Proof that Supernormality implies normality

Let $\lambda > 0$ real number.

Let $a_k(\lambda) = \frac{e^{-\lambda} \lambda^k}{k!}$ for $k \geq 0$

Fact 1 $\sum_{k \geq 0} a_k(\lambda) = e^{-\lambda} \sum_{k \geq 0} \frac{\lambda^k}{k!} = e^{-\lambda} e^{\lambda} = 1$

Fact 2 $\sum_{k \geq 0} k\, a_k(\lambda) = e^{-\lambda} \sum_{k \geq 1} \frac{\lambda^k}{(k-1)!} = \lambda e^{-\lambda} \sum_{k \geq 0} \frac{\lambda^k}{k!} = \lambda$

Let x be a sequence over a binary alphabet.

For u, w words $|w|_u$ is the number of occurrences of u in w

Let $A_{k,n}(\lambda) = \frac{\#\{w : |w| = n \quad |x[1..\lambda 2^n + n]|_w = k\}}{2^n}$

Rq $x[1..t]$ is $x[1..\lfloor t \rfloor]$ if t is not an integer.

Def x is λ-supernormal if

$$\lim_{n \to +\infty} A_{k,n}(\lambda) = a_k(\lambda) \quad \text{for each integer } k \geq 0.$$

Fact 1 $\sum_{k \geq 0} A_{k,n}(\lambda) = 1$

Fact 2 $A_{k,n}(\lambda) = 0$ if $k > \lambda 2^n$

Let $\varepsilon > 0$ be chosen

let k_0 be chosen such that $\frac{1}{\lambda}\sum_{k>k_0} k a_k(\lambda) < \frac{\varepsilon}{2}$

let n_0 be chosen such that

$$\forall\, 0 \le i \le k_0-1 \quad \forall n > n_0 \quad |A_{i,n}(\lambda) - a_i(\lambda)| \le \frac{\varepsilon\lambda}{2k_0(k_0+1)}$$

Rq: $\sum_{i=0}^{k_0-1} k a_k(\lambda) = \lambda - \sum_{k \ge k_0} k a_k(\lambda) \ge \lambda(1-\varepsilon/2)$

Consider the positions from 1 to $\lfloor \lambda 2^n \rfloor$ in x

A position is "blamed" if the string of length n starting at that position occurs more than k_0 in $x[1..\lfloor\lambda 2^n\rfloor + n]$

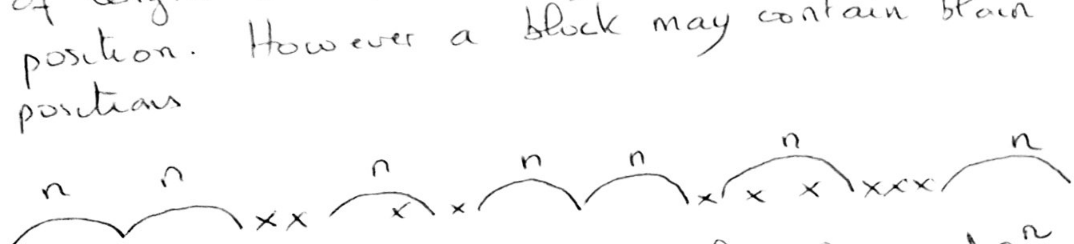
The number of non-blamed positions is

$$\sum_{i=0}^{k-1} i\, A_{i,n}(\lambda)\, 2^n \ge 2^n \sum_{i=0}^{k-1} i \left(a_i(\lambda) - \frac{\varepsilon\lambda}{2k_0(k_0+1)}\right)$$

$$\ge 2^n \sum_{i=0}^{k-1} i a_i(\lambda) - \frac{2^n \varepsilon\lambda}{2}$$

$$\ge 2^n\lambda\left(1-\frac{\varepsilon}{2}\right) - \frac{2^n\varepsilon\lambda}{2} = 2^n\lambda(1-\varepsilon)$$

We cover the positions of $1..\lambda 2^n$ by blocks of length such that blocks do not start at blamed position. However a block may contain blam positions



The number of left positions is less than $\varepsilon\lambda 2^n$

~~The~~ For a fixed word w, the number of bad blocs is a o(number of possible block). Since each block can at most occur k_0-1 times $\rightarrow$ ok

Hot spot lemma is needed to conclude.