Estimation of Multivariate PDF using the Maximum Entropy Principle

1- Define the handle function for marginal pdf only with Global Constraints

$$P_k(\underline{x}) = e^{-1+\lambda_1+\lambda_2 r_2+\lambda_3 r_3+\lambda_4 r_4} \qquad (1)$$

2- Define the handle function for joint pdf including all Constraints.

$$p(\underline{x}) = e^{-1+\lambda_1+\lambda_2 r_2(\underline{x})+\lambda_3 r_3(\underline{x})+\lambda_4 r_4(\underline{x})+\lambda_5 q(\underline{x})} \qquad (2)$$

where q is a <u>Gaussian</u> pdf.

3- Get the Initial $\underline{\lambda}$ and $\underline{\alpha}$ with global constraints P_K using sample averages.

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} \quad \text{AND} \quad \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} \qquad (3)$$

4- Set $\lambda_5 = 0$ and $\alpha_5 = \frac{1}{T}\sum q(x)$ (for our problem)

5- EVACUATE $E\{\underline{r}-\underline{\alpha}\}$.

$$E\{\underline{r}-\underline{\alpha}\} = \begin{bmatrix} E\{r_1\}-\alpha_1 \\ \\ E\{r_M\}-\alpha_M \end{bmatrix} = \begin{bmatrix} \int\cdots\iint r_1 p - \alpha_1 \\ \\ \int\cdots\iint r_M p - \alpha_M \end{bmatrix} \qquad (4)$$

where M is the number of all Constraints.

6- FROM [1] DIANNE P. O'LEARY, " SCIENTIFIC COMPUTING WITH CASE STUDIES", WE USE THE MONTE CARLO INTEGRATION USING QUASI-RANDOM NUMBERS METHOD TO EVALUATE THE FOLLOWING INTEGRALS.

FOR $r_1(\underline{x})$,

$$E\{r_1 - \alpha_1\} = \iint \cdots \int r_1(\underline{x})\, p(\underline{x}) \qquad (5)$$

FOR $r_2(\underline{x})$,

$$E\{r_2 - \alpha_2\} = \iint \cdots \int r_2(\underline{x})\, p(\underline{x}) \qquad (6)$$

FOR $r_3(\underline{x})$,

$$E\{r_3 - \alpha_3\} = \iint \cdots \int r_3(\underline{x})\, p(\underline{x}) \qquad (7)$$

FOR $r_4(\underline{x})$

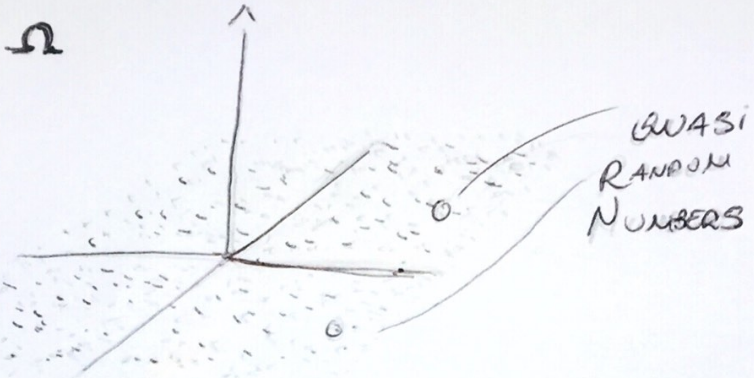
$$E\{r_4 - \alpha_4\} = \iint \cdots \int r_4(\underline{x})\, p(\underline{x}) \qquad (8)$$

FOR $r_5(\underline{x}) = q(\underline{x})$,

$$E\{q - \alpha_5\} = \iint \cdots \int q(\underline{x})\, p(\underline{x}) \qquad (9)$$

8- THE QUASI-RANDOM NUMBERS FROM THE VAN DER CORPUT SEQUENCE GIVE A SEQUENCE THAT RATHER UNIFORMLY COVERS Ω WITH SAMPLES

(FOR OUR PROBLEM)



LET D BE THE SET CONTAINING ALL QUASI-RANDOM POINTS COVERING Ω.

9 - WE CAN APPROXIMATE MULTIDIMENSIONAL INTEGRALS BY AVERAGE FUNCTION AS FOLLOWS,

FOR $E\{r_i\} = \iint \cdots \int r_i(\underline{x})\, P(\underline{x})$, $i = 1, \dots, M$. WE SET THE CONSTRAINTS TIMES THE JOINT pdf IN D AND APPROXIMATE THE MULTIDIMENSIONAL INTEGRAL BY THE AVERAGE FUNCTION TIMES THE RANGE (lenth)

$$\begin{bmatrix} \iint \cdots \int r_1 P \\ \vdots \\ \iint \cdots \int r_M P \end{bmatrix} = \begin{bmatrix} r_1(D)\, e^{-1+\lambda_1+\lambda_2 r_2(D)+\lambda_3 r_3(D)+\lambda_4 r_4(D)+\lambda_5 g(D)} \\ \vdots \\ r_M(D)\, e^{-1+\lambda_1+\lambda_2 r_2(D)+\lambda_3 r_3(D)+\lambda_4 r_4(D)+\lambda_5 g(D)} \end{bmatrix} =$$

D IS K-BY-$\tilde{T}$, K IS THE DIMENSION OF SPACE AND $\tilde{T}$ IS THE NUMBER OF SAMPLE SIZE OF QUASI-RANDOM POINTS.

$$= \begin{bmatrix} \underline{QMC}_{\Gamma_1} \\ \vdots \\ \underline{QMC}_{\Gamma_M} \end{bmatrix} \xRightarrow{\text{APLYING AVERAGE FUNCTION}} \begin{bmatrix} \left(\frac{\sum^{\tilde{T}} \underline{QMC}_{\Gamma_1}}{\tilde{T}}\right) \ell \\ \vdots \\ \left(\frac{\sum^{\tilde{T}} \underline{QMC}_{\Gamma_M}}{\tilde{T}}\right) \ell \end{bmatrix} \cong \begin{bmatrix} \iiint \cdots \int \Gamma_1 P \\ \vdots \\ \iiint \cdots \int \Gamma_M P \end{bmatrix}$$

$$\begin{bmatrix} E\{\Gamma_1 - \alpha_1\} \\ \vdots \\ E\{\Gamma_M - \alpha_M\} \end{bmatrix} = \begin{bmatrix} \iiint \cdots \int \Gamma_1 P - \alpha_1 \\ \vdots \\ \iiint \cdots \int \Gamma_M P - \alpha_M \end{bmatrix} \approx \begin{bmatrix} \left(\frac{\sum^{\tilde{T}} \underline{QMC}_{\Gamma_1}}{\tilde{T}}\right) \ell - \alpha_1 \\ \vdots \\ \left(\frac{\sum^{\tilde{T}} \underline{QMC}_{\Gamma_M}}{\tilde{T}}\right) \ell - \alpha_M \end{bmatrix}$$