Fundação Getulio Vargas
Estruturas de Dados e Algoritmos
Prof. Jorge Poco

**Homework 1**
**Due:**: March 3, 2019

# Homework 1

## 1 Induction

Answers should be written in this document.

1. Prove by Induction that: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \forall n \geq 0$

2. Prove by Induction that: $\forall n \geq 7$ it is true $3^n < n!$

3. Prove by Induction that $\forall n \geq 0$

$$\left\lceil \frac{n}{2} \right\rceil = \begin{cases} \frac{n}{2} & \text{si } n \text{ es par} \\ \frac{n+1}{2} & \text{si } n \text{ es impar} \end{cases}$$

4. Prove by induction that a number is divisible by 3 if and only if the sum of its digits is divisible by 3.

5. Prove that any integer greater than 59 can be formed using only 7 and 11 cent coins.

6. Prove by induction that $F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$

7. Prove by induction in $n$ that $\sum_{m=0}^{n} \binom{n}{m} = 2^n$

8. Prove by induction that a graph with $n$ vertices can have at most $\frac{n(n-1)}{2}$ edges.

9. Prove by induction that a complete binary tree[1] with $n$ has $2^n - 1$ vertices.

10. A polygon is convex if each pair of points in the polygon can be joined by a straight line that does not leave the polygon. Prove by induction in $n > 3$ that the sum of the angles of a polygon of $n$ vertices is $180(n-2)$.

## 2 Insertion Sort vs Mergesort

Implement the insertion sort and merge sort using the template *test.py* (use Python 3.X). You must submit your code as an attached file. Graphs and descriptions must be included in this document.

---

[1]http://web.cecs.pdx.edu/ sheard/course/Cs163/Doc/FullvsComplete.html

## 2.1   Random Order

1. Create 10 sets of numbers in random order. The sets must have {10k, 20k, 30k, ..., 100k} numbers.

2. Sort these numbers using the 2 algorithms and calculate the time each algorithm takes for each set of numbers.

3. Generate a graph (using excel or another tool) showing a *linechart*, where the x-axis is the "number of elements", and the y-axis is the time that the algorithm takes. This graphic must have 2 lines of different colors with its legend.

4. Write a small paragraph describing the results.

## 2.2   Ascending Order

Do the same experiment when the numbers are ordered in ascending order.

## 2.3   Descending Order

Do the same experiment when the numbers are ordered in descending order.