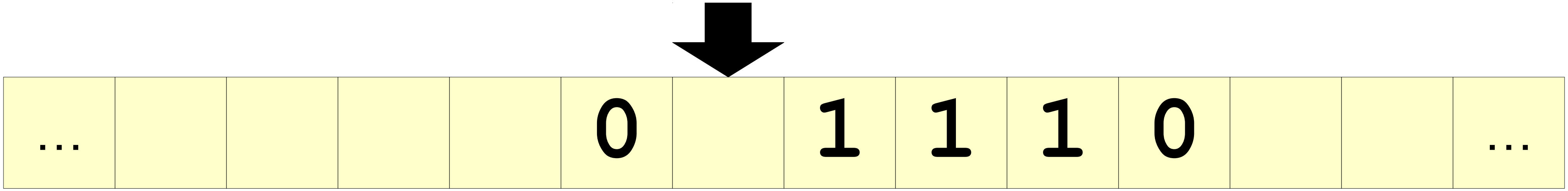
…					0		1	1	1	0			…

A Leap of Faith



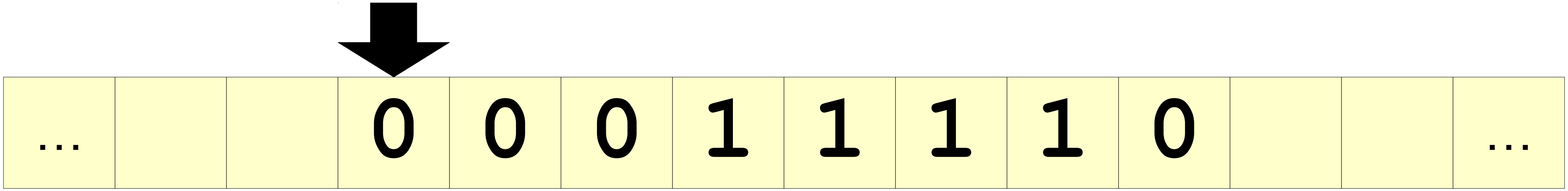
…					0		1	1	1	0			…

A Leap of Faith

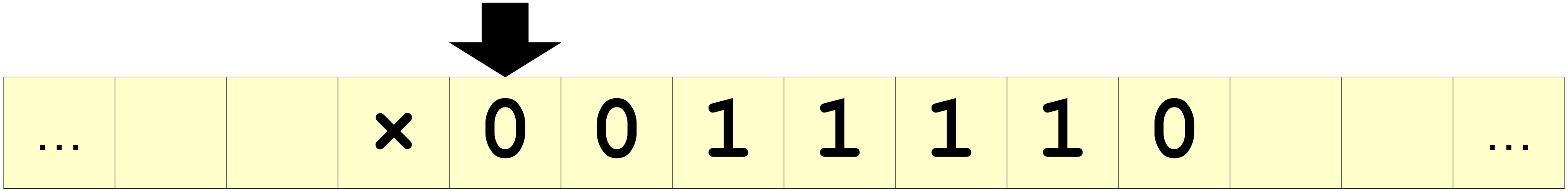


How do we know that this blank isn't one of the infinitely many blanks after our input string?

The Solution

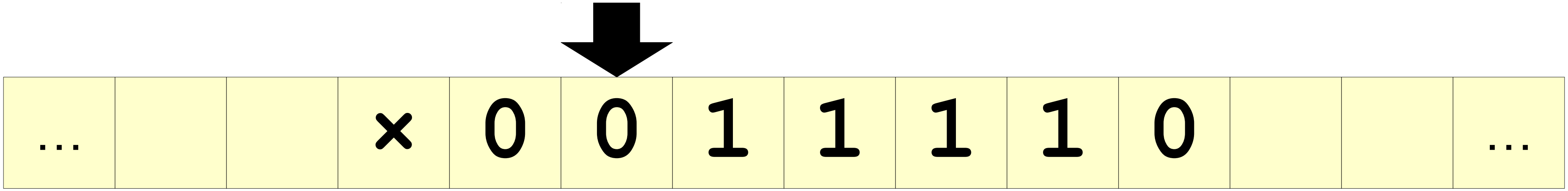


The Solution



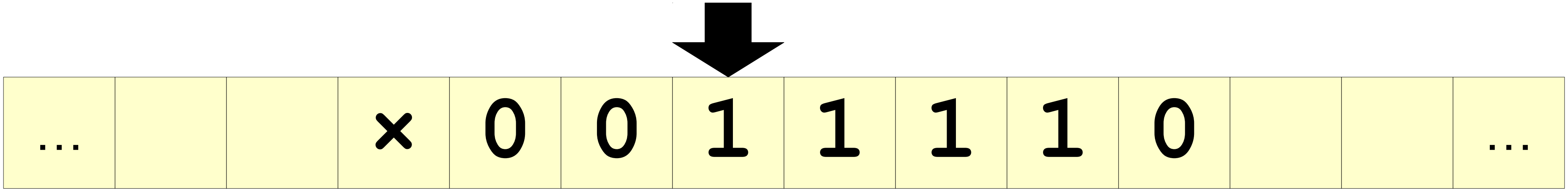
…			×	0	0	1	1	1	1	0			…

The Solution

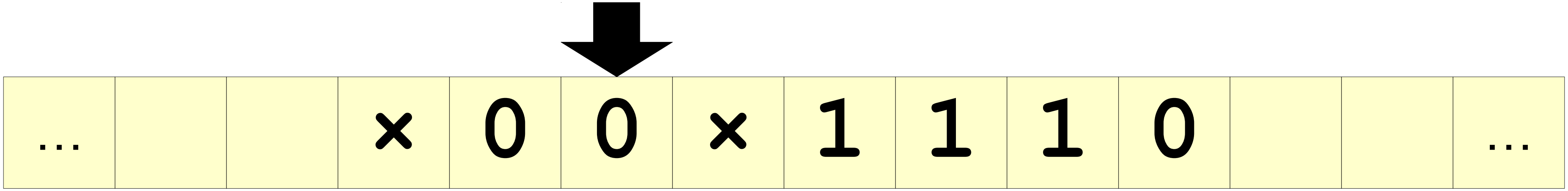


…			×	0	0	1	1	1	1	0			…

The Solution

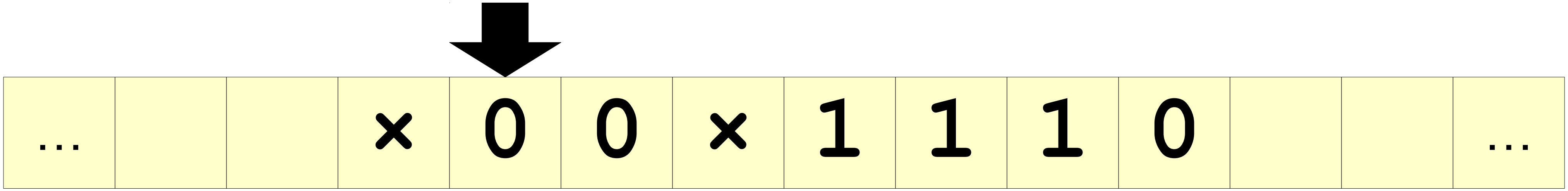


The Solution

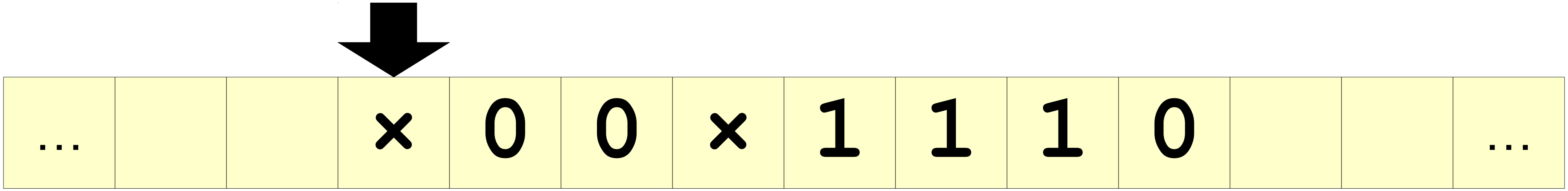


…			×	0	0	×	1	1	1	0			…

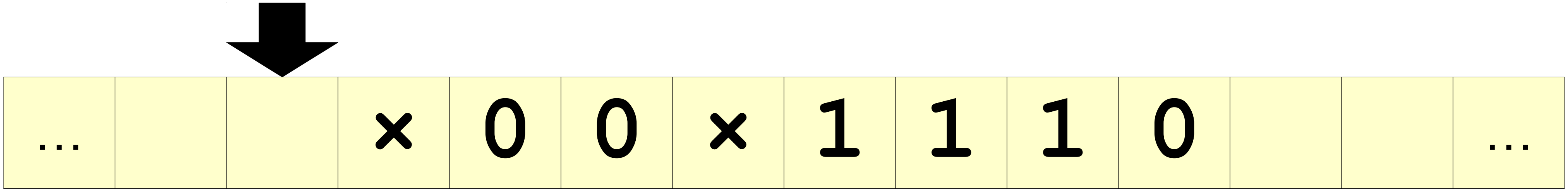
The Solution



The Solution

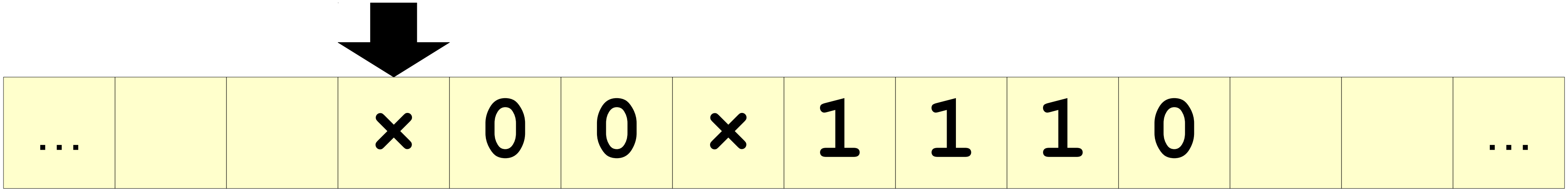


The Solution

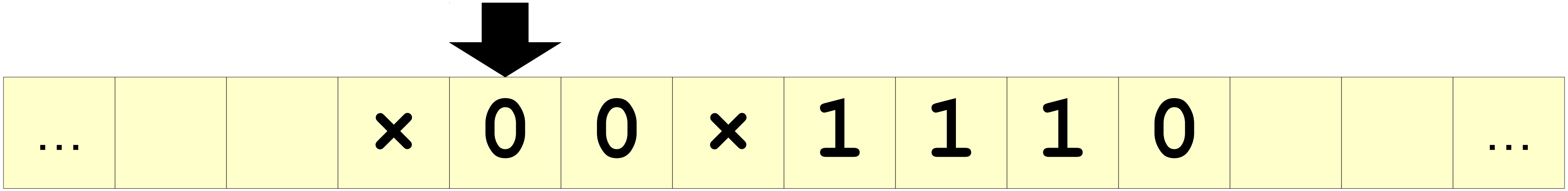


…			×	0	0	×	1	1	1	0			…

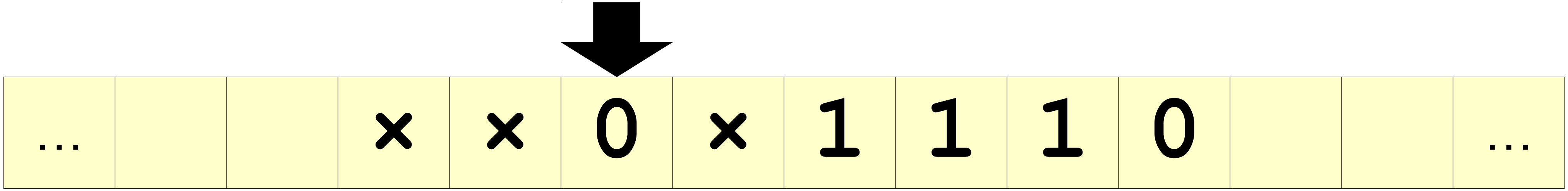
The Solution



The Solution

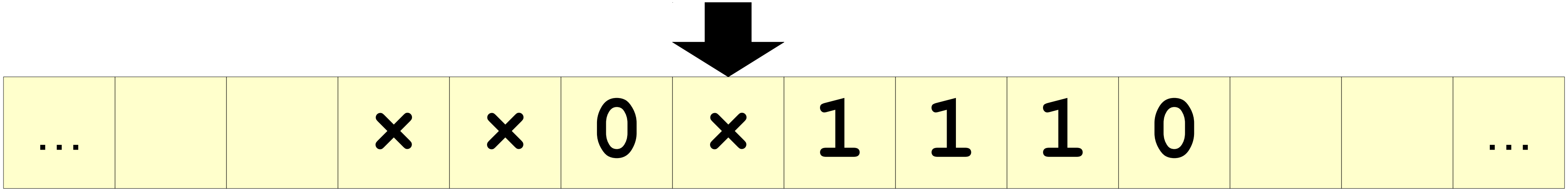


The Solution

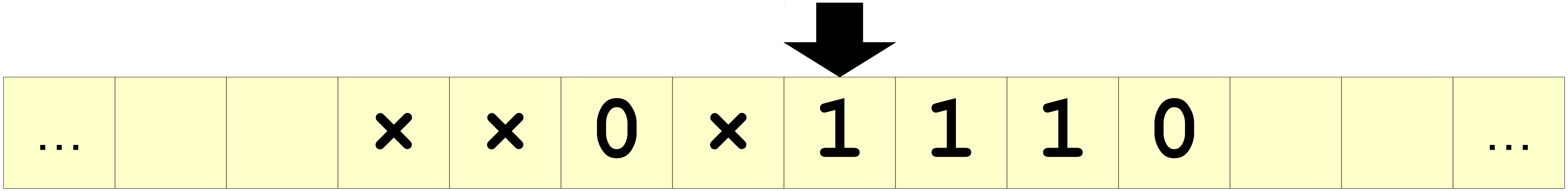


…			×	×	0	×	1	1	1	0			…

The Solution

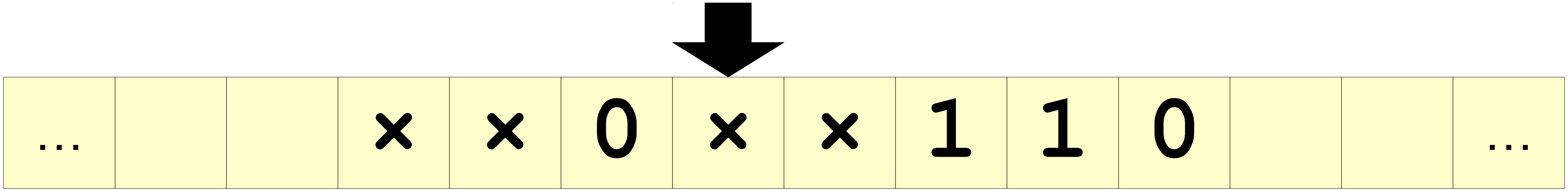


The Solution

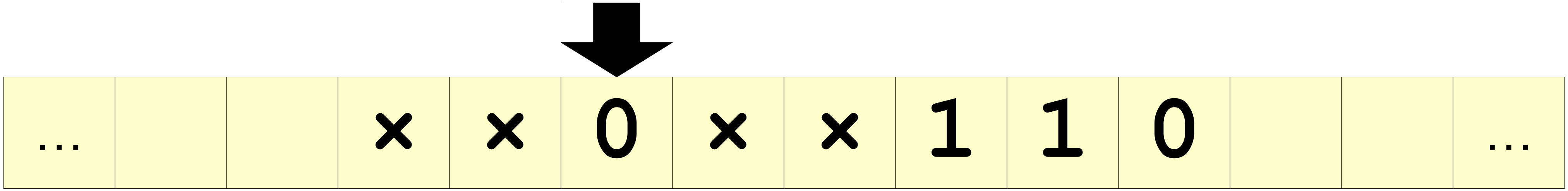


…			×	×	0	×	1	1	1	0			…

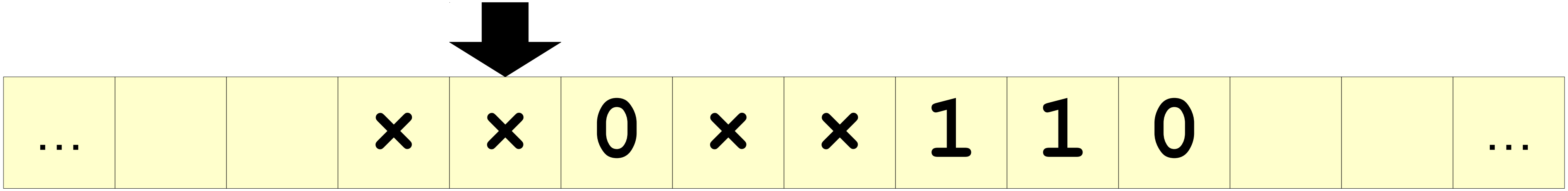
The Solution



The Solution

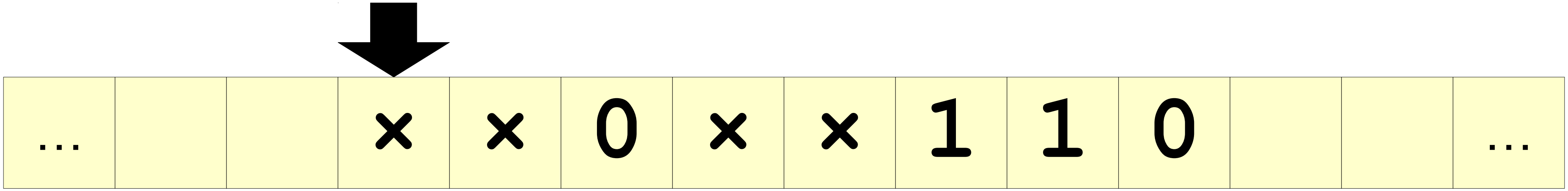


The Solution

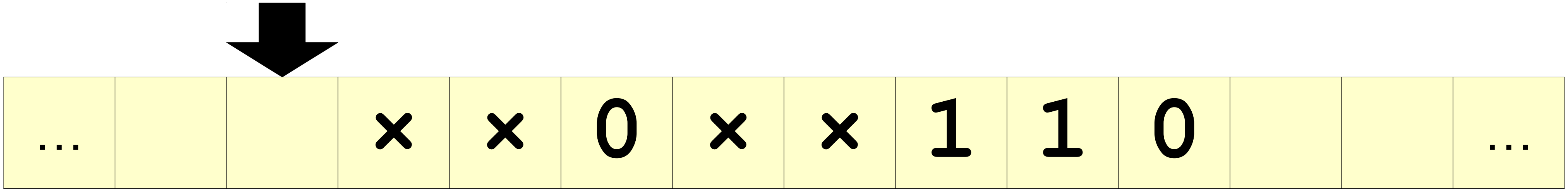


…			×	×	0	×	×	1	1	0			…

The Solution

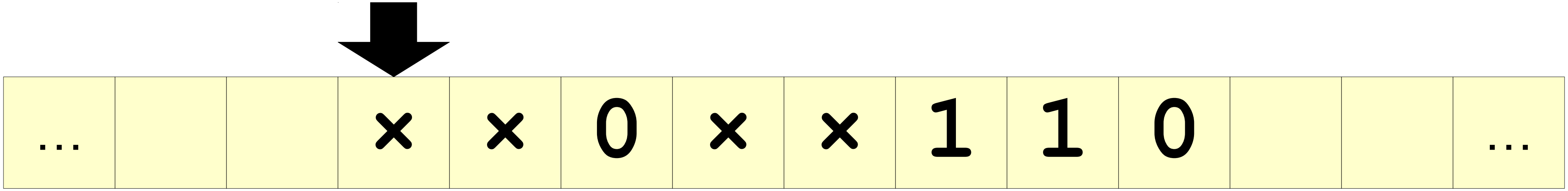


The Solution



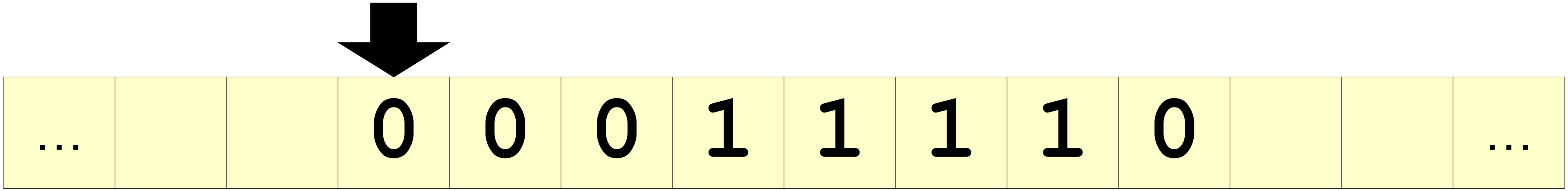
…			×	×	0	×	×	1	1	0			…

The Solution



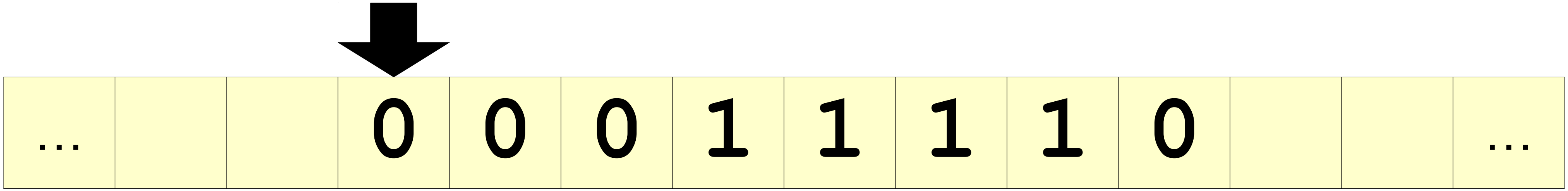
A Different Idea

A Different Strategy



…			0	0	0	1	1	1	1	0			…

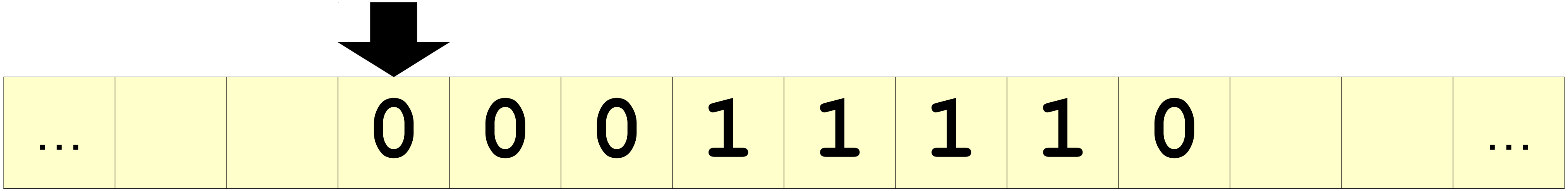
A Different Strategy



…			0	0	0	1	1	1	1	0			…

Could we sort the characters of this string?

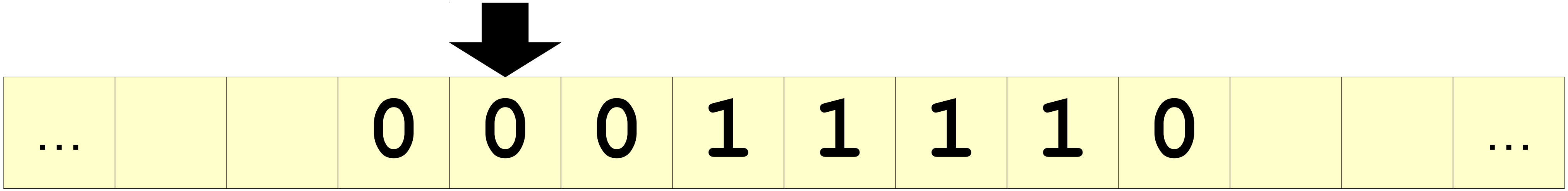
A Different Strategy



…			0	0	0	1	1	1	1	0			…

Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

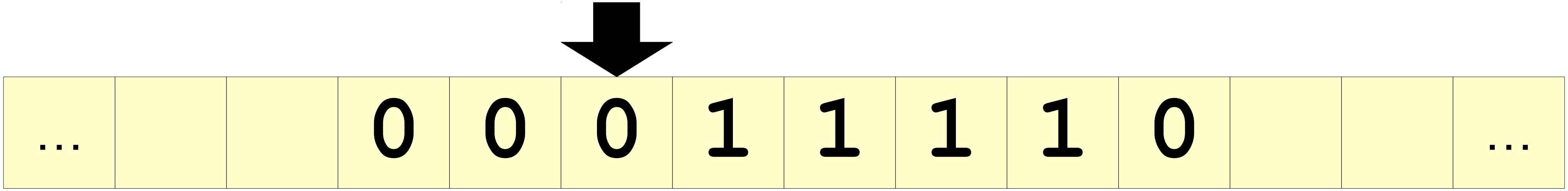
A Different Strategy



…			0	0	0	1	1	1	1	0			…

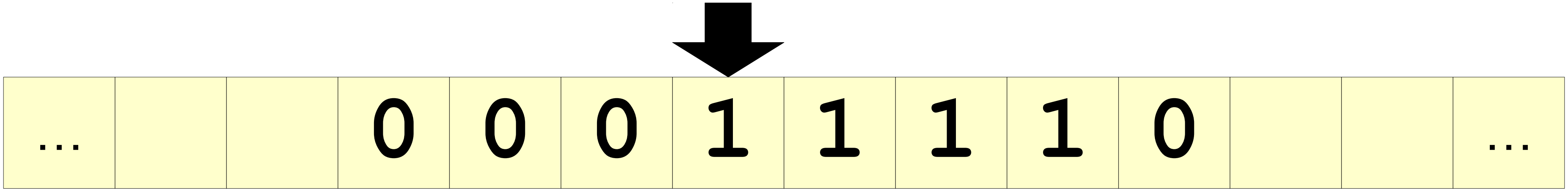
Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

A Different Strategy



Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

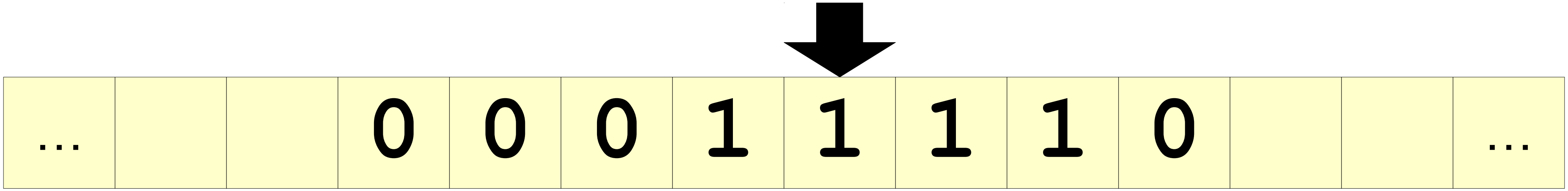
A Different Strategy



…			0	0	0	1	1	1	1	0			…

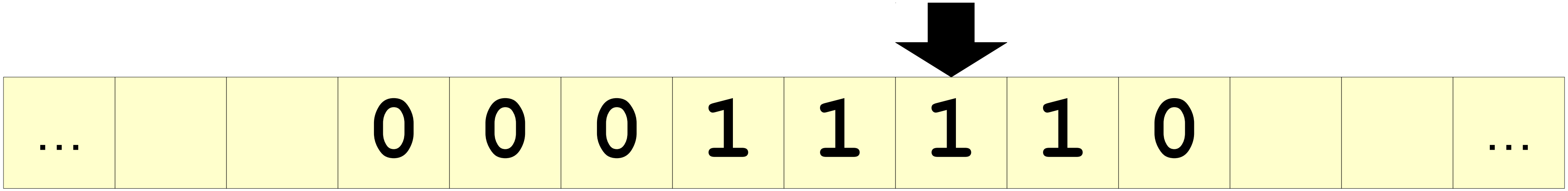
Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

A Different Strategy



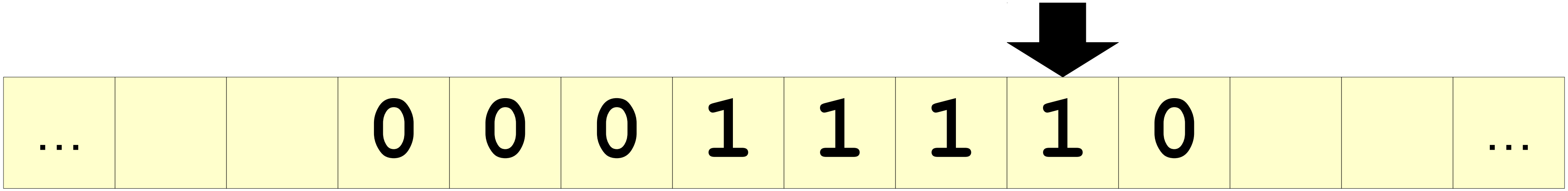
Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

A Different Strategy



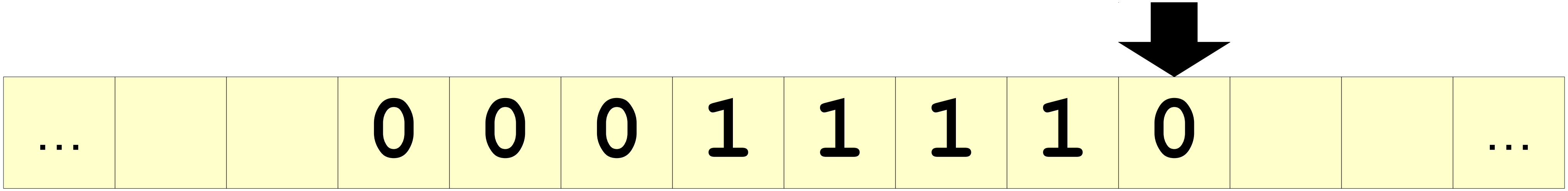
Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

A Different Strategy



Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

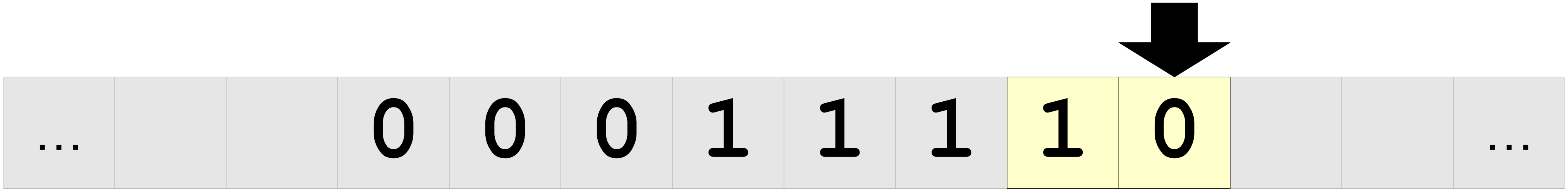
A Different Strategy



…			0	0	0	1	1	1	1	0			…

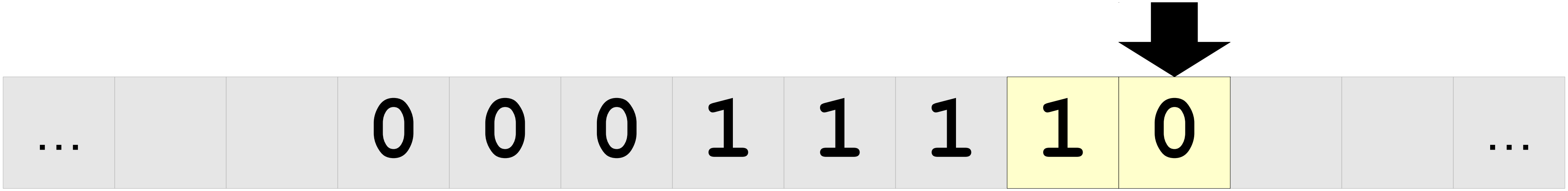
Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

A Different Strategy



Observation 1: A string of 0s and 1s is sorted if it matches the regex 0*1*.

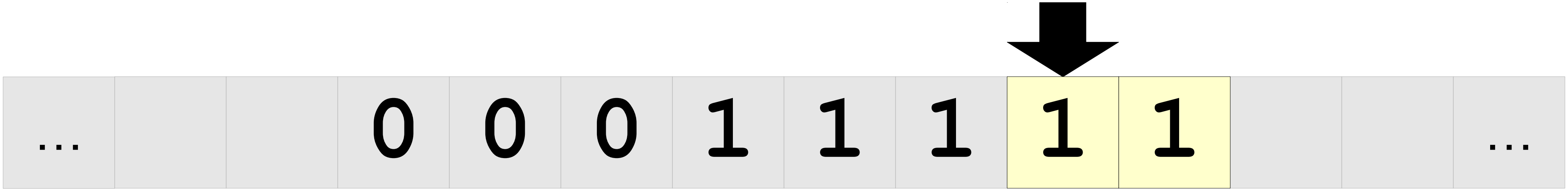
A Different Strategy



…			0	0	0	1	1	1	1	0			…

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.

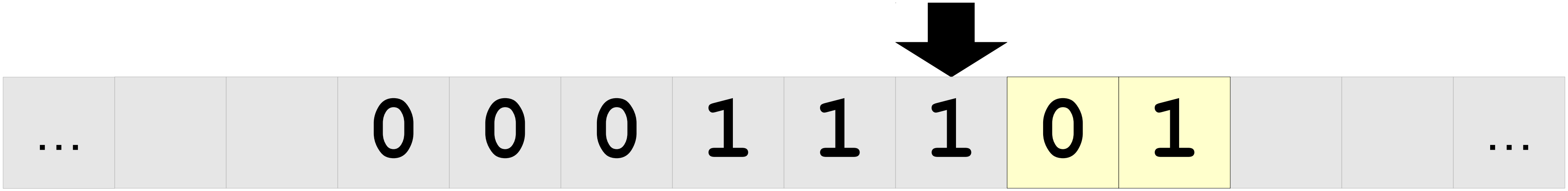
A Different Strategy



…			0	0	0	1	1	1	1	1			…

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.

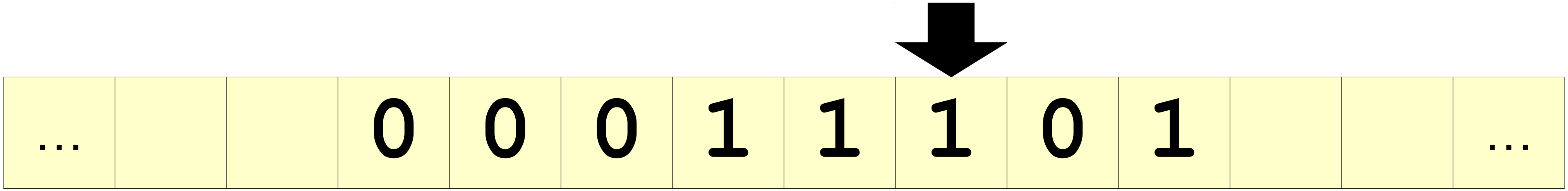
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.

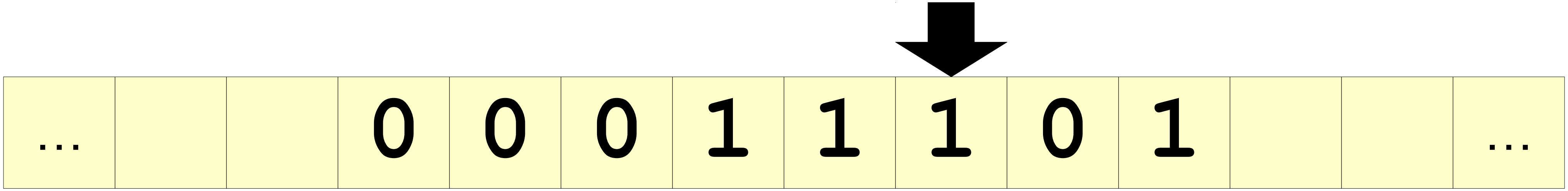
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Observation 2: A string of 0s and 1s is not sorted if it contains 10 as a substring.

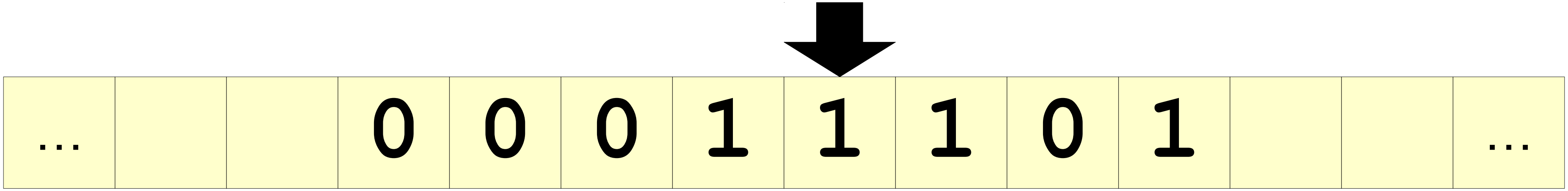
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

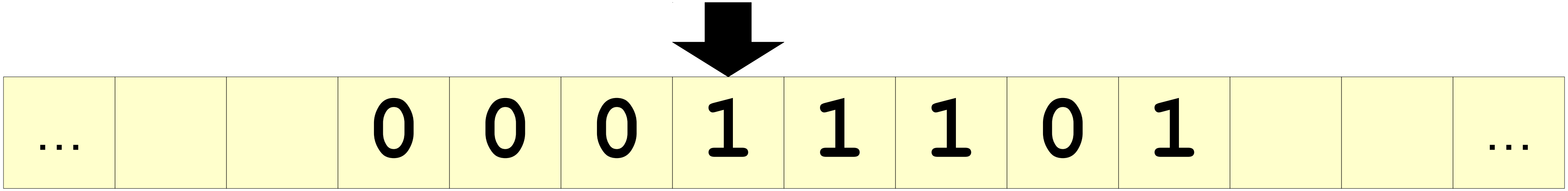
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

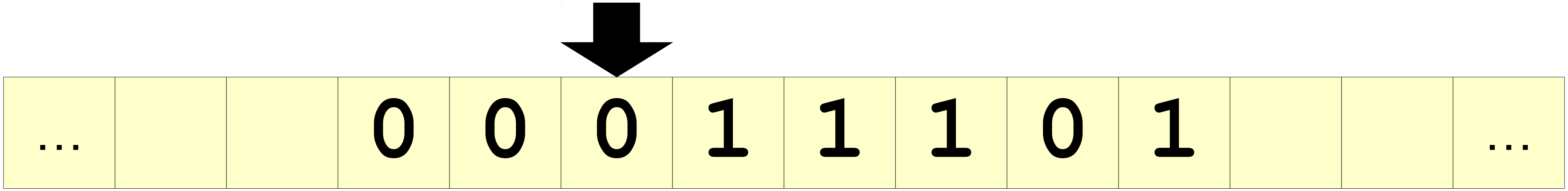
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

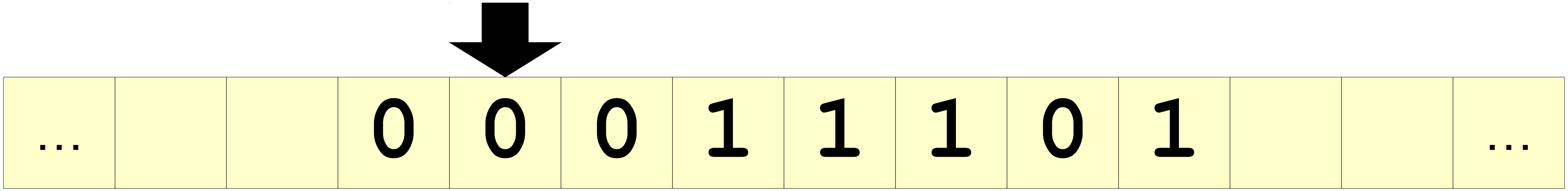
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

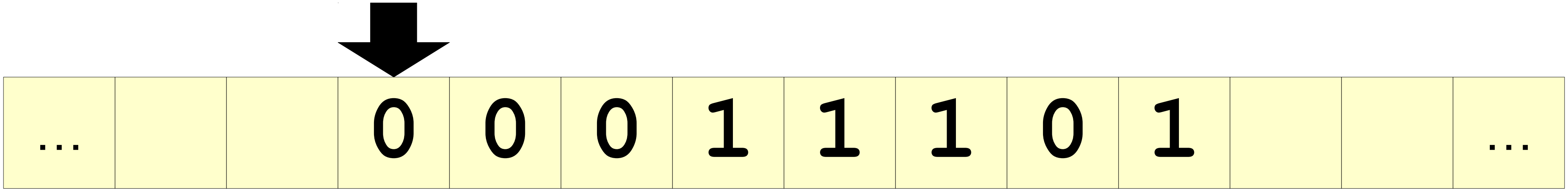
A Different Strategy



…			0	0	0	1	1	1	0	1			…

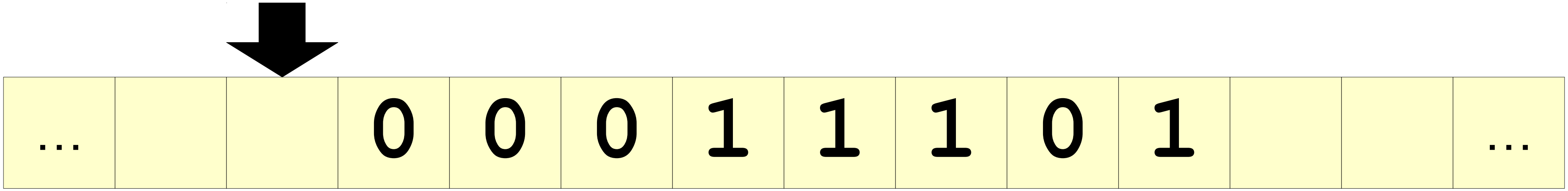
Idea: Repeatedly find a copy of 10 and replace it with 01.

A Different Strategy



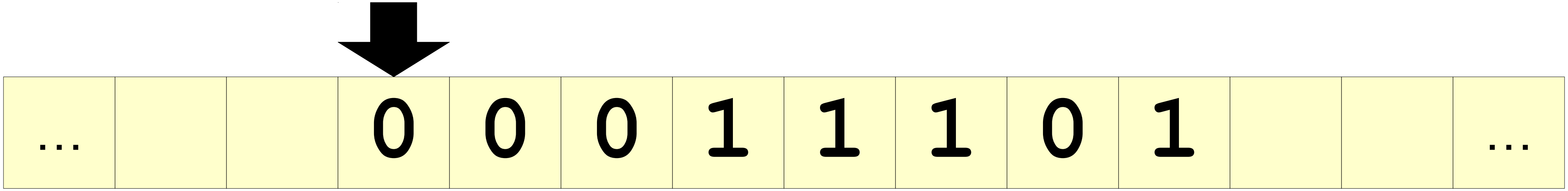
Idea: Repeatedly find a copy of 10 and replace it with 01.

A Different Strategy



Idea: Repeatedly find a copy of 10 and replace it with 01.

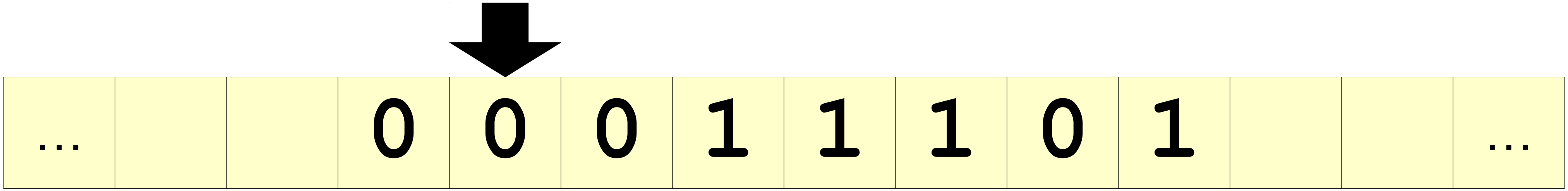
A Different Strategy



… 0 0 0 1 1 1 0 1 …

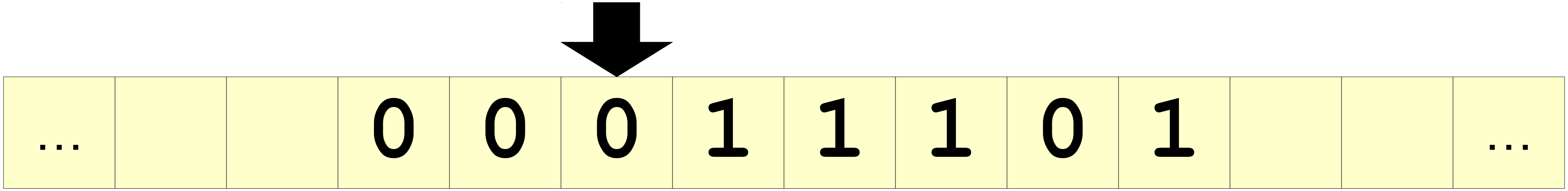
Idea: Repeatedly find a copy of 10 and replace it with 01.

A Different Strategy



Idea: Repeatedly find a copy of 10 and replace it with 01.

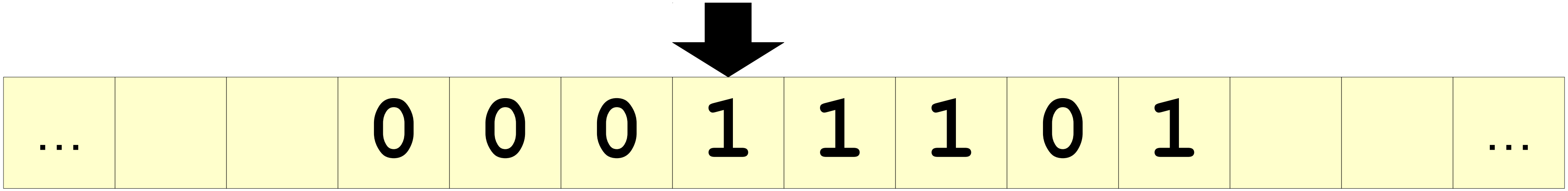
A Different Strategy



…			0	0	0	1	1	1	0	1			…

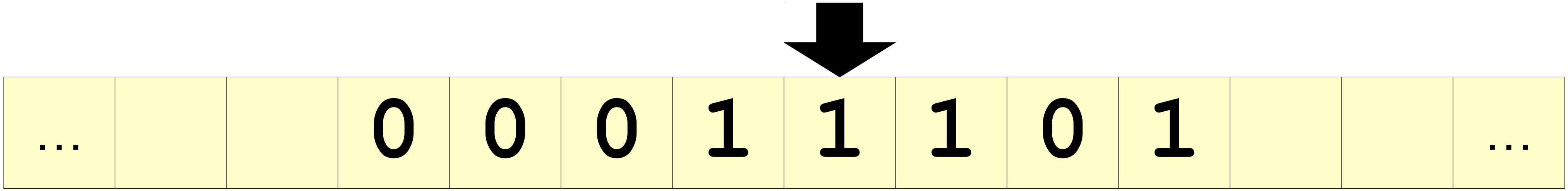
Idea: Repeatedly find a copy of 10 and replace it with 01.

A Different Strategy



Idea: Repeatedly find a copy of 10 and replace it with 01.

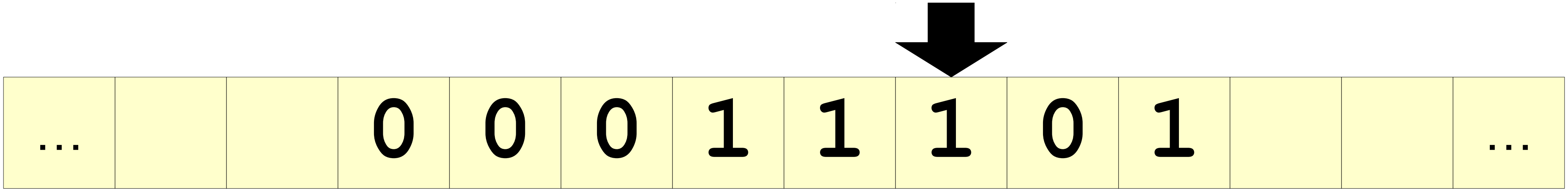
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

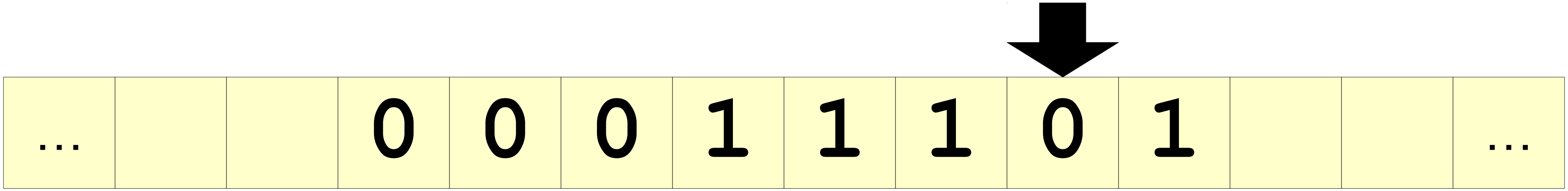
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

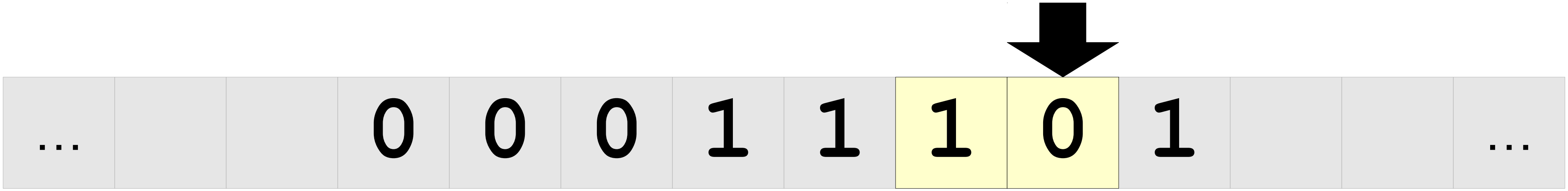
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

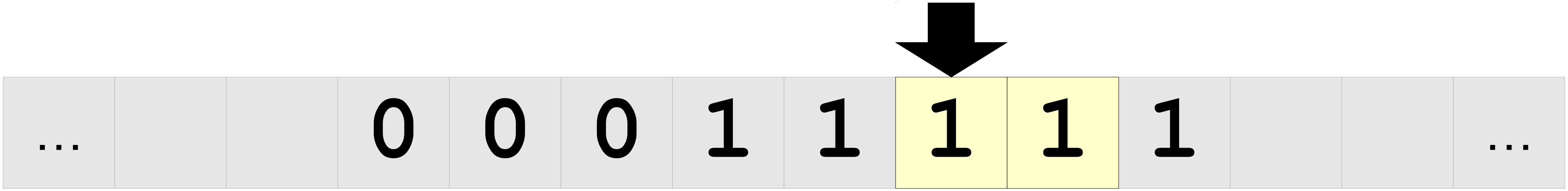
A Different Strategy



…			0	0	0	1	1	1	0	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

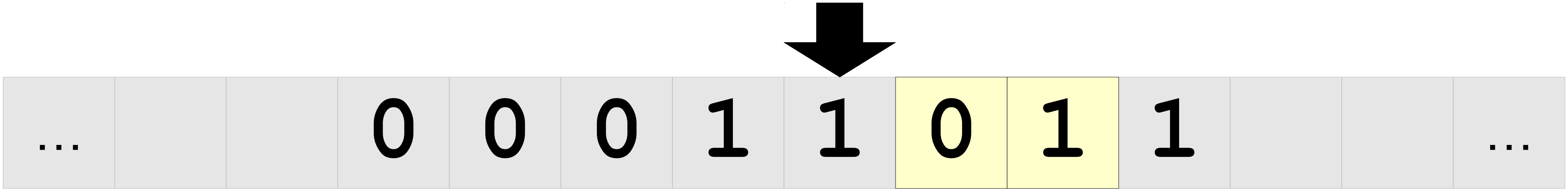
A Different Strategy



…			0	0	0	1	1	1	1	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

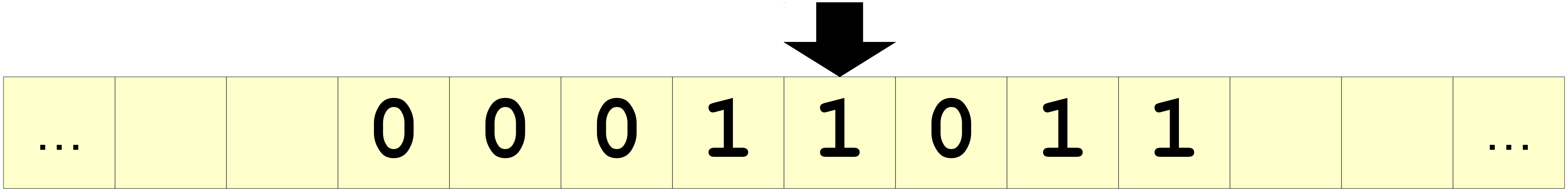
A Different Strategy



…			0	0	0	1	1	0	1	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

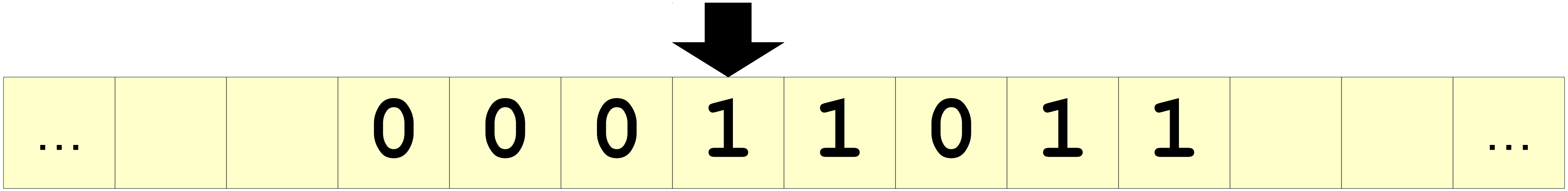
A Different Strategy



…			0	0	0	1	1	0	1	1			…

Idea: Repeatedly find a copy of 10 and replace it with 01.

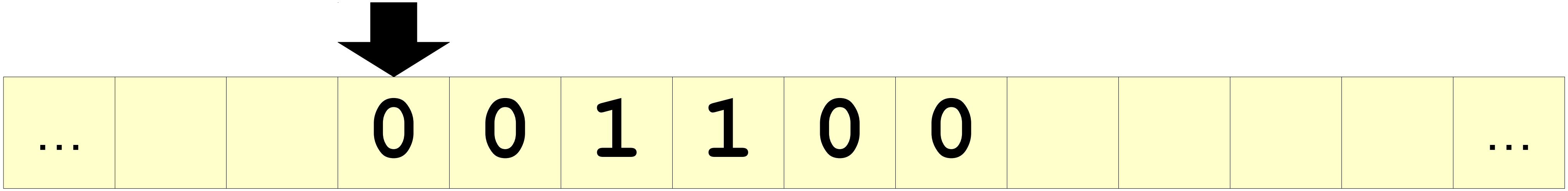
A Different Strategy

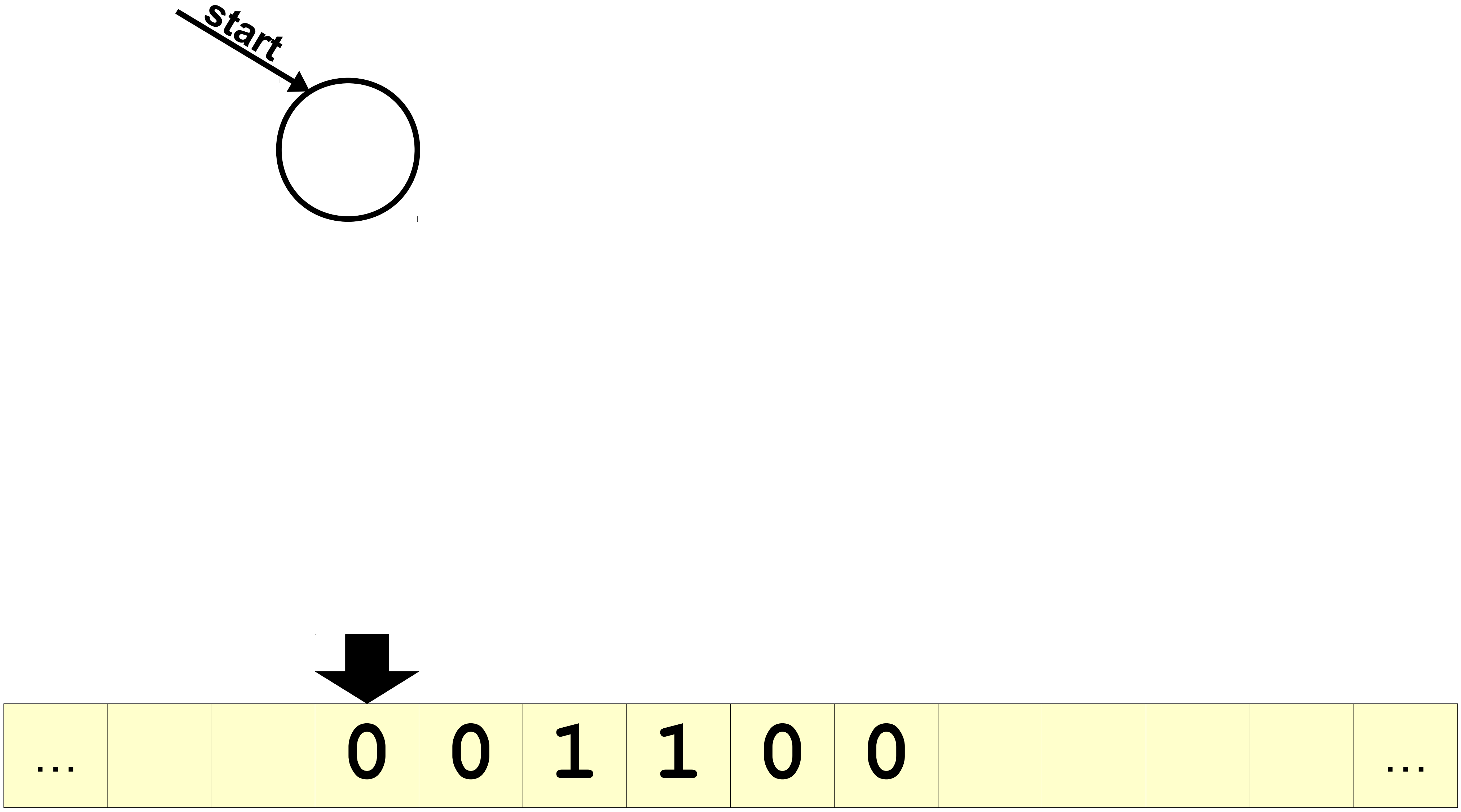


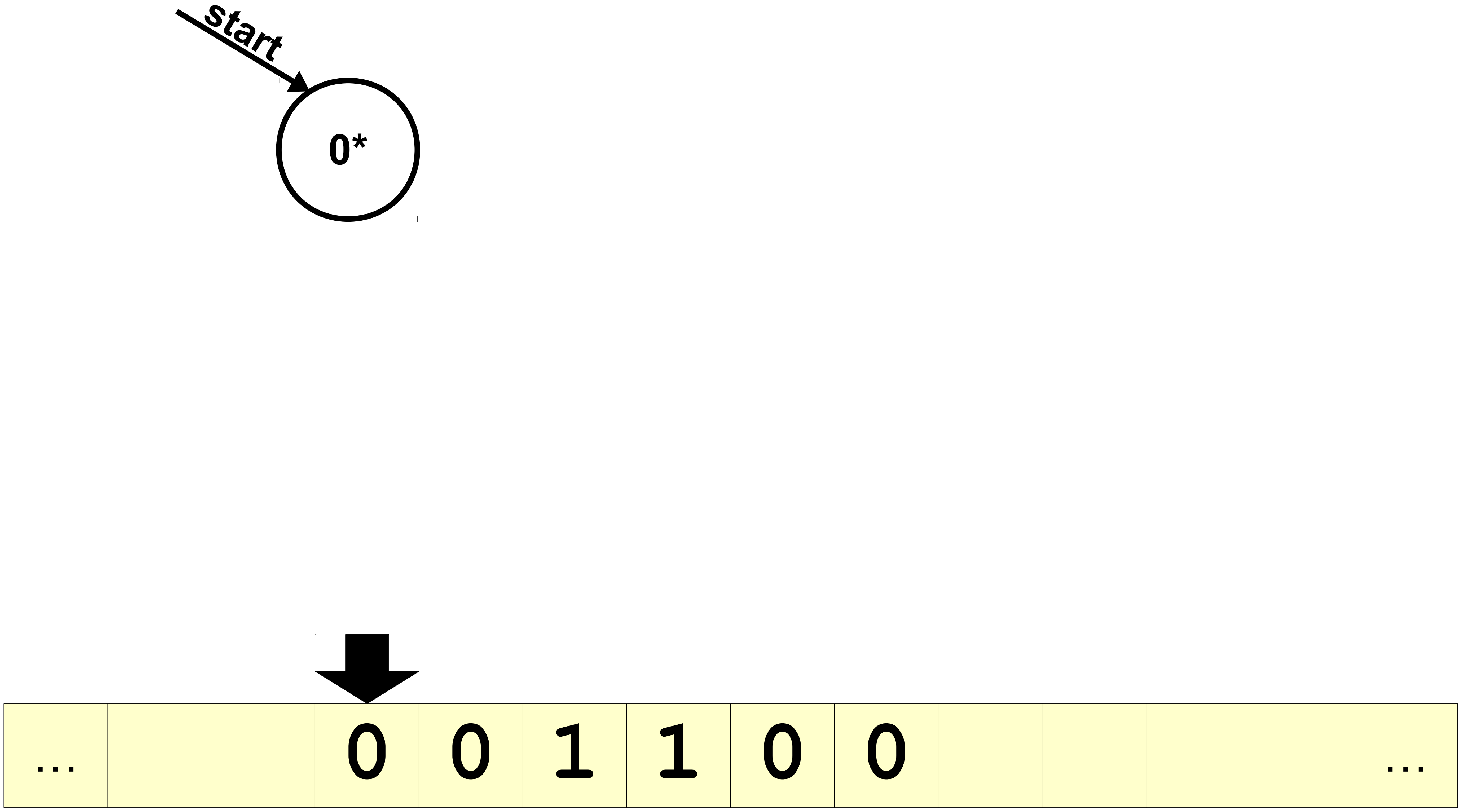
…			0	0	0	1	1	0	1	1			…

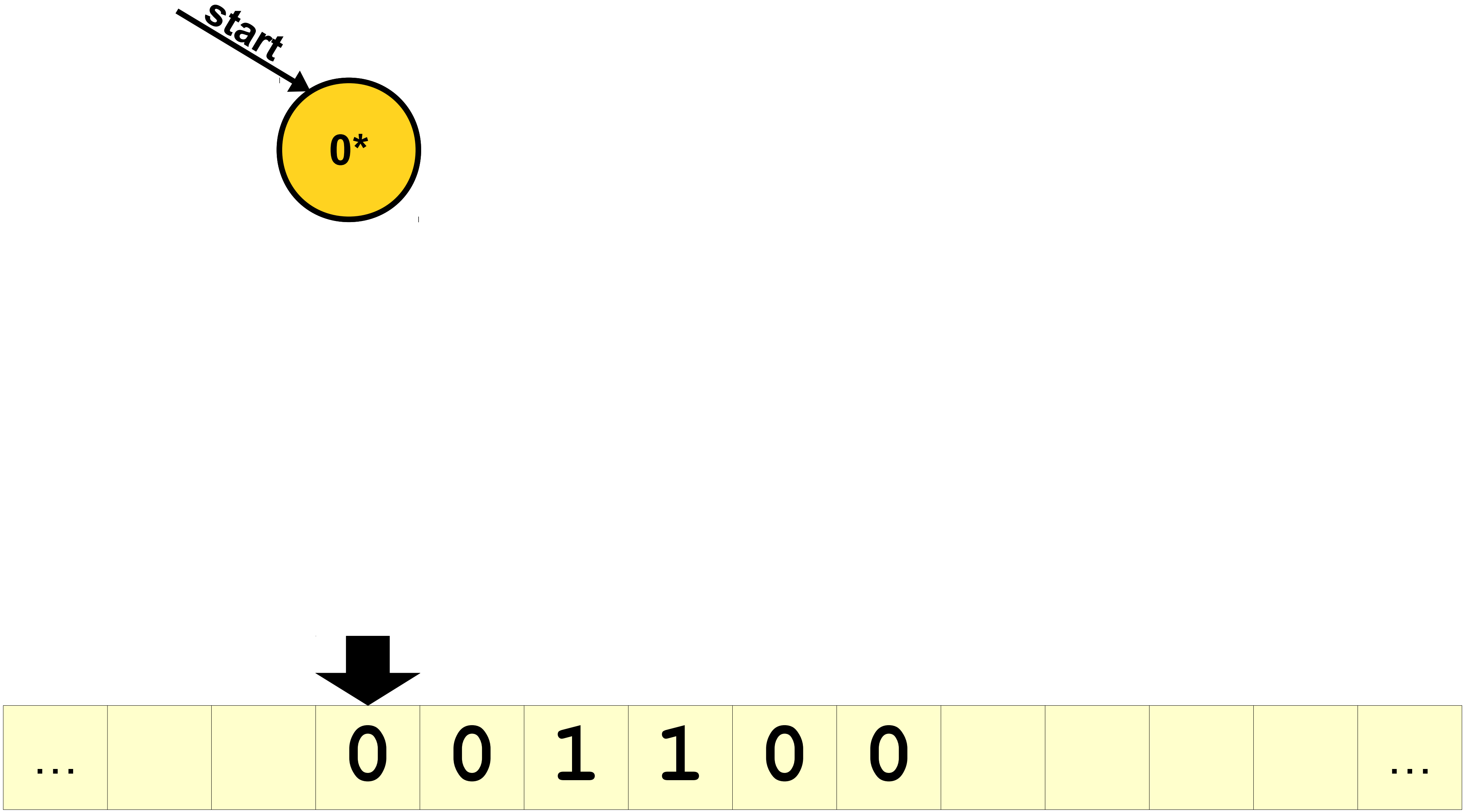
Idea: Repeatedly find a copy of 10 and replace it with 01.

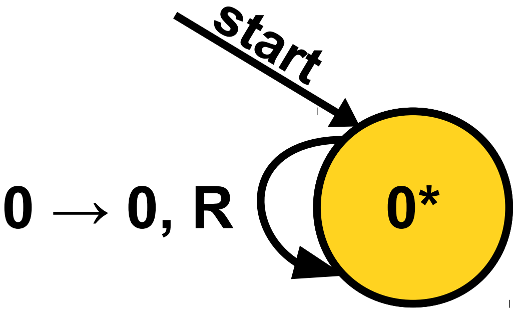
Let's Build It!

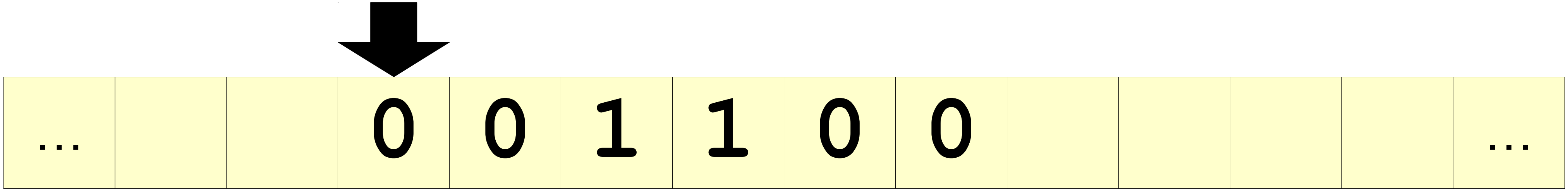

...
0
0
1
1
0
0
...


start
…
0
0
1
1
0
0
…

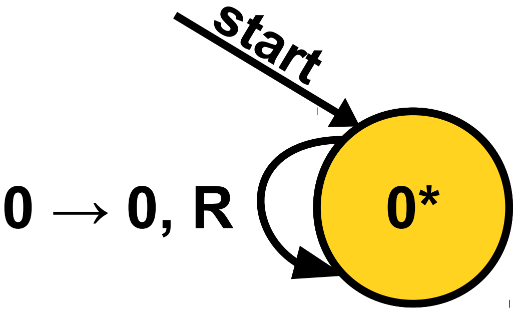

start
0*
…
0
0
1
1
0
0
…

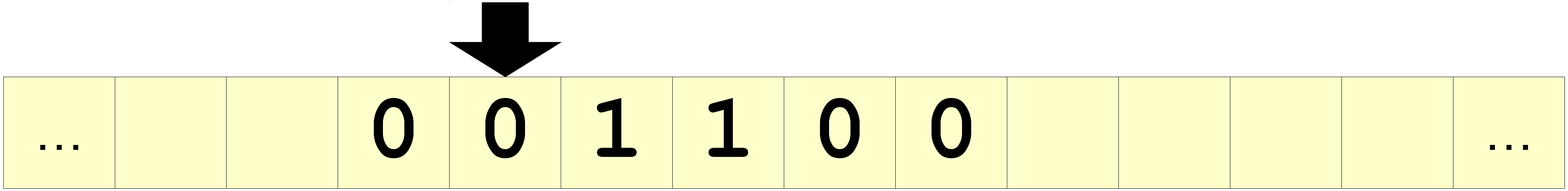

start
0*
...
0
0
1
1
0
0
...

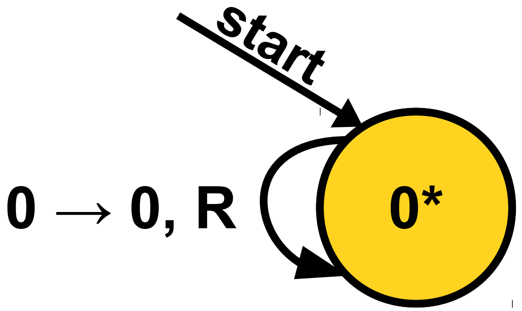


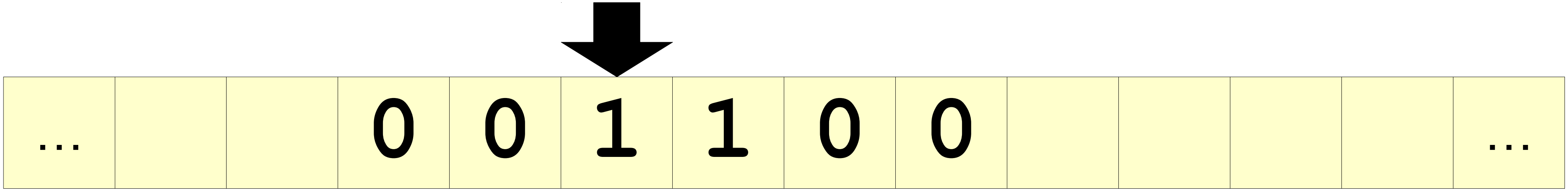


…			0	0	1	1	0	0					…

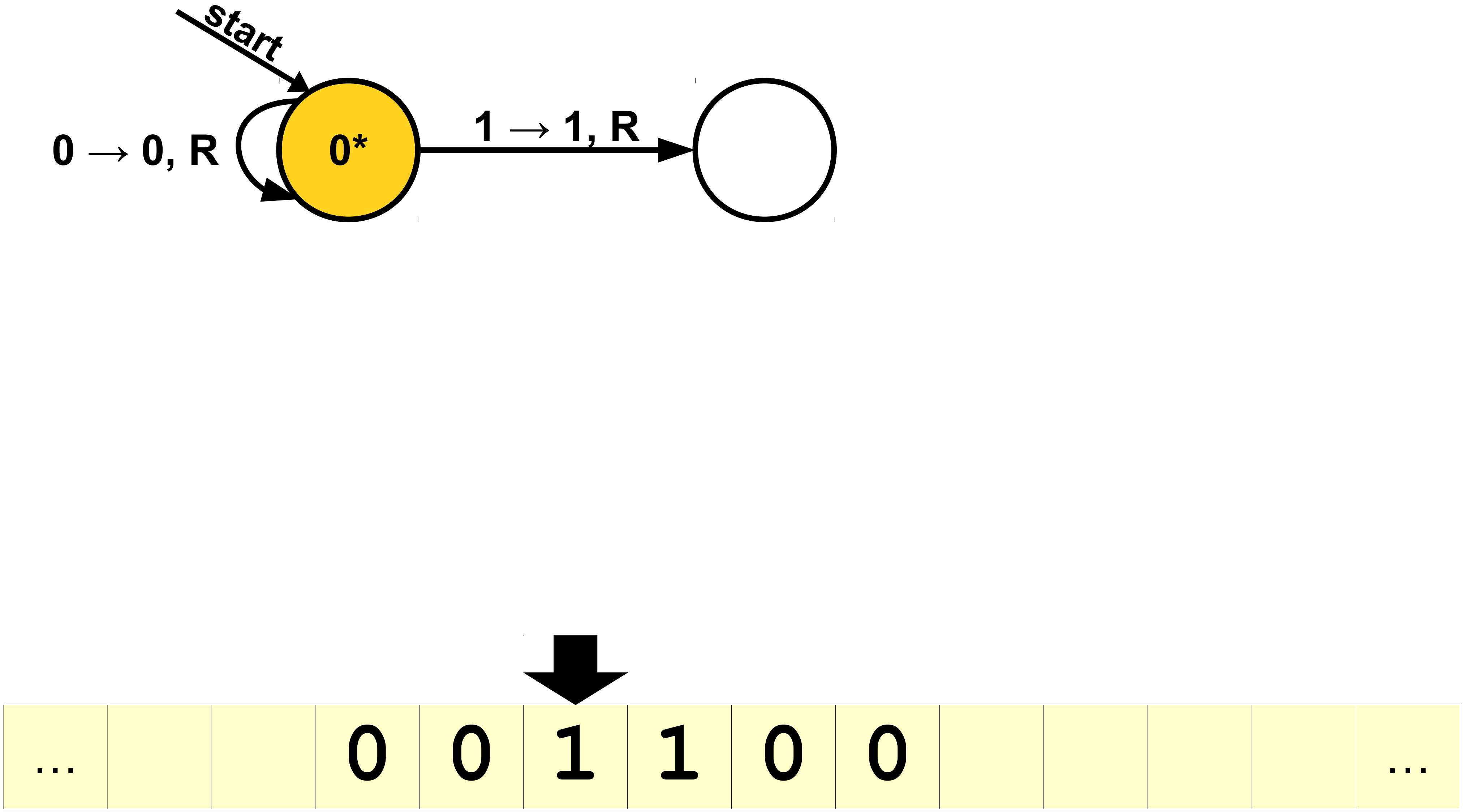

start
0 → 0, R
0*

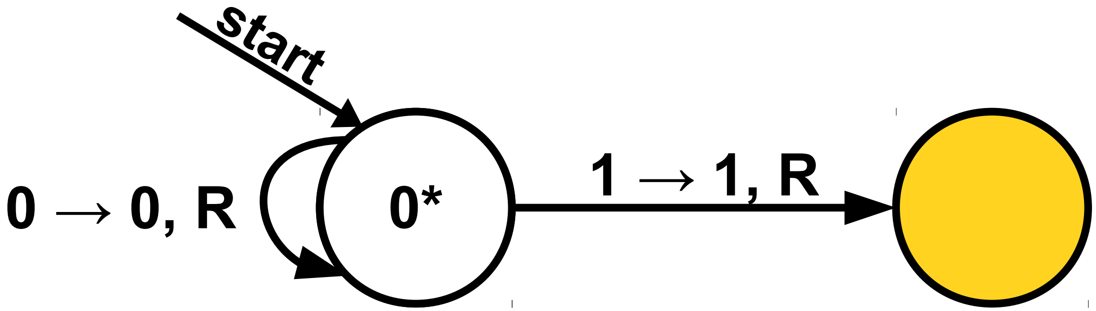

…
0
0
1
1
0
0
…

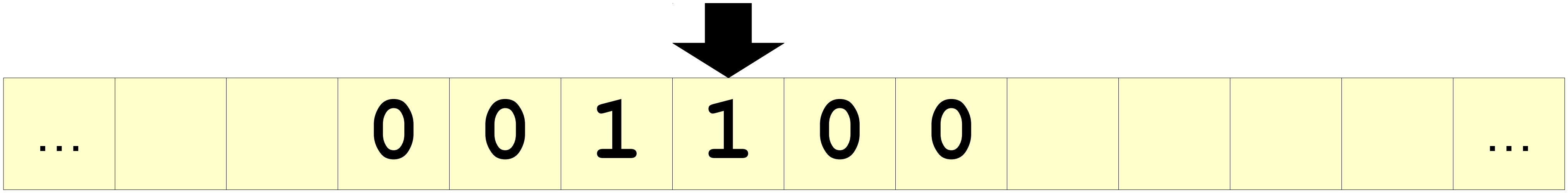


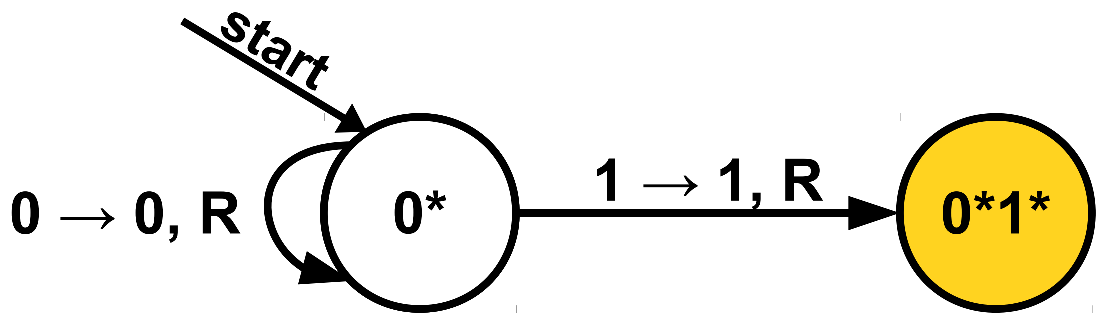


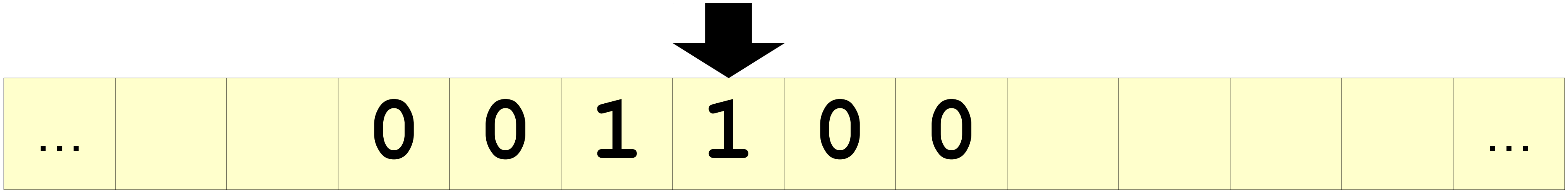
…			0	0	1	1	0	0					…


start
0 → 0, R
0*
1 → 1, R
…
0
0
1
1
0
0
…

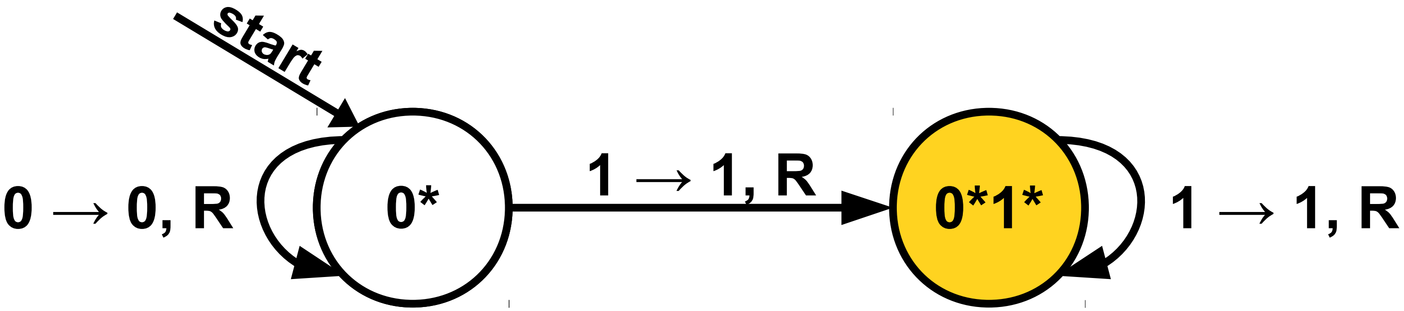

start
0 → 0, R
0*
1 → 1, R

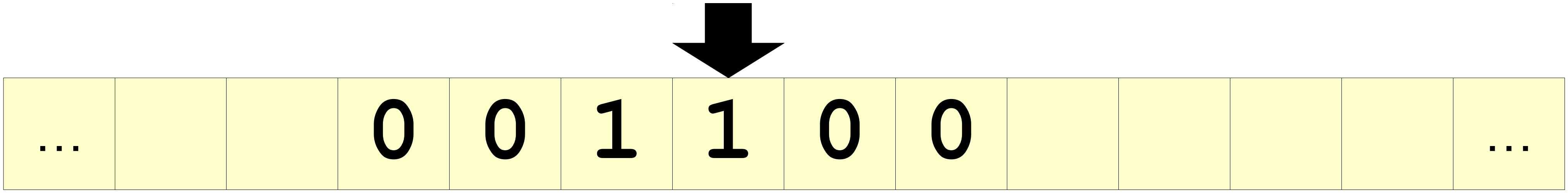

…
0
0
1
1
0
0
…



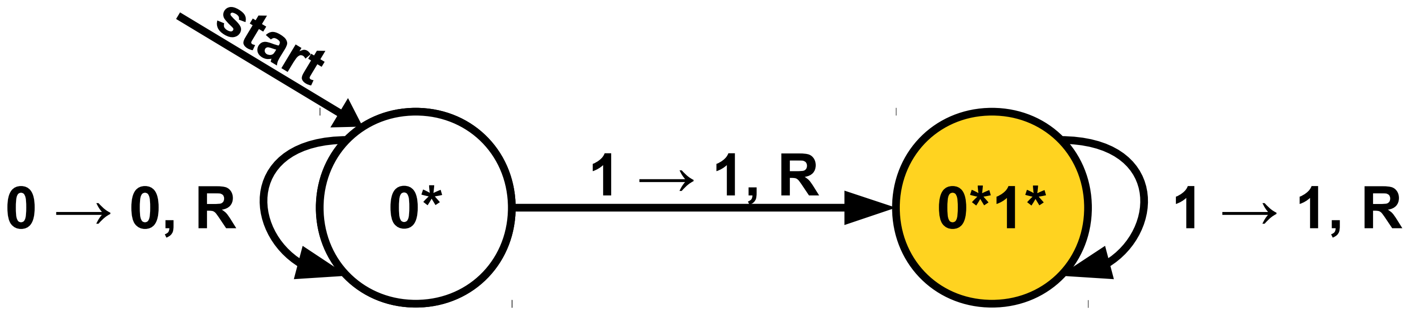


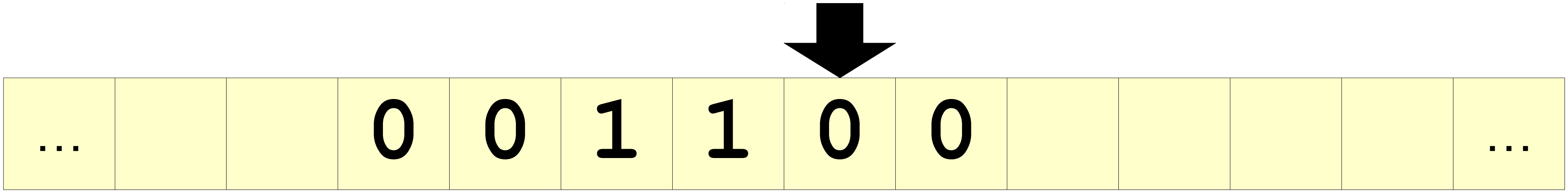
…			0	0	1	1	0	0					…

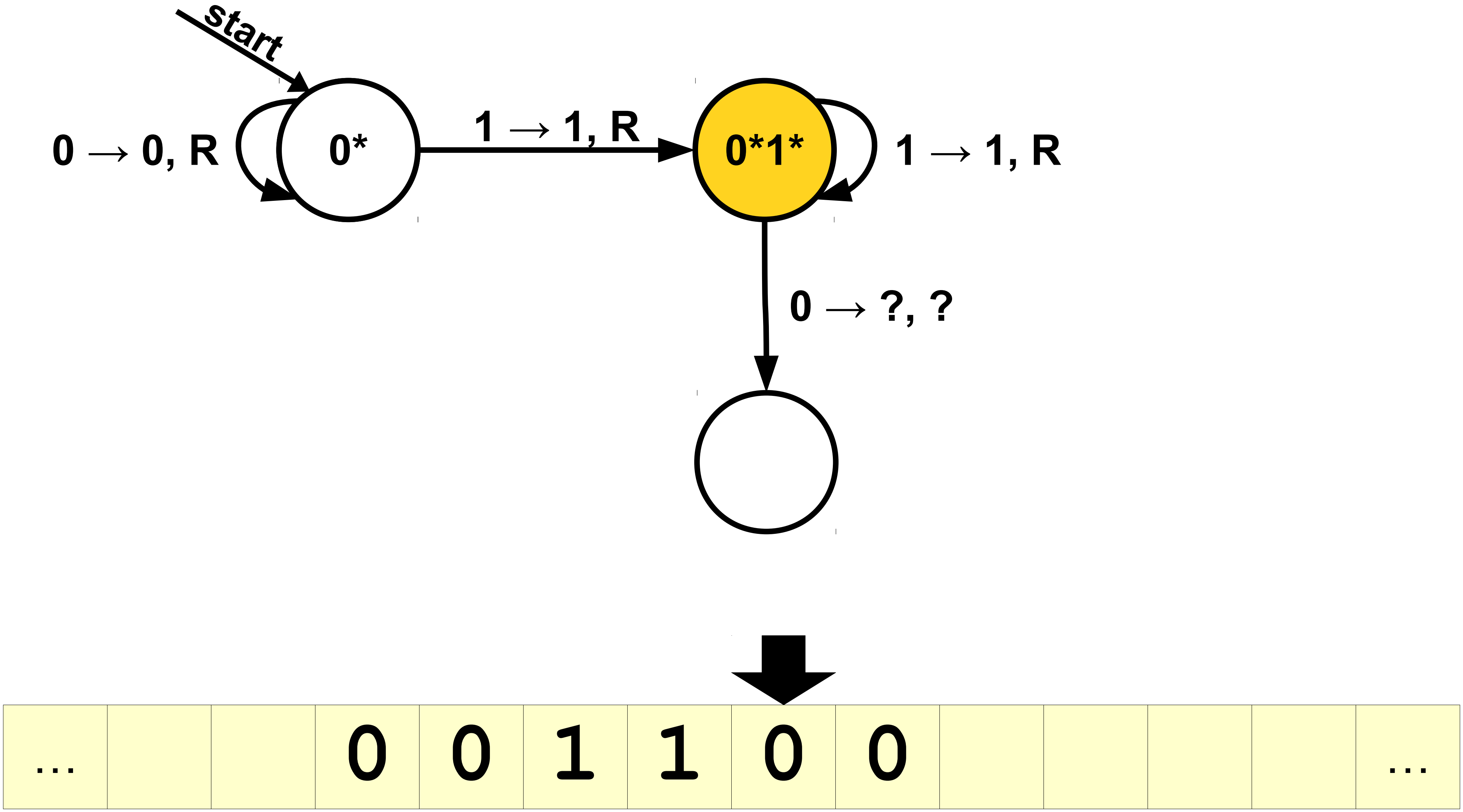


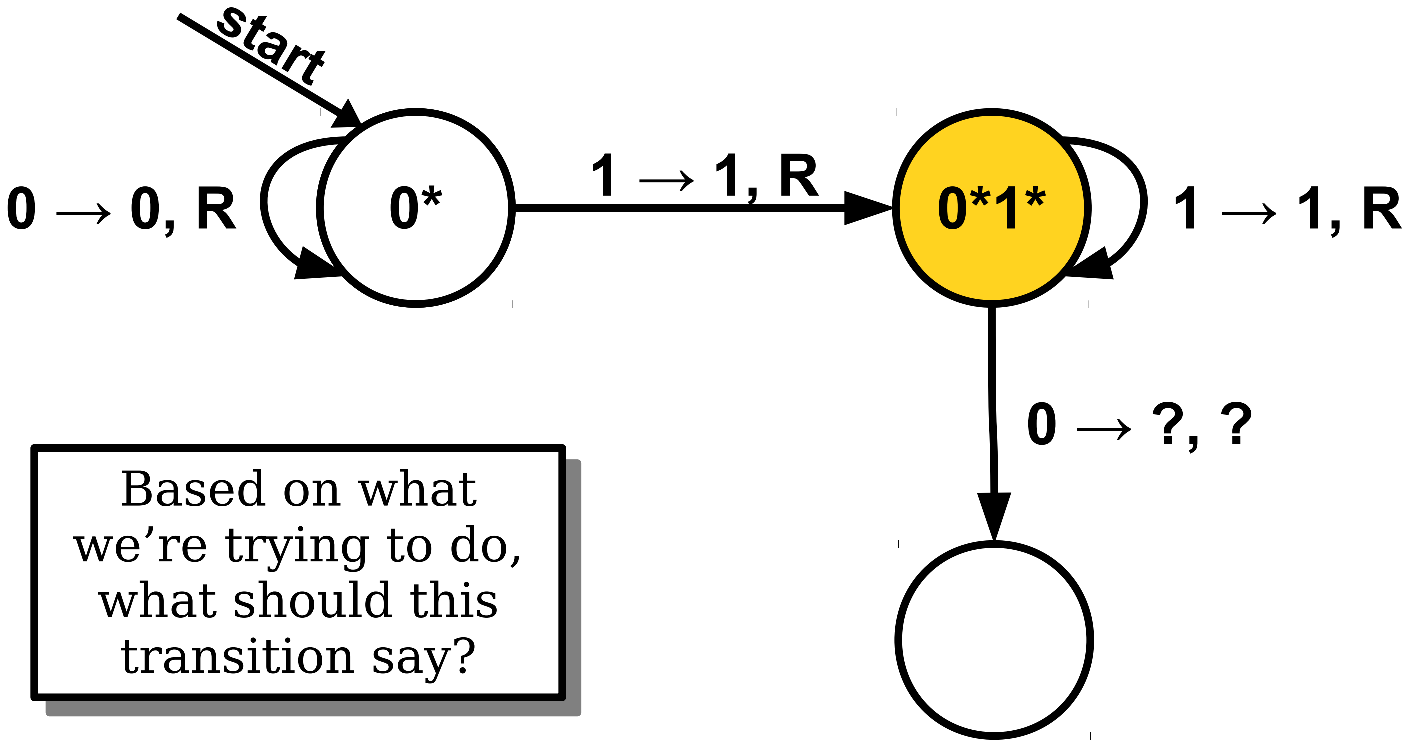


…			0	0	1	1	0	0					…

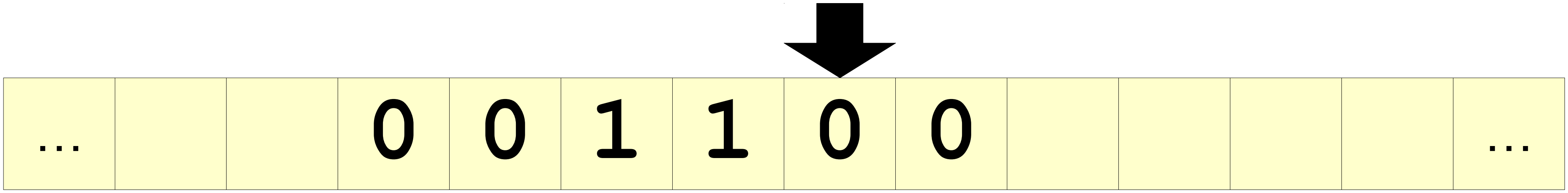

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R

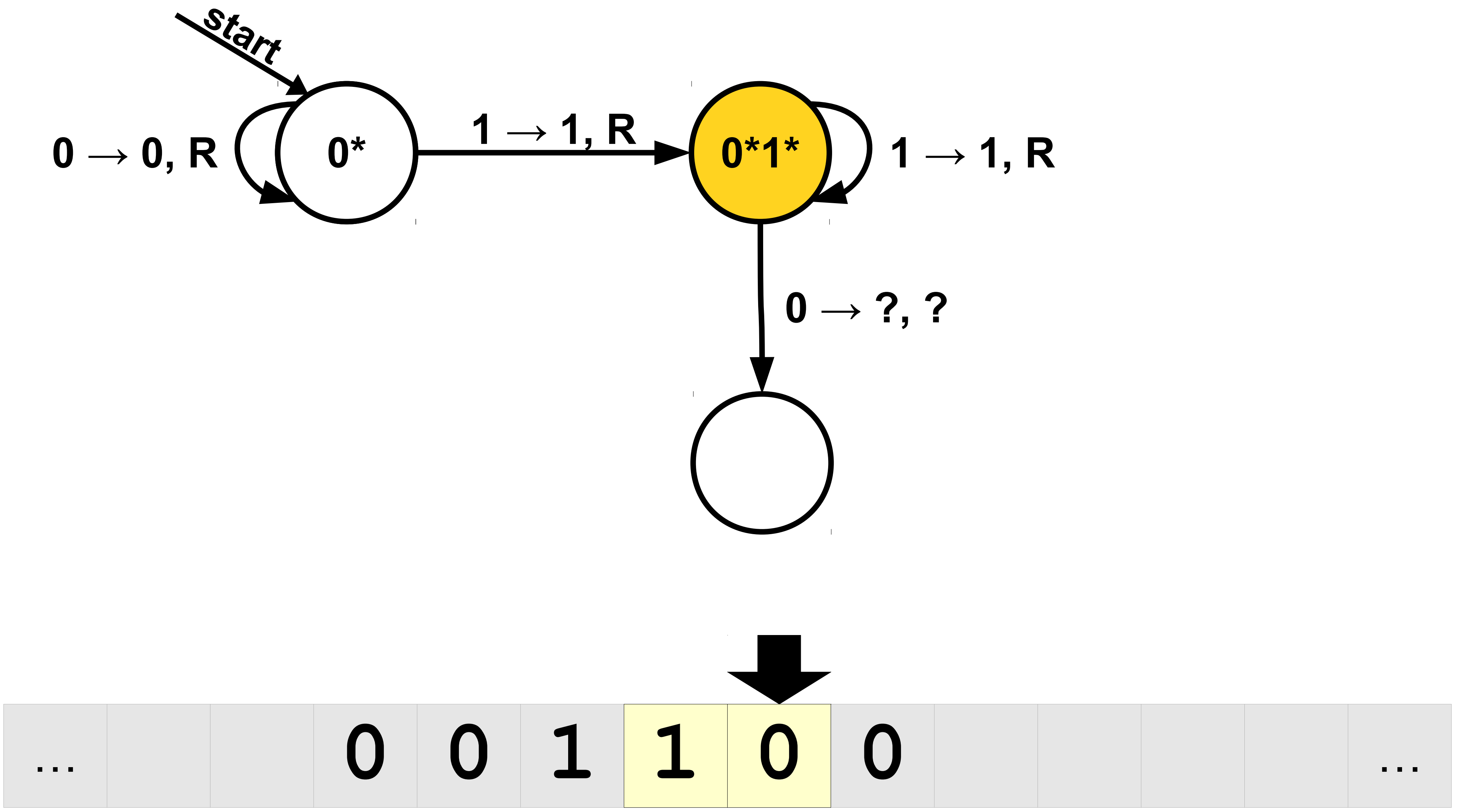

…
0
0
1
1
0
0
…

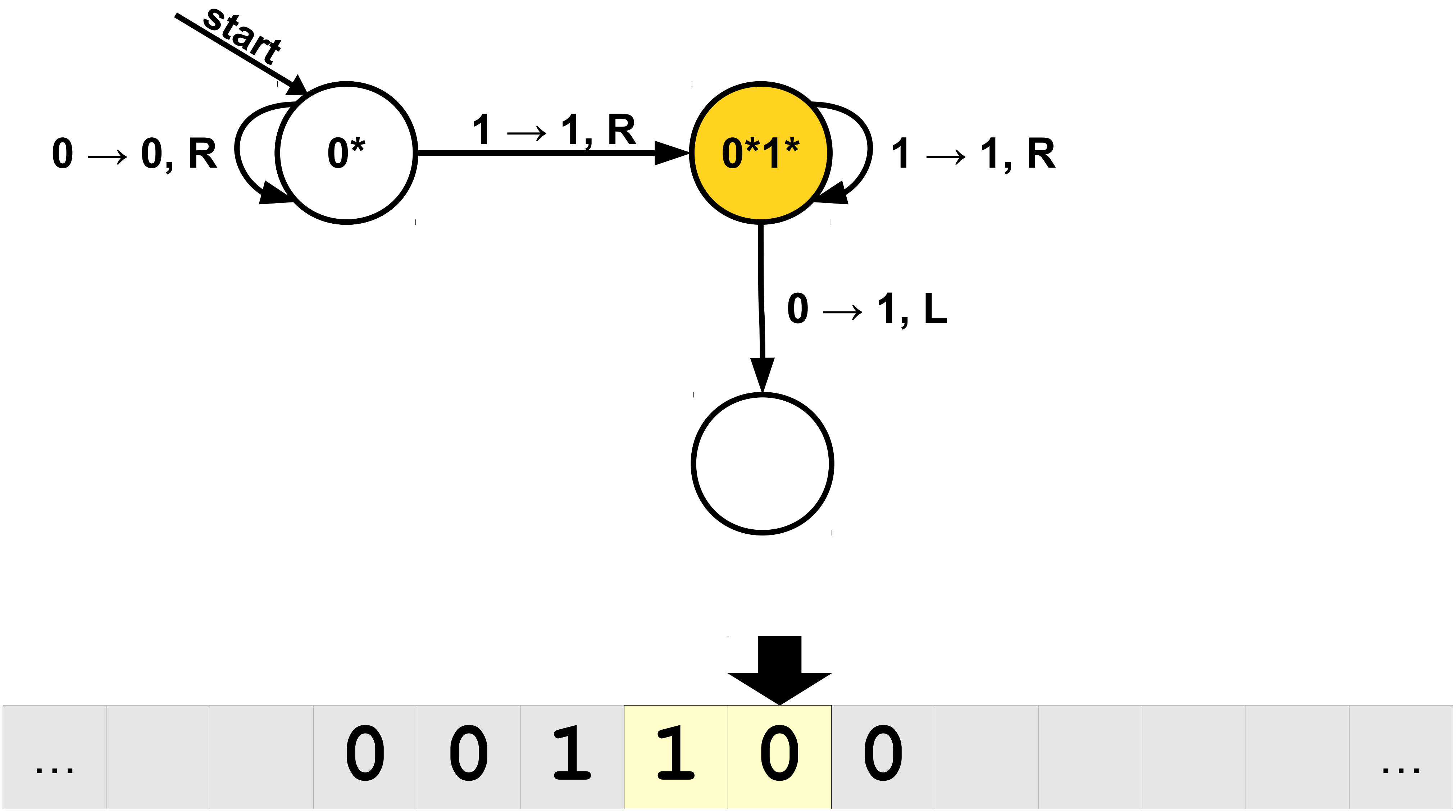

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
0 → ?, ?
…
0
0
1
1
0
0
…

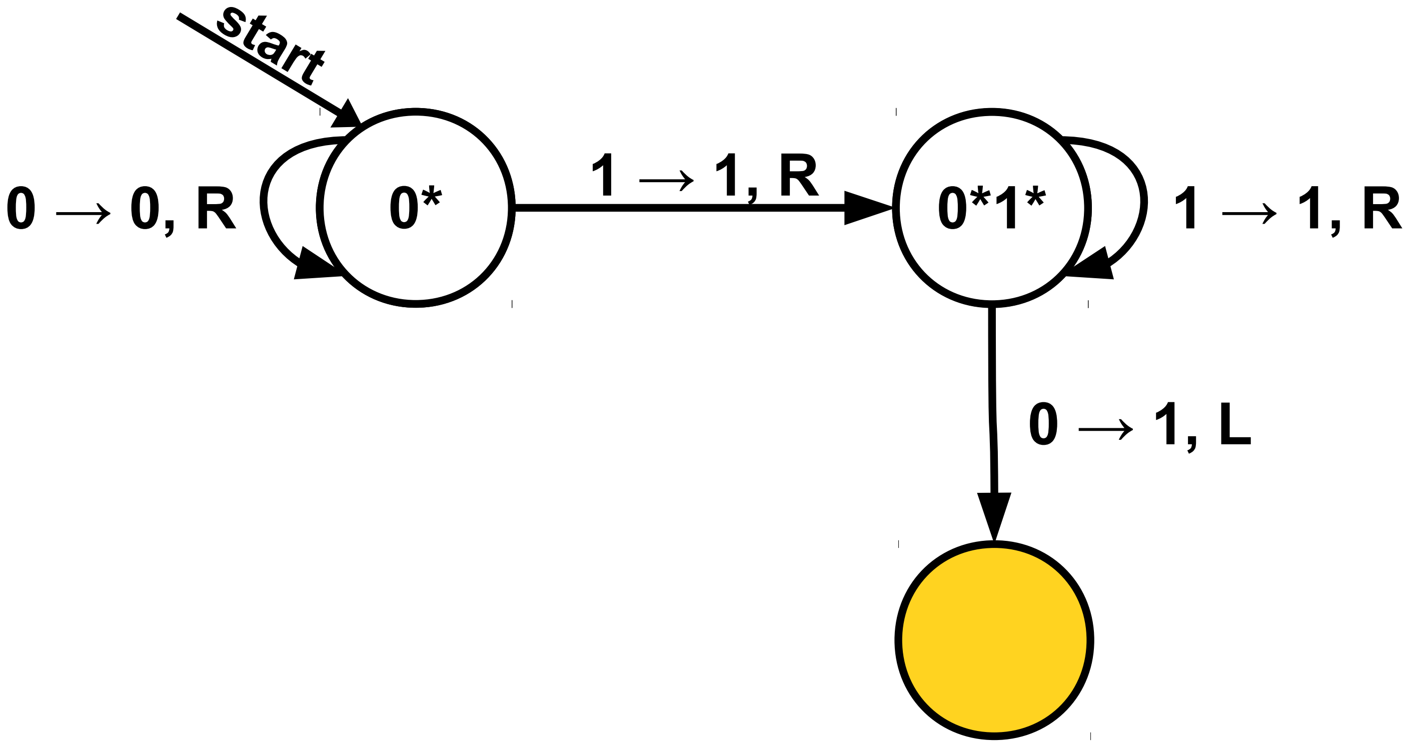


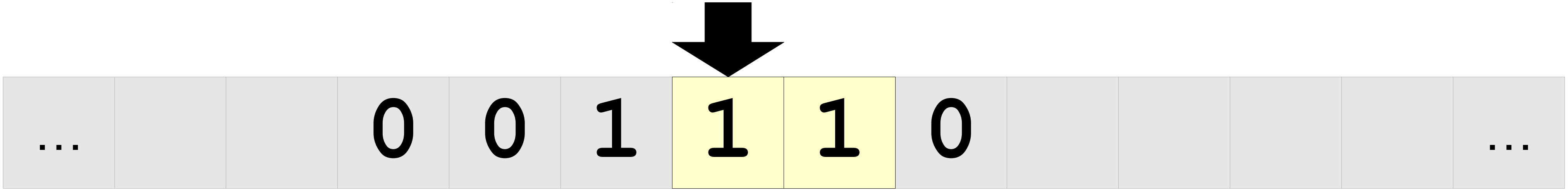
Based on what we’re trying to do, what should this transition say?



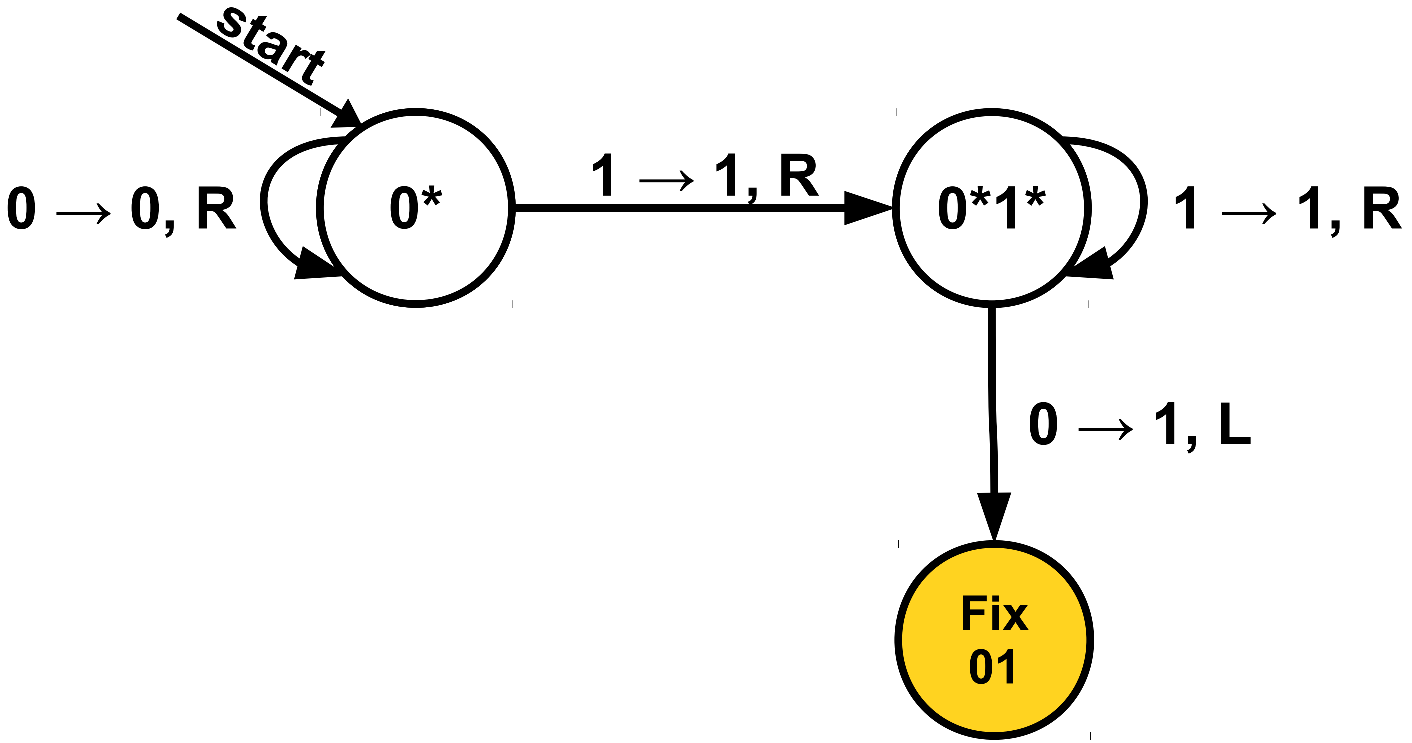

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
0 → ?, ?
…
0
0
1
1
0
0
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
…
0
0
1
1
0
0
…





…			0	0	1	1	1	0					…



start

0 → 0, R

0*

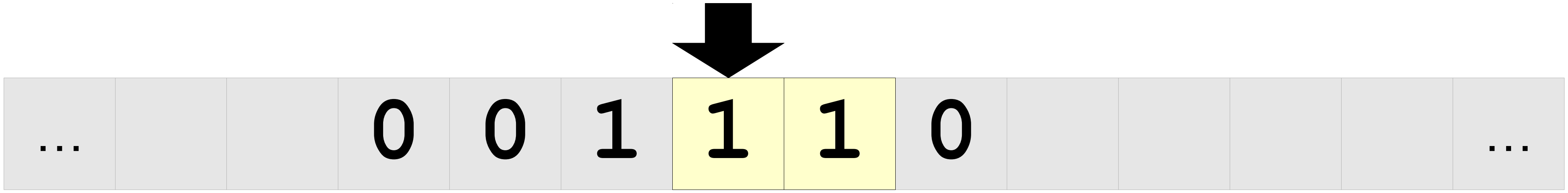
1 → 1, R

0*1*

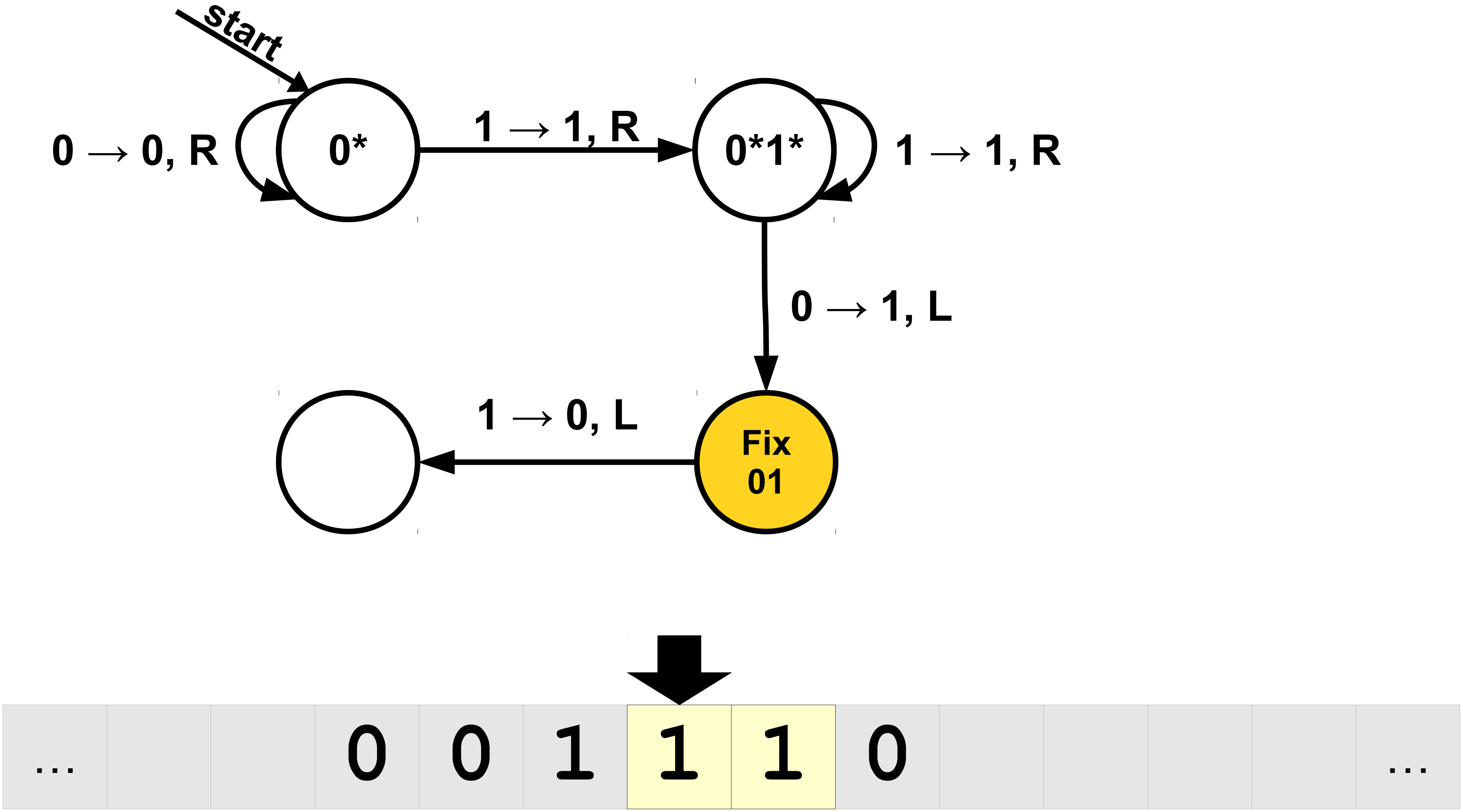
1 → 1, R

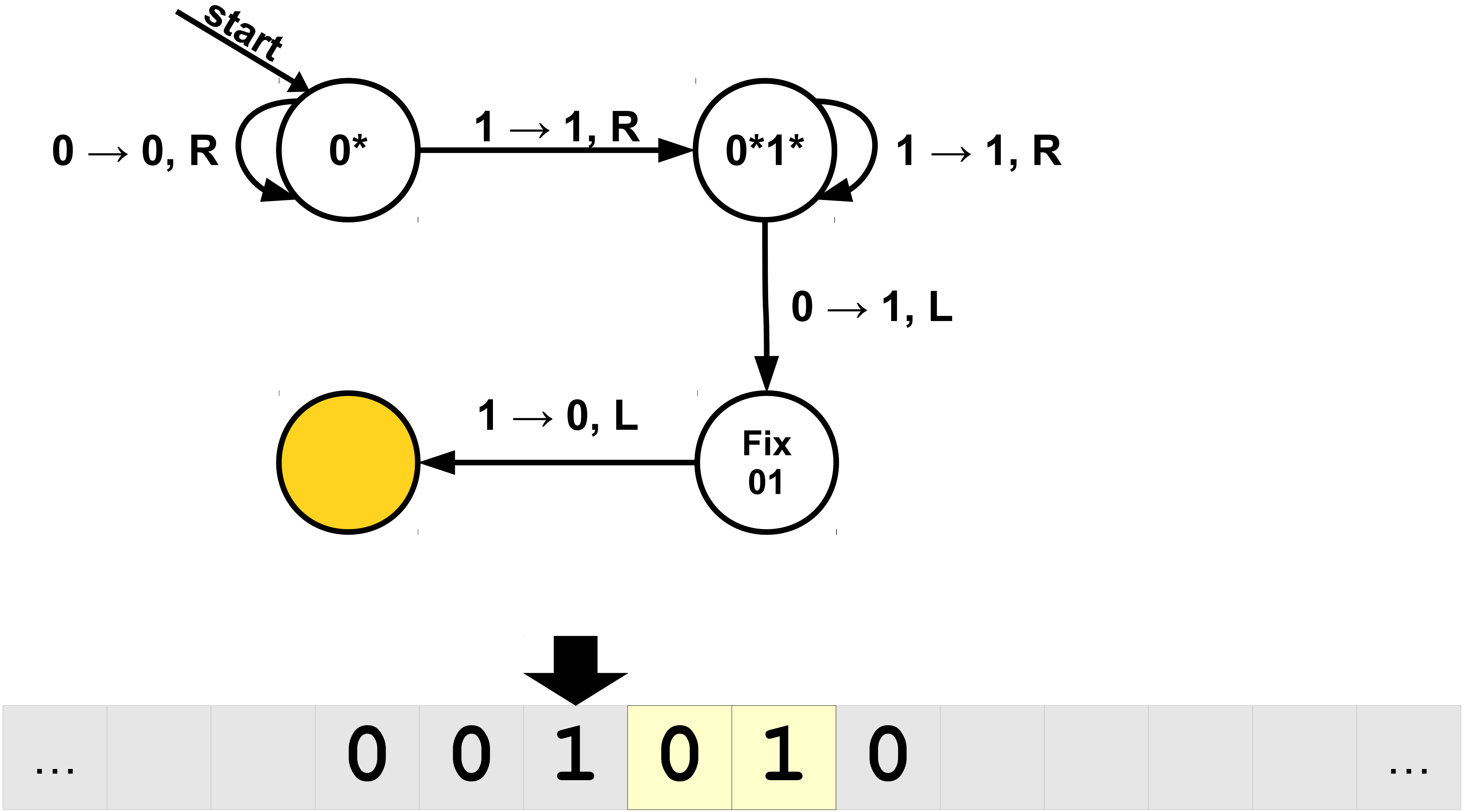
0 → 1, L

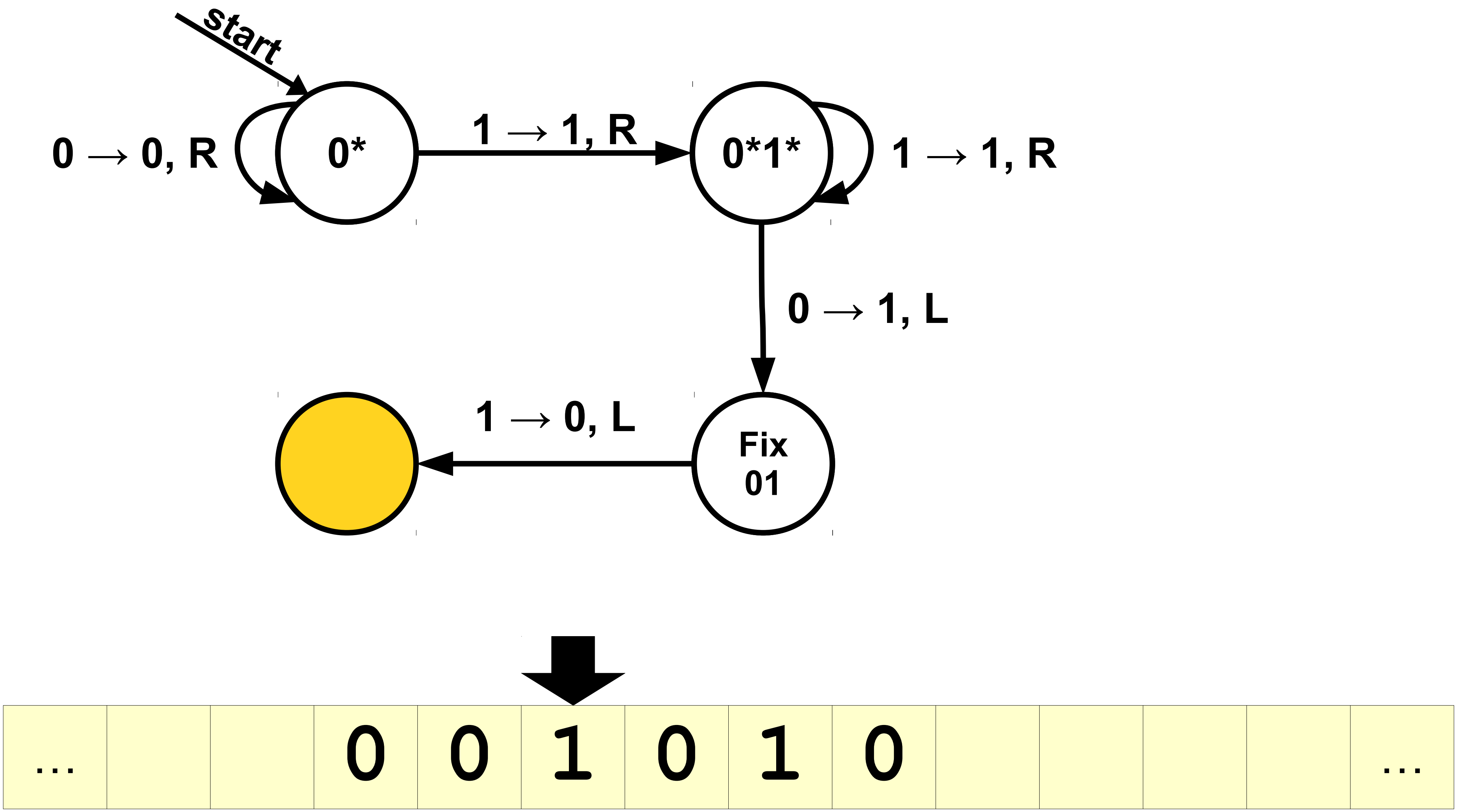
Fix 01

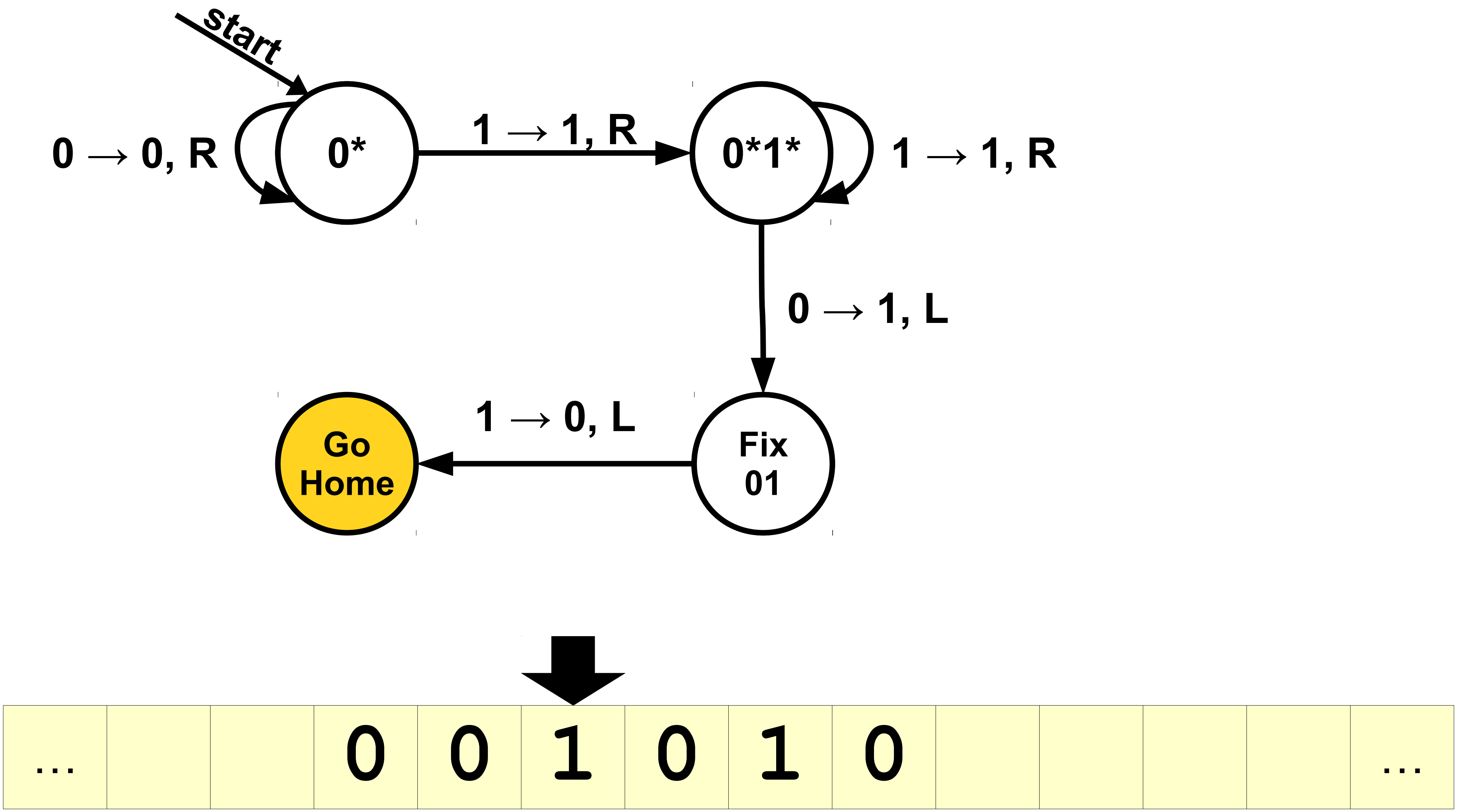


…			0	0	1	1	1	0					…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
1 → 0, L
Fix
01
…
0
0
1
1
1
0
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
1 → 0, L
Fix
01
…
0
0
1
0
1
0
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
1 → 0, L
Fix 01
… 0 0 1 0 1 0 …



start

0*

0 → 0, R

1 → 1, R

0*1*

1 → 1, R

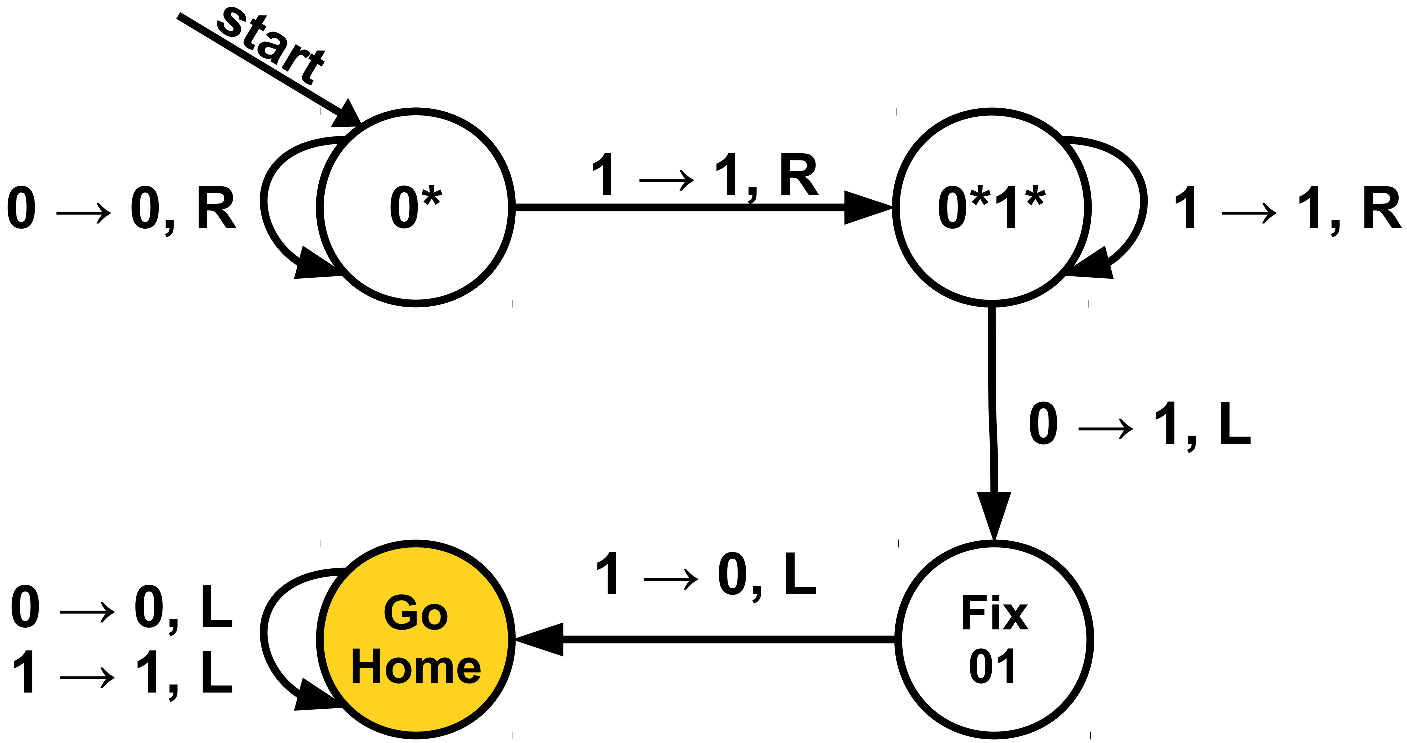
0 → 1, L

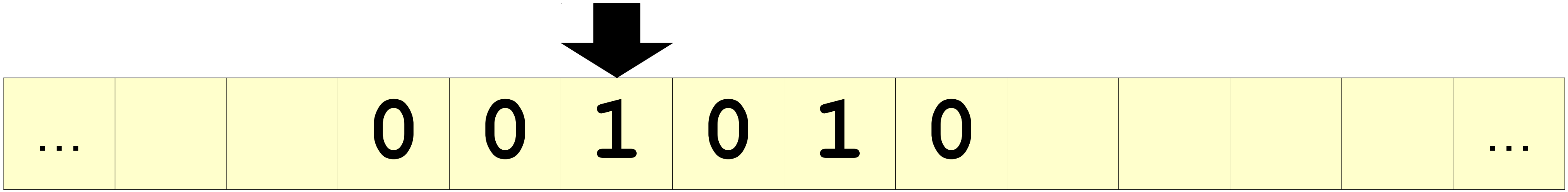
Fix 01

1 → 0, L

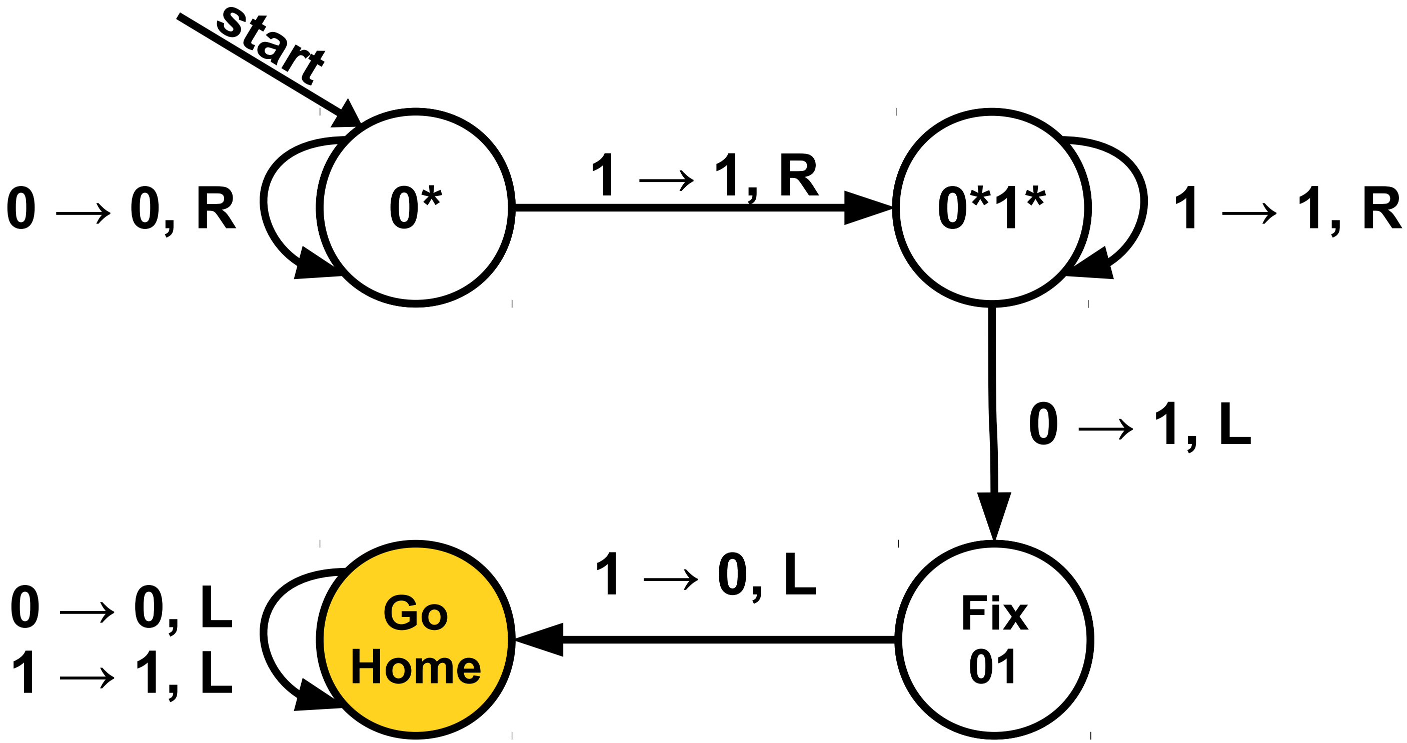
Go Home

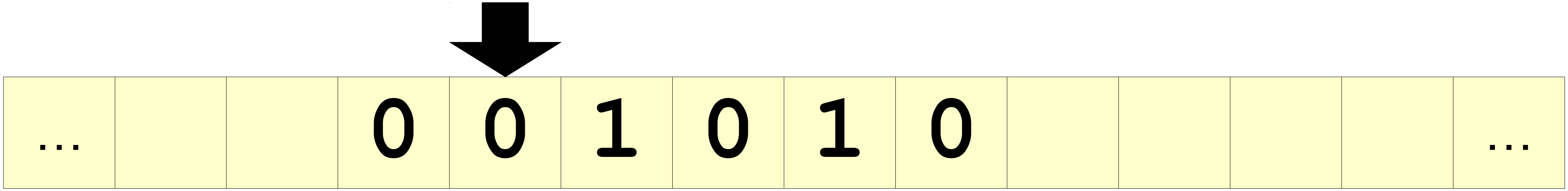
…			0	0	1	0	1	0					…



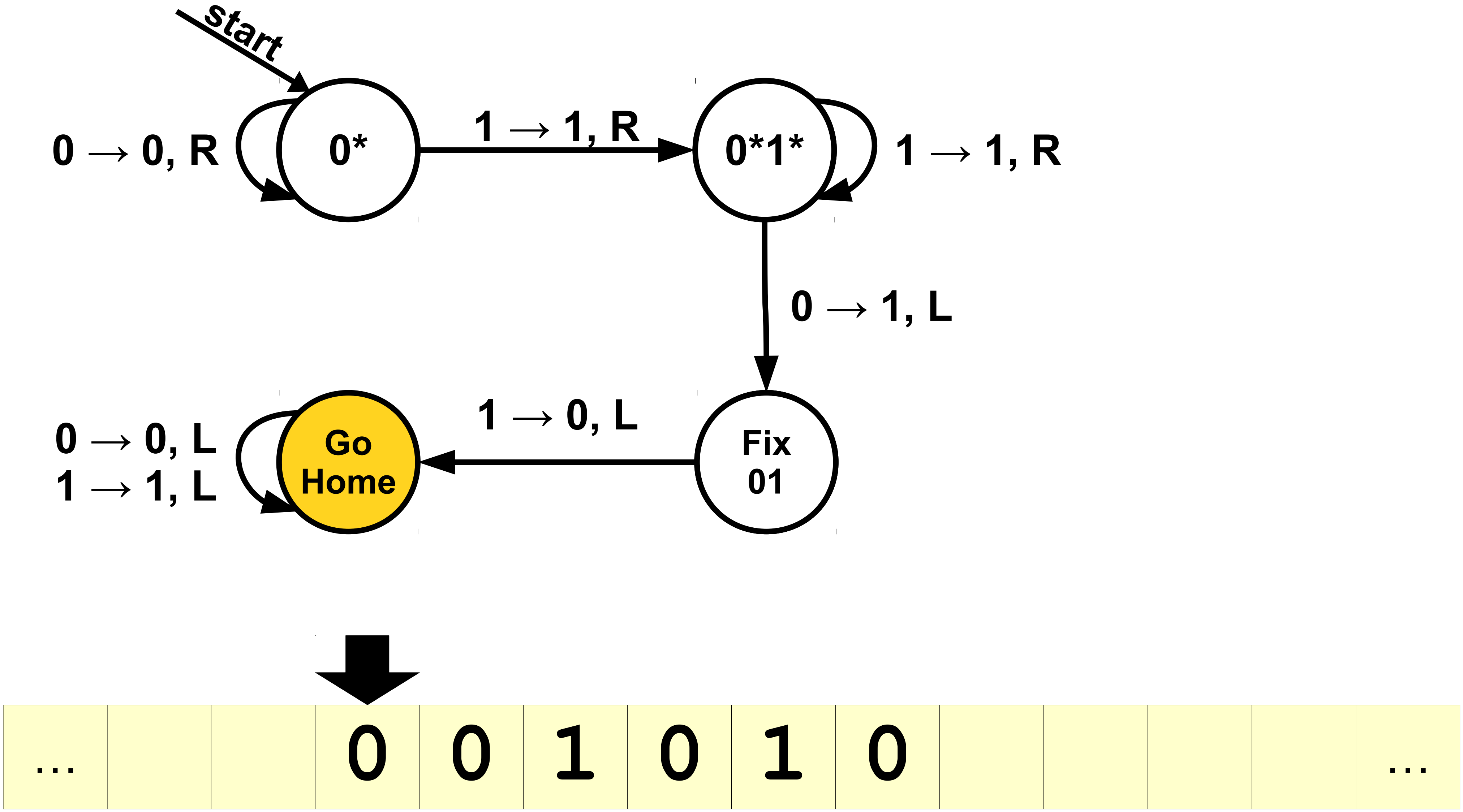


…			0	0	1	0	1	0					…

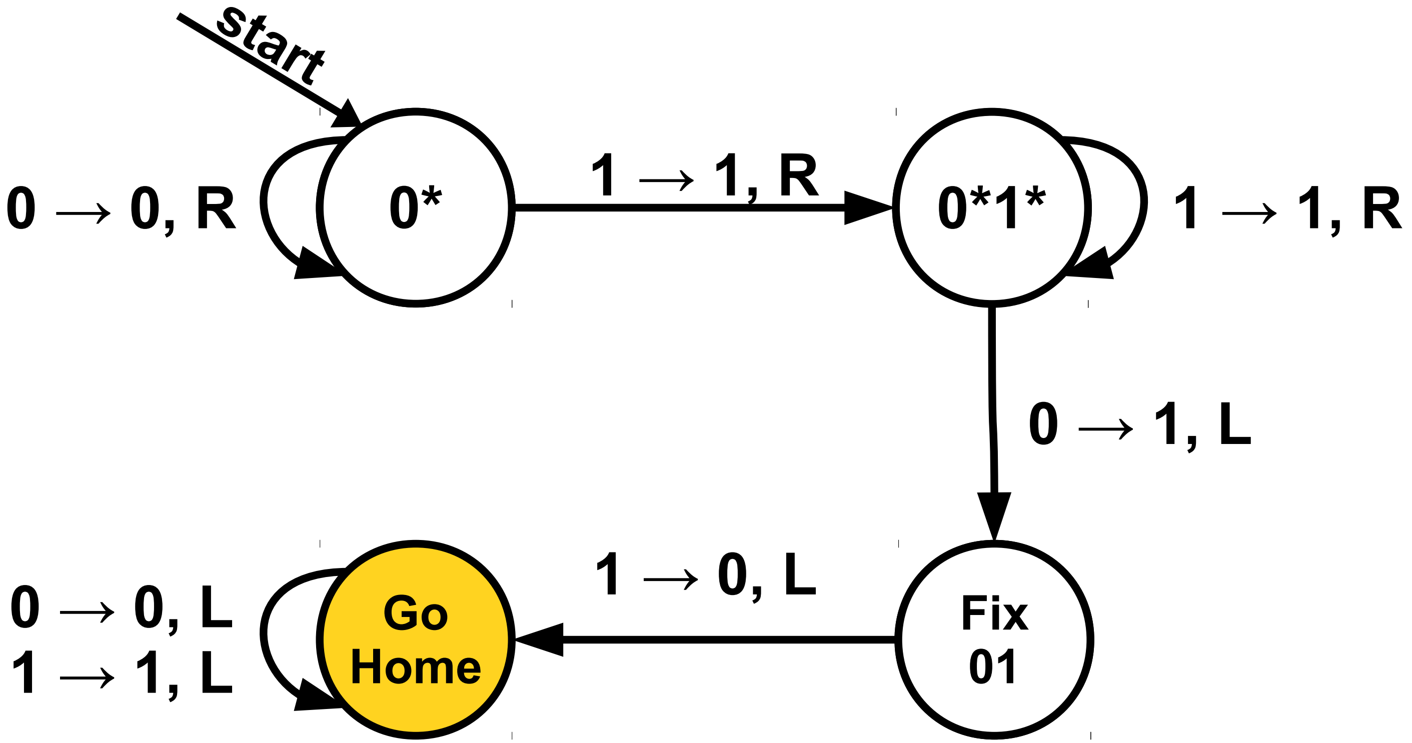


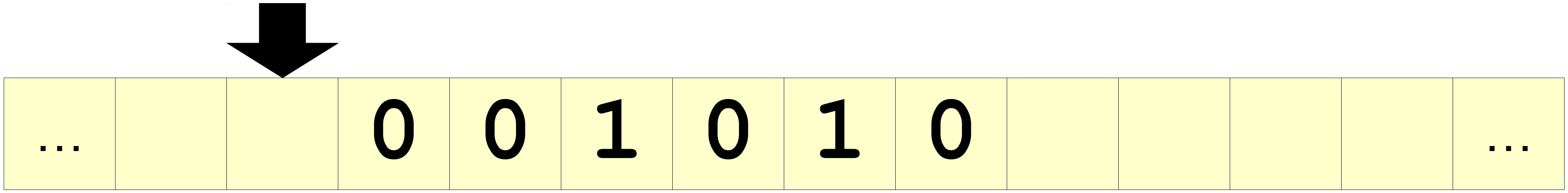


…			0	0	1	0	1	0					…

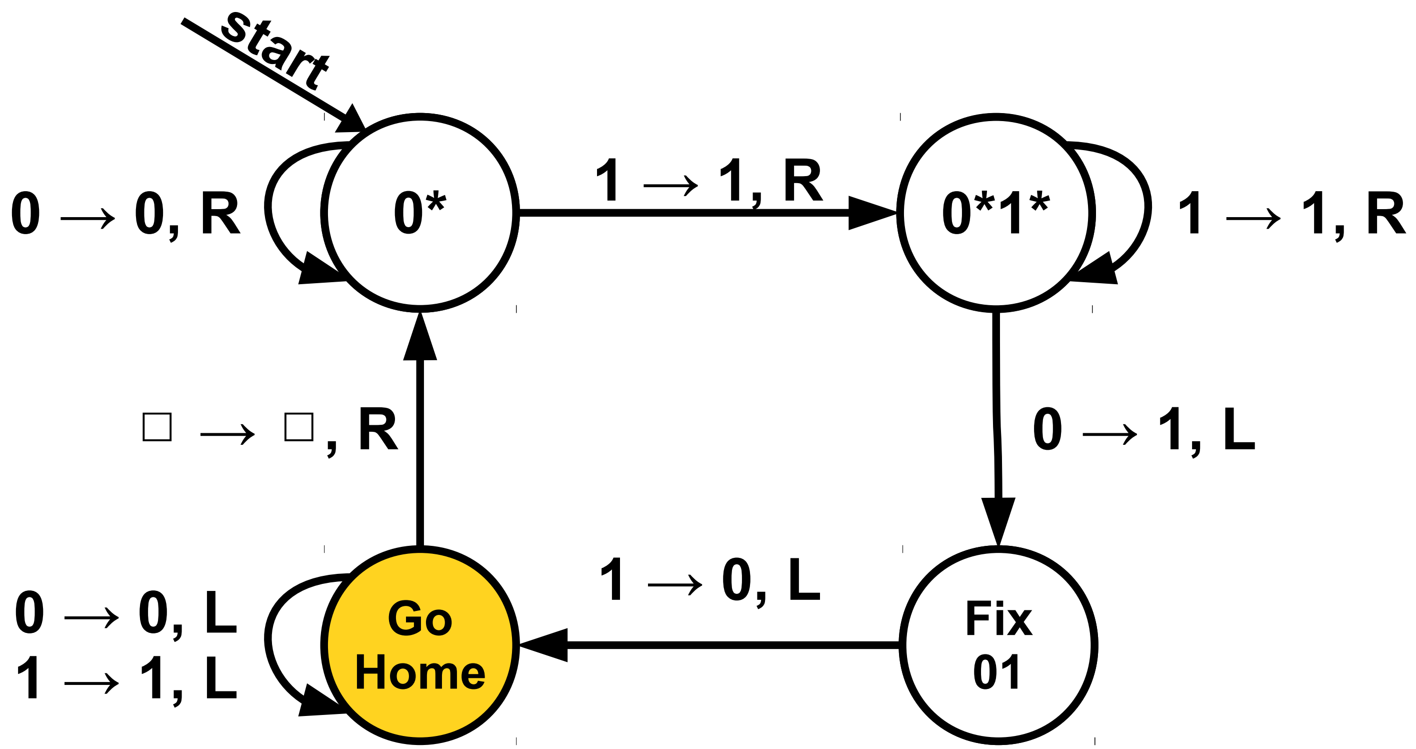


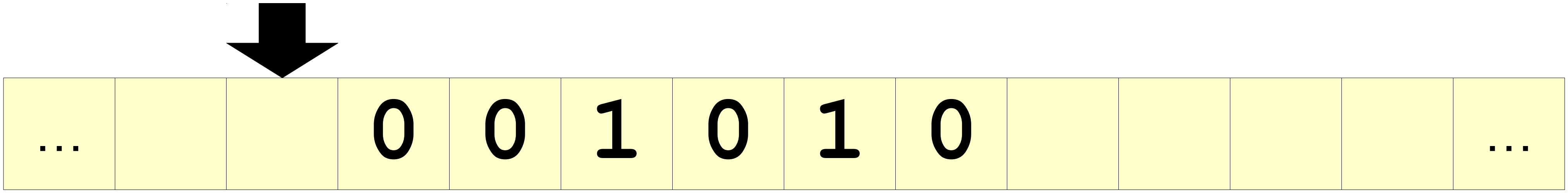
…			0	0	1	0	1	0					…

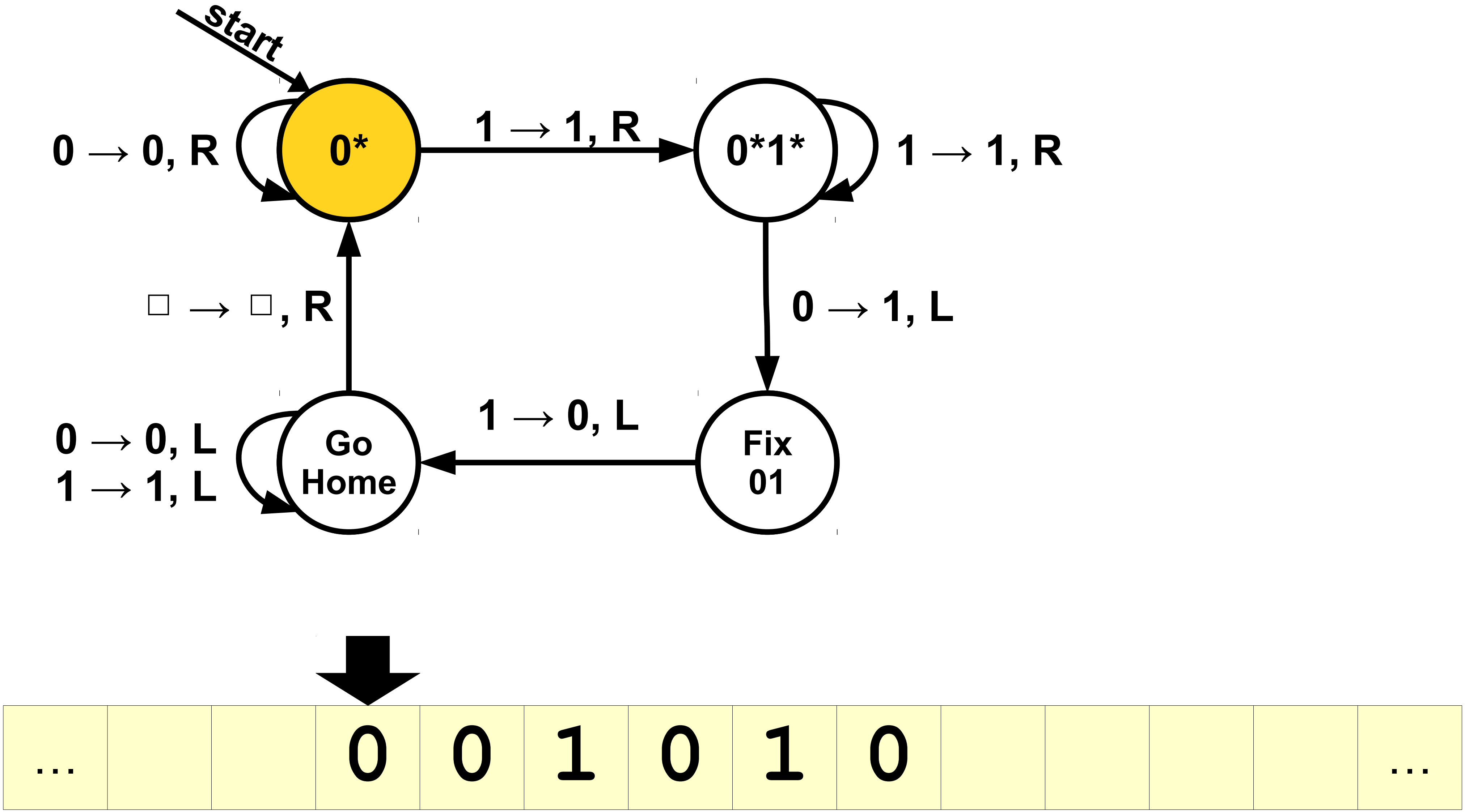




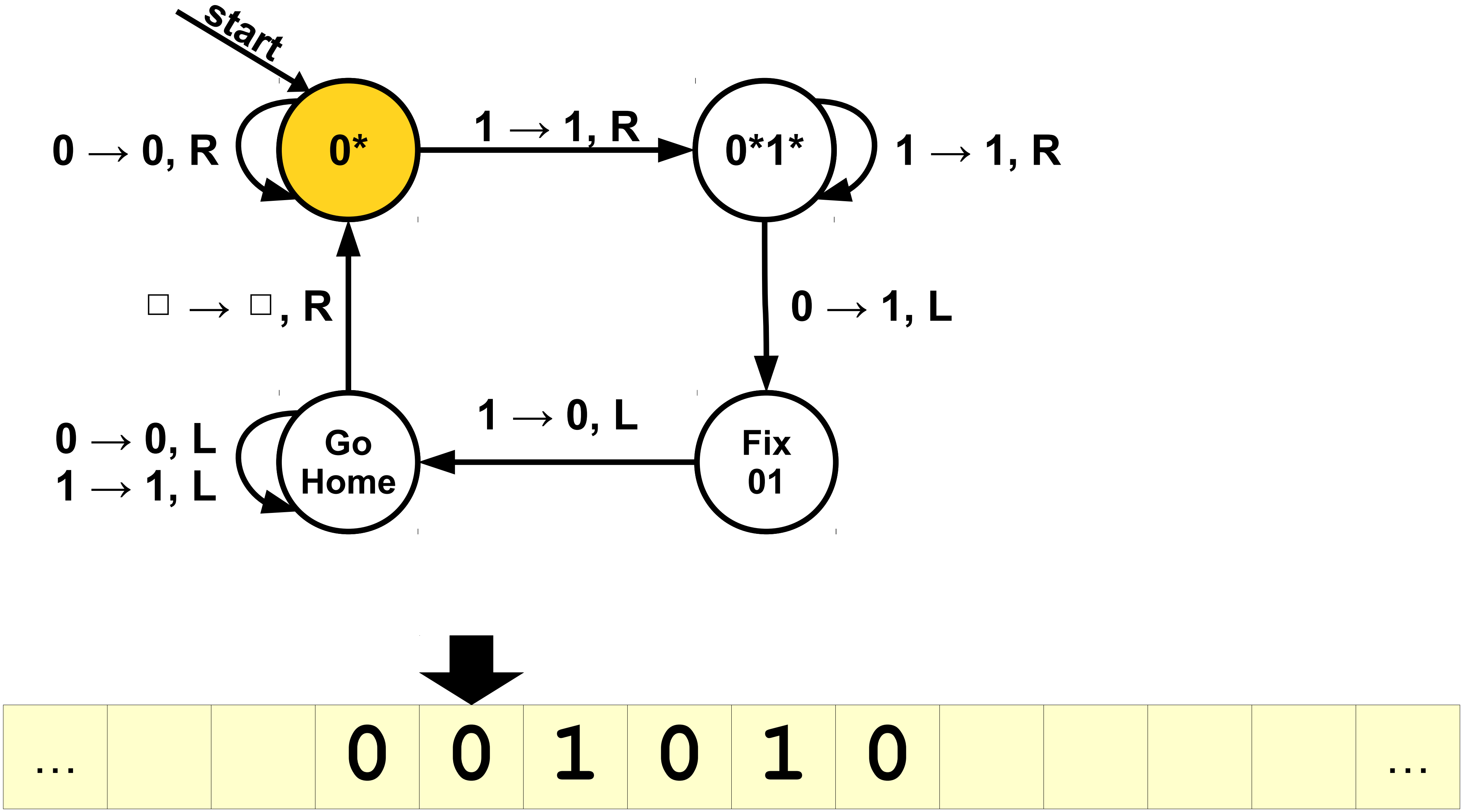
…			0	0	1	0	1	0					…

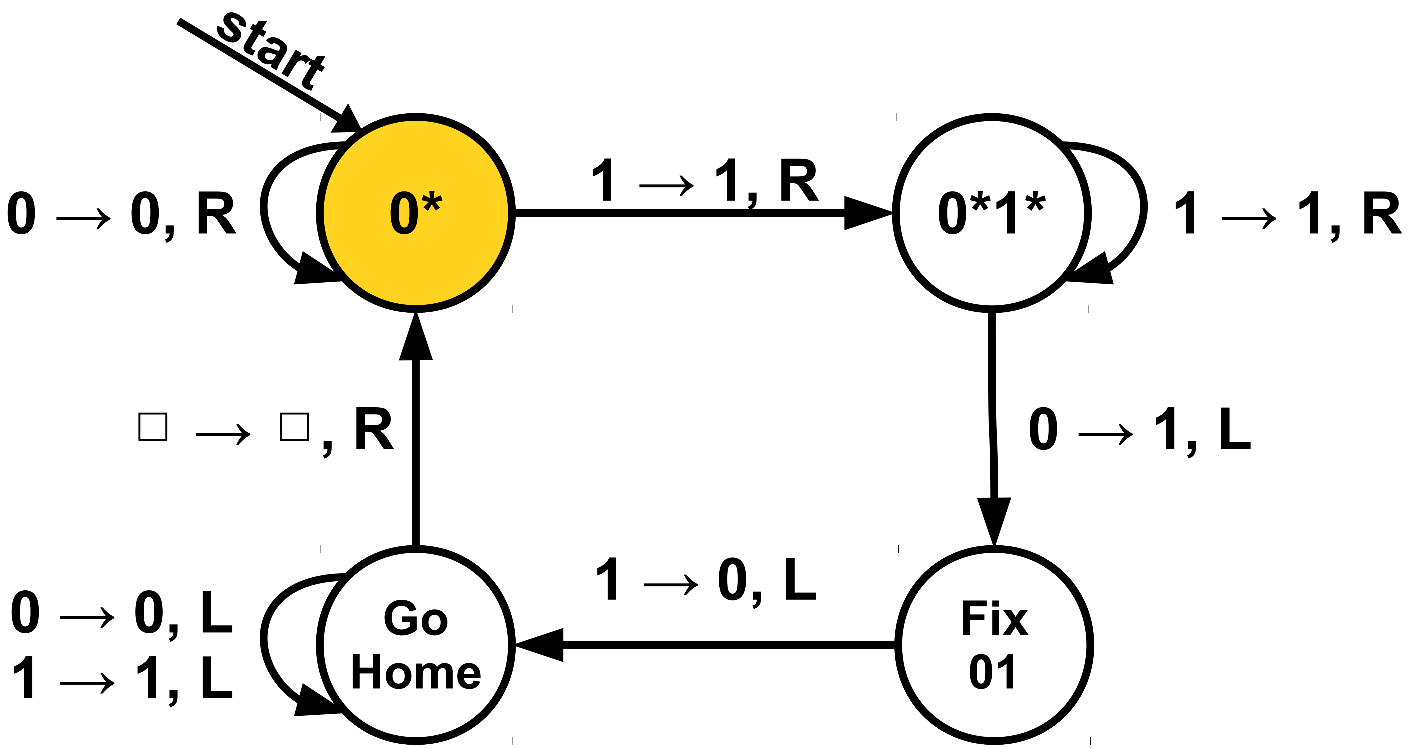

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01


…
0
0
1
0
1
0
…



…			0	0	1	0	1	0					…


start
0*
0 → 0, R
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
Fix 01
1 → 0, L
Go Home
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
1
0
1
0
…



start

0 → 0, R

0*

1 → 1, R

0*1*

1 → 1, R

□ → □, R

0 → 1, L

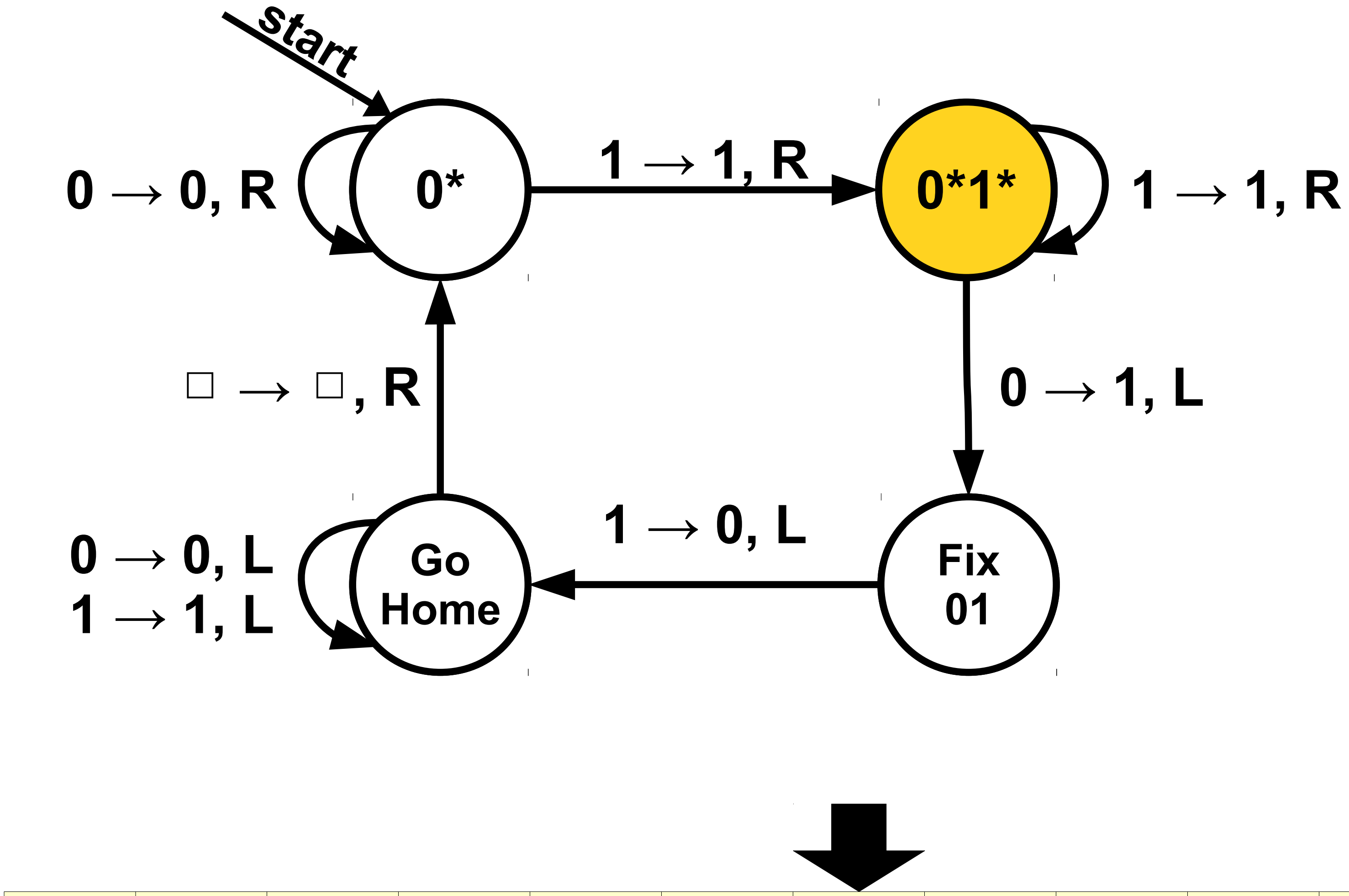
0 → 0, L
1 → 1, L

Go Home

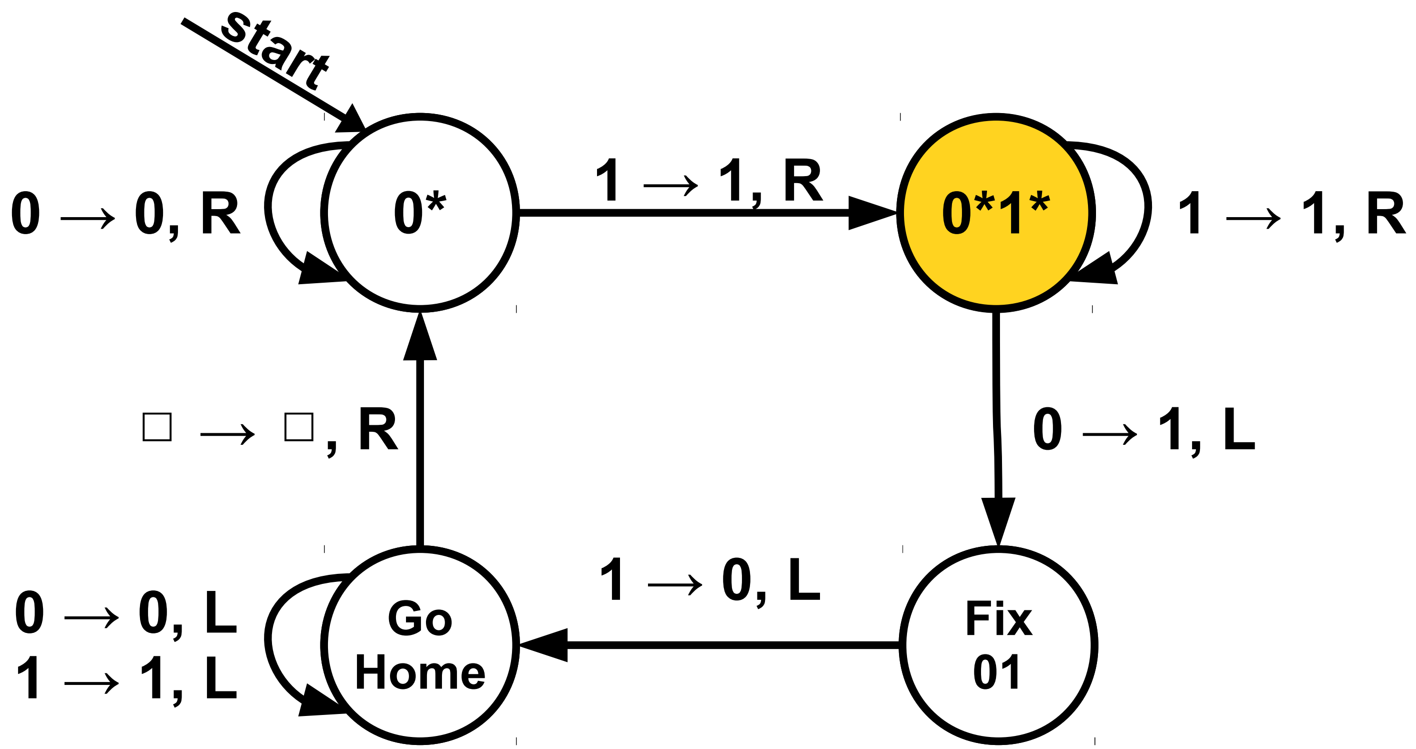
1 → 0, L

Fix 01

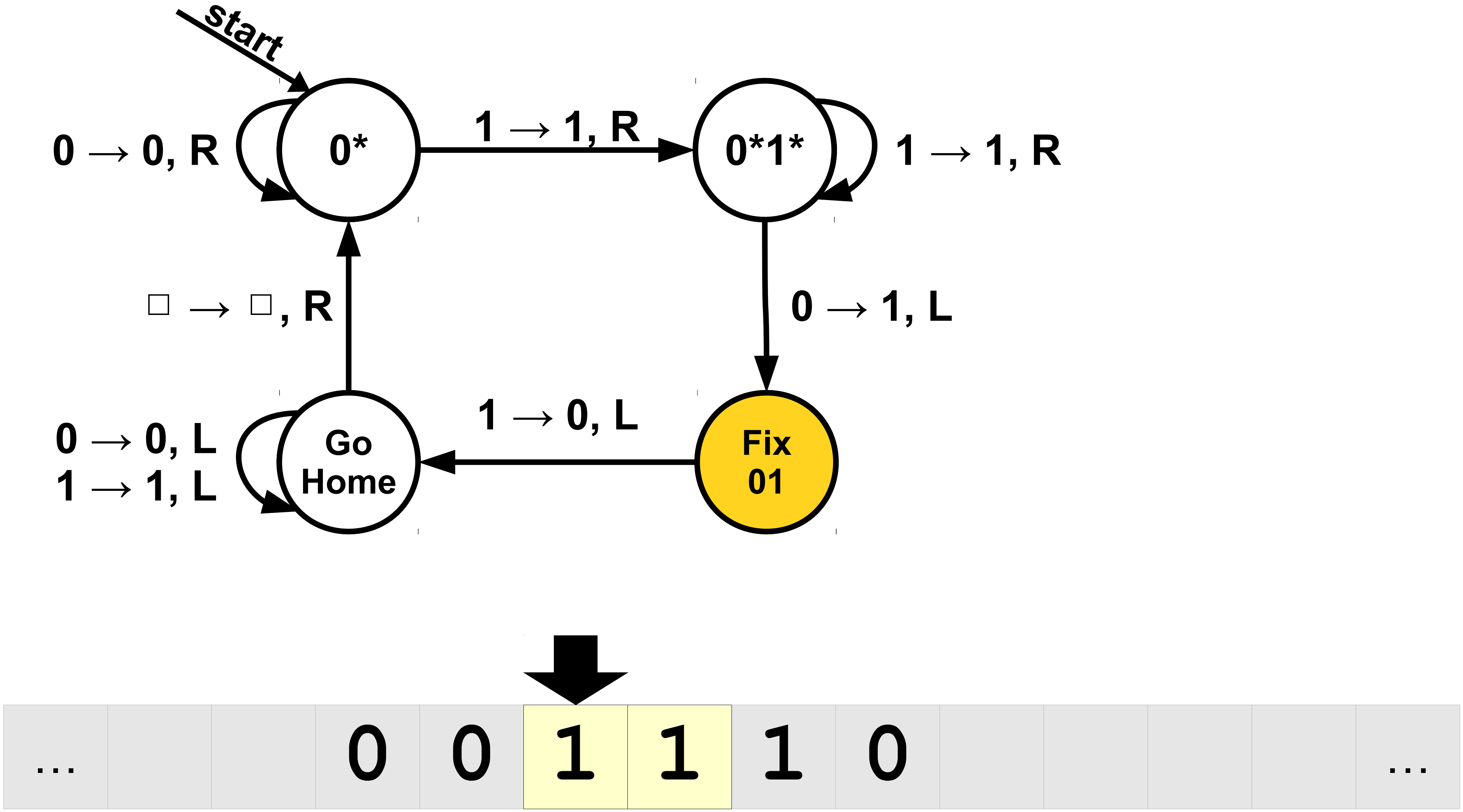
…			0	0	1	0	1	0					…

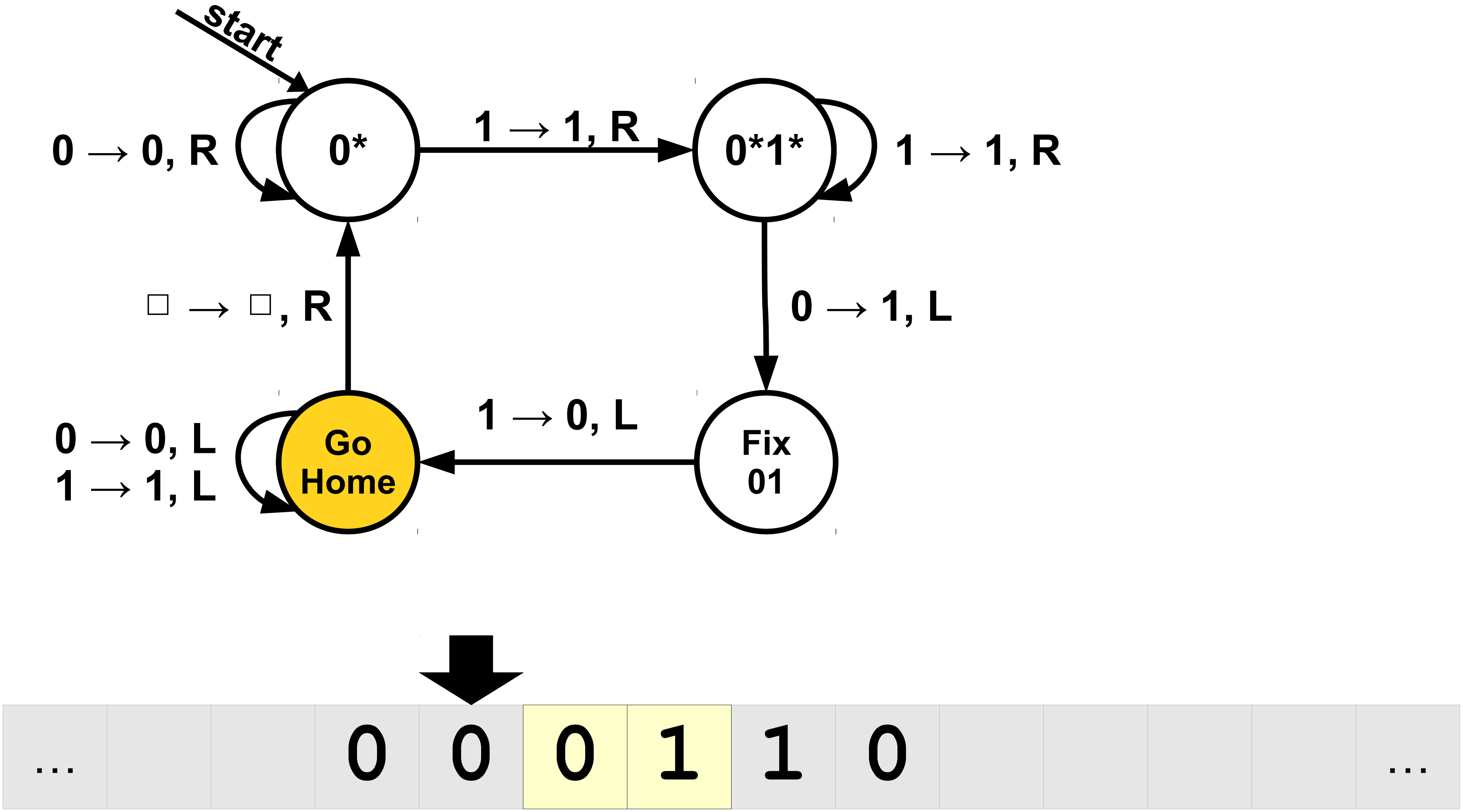


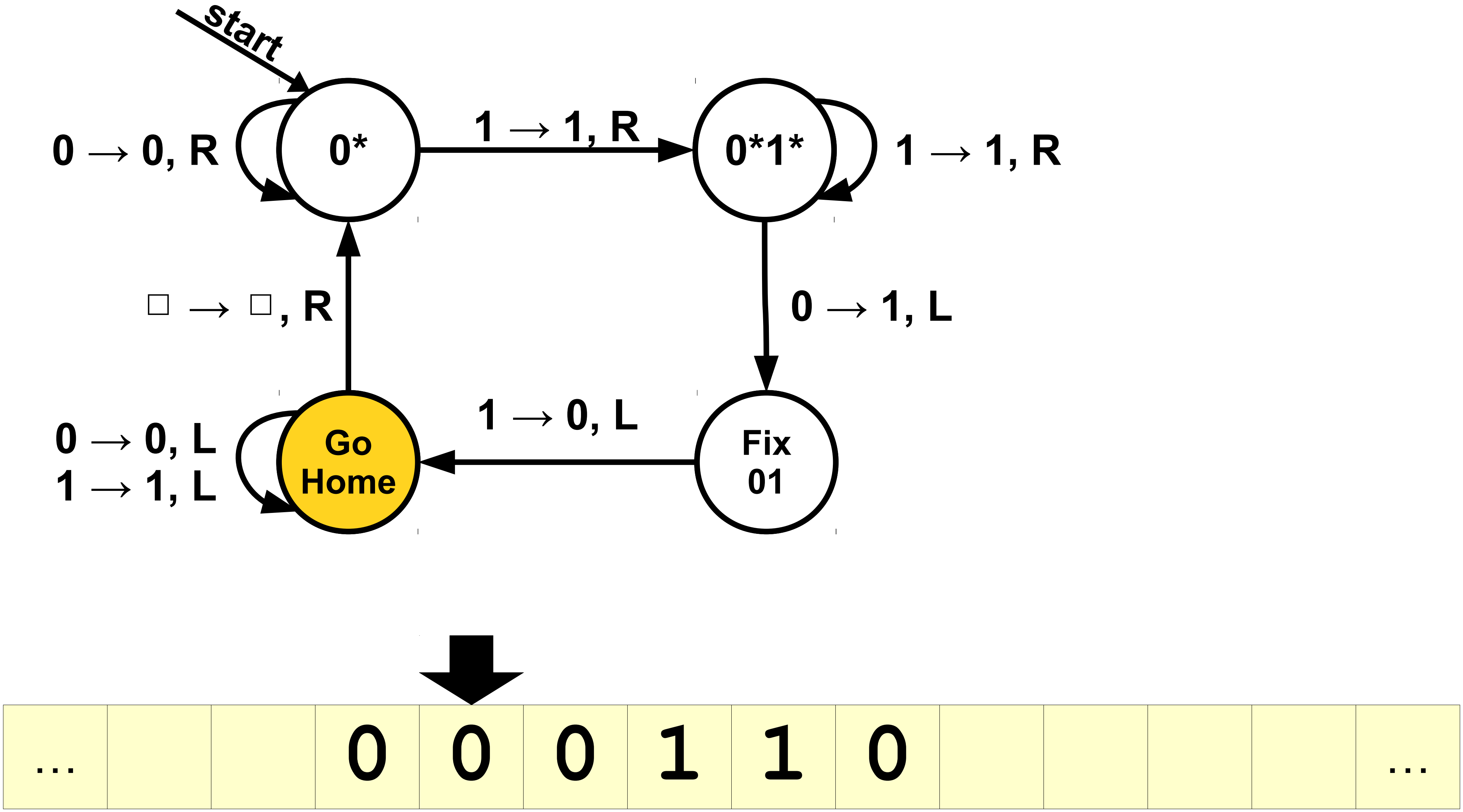
…			0	0	1	0	1	0					…



…			0	0	1	0	1	0					…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
1
1
1
0
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go
Home
1 → 0, L
Fix
01
… 0 0 0 1 1 0 …


start
0*
0 → 0, R
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
□ → □, R
Fix
01
1 → 0, L
Go
Home
0 → 0, L
1 → 1, L
…
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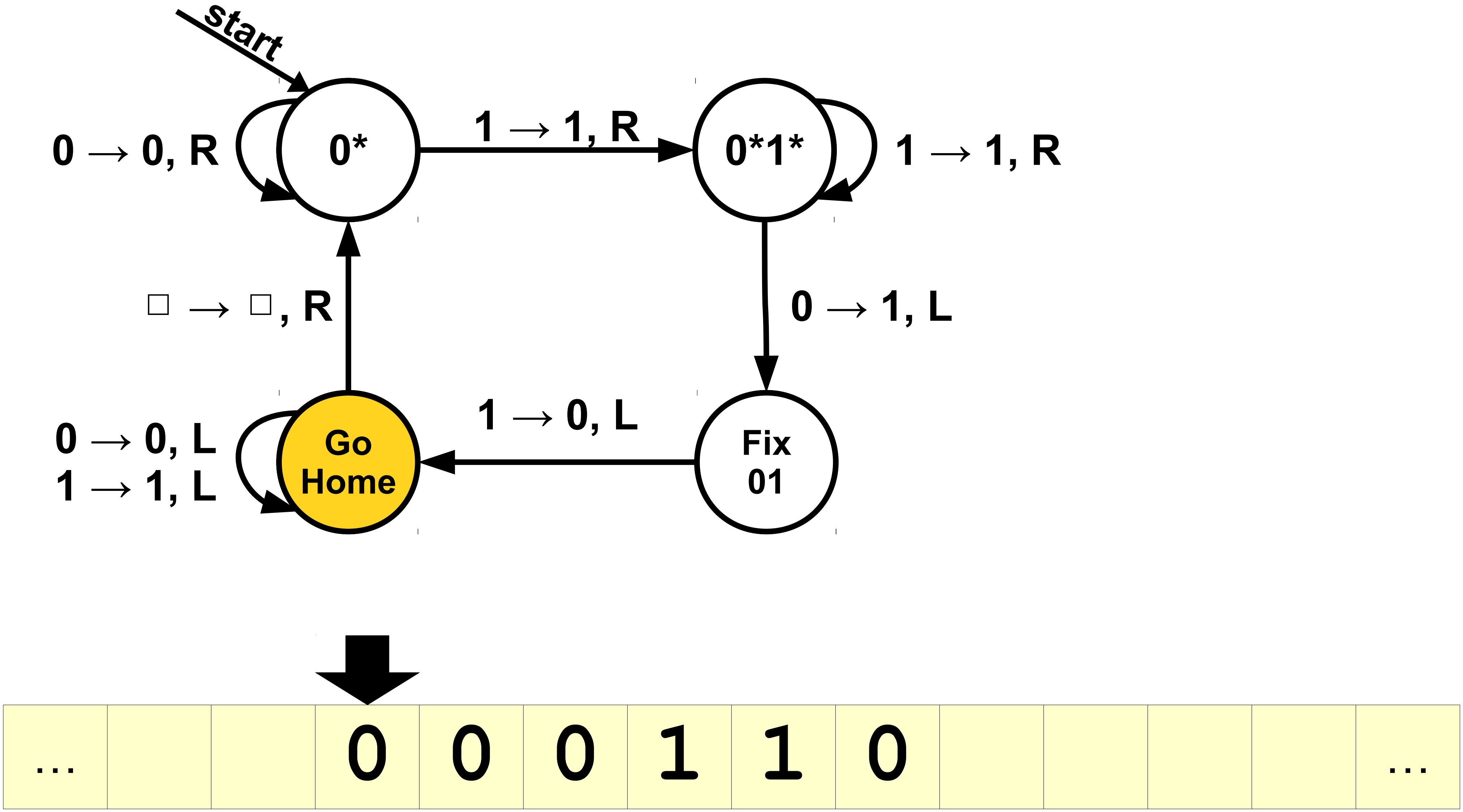
0
0
1
1
0
…


start
0*
0*1*
Fix 01
Go Home
0 → 0, R
1 → 1, R
1 → 1, R
0 → 1, L
1 → 0, L
0 → 0, L
1 → 1, L
□ → □, R
…
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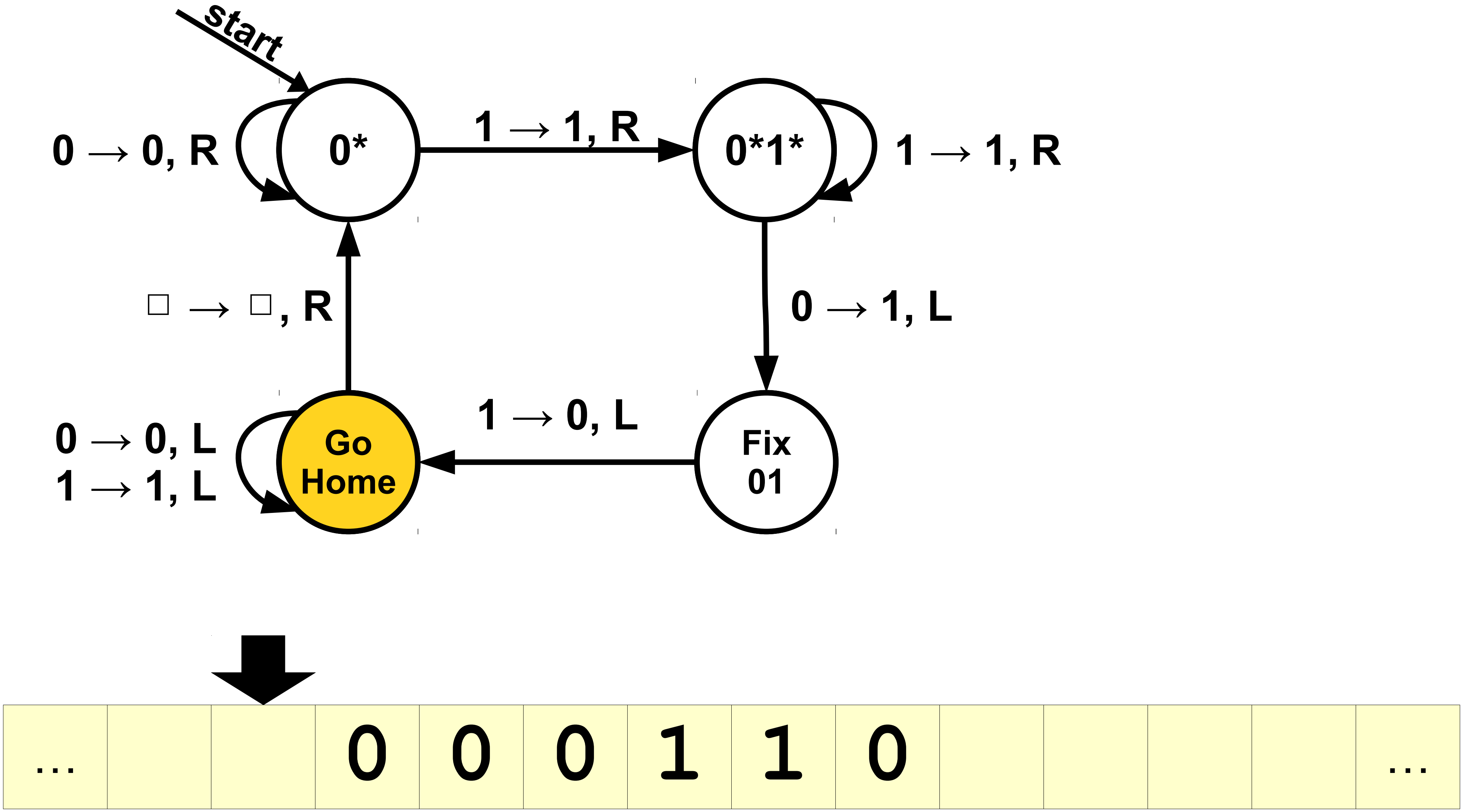
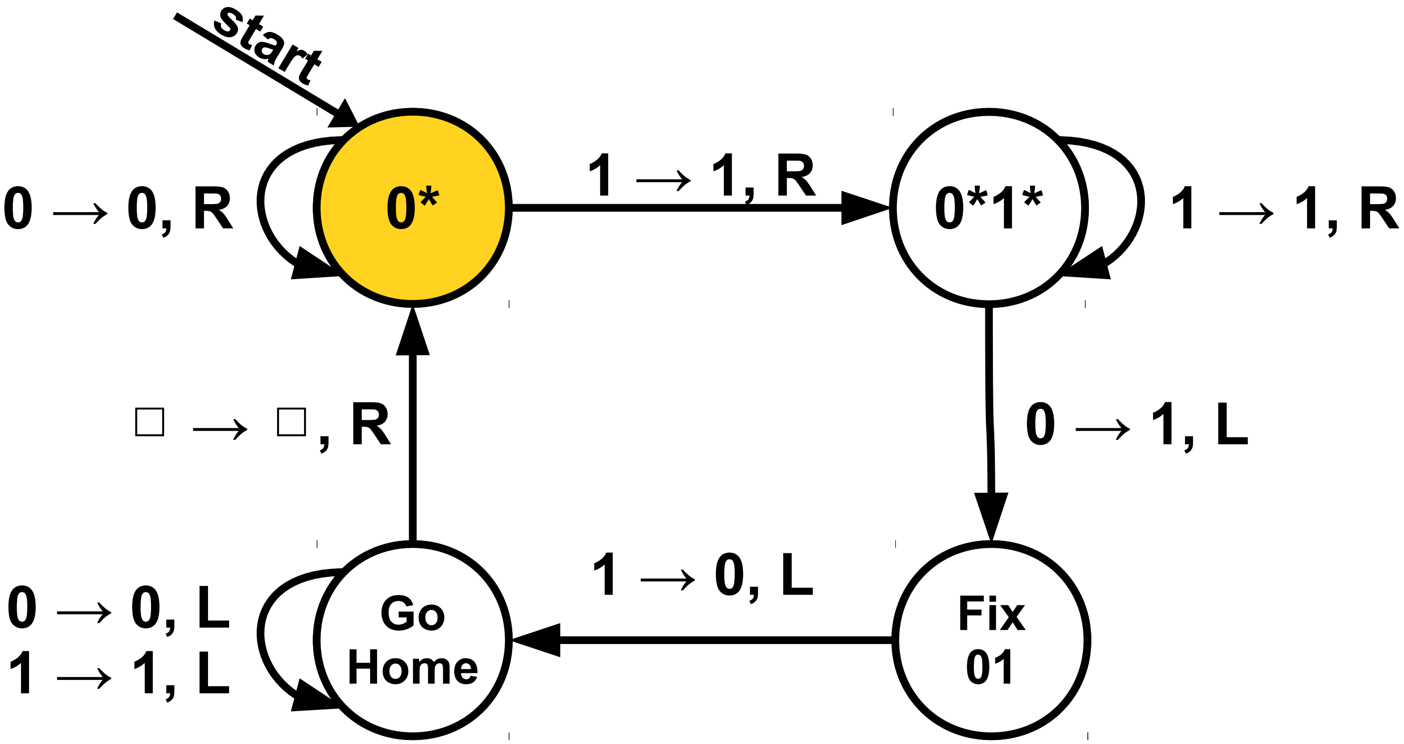
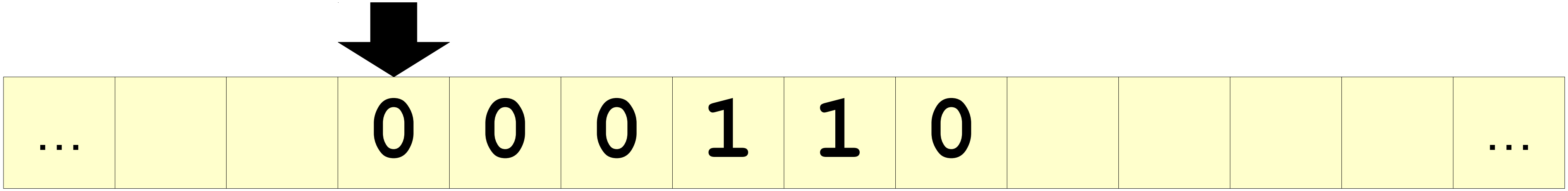
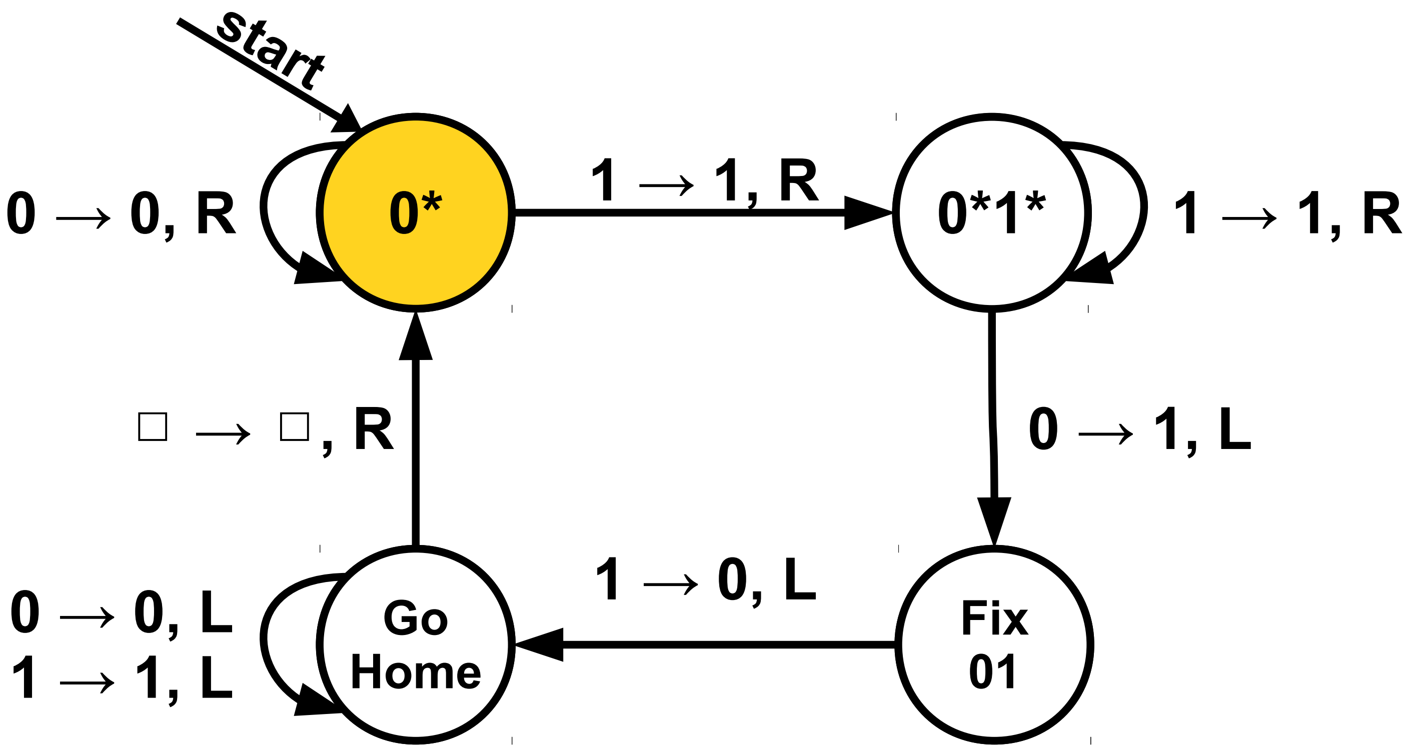
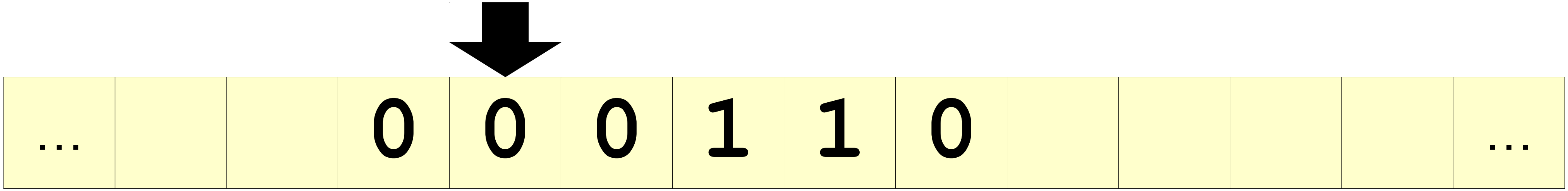
0
0
1
1
0
…

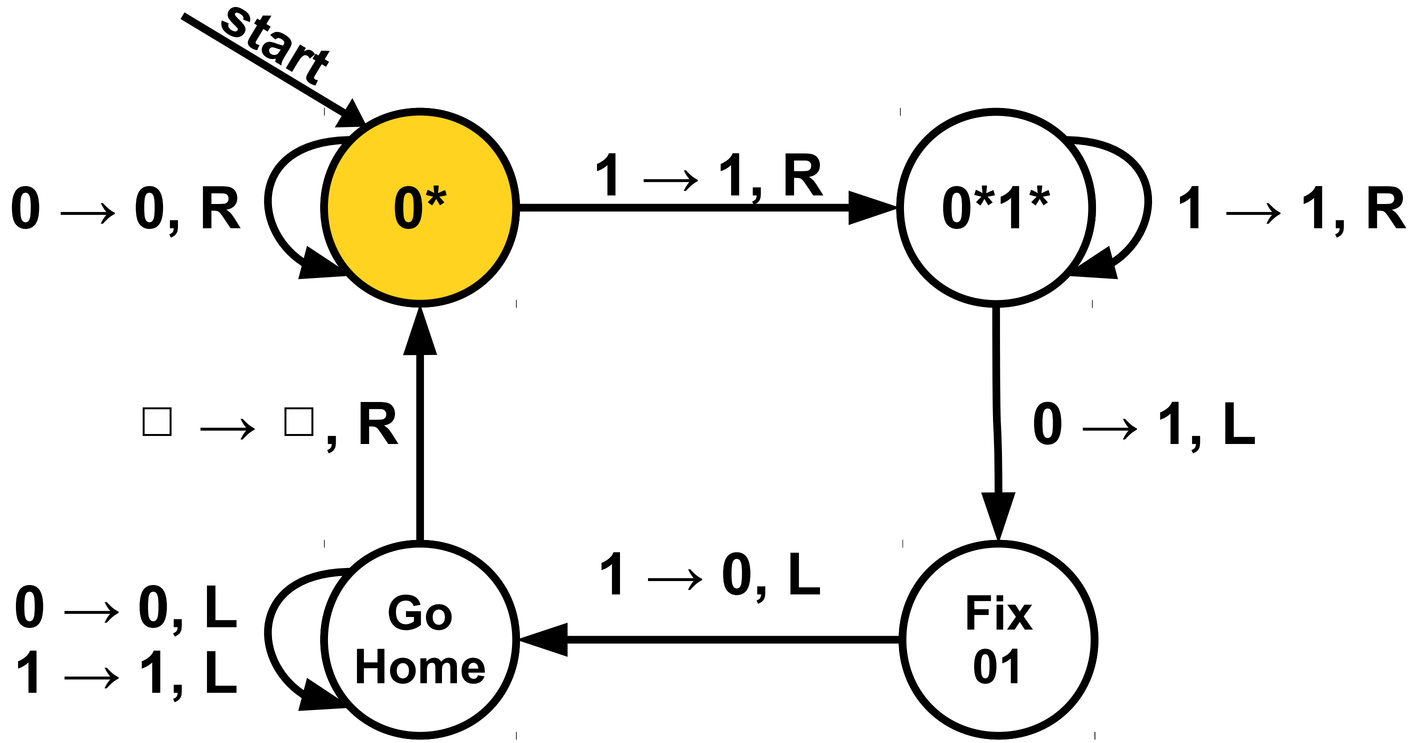

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
1
1
0
…

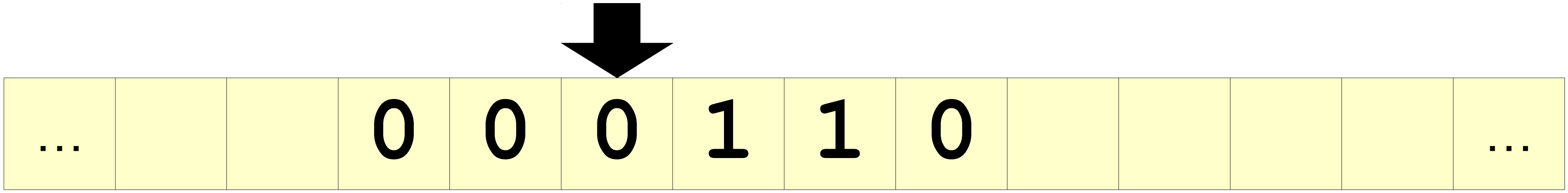

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01


…
0
0
0
1
1
0
…

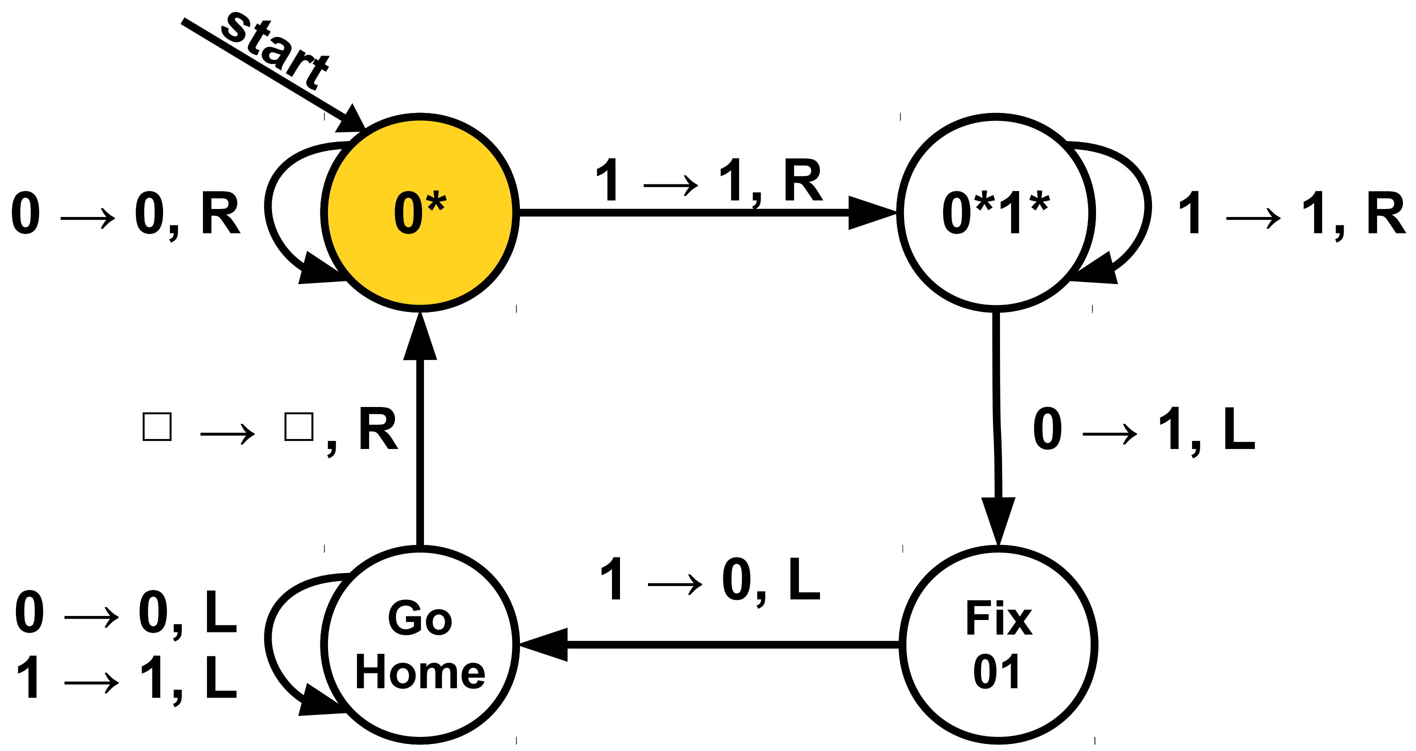

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01

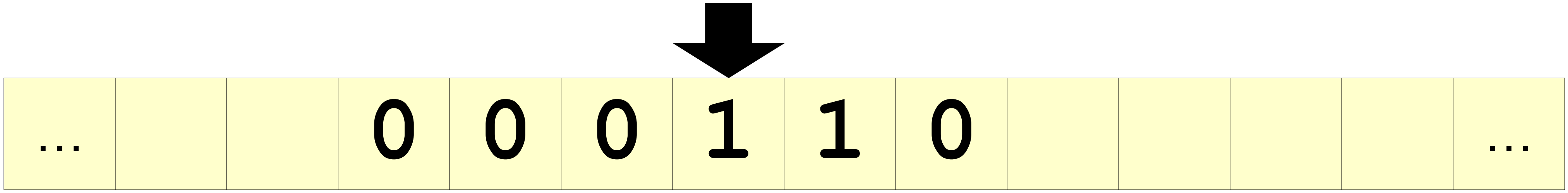

…
0
0
0
1
1
0
…



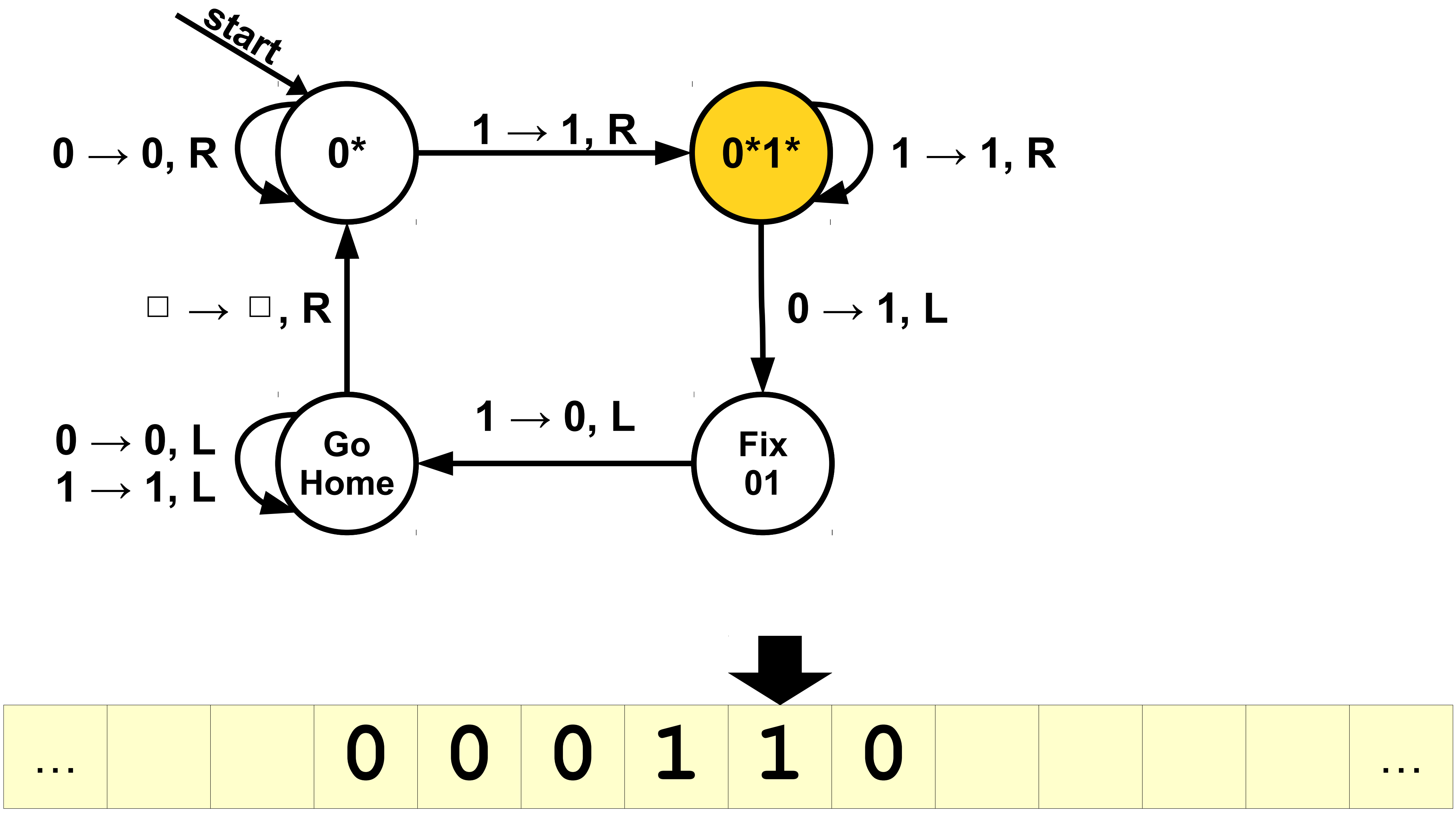


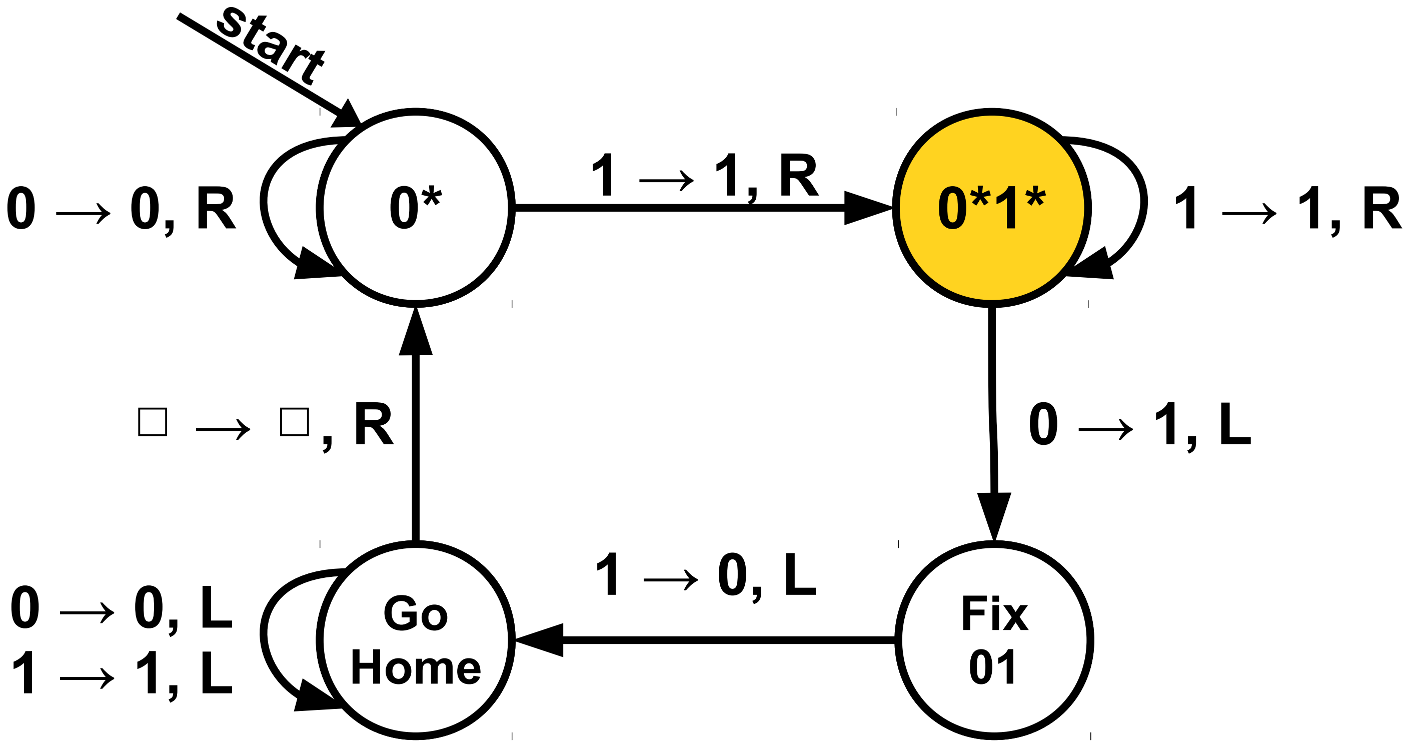
…			0	0	0	1	1	0					…

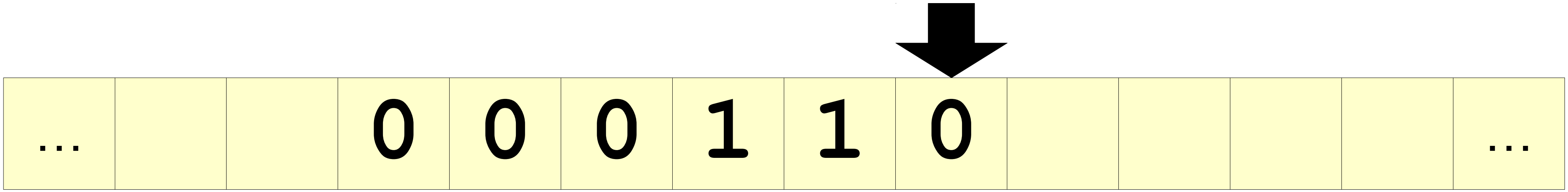




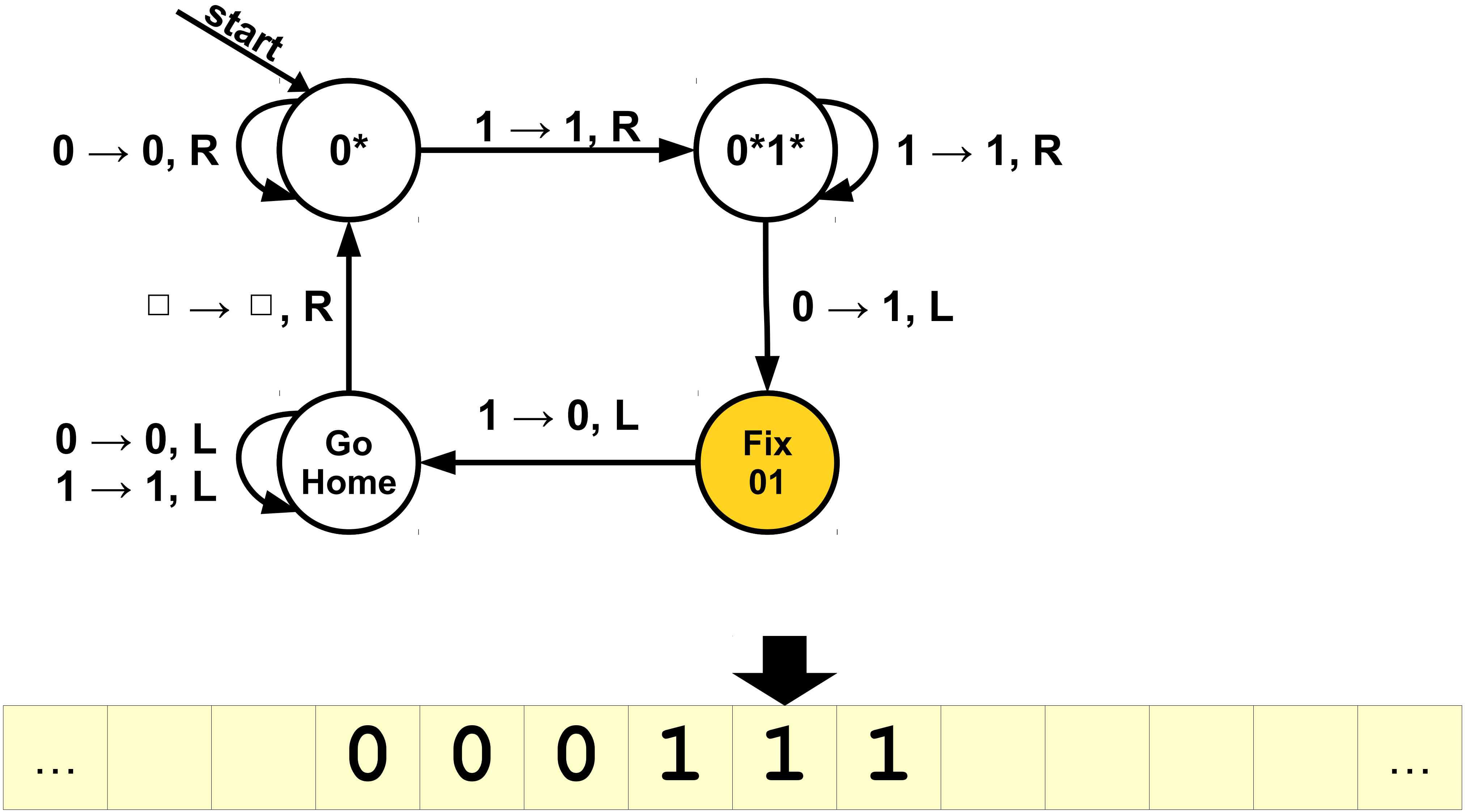
…			0	0	0	1	1	0					…

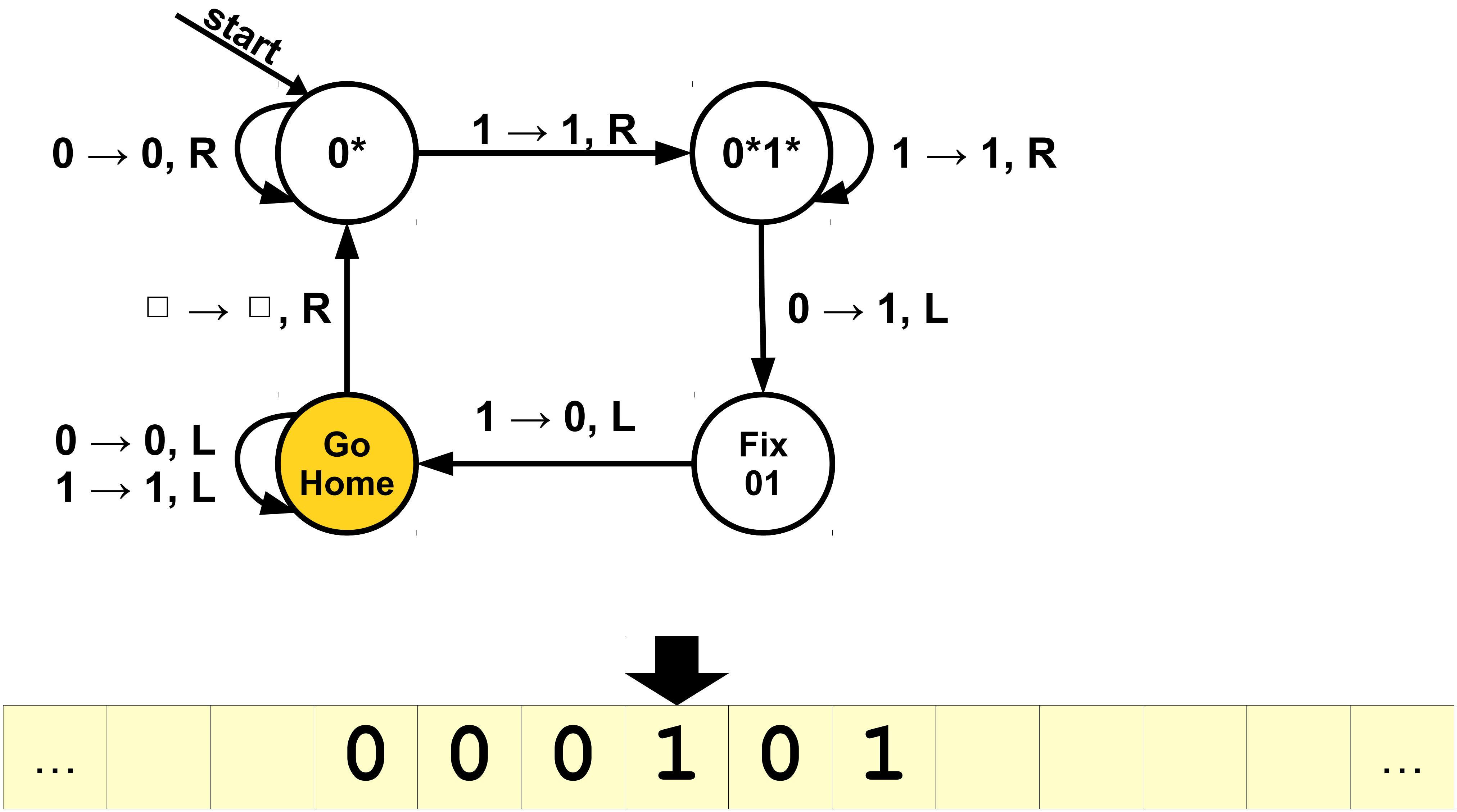

start
0*
0 → 0, R
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
□ → □, R
Fix 01
1 → 0, L
Go Home
0 → 0, L
1 → 1, L
...
0
0
0
1
1
0
...

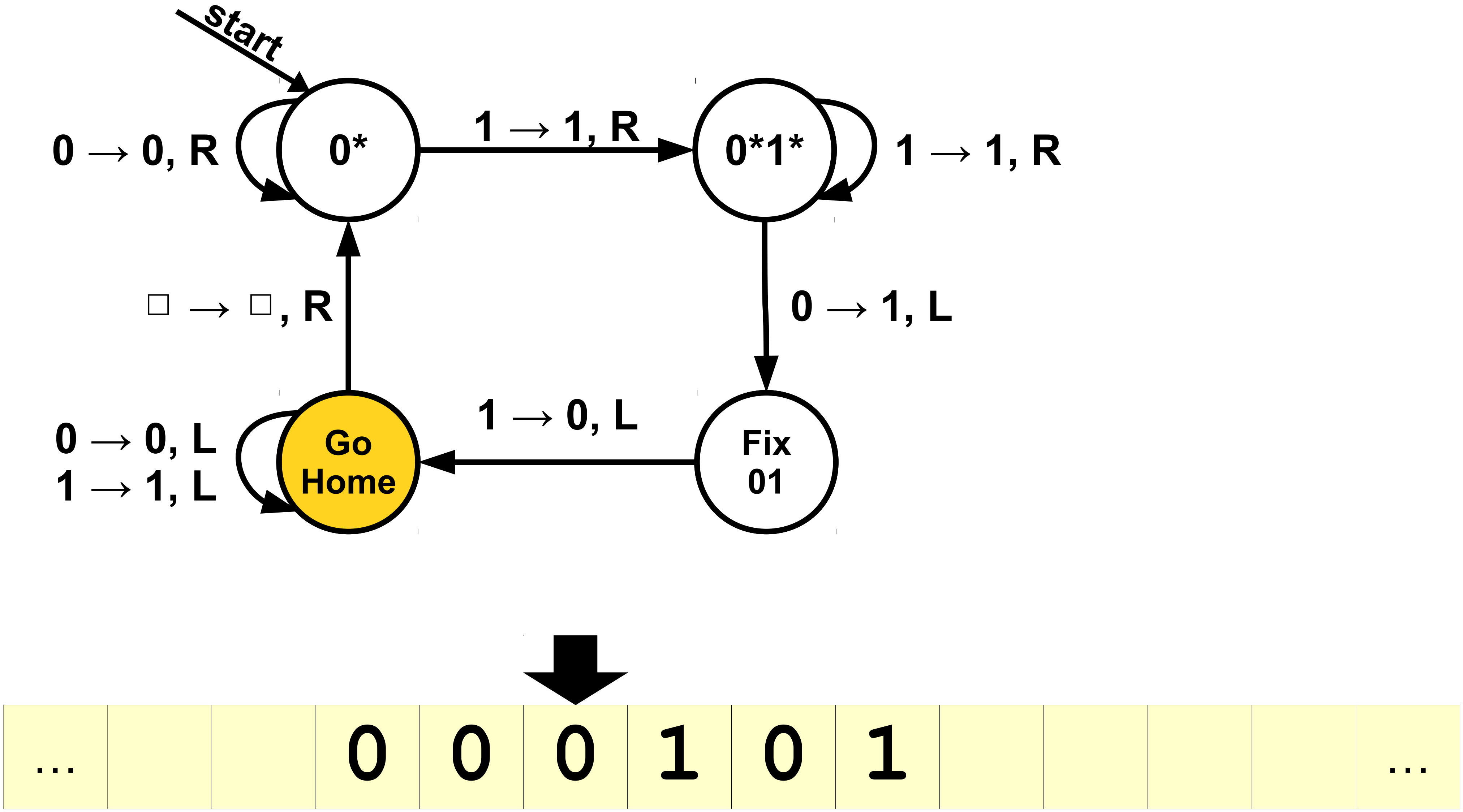


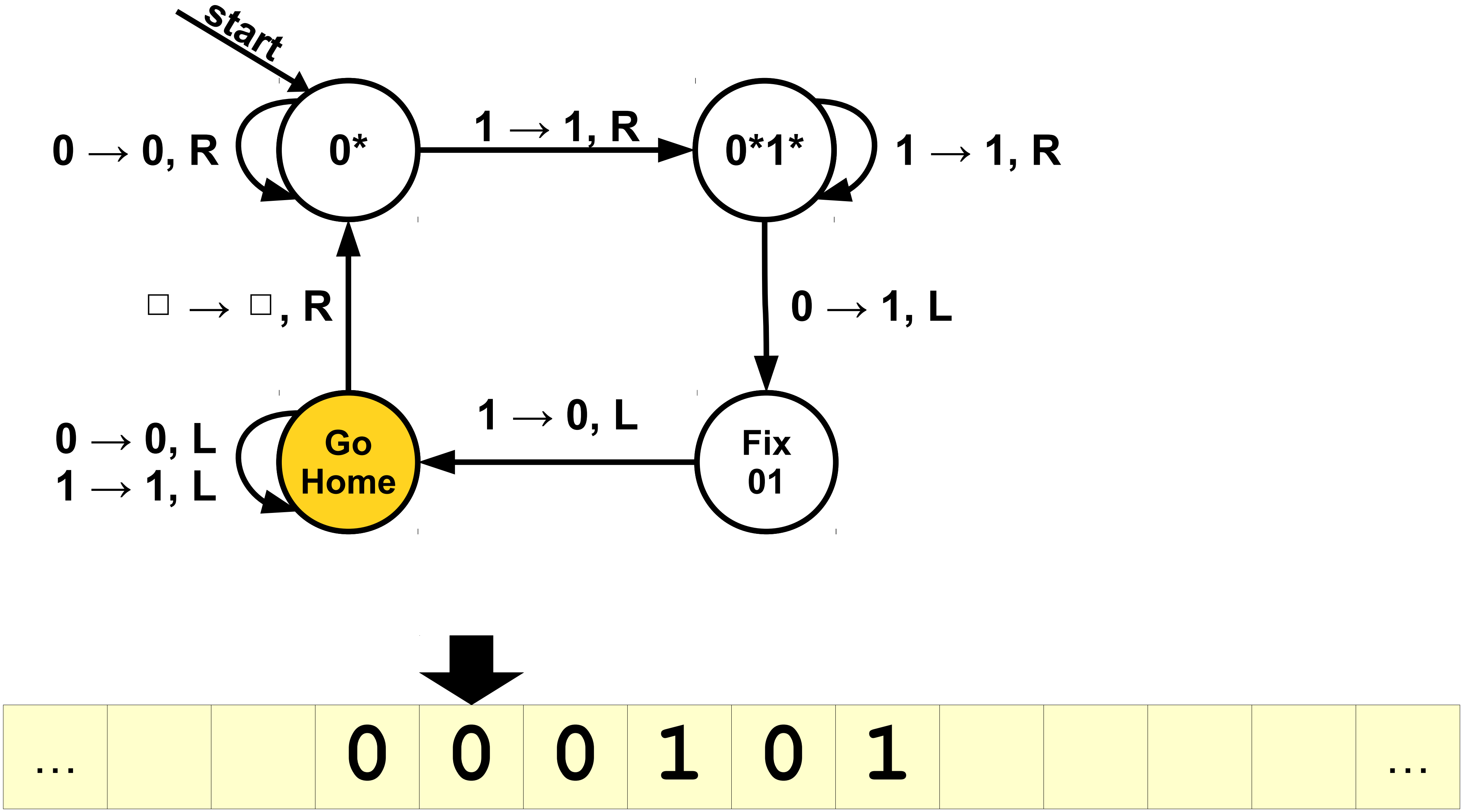


…			0	0	0	1	1	0					…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
1
1
1
…


start
0*
0*1*
Go Home
Fix 01
0 → 0, R
1 → 1, R
1 → 1, R
0 → 1, L
1 → 0, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
0
1
0
1
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
… 0 0 0 1 0 1 …


start
0*
0 → 0, R
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
□ → □, R
Fix
01
1 → 0, L
Go
Home
0 → 0, L
1 → 1, L
…
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0
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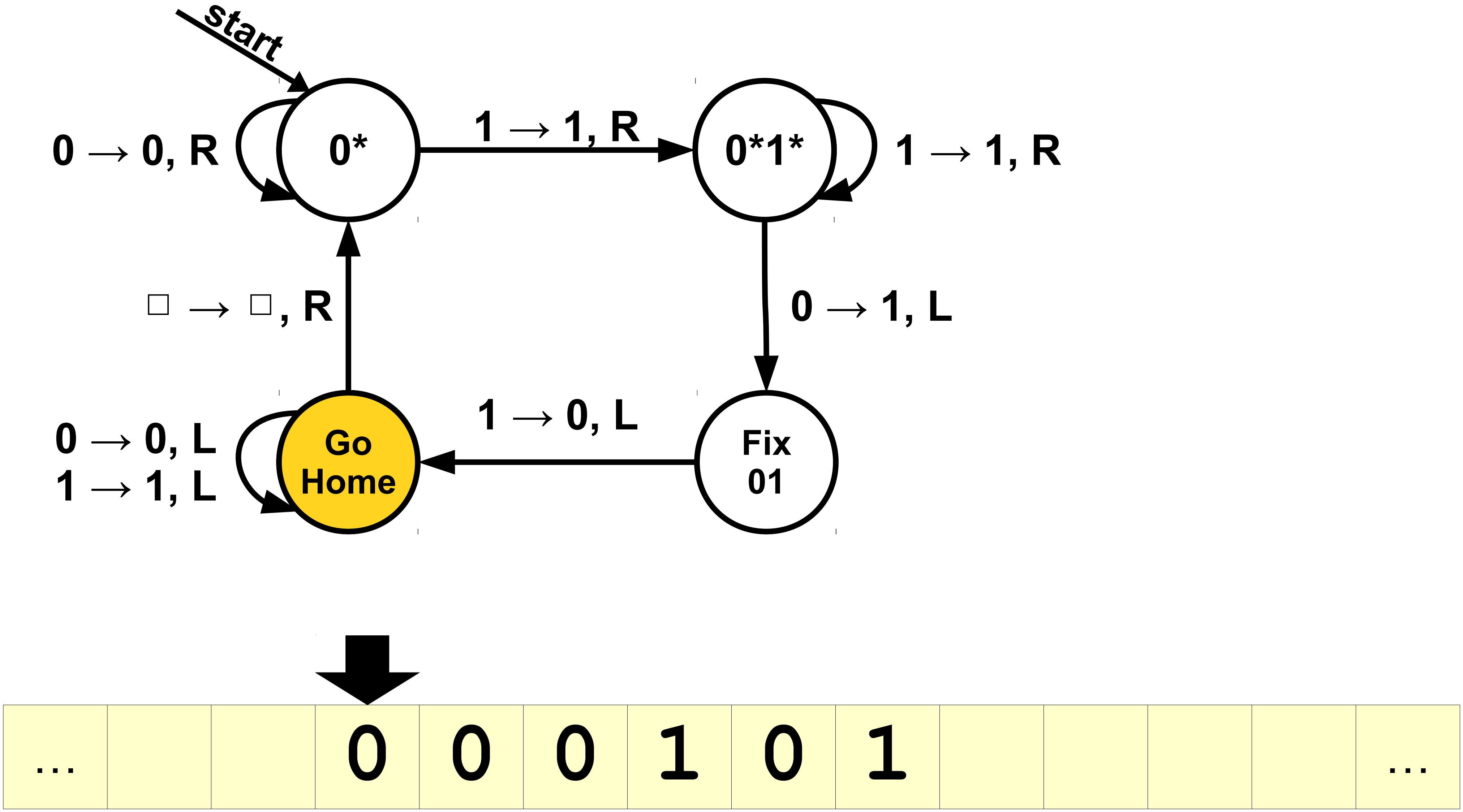
1
0
1
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
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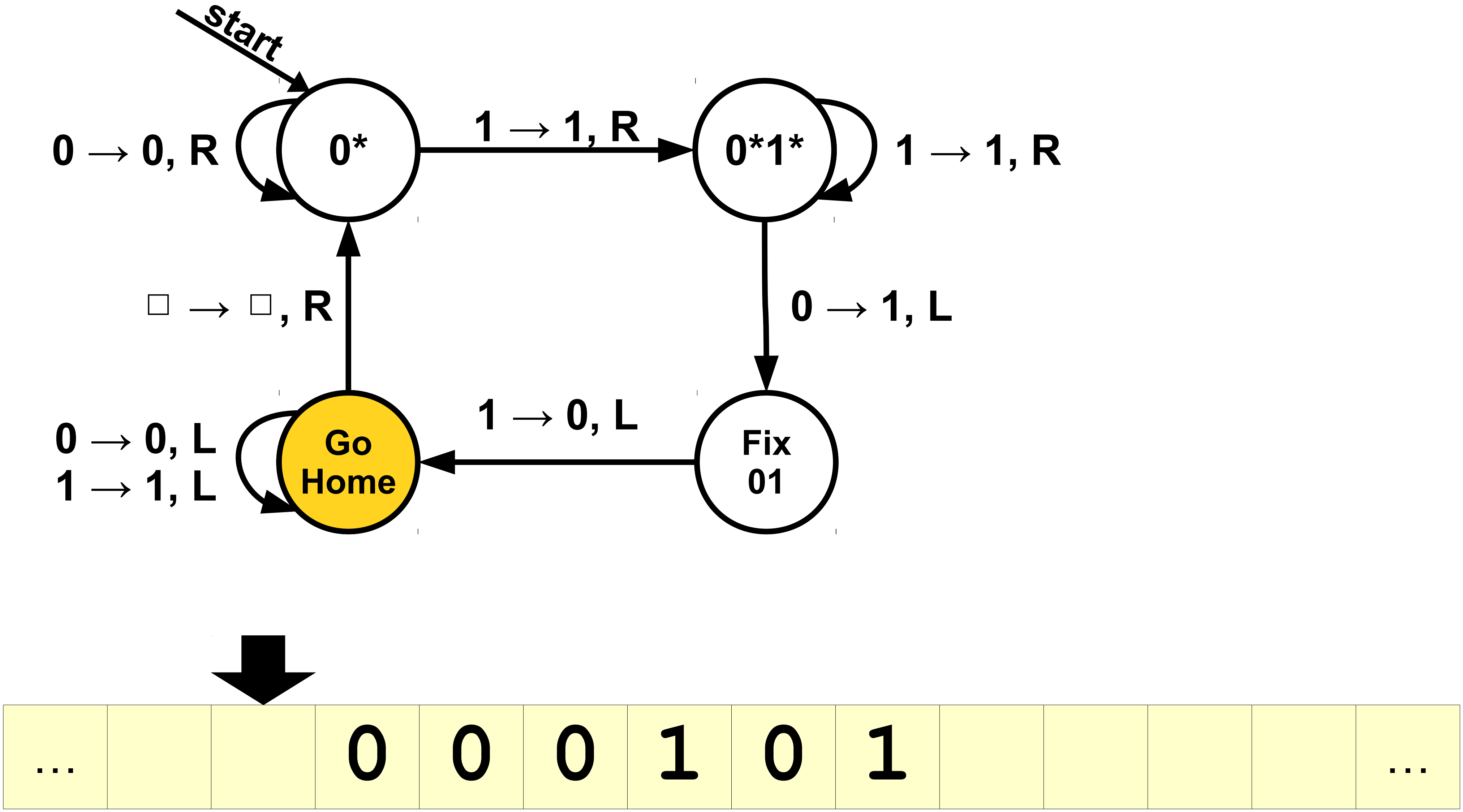
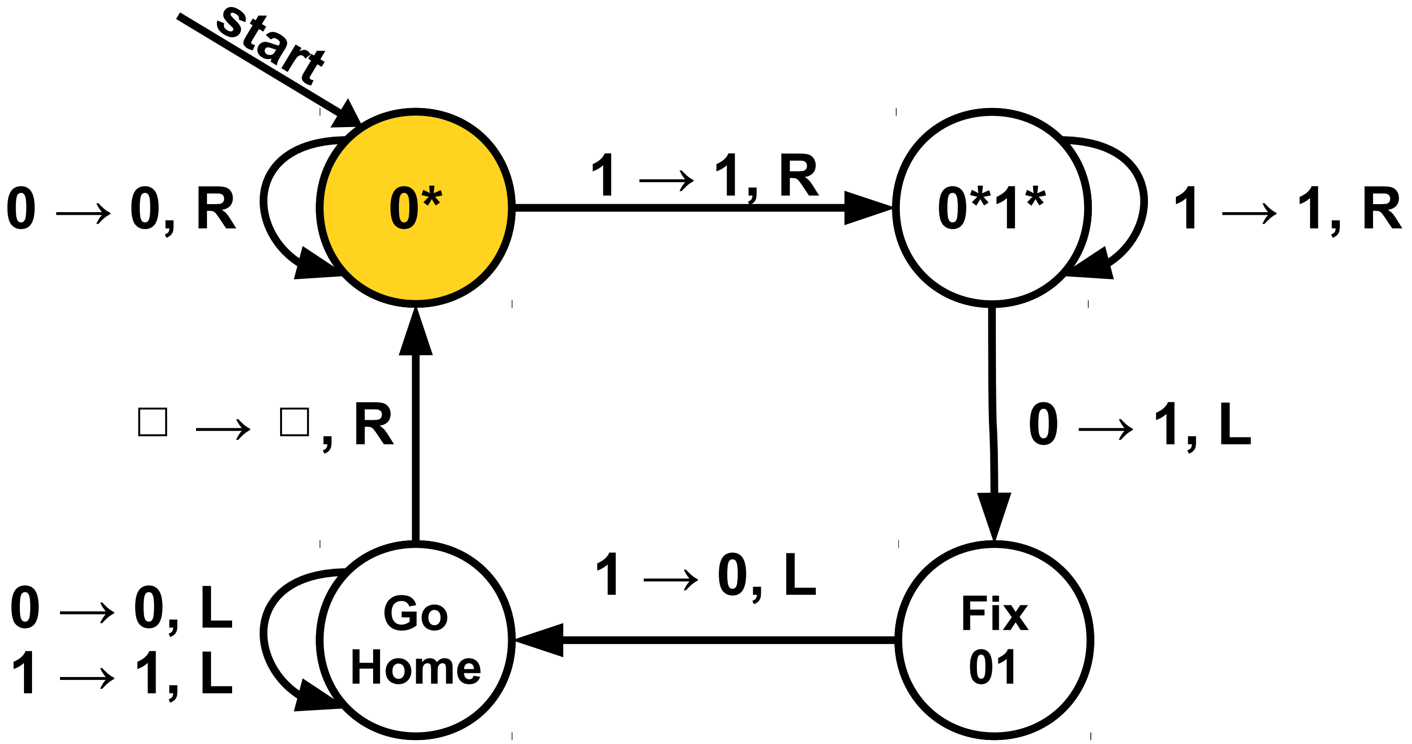
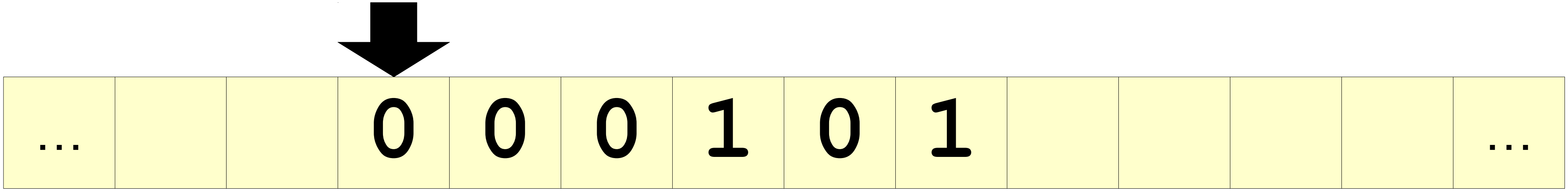
0
1
0
1
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
1
0
1
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01


…
0
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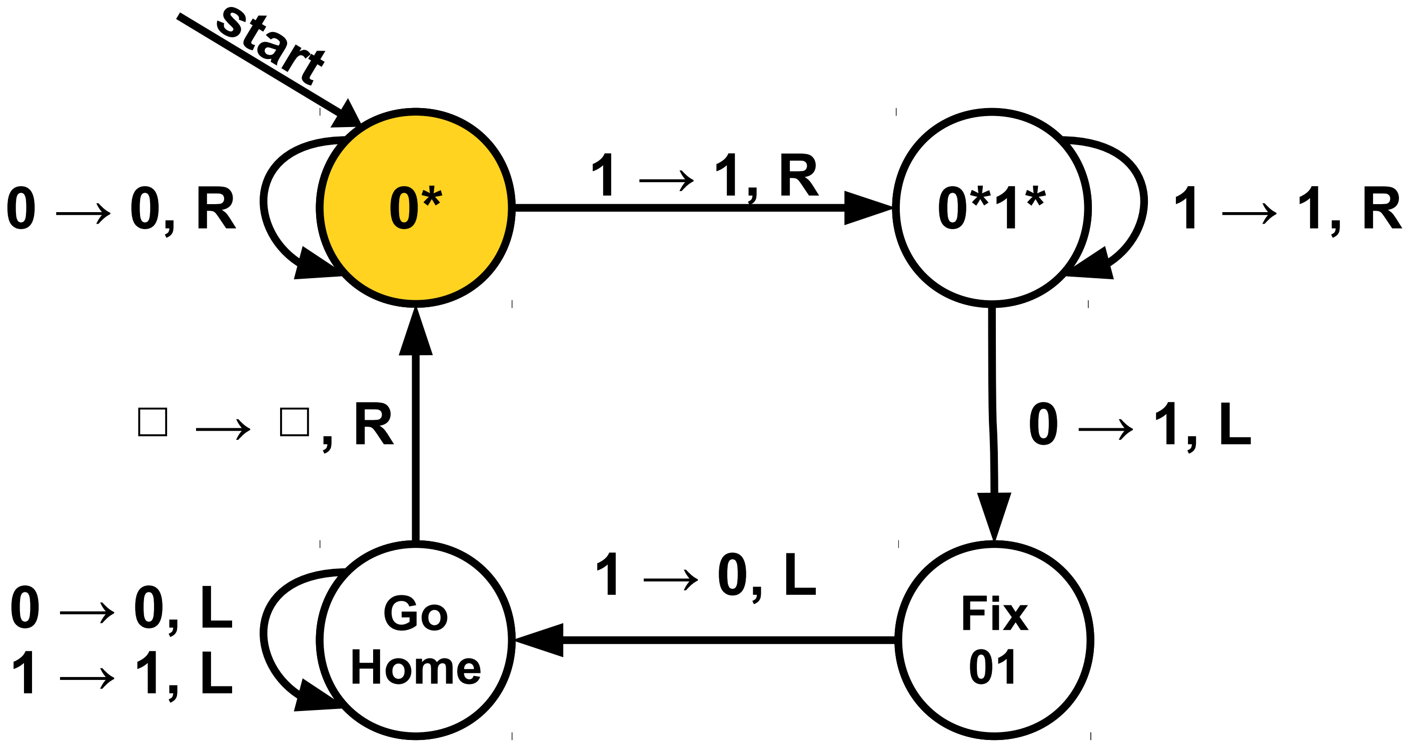
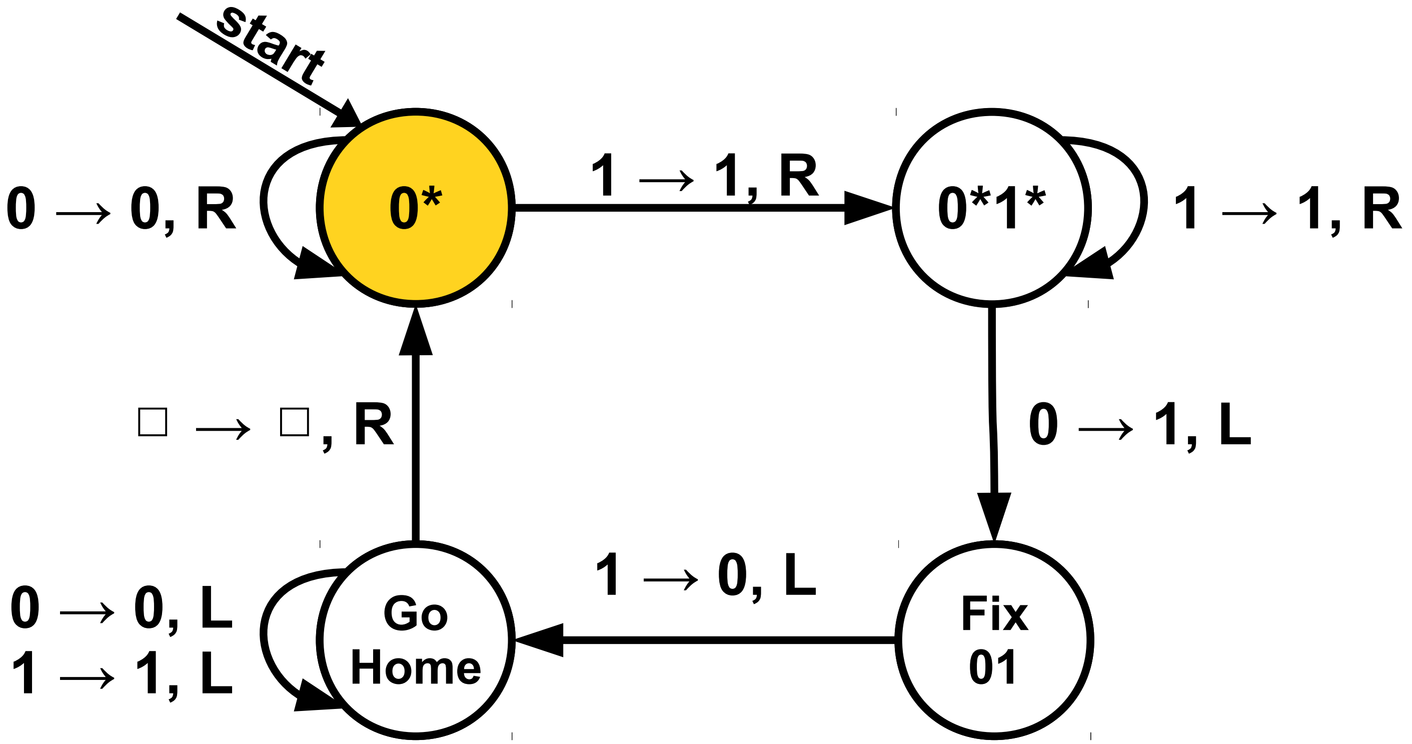
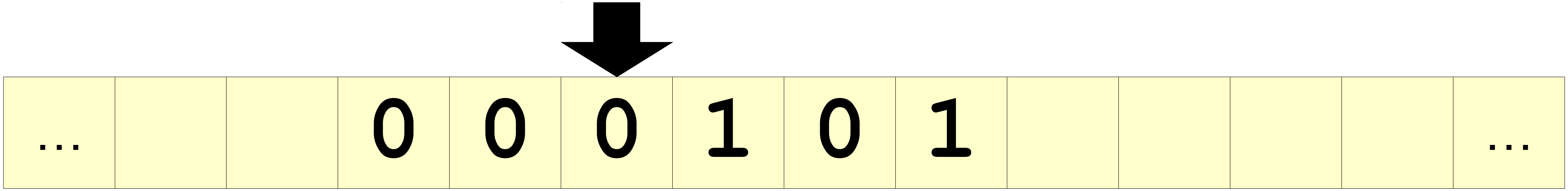
1
0
1
…

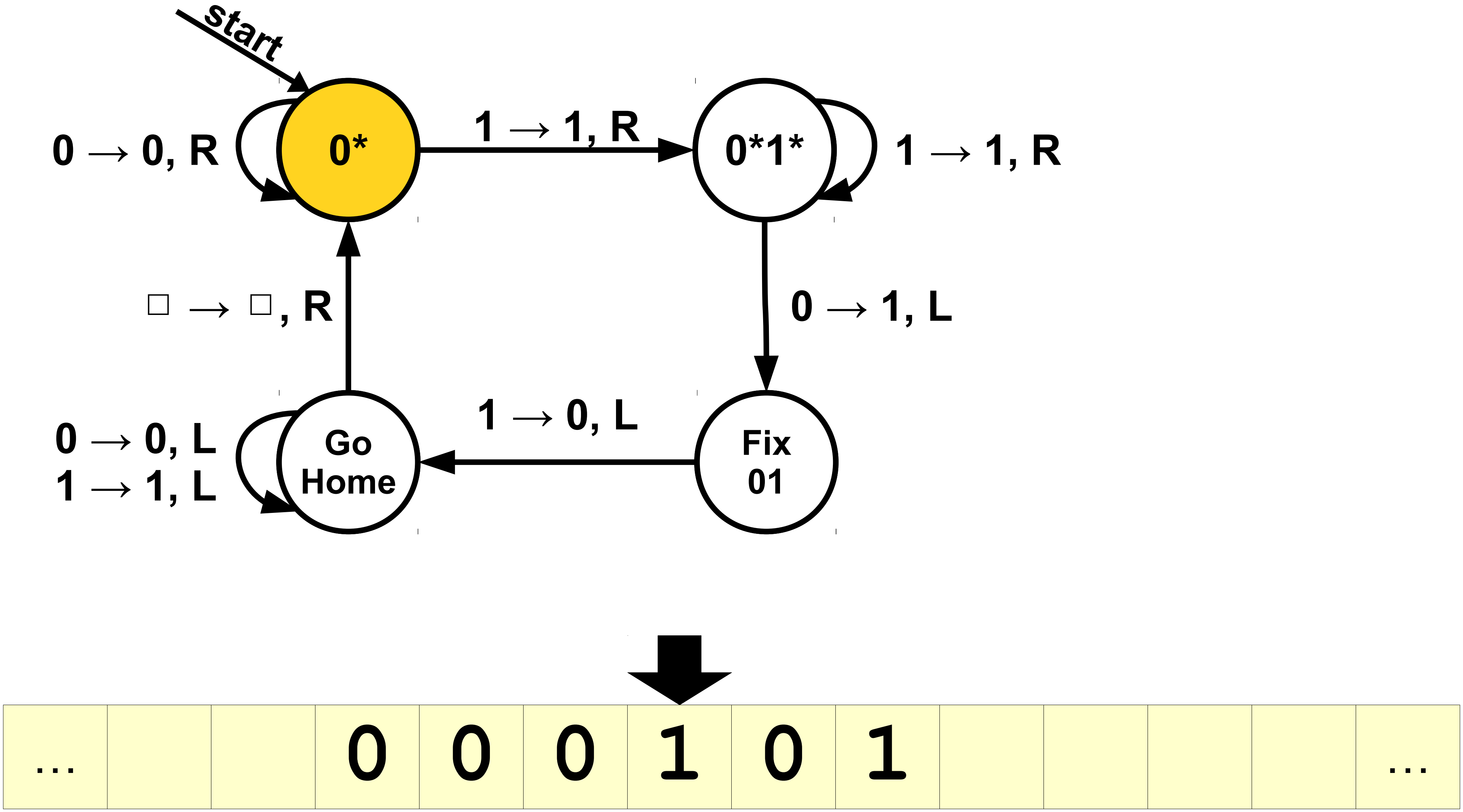


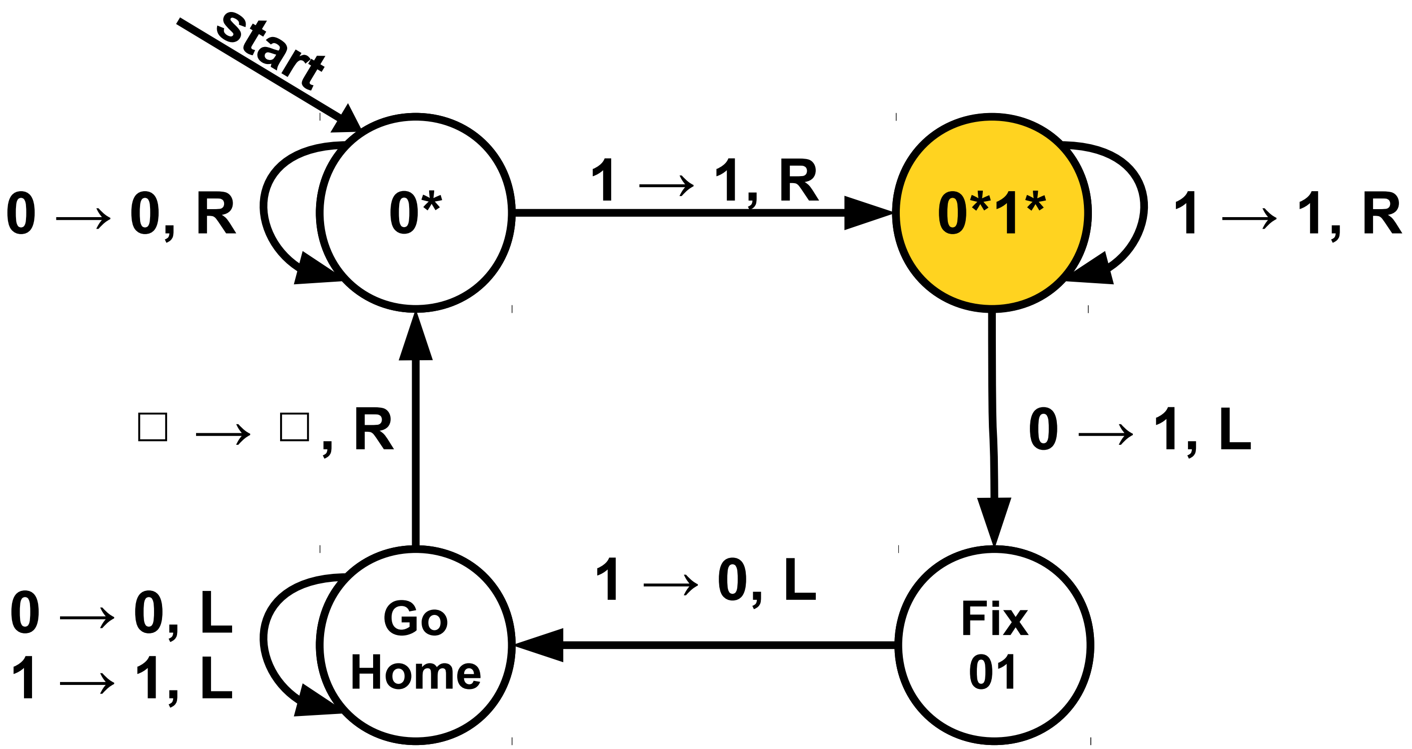
…			0	0	0	1	0	1					…

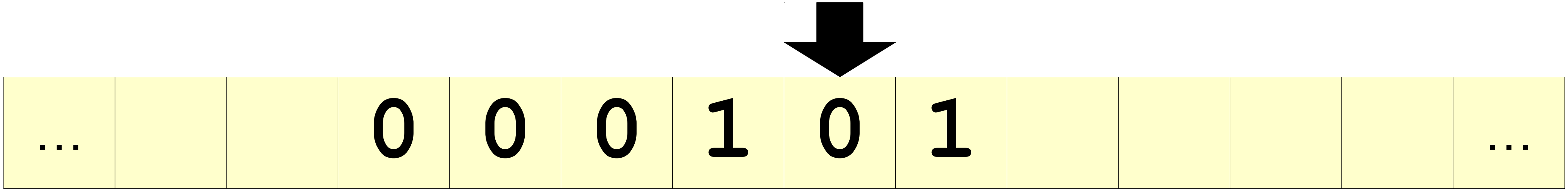


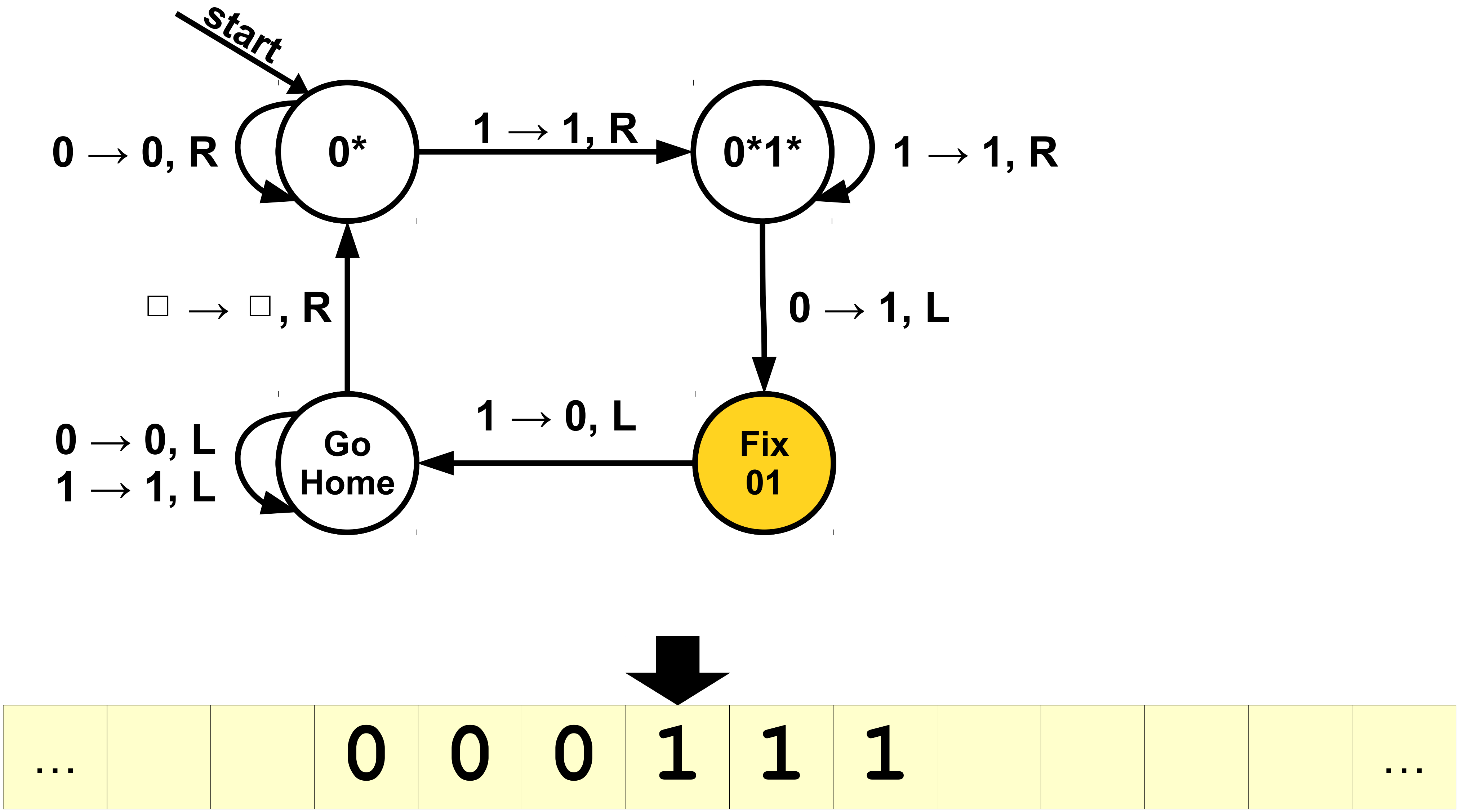


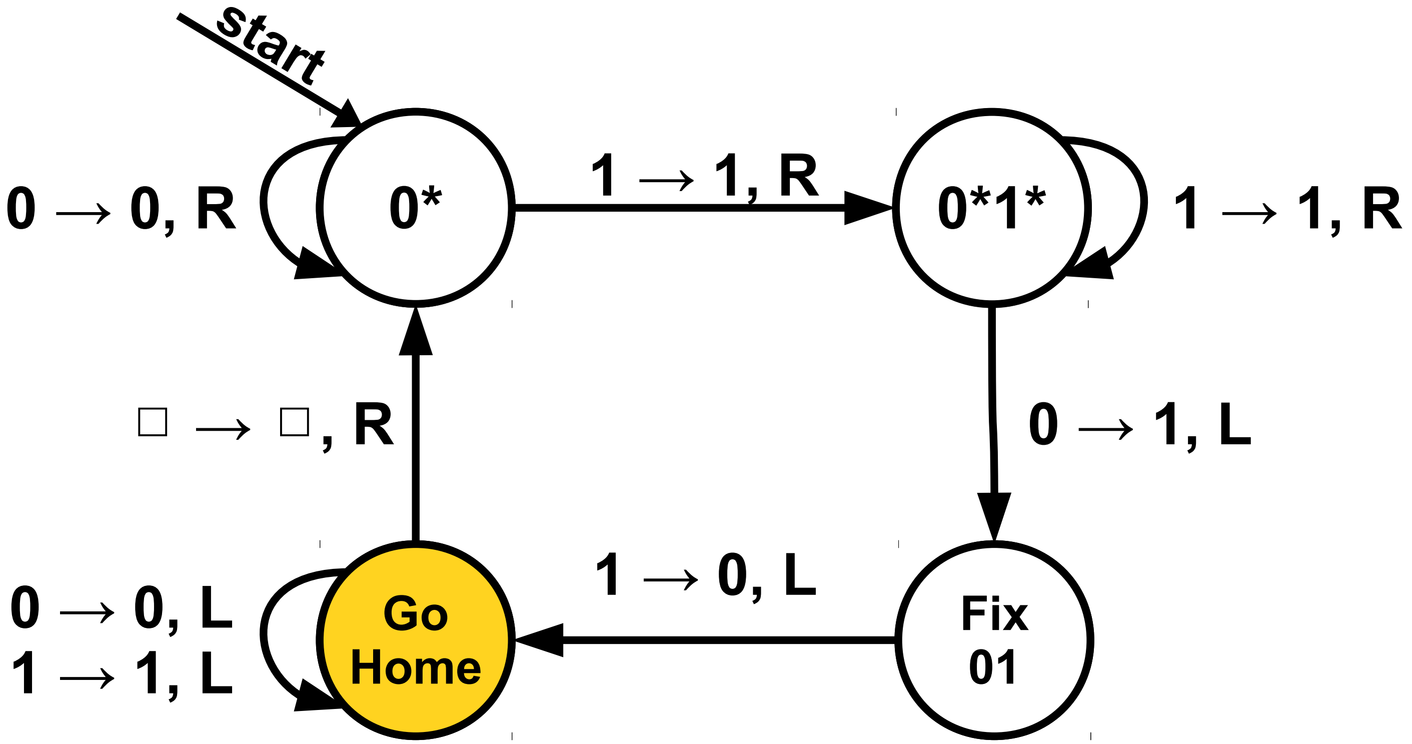
…			0	0	0	1	0	1					…

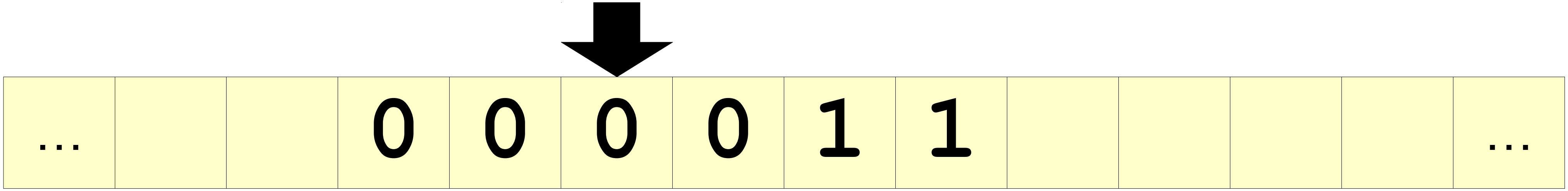

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
1
0
1
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
1 → 0, L
0 → 0, L
1 → 1, L
Go Home
Fix 01

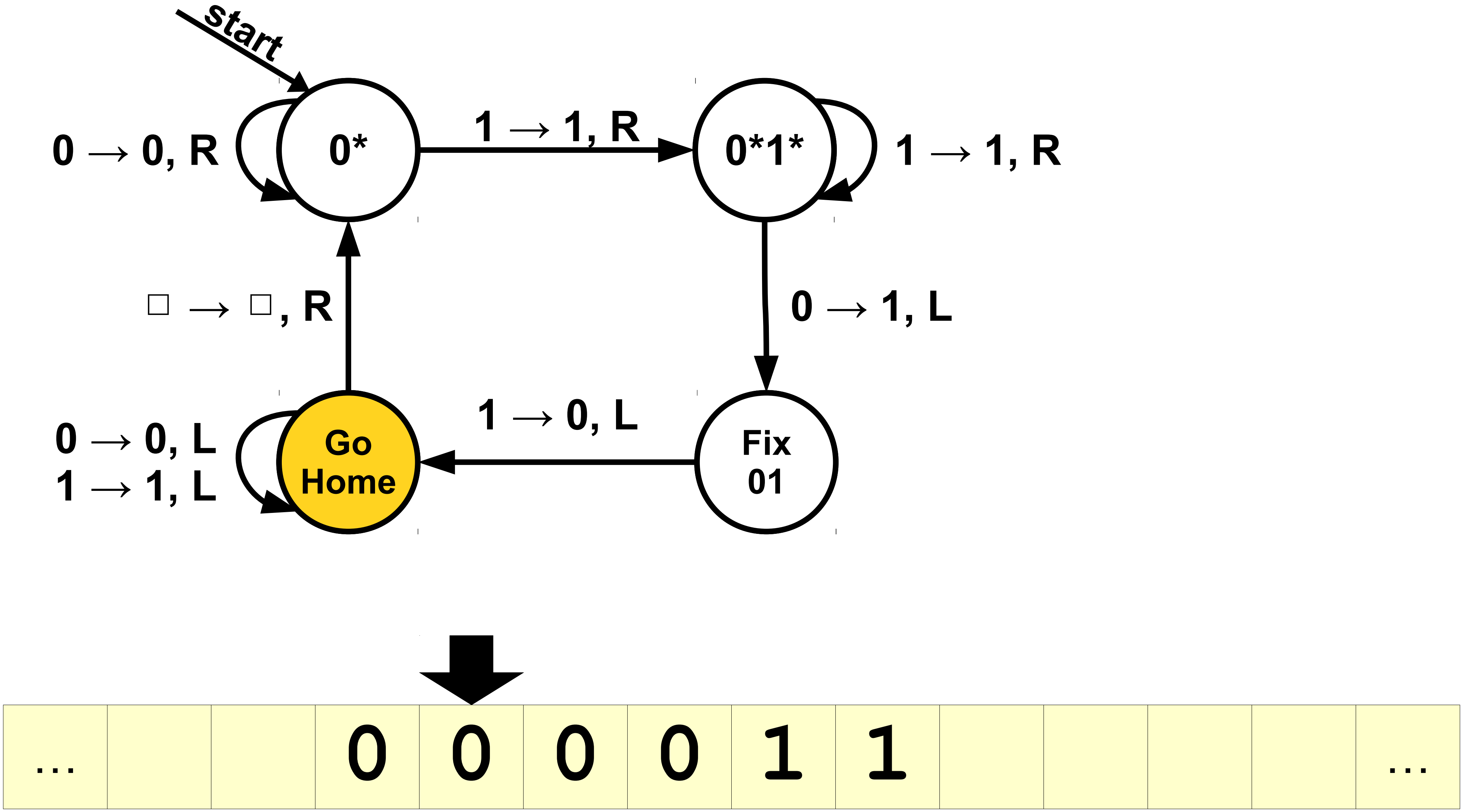

…
0
0
0
1
0
1
…

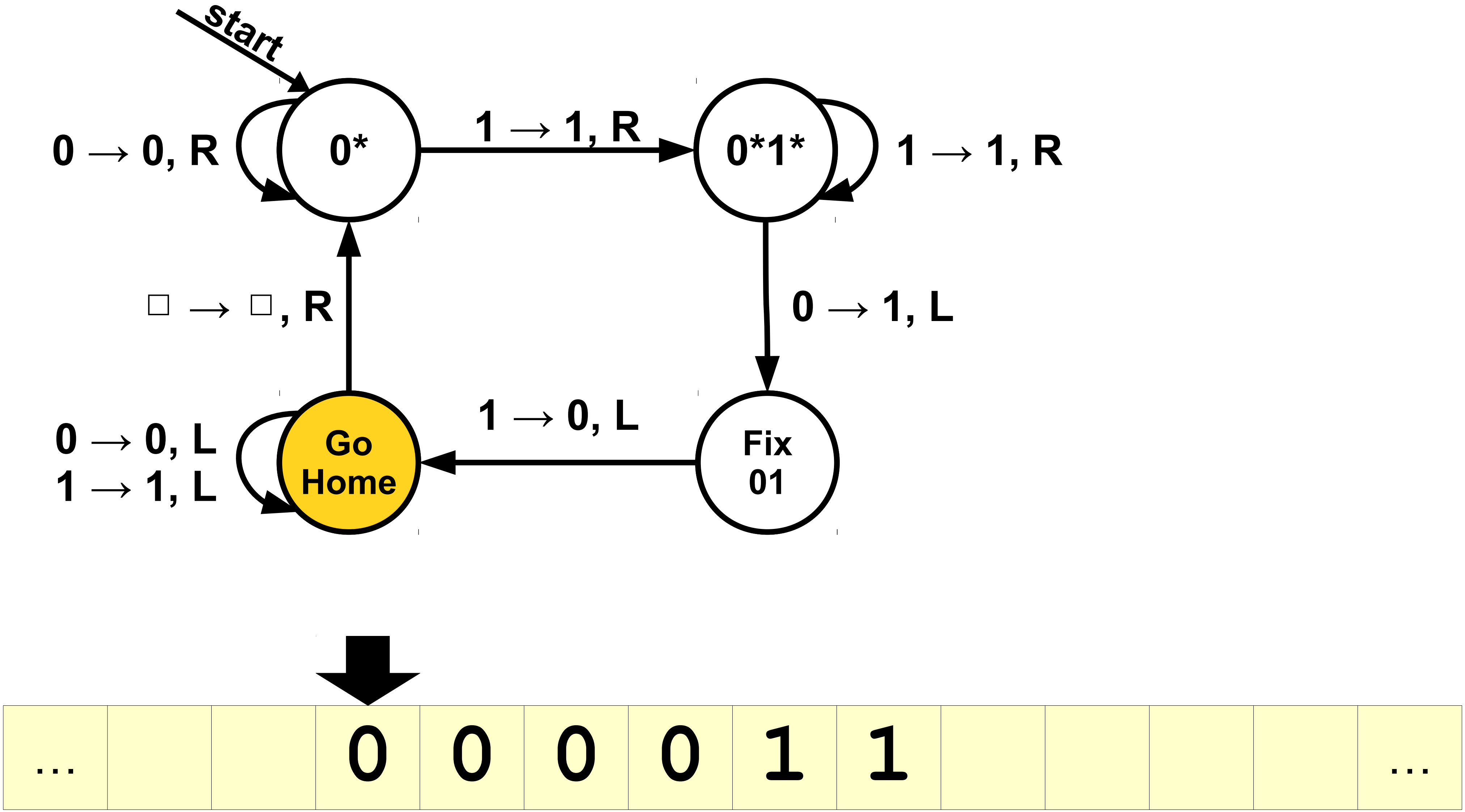

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
1
1
1
…

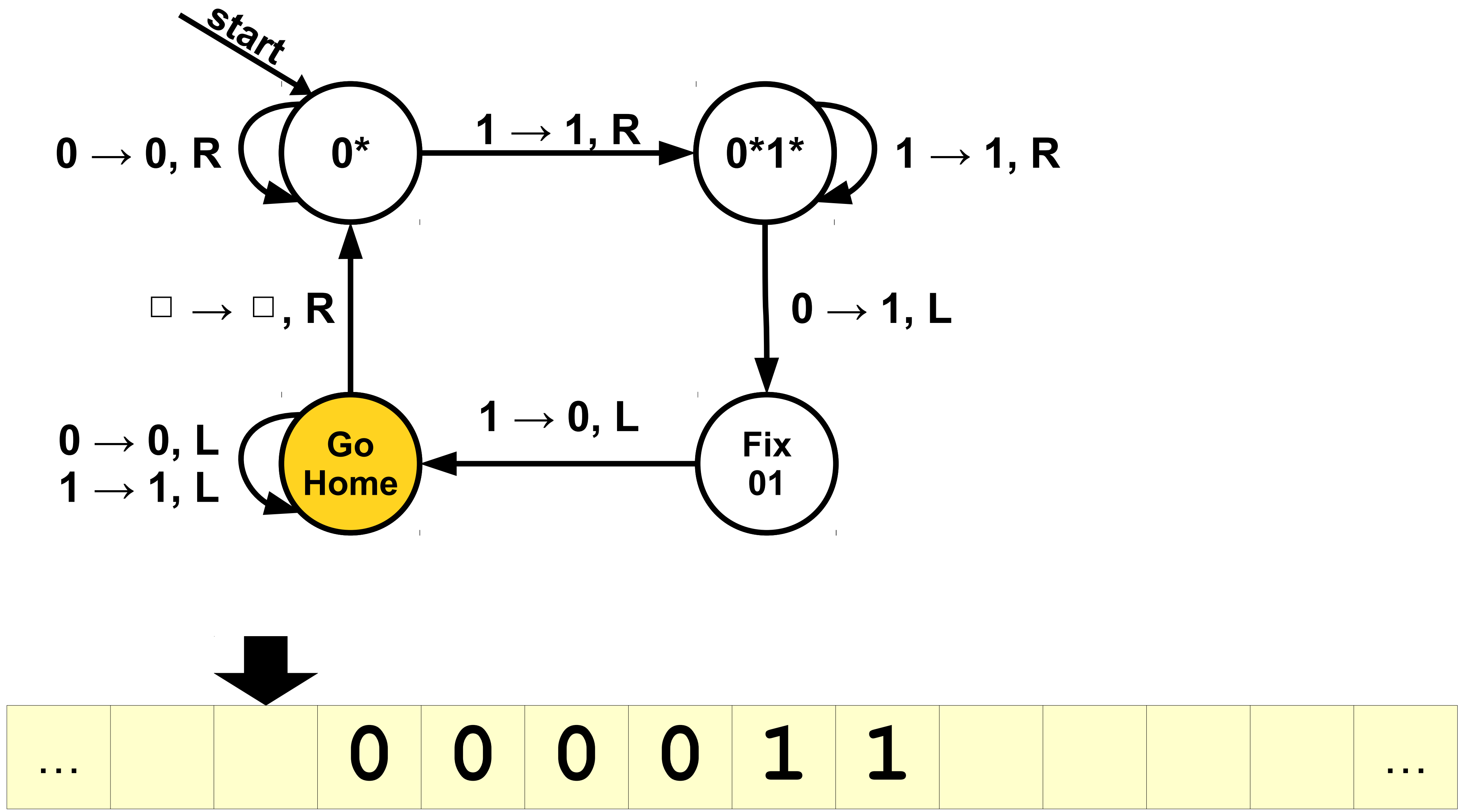




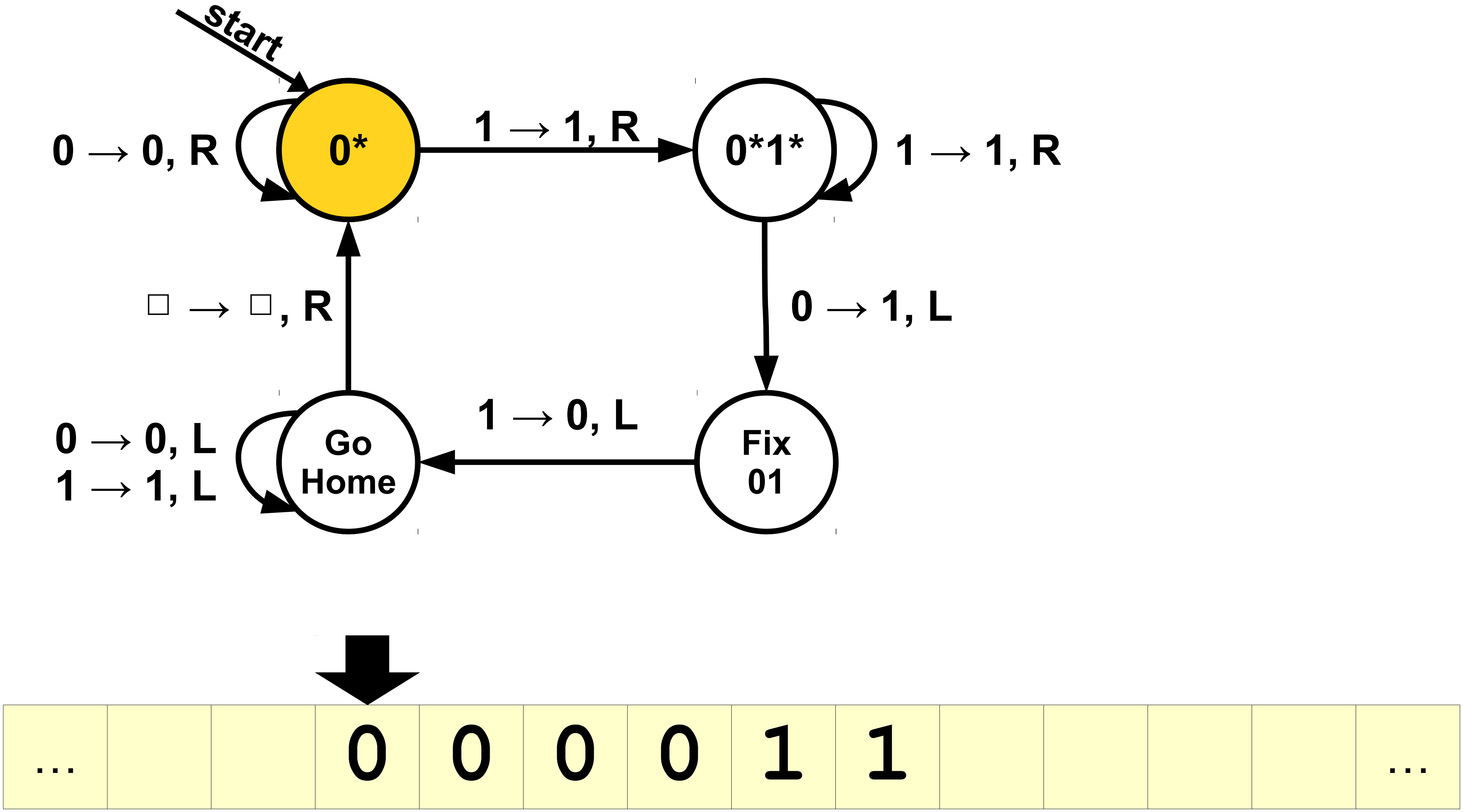
…			0	0	0	0	1	1					…

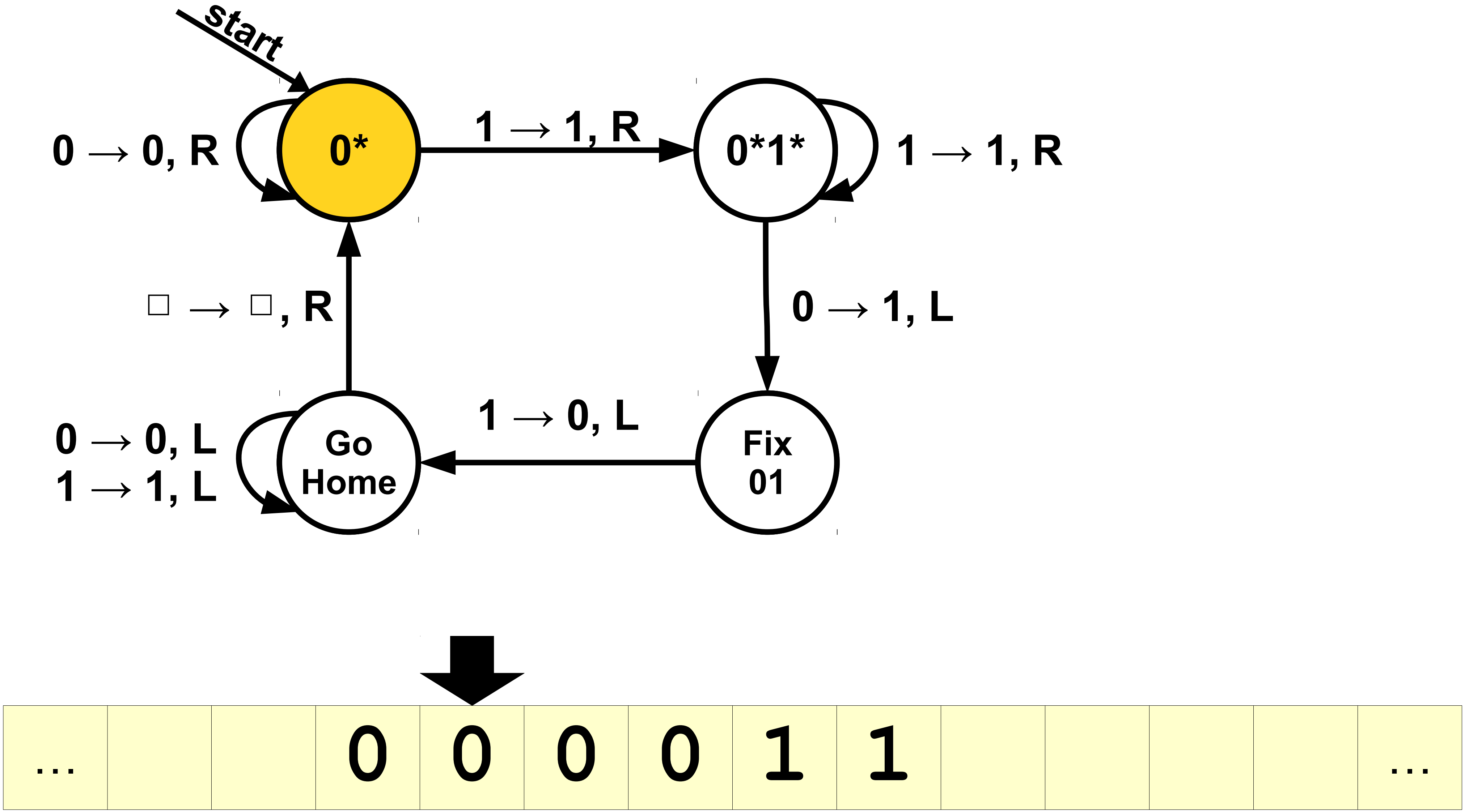

start
0*
0*1*
Fix 01
Go Home
0 → 0, R
1 → 1, R
1 → 1, R
0 → 1, L
1 → 0, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
0
0
1
1
…

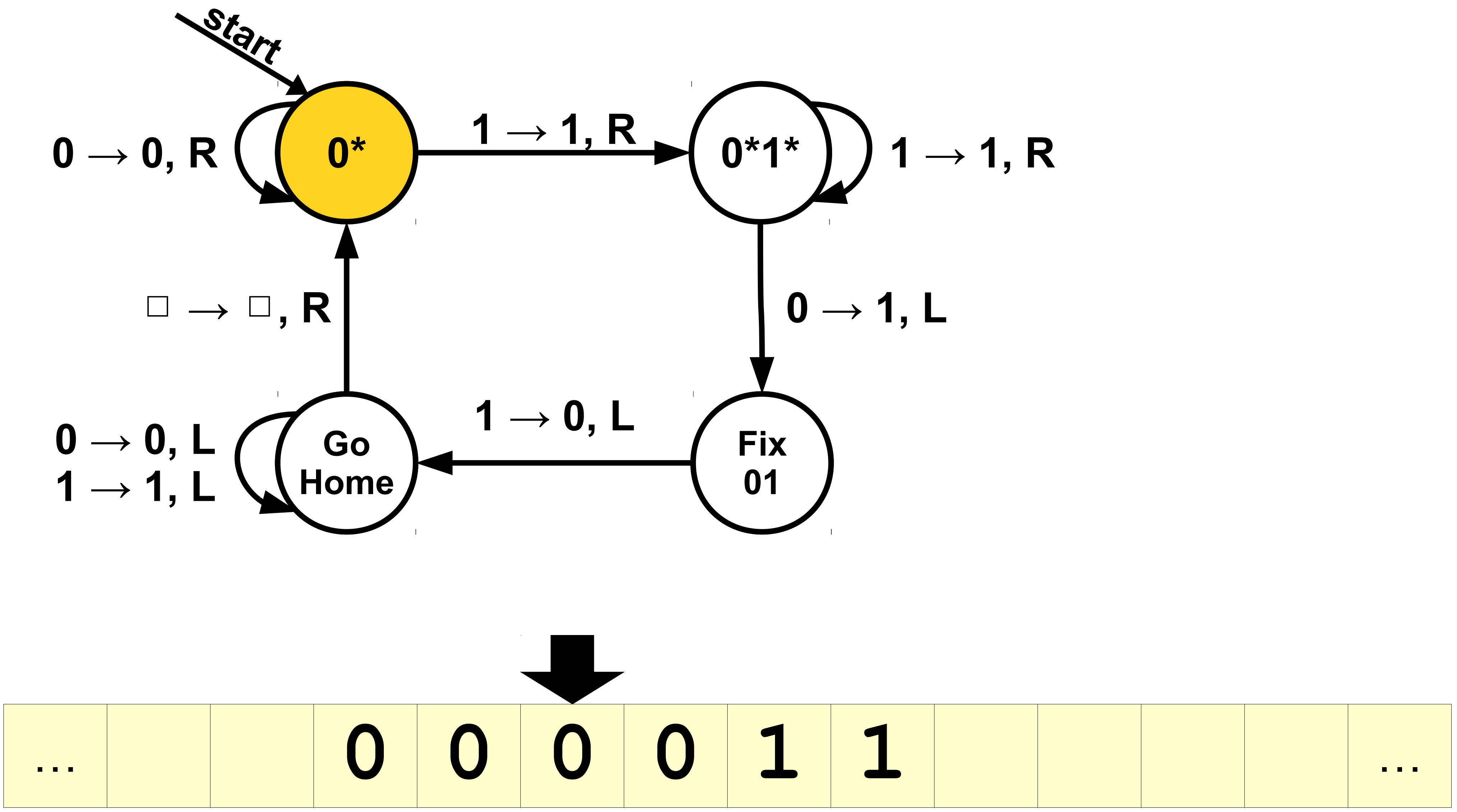

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

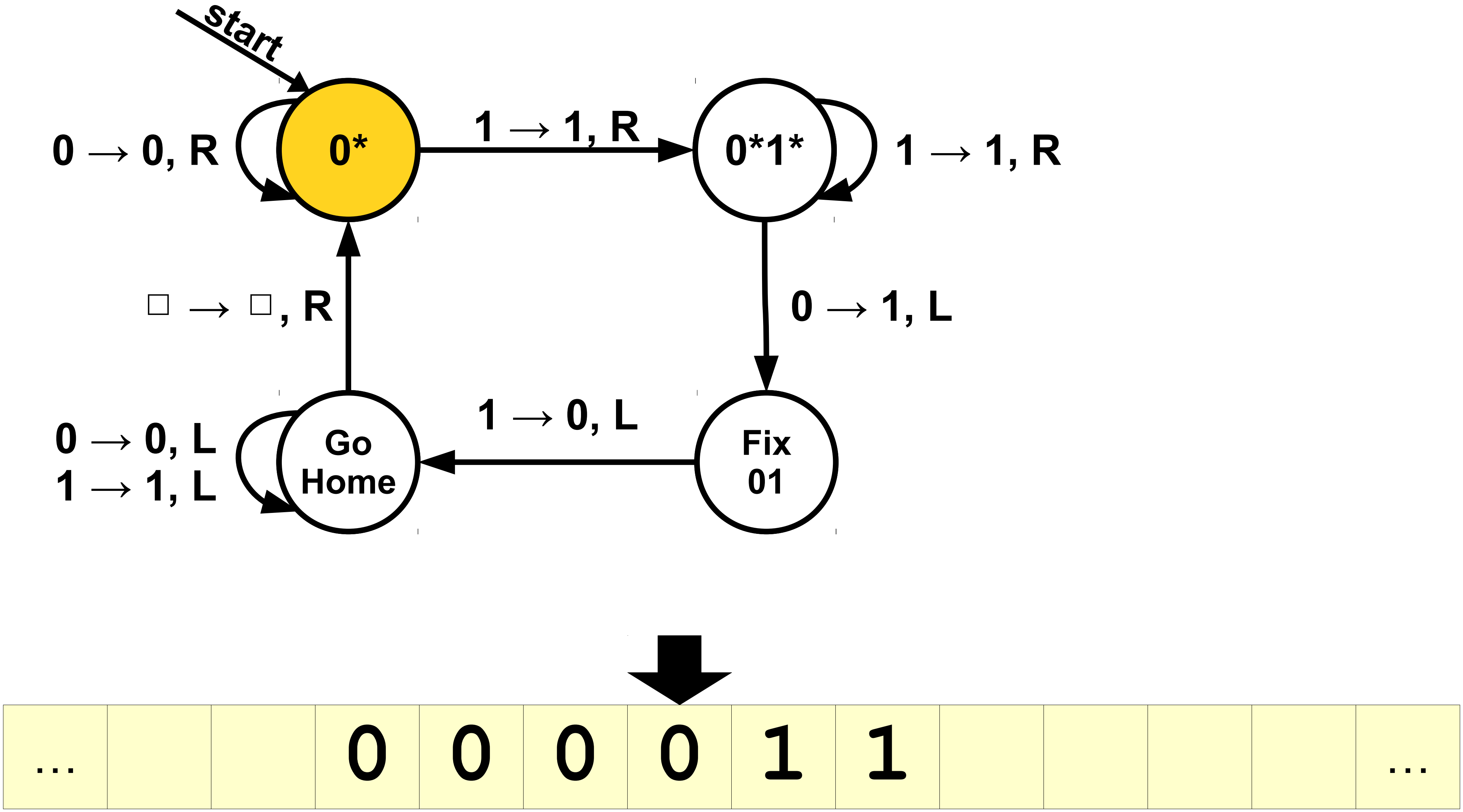


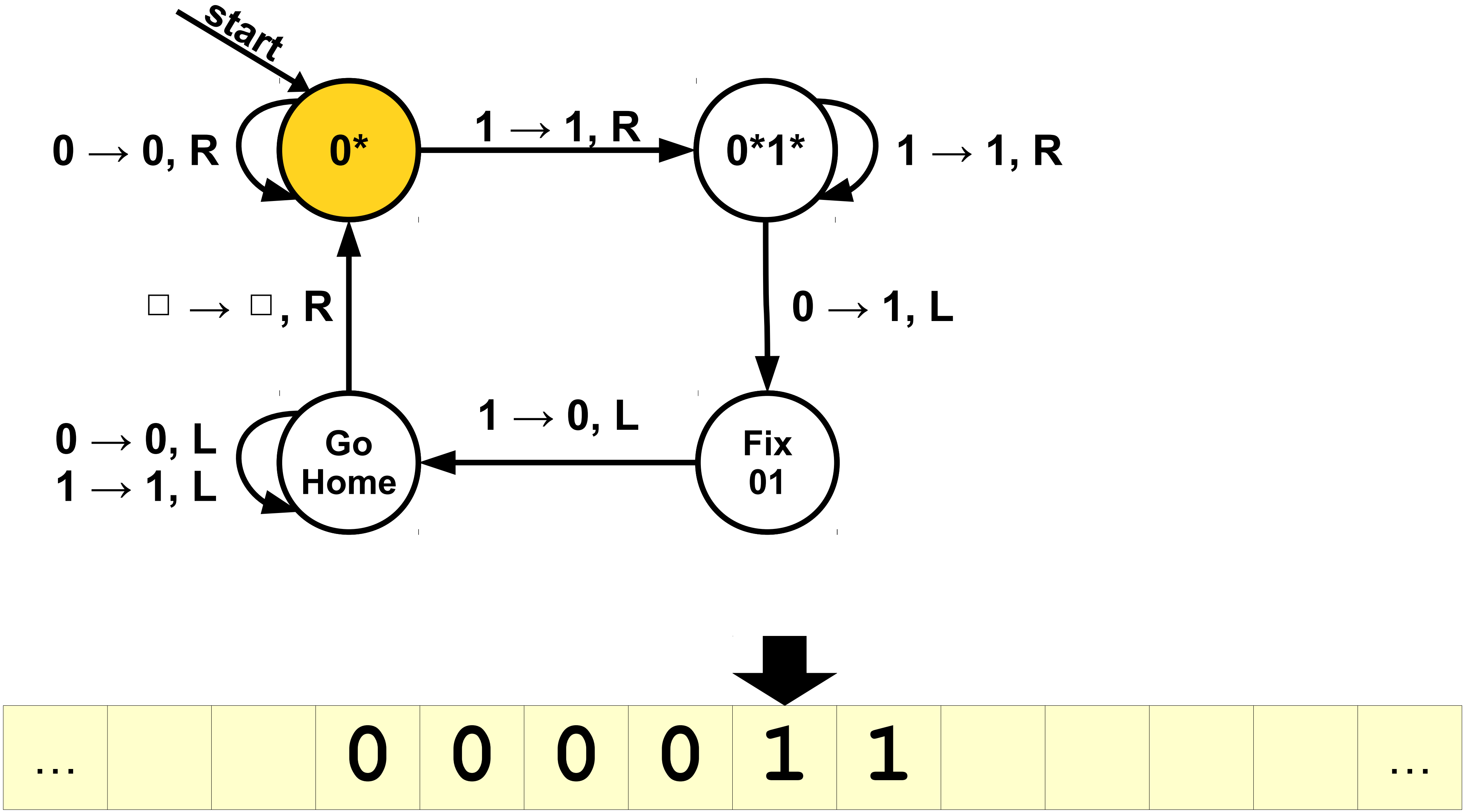
…			0	0	0	0	1	1					…


start
0*
0 → 0, R
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
Fix
01
1 → 0, L
Go
Home
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
0
0
1
1
…

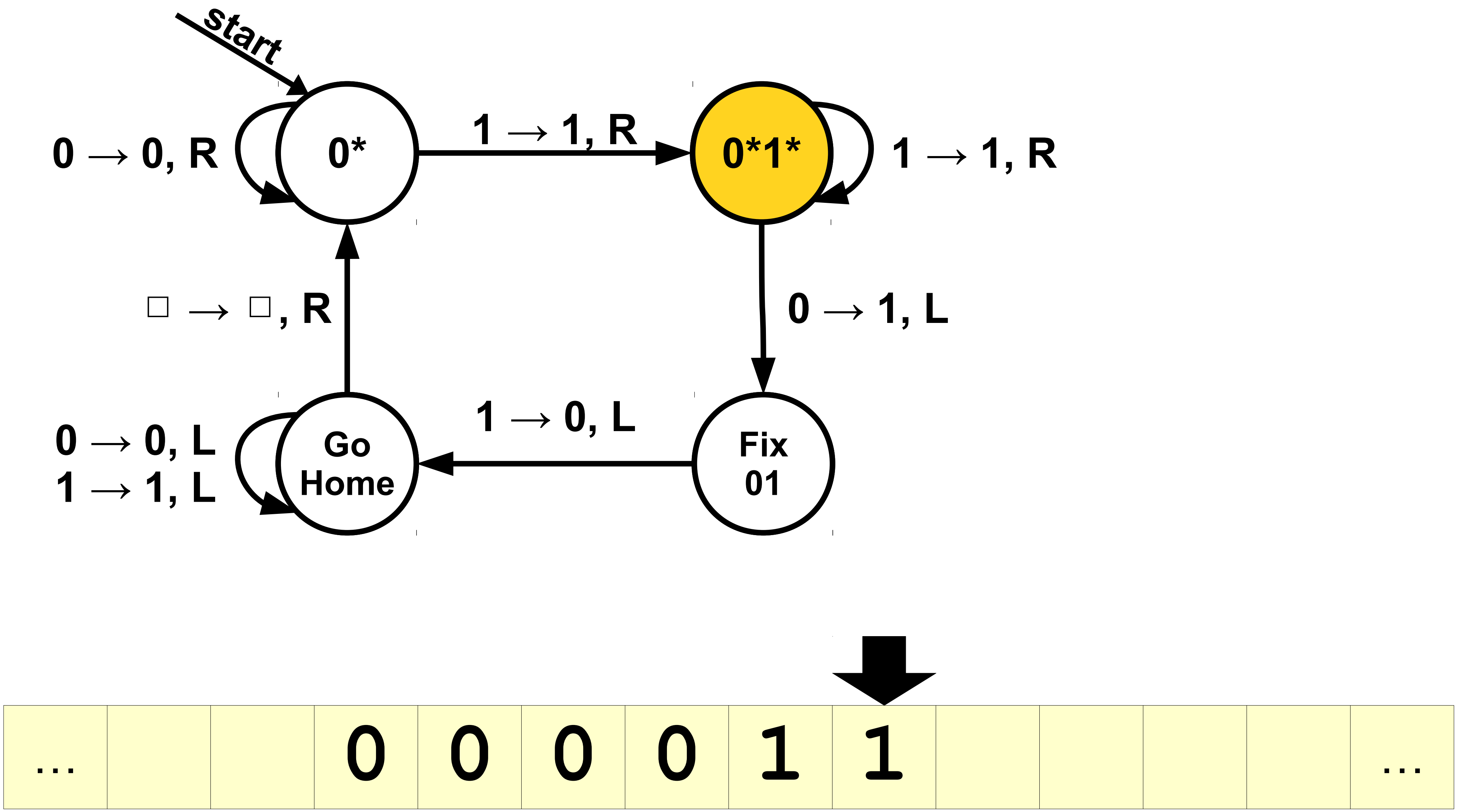

start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…


start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

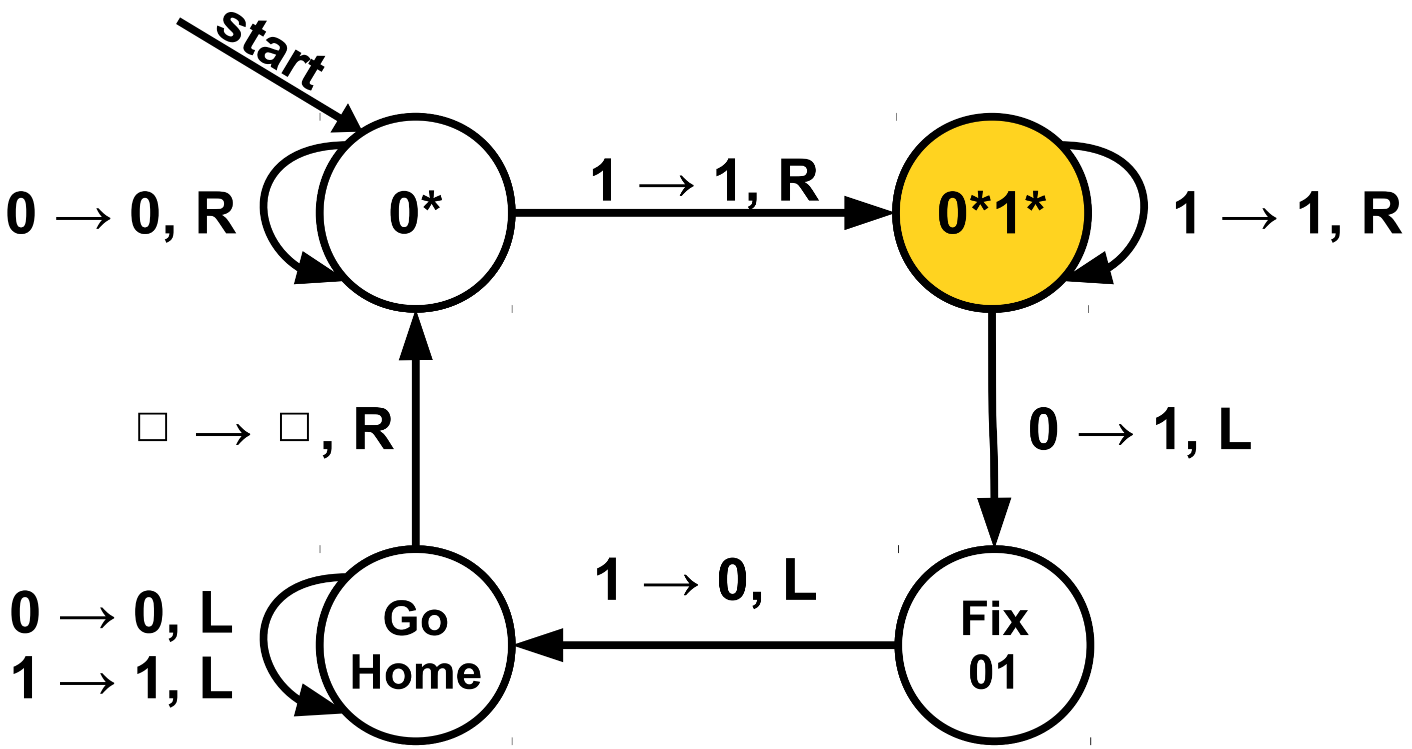

start
0*
0 → 0, R
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
Fix
01
1 → 0, L
Go
Home
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
0
0
1
1
…

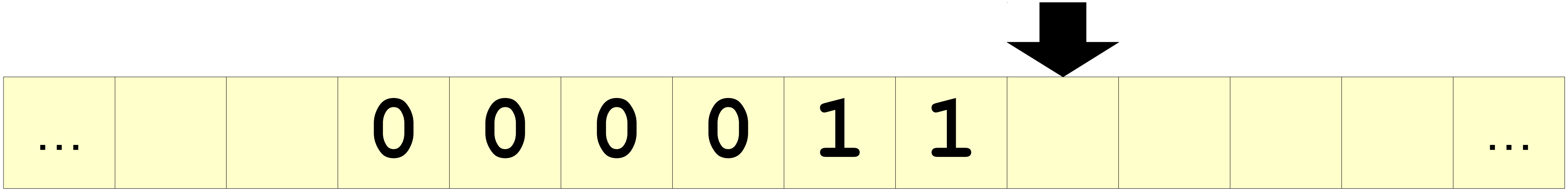


…			0	0	0	0	1	1					…



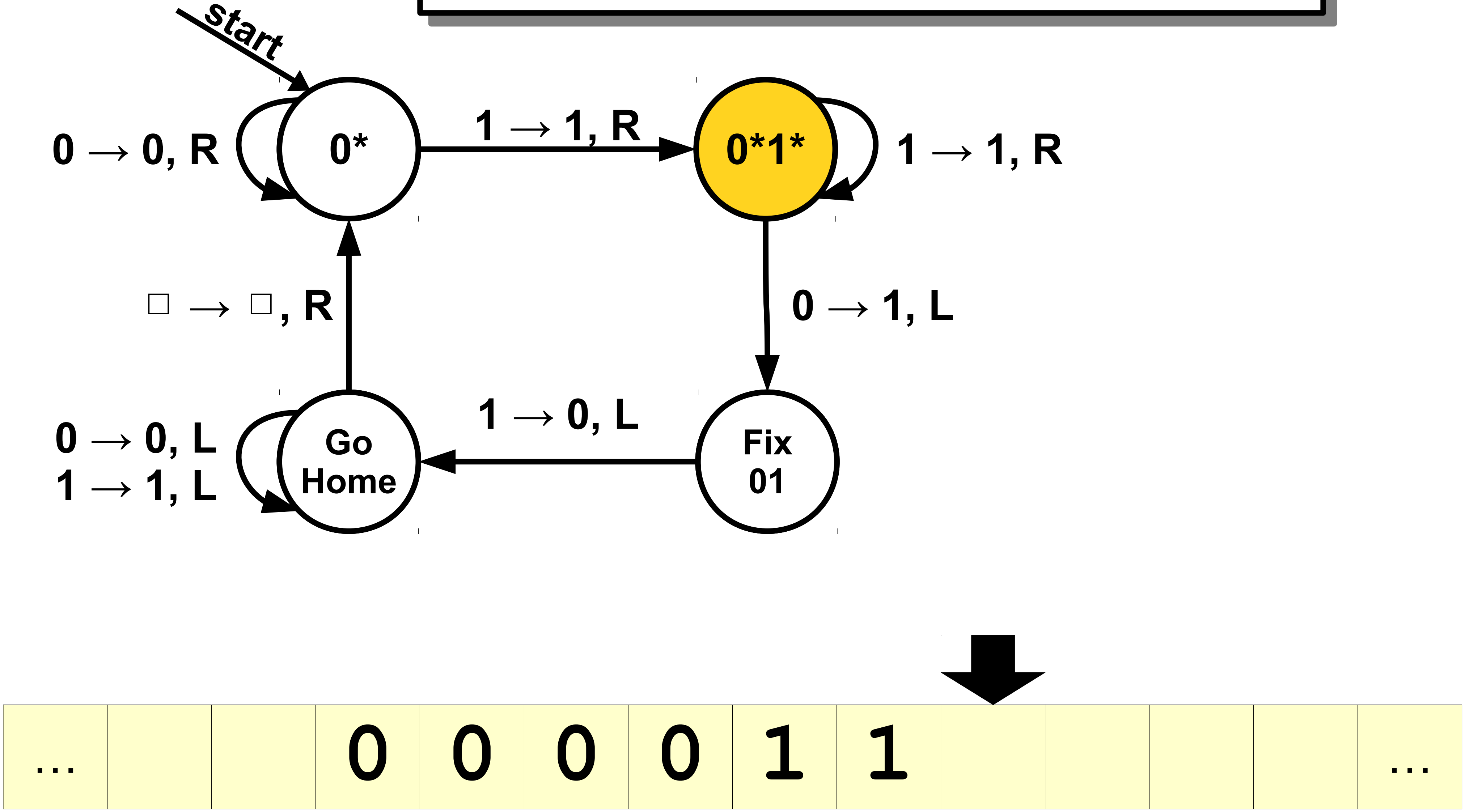
…			0	0	0	0	1	1					…

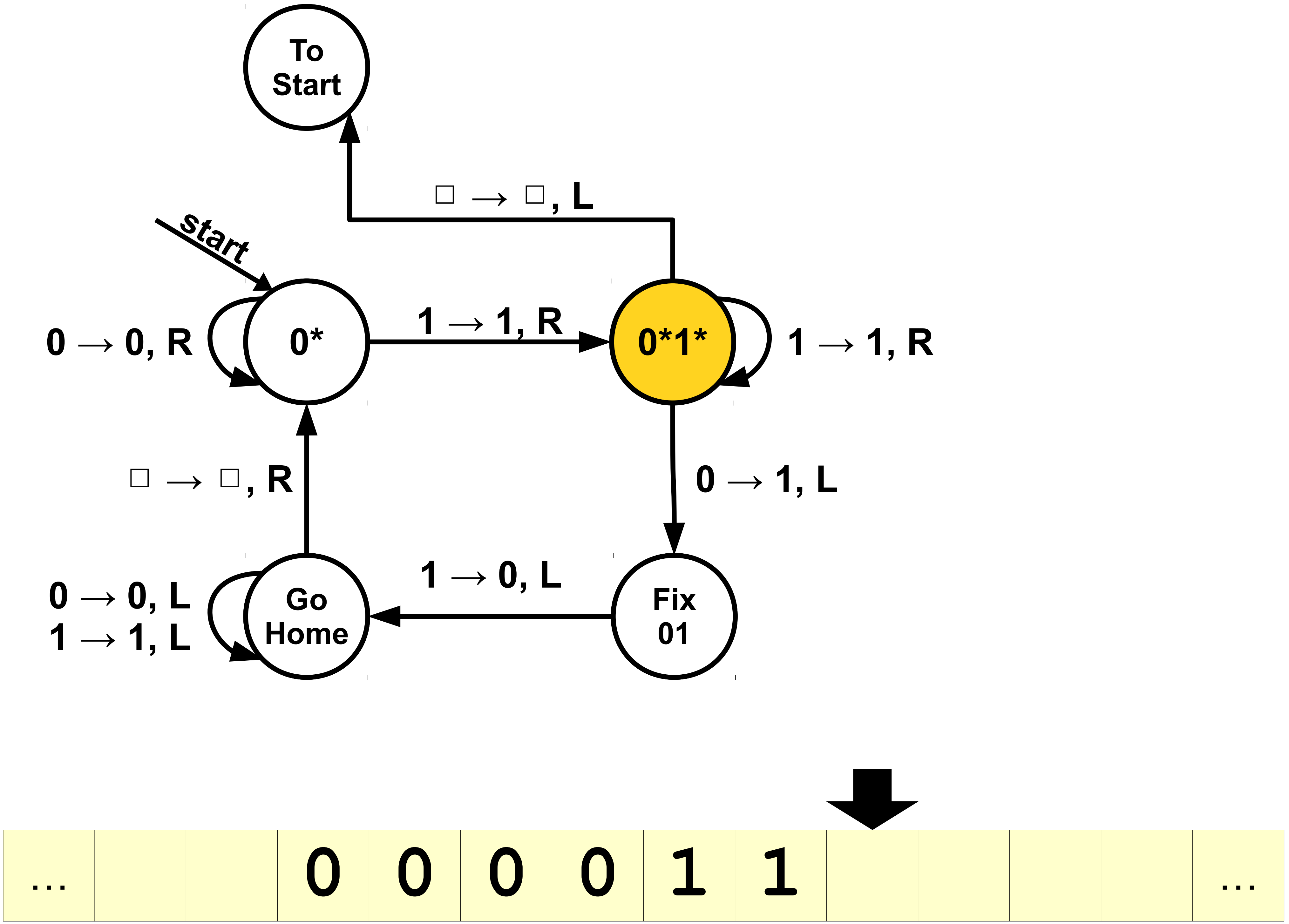


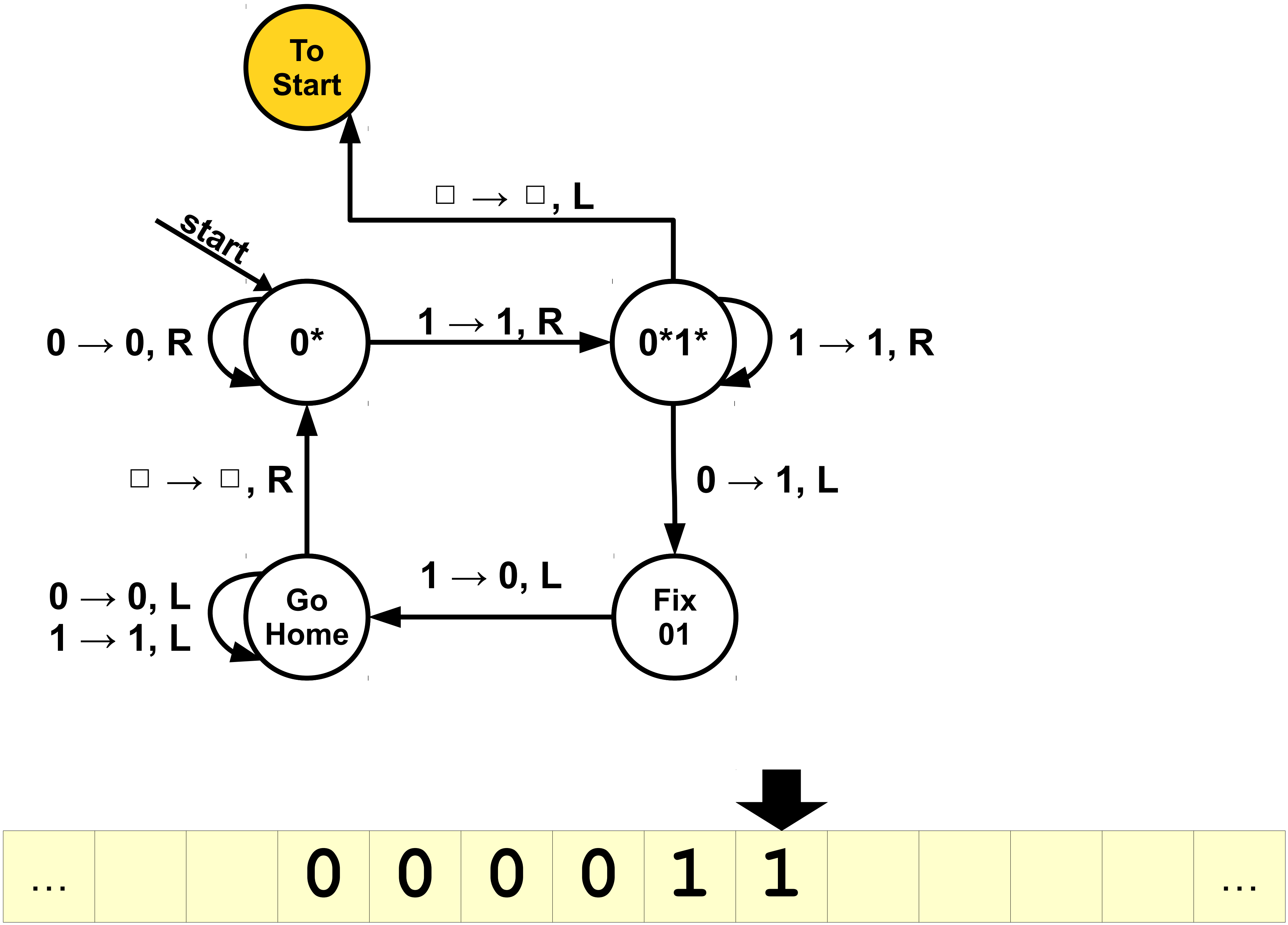


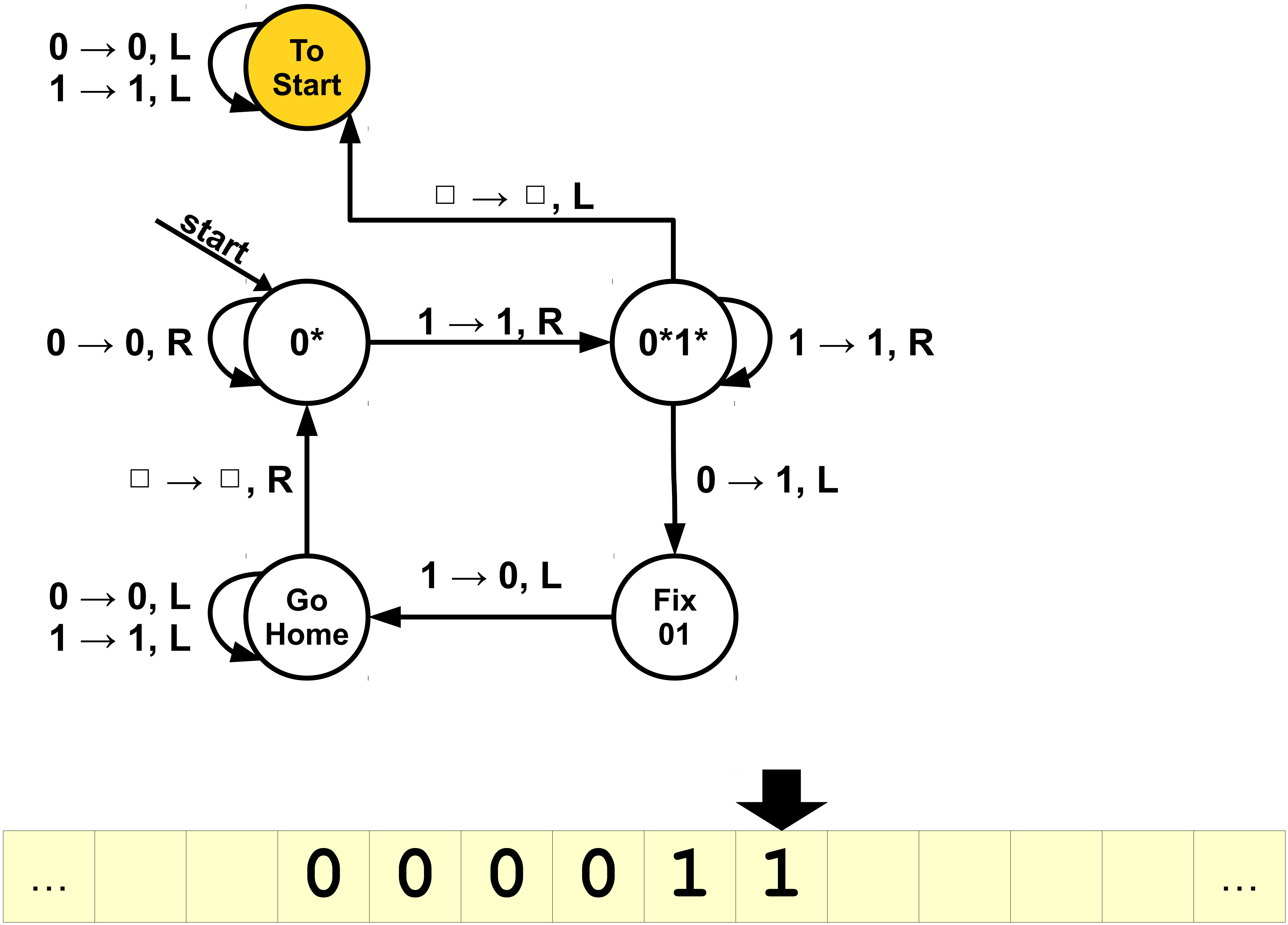
…			0	0	0	0	1	1					…

Our ultimate goal here was to sort everything so we could hand it off to the machine to check for 0^n1^n. Let's rewind the tape head back to the start.

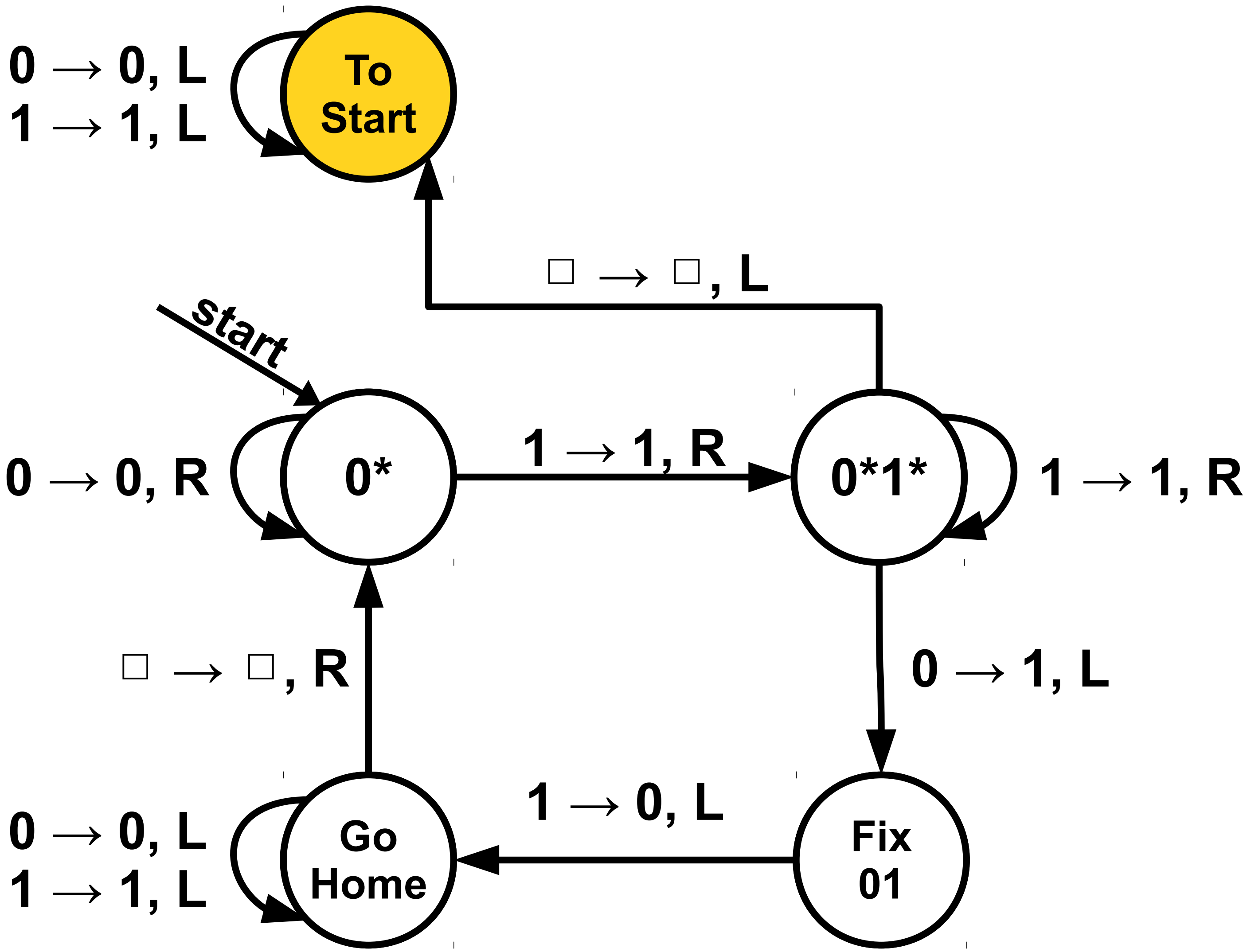



To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

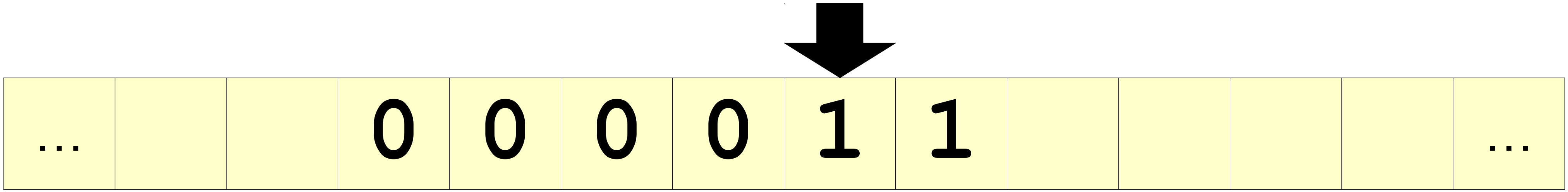

To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…



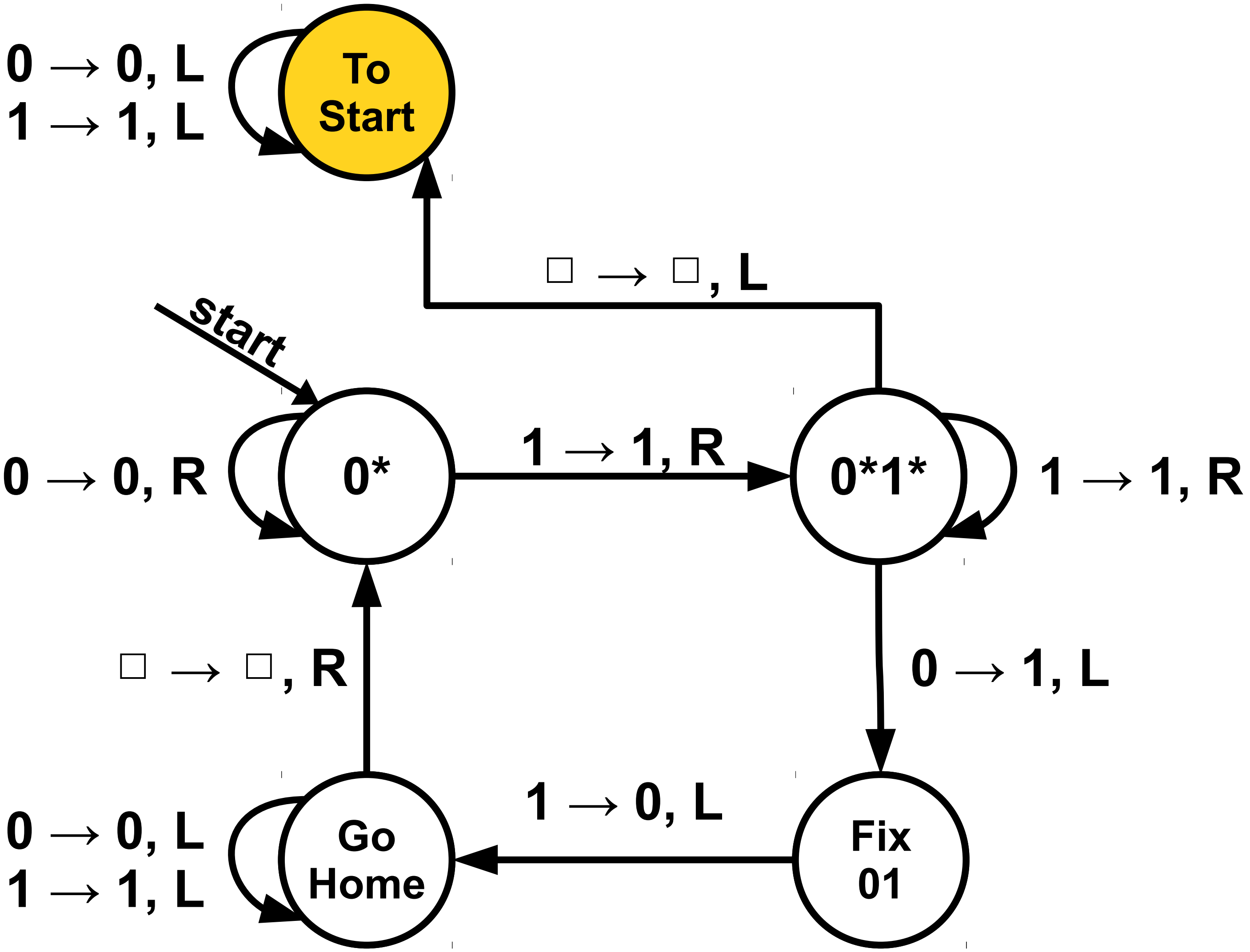
0 → 0, L
1 → 1, L
To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

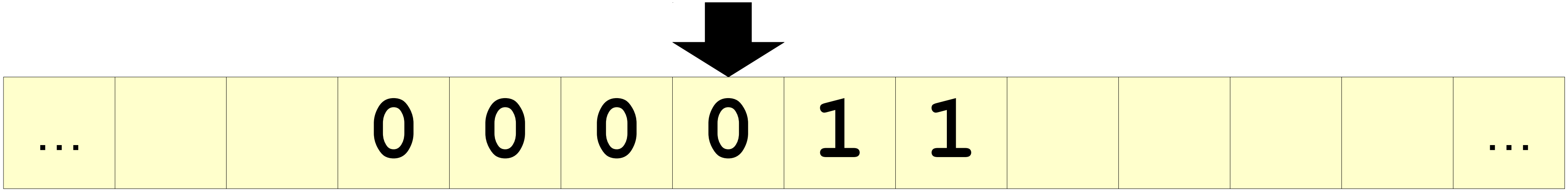


To Start
0 → 0, L
1 → 1, L
□ → □, L
start
0*
0 → 0, R
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
Go Home
0 → 0, L
1 → 1, L
1 → 0, L
Fix 01

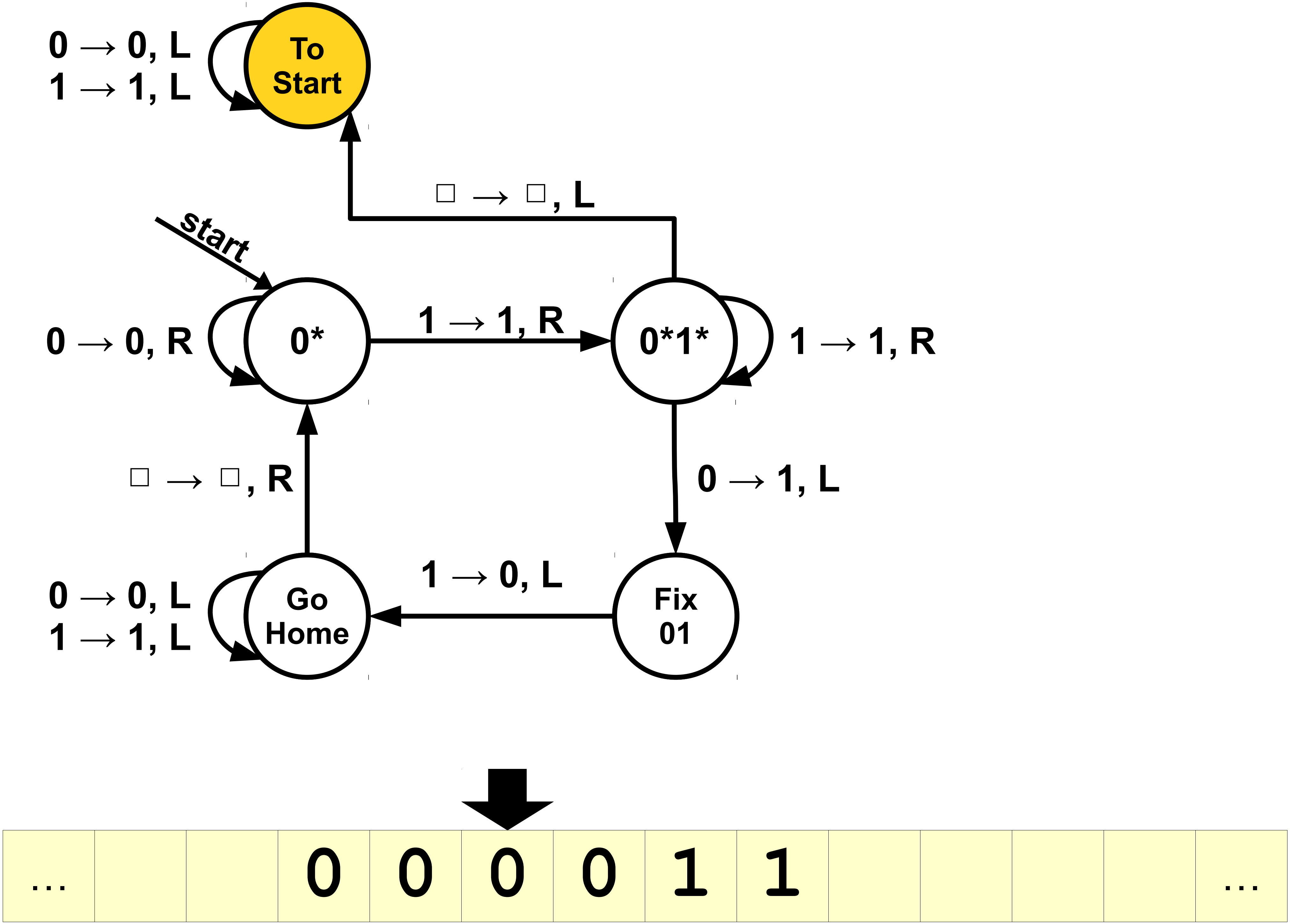


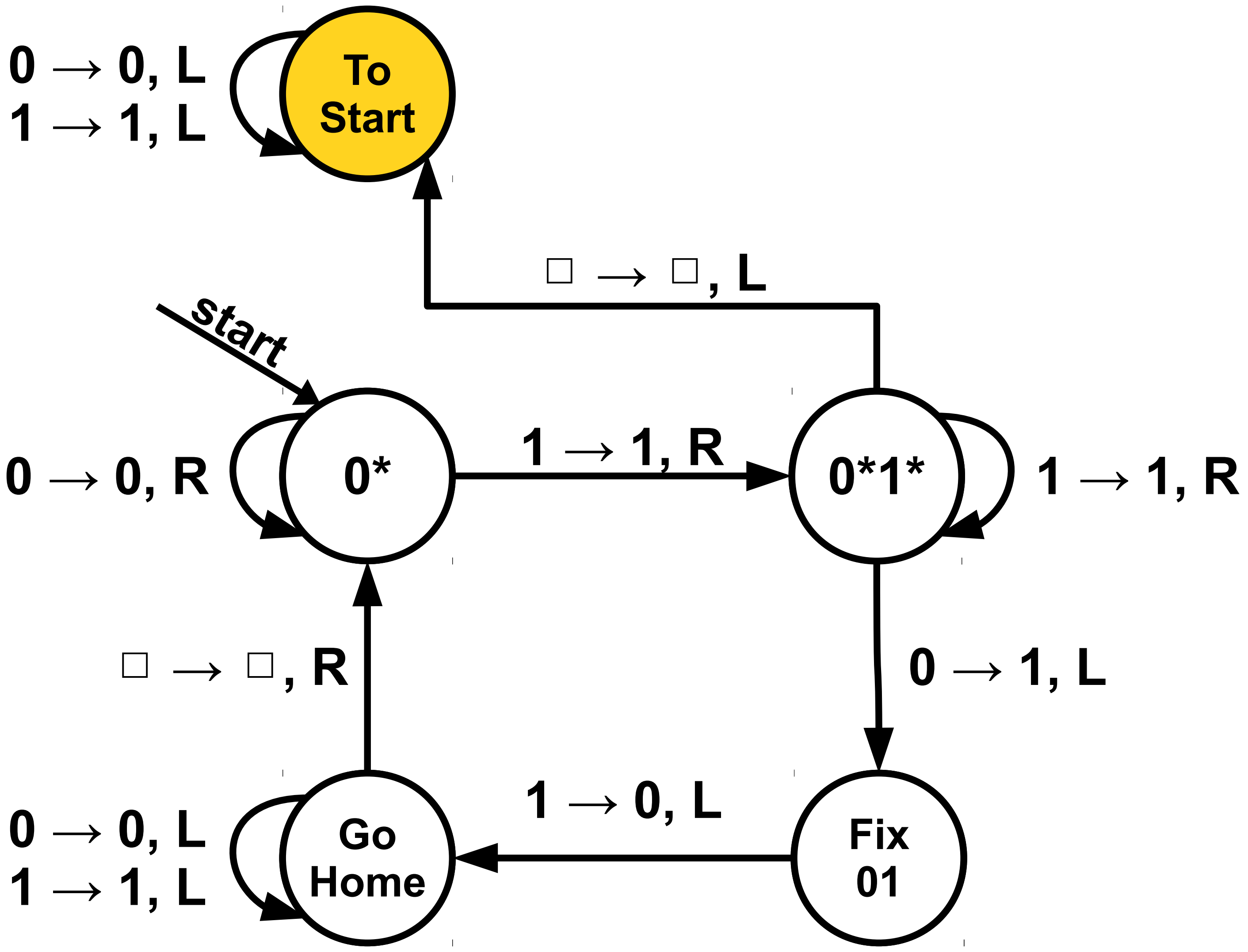
…			0	0	0	0	1	1					…

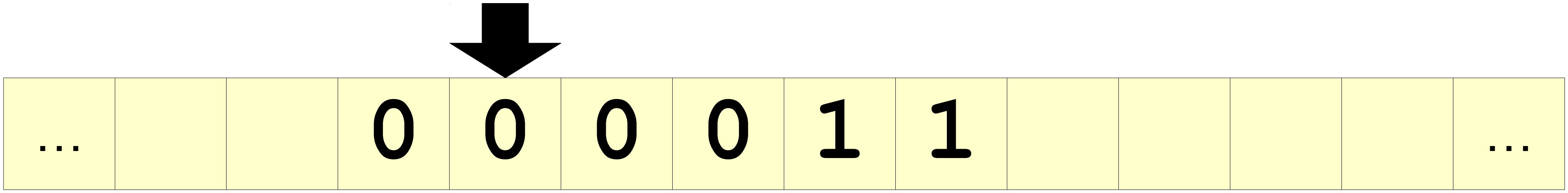

0 → 0, L
1 → 1, L
To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01

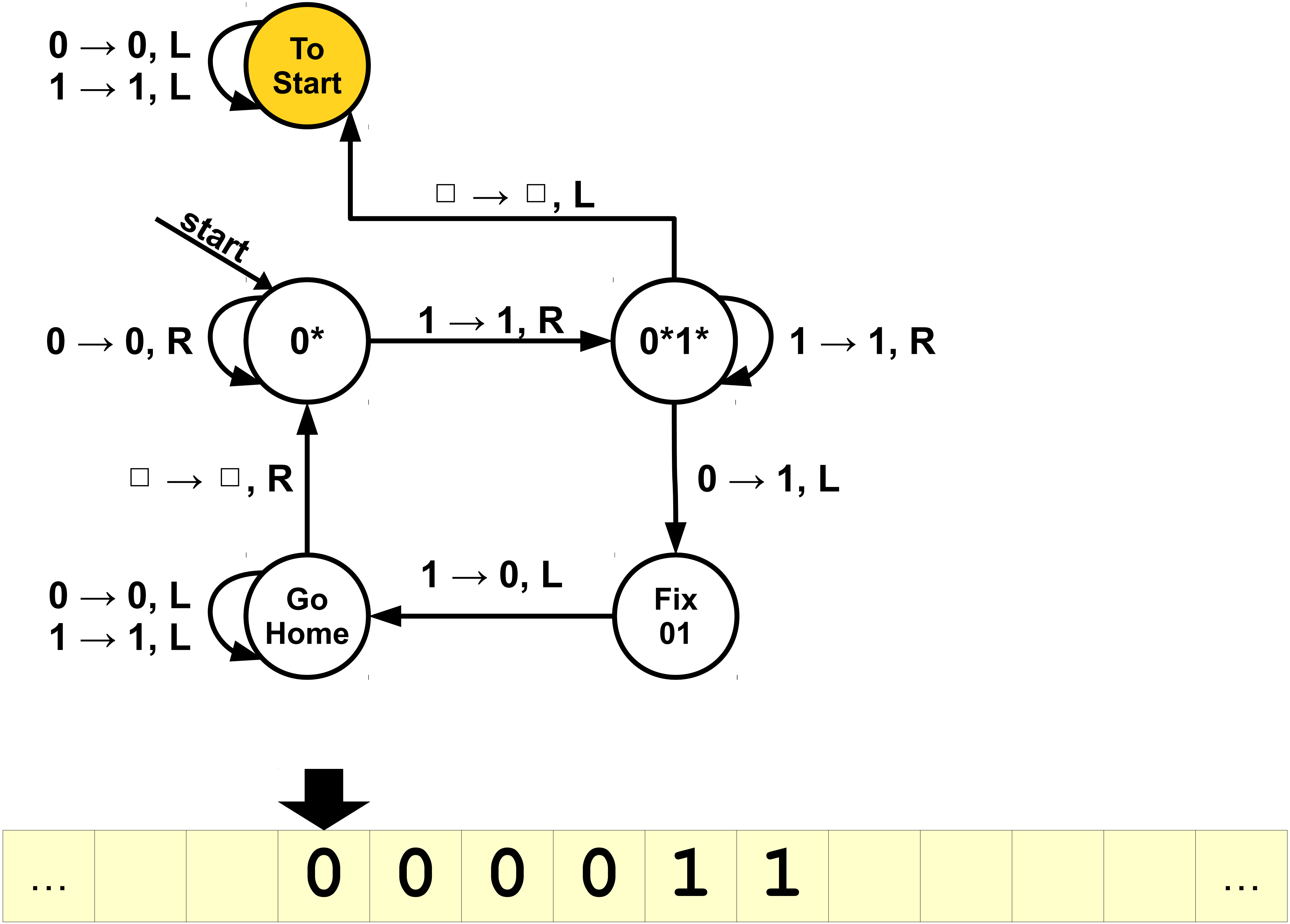


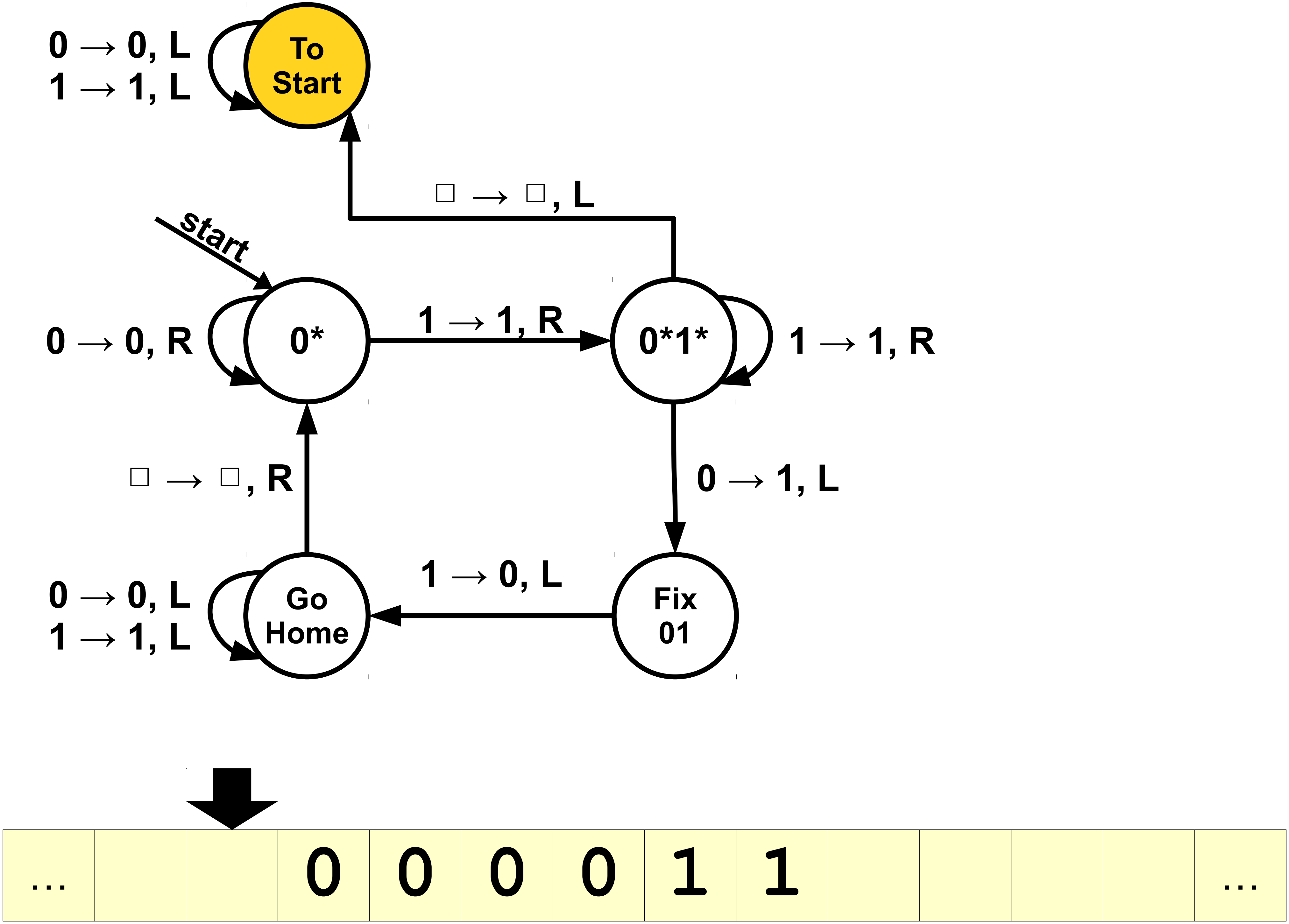
…			0	0	0	0	1	1					…

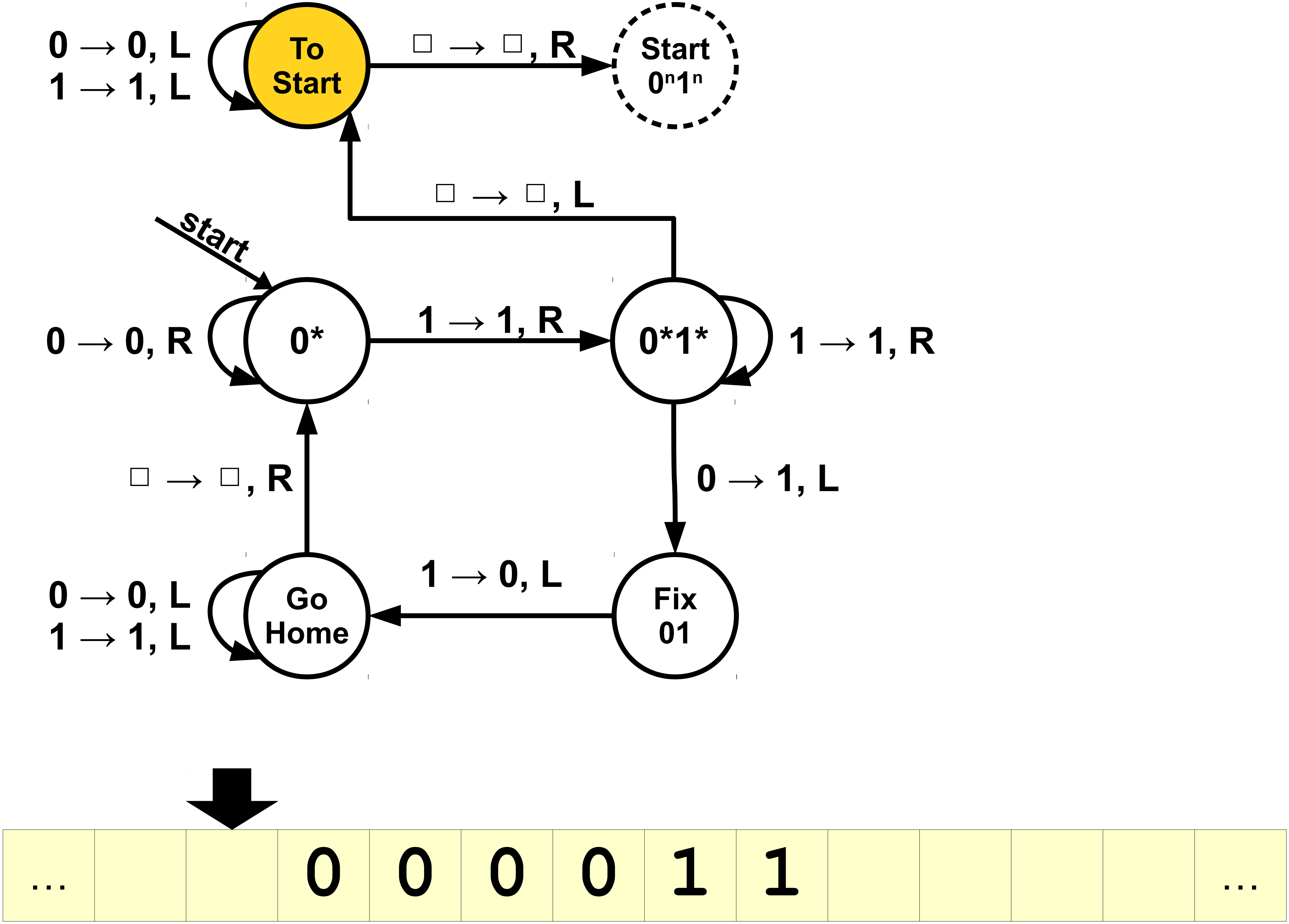

0 → 0, L
1 → 1, L
To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

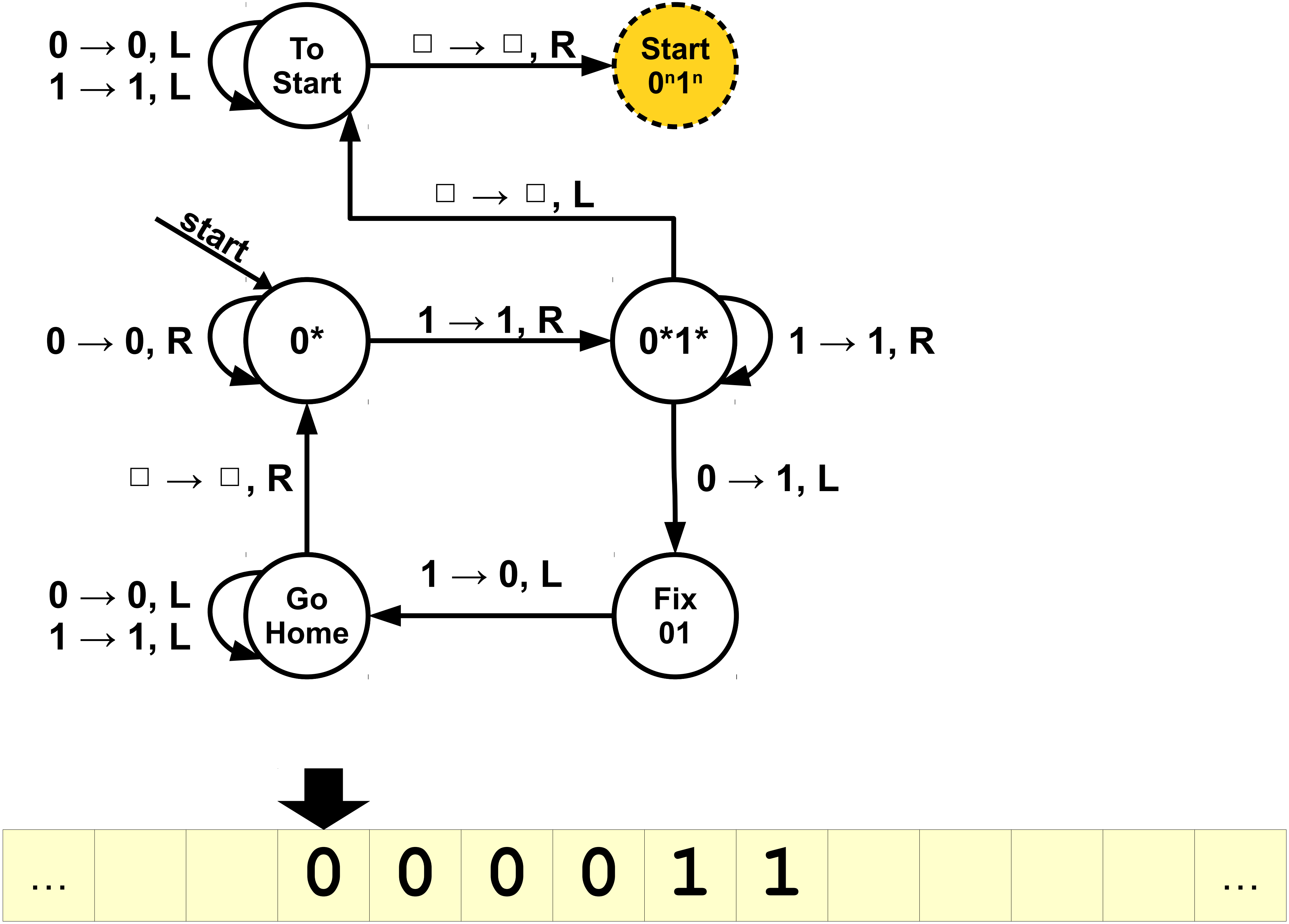

0 → 0, L
1 → 1, L
To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01

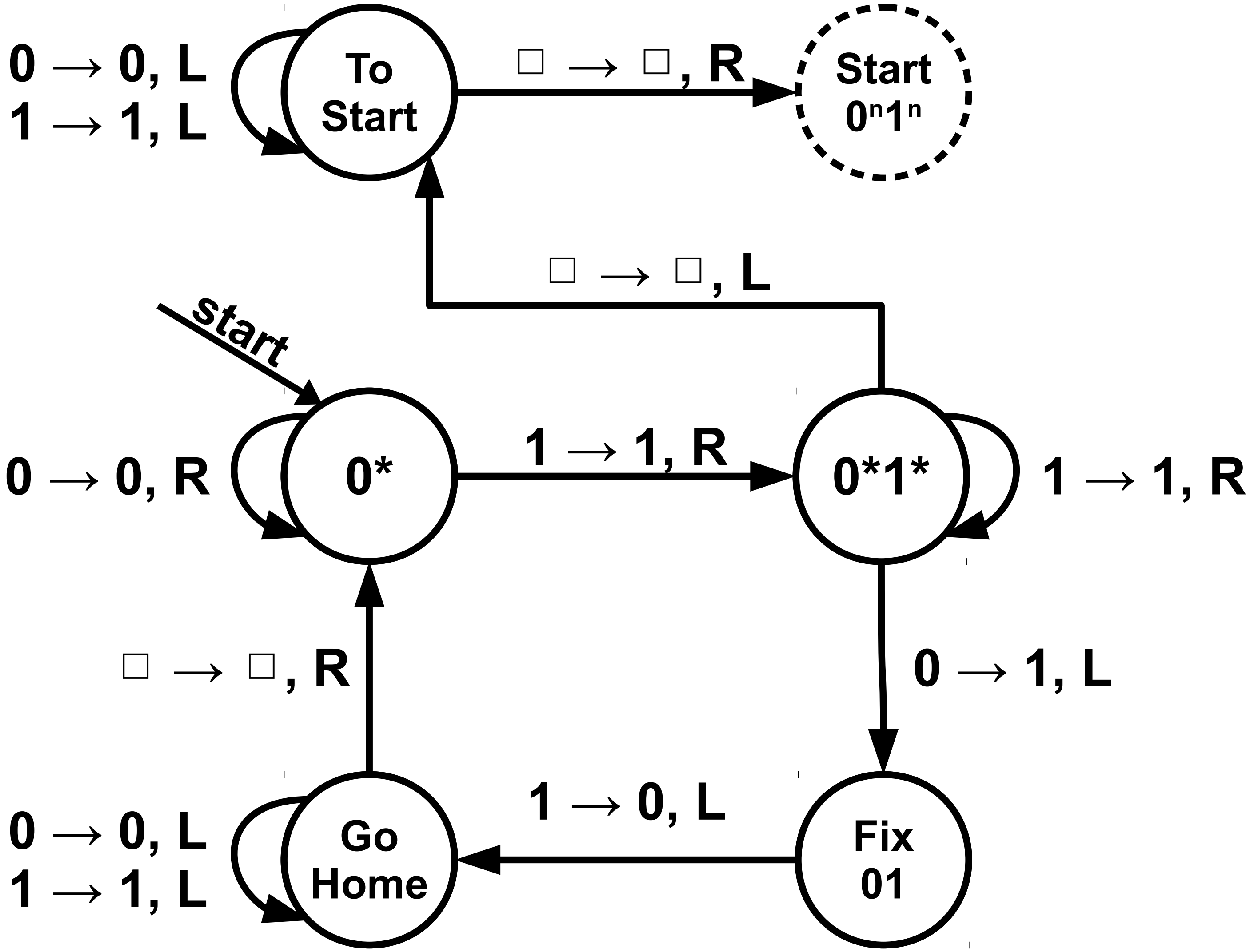

…
0
0
0
0
1
1
…

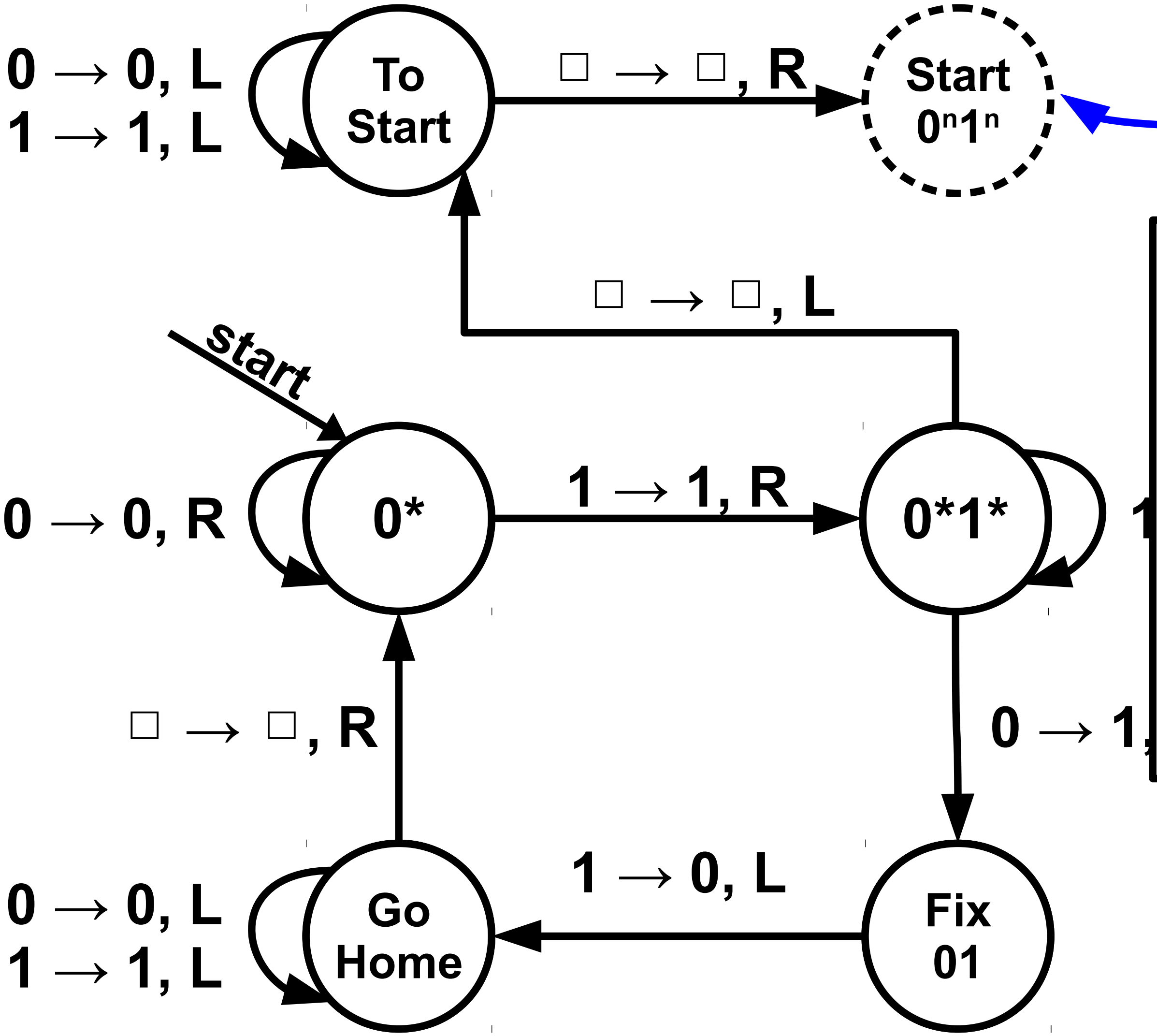

0 → 0, L
1 → 1, L
To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…


0 → 0, L
1 → 1, L
To Start
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

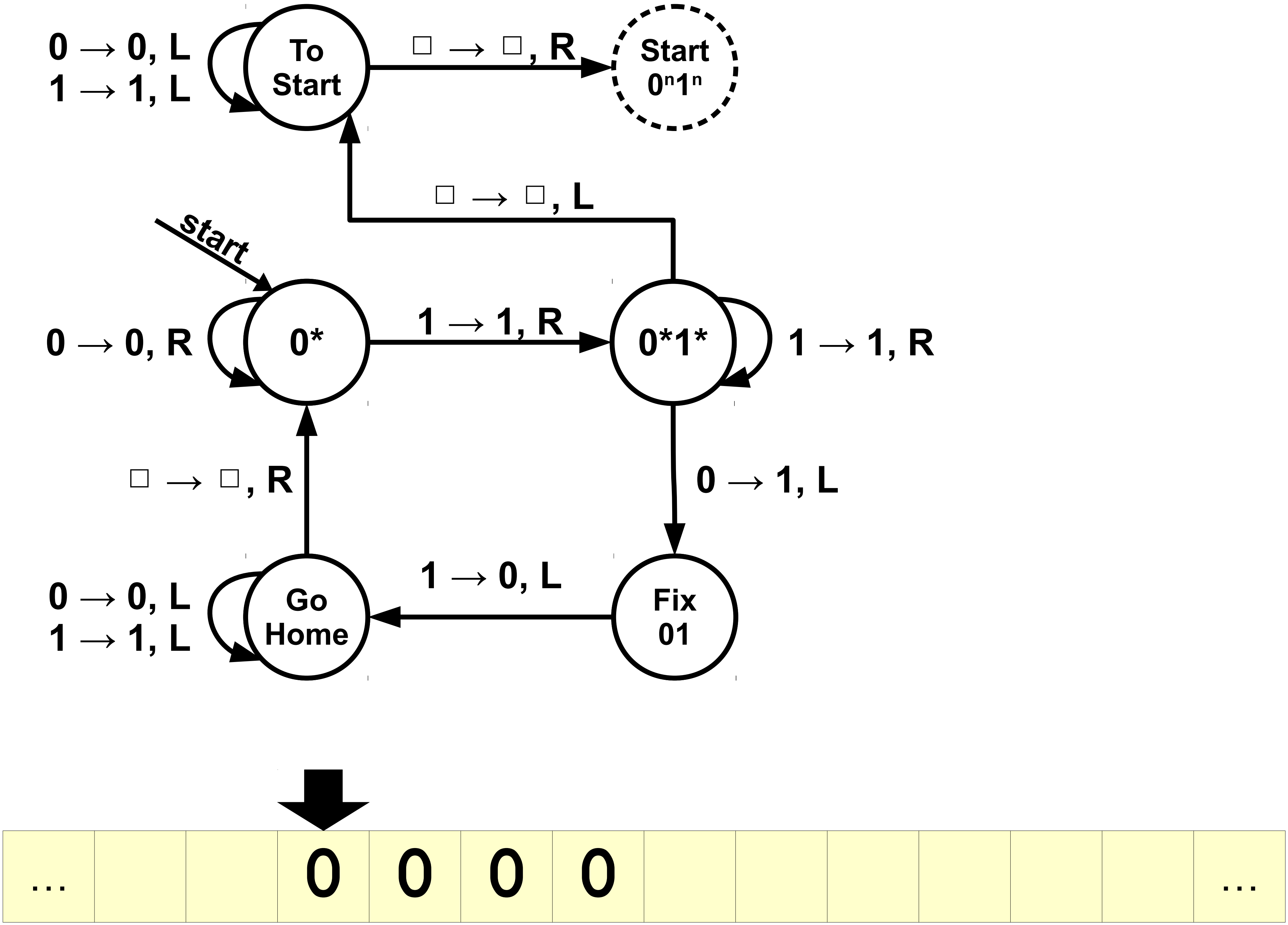

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0^n1^n
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

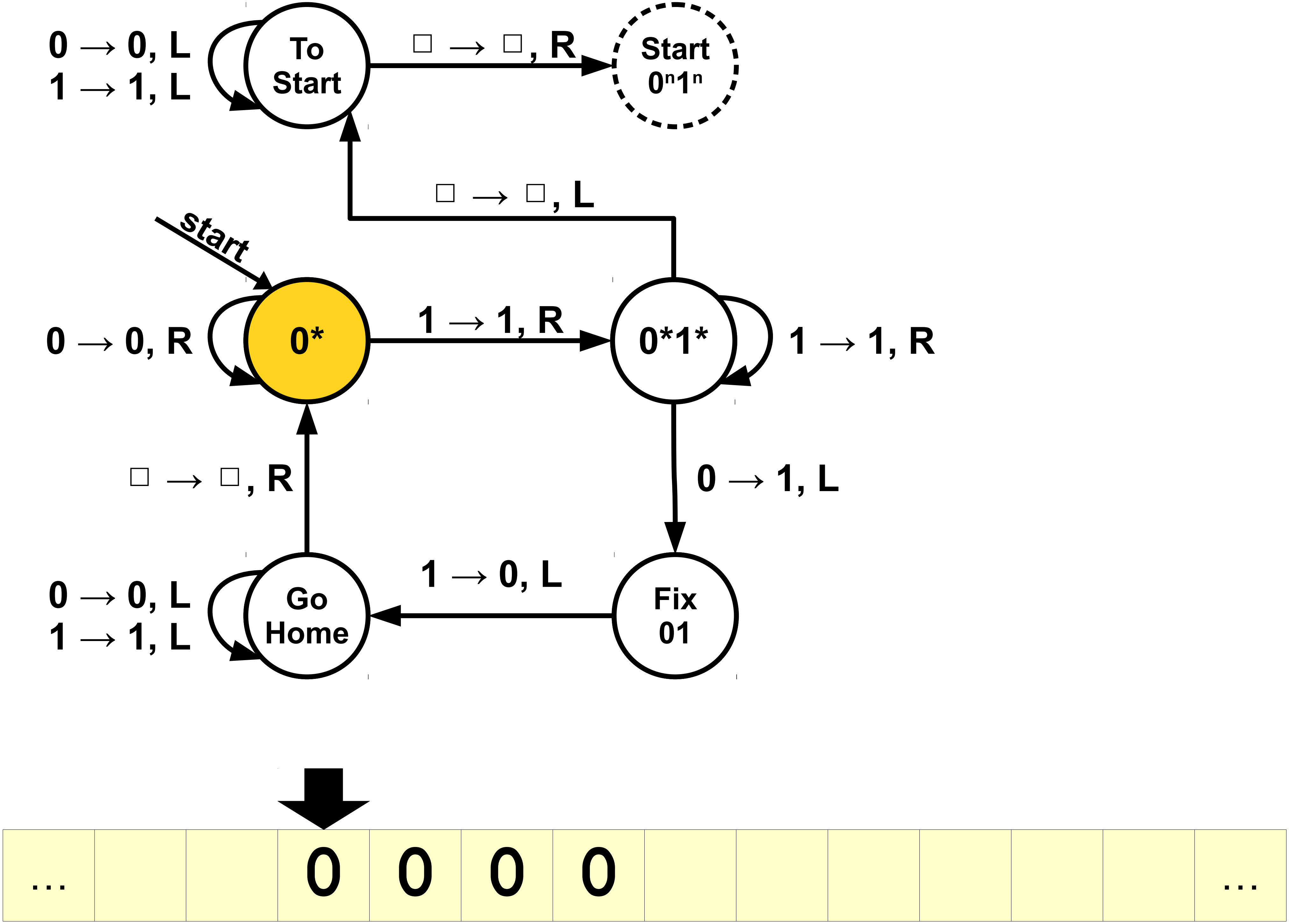

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
1
1
…

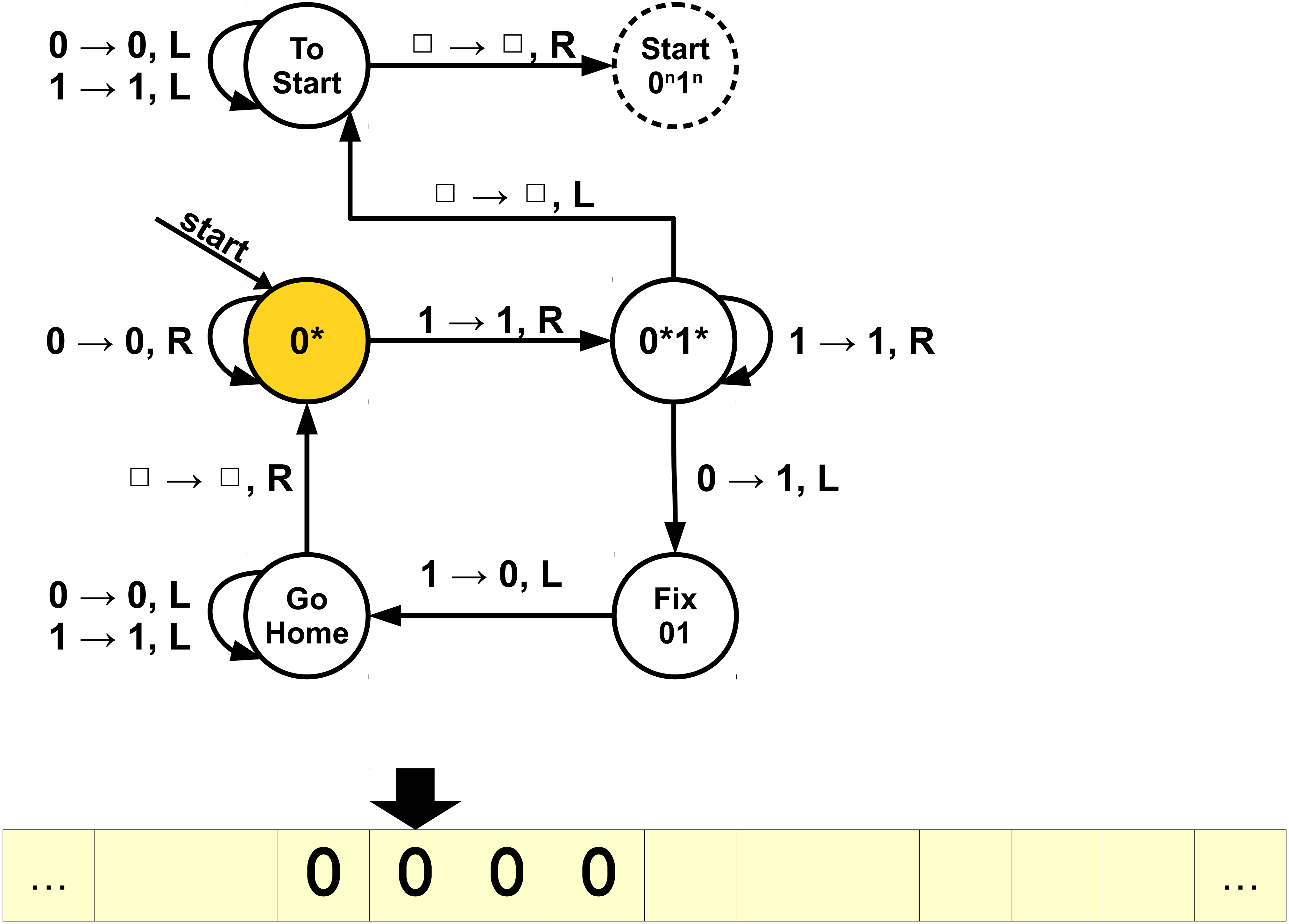

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
0 → 1, L
□ → □, R
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01

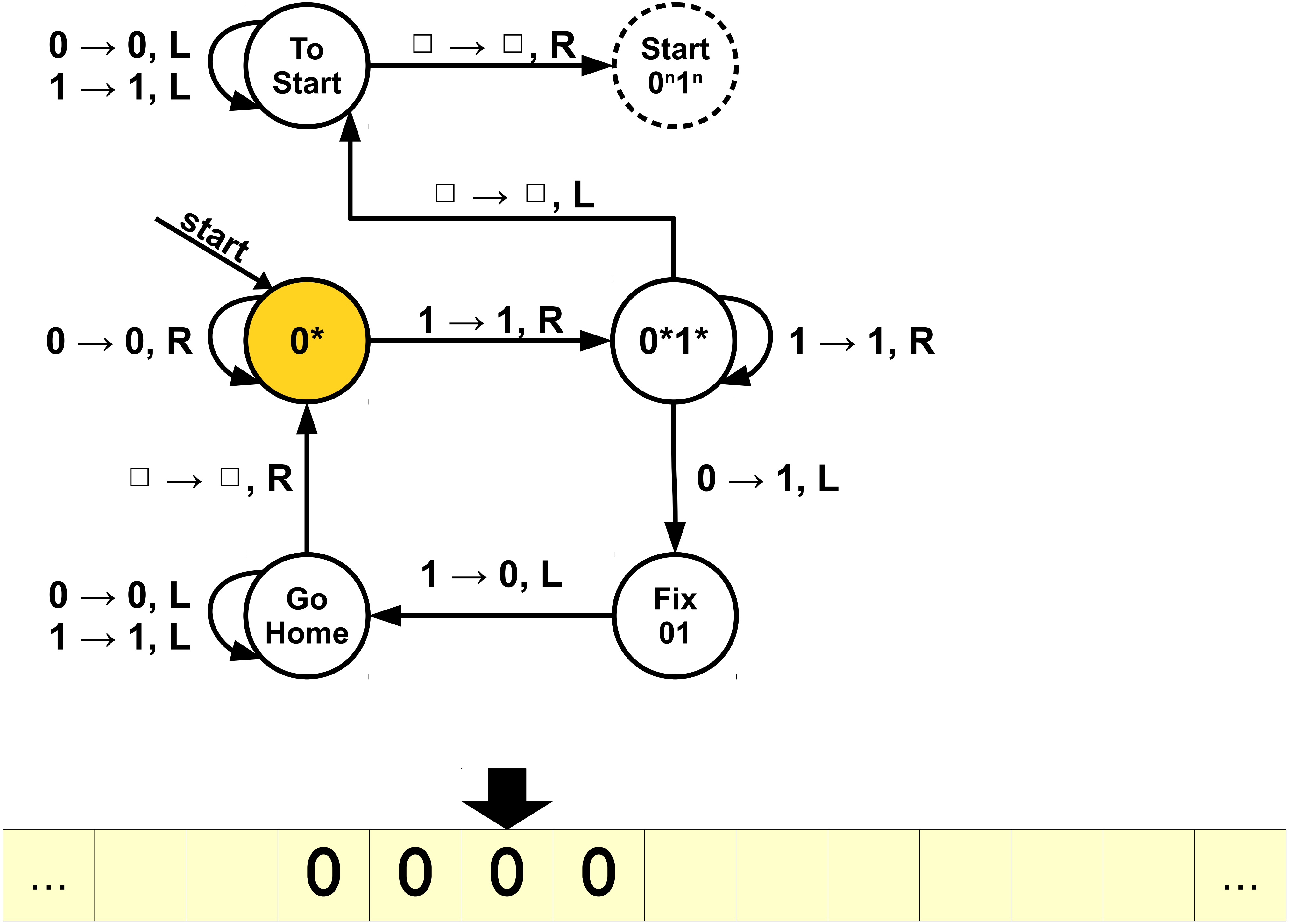


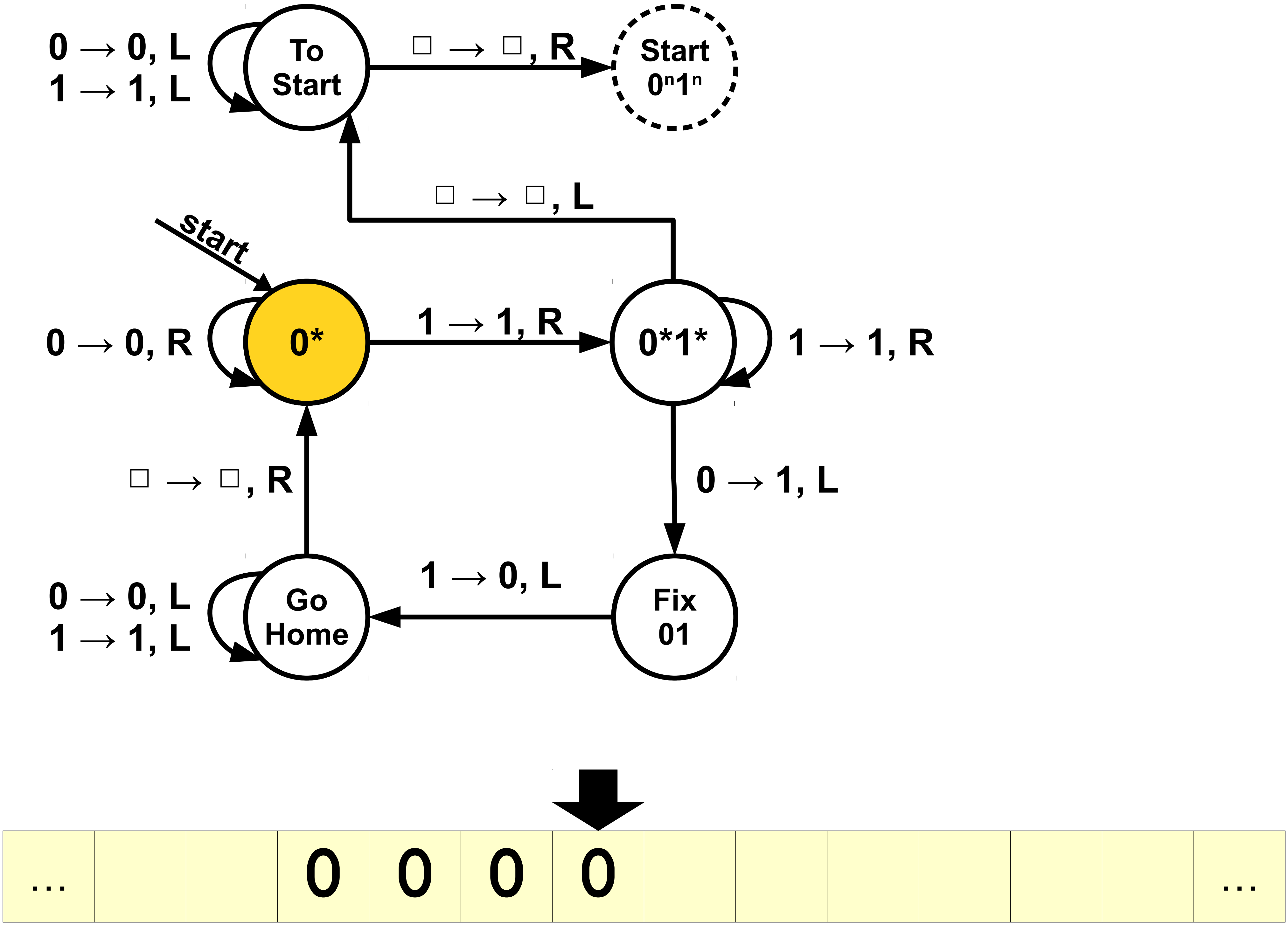
This is just a placeholder. Imagine snapping in the entire TM for 0^n1^n into this diagram, putting the start state in the dashed area.

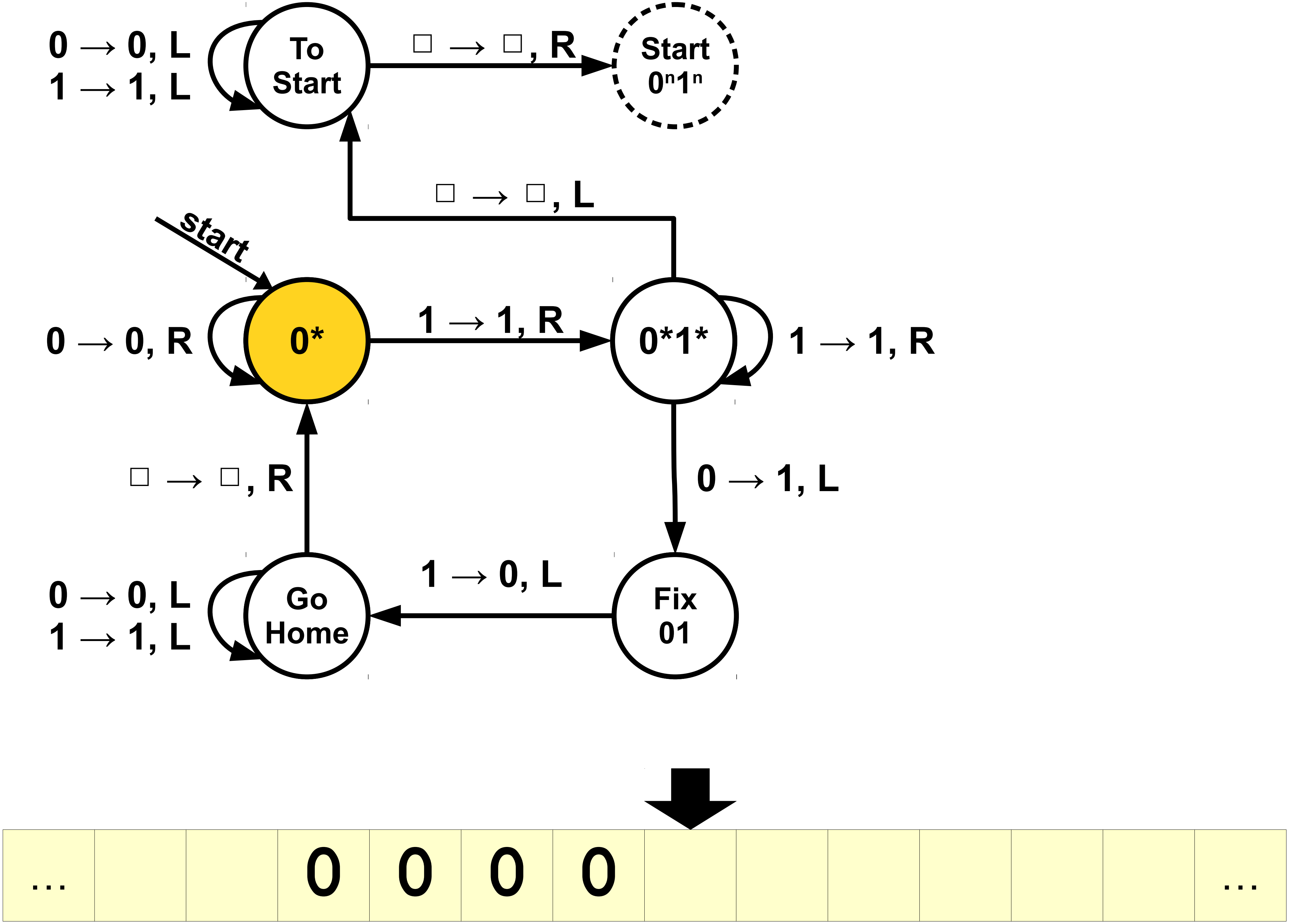

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
1 → 0, L
0 → 0, L
1 → 1, L
Go Home
Fix 01
…
0
0
0
0
…

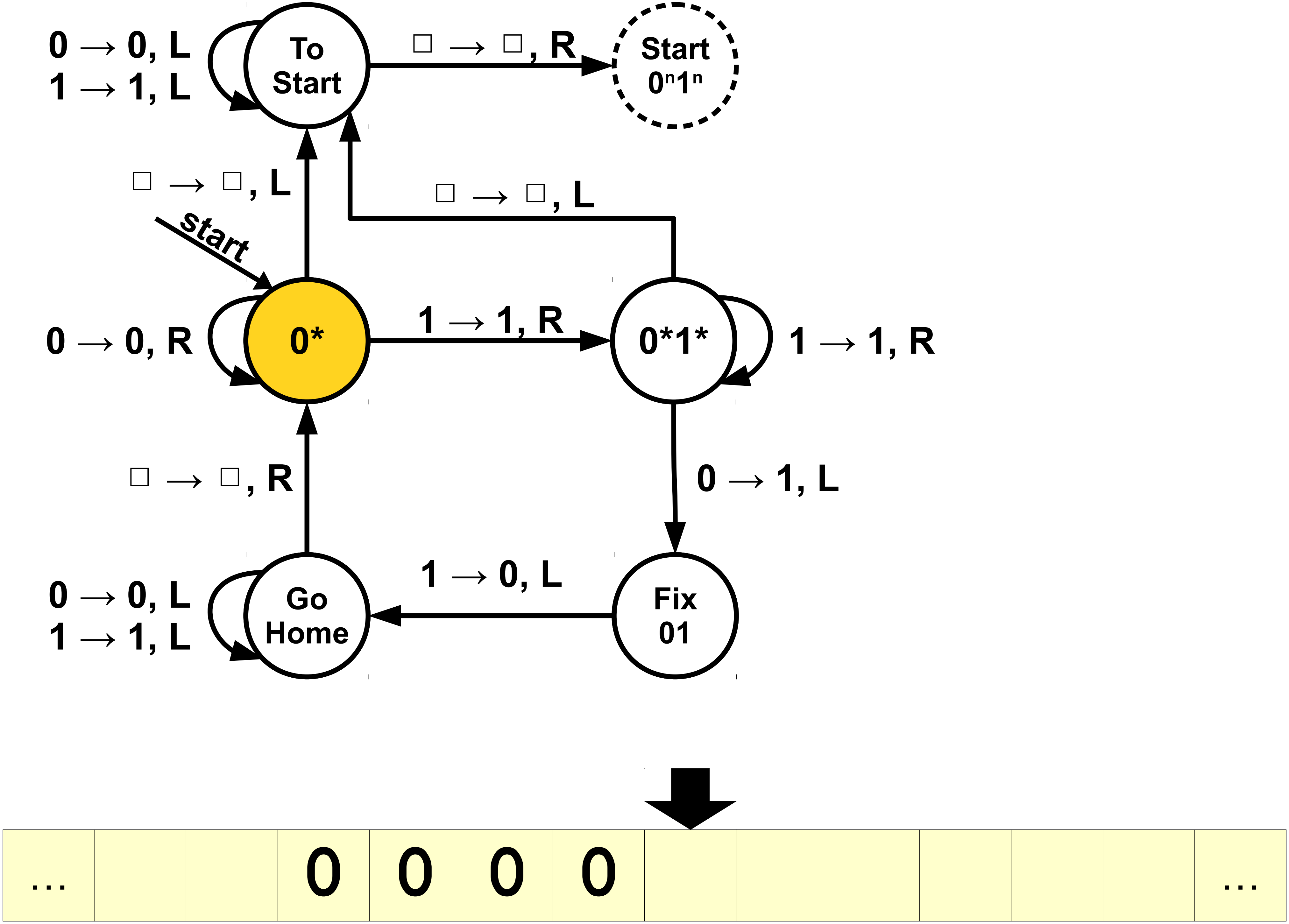

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…

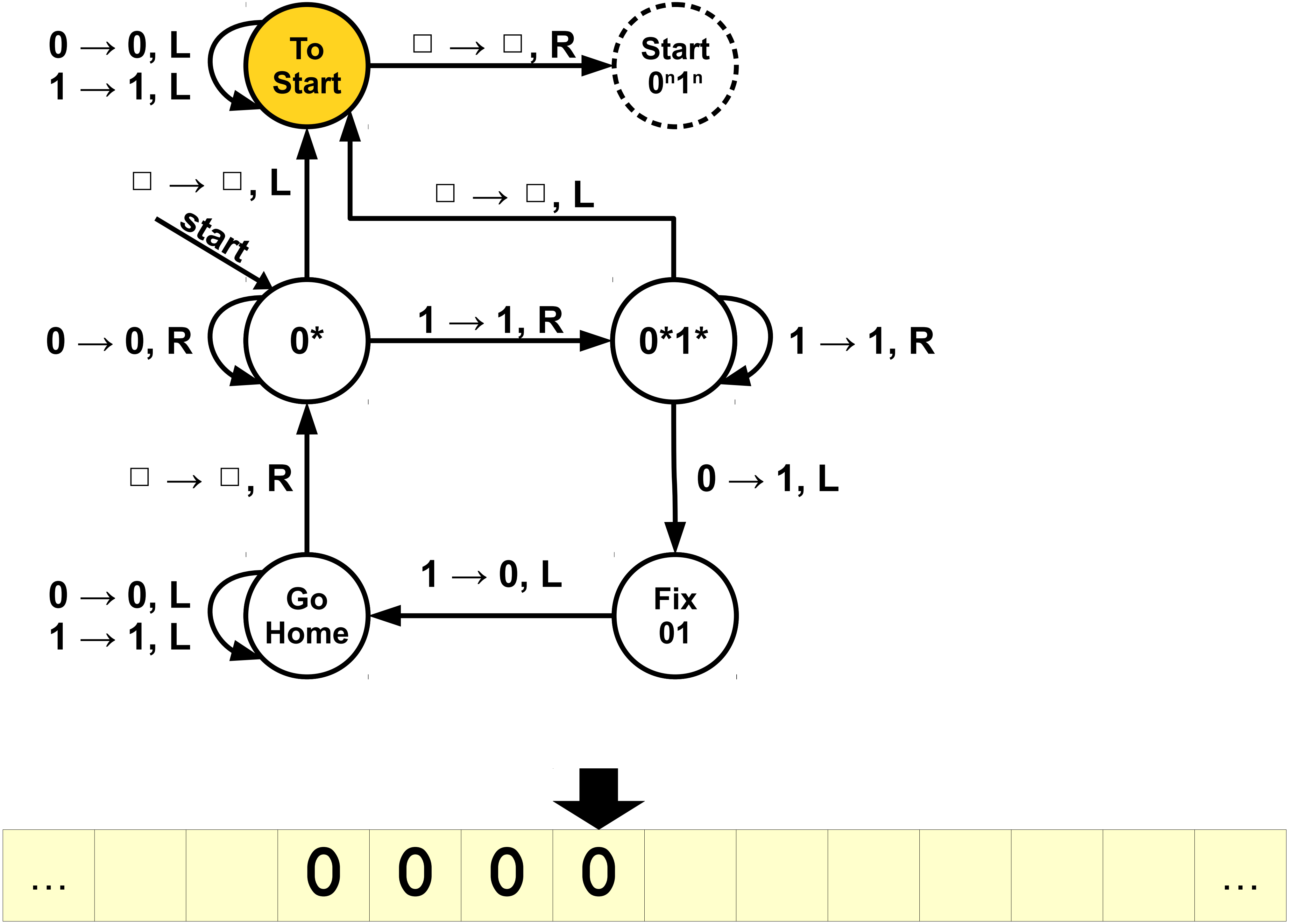

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…

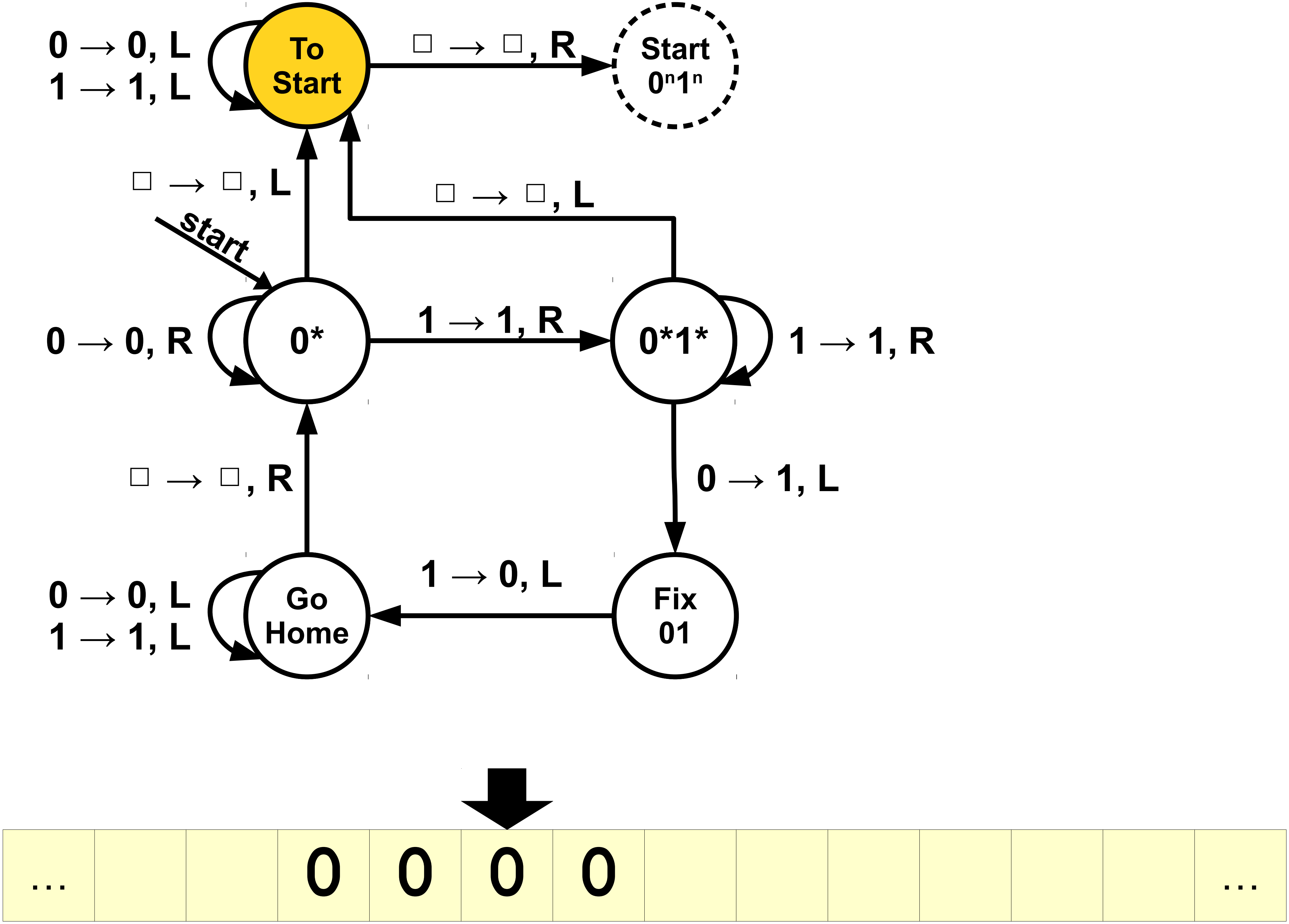

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
1 → 0, L
0 → 0, L
1 → 1, L
Go Home
Fix 01
…
0
0
0
0
…

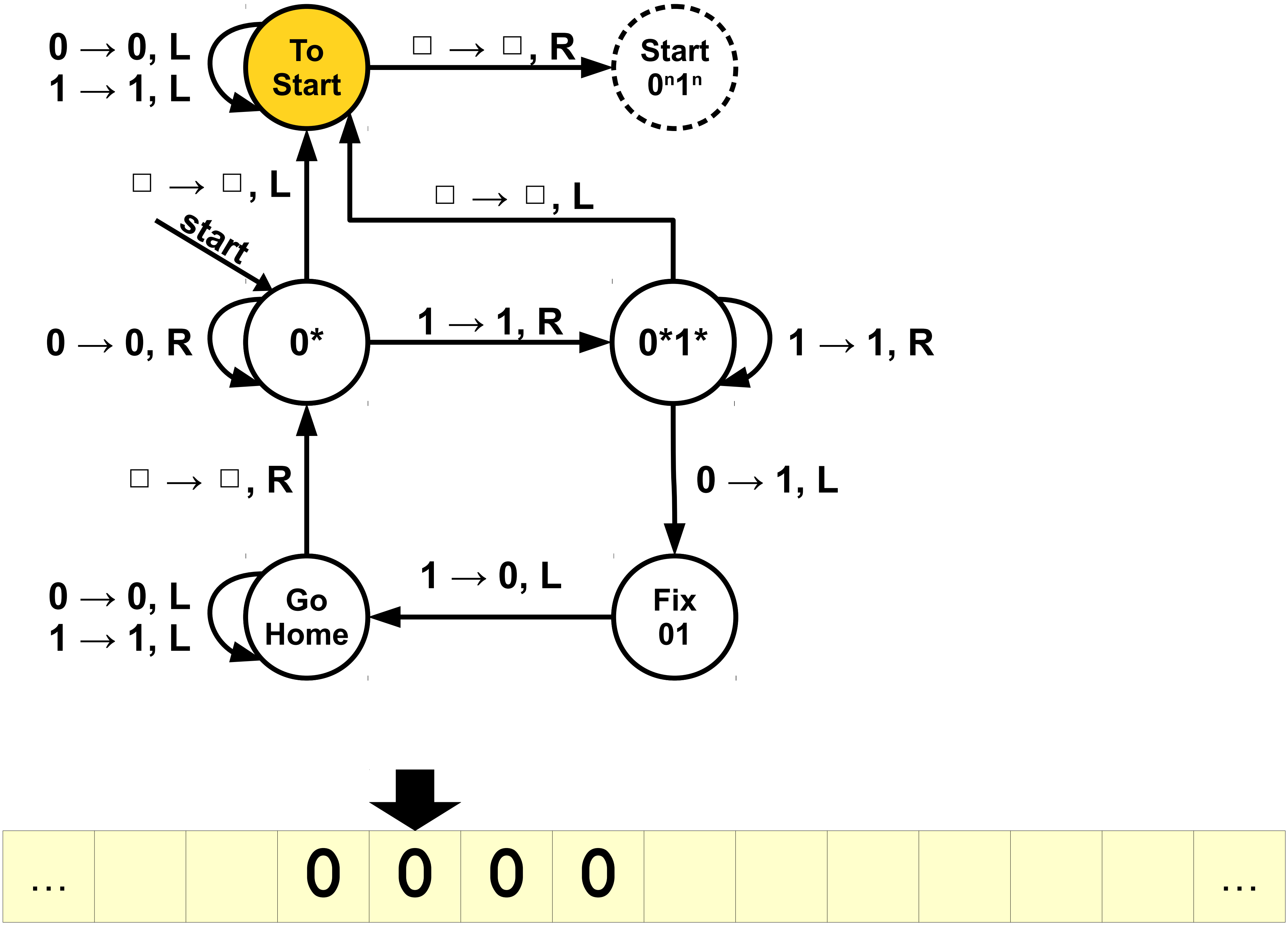

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…

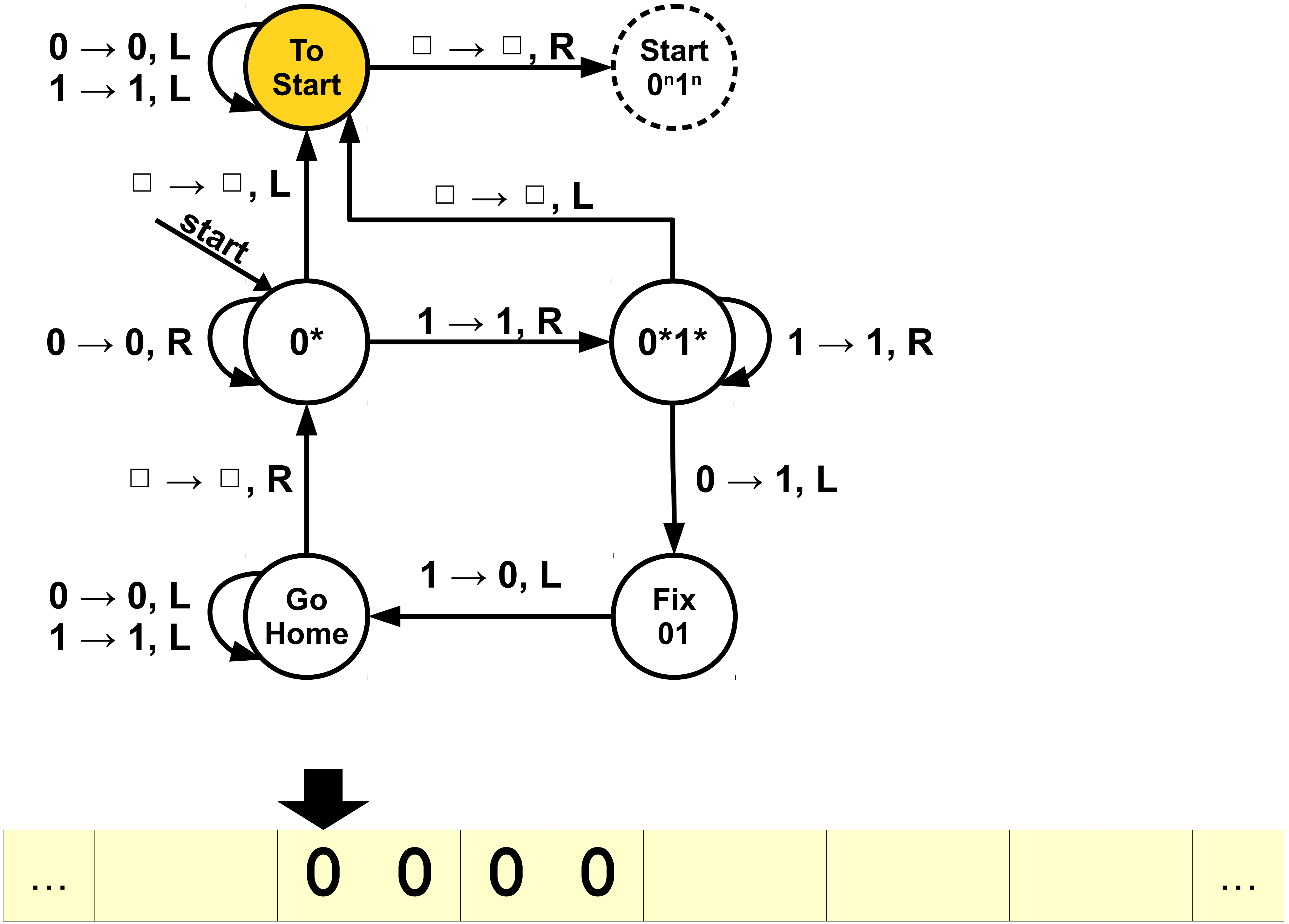

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…

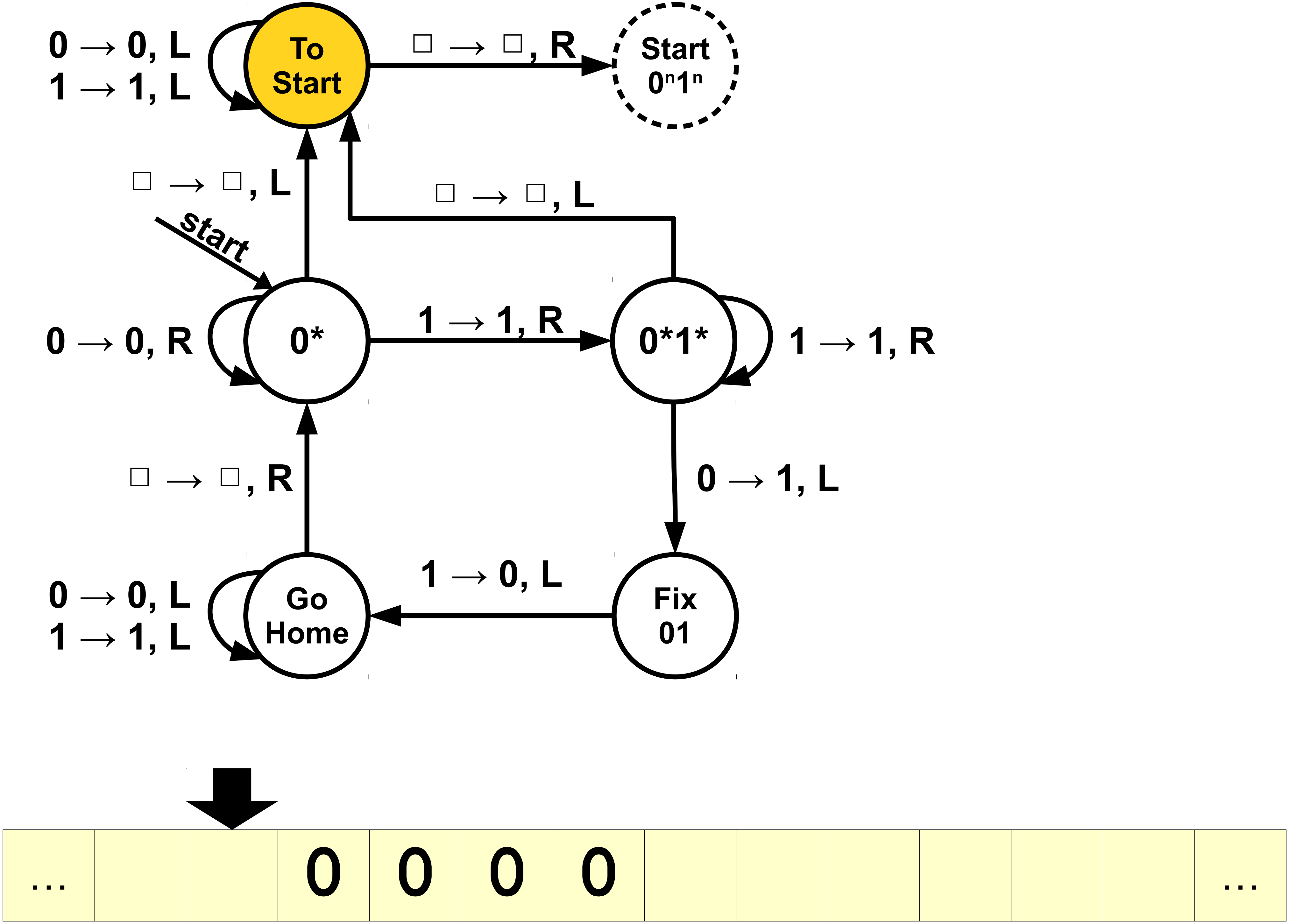

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…

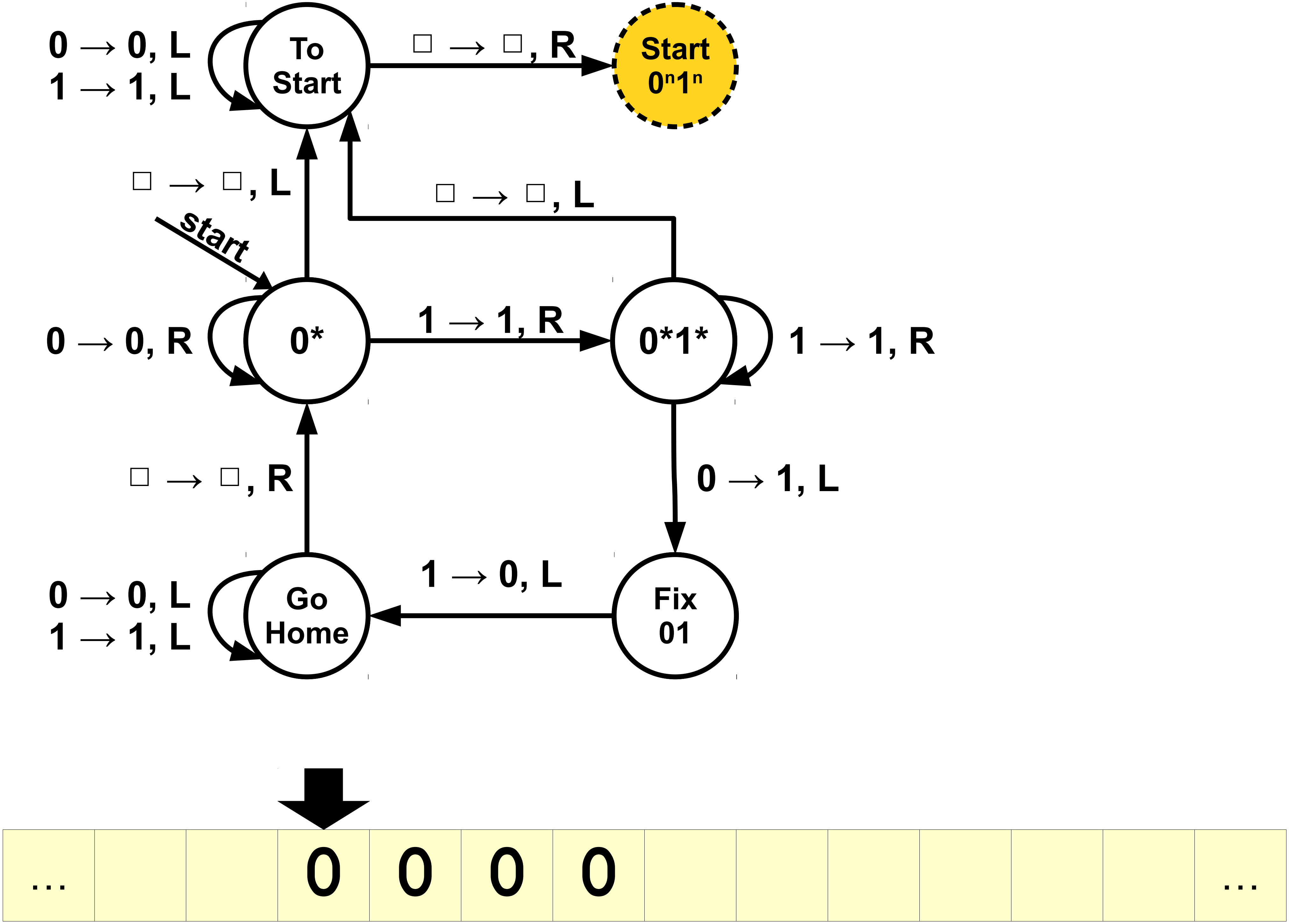

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
□ → □, L
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…

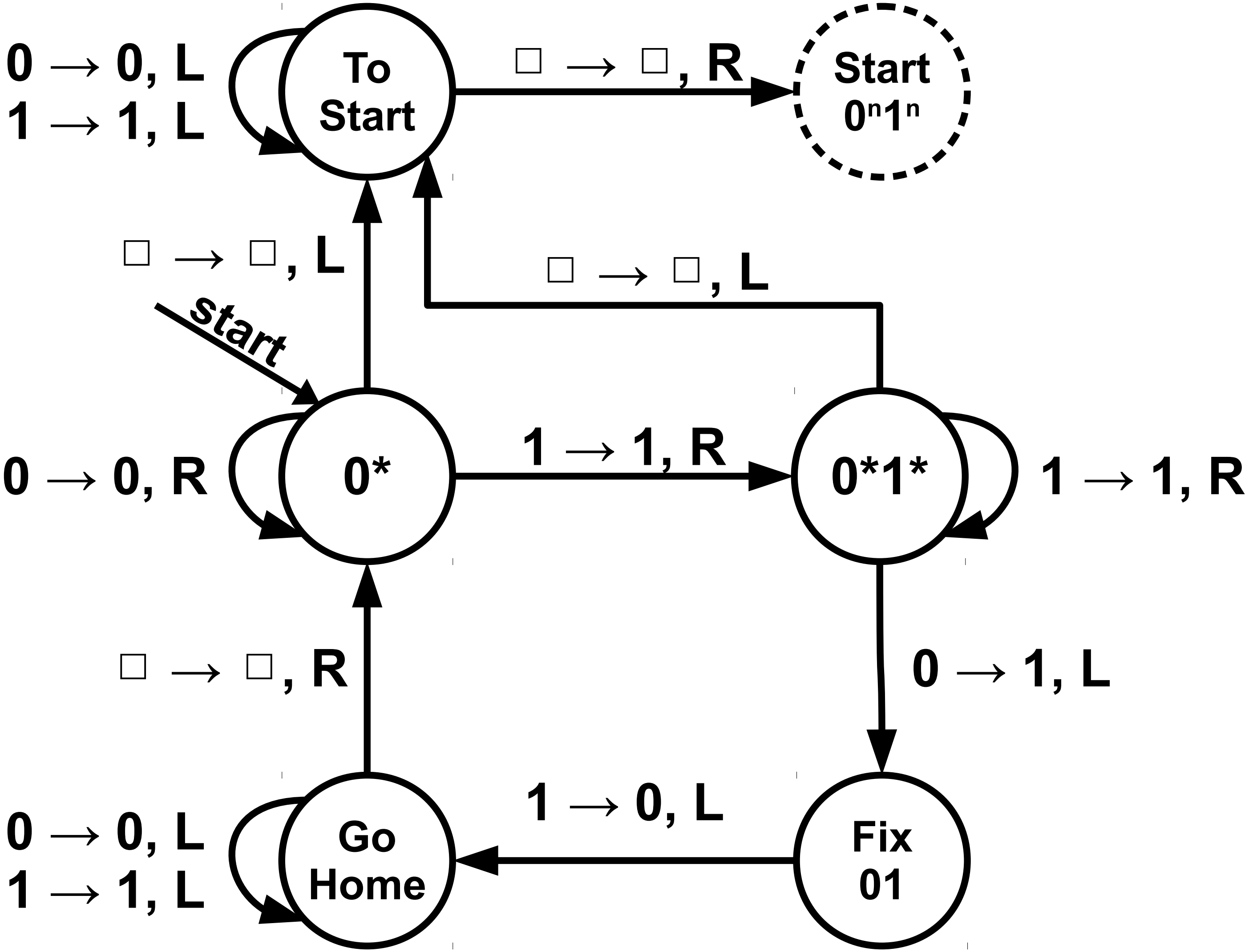

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…

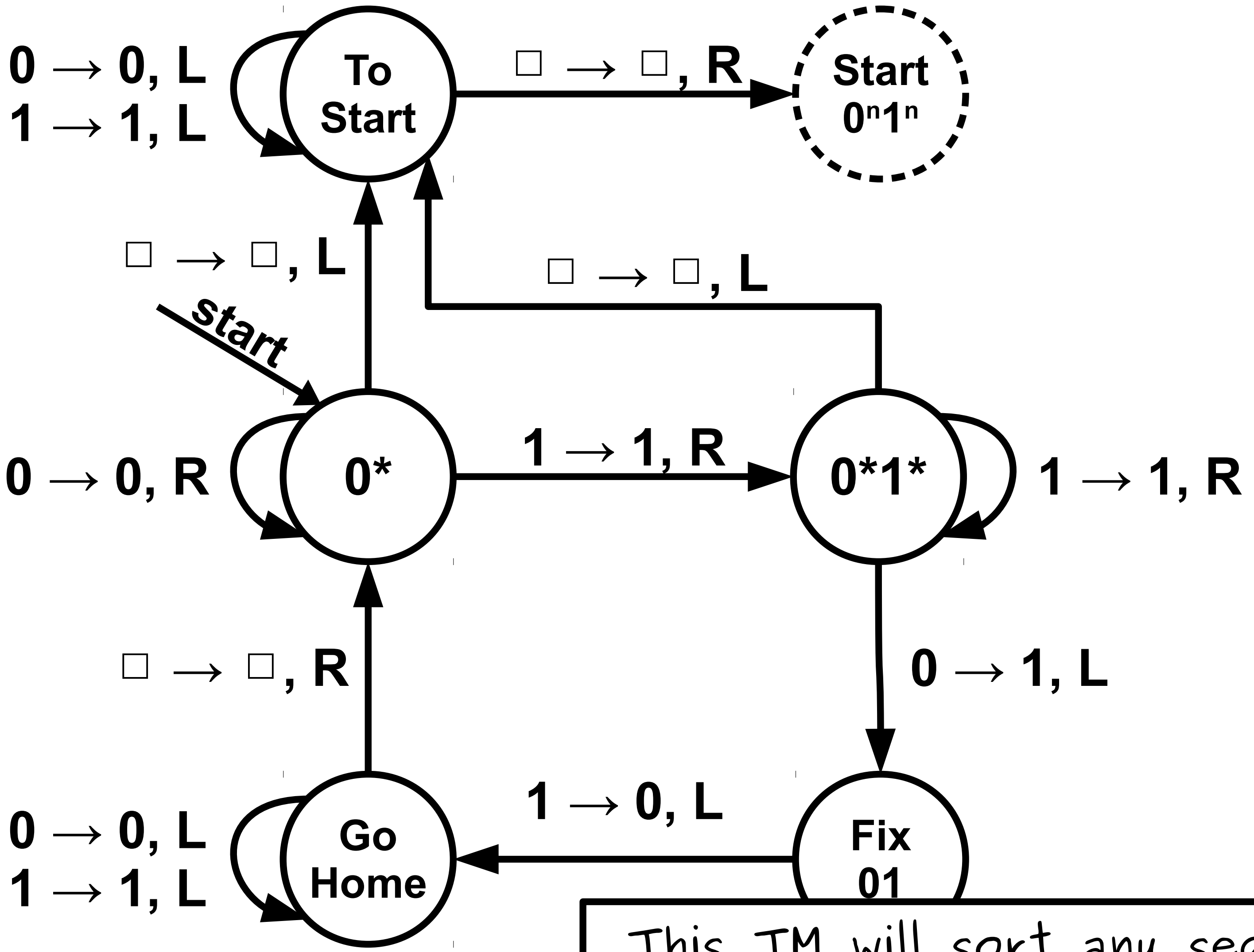

0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
□ → □, L
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…


To Start
Start 0ⁿ1ⁿ
0*
0*1*
Go Home
Fix 01
start
0 → 0, L
1 → 1, L
□ → □, R
□ → □, L
□ → □, L
0 → 0, R
1 → 1, R
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
1 → 0, L
…
0
0
0
0
…


0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
start
□ → □, L
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…


0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0ⁿ1ⁿ
□ → □, L
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01
…
0
0
0
0
…


0 → 0, L
1 → 1, L
To Start
□ → □, R
Start 0^n1^n
□ → □, L
□ → □, L
start
0 → 0, R
0*
1 → 1, R
0*1*
1 → 1, R
□ → □, R
0 → 1, L
0 → 0, L
1 → 1, L
Go Home
1 → 0, L
Fix 01



This TM will sort any sequence of 0s and 1s, but it might take a while.

Fun problem: design a TM that sorts a string of 0s and 1s, but does so while taking way fewer steps than this machine.

TM Subroutines

- A ***TM subroutine*** is a Turing machine that, instead of accepting or rejecting an input, does some sort of processing job.
- TM subroutines let us compose larger TMs out of smaller TMs, just as you'd write a larger program using lots of smaller helper functions.
- Here, we saw a TM subroutine that sorts a sequence of 0s and 1s into ascending order.

TM Subroutines

- Typically, when a subroutine is done running, you have it enter a state marked “done” with a dashed line around it.
- When we're composing multiple subroutines together – which we'll do in a bit – the idea is that we'll snap in some real state for the “done” state.

What other subroutines can we make?

Next Time

- ***Arithmetic Subroutines***
  - What else can we do with TMs?
- ***The Church-Turing Thesis***
  - A scientific hypothesis about computation.
- ***R and RE Languages***
  - What does it mean to solve a problem?