Framework.

1. Define all functions & their integrals & derivatives.
2. Define handle functions for joint pdf including all constraints
3. Define handle function for marginal pdf with only global constraints.
4. Preprocess data.
5. Get initial λ & α with only global constraints. Note that since we don't have $p(x)$ yet we use sample averages. B is Jacobian.
6. Details about 5.

We have $p(\underline{x}) = e^{-1+\lambda_0+\lambda_1 r_1(\underline{x})+\lambda_2 r_2(\underline{x})+\lambda_3 r_3(\underline{x})}$

$$\int_\Omega p(\underline{x}) = \int_\Omega e^{-1+\lambda_0}\cdot e^{\lambda_1 r_1(\underline{x})+\lambda_2 r_2(\underline{x})+\lambda_3 r_3(\underline{x})}$$

$$1 = e^{-1+\lambda_0}\cdot\int_\Omega e^{\lambda_1 r_1(\underline{x})+\lambda_2 r_2(\underline{x})+\lambda_3 r_3(\underline{x})}$$

$$\left(\tilde{p}(\underline{x}) = p_{X_1}(x_1)\cdots p_{X_k}(x_k)\right) \quad 1 = e^{-1+\lambda_0} * \int\cdots\int e^{\lambda_1 r_1(\underline{x})+\lambda_2 r_2(\underline{x})+\lambda_3 r_3(\underline{x})}$$

under assumption that $X_1,\dots,X_k$ are i.i.d.

$$1 = e^{-1+\lambda_0}\underbrace{\left(\int e^{\lambda_1 r_1(x)+\lambda_2 r_2(x)+\lambda_3 r_3(x)}\right)^K}_{\left(\tilde{p}_x(x)\right)^K}$$

?**Only when use moments.**

$$\Rightarrow \boxed{\lambda_0 = 1 - K\cdot\ln\tilde{p}_x(x)} \Rightarrow \underline{\lambda} = \begin{bmatrix}\lambda_0\\ \lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix} \quad \& \quad \underline{\alpha} = \begin{bmatrix}1\\ \vdots\\ \alpha_4\end{bmatrix}$$

7. Note:

$$p(\underline{x}) = e^{\cancel{\quad}+\lambda_1 r_1(\underline{x})+\lambda_2 r_2(\underline{x})+\lambda_3 r_3(\underline{x})}$$

$$P_1(x) = e^{\lambda_1 r(x)+\lambda_2 r(x)^2+\lambda_3(x^3)}$$

<u>Question</u>:

$$E\{\underline{r}-\underline{\alpha}\} = \begin{bmatrix}E\{r_1\}-\alpha_1\\ \vdots\\ E\{r_k\}-\alpha_k\end{bmatrix} = \overbrace{\begin{bmatrix}\int\int\cdots\int r_1 p(\underline{x})\\ \vdots\\ \int\int\cdots\int r_k p(\underline{x})\end{bmatrix}}^{\int rp\cdots\int rp}$$

If we have joint moments & $X_1,\dots,X_k$ are ind. $\Rightarrow$

$$E\{X_1\cdots X_k\} \stackrel{?}{=} E\{X_1\}E\{X_2\}\cdots E\{X_k\}$$

If we make it (for constraint 4)

$$\int \cancel{\quad}\,[1\ 1\ \dots\ 1]\begin{bmatrix}g(x_1)\\ \vdots\\ g(x_k)\end{bmatrix} p(x_1,\dots,x_k) \stackrel{k=2}{=} \underbrace{\int p(x_2)}_{=1}\int g(x_2)\cdot p(x_2)$$

$$= \int g(x_1)p(x_1,x_2) + g(x_2)\cdot p(x_1,x_2) = \int\int g(x_1)\cdot p(x_1)\cdot p(x_2) + \int\int g(x_2)\cdot p(x_1)p_2(x_2)$$

8. $\tilde{p}_k \to$ without const. $\to \int_{-\infty}^{\infty} \tilde{p}_x$, $\int_{low}^{high} \tilde{p}_x$

9. $y_i = r_i \tilde{P}_x = E\{r_i\}$

$$p = e^{-1+\lambda_0+\lambda_1 r_1+\lambda_2 r_2+\lambda_3 r_3+\lambda_5 g} = \underbrace{e^{-1+\lambda_0+\lambda_1 r_1+\lambda_2 r_2+\lambda_3 r_3}}_{e^{-1+\lambda_0}\cdot\prod_{k=1}^{K} P_k(x)} \cdot e^{\lambda_5 g} \cdot e^{\lambda_5 g}$$

$$= e^{-1+\lambda_0} \cdot P_x^{K}(x) \cdot e^{\lambda_5 g}$$

$$E\{r_i\} = \int r_i \cdot P = \int r_i \cdot e^{-1+\lambda_0} \cdot P_x^{K}(x) \cdot e^{\lambda_5 g} = e^{-1+\lambda_0} \cdot \int r_i P_x^{K}(x) \cdot e^{\lambda_5 g}$$

$$p = e^{\lambda_5 g} \cdot \tilde{p}$$

$$E\{r_i\} = \int_{\Omega} r_i P = \int_{\Omega_\gamma} r_i P + \int_{\Omega_\gamma^c} r_i P =$$

$$= \int_{\Omega_\gamma} r_i e^{\lambda_5 g} \tilde{p} + \int_{\Omega_\gamma^c} r_i e^{\lambda_5 g} \cdot \tilde{p} = \int_{\Omega_\gamma} r_i e^{\lambda_5} \tilde{p} + \int_{\Omega_\gamma^c} r_i \tilde{p} =$$

$$= e^{\lambda_5} \int_{\Omega_\gamma} r_i \tilde{p} + \left(\int_{\Omega} r_i \tilde{p} - \int_{\Omega_\gamma} r_i \tilde{p} \right) = (e^{\lambda_5} - 1) \int_{\Omega_\gamma} \overset{r_i e^{-1+\lambda_0} \cdot \tilde{p}'}{r_i \tilde{p}} + \int_{\Omega} \overset{r_i e^{-1+\lambda_0} \cdot \tilde{p}'}{r_i \tilde{p}}$$

$$= (e^{\lambda_5} - 1)(e^{-1+\lambda_0}) \left[\int_{low}^{hig} r_i \tilde{p}' \right]^K + (e^{-1+\lambda_0}) \left[\int_{-\infty}^{\infty} r_i \tilde{p}' \right]^K =$$

$$= (e^{\lambda_5} - 1) \left[\int_{low}^{hig} r_i \, e^{-1+\lambda_0} \cdot \tilde{p}' \right]^K + \left[\int_{-\infty}^{\infty} r_i \, e^{-1+\lambda_0} \cdot \tilde{p}' \right]^K$$

$$= (e^{\lambda_5} - 1) \cdot (e^{-1+\lambda_0}) \left[\int_{low}^{hi} r_i \tilde{p}' \right]^K + (e^{-1+\lambda_0}) \left[\int_{-\infty}^{\infty} r_i \tilde{p}' \right]^K$$

$$= (e^{-1+\lambda_0}) \left[(e^{\lambda_5} - 1) \int_{l}^{h} r_i \tilde{p}'^{K} + \int_{-\infty}^{\infty} r_i \tilde{p}'^{K} \right]$$