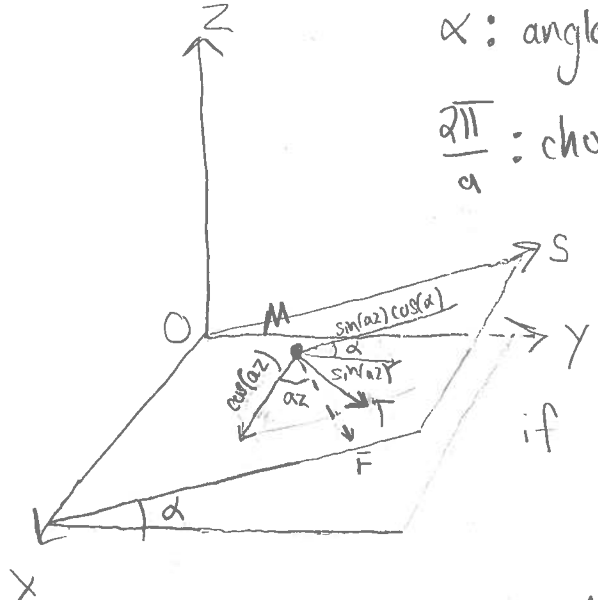
Case 1: No longitudinal component, DNA fibre orientation: $\vec{f} = (\cos(az), \sin(az), 0)$

↑ longitudinal component

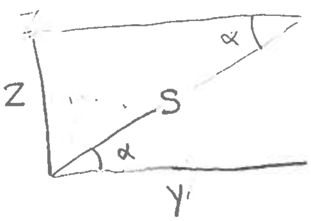


α : angle between image plane & cholesteric disk

$\frac{2\pi}{a}$: cholesteric pitch

MF : actual orientation of DNA fibre at M (in Oxy plane)

MT : how MF appears in image plane (Oxs plane)



if $dx = \cos(az)$

$ds = \sin(az)\cos(\alpha)$

$z = s\sin(\alpha)$

then solving $\frac{dx}{ds} = \frac{\cos(az)}{\sin(az)\cos(\alpha)} = \frac{\cos(a s \sin(\alpha))}{\sin(a s \sin(\alpha))\cos(\alpha)}$

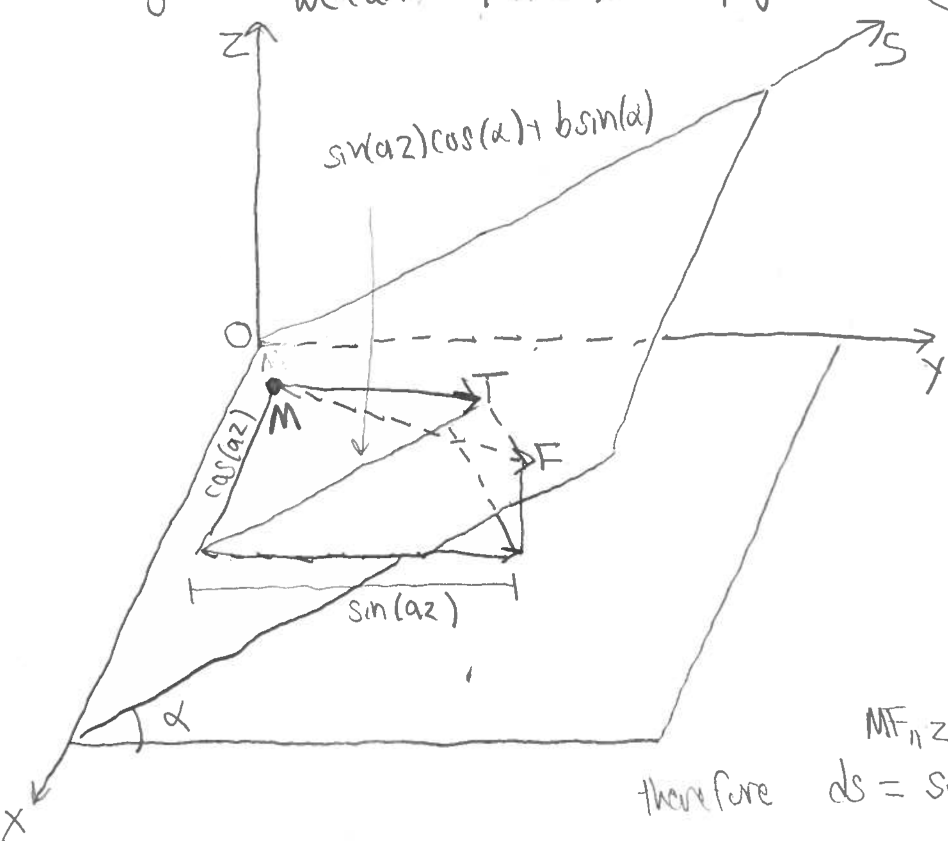
gives the arc traced out by DNA fibres in the image phone

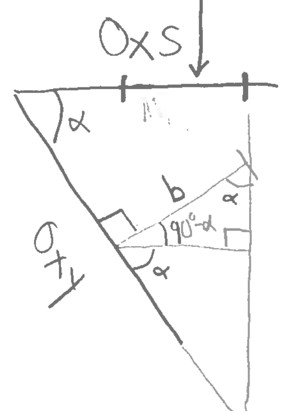
the solution is: $x(s) = x_0 + \frac{1}{a\sin(\alpha)\cos(\alpha)} \ln\left(\left|\sin(s a \sin(\alpha))\right|\right)$

Case 2 Yes longitudinal component, DNA fibre orientation: $\vec{f} = (\cos(az), \sin(az), b)$ magnitude b

we can independently sum the projection of $\underbrace{MF_{\perp}z \text{ onto } Oxs}_{}$ & $\underbrace{MF_{\parallel}z \text{ onto } O}_{}$

same as case 1: $\sin(az)\cos(\alpha)$





$MF_{\parallel}z$ onto $Oxs = b\sin(\alpha)$

therefore $ds = \sin(az)\cos(\alpha) + b\sin(\alpha)$

are traced out by DNA fibres in the image plane

$$\frac{dx}{ds} = \frac{\cos(az)}{\sin(az)\cos(\alpha) + b\sin(\alpha)}, \quad z = s\sin(\alpha)$$

$$= \frac{\cos(a s \sin(\alpha))}{\sin(a s \sin(\alpha))\cos(\alpha) + b\sin(\alpha)}$$

the solution is

$$x(s) = x_0 + \frac{1}{a\sin(\alpha)\cos(\alpha)} \ln\left(\left|\sin(s a \sin(\alpha)) + b\tan(\alpha)\right|\right)$$