- Notes from APL Lab Manual

Knot Theory

- A knot is a closed curve in space w/ no self intersections
- Trivial knot: a loop
- ~~Tr~~
- Non-trivial knots of ~~and~~ one dimensional objects exist only in 3D
- Knots are denoted as C_k
  - C = minimum number of intersections when projecting knot onto a plane
  - k = a cardinal number to distinguish between topologically different knots w/ the same C

- In the experiment, we use a ball chain of hollow spheres connected by rods
- These can be treated as chemical bonds
- The balls are free to slide along the rods
- The chain displays diffuse behaviour while vibrating on the metal plate due to its random motion

- The 3_1 chain is said to become uncrossed when the number of crossings goes from 3 to ~~*~~ 2
- For a knot in the center of the chain, the average unknotting time is

$$\tau_{avg} = \tau_0 (N - N_0)^{\delta}$$

number of beads on → N

N_0 ← min number of beads in knot (const.)

Basic Experiment

- Measure τ for at least one set of experimental conditions w/ a trefoil (31) knot
- Use chains of same material & bead sizes but different lengths
- knot initially in middle of chain
- First choice: yellow brass chain w/ 2.4mm ~~???~~ diameter beads

- The dynamics of the bouncing balls can ~~vat~~ vary from periodic to chaotic depending on freq./amplitude of shaker
- Shaker has resonant freq ~20 Hz
- Cannot do sinusoidal motion for <10 Hz or large frequencies

- To find suitable conditions, look for a freq. 11–19 Hz that produces random chain motion

- Trefoil knot is asymmetric: one end passes under a loop and one end passes over
- This may cause the motion of the crossing points to be asymmetric. Note which end unknots each time

- Also examine ~~???~~ survival probability S3 from Ben-Naim et. Al. and describe qualitatively how the unknotting process is affected by experimental conditions

- The survival probability of a knot is the probability that the knot still exists at time t, denoted by $S(t,N)$
- The exit time distribution is

$$R(t,N) = -\frac{d}{dt} S(t,N)$$

and

$$\tau = \int dt\, t R(t,N)$$

- Ben-Naim suggests that $S(t,N) = f(z)$ w/ $z = t/\tau$
- I'll have to read more about this later
- I don't know how to measure this

Data Analysis

- The motion should be random so we should take multiple ~~ms~~ trials ~~of~~ for each length. For M trials, ~~o~~

$$\tau_{avg} = \frac{1}{M}\sum_{j=1}^{M} \tau_j$$

$$\sigma_\tau = \sqrt{\frac{1}{M}\sum_{j=1}^{M}(\tau_j - \tau_{avg})^2}$$

- The uncertainty in the mean is

$$\sigma_{avg} = \frac{\sigma_\tau}{\sqrt{M}}$$

Equipment
- The apparatus is an alluminum plate attatched to a magnetically driven speaker, driven by a signal generator
- Generates sinusoidal oscillation

- Accelerometer at bottom of plate, accel. proportional to plate output voltage, measured by oscilloscope
- Make sure max accel. is constant

Fri, March 10

- Showed up and identified equipment
- Shaker, speaker, signal generator, oscilloscope

- Turned on oscilloscope & freq generator, set freq. to 11 Hz. Left amp at 4 Vpp?
- Turned on speaker
- Started oscillating

Vpp is the peak-to-peak amplitude

- Oscilloscope varies from sinusoidal to just noise
- Do the speaker vol knobs do anything?
- Two channels on speaker

- Is freq too small? No. It was just the auto range and small amplitude
- The knob on the speaker sets the amplitude

- What about the amplitude in the Freq generator
- I'll just leave it a 4 Vpp for now
- Talked w/ lab technician, both Vpp on ~~&~~ signal generator and speaker ~~have~~ knob set amplitude
- I will leave 1 Vpp on freq generator since speaker has a lot of power

- I will be working w/ YB chains (0.1 ~~mm~~ cm balls?)
- I cut off a length of 30 balls and placed on plate unknotted

Fri, March 10

~~Peak~~ PP
- PP amplitude a const 1.24 ± 0.01 V on oscilloscope
- The chain tends to drift to side of plate closer to me for 3 trials. Fix legs

- Prof. John came in, we had a discussion about stochastic mech: study random walk of crossing points
- N_0 is the minimum number of beads required to make a knot. A constant, dependent on Ck and bead
- Take ~30 trials for each set of conditions
- You want to make the experiment easy to redo. Mark center of knot, left end, and right end w/ sharpie
- For survival probability, τ / τ_{avg} for each set of conditions, and sort the array! ?

- This is easier to do for lower amp and higher freq. 912 mV, 18 Hz
- Played w/ legs until shaker can go for 10+ sec w/o noticible drift

- Tied a Ck knot, looks like $N_0 = 24 \pm 1$
~~Minimum~~
- Cut a ~~44~~ N=44 chain. Smaller than this should be very hard to knot
- Nevermind, too hard to knot.
- Start w/ N=80
~~Back to~~

Fri, March 10

- Random mot^n at 13 Hz, ~1 Volt pp osc.
- Tied a 31 knot on N=80 chain
- Marked the center w/ red pen

- I tied the wrong knot initially
- $N_0 = 19 \pm 1$ for trefoil "
- Placed knot on center of shaker and started it w/ stop watch
- Let U denote under, unknotted first and O denote over loop

U = passes under loop
O = passes over

Table 1: N = 80 unknotting times

- very hard to tell if U or O. After 4th measurement, I marked O end w/ black sharpie
- Uncertainty at least 1s so only mark up to sec

τ (s)	Unknotted side	τ (s)	Unknotted side
11 ~~11.14~~	O	5	U
18 ~~18.15~~	U	4	O
9 ~~9.09~~	U	6	U
24 ~~23.75~~	O	8	U
6 ~~5.56~~	U	26	O
18 ~~18.37~~	U	7	U
61 ~~[illegible]~~	O	6	U
8 ~~7.79~~	O	10	O
7	U	14	O
12	O	9	U
7	U	44	U
7	U	14	O
5	O	17 ~~14~~	O
24	U	7	O
25	O	7	O

Fri, March 10

•Notes:
- It is hard to tell exactly when the knot does at this amplitude. Large uncertainty in τ, and more likely that I overestimate τ rather than underestimate. Usually I see extra I think
- ~~I~~ for really long times (20s+) It is possible that the chain drifts and interacts w/ walls of plate. This may affect results
- I placed chain in center of the shaker, w/ arms extending straight. Over to right
- Measured τ w/ phone stop watch

•Next chain: N = ~~40~~ 120
•Once again, marked O end in black, center in red
•Repeat process

changed this to ~~red~~ black sharpie too

Table 2:

τ (s)	Unknotted side
8	O
24	O
39	U
27	O
12	O
26	U
76	O
15	O
27	O

Switched to stop watch here

•Too much self crossing?
•Talked to Prof. John. I want just enough amplitude so that the knot moves around, but not so much that it

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self crosses. Throw away data on p. 7-8
•Start w/ the upper & lower bounds of measurement & subdivide
•Take data until standard error becomes small
* •Around 840 mV seems appropriate. Still 13 Hz

N = 40 — Table 3

τ (s)	Unknotted
11.43	U
3.93	U
9.47	U
13.47	O
6.31	U
2.34	U
9.90	O
3.09	~~O~~ U
6.69	U
3.97	U
~~—~~	
4.47	U
7.47	U
4.90	U
9.90	U
3.63	U

Knot got stuck on edge

τ (s)	Unknotted
5.22	U
6.85	O
4.25	U
12.66	O
6.19	U
4.38	U
3.00	U
6.28	U
4.68	U
3.65	U
3.06	U
3.53	U
2.87	U
6.52	U
11.24	U

•That was a lot better. Knot never flipped over so U and O ~~[illegible]~~ sides remained the same throughout motion

Fri, March 10

• For the high end, I'll use N = 120

T(s)	Unknotted
21.53	U
53.40	U
296.07	U
105.44	U
352.32	U
22.13	U
123.53	U
40.91	O
86.09	U
78.03	U
48.91	U
125.96	O
123.69	U
~~10~~ 63.72	U
49.44	U

Table 4: 15 unknotting times for N = 120 w/ Vpp = 840 mV freq = 13 Hz

• For both table 3 and 4, the knots tend to unknot from the side which passes under the loop

• I put each data set (Tbl 3 & 4) in their own ~~data sets~~ excel files and performed data analysis as on pg. 3

• Now I can add data points and automatically update the $T_{avg} \pm \sigma_{standard}$

• ~~I want to~~ ~~I~~ • This results in

$T_{avg}(N=40) = 6.2s \pm 0.6s$ (% error = 0.09) ← here I mean err/mean

$T_{avg}(N=120) =$ ~~1~~ ~~120~~ $116 \pm 23 s$ (% error = 0.20) ← this was not right?

• I think it is reasonable to shoot for % error < 0.1

• Since it drops $\sqrt{\text{length of dataset}}$, I need 4 times more trials to get the percent error in $T_{avg}(N=120)$ to be 0.1

• 60 trials

• I ~~think~~ think, 30 trials for now, then study other Ns before returning

Tues, March 14

• Arrived at lab and turned on all equipment

• Reset amplitude to 840 Vpp (1.100 Vpp on signal generator) ← mV, oscilloscope

• Freq to 13 Hz (actual range is 840 - 856 Vpp)

• Back to taking data.

• Tied loop in middle of N = 120 chain, w/ the side passing over the loop still marked in black ~~spa~~ sharpie

• Placed knot in center of shaker and started

T(s)	Unknotted
19.72	O
76.13	U
33.50	O
41.44	O
23.41	O
58.28	O
87.19	U
27.16	U
63.31	U
60.28	U
16.09	O
32.66	U
21.12	U
17.28	U
49.78	U

Table 5: 15 more unknotting times for N = 120 w/ Vpp = 840 mV freq = 13 Hz

• A lot more Os today

• A lot more shorter times?

• A lot more self intersection?

• The voltage on the oscilloscope tends to vary by a bit. 10-20 mV ~~volts~~ variation

• This time, it ~~varied~~ seemed like 840 mV was at the bottom of the variation, not the center. I will take a few more data points at a slightly lower Vpp, w/ 840 mV at center and see the difference

Tues, March 14th, 2023

T (s)	unknotted
19.28	U
150.93	O
47.04	U
173.03	O
155.47	U
108.63	O
32.13	O
118.08	O
158.53	O
49.28	U
159.94	U
33.78	U
26.75	O
56.25	O
52.25	U

Table 6: 15 unknotting times for N=120 w/ Vpp = 840 mV and freq = 13 Hz

- Combined w/ table 4, ~~Tst~~ $T_{avg}(N=120)$ = ~~116 ± 14~~ s
- STD is 76 seconds (10:43 AM)

$T_{avg}(N=120) = 100 \pm 14$ s (Mar 16, 3:06)

10:20
- After 5 attempts, this seemed a lot more similar to the conditions I had last time
- No self intersection in the vibrating chain, and longer times

* • Vpp on oscilloscope oscillates from 832 - 848 mV, centered at 840 mV. Ocassionally varies as much as 824 - 856mV. I will replicate this from now on
- Amp 1 = 1.094 Vpp on signal generator. Use this if nobody touches knob on speaker.

Throw away table 5 :c

10:32 • Observation: suppose it takes about ~1s for me to react to the unknotting.

Tues, March 14th

- This would account for ~1s bias in all of my measurements. This wont be a problem for large N w/ times of 1 minute +. But what about small N? How can I account for this bias?
- I could shift my dataset by some constant value to attempt to improve my fit. Maybe try 1s or 0.5s
- Consider this later in analysis phase

- Time to take measurements for N = 80

T (s)	unknotted	T (s)	unknotted
15.53	U	17.06	U
8.00	U	13.21	U
19.13	U	6.65	U
12.46	U	35.28	O
18.72	U	28.13	U
13.85	U	17.03	U
35.90	U	19.15	U
11.78	U	23.28	U
9.88	U	11.88	U
17.31	U	31.46	O
22.38	U	11.50	U
27.97	U	10.94	U
15.56	U	10.97	U
22.57	U	26.13	U
23.13	O	13.75	U

Table 7: 30 unknotting times for N=80 w/ Vpp = 840mV and freq = 13 Hz

$T_{avg}(N=80) = 17 \pm 1$ s

STD = 8 s

Tues, March 14th

•Next, repeat the process for N=60

T (s)	Unknotted	T (s)	Unknotted
~~19~~ 14.91	U	8.72	U
8.63	U	7.34	U
6.12	U	18.16	U
22.53	U	13.69	U
10.56	U	21.06	U
13.40	O	9.75	U
23.16	O	9.65	U
9.72	U	15.72	U
7.75	U	8.28	U
4.94	U	9.22	O
10.81	U	4.85	U
6.88	U	15.57	U
10.56	U	24.25	O
24.34	U	11.69	U
38.47	O	18.18	U

Table 8: 30 unknotting times for N=60 w/ Vpp = 840mV and Freq = 13 Hz

$\tau_{avg}(N=60) = 12 \pm 1$ s

STD = 7 s

•Plotting the data I've found so far, does not appear quadratic for N<80.

•I will investigate larger Ns

•Maybe N = 100 - 200 range?

•In particular, N = 80 seems to have a much smaller τ_{avg} than the model predicts.

Thurs. March 16th,

•Last time, I found that N=60 and N=80 seemed to disagree w/ the model

-The model also predicts that $\sigma/\tau_{avg} \simeq 0.63$, independent of length. I found

$N=40 \Rightarrow \sigma/\tau_{avg} = 0.50$

$N=60 \Rightarrow \sigma/\tau_{avg} = 0.61$

$N=80 \Rightarrow \sigma/\tau_{avg} = 0.45$

$N=120 \Rightarrow \sigma/\tau_{avg} = 0.66$

• N=40 disagreement makes sense. I estimated there is ~1 sec bias, which means $\tau_{avg}(N=40)$ is 16% bigger than it should be

•Why does N=80 disagree? I will remeasure this now

•Turned on ~~equipmen~~ equipment. Set Vpp to 832-848mV

T (s)	Unknotted	T (s)	Unknotted	T (s)	Unknotted
43.12	U	14.54	U	18.03	U
70.62	U	15.72	U	38.56	U
63.37	O	21.19	U	28.03	U
53.78	U	100.16	U	17.32	U
136.15	U	72.97	U	17.56	U
116.13	U	15.03	U	30.85	U
21.06	U	38.16	O		
63.22	U	97.75	U		
28.72	U	18.72	U		
20.91	U	13.13	O		
32.75	U	25.75	U		
30.37	O	15.13	U		

Table 9: 30 unknotting times for N=80

$\tau_{avg}(N=80) =$ ~~43 ± 8 s~~ $\rightarrow 43 \pm 6$ s

(Mar 16, 3:06)

Thurs, Mar 16

12:30 • After just 5 N=80 trials, it is clear something in table 7 is wrong
- It is possible I did not pull the knot tight enough. It seems that the knot spends a long time loosening before it can travel

• I will test this by pulling the knot until it is very small, but not tight (for N=80)

• ~~First a~~ Times: 13.63 sec, 26.44 sec

• That definitely seems to be the culprit.

~~to make sure today's con~~

• I had one trial I needed to discard because the ~~be~~ knot got stuck in periodic behavior after getting stuck on wall of shaker

~~Th~~

• It is hard to ~~get~~ repeat the same conditions for this experiment

• The motion is very ~~sensit~~ sensitive to amplitude, but the oscilloscope amplitude varies a lot

• It also seems that the knot is easier to tie tight w/ time

Thurs, Mar ~~17~~ 16th

• Retry N=60 as well then

T (s)	Unknotted
6.81	U
9.44	O
20.68	U
~~14.~~ 17.43	U
14.69	U

Table 10: ~~30~~ 5 unknotting times for N=60.

This agrees w/ table 8

• Putting together w/ table 8, $T_{avg}(N=60)$ = ~~12 ± 4 s~~ 14 ± 1 s

(Mar 16, 3:05)

• Now, move on to N = 100

T (s)	Unknotted	T (s)	Unknotted
80.44	U	27.37	O
123.25	U	19.50	U
55.34	U	60.16	U
31.81	O	81.91	U
121.09	U	33.16	U
118.72	U	91.93	U
87.28	U	50.63	U
69.94	U	43.19	U
35.84	U	57.69	U
44.94	U	76.18	O
34.03	U	14.75	U
88.06	U	44.04	O
~~86~~ 106.32	U	23.75	U
36.60	U	27.19	U
60.94	U	30.50	U

Table 10: 30 unknotting times for N=100

$T_{avg}(N=100) = 54 \pm 6$ s

• I noticed some errors on excel sheet math, will go back and fix (Mar 16, 5:05)

Thurs, March 16th

- I believe my analysis at the beginning of page 16 is wrong
- I only took 2 times to prove my hypothesis ~~of~~ that the chain was not tied tightly enough. It ~~is also~~ is likely that it was just statistical fluctuation. ~~Combi~~ I combined the data from tables 7 and 9 to get

$T_{avg}(N=80) = \cancel{30 \pm}$
$= 31 \pm 3$ s

- This seems to agree more w/ the model. I should take more measurements for N=120 as well then

Fri, March 17th

- Arrived at lab, turned on equipment
- Set freq to ~~11~~ 13hz, amp to ~~.~~840 mV Vpp
- Did a test run to make sure the chain doesn't self intersect
- Now, get 30 more times for N=120

	T (s)	Unknotted	T (s)	Unknotted
	33.47	U	52.60	O
	114.22	U	35.44	O
	66.31	U	116.15	U
	84.62	U	28.22	U
	75.72	O	91.31	U
	35.62	U	104.84	U
	116.78	U	80.57	O
	107.35	U	18.12	O
	115.06	U	61.42	U
weird →	349.50	U	134.66	U
	12.59	U	66.44	U
	31.75	U	29.44	U
	42.25	O	95.15	U
	88.82	U	50.50	O
	32.44	U	200.56	U

Table 11: 30 Unknotting times for N=120

- Combine w/ data for table 8 & 6,

$T_{avg}(N=120) = 91 \pm 9$ s

- I threw away ~~a trial~~ 3 trials bcs the knot flipped over

Fri, March 17th

- seaborn.distplot
- Talked w/ prof. John
- He said to plot histograms for each data point
- Survival probability is cummulative histogram. Plot length_arr-~~cnd~~ against time
- Get one more datapoint, N=140

τ(s)	Unknotting	τ(s)	Unknotting	τ(s)	Unknotting
179.19	U	52.22	U	44.78	U
144.81	O	91.97	O	103.25	U
62.25	O	157.91	O	81.84	U
404.56	O	193.17	U	58.53	U
59.94	U	60.10	U	89.56	U
148.53	U	236.03	O	32.56	O
35.63	U	80.75	O	217.19	O
125.75	U	72.50	O	1[illegible]4.94	U
46.14	U	210.68	U	36.59	U
20.97	U	72.22	O	67.31	U
32.50	O	136.22	O		
34.63	U	79.38	O		
58.56	U	606.78	U		
66.84	O	18.97	U		
56.09	U	42.75	U		
36.13	U	198.78	U		
63.64	U	190.03	O		
63.59	U	27.75	U		
28.10	U	24.04	U		
142.22 ~~122.22~~	U	37.31	U		

Table 12: 50 unknotting times for N=140

$\tau_{avg}(N=140) = 108 \pm 15\,s$

- 15 Os!! That's pretty strange

Sat, March 18th

Summary and Analysis

- Below is a summary of my data for finding δ

Table 13

N (Length)	Mean Unknotting Times (s)	Standard Deviation	Standard Err
40	6.2 s	3.1 s	0.6 s
60	14 s	7 s	1 s
80	31 s	27 s	3 s
100	59 s	31 s	6 s
120	91 s	71 s	9 s
140	109 s	106 s	16 s

- Note that tables ~~1, 4~~ 1, 2, ~~4~~ 5 were ommitted for the acceleration amplitudes being too large
- I will fit this data w/

$$\tau_{avg} = \tau_0 (N - N_0)^{\delta} + b$$

- Since I only have 6 data points, I will use N_0 and b as known parameters

- Earlier, I said $N_0 = 19$. However, I counted the crossing points of the chain in this estimate. These should not be include. N_0 should refer th. to the interior of the knot
- A more accurate estimate is $N_0 = 16$

- b is used to account for the systematic error discussed in pages 12-13
- It takes time for me to notice the knot has opened and to stop the timer. Thus I tend to over estimate the unknotting time by about 1s. So I take b=1s

Sat, March 18th

- With this, I can perform a least squares fit for δ and τ_0 resulting in

$$\delta = 2.0 \pm 0.1$$
$$\tau_0 = (7 \pm 3)\cdot 10^{-3}\ s$$

- This value of δ agrees w/ theory. The goodness of fit:

$$\chi_{red}^2 = 1.6$$
$$CDF = 17\%$$

- My results show that the model agrees w/ reality
- Plot ~~bellow~~ below

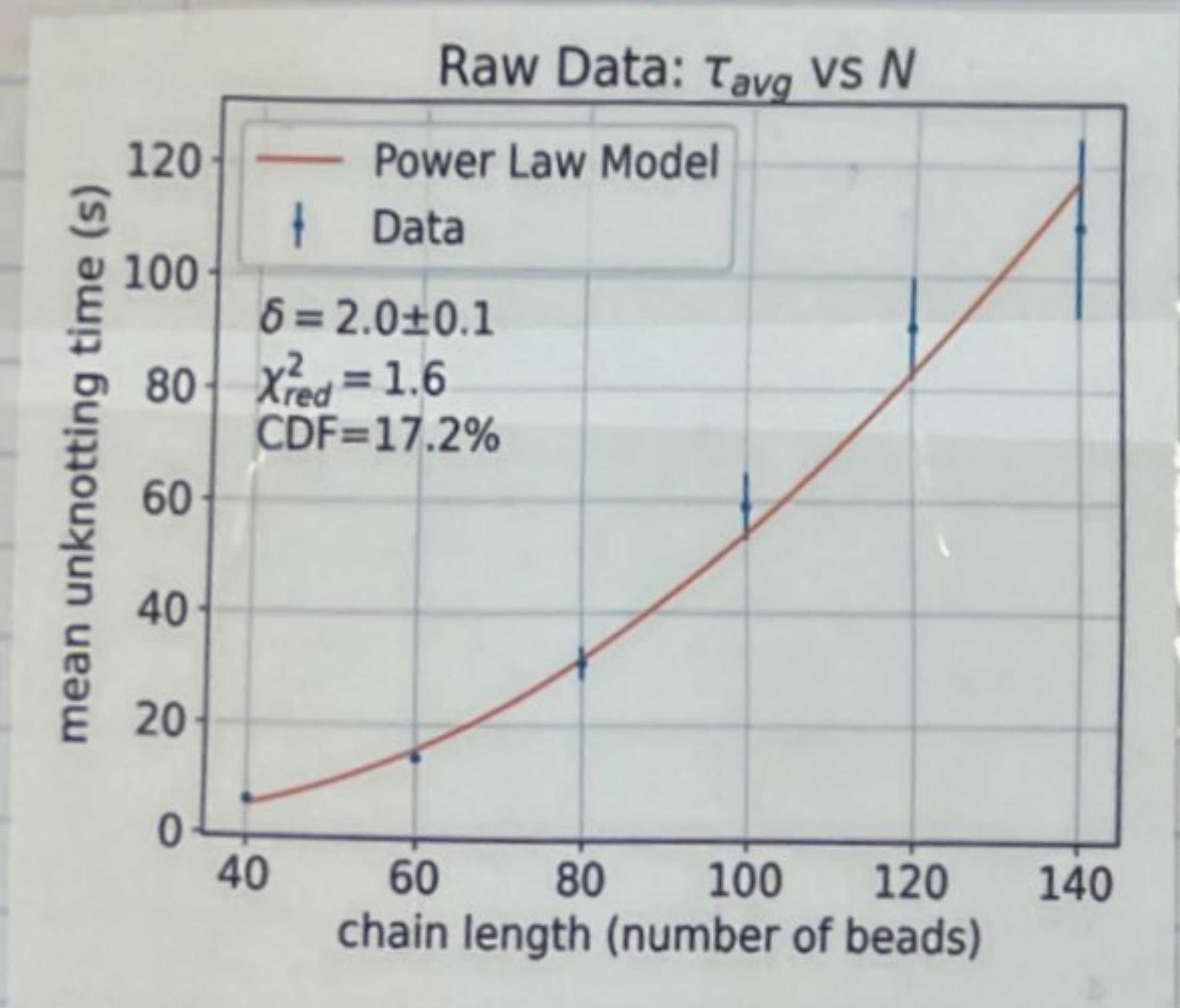


Fig 1: Average opening time vs N ~~for~~. Raw data

- Blue is data, orange is best fit

- According to Ben-Naim, $\tau_0 = \tau_s / D$, where $\tau_s = 0.056213$ is the theoretical, dimensionless mean exit time
- Image generated using get_delta.py
- D is the diffusion constant / hopping rate. My result has

$$D = 8 \pm 4\ s^{-1}$$

- This makes sense. The chain was vibrating at 13hz, and did not ~~go~~ hop a bead ~~during~~ for a fraction of the oscillations, so having a hopping rate slightly lower than the freq, makes sense

Sat March 18th

- The survival probability $S(t,N)$ can be measured like so: For a fixed length N, and for time t, the empirical value of $S(t,N)$ ~~at~~ is the fraction of survival times in the data set greater than t.
- Plotting this against t/τ_{avg} for each data set for different Ns shows that they all follow the same characteristic curve.

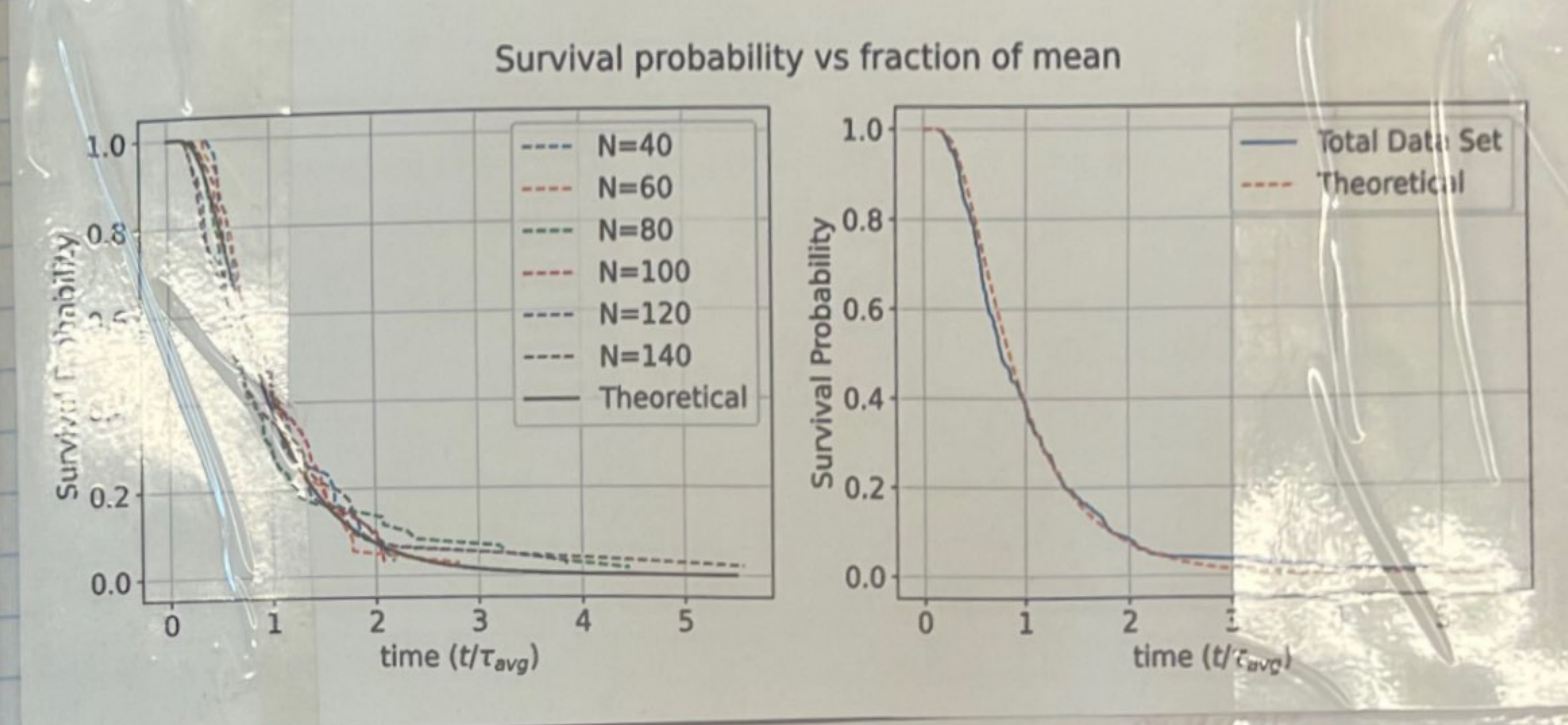


Fig 2: Survival probability vs t/τ_{avg}. On left, we have the survival probability for each independent chain length and the theoretical curve. On the right, we have the survival probability computed from the entire data set

- Analysis on page 44
- Images generated using survival.py

This page is wrong !!

Sat, March 18th

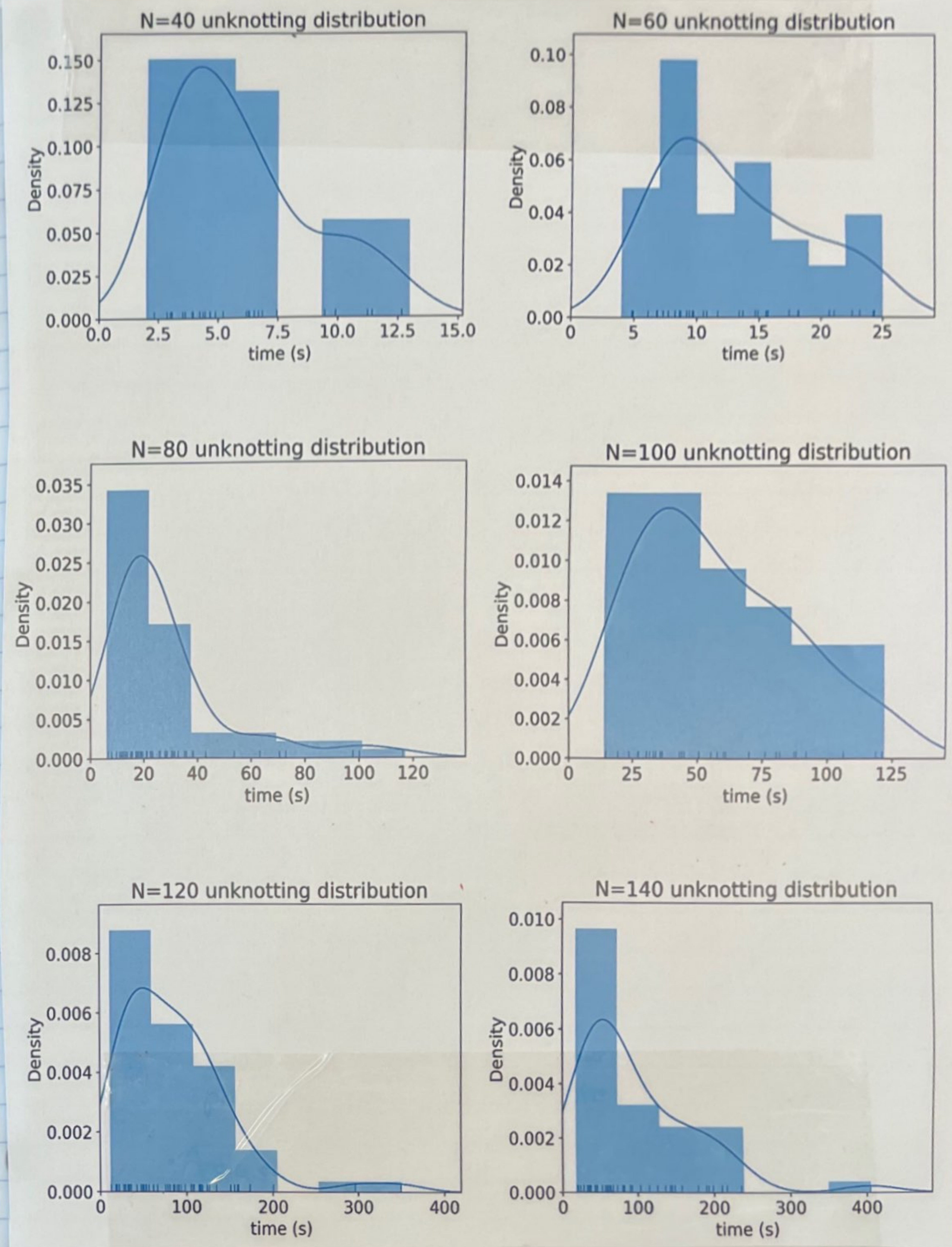
- I mentioned on pg 19 that I discarded some trials because the knot flipped over. However, I did not do this for other trials
- To study the effects of such points on the model (points w/ anomalously high untangling times), I can plot a histogram for each data set and omit anomalous data points.
- I ommited the highest 3 data points for N = 80, 120
- Ommited highest data point for N = 140
- The ~~first~~ result is a slightly worse fit.

$\delta = 1.9 \pm 0.1$ $\qquad$ $\chi_{red}^2 = 2.8$

$\tau_0 = (12 \pm 6) \cdot 10^{-3}$ s $\qquad$ CDF = 2.5%

- I can conclude that these anomalous points are important to the distribution and should be included

Fig 3: Next page: The distributions of opening times of opening times for each chain length
- Note the Gaussian peaks w/ long tails
- A kernel density estimate & rug plot is included for each distribution
- Images generated using histograms.py



Sat, March 18th

Sat, March 18th

Important Notes

Qualitative Knot Behaviour:

- I found that the knot greatly preferred to open towards the side which passed under the loop. If the knot flipped, it tended to travel to the side which was now under the loop, which would increase the unknotting time

- The knot tended to start by growing a bit from its ~~natural~~ minimal size, before all the crossing points began to travel towards the end of the chain which passed under the loop
- For longer chains, the knot could spend a long time in this state, sometimes even tightening down to its smallest state again. When this happened, its random walk would come almost to a stop, greatly increasing its unknotting time
- Occasionally, the crossing points would travel in opposite directions and the knot would grow slowly to a maximum size before opening

Sat March 18th

- I made a mistake on pg 24
- When generating my histograms, I wanted to Sturge's
- Rule to plot my histogram. i.e.

  ~~plot~~ num bins = $\lceil 1 + \log_2(\text{num data points}) \rceil$
- I then removed the anomalously large data points from the distribution. I did this by looking at the histograms and excluding those large times which did not come in clusters. i.e. The largest k ~~p~~ times. For each data set, this was

N	k	New Mean (s)
40	5	4.5 ± 0.4
60	0	14 ± 1
80	8	25 ± 2
100	4	50 ± 5
120	3	76 ± 6
140	1	91 ± 9

- The result was $\left\{ \begin{array}{l} \delta = 2.01 \pm 0.07 \\ \tau_0 = (6 \pm 1) \cdot 10^{-3}\,s \end{array} \right.$ $\left\{ \begin{array}{l} \chi^2_{red} = 1.04 \\ CDF = 38.4\% \end{array} \right.$

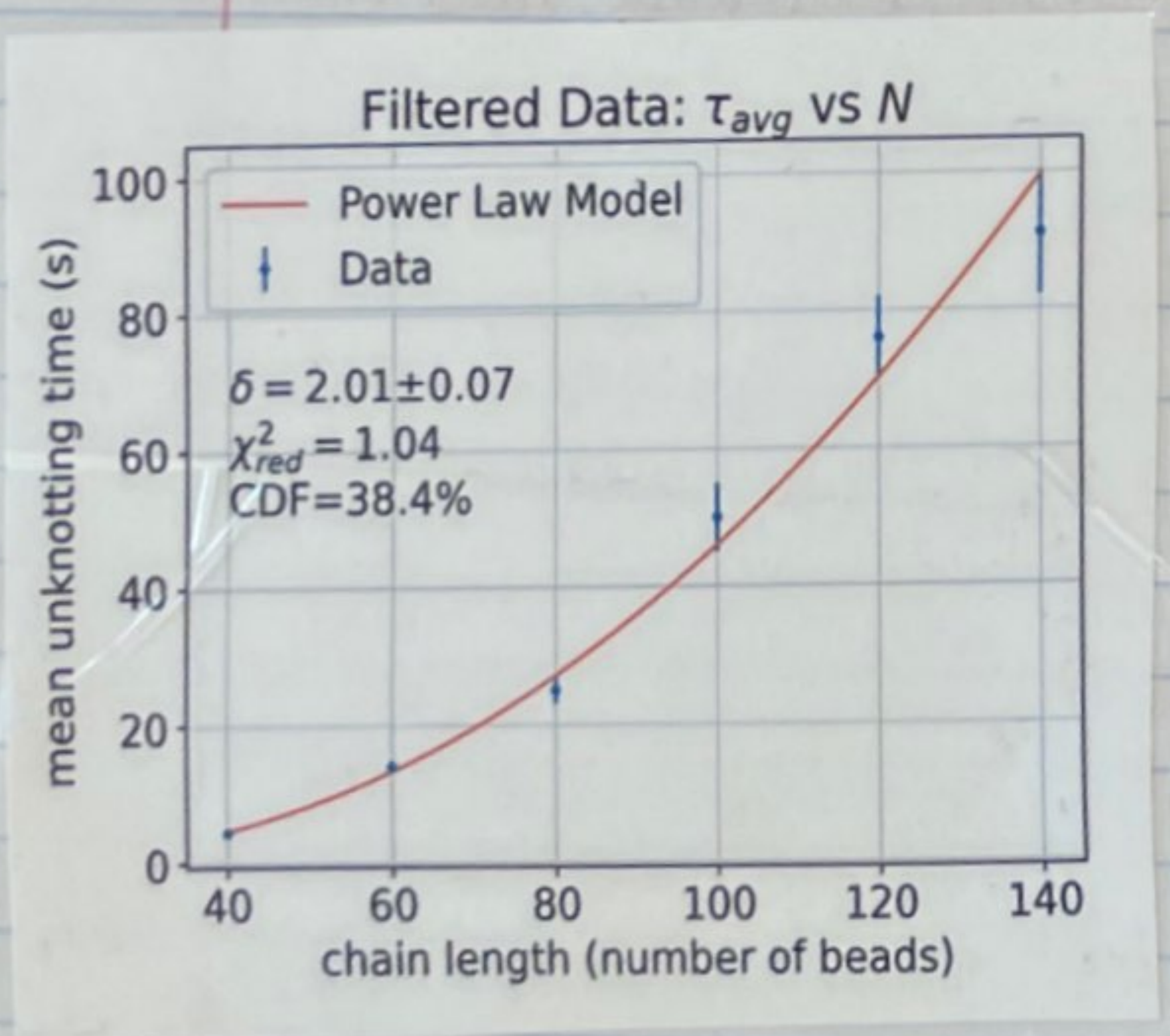


Fig 4: Average opening time vs N, for the filtered data

- Image generated using get_delta.py
- The resulting diffusivity is $D = 9 \pm 3\ s^{-1}$

- Removing the tail of the gaussian therefore improved my results
- The long tail mostly consisted of trials where the knot flipped, or the chain ~~x~~ created a new self intersection. This violates Ben-Naim's assumptions, so it makes sense that the data improved

~~Sun~~ Mon, March 20th

Radius of Gyration:

- The radius of gyration of ~~a~~ the chain ~~of~~ of length N is

$$R_g = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\vec{r}_i - \vec{R}_c)^2}$$

where $\vec{r}_i$ is the position of the ith bead and $\vec{R}_c$ given by

$$\vec{R}_c = \frac{1}{N}\sum_{i=1}^{N}\vec{r}_i$$

is the chain's center of mass

- The maximum radius of gyration occurs when the chain is in a straight line and is equal to

$$R_{g,max} = \cancel{R_c} = \frac{L}{2\sqrt{3}}$$

- Question: Does the radius of gyration approach a certain value $R_g/R_{g,max}$? If so, how does it depend on the physical properties of the chain and shaking process?

Tue, March 21st

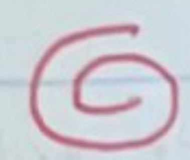
- I will use the python code ~~real time ROG~~ Real Time_ROG
- No, code ~~Simple_ROG~~ Simple ROG which measures the ROG of a light chain on a dark background
- Start equipment w/ V_{pp} = 840 mV & freq = 13 Hz
- Use N=120, 1mm YB chain for test video
- Nevermind, forget all that
- A custom code will be developed which sends the video to grayscale, inverts it, ~~does a~~ uses a threshold value to determine which pixels are part of the chain, and then calculates the ROG for each frame
- Today I am using a webcam that films at 30 fps, so I will use freq = 15 Hz so the chain is at the same height each frame
- I want to investigate the dependence of asymptotic ROG on chain length
- Start w/ N=140 in YB 1mm chain
- The chain keeps drifting out of frame
- At first, I tried to start the chain in a straight line (maximum ROG) but the transient motion takes a long time to end this way
- Regardless, for both max ROG start and a random ~~initi~~ initial condition the final shape is

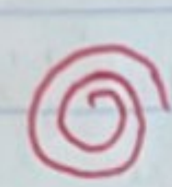
Tues, March 21st

- I will ~~recod~~ record each trial ~~for~~ for about a ~~minute~~ minute after transient ends
- I must cut new chains: The old ones are marked in black sharpie which could interfere w/ imageJ software during thresholding

- Again, freq = 15 Hz. Amp = ~~840 mV~~ 832 mV

- For N=40, final shape C

- For N=60, final shape 

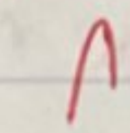
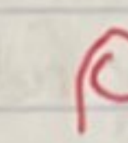
- For N=80, never settled into a final, stable state
- Oh wait! Spiral 

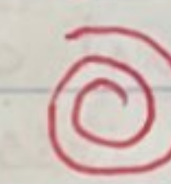
- Took several ~~min~~ minutes to form

- The length of the transient ROG time seems to grow very quickly w/ chain length. If I can confirm that the final result is always a spiral, I can start the chain in a spiral position in the future to speed things along

Fri, March 24th

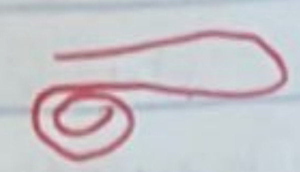
- Last time, plate was reflecting too much light which made thresholding difficult
- Today, I placed a black drop cloth and tarp light over the plate to reduce the reflected light and improve contrast between plate and chain

- I will also start chain in a compactified form, randomly oriented to reduce time required for equilibrium
- Start w/ N=40
- Of course Vpp = 840 mV, Freq = 15 hz
- About 2 minutes like this 
- Saved to trial1.mp4
- Next, N=60
- About 2 minutes of footage like this 
- Saved to trial2.mp4
- There seems to be a double image of the chain in the video
- From its compactified pos^n, equilibrium almost instantly

- Next, N=80
- After 10 mins, still no equilibrium
- I had to throw away this time because it takes up too much space
- Restart w/ random, compact orientation
- Final spiral 

- Saved to trial3.mp4

Fri, March 24th

- N=100 next
- Stuck like this  for 40 MINUTES

- Saved to trial4.mp4

This was for N=120

- The chain spiraled, but w/ a twist, and then opened again before re-spiralling
- After 47 minutes, we got a good spiral
- ~~The~~ Saved to trial 5
- These first 5 trials serve as evidence that the long term behaviour is a ~~loose~~ spiral. I will use this ~~to~~ as justification to start future trials in a loose spiral formation to save time

- Loose spiral for N=140. That still resulted in over 20 mins time because the spiral had slight deformities and unraveled
- Saved to ~~N=60~~ trial 6
- Camera frame has height 1080 and width 1920

- Took an image of a sticker triangle on shaker to get pixel to cm conversion
- Conversion rate: 22.7 cm/1080 px
- I got this by putting the image in an image tracker, setting the cm scale using the triangle, and then measuring the height of the ~~screen~~ image, which I know to be 1080 px

Fri, March 24th

- A rough model for what is going on:

- The chain wants to spiral into a disk and fill its area of radius R

- The radius of gyration of a solid disk is (using R_g as the ~~m~~ root mean square distance from centre of mass)

$$R_g = \sqrt{\frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R r^2 (r\,dr\,d\theta)}$$

$$= \sqrt{\frac{2\pi}{\pi R^2}\cdot\frac{R^4}{4}}$$

$$= \frac{R}{\sqrt{2}}$$

- Now, the area of the circle is filled by N radius a beads ~~as~~ so

$$\pi R^2 = \pi a^2 \cdot N$$

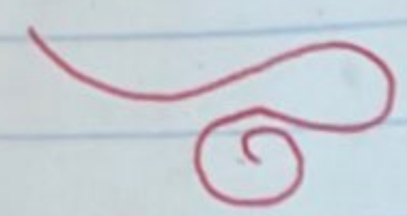
$$\Rightarrow 2\pi R_g^2 = \pi a^2 \cdot N$$

$$\Rightarrow R_g = a\sqrt{\frac{N}{2}} \quad **$$

- I should expect $R_g \propto N^{1/2}$ for any bead size
- This model assumes large N

Fri, March 24th

Early analysis:
- I converted each video trial 1-6 to a folder of ~~slides~~ images, 1 frame every 2 seconds
- Calculated ROG in each image
- Took the last minute of footage ~~for~~ (30 frames) for each trial, when the chain is at equilibrium and plotted mean rog vs N, w/ standard error in the mean as y-error
- Trial 4 is clearly an outlier. The ROG is way too big and the final shape was something like



- I need to redo this trial

Tues, March 28th

- Turned on equipment. freq = 15hz, Amp = ~840mV Vpp
- Put black drop cloth back over shaker.
- Redo N=100, but start chain in a loose spiral to save time
- It appears that the equilibrium ROG decreases until N~30
- It is possible I did not give N=40, 60 enough time as they were very loose spirals
- Computer froze as soon as video ended. Data may be lost :(
- I connected the camera to my laptop instead, the lab PC is no good
- Since I hadn't touched the chain, I recorded a short 30 sec video again to get the equilibrium ROG data for N=100
- Saved to trial 7_discarded
- Video too short, need to start over, I will do this after N=60
- For N=60, started in a tight, perfect spiral and it opened up again
- Saved to trial 7

After plotting, Rog seems to be increasing at the end of video. Not enough time to loosen →

- N=100 again, in a tight spiral this time
- Saved to trial 8

- Cut N=50, put in a spiral and it opened!
- Saved to trial 9

Tues, March 28th

- Cut N=55 and it opened after a couple minutes
- Saved to trial 10

- The point where spirals can form is ~60 balls

~~Next, I will do N=160~~

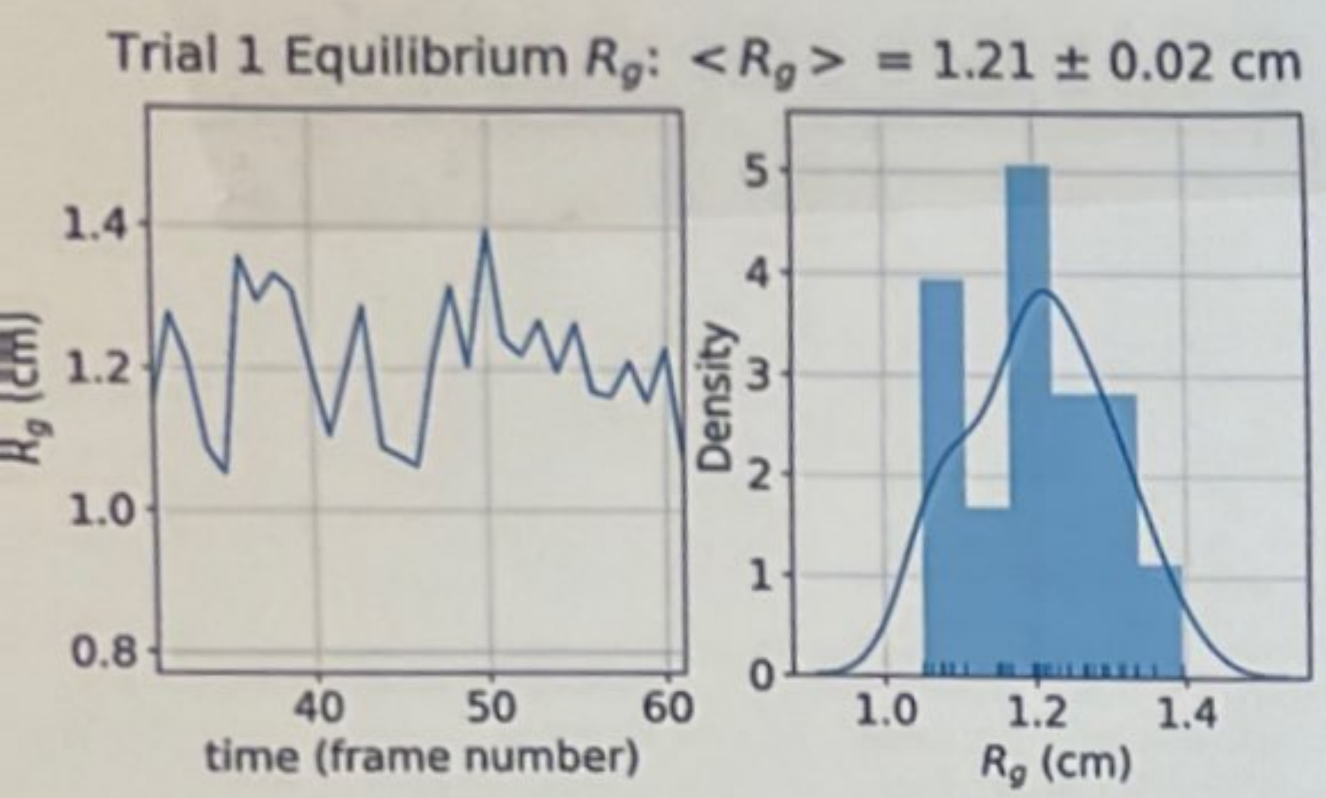
- Redo N=100. After plotting ROG vs time, I saw trial 8 had an increasing ROG still, not equilibrium
- The chain expanded to a very loose spiral and remained in that position
- Saved to trial 11

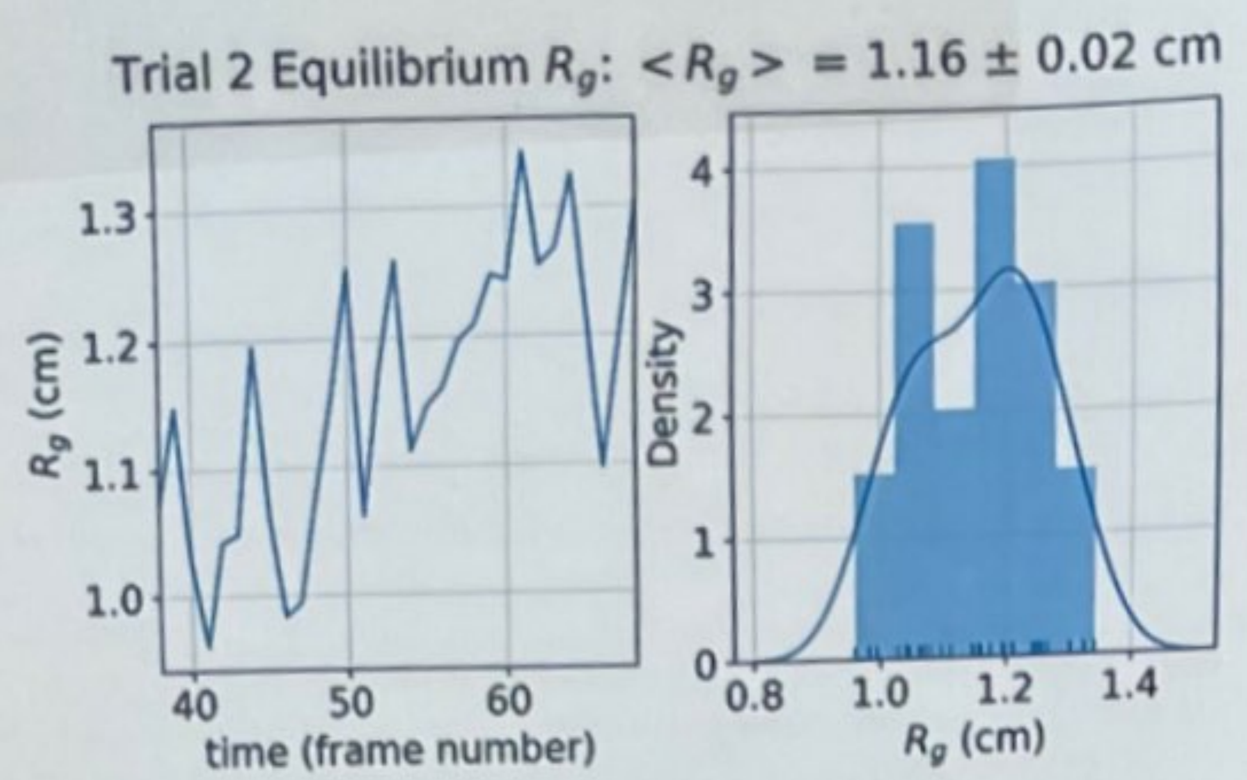
- Now repeat for N=160
- I started this trial in a spiral formation but it unraveled. I think it was ~~to~~ trying to start a double spiral at this length. Did not have time to see the final result

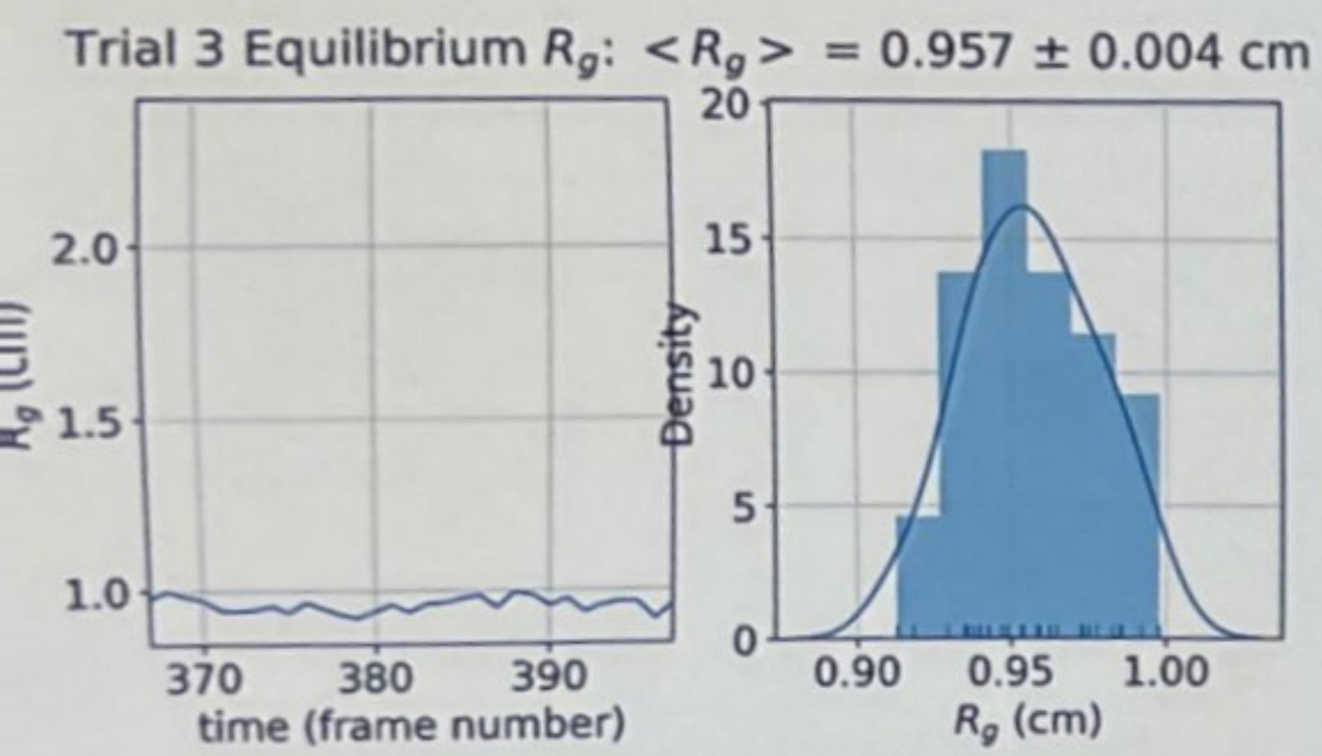
Fri, March 31st

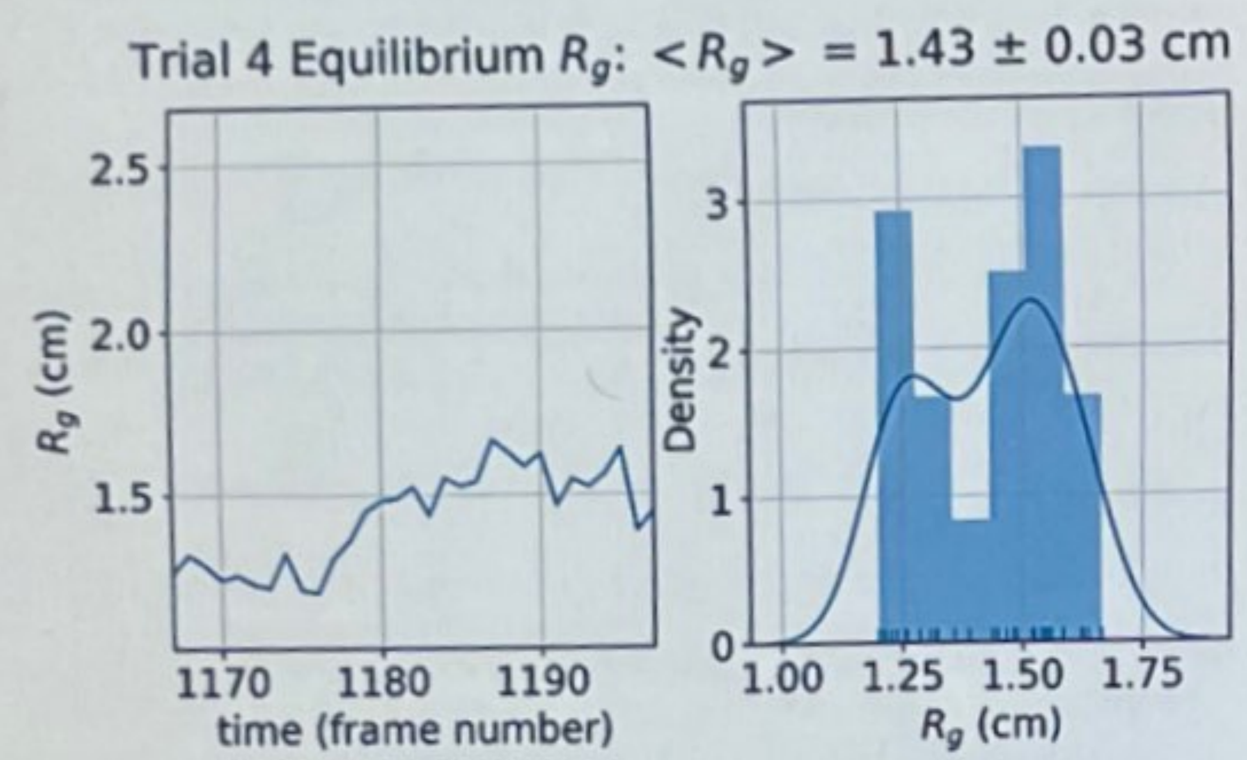
- Repeat N=160, but w/ a random compactified initial pos^n instead of a spiral (10:00 AM)

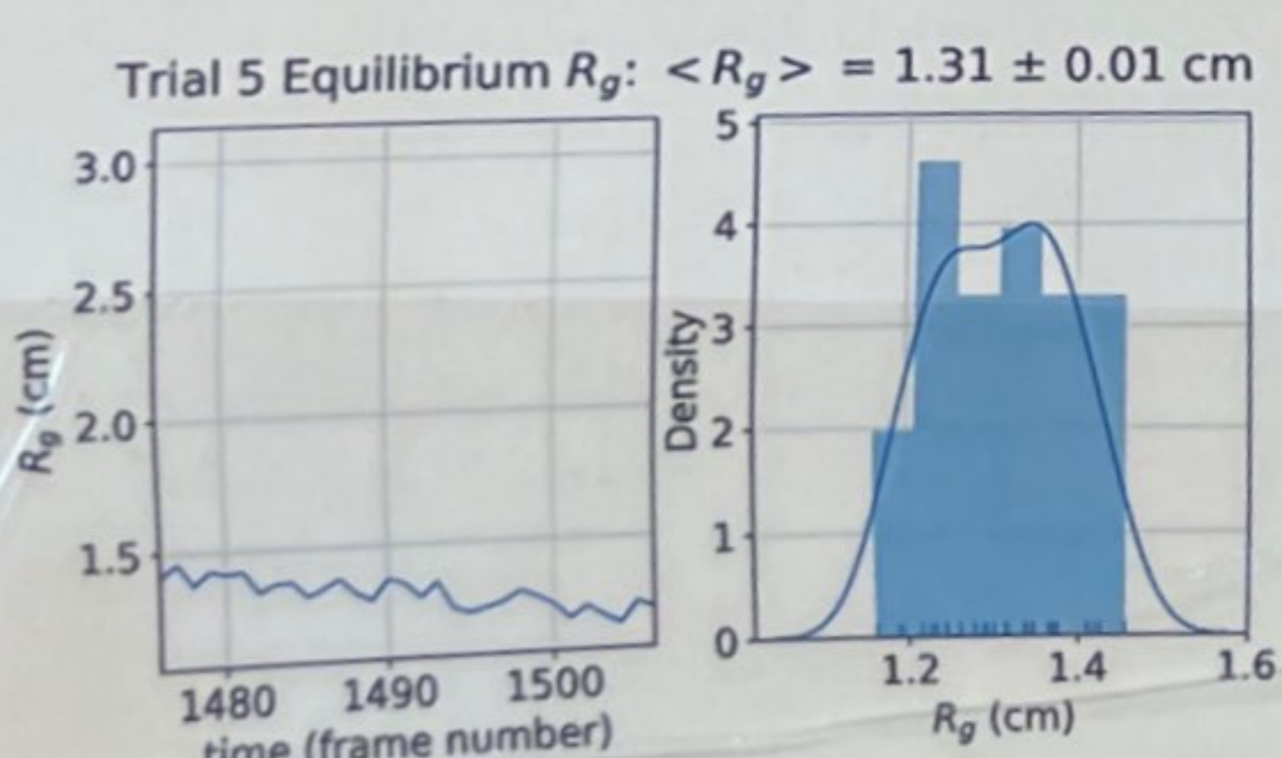
- Did not have time to finish this either

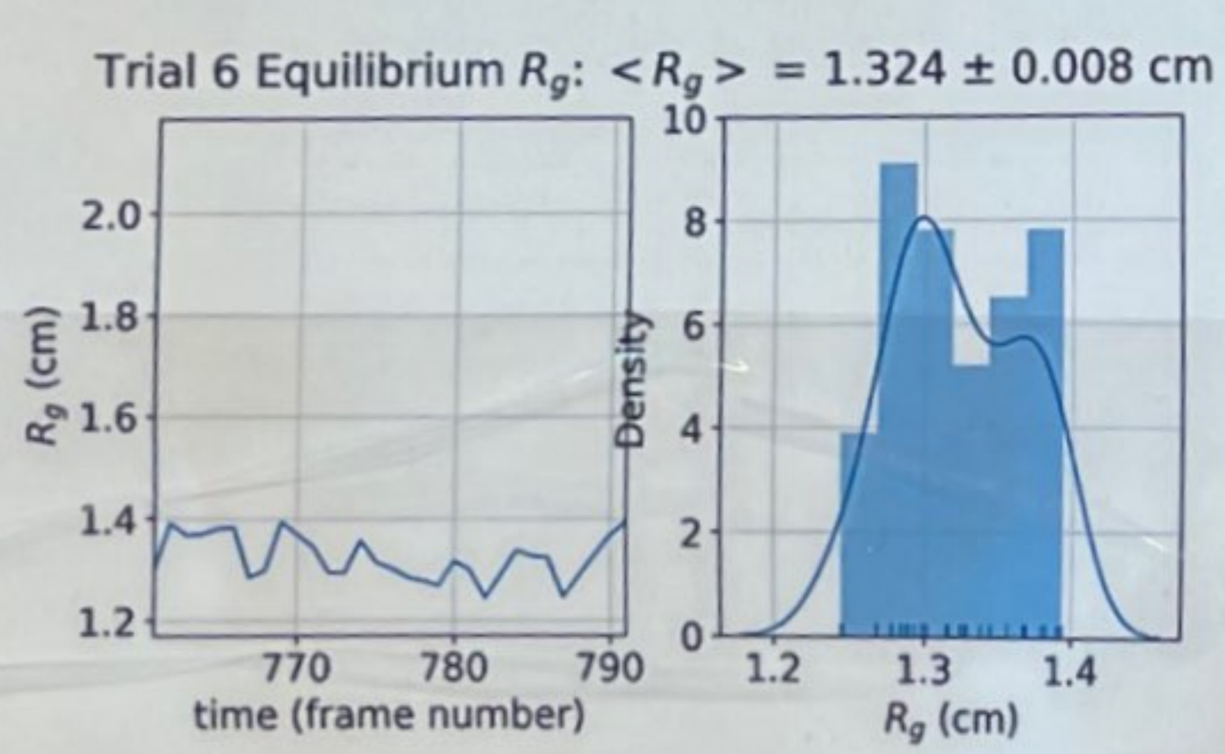


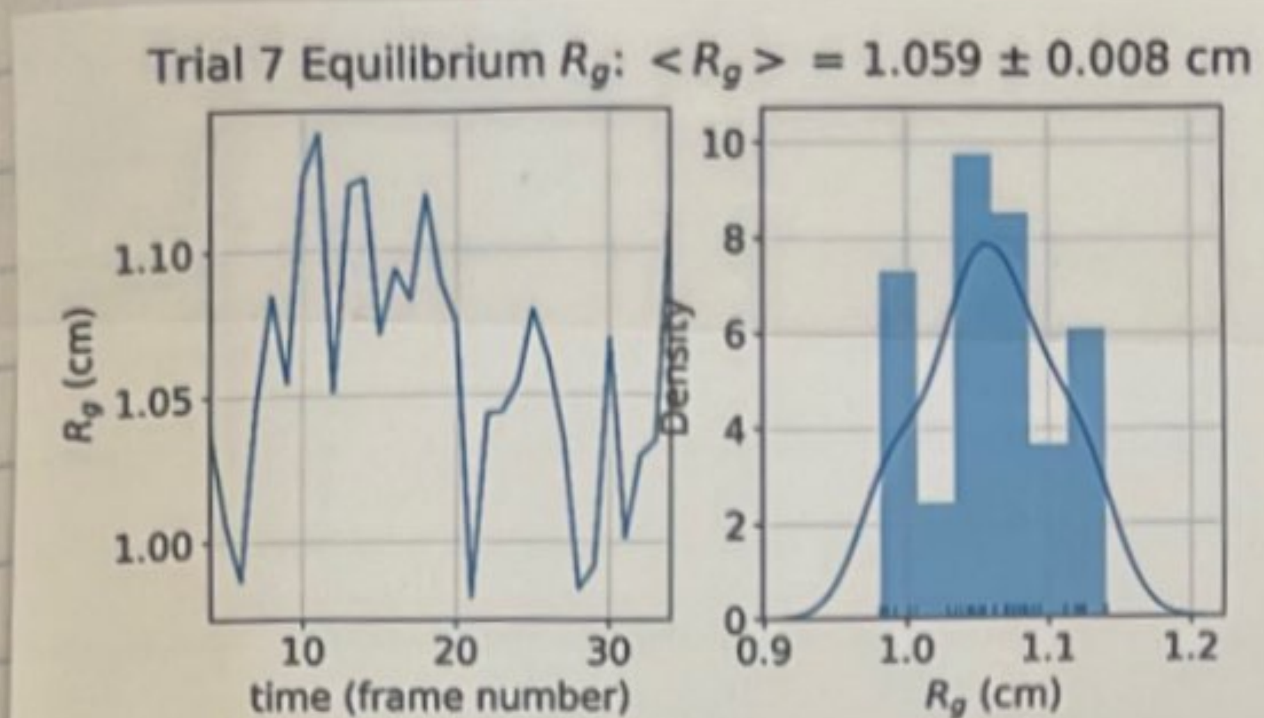


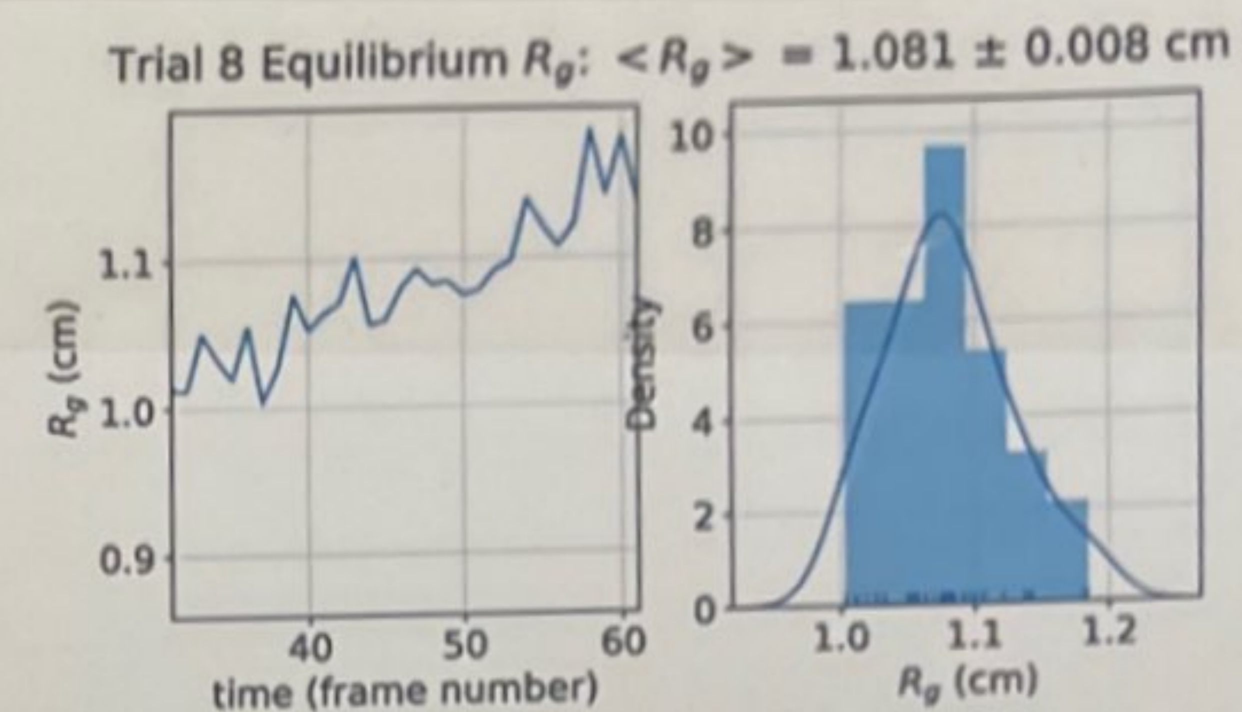


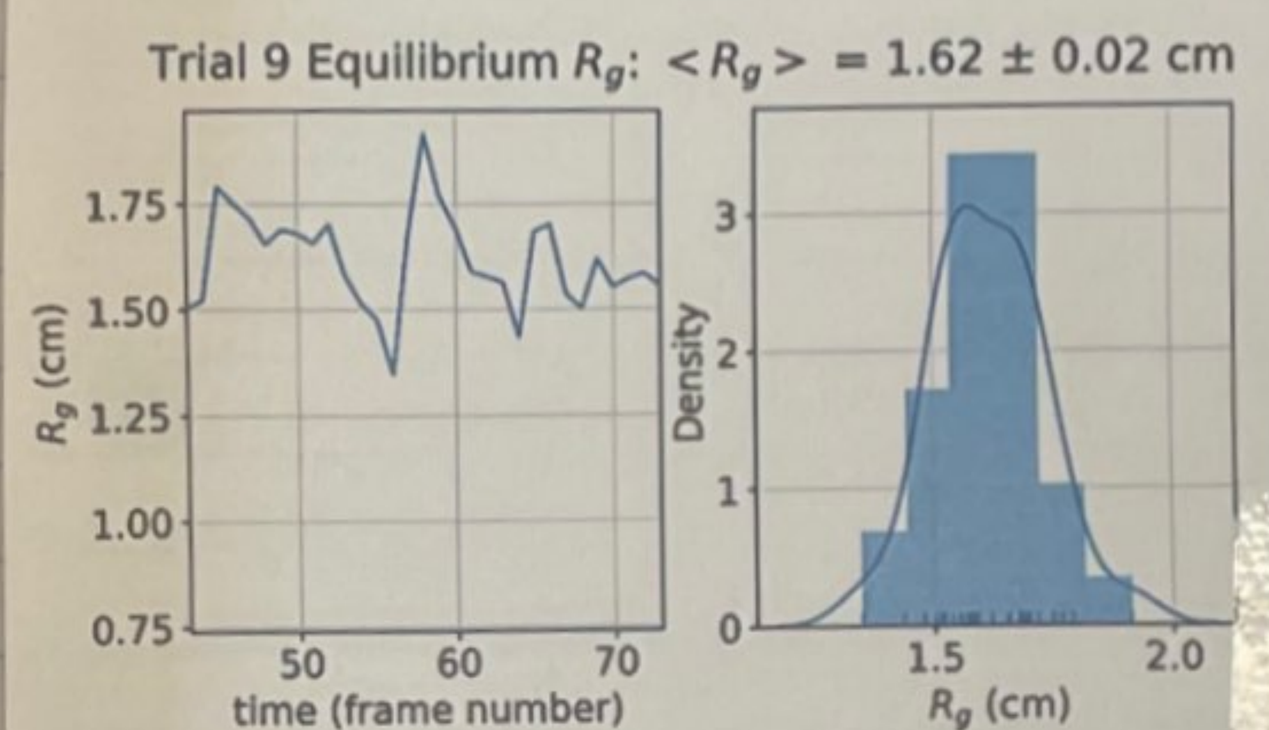


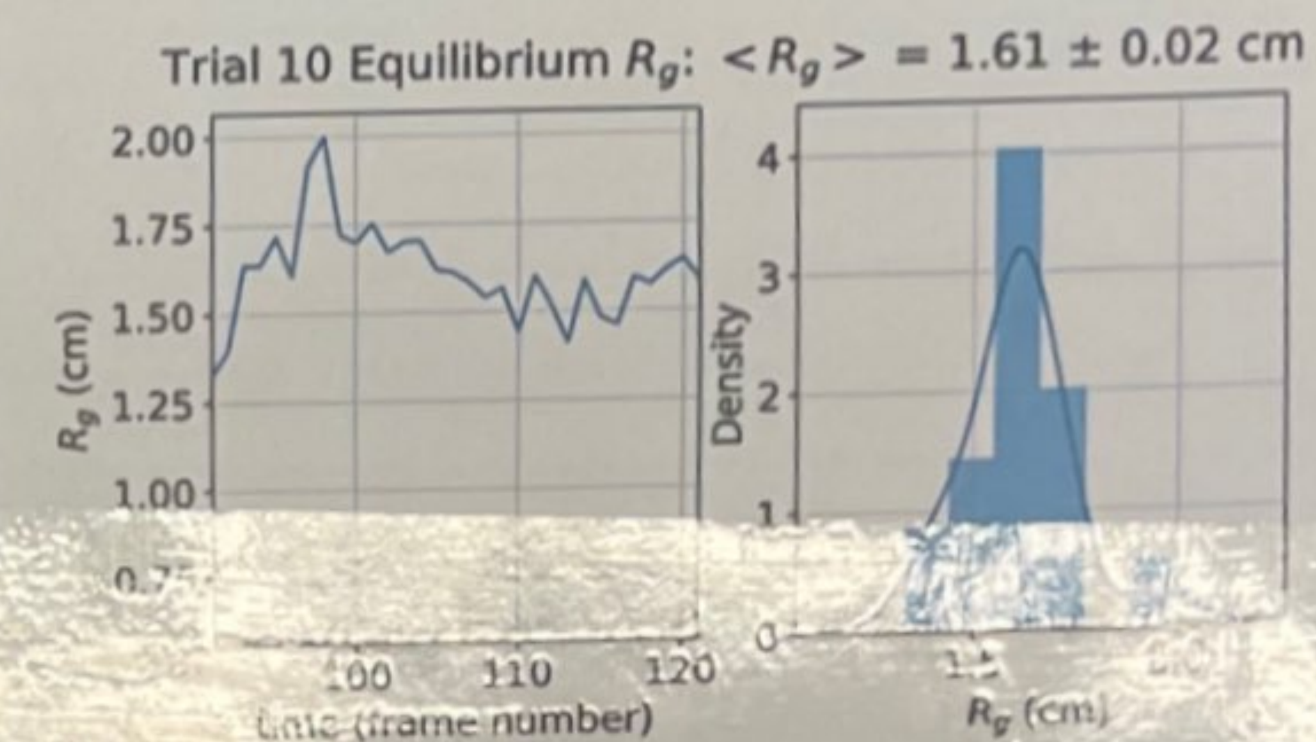












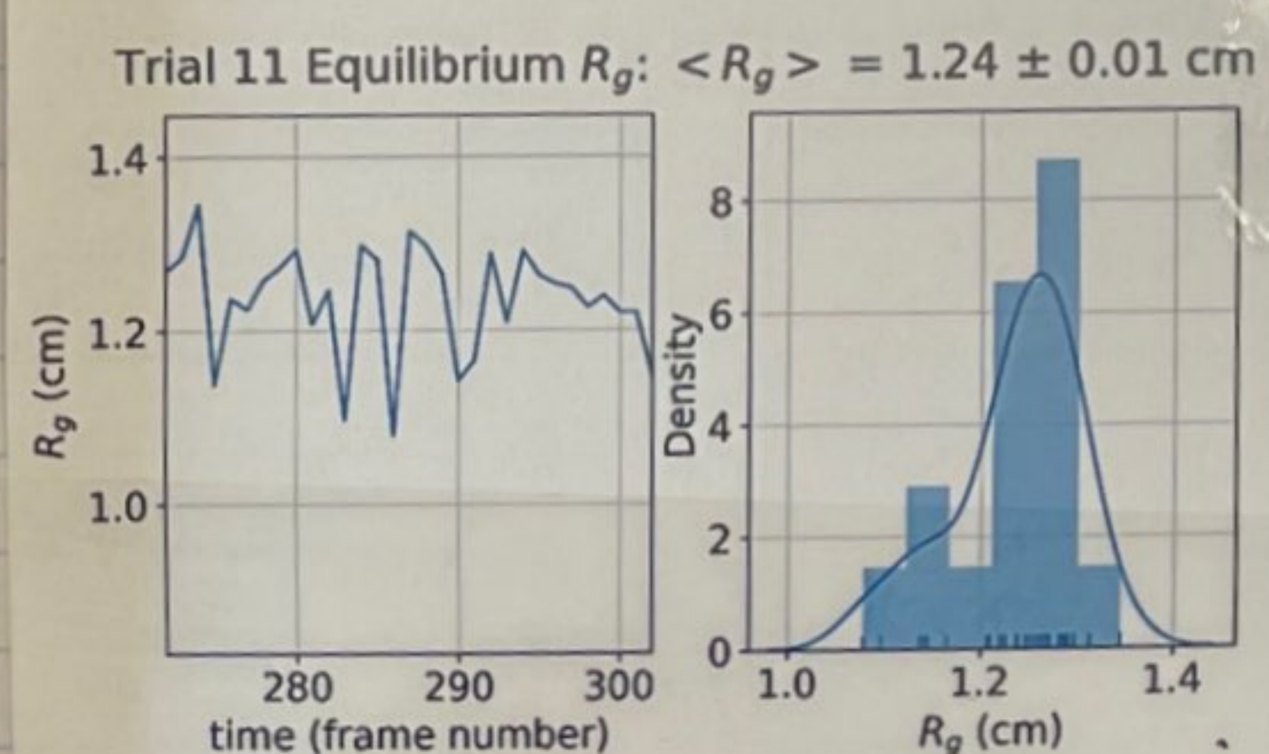


Fig 5: The last 60 seconds of each ROG trial, and the corresponding ROG distributions

- Trials 2, 4, 8 were discarded and not used in the analysis
- Images generated using plot_rog.py

NEXT PAGE

NEXT PAGE

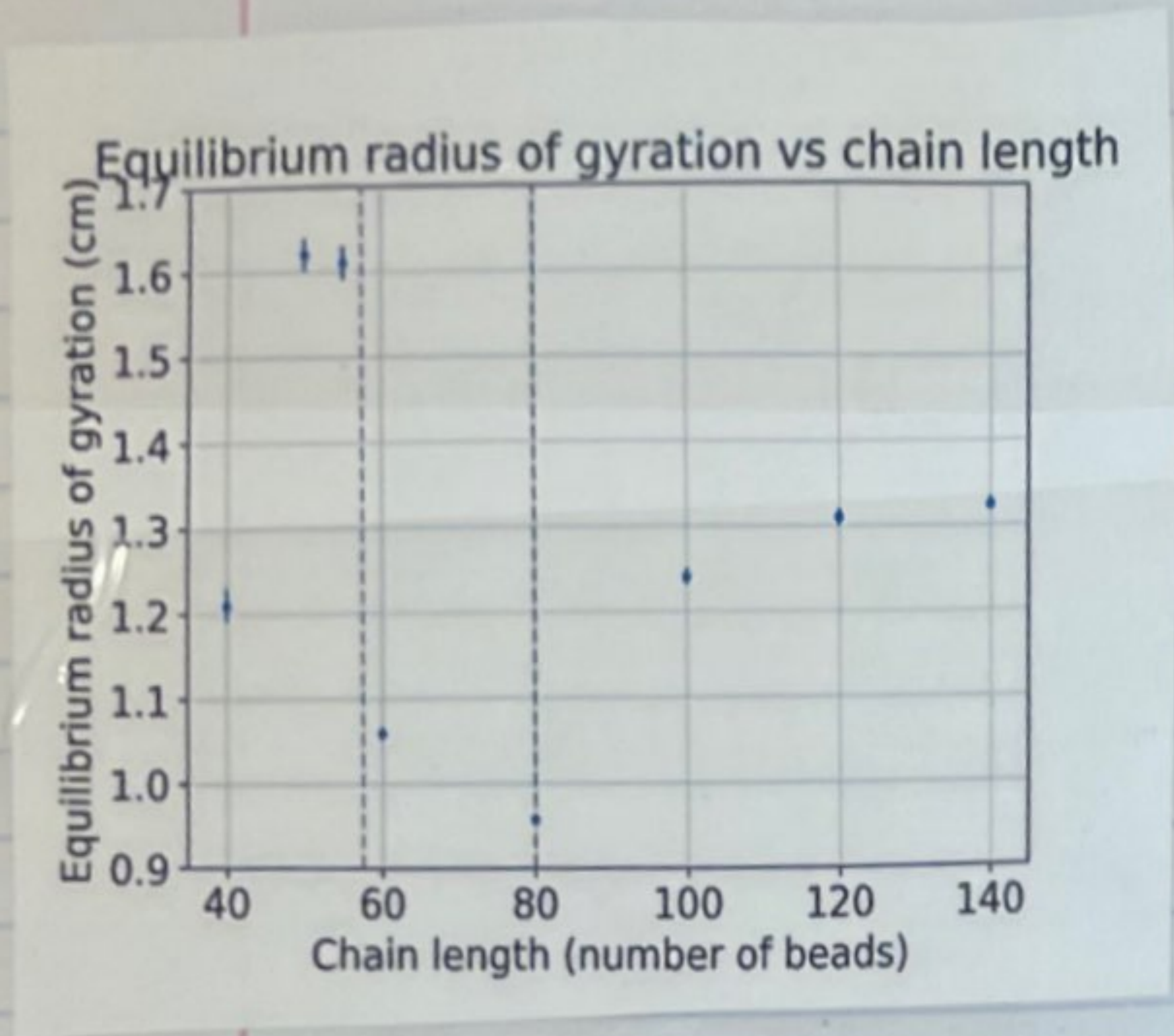


Fig 6: Equilibrium radius of gyration vs N. Each data point is the mean radius of gyration over the last 60 seconds of chain motion in a given trial. The error bars are given by the standard error in the mean
- The three black lines seperate the three regimes of equilibrium motion
- fit-pos.py for image plotting

- The equilibrium radius of gyration seemed to fall into 3 regimes

Regime 1: N<60
- This regime saw chains too short to form natural spirals
- These chains prefered an equilibrium U shape, and as a result, were much more inclined to have larger variations in equilibrium Rg due to having a more flexible shape than a spiral. Hence the large error bars when compared to ~~spira~~ other regimes
- This regime naturally saw a large increase in Rg w/ chain length
- The chain links have a maximum bending angle. For short chains, most of the links must be near this maximum bend to produce a spiral, which is a very unlikely state for the system to find itself in

- Even when starting in a spiral, chains in this regime would just open, as seen in trial 9

Regime 2: 60<N<80
- This regime saw chains forming loose spirals
- Increasing N saw the mass of the outer loops of chain increase ~~in this re~~
- This made it less likely for the spiral to open in ~~that~~ random motion, and meant that for larger N, the chain would be more likely to seek to fill its area
- The result is Rg decreasing w/ N

Regime 3: N>80
- This regime saw area filling spirals
- The large mass of the outer loops would trap the inner loops in an decreasing space until the area was full
- Because the spirals could no longer tighten, the radius of gyration had to grow with N. This regime behaves according to the model I described in page 35, and appears to grow as $R_g \propto \sqrt{N}$
- More data is needed to perform an appropriate fit

- Investigation for N=160 saw that the initial spiral was opened. The chain's motion continued for hours w/ no equilibrium. This suggests a fourth regime, perhaps the

Analysis of Survival Probability:

- The Kolmogorov Smirnov Test can be used to determine if two data sets came from the same distribution

- The ~~scipy~~ scipy.stats function ktest, given two arrays A and B tests the following null-hypothesis: The distributions A and B are identical

- Because the p-value is the probability that the null hypothesis is true, we actually want a p-value close to 1

- Running ktest on each distribution pair on page 23, results a p-value 1.0 for every pair
- This is evidence that every distribution there actually comes from the same curve

- Then, the statistics of opening times can be analyzed w/o considering chain length by rescaling the mean times for each chain to unity

- ~~Sy~~ Summary and ~~and~~ abstract in ~~KNOT_Abstract~~ KNOT_Abstract_And_Summary.pdf