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Stochastic programming models

Maintenance scheduling model

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Abstract A maintenance schedule for each power generating units in a electrical system is essential for reliable and secure operation. This work presents a model capable of planning the optimal maintenance schedule for a whole integrated system. The criteria is to maximize the reserve given that each generating unit has at least a given number of maintenance periods per year.

Since the reserve depends on generation scenarios of the renewable sources, the problem has a stochastic nature. So instead of maximizing the average reserve, the methodology maximize a convex combination between CVaR and average reserve.

Keywords Power Systems Operation · Maintenance · Scheduling · Stochastic Optimization

1 Introduction

The main objective of the model is to maximize the smallest reserve among the stages considered in the system, given that each unit of each plant must be in maintenance for, at least, a month per year. In the case of a weekly schedule, the maintenance constraint will force 4 weeks of maintenance among the year, not necessarily sequential.

The total demand considered in the problem must reflect the renewable generation scenarios. This is done by subtracting, for each generation scenarios and for each stage, the generation for each renewable source, resulting in a net demand. Since the renewable generation is a random variable, the net demand and the reserve are also random variables, with an unknown distribution.

Since the variables of the problem are stochastic, the optimization also needs to be stochastic. The objective function consists in maximize a convex combination between CVaR and the mean reserve, since the reserve is a random variable.

The output of such optimization program can be used for operation planning methods such as SDDP [1].

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2 Basic Notation

2.1 Sets

H is the set of hydro plants
J is the set of thermal plants
R is the set of renewable energy plants (solar and wind power)
T is the set of stages
S is the set of generation scenarios
U_h is the set of hydro plant j units.
U_j is the set of thermal plant j units.

2.2 Constants

$\bar{g}_j^t$ is the total capacity of the thermal plant j in stage t
$\bar{g}_h^t$ is the total capacity of the hydro plant h in stage t
u_j is the number of units of the thermal plant j in stage t
u_h is the number of units of the hydro plant h in stage t
ψ_j^t is the ongoing corrective maintenance of the thermal plant j in stage t
ψ_h^t is the ongoing corrective maintenance of the hydro plant h in stage t
$r_{l,s}^t$ is the generation of the renewable source l, in scenario s, in stage t
d^t is the total demand in stage t
d_s^t is the net demand in scenario s, in stage t
β is the significance level of the CVaR used in the optimization.
$\lambda \in [0, 1]$ is the coefficient of the convex combination between CVaR and mean of the reserve.
k indicates whether the problem is monthly ($k = 1$) or weekly ($k = 4$).
N_S is the number of renewable generation scenarios.
N_t is the total number of stages.

2.3 Decision Variables

$\alpha_{u,j}^t$ binary variable. If $\alpha_{u,j}^t = 1$, unit u of thermal plant j is available in stage t. If $\alpha_{u,j}^t = 0$, then it is in maintenance.
$\alpha_{u,h}^t$ binary variable. If $\alpha_{u,h}^t = 1$, unit u of hydro plant h is available in stage t. If $\alpha_{u,h}^t = 0$, then it is in maintenance.
δ_s auxiliary variable represents the smallest reserve among the stages of the year for each scenario s.
z auxiliary variable in the CVaR calculus. Represents the VAR of the distribution of δ_S.
μ_s auxiliary variable in the CVaR calculus. Represents the distance between each sample δ_s and VaR.

3 Methodology

3.1 Net demand

The net demand is represented by the difference between the total demand of each stage and the generation of each stage per scenario:

$$d_s^t = d^t - \sum_{l \in R} r_{l,s}^t \quad \forall (t, s) \in T \times S \tag{1}$$

3.2 Optimization Model

$$\text{Maximize} \quad \lambda\, CVaR_\beta + (1-\lambda)\,\frac{1}{N_S}\sum_{s\in S}\delta_s \tag{2}$$

subject to

$$\delta_s \le \frac{1}{d^t}\Bigg(\sum_{h\in H}\sum_{u\in U_h}\left[\left(1-\psi_h^t\right)\frac{\bar{g}_h^t}{u_h}\alpha_{u,h}^t\right] \tag{3}$$

$$+\sum_{j\in J}\sum_{u\in U_j}\left[\left(1-\psi_j^t\right)\frac{\bar{g}_j^t}{u_j}\alpha_{u,j}^t\right]-d_s^t\Bigg),$$

$$\forall (t,s)\in T\times S \tag{4}$$

$$CVaR_\beta = z + \frac{1}{N_S}\frac{1}{1-\beta}\sum_{s\in S}\mu_s \tag{5}$$

$$\mu_s + z \le \delta_s, \qquad \forall s\in S \tag{6}$$

$$\mu_s \le 0, \qquad \forall s\in S \tag{7}$$

$$\sum_{t\in T}\alpha_{u,h}^t = N_t - k, \qquad \forall u\in U_h, h\in H \tag{8}$$

$$\sum_{t\in T}\alpha_{u,j}^t = N_t - k, \qquad \forall u\in U_j, j\in J \tag{9}$$

$$\alpha_{u,h}^t,\ \alpha_{u,j}^t\ \in\{0,1\}, \qquad \forall u\in U_j, h\in H, t\in T \tag{10}$$

$$\alpha_{u,h}^t,\ \alpha_{u,j}^t\ \in\{0,1\}, \qquad \forall u\in U_h, j\in J, t\in T \tag{11}$$

The objective function is the convex combination of CVaR and the expected value of the reserve.

In constraints (4), we see the reserve for each scenario s in the right hand side. Can be viewed as $\delta_s = min_t\left(\delta_s^t\right)$, where δ_s^t is the reserve for each scenario and each stage.

Constraints (5) and (6) implement the calculation of the CVaR for the distribution δ_s.

Constraints (8) and (9) are the maintenance constraints, where the units are restricted to have k maintenance periods per N_t stages.

4 Results

4.1 Monthly stages

The methodology was implemented into a simplified Uruguay systems data base, where it has considered 4 hydro plants (Salto Grande with 7 units, Terra with 4, Baygorria with 3 and Palmar with 3) and 4 Thermal plants (La tablada with 2 units, Maldonado with 1, Jose Battle with 1 and Punta Del Tigre with 1). In this example, $\lambda = 70\%$ and $\beta = 10\%$. The stages are monthly and the study was implemented for the year of 2017.

Tables 1 and 2 show the schedule plan optimized by the model for the whole year. Figure 1 presents the comparison of the total generation available with the total expected demand per stage for the year of 2017.

Equations (13) and (14) define how percent demand ($d_t^{\%}$) and percent generation ($G_t^{\%}$) are obtained in Figure 1.

$$G_t = \sum_{h\in H}\sum_{u\in U_h}\left[\left(1-\psi_h^t\right)\frac{\bar{g}_h^t}{u_h}\alpha_{u,h}^t\right] + \sum_{j\in J}\sum_{u\in U_j}\left[\left(1-\psi_j^t\right)\frac{\bar{g}_j^t}{u_j}\alpha_{u,j}^t\right] \tag{12}$$

$$G_t^{\%} = \frac{G_t}{max_t\left(G_t\right)} \tag{13}$$

$$d_t^{\%} = \frac{d_t}{max_t\,(d_t)} \tag{14}$$

Table 1 Hydro plants schedule results

Hydro Plant	Unit	Jan	Fev	Mar	Apr	May	Jun	Jul	Ago	Sep	Out	Nov	Dez
Salto Grande	1												■
Salto Grande	2					■							
Salto Grande	3			■									
Salto Grande	4				■								
Salto Grande	5			■									
Salto Grande	6											■	
Salto Grande	7				■								
Terra	1							■					
Terra	2							■					
Terra	3							■					
Terra	4							■					
Baygorria	1			■									
Baygorria	2					■							
Baygorria	3		■										
Palmar	1		■										
Palmar	2	■											
Palmar	3	■											

Table 2 Thermal plants schedule results

Thermal Plant	Unit	Jan	Fev	Mar	Apr	May	Jun	Jul	Ago	Sep	Out	Nov	Dez
La Tablada	1												■
La Tablada	2											■	
Maldonado	1										■		
Jose Batlle	1									■			
Punta del Tigre	1								■				

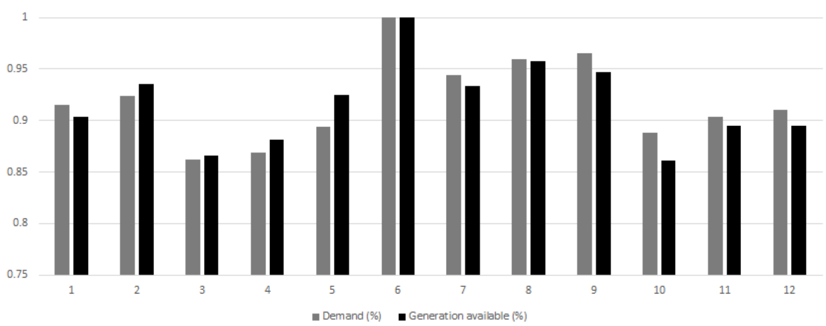

Fig. 1 Generation available and demand per stage

4.2 Weekly stages

The methodology was also implemented for a Morocco data base with weekly stages. We considered 1 hydro plant (Afourer with 2 units) and 1 thermal plant (Mohammedia with 1 unit). In this example, $\lambda = 70\%$ and $\beta = 10\%$. The study was implemented for the year of 2017.

Tables 3 and 4 show the schedule plan optimized by the model for the whole year by week.

Figure 2 shows the comparison of the monthly mean of total generation available with the total expected value of demand per month for the year of 2017. Since the stages are weekly, the monthly result must be taken by the mean of a group of stages that represents the required month.

Equations (13) and (14) show how percentage demand and percentage generation are obtained.

Table 3 Hydro plant Afourer schedule

Hydro	**Unit 1**					**Unit 2**				
Month	**Week 1**	**Week 2**	**Week 3**	**Week 4**	**Week 5**	**Week 1**	**Week 2**	**Week 3**	**Week 4**	**Week 5**
Jan										
Fev										
Mar										
Apr										
May										
Jun										
Jul										
Ago										
Sep										
Out										
Nov										
Dez										

Table 4 Thermal plant Mohammedia schedule

Month	**Week 1**	**Week 2**	**Week 3**	**Week 4**	**Week 5**
Jan					
Fev					
Mar					
Apr					
May					
Jun					
Jul					
Ago					
Sep					
Out					
Nov					
Dez					

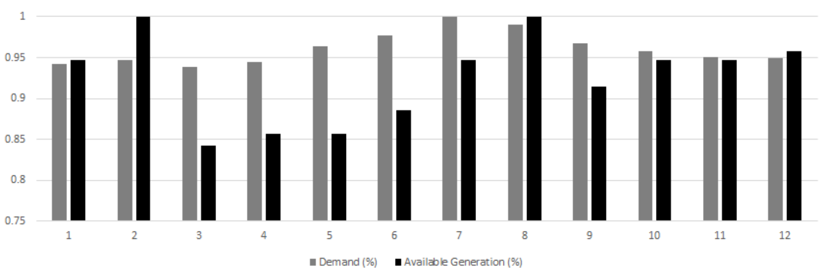

Fig. 2 Mean generation available and mean demand per month

5 Conclusions

Since constraints (8) e (9) restrict a certain number of maintenance per year (1 for monthly stages and 4 for weekly stages), the results shows that the model try to allocate all maintenance required to the periods where the total demand is small. Figures 1 and 2 shows that for stages where the demand is at its peak, there isn't a planned maintenance, decreasing the risk of not having available generation in periods of high demand.

References

1. M. V. Pereira and L. M. Pinto, "Multi-stage stochastic optimization applied to energy planning," *Mathematical Programming*, vol. 52, no. 1-3, pp. 359–375, 1991.