1. Almost every word is Supernormal

Definition 1.1. Let $Bad^{+}(A, N, k, \epsilon)$ be the set of words of length N such that the number of digits occurring k times is more than the expected amount, or what is to say:

$$Bad^{+}(A, N, k, \epsilon) = \{w \in A^N : |\{d \in A : |w|_d = k\}| > e^{-\lambda}\frac{\lambda^k}{k!}|A| + \epsilon|A|\}$$

Definition 1.2. Let $Bad^{-}(A, N, k, \epsilon)$ be the set of words of length N such that the number of digits occurring k times is less than the expected amount, or what is to say:

$$Bad^{+}(A, N, k, \epsilon) = \{w \in A^N : |\{d \in A : |w|_d = k\}| < e^{-\lambda}\frac{\lambda^k}{k!}|A| - \epsilon|A|\}$$

Definition 1.3. Let $Bad(A, N, k, \epsilon)$ be the set of words of length N such that the number of digits occurring k times is different from the expected amount. Or what is to say:

$$Bad(A, N, k, \epsilon) = Bad^{-}(A, N, k, \epsilon) \cup Bad^{+}(A, N, k, \epsilon)$$

1.1. Bounding Bad^{+}

Let $Q^{+} = e^{-\lambda}\frac{\lambda^k}{k!}|A| + \epsilon|A|$. First it is important to notice that:

$$|Bad^{+}(A, N, k, \epsilon)| \leq \underbrace{\prod_{i=0}^{Q^{+}-1}\binom{N-ki}{k}}_{1} \cdot \underbrace{|A|(|A|-1)(|A|-2)\cdots(|A|-Q^{+}+1)}_{2} \cdot \underbrace{(A-Q^{+})^{N-Q^{+}k}}_{3}$$

Where:

- 1) is for choosing the places to put the symbols from Q^{+}
- 2) is for picking the Q^{+} symbols that appear too much
- 3) is for placing the symbols that do not appear k times in unoccupied possitions.

Now, this term is bigger or equal than Bad^{+} because this accounts for Q^{+} but it could also be $Q^{+}+1$,$Q^{+}+2$, etc.

$$|Bad^{+}(A, N, k, \epsilon)| \leq \prod_{i=0}^{Q^{+}-1}\binom{N-ki}{k} \cdot |A|(|A|-1)(|A|-2)\cdots(|A|-Q^{+}+1) \cdot (|A|-Q^{+})^{N-Qk} =$$

$$\binom{N}{k}\binom{N-k}{k}\binom{N-2k}{k}\cdots\binom{N-Q^{+}k-k}{k} \cdot \frac{|A|!}{(|A|-Q^{+})!} \cdot (|A|-Q^{+})^{N-Qk} =$$

$$\frac{N!}{k!(N-k)!}\frac{(N-k)!}{k!(N-2k)!}\frac{(N-k2)!}{k!(N-3k)!}\cdots\frac{(N-Q^{+}k-k)!}{k!(N-Q^{+}k-2k)!} \cdot \frac{|A|!}{(|A|-Q^{+})!} \cdot (|A|-Q^{+})^{N-Qk} =$$

$$\frac{N!}{k!^{Q^{+}-1}(N-Q^{+}k-2k)!} \cdot \frac{|A|!}{(|A|-Q^{+})!} \cdot (|A|-Q^{+})^{N-Qk}$$

1.2. Bounding Bad^{-}

In the case of Bad^{-}, we define $Q^{-} = e^{-\lambda}\frac{\lambda^k}{k!}|A| - \epsilon|A|$ We will see that in this case it is not the same if $k = 0$ than if it is different.

$$|Bad^{-}(A, N, k, \epsilon)| = \sum_{j=0}^{Q^{-}}\left[\underbrace{\binom{|A|}{j}}_{1} \cdot \underbrace{\prod_{i=0}^{j-1}\binom{N-ki}{k}}_{2}\underbrace{C^{w}_{|A|-j}(N-jk, \hat{k})}_{3} \cdot \underbrace{(N-kj)!}_{4}\right]$$

Where:

- 1) is for picking the j symbols that occur exactly k times
- 2) is for choosing the places to put these j symbols
- 3) is the amount of weak compositions of $N-jk$ in $|A|-j$ parts with the restriction of having no parts equal to k
- 4) is for putting each of the compositions in postions.

Remark 1. In the case where $k=0$ the restriction for the number of weak compositions yields exactly the number of strong compositions of N in $A-j$ parts and it needs to be treated differently.

Fact 1. The number of weak compoitions of m into exactly p parts is $\binom{m+p-1}{m} = \binom{m+p-1}{p-1}$

1.2.1. $K \neq 0$

$$|Bad^{-}A, N, k, \epsilon)| = \sum_{j=0}^{Q^-}\left[\binom{|A|}{j}\cdot\prod_{i=0}^{j-1}\binom{N-ki}{k}C^w_{a-j}(N-jk,\hat{k})\cdot(N-kj)!\right] \leq$$

$$\sum_{j=0}^{Q^-}\left[\binom{|A|}{j}\cdot\prod_{i=0}^{j-1}\binom{N-ki}{k}\binom{N-jk+|A|-j-1}{N-jk}\cdot(N-kj)!\right] =$$

$$\sum_{j=0}^{Q^-}\left[\binom{|A|}{j}\cdot\prod_{i=0}^{j-1}\binom{N-ki}{k}\frac{(N-jk+|A|-j-1)!}{(N-jk)!(|A|-j-1)!}\cdot(N-kj)!\right] =$$

$$\sum_{j=0}^{Q^-}\left[\binom{|A|}{j}\cdot\prod_{i=0}^{j-1}\binom{N-ki}{k}\frac{(N-jk+|A|-j-1)!}{(|A|-j-1)!}\right] =$$

$$\sum_{j=0}^{Q^-}\left[\binom{|A|}{j}\cdot\frac{N!}{k!(N-k)!}\frac{(N-k)!}{k!(N-2k)!}\cdots\frac{(N-kj-k)!}{k!(N-kj-2k)!}\frac{(N-jk+|A|-j-1)!}{(|A|-j-1)!}\right] =$$

$$\sum_{j=0}^{Q^-}\left[\binom{|A|}{j}\cdot\frac{N!}{k!^{j-1}(N-kj-2k)!}\frac{(N-jk+|A|-j-1)!}{(|A|-j-1)!}\right] =$$

1.2.2. $K = 0$

$$\sum_{j=0}^{Q^-}\binom{N-1}{A-j-1}\cdot\binom{A}{j}\cdot N! =$$

$$Q^-N!\sum_{j=0}^{Q^-}\binom{N-1}{A-j-1}\cdot\binom{A}{j} =$$

$$Q^-N!\sum_{j=0}^{Q^-}\binom{N-1}{N-j-1}\cdot\binom{N}{j} =$$

$$Q^-N!\sum_{j=0}^{Q^-}\binom{N-1}{j}\cdot\binom{N}{j} \leq$$

$$Q^-N!\sum_{j=0}^{Q^-}\binom{N}{j}^2 =$$

$$Q^-N!\left(\frac{N!}{(N!0!)}+\frac{N!}{(N-1!1!)}+\cdots+\frac{N!}{(N-Q^-)!Q^-!}\right)^2 =$$

$$Q^-N!\left(\frac{N!Q^-}{\sum_{j=0}^{Q^-}(N-j)!j!}\right)^2 \leq$$