

Lista 3

1. Problemas 3.11 e 3.12 da referência.
2. Problem 1: Let $X = (X_1, X_2) \sim N_d(\mu, \Sigma)$, where $X_1 = (X_{1,1}, \dots, X_{1,p})$, $X_2 = (X_{2,1}, \dots, X_{2,q})$ and $d = p + q$. Also, denote $\mu = (\mu_1, \mu_2)$ with $\mu_1 \in \mathbb{R}^p$ and $\mu_2 \in \mathbb{R}^q$; and $\Sigma = \begin{pmatrix} \Sigma_{1,1} & \Sigma_{1,2} \\ \Sigma_{2,1} & \Sigma_{2,2} \end{pmatrix}$. Show that
 - (a) $X_1 \sim N_p(\mu_1, \Sigma_{1,1})$.
 - (b) $X_1|X_2 = x_2 \sim N_p(\mu_{1|2}, \Sigma_{1|2})$, where $\mu_{1|2} = \mu_1 + \Sigma_{1,2}\Sigma_{2,2}^{-1}(x_2 - \mu_2)$ and $\Sigma_{1|2} = \Sigma_{1,1} - \Sigma_{1,2}\Sigma_{2,2}^{-1}\Sigma_{2,1}$.
3. Problem 2: Let $X \sim Exp(r)$, i.e. $f_X(x) = re^{-rx}$ for $x > 0$ and $Y \sim Pareto(\alpha)$, i.e., $f_Y(y) = (1 + \frac{y}{\alpha})^{-(1+\alpha)}$, for $y > 0$. Couple these two random variables with a Gumbel copula with parameter θ , i.e.,

$$C_{Gu}(u, v|\theta) = \exp \left\{ -((- \ln u)^\theta + (- \ln v)^\theta)^{1/\theta} \right\}, \quad 1 \leq \theta < \infty.$$

- (a) Derive the joint cdf of X_1 and X_2 .
- (b) What happens to the dependence structure when $\theta = 1$? And when $\theta \rightarrow \infty$.
- (c) Use the package `copula` (see the slides!) and generate data from the joint distribution of X_1 and X_2 . The help of the function `mvdc` will be helpful.

Referência: Carmona, René. Statistical analysis of financial data in R, 2013.