S-109A Introduction to Data Science

Homework 3 - Forecasting Bike Sharing Usage

INSTRUCTIONS

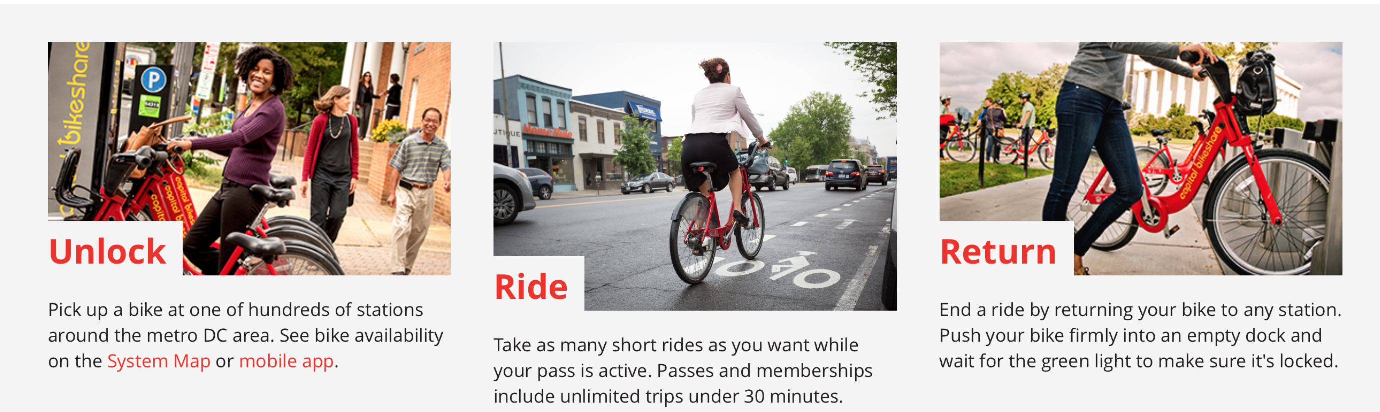
- To submit your assignment follow the instructions given in Classroom.
- Restart the kernel and run the whole notebook again before you submit.
- If you submit individually and you have worked with someone, please include the name of your [one] partner below.

Names of people you have worked with goes here:

In [1]:

```
## RUN THIS CELL TO GET THE RIGHT FORMATTING
from IPython.core.display import HTML
def css_styling():
    styles = open("style/cs109.css", "r").read()
    return HTML(styles)
css_styling()
```

Out[1]:



Main Theme: Multiple Linear Regression, Subset Selection, Polynomial Regression

Overview

You are hired by the administrators of the [Capital Bikeshare program (https://www.capitalbikeshare.com)](https://www.capitalbikeshare.com) program in Washington D.C., to **help them predict the hourly demand for rental bikes** and **give them suggestions on how to increase their revenue**. You will prepare a small report for them.

The hourly demand information would be useful in planning the number of bikes that need to be available in the system on any given hour of the day, and also in monitoring traffic in the city. It costs the program money if bike stations are full and bikes cannot be returned, or empty and there are no bikes available. You will use multiple linear regression and polynomial regression and will explore techniques for subset selection. The goal is to build a regression model that can predict the total number of bike rentals in a given hour of the day, based on attributes about the hour and the day.

An example of a suggestion to increase revenue might be to offer discounts during certain times of the day either during holidays or non-holidays. Your suggestions will depend on your observations of the seasonality of ridership.

The data for this problem were collected from the Capital Bikeshare program over the course of two years (2011 and 2012).

Use only the libraries below:

In [2]:

```
import numpy as np
import pandas as pd
import matplotlib
import matplotlib.pyplot as plt

import statsmodels.api as sm
from statsmodels.api import OLS

from sklearn import preprocessing
from sklearn.preprocessing import PolynomialFeatures
from sklearn.metrics import r2_score
from sklearn.model_selection import train_test_split

from pandas.plotting import scatter_matrix

import seaborn as sns


%matplotlib inline
```

Data Exploration & Preprocessing, Multiple Linear Regression, Subset Selection

Overview

The initial data set is provided in the file `data/BSS_hour_raw.csv` . You will add some features that will help us with the analysis and then separate it into training and test sets. Each row in this file contains 12 attributes and each entry represents one hour of a 24-hour day with its weather, etc, and the number of rental rides for that day divided in categories according to if they were made by registered or casual riders. Those attributes are the following:

- `dteday` (date in the format YYYY-MM-DD, e.g. 2011-01-01)
- `season` (1 = winter, 2 = spring, 3 = summer, 4 = fall)
- `hour` (0 for 12 midnight, 1 for 1:00am, 23 for 11:00pm)
- `weekday` (0 through 6, with 0 denoting Sunday)

- `holiday` (1 = the day is a holiday, 0 = otherwise)
- `weather`
  - 1: Clear, Few clouds, Partly cloudy, Partly cloudy
  - 2: Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist
  - 3: Light Snow, Light Rain + Thunderstorm
  - 4: Heavy Rain + Thunderstorm + Mist, Snow + Fog
- `temp` (temperature in Celsius)
- `atemp` (apparent temperature, or relative outdoor temperature, in Celsius)
- `hum` (relative humidity)
- `windspeed` (wind speed)
- `casual` (number of rides that day made by casual riders, not registered in the system)
- `registered` (number of rides that day made by registered riders)

General Hints

- Use pandas .describe() to see statistics for the dataset.
- When performing manipulations on column data it is useful and often more efficient to write a function and apply this function to the column as a whole without the need for iterating through the elements.
- A scatterplot matrix or correlation matrix are both good ways to see dependencies between multiple variables.
- For Question 2, a very useful pandas method is .groupby(). Make sure you aggregate the rest of the columns in a meaningful way. Print the dataframe to make sure all variables/columns are there!

Resources

[http://pandas.pydata.org/pandas-docs/stable/generated/pandas.to_datetime.html (http://pandas.pydata.org/pandas-docs/stable/generated/pandas.to_datetime.html)](http://pandas.pydata.org/pandas-docs/stable/generated/pandas.to_datetime.html)

Question 1: Explore how Bike Ridership varies with Hour of the Day

Learn your Domain and Perform a bit of Feature Engineering

1.1 Load the dataset from the csv file `data/BSS_hour_raw.csv` into a pandas dataframe that you name `bikes_df`. Do any of the variables' ranges or averages seem suspect? Do the data types make sense?

1.2 Notice that the variable in column `dteday` is a pandas `object`, which is **not** useful when you want to extract the elements of the date such as the year, month, and day. Convert `dteday` into a `datetime` object to prepare it for later analysis.

1.3 Create three new columns in the dataframe:

- `year` with 0 for 2011 and 1 for 2012.
- `month` with 1 through 12, with 1 denoting Jan.
- `counts` with the total number of bike rentals for that day (this is the response variable for later).

1.4 Use visualization to inspect and comment on how **casual** rentals and **registered** rentals vary with the `hour`.

1.5 Use the variable `holiday` to show how **holidays** affect the relationship in question 1.4. What do you observe?

1.6 Use visualization to show how **weather** affects **casual** and **registered** rentals. What do you observe?

Answers

1.1 Load the dataset from the csv file ...

In [3]:

```
# your code here

bikes_df = pd.read_csv('data/BSS_hour_raw.csv', parse_dates=['dteday'])
bikes_df.sample(5)
```

Out[3]:

	dteday	season	hour	holiday	weekday	workingday	weather	temp	atemp	hum	wi
8810	2012-01-07	1	22	0	6	0	1	0.44	0.4394	0.38	
441	2011-01-20	1	10	0	4	1	1	0.26	0.2273	0.48	
8024	2011-12-06	4	0	0	2	1	2	0.50	0.4848	0.77	
5635	2011-08-27	3	17	0	6	0	3	0.64	0.5758	0.89	
14271	2012-08-22	3	20	0	3	1	1	0.64	0.6061	0.73	

In [4]:

```
bikes_df.describe()
```

Out[4]:

	season	hour	holiday	weekday	workingday	weather	
count	17379.000000	17379.000000	17379.000000	17379.000000	17379.000000	17379.000000	17
mean	2.501640	11.546752	0.028770	3.003683	0.682721	1.425283	
std	1.106918	6.914405	0.167165	2.005771	0.465431	0.639357	
min	1.000000	0.000000	0.000000	0.000000	0.000000	1.000000	
25%	2.000000	6.000000	0.000000	1.000000	0.000000	1.000000	
50%	3.000000	12.000000	0.000000	3.000000	1.000000	1.000000	
75%	3.000000	18.000000	0.000000	5.000000	1.000000	2.000000	
max	4.000000	23.000000	1.000000	6.000000	1.000000	4.000000	

In [5]:

```
bikes_df.dtypes
```

Out[5]:

```
dteday        datetime64[ns]
season                 int64
hour                   int64
holiday                int64
weekday                int64
workingday             int64
weather                int64
temp                 float64
atemp                float64
hum                  float64
windspeed            float64
casual                 int64
registered             int64
dtype: object
```

In [6]:

```
print(np.min(bikes_df.dteday))
print(np.mean(bikes_df.dteday))
print(np.max(bikes_df.dteday))
```

```
2011-01-01 00:00:00
2012-01-02 04:08:34.552045568
2012-12-31 00:00:00
```

`season` makes sense because its min is 1, its max is 4 and its mean is 2.5, suggesting data is distributed all seasons equally. The same for `hour`, `weekday` and `dteday`. For `holiday`, the min is 0 and the max is 1. It's mean is around 0.03, what suggests data is distributed between few holidays and many non-holidays. The same for `workingday` and `weather`. For `temp`, `atemp`, `hum` and `windspeed`, data seems to be normalized, that is, between 0 and 1. The number of rides (`casual` and `registered`) also seems reasonable.

All data types make sense. They're all quantitative.

1.2 Notice that the variable in column ....

In [7]:

```
# your code here

# It's easier to do this direct with read_csv, as already done.
```

1.3 Create three new columns ...

In [8]:

```
# your code here

bikes_df['year'] = bikes_df.dteday.dt.year.apply(lambda x: 0 if x == 2011 else 1)
bikes_df['month'] = bikes_df.dteday.dt.month
bikes_df['counts'] = bikes_df.casual + bikes_df.registered
bikes_df.sample(5)
```

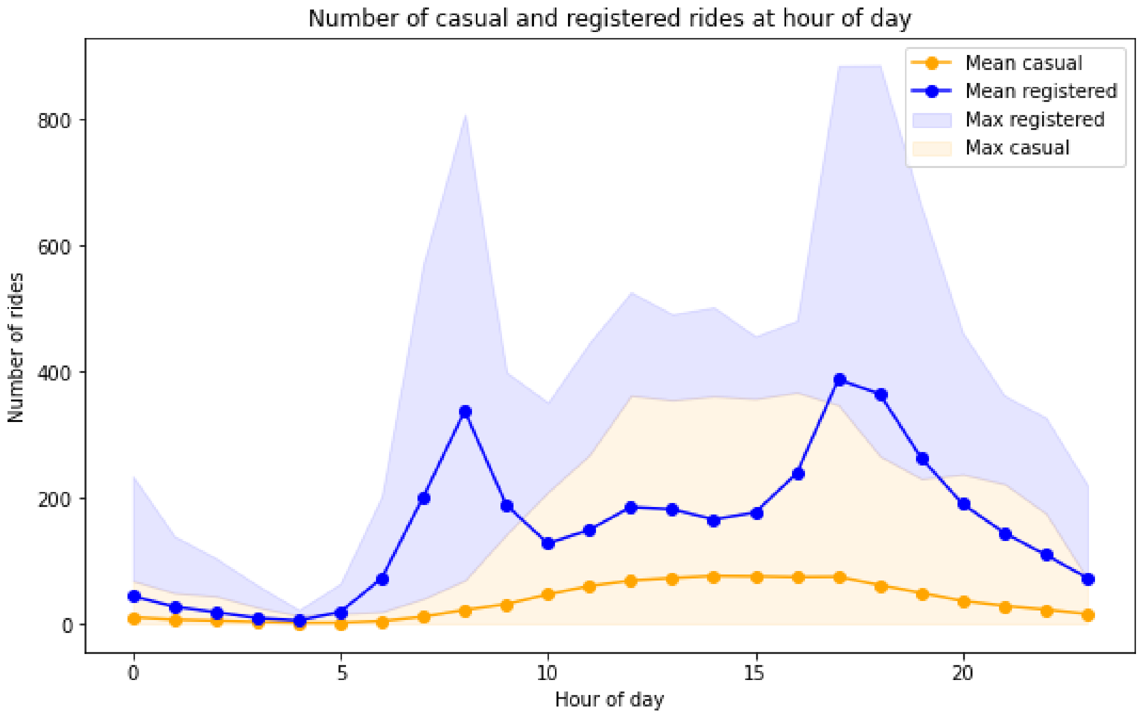
Out[8]:

	dteday	season	hour	holiday	weekday	workingday	weather	temp	atemp	hum	wir
3675	2011-06-07	2	1	0	2	1	1	0.62	0.6061	0.69	
11327	2012-04-22	2	4	0	0	0	2	0.44	0.4394	0.77	
4734	2011-07-21	3	4	0	4	1	2	0.72	0.7121	0.84	
8181	2011-12-12	4	13	0	1	1	2	0.30	0.3182	0.52	
3758	2011-06-10	2	12	0	5	1	1	0.84	0.7576	0.44	

1.4 Use visualization to inspect and comment on how casual rentals and registered rentals vary with the hour.

In [9]:

```
# your code here

casual_mean = bikes_df[['hour', 'casual']].groupby('hour').mean()
casual_max = bikes_df[['hour', 'casual']].groupby('hour').max()
registered_mean = bikes_df[['hour', 'registered']].groupby('hour').mean()
registered_max = bikes_df[['hour', 'registered']].groupby('hour').max()

fig, ax = plt.subplots(figsize=(10, 6))
ax.fill_between(
    registered_max.index,
    registered_max.registered,
    casual_max.casual,
    alpha=0.1,
    color='blue',
    label='Max registered'
)
ax.fill_between(
    casual_max.index,
    casual_max.casual,
    alpha=0.1,
    color='orange',
    label='Max casual'
)
ax.plot(
    casual_mean.index,
    casual_mean.casual,
    marker='o',
    color='orange',
    label='Mean casual'
)
ax.plot(
    registered_mean.index,
    registered_mean.registered,
    marker='o',
    color='blue',
    label='Mean registered'
)
ax.legend()
ax.set_title('Number of casual and registered rides at hour of day')
ax.set_xlabel('Hour of day')
ax.set_ylabel('Number of rides')
plt.show()
```

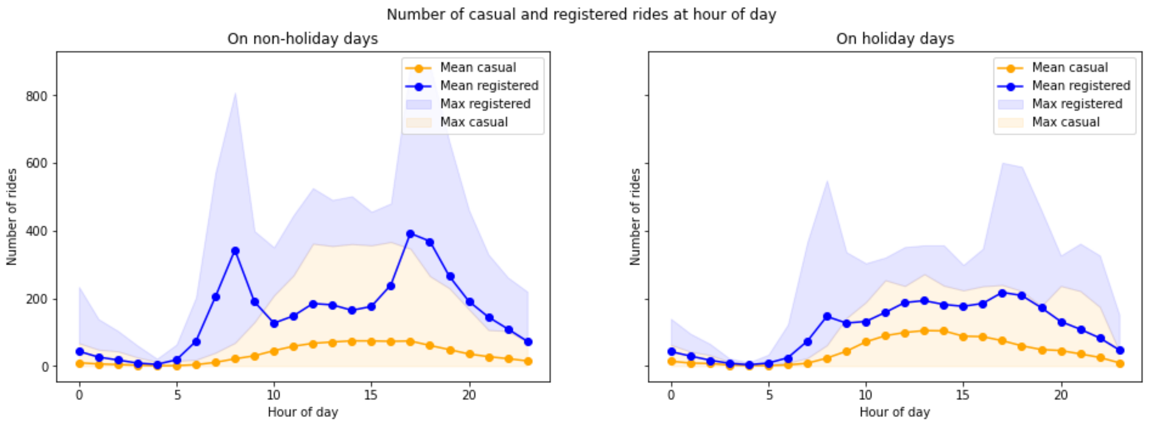


Both registered and casual curves share the same trend: rides begin in the morning, increase until afternoon, decreases at night and almost vanish at dawn. Despite this, registered rides have peaks in the beggining of the morning and at the end of afternoon. One hypothesis for this is because these are commuting time, that is, time of day where people are going to (or coming back from) work or school. Maybe it affects only registered rides because people who use bike to commute in general is registered: it's cheaper to ride bike everyday if you are registered.

1.5 Use the variable holiday to show how holidays affect the relationship in question 1.4. What do you observe?

In [10]:

```
# your code here

fig, axes = plt.subplots(1, 2, figsize=(16, 5), sharex=True, sharey=True)

for holiday in range(2):

    casual_mean = bikes_df[bikes_df.holiday == holiday][['hour', 'casual']].groupby
    casual_max = bikes_df[bikes_df.holiday == holiday][['hour', 'casual']].groupby(
    registered_mean = bikes_df[bikes_df.holiday == holiday][['hour', 'registered']]
    registered_max = bikes_df[bikes_df.holiday == holiday][['hour', 'registered']].
    axes[holiday].fill_between(
        registered_max.index,
        registered_max.registered,
        casual_max.casual,
        alpha=0.1,
        color='blue',
        label='Max registered'
    )
    axes[holiday].fill_between(
        casual_max.index,
        casual_max.casual,
        alpha=0.1,
        color='orange',
        label='Max casual'
    )
    axes[holiday].plot(
        casual_mean.index,
        casual_mean.casual,
        marker='o',
        color='orange',
        label='Mean casual'
    )
    axes[holiday].plot(
        registered_mean.index,
        registered_mean.registered,
        marker='o',
        color='blue',
        label='Mean registered'
    )
    axes[holiday].legend()
    if not holiday:
        axes[0].set_title('On non-holiday days')
    else:
        axes[1].set_title('On holiday days')
    axes[holiday].set_xlabel('Hour of day')
    axes[holiday].set_ylabel('Number of rides')

fig.suptitle("Number of casual and registered rides at hour of day")
plt.show()
```

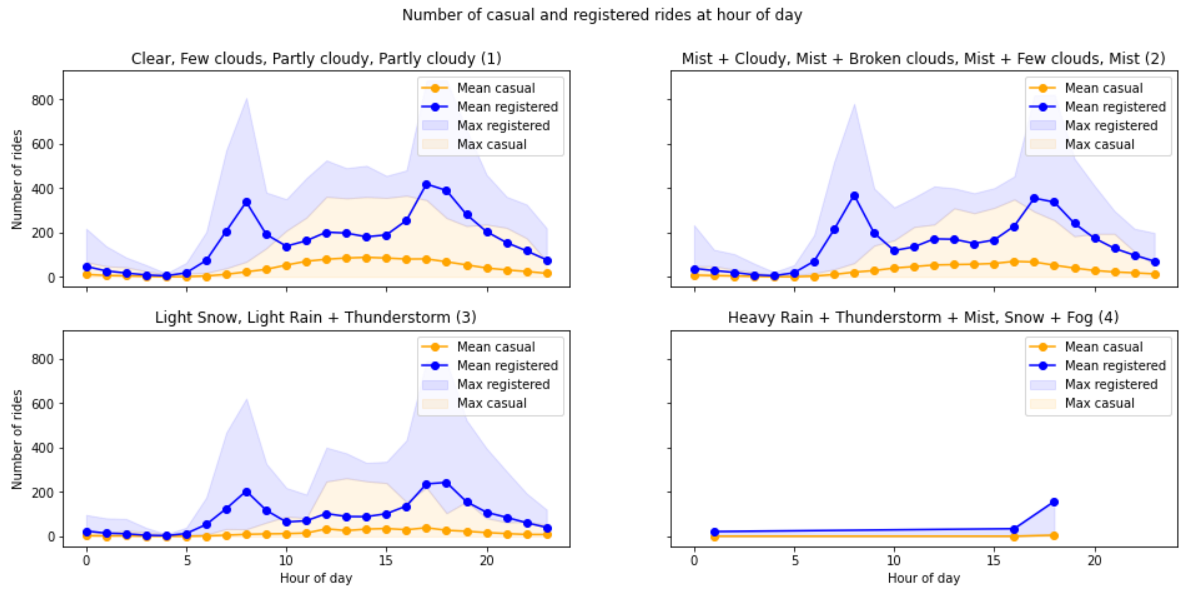


Both curves still share the same trends. However, it seems casual rides increase, maybe because recreation during holidays, and the peaks of registered rides decrease, most likely because less people is working during these days. We also see that in both curves there's a peak in max rides (the max number of rides at that hour of day) during the night (around 21:00), which doesn't appear on non-holiday days. It doesn't seem to affect the mean, maybe it's a specific day or set of days.

1.6 Use visualization to show how weather affects casual and registered rentals. What do you observe?

In [11]:

```
# your code here

fig, axes = plt.subplots(2, 2, figsize=(16, 7), sharex=True, sharey=True)

for weather in range(1, 5):
    ax = axes.reshape(-1)[weather-1]

    casual_mean = bikes_df[bikes_df.weather == weather][['hour', 'casual']].groupby
    casual_max = bikes_df[bikes_df.weather == weather][['hour', 'casual']].groupby(
    registered_mean = bikes_df[bikes_df.weather == weather][['hour', 'registered']]
    registered_max = bikes_df[bikes_df.weather == weather][['hour', 'registered']].
    ax.fill_between(
        registered_max.index,
        registered_max.registered,
        casual_max.casual,
        alpha=0.1,
        color='blue',
        label='Max registered'
    )
    ax.fill_between(
        casual_max.index,
        casual_max.casual,
        alpha=0.1,
        color='orange',
        label='Max casual'
    )
    ax.plot(
        casual_mean.index,
        casual_mean.casual,
        marker='o',
        color='orange',
        label='Mean casual'
    )
    ax.plot(
        registered_mean.index,
        registered_mean.registered,
        marker='o',
        color='blue',
        label='Mean registered'
    )
    ax.legend()

    if weather == 1:
        ax.set_title('Clear, Few clouds, Partly cloudy, Partly cloudy ({})'.format(
    elif weather == 2:
        ax.set_title('Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist 
    elif weather == 3:
        ax.set_title('Light Snow, Light Rain + Thunderstorm ({})'.format(weather))
    elif weather == 4:
        ax.set_title('Heavy Rain + Thunderstorm + Mist, Snow + Fog ({})'.format(wea

    if weather in [3, 4]:
        ax.set_xlabel('Hour of day')
    if weather in [1, 3]:
        ax.set_ylabel('Number of rides')

fig.suptitle("Number of casual and registered rides at hour of day")
plt.show()
```



Let's compare the graphs with the first one (the one with clear sky or with clouds). It seems mist makes casual curve decrease, but it doesn't affect registered curve. Maybe it's because registered rides is most likely associated with people who needs the bike to get to work or home, and casual is most likely to be associated with recreation. Snow, rain and thunderstorm make both curves to decrease. But they still seem to share the same trend and peaks. For the graph 4, we don't have enough data. It's possibly because people don't ride bikes in adverse climate conditions, but we can't discard dataset problems too.

Question 2: Explore Seasonality on Bike Ridership.

Seasonality and weather

Now let's examine the effect of weather and time of the year. For example, you want to see how ridership varies with season of the year.

2.1 Make a new dataframe with the following subset of attributes from the previous dataset and with each entry being **ONE** day:

- `dteday` , the timestamp for that day (fine to set to noon or any other time)
- `weekday` , the day of the week
- `weather` , the most severe weather that day
- `season` , the season that day falls in
- `temp` , the average temperature
- `atemp` , the average atemp that day
- `windspeed` , the average windspeed that day
- `hum` , the average humidity that day
- `casual` , the **total** number of rentals by casual users
- `registered` , the **total** number of rentals by registered users
- `counts` , the **total** number of rentals

Name this dataframe `bikes_by_day` and use it for all of Question 2.

2.2 How does **season** affect the number of bike rentals for **casual riders** or **registered riders** per day? Use the variable `season` for this question. Comment on your observations.

2.3 What percentage of rentals are made by casual riders or registered riders for each day of the week? Comment on any patterns you see and give a possible explanation.

2.4 How is the **distribution of total number of bike rentals** different for sunny days vs cloudy days?

2.5 Visualize how the **total number of rides** per day varies with the **season**. Do you see any **outliers**? (We define an outlier as a value 1.5 times the IQR above the 75th percentile or 1.5 times the IQR below the 25th percentiles. This is the same rule used by pyplot's boxplot function). If you see any outliers, identify those dates and investigate if they are a chance occurence, an error in the data collection, or an important event.

HINT

- Use .copy() when creating the new dataframe, so you leave the original untouched. We will come back to it later.
- Use .groupby() to creat the new dataframe. You will have to make some choice on how to aggregate the variables.

Answers

2.1 Make a new dataframe with the following subset ...

In [12]:

```
# your code here

bikes_by_day = bikes_df.groupby('dteday').agg({
    'weekday': 'mean',
    'weather': 'max',
    'season': 'mean',
    'temp': 'mean',
    'atemp': 'mean',
    'windspeed': 'mean',
    'hum': 'mean',
    'casual': 'sum',
    'registered': 'sum',
    'counts': 'sum'
}).copy()
bikes_by_day.sample(5)
```

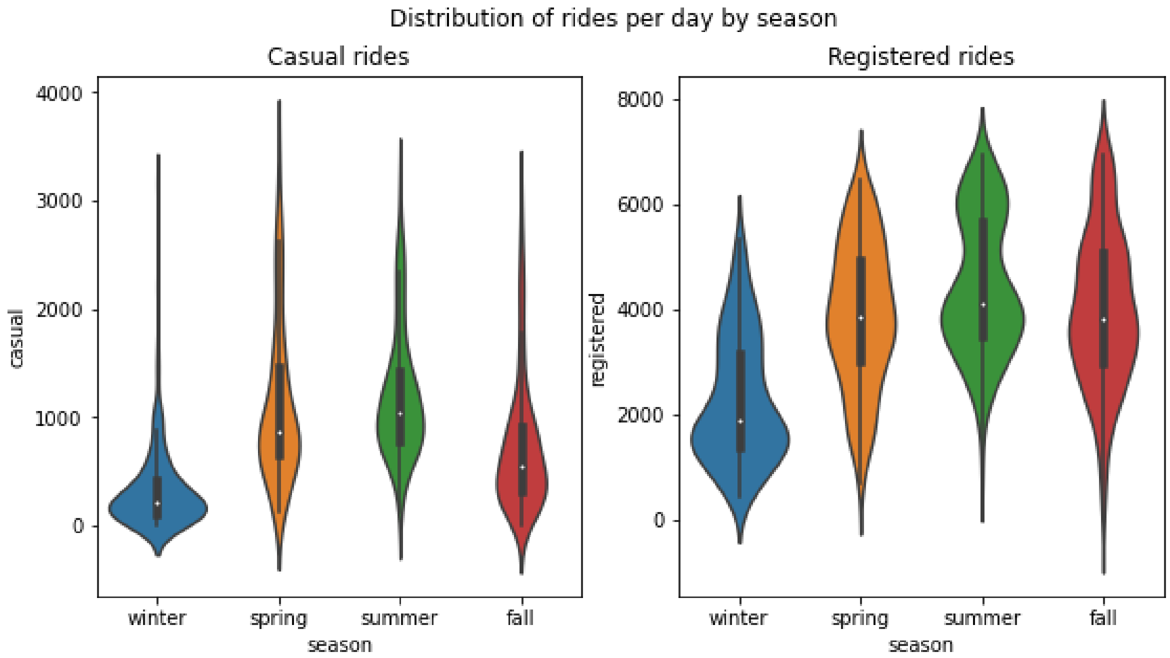
Out[12]:

	weekday	weather	season	temp	atemp	windspeed	hum	casual	registere
dteday									
2012-01-20	5	3	1	0.217500	0.220958	0.202750	0.450000	115	304
2011-03-12	6	1	1	0.329167	0.325750	0.220775	0.594583	724	140
2011-08-15	1	3	3	0.665833	0.616167	0.208954	0.712083	775	356
2012-07-30	1	2	3	0.730833	0.684987	0.153617	0.668750	1040	606
2011-03-05	6	2	1	0.384167	0.378779	0.251871	0.789167	640	143

2.2 How does season affect the number of bike ...

In [13]:

```
# your code here

fig, axes = plt.subplots(1, 2, sharey=False, sharex=True, figsize=(10, 5))
sns.violinplot(x='season', y='casual', data=bikes_by_day, ax=axes[0])
fig.suptitle("Distribution of rides per day by season")
axes[0].set_title('Casual rides')
axes[0].set_xticklabels(['winter', 'spring', 'summer', 'fall'])
sns.violinplot(x='season', y='registered', data=bikes_by_day, ax=axes[1])
axes[1].set_title('Registered rides')
axes[1].set_xticklabels(['winter', 'spring', 'summer', 'fall'])
plt.show()
```



It seems people dislike riding bikes in the winter, for both casual and registered rides. In both cases, summer appears to have more rides, but it's close to spring and fall.

2.3 What percentage of rentals are made by casual riders or registered riders ...

In [14]:

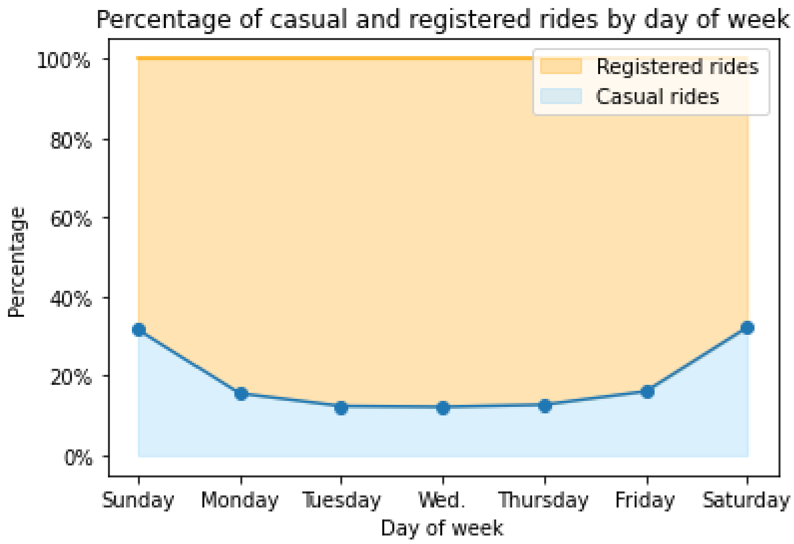
```
# your code here

df = bikes_by_day.groupby('weekday').agg({
    'casual': 'sum',
    'registered': 'sum',
    'counts': 'sum'
})
df.casual /= df.counts
df.registered /= df.counts
df[['casual', 'registered']]
```

Out[14]:

	casual	registered
weekday		
0	0.316469	0.683531
1	0.155397	0.844603
2	0.123304	0.876696
3	0.121170	0.878830
4	0.126619	0.873381
5	0.160393	0.839607
6	0.321996	0.678004

In [15]:

```
fig, ax = plt.subplots()
ax.plot(df.index, [1]*len(df), color='orange')
ax.fill_between(df.index, [1]*len(df), df.casual, alpha=0.3, label='Registered ride
ax.plot(df.index, df.casual, marker='o')
ax.fill_between(df.index, df.casual, alpha=0.3, label='Casual rides', color='lights
ax.set_title('Percentage of casual and registered rides by day of week')
ax.set_xlabel('Day of week')
ax.set_ylabel('Percentage')
ax.legend(loc='upper right')
ax.set_yticklabels(['fill', '0%', '20%', '40%', '60%', '80%', '100%'])
ax.set_xticklabels(['fill', 'Sunday', 'Monday', 'Tuesday', 'Wed.', 'Thursday', 'Fri
plt.show()
```

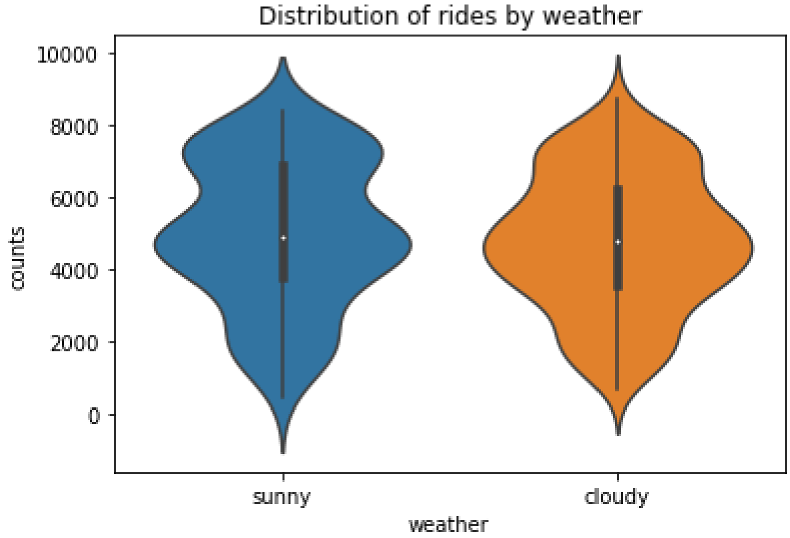


We can see that, in the weekends, casual rides increase their share. Maybe it's because: less people is going to work or school during these days, and registered rides is most likely to be used for commute; people who use bikes for recreation tend to do it in weekends, most likely with casual rides.

2.4 How is the distribution of total number of bike rentals different ...

In [16]:

```
# your code here
fig, ax = plt.subplots()
sns.violinplot(
    x='weather',
    y='counts',
    data=bikes_by_day[(bikes_by_day.weather == 1) | (bikes_by_day.weather == 2)],
    ax=ax
)
ax.set_title('Distribution of rides by weather')
ax.set_xticklabels(['sunny', 'cloudy'])
plt.show()
```

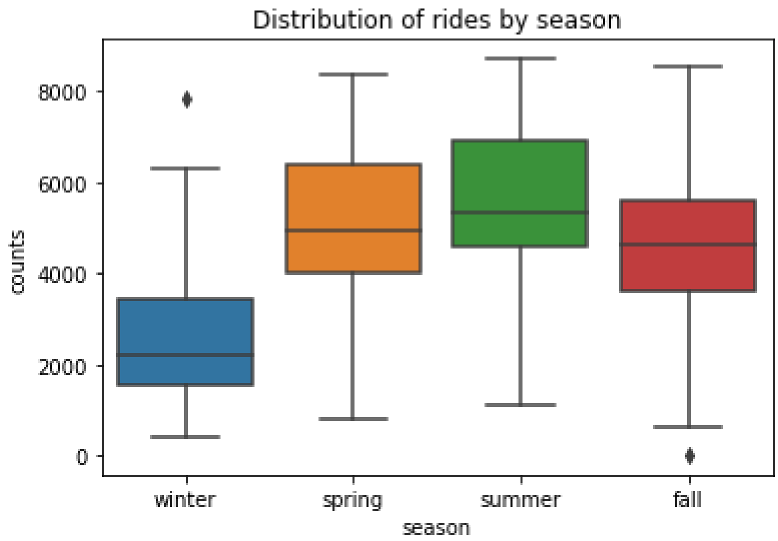


In general, sunny days has slightly more rides than cloudy days, because of greater median and quantiles.

2.5 Visualize how the total number of rides per day ...

In [17]:

```
# your code here

fig, ax = plt.subplots()
sns.boxplot(x='season', y='counts', data=bikes_by_day, ax=ax)
ax.set_title("Distribution of rides by season")
ax.set_xticklabels(['winter', 'spring', 'summer', 'fall'])
plt.show()
```



In [18]:

```
# Outlier in the winter
bikes_by_day[(bikes_by_day.counts > 7000) & (bikes_by_day.season == 1)]
```

Out[18]:

	weekday	weather	season	temp	atemp	windspeed	hum	casual	registere
dteday									
2012-03-17	6	2	1	0.514167	0.505046	0.110704	0.755833	3155	468

In [19]:

```
# Verify this outlier
bikes_df[bikes_df.dteday == '2012-03-17']
```

Out[19]:

	dteday	season	hour	holiday	weekday	workingday	weather	temp	atemp	hum	wi
10461	2012-03-17	1	0	0	6	0	2	0.44	0.4394	0.94	
10462	2012-03-17	1	1	0	6	0	2	0.44	0.4394	0.94	
10463	2012-03-17	1	2	0	6	0	2	0.44	0.4394	0.88	
10464	2012-03-17	1	3	0	6	0	2	0.44	0.4394	0.88	
10465	2012-03-17	1	4	0	6	0	2	0.42	0.4242	0.94	
10466	2012-03-17	1	5	0	6	0	2	0.42	0.4242	0.94	
10467	2012-03-17	1	6	0	6	0	2	0.42	0.4242	0.94	
10468	2012-03-17	1	7	0	6	0	2	0.40	0.4091	1.00	
10469	2012-03-17	1	8	0	6	0	2	0.42	0.4242	0.94	
10470	2012-03-17	1	9	0	6	0	2	0.44	0.4394	0.88	
10471	2012-03-17	1	10	0	6	0	2	0.50	0.4848	0.77	
10472	2012-03-17	1	11	0	6	0	2	0.52	0.5000	0.77	
10473	2012-03-17	1	12	0	6	0	1	0.56	0.5303	0.68	
10474	2012-03-17	1	13	0	6	0	1	0.60	0.6061	0.60	
10475	2012-03-17	1	14	0	6	0	1	0.62	0.6212	0.53	
10476	2012-03-17	1	15	0	6	0	1	0.64	0.6212	0.53	
10477	2012-03-17	1	16	0	6	0	1	0.64	0.6212	0.50	
10478	2012-03-17	1	17	0	6	0	1	0.64	0.6212	0.50	
10479	2012-03-17	1	18	0	6	0	1	0.62	0.6212	0.57	
10480	2012-03-17	1	19	0	6	0	1	0.58	0.5455	0.64	
10481	2012-03-17	1	20	0	6	0	1	0.56	0.5303	0.64	
10482	2012-03-17	1	21	0	6	0	1	0.54	0.5152	0.68	

	dteday	season	hour	holiday	weekday	workingday	weather	temp	atemp	hum	wi
10483	2012-03-17	1	22	0	6	0	1	0.54	0.5152	0.68	
10484	2012-03-17	1	23	0	6	0	1	0.50	0.4848	0.77	

In [20]:

```
# Outlier in the fall
bikes_by_day[(bikes_by_day.counts < 500) & (bikes_by_day.season == 4)]
```

Out[20]:

	weekday	weather	season	temp	atemp	windspeed	hum	casual	registered	counts
dteday										
2012-10-29	1	3	4	0.44	0.4394	0.3582	0.88	2	20	22

In [21]:

```
# Verify this outlier
bikes_df[bikes_df.dteday == '2012-10-29']
```

Out[21]:

	dteday	season	hour	holiday	weekday	workingday	weather	temp	atemp	hum	wi
15883	2012-10-29	4	0	0	1	1	3	0.44	0.4394	0.88	

We see the same distributions of rides than we saw in casual and registered rides before: less rides in the winter, summer with more rides, but spring and fall close to summer.

There're two outliers, as can be seen in the graph. Searching for them, we find:

- 2012-03-17 had more rides than winter is usual to have. Besides this day being Saturday with good climate conditions for riding, it was St. Patricks Day, with so president [Obama going to a bar to celebrate (https://www.huffpostbrasil.com/entry/obama-st-patricks-day_n_1355563?ri18n=true)](https://www.huffpostbrasil.com/entry/obama-st-patricks-day_n_1355563?ri18n=true).
- 2012-10-29 had less rides than usually for a day in the fall. Although it was Monday, it had rides only in the first hour. It's because Washington DC [shut down because of hurricane Sandy (https://www.theguardian.com/world/2012/oct/29/washington-dc-shutdown-hurricane-sandy)](https://www.theguardian.com/world/2012/oct/29/washington-dc-shutdown-hurricane-sandy).

Question 3: Prepare the data for Regression

3.1 Visualize and describe inter-dependencies among the following variables: `weekday` , `season` , `month` , `weather` , `temp` , `atemp` , `hum` , `windspeed` , `casual` , `registered` , `counts` . Note and comment on any strongly related variables.

3.2 Convert the categorical attributes into multiple binary attributes using **one-hot encoding**.

3.3 Split the initial `bikes_df` dataset (with hourly data about rentals) into train and test sets. Do this in a 'stratified' fashion, ensuring that all months are equally represented in each set. Explain your choice for a splitting algorithm. We ask you to create your train and test sets, but for consistency and easy checking we ask that, for the rest of this problem set, you use the train and test set provided in the question below.

3.4 Read `data/BSS_train.csv` and `data/BSS_test.csv` into dataframes `BSS_train` and `BSS_test` , respectively. After checking your train and test datasets for accuracy, remove the `dteday` column from both train and test dataset. We do not need it, and its format cannot be used for analysis. Also, remove any predictors that would make predicting the `count` trivial.

3.5 Calculate the **Pearson correlation** coefficients between all the features. Visualize the matrix using a heatmap. Which predictors have a positive correlation with the number of bike rentals? For categorical attributes, you should use each binary predictor resulting from one-hot encoding to compute their correlations. Identify pairs of predictors with collinearity >0.7.

Hints:

- You may use the `np.corrcoef` function to compute the correlation matrix for a data set (do not forget to transpose the data matrix). You may use `plt.pcolor` function to visualize the correlation matrix.

Answers

3.1 Visualize and describe inter-dependencies ...

In [22]:

```
columns = ['weekday', 'season', 'month', 'weather', 'temp', 'atemp', 'hum', 'windsp
matrix = bikes_df[columns].to_numpy()
correlations = np.corrcoef(matrix.transpose())
corr_df = pd.DataFrame(
    correlations,
    columns=columns,
    index=columns
)
corr_df.style.applymap(lambda x: 'background-color: green' if np.abs(x) > 0.6 and n
```

Out[22]:

	weekday	season	month	weather	temp	atemp	hum	windspe
weekday	1.000000	-0.002335	0.010400	0.003311	-0.001795	-0.008821	-0.037158	0.0115
season	-0.002335	1.000000	0.830386	-0.014524	0.312025	0.319380	0.150625	-0.1497
month	0.010400	0.830386	1.000000	0.005400	0.201691	0.208096	0.164411	-0.1353
weather	0.003311	-0.014524	0.005400	1.000000	-0.102640	-0.105563	0.418130	0.0262
temp	-0.001795	0.312025	0.201691	-0.102640	1.000000	0.987672	-0.069881	-0.0231
atemp	-0.008821	0.319380	0.208096	-0.105563	0.987672	1.000000	-0.051918	-0.0623
hum	-0.037158	0.150625	0.164411	0.418130	-0.069881	-0.051918	1.000000	-0.2901
windspeed	0.011502	-0.149773	-0.135386	0.026226	-0.023125	-0.062336	-0.290105	1.0000
casual	0.032721	0.120206	0.068457	-0.152628	0.459616	0.454080	-0.347028	0.0902
registered	0.021578	0.174226	0.122273	-0.120966	0.335361	0.332559	-0.273933	0.0823
counts	0.026900	0.178056	0.120638	-0.142426	0.404772	0.400929	-0.322911	0.0932

There exists strong correlations between `month` and `season` (which is obvious), between `temp` and `atemp` (which is expected too), between `casual` and `counts` (which is obvious too), and between `counts` and `registered`.

3.2 Convert the categorical attributes ....

In [23]:

```
# your code here

cat_bikes_df = pd.get_dummies(bikes_df, columns=['season', 'weekday', 'weather', 'm
cat_bikes_df.sample(5)
```

Out[23]:

	dteday	hour	holiday	workingday	temp	atemp	hum	windspeed	casual	registered
15894	2012-10-30	23	0	1	0.30	0.3030	0.81	0.1343	3	36
6222	2011-09-21	20	0	1	0.62	0.5455	0.94	0.0000	11	149
11472	2012-04-28	5	0	0	0.34	0.3333	0.46	0.1343	2	3
8738	2012-01-04	22	0	1	0.20	0.2273	0.47	0.0896	4	48
2633	2011-04-24	15	0	0	0.66	0.6212	0.61	0.2537	167	202

5 rows × 35 columns

3.3 Split the initial bikes_df dataset...

In [24]:

```
# your code here

train, test = train_test_split(bikes_df, stratify=bikes_df.month)
```

`train_test_split` is a good function from `sklearn` to split data into train and test. We want to stratify the data with the months because we don't want train or test data to have sub or super representation of some month, something that can affect our models.

3.4 Read `data/BSS_train.csv` and `data/BSS_test.csv` into ...

In [25]:

```
# your code here

BSS_train = pd.read_csv('data/BSS_train.csv')
BSS_test = pd.read_csv('data/BSS_test.csv')
```

In [26]:

```
# Sample seems to be accurated
BSS_train.sample(5)
```

Out[26]:

	Unnamed: 0	dteday	hour	holiday	year	workingday	temp	atemp	hum	windspeed	..
921	1176	2011-02-21	19	1	0	0	0.24	0.2121	0.87	0.3582	..
797	1020	2011-02-15	5	0	0	1	0.22	0.1818	0.32	0.4627	..
12961	16225	2012-11-13	19	0	1	1	0.32	0.2879	0.49	0.3582	..
13888	17359	2012-12-31	4	0	1	1	0.14	0.1667	0.69	0.1045	..
9510	11915	2012-05-16	16	0	1	1	0.72	0.6515	0.45	0.1045	..

5 rows × 36 columns

In [27]:

```
# Variables seem to be accurated
BSS_train.columns
```

Out[27]:

```
Index(['Unnamed: 0', 'dteday', 'hour', 'holiday', 'year', 'workingda
y', 'temp',
       'atemp', 'hum', 'windspeed', 'casual', 'registered', 'counts',
'spring',
       'summer', 'fall', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Au
g',
       'Sept', 'Oct', 'Nov', 'Dec', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri',
'Sat',
       'Cloudy', 'Snow', 'Storm'],
      dtype='object')
```

In [28]:

```
# Describe seems to be accurated
BSS_train.describe().transpose()
```

Out[28]:

	count	mean	std	min	25%	50%	75%	
Unnamed: 0	13903.0	8707.333166	5012.167209	0.00	4360.5000	8721.0000	13051.5000	1737
hour	13903.0	11.529454	6.917884	0.00	6.0000	11.0000	18.0000	2
holiday	13903.0	0.029274	0.168580	0.00	0.0000	0.0000	0.0000	
year	13903.0	0.504567	0.499997	0.00	0.0000	1.0000	1.0000	
workingday	13903.0	0.680644	0.466244	0.00	0.0000	1.0000	1.0000	
temp	13903.0	0.496732	0.192699	0.02	0.3400	0.5000	0.6600	
atemp	13903.0	0.475426	0.171951	0.00	0.3333	0.4848	0.6212	
hum	13903.0	0.629051	0.193100	0.00	0.4800	0.6300	0.7800	
windspeed	13903.0	0.190025	0.122009	0.00	0.1045	0.1940	0.2537	
casual	13903.0	35.414515	48.838431	0.00	4.0000	16.0000	48.0000	36
registered	13903.0	152.470977	150.301784	0.00	33.0000	115.0000	218.0000	88
counts	13903.0	187.885492	180.113476	1.00	39.0000	141.0000	280.0000	97
spring	13903.0	0.253183	0.434850	0.00	0.0000	0.0000	1.0000	
summer	13903.0	0.258865	0.438027	0.00	0.0000	0.0000	1.0000	
fall	13903.0	0.244623	0.429879	0.00	0.0000	0.0000	0.0000	
Feb	13903.0	0.077178	0.266883	0.00	0.0000	0.0000	0.0000	
Mar	13903.0	0.084730	0.278489	0.00	0.0000	0.0000	0.0000	
Apr	13903.0	0.082716	0.275462	0.00	0.0000	0.0000	0.0000	
May	13903.0	0.085593	0.279772	0.00	0.0000	0.0000	0.0000	
Jun	13903.0	0.082860	0.275680	0.00	0.0000	0.0000	0.0000	
Jul	13903.0	0.085593	0.279772	0.00	0.0000	0.0000	0.0000	
Aug	13903.0	0.084874	0.278704	0.00	0.0000	0.0000	0.0000	
Sept	13903.0	0.082716	0.275462	0.00	0.0000	0.0000	0.0000	
Oct	13903.0	0.083507	0.276657	0.00	0.0000	0.0000	0.0000	
Nov	13903.0	0.082716	0.275462	0.00	0.0000	0.0000	0.0000	
Dec	13903.0	0.085305	0.279346	0.00	0.0000	0.0000	0.0000	
Mon	13903.0	0.140977	0.348010	0.00	0.0000	0.0000	0.0000	
Tue	13903.0	0.141121	0.348158	0.00	0.0000	0.0000	0.0000	
Wed	13903.0	0.144861	0.351973	0.00	0.0000	0.0000	0.0000	
Thu	13903.0	0.140833	0.347862	0.00	0.0000	0.0000	0.0000	
Fri	13903.0	0.142128	0.349194	0.00	0.0000	0.0000	0.0000	
Sat	13903.0	0.145508	0.352625	0.00	0.0000	0.0000	0.0000	
Cloudy	13903.0	0.260375	0.438855	0.00	0.0000	0.0000	1.0000	

	count	mean	std	min	25%	50%	75%
Snow	13903.0	0.082860	0.275680	0.00	0.0000	0.0000	0.0000
Storm	13903.0	0.000144	0.011993	0.00	0.0000	0.0000	0.0000

In [29]:

```
# The same for test data
BSS_test.sample(5)
```

Out[29]:

	Unnamed: 0	dteday	hour	holiday	year	workingday	temp	atemp	hum	windspeed	...
1449	7164	2011-10-31	3	0	0	1	0.24	0.2576	0.87	0.1045	...
3372	16883	2012-12-11	6	0	1	1	0.34	0.3030	0.71	0.2985	...
2483	12349	2012-06-03	18	0	1	0	0.70	0.6364	0.34	0.2537	...
130	610	2011-01-28	16	0	0	1	0.22	0.2727	0.80	0.0000	...
627	3019	2011-05-10	17	0	0	1	0.64	0.6212	0.33	0.0000	...

5 rows × 36 columns

In [30]:

```
BSS_test.columns
```

Out[30]:

```
Index(['Unnamed: 0', 'dteday', 'hour', 'holiday', 'year', 'workingda
y', 'temp',
       'atemp', 'hum', 'windspeed', 'casual', 'registered', 'counts',
'spring',
       'summer', 'fall', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Au
g',
       'Sept', 'Oct', 'Nov', 'Dec', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri',
'Sat',
       'Cloudy', 'Snow', 'Storm'],
      dtype='object')
```

In [31]:

```
BSS_test.describe().transpose()
```

Out[31]:

	count	mean	std	min	25%	50%	75%	
Unnamed: 0	3476.0	8615.672612	5036.486147	6.0000	4270.5000	8562.5000	12957.2500	17376
hour	3476.0	11.615938	6.901033	0.0000	6.0000	12.0000	17.0000	23
holiday	3476.0	0.026755	0.161389	0.0000	0.0000	0.0000	0.0000	1
year	3476.0	0.494534	0.500042	0.0000	0.0000	0.0000	1.0000	1
workingday	3476.0	0.691024	0.462138	0.0000	0.0000	1.0000	1.0000	1
temp	3476.0	0.498009	0.192006	0.0200	0.3400	0.5000	0.6600	0
atemp	3476.0	0.477173	0.171464	0.0303	0.3333	0.4848	0.6212	0
hum	3476.0	0.619940	0.192104	0.0000	0.4700	0.6200	0.7800	1
windspeed	3476.0	0.190389	0.123672	0.0000	0.1045	0.1940	0.2537	0
casual	3476.0	36.722957	51.122565	0.0000	4.0000	17.0000	48.2500	354
registered	3476.0	159.050058	155.418060	0.0000	37.0000	119.0000	227.2500	876
counts	3476.0	195.773015	186.289553	1.0000	44.0000	147.5000	285.2500	970
spring	3476.0	0.255754	0.436347	0.0000	0.0000	0.0000	1.0000	1
summer	3476.0	0.258055	0.437627	0.0000	0.0000	0.0000	1.0000	1
fall	3476.0	0.239068	0.426576	0.0000	0.0000	0.0000	0.0000	1
Feb	3476.0	0.077100	0.266789	0.0000	0.0000	0.0000	0.0000	1
Mar	3476.0	0.084868	0.278725	0.0000	0.0000	0.0000	0.0000	1
Apr	3476.0	0.082566	0.275265	0.0000	0.0000	0.0000	0.0000	1
May	3476.0	0.085731	0.280006	0.0000	0.0000	0.0000	0.0000	1
Jun	3476.0	0.082854	0.275701	0.0000	0.0000	0.0000	0.0000	1
Jul	3476.0	0.085731	0.280006	0.0000	0.0000	0.0000	0.0000	1
Aug	3476.0	0.084868	0.278725	0.0000	0.0000	0.0000	0.0000	1
Sept	3476.0	0.082566	0.275265	0.0000	0.0000	0.0000	0.0000	1
Oct	3476.0	0.083429	0.276570	0.0000	0.0000	0.0000	0.0000	1
Nov	3476.0	0.082566	0.275265	0.0000	0.0000	0.0000	0.0000	1
Dec	3476.0	0.085443	0.279580	0.0000	0.0000	0.0000	0.0000	1
Mon	3476.0	0.149310	0.356445	0.0000	0.0000	0.0000	0.0000	1
Tue	3476.0	0.141254	0.348334	0.0000	0.0000	0.0000	0.0000	1
Wed	3476.0	0.132624	0.339216	0.0000	0.0000	0.0000	0.0000	1
Thu	3476.0	0.147583	0.354738	0.0000	0.0000	0.0000	0.0000	1
Fri	3476.0	0.147008	0.354165	0.0000	0.0000	0.0000	0.0000	1
Sat	3476.0	0.140679	0.347740	0.0000	0.0000	0.0000	0.0000	1
Cloudy	3476.0	0.265823	0.441834	0.0000	0.0000	0.0000	1.0000	1
Snow	3476.0	0.076812	0.266332	0.0000	0.0000	0.0000	0.0000	1

	count	mean	std	min	25%	50%	75%	
Storm	3476.0	0.000288	0.016961	0.0000	0.0000	0.0000	0.0000	1

In [32]:

```
# Drop some columns
BSS_train.drop(columns=['Unnamed: 0', 'dteday', 'casual', 'registered'], inplace=Tr
BSS_test.drop(columns=['Unnamed: 0', 'dteday', 'casual', 'registered'], inplace=Tru
```

In [33]:

```
# The test_size
len(BSS_test)/(len(BSS_test) + len(BSS_train))
```

Out[33]:

```
0.20001150814201046
```

In [34]:

```
# Percentage of February months in test data (very close to test_size)
(BSS_test.Feb == 1).sum()/((BSS_test.Feb == 1).sum() + (BSS_train.Feb == 1).sum())
```

Out[34]:

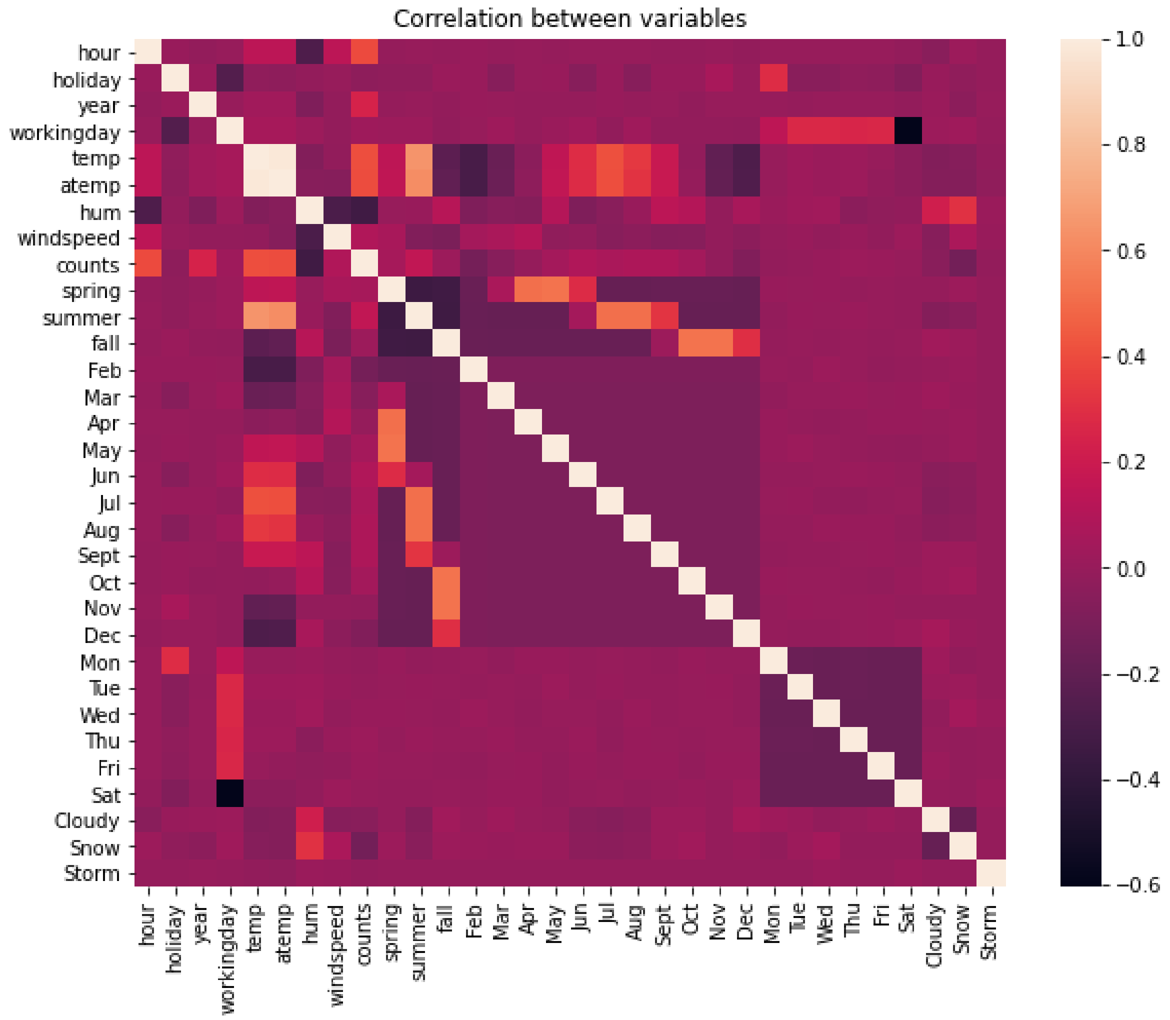
```
0.19985085756897839
```

Train and test data seem to be accurated, because a sample, the column names and the describe of the DataFrame seem to be reasonable. Besides that, data seems to be stratified too (look at the sanity check with February month). The columns `Unnamed: 0`, `dteday` were removed, as well as the columns `casual` and `registered`, because they would make the prediction of `counts` trivial.

3.5 Calculate the Pearson correlation ....

In [35]:

```
# your code here

# The heatmap

correlations = np.corrcoef(BSS_train.to_numpy().transpose())
df = pd.DataFrame(
    correlations,
    columns=BSS_train.columns,
    index=BSS_train.columns
)
fig, ax = plt.subplots(figsize=(10, 8))
sns.heatmap(df, ax=ax)
ax.set_title('Correlation between variables')
plt.show()
```



In [36]:

```
# The positive correlations with count
df2 = df.iloc[df.counts.argsort()].iloc[::-1]
df2[df2.counts > 0][['counts']]
```

Out[36]:

	counts
counts	1.000000
temp	0.406155
atemp	0.401119
hour	0.394167
year	0.243886
summer	0.159319
Jun	0.094448
windspeed	0.093981
Aug	0.085847
Sept	0.080225
Jul	0.073333
spring	0.058418
May	0.050471
Oct	0.046657
workingday	0.029534
fall	0.022531
Thu	0.018731
Fri	0.015080
Wed	0.006535
Sat	0.000878

In [37]:

```
# Pairs of predictors with abs(correlation) > 0.7
np.where((np.abs(df.to_numpy()) > 0.7) & (np.round(df.to_numpy(), 6) != 1))
```

Out[37]:

```
(array([4, 5]), array([5, 4]))
```

In [38]:

```
# It's obvious:
print(df.iloc[4, 5], df.index[4], df.columns[5])
```

```
0.987407710819568 temp atemp
```

`temp`, `atemp`, `hour` and `year` are the variables with the strongest positive correlations with `counts`. The only pair of variables with correlation > 0.7 (in absolute value) is `temp` and `atemp`.

Question 4: Multiple Linear Regression

4.1 Use statsmodels to fit a multiple linear regression model to the training set using all the predictors (no interactions or polynomial terms), and report its R^2 score on the train and test sets.

4.2 Find out which of estimated coefficients are statistically significant at a significance level of 5% (p-value < 0.05). Comment on the results.

4.3 Make a plot of residuals of the fitted model $e = y - \hat{y}$ as a function of the predicted value $\hat{y}$. Note that this is slightly different from the residual plot for simple linear regression. Draw a horizontal line denoting the zero residual value on the Y-axis. Does the plot reveal a non-linear relationship between the predictors and response? What does the plot convey about the variance of the error terms?

Answers

4.1 Use statsmodels to fit a ...

In [39]:

```
# your code here
X_train = sm.add_constant(BSS_train.drop(columns='counts').to_numpy())
X_test = sm.add_constant(BSS_test.drop(columns='counts').to_numpy())
y_train = BSS_train.counts.to_numpy()
y_test = BSS_test.counts.to_numpy()

model = sm.OLS(y_train, X_train).fit()
print('Train data R² score: {:.4f}\nTest data R² score:  {:.4f}'.format(
    r2_score(y_train, model.predict()),
    r2_score(y_test, model.predict(X_test))
))
```

```
Train data R² score: 0.4065
Test data R² score:  0.4064
```

4.2 Find out which of estimated coefficients ...

In [40]:

```
# your code here

model.summary()
```

Out[40]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.407
Model:	OLS	**Adj. R-squared:**	0.405
Method:	Least Squares	**F-statistic:**	316.8
Date:	Mon, 20 Jul 2020	**Prob (F-statistic):**	0.00
Time:	23:10:04	**Log-Likelihood:**	-88306.
No. Observations:	13903	**AIC:**	1.767e+05
Df Residuals:	13872	**BIC:**	1.769e+05
Df Model:	30		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-21.0830	8.641	-2.440	0.015	-38.020	-4.146
x1	7.2214	0.184	39.144	0.000	6.860	7.583
x2	-18.0958	6.597	-2.743	0.006	-31.027	-5.165
x3	76.3519	2.380	32.084	0.000	71.687	81.017
x4	11.3178	2.751	4.114	0.000	5.926	16.710
x5	333.2482	44.162	7.546	0.000	246.684	419.812
x6	74.6312	46.207	1.615	0.106	-15.940	165.202
x7	-205.4959	7.801	-26.343	0.000	-220.786	-190.205
x8	22.5168	10.753	2.094	0.036	1.439	43.595
x9	43.1541	7.417	5.818	0.000	28.615	57.693
x10	29.5426	8.773	3.367	0.001	12.346	46.739
x11	68.5953	7.492	9.156	0.000	53.911	83.280
x12	-7.6430	5.966	-1.281	0.200	-19.336	4.050
x13	-11.6737	6.665	-1.752	0.080	-24.737	1.390
x14	-41.5244	9.878	-4.204	0.000	-60.886	-22.163
x15	-33.2927	10.543	-3.158	0.002	-53.958	-12.628
x16	-65.8039	10.716	-6.141	0.000	-86.809	-44.799
x17	-93.4805	12.086	-7.734	0.000	-117.171	-69.789
x18	-59.2081	11.832	-5.004	0.000	-82.401	-36.015
x19	-16.0517	10.575	-1.518	0.129	-36.780	4.676
x20	-16.1602	9.865	-1.638	0.101	-35.497	3.177
x21	-25.8732	9.527	-2.716	0.007	-44.547	-7.199

x22	-10.2043	7.614	-1.340	0.180	-25.128	4.719
x23	-2.6601	2.978	-0.893	0.372	-8.498	3.177
x24	-6.1425	3.208	-1.915	0.056	-12.430	0.145
x25	2.2964	3.183	0.721	0.471	-3.943	8.536
x26	-3.1611	3.185	-0.993	0.321	-9.404	3.082
x27	2.8892	3.186	0.907	0.364	-3.355	9.133
x28	14.9459	4.382	3.411	0.001	6.357	23.535
x29	6.7868	2.900	2.341	0.019	1.103	12.470
x30	-28.2859	4.819	-5.870	0.000	-37.731	-18.841
x31	42.3569	98.377	0.431	0.667	-150.475	235.189

Omnibus:	2831.359	**Durbin-Watson:**	0.755
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5657.789
Skew:	1.224	**Prob(JB):**	0.00
Kurtosis:	4.943	**Cond. No.**	1.17e+16

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 1.87e-26. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

In [41]:

```
# The statistically significant (with level of 5%) estimated coefficients

list(np.array(['intercept']+list(BSS_train.drop(columns='counts').columns))[model.p
```

Out[41]:

```
['intercept',
 'hour',
 'holiday',
 'year',
 'workingday',
 'temp',
 'hum',
 'windspeed',
 'spring',
 'summer',
 'fall',
 'Apr',
 'May',
 'Jun',
 'Jul',
 'Aug',
 'Nov',
 'Sat',
 'Cloudy',
 'Snow']
```

In [42]:

```
# The statistically insignificant (with level of 5%) estimated coefficients

list(np.array(['intercept']+list(BSS_train.drop(columns='counts').columns))[model.p
```

Out[42]:

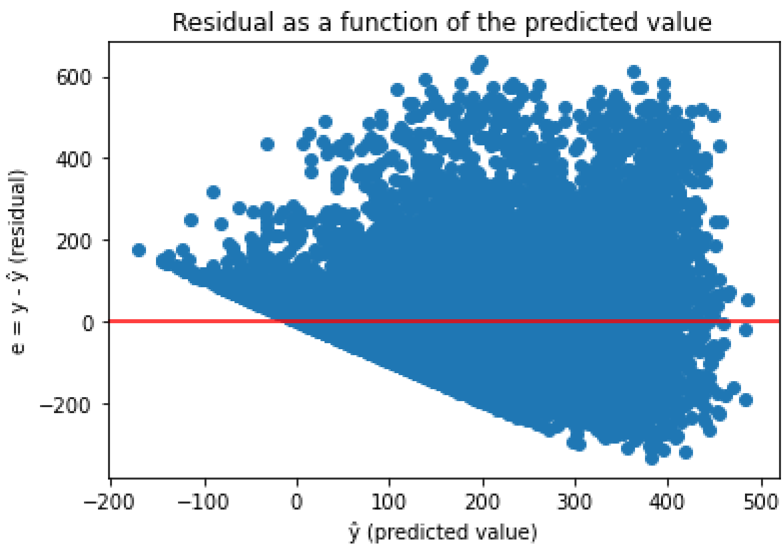
```
['atemp',
 'Feb',
 'Mar',
 'Sept',
 'Oct',
 'Dec',
 'Mon',
 'Tue',
 'Wed',
 'Thu',
 'Fri',
 'Storm']
```

We see that not all features have coefficients statistically significant (with level of 5%), but the majority of them have. `atemp` has statistically insignificant coefficient (maybe because there's already `temp` ?), many months and `Storm` (maybe because of few data?).

4.3 Make a plot of residuals of the fitted ...

In [43]:

```
plt.scatter(model.predict(), model.resid)
plt.xlabel('ŷ (predicted value)')
plt.ylabel('e = y - ŷ (residual)')
plt.title('Residual as a function of the predicted value')
plt.axhline(0, color='red')
plt.show()
```



If `counts` variable were a linear function of the other variables, the residuals shoud have the same variance. That is, the residuals should have the same distribution, independently of the predicted value. It's not what we see above, because variance is changing. It reveals a non-linear relationship.

Question 5: Subset Selection

5.1 Implement forward step-wise selection to select a minimal subset of predictors that are related to the response variable:

We require that you implement the method **from scratch**. You may use the Bayesian Information Criterion (BIC) to choose the subset size in each method.

5.2 Do these methods eliminate one or more of the colinear predictors (if any) identified in Question 3.5? If so, which ones. Briefly explain (3 or fewer sentences) why you think this may be the case.

5.3 Fit the linear regression model using the identified subset of predictors to the training set. How do the test R^2 scores for the fitted models compare with the model fitted in Question 4 using all predictors?

Answers

5.1 Implement forward step-wise ....

In [44]:

```
# your code here

def forward_selection(df_train, target):
    models = []
    variables = list(df_train.drop(columns=target).columns)
    J = len(variables)
    y_train = df_train[target].to_numpy()
    n = len(y_train)

    X_train = np.ones((n, 1))
    model_0 = sm.OLS(y_train, X_train).fit()
    mse = np.mean((y_train - model_0.predict())**2)
    models.append({'model': model_0, 'variables': [], 'bic': model_0.bic})

    for k in range(1, J+1):
        model_candidates = []
        for variable in variables:
            predictors = models[-1]['variables'].copy()
            predictors.append(variable)
            X_train = df_train[predictors].to_numpy()
            X_train = sm.add_constant(X_train)
            model = sm.OLS(y_train, X_train).fit()
            mse = np.mean((y_train - model.predict())**2)
            model_candidates.append({'model': model, 'variables': predictors, 'bic'
        model = min(model_candidates, key=lambda x: x['bic'])
        models.append(model)
        variables.remove(model['variables'][-1])

    return min(models, key=lambda x: x['bic'])

my_model = forward_selection(BSS_train, 'counts')
my_model
```

Out[44]:

```
{'model': <statsmodels.regression.linear_model.RegressionResultsWrappe
r at 0x7fde9bf69730>,
 'variables': ['temp',
  'hour',
  'year',
  'hum',
  'fall',
  'Jul',
  'Snow',
  'Aug',
  'Jun',
  'holiday',
  'spring'],
 'bic': 176790.38694346516}
```

The selected subset is: `temp`, `hour`, `year`, `hum`, `fall`, `Jul`, `Snow`, `Aug`, `Jun`, `holiday`, `spring`.

5.2 Do these methods eliminate ...

The coefficients positive related to `counts` that were eliminated are: `atemp` , `summer` , `windspeed` , `Sept` , `May` , `Oct` , `workingday` , `Thu` , `Fri` , `Wed` , `Sat` .

Maybe it happened because some of them has, despite of positive correlation, week correlation, and there are negative correlated coefficients that are stronger. Maybe they're already represented by another feature.

5.3 In each case, fit linear regression ...

In [45]:

```
# your code here

my_X_test = sm.add_constant(BSS_test[my_model['variables']].to_numpy())
r2_score(y_test, my_model['model'].predict(my_X_test))
```

Out[45]:

```
0.40469087705369555
```

The test R^2 score with all predictors was 0.4064. So the new R^2 score is less than that one.

It's kind of expected. The R^2 score for train data with all predictors was 0.4065, very close to the test one. The R^2 for train data with all predictors is an upperbound for R^2 in train data (imagine linear regression with less predictors as putting the eliminated coefficients as zero), and it's expected to see R^2 on test data less than on train data. So it's reasonable to see this new R^2 on test data less than the old one.

Question 6: Polynomial Regression

We will now try to improve the performance of the regression model by including higher-order polynomial terms.

6.1 For each continuous predictor X_j, include additional polynomial terms X_j^2, X_j^3, and X_j^4, and fit a polynomial regression model to the expanded training set. How does the R^2 of this model on the test set compare with that of the linear model fitted in the previous question? Using a t-tests, find out which of the estimated coefficients for the polynomial terms are statistically significant at a significance level of 5%.

In [46]:

```
# your code here

features = list(BSS_train.drop(columns='counts').columns)
continuous = [False]*len(features)
continuous[4:8] = [True]*4

transform = PolynomialFeatures(4, include_bias=False)

X_train_transformed = np.ones((X_train.shape[0], 1))
for idx, feature in enumerate(features):
    if continuous[idx]:
        transformed = transform.fit_transform(BSS_train[feature].to_numpy()[..., np
        X_train_transformed = np.concatenate((X_train_transformed, transformed), ax
    else:
        not_transformed = BSS_train[feature].to_numpy()[..., np.newaxis]
        X_train_transformed = np.concatenate((X_train_transformed, not_transformed)

X_test_transformed = np.ones((X_test.shape[0], 1))
for idx, feature in enumerate(features):
    if continuous[idx]:
        transformed = transform.fit_transform(BSS_test[feature].to_numpy()[..., np.
        X_test_transformed = np.concatenate((X_test_transformed, transformed), axis
    else:
        not_transformed = BSS_test[feature].to_numpy()[..., np.newaxis]
        X_test_transformed = np.concatenate((X_test_transformed, not_transformed),


model = sm.OLS(y_train, X_train_transformed).fit()

print('Test data R² score: {:.4f}'.format(r2_score(y_test, model.predict(X_test_tra
```

```
Test data R² score: 0.4203
```

In [47]:

```
# Statistically significant (level of 5%) coefficients

polynomial_features = ['intercept']
for idx, feature in enumerate(features):
    if continuous[idx]:
        for i in range(1, 5):
            polynomial_features.append('{}^{}'.format(feature, i))
    else:
        polynomial_features.append('{}'.format(feature))
print(list(np.array(polynomial_features)[model.pvalues < 0.05]))
```

```
['intercept', 'hour', 'holiday', 'year', 'workingday', 'temp^2', 'temp
^3', 'temp^4', 'hum^1', 'hum^2', 'hum^3', 'hum^4', 'windspeed^2', 'win
dspeed^3', 'windspeed^4', 'spring', 'summer', 'fall', 'May', 'Jun', 'J
ul', 'Aug', 'Tue', 'Sat', 'Cloudy', 'Snow']
```

We achieved the best test R^2 score: 0.4203. The list of statistically significant coefficients are list above. Almost all features selected in the previous (`temp` , `hour` , `year` , `hum` , `fall` , `Jul` , `Snow` , `Aug` , `Jun` , `holiday` , `spring`) question have a term (or a polynomial term) with statistically significance (level of 5%) above.

Written Report to the Administrators

Question 7

Write a short summary report, intended for the administrators of the company, to address two major points (can be written as two large paragraphs):

1. How to predict ridership well (which variables are important, when is ridership highest/lowest, etc.).
2. Suggestions on how to increase the system revenue (what additional services to provide, when to give discounts, etc.).

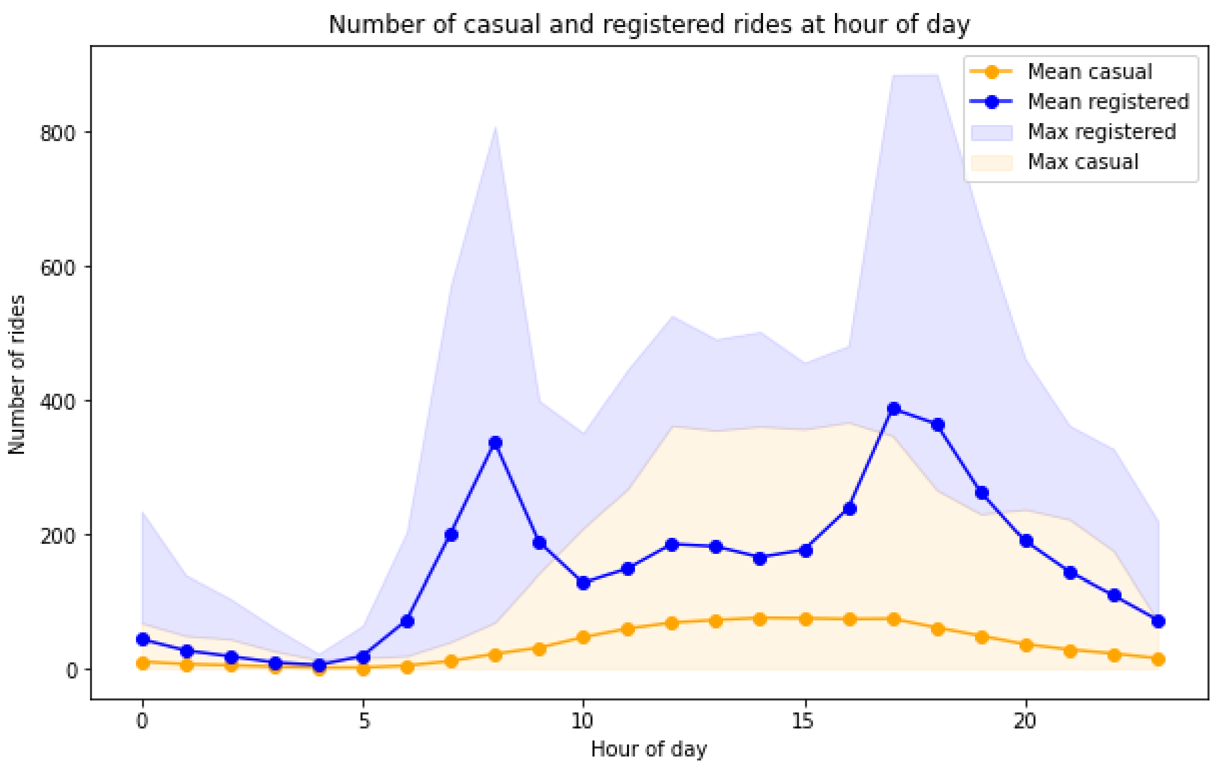
Include your report below. The report should not be longer than 300 words and should include a maximum of 3 figures.

Answers

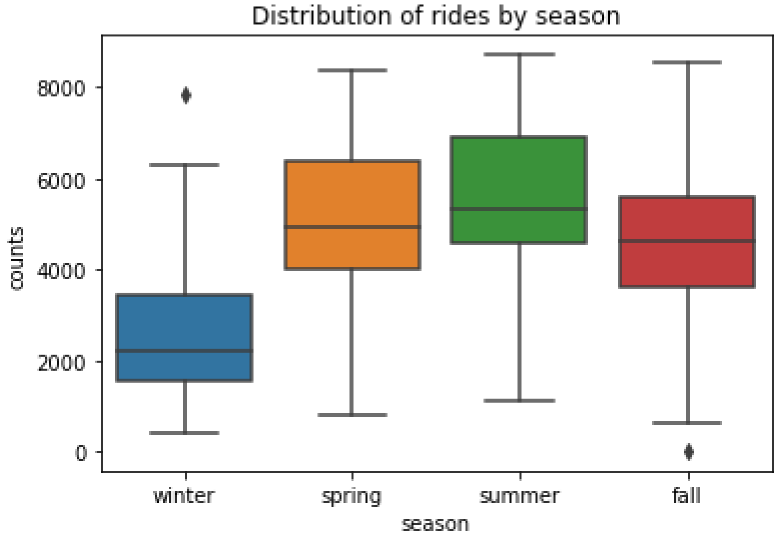
7

Prediction of number of rides is a very important task when dealing with bike sharing services. We found some interesting insights in our analysis.

The most part or the rides is done by registered clientes in commute time, that is, people is using for going to work or school.



In general, the trend is: rides begin with the morning, end with the night and vanish at dawn, with peaks of use in commute times. On holidays, we also see that, but these peaks are shorter and casual rides are greater. We also see that weather doesn't affect as much as we could think: people still ride bikes in snow and thunderstorm days, although the number is less. People ride bikes less in winter, and in the other seasons the numbers are very close.



During weekdays, approx. 15% of the rides are casual rides, against 35% in weekends.

When dealing with linear regression models, a good subset of features is `temp` , `hour` , `year` , `hum` , `fall` , `Jul` , `Snow` , `Aug` , `Jun` , `holiday` , `spring` . But we have to say that we observed non-linear relationships with data, so maybe it's better to use polynomial regressions, which we observe to perform better.

Our tips are based in three aspects:

- **Raise rides when there're few of them.** In the winter, people use less the bikes, so it's a good ideia to offer discounts during this season.
- **Take advantage of loyalty.** We saw people use your service a lot for commute, that is, people use it everyday. Offer discounts and advantages for these users is a good idea, in order to attract more people and to loyalty them.
- **Offer more bikes when it's being heavy used.** It happens during commute time and in seasons like the summer, and it’s good to make sure there’s no shortage of bicycles.