Homework 8

Topological Data Analysis with Persistent Homology

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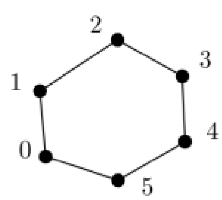
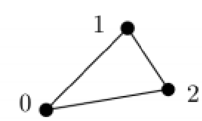
Exercise 48. *Let* $K = [0], [1], [2], [3], [4], [5], [0, 1], [1, 2], [2, 3], [3, 4], [4, 5], [5, 0]$ *and* $L = [0], [1], [2], [0, 1], [1, 2], [2, 0]$. *Consider the simplical map* $f \colon i \mapsto i$ modulo 3. *Show that the induced map* $(f_1)_*$ *is zero.*





There are only two 1-cycles of K: $Z_1(K) = \{0, [0, 1] + [1, 2] + [2, 3] + [3, 4] + [4, 5] + [5, 0]\}$. Because there are no simplices of dimension 2, the 1-boundary of K, L are $B_1(K) = B_1(L) = \{0\}$. So we end up with the two elements of $Z_1(K)$ being the two representants of equivalence classes of $H_1(K) = \{0 + \{0\}, [0, 1] + [1, 2] + [2, 3] + [3, 4] + [4, 5] + [5, 0] + \{0\}\}$.

Let's applic $(f_1)_*$ in our homology group:

$$(f_1)_*(0 + B_1(K)) = f_1(0) + B_1(L) = \{0\}$$

$$\begin{aligned}
(f_1)_*([0, 1] + [1, 2] + [2, 3] + [3, 4] + [4, 5] + [5, 0] + B_1(K)) &= f_1([0, 1] + [1, 2] + [2, 3] + [3, 4] + [4, 5] + [5, 0]) + B_1(L) \\
&= [0, 1] + [1, 2] + [2, 0] + [0, 1] + [1, 2] + [2, 0] + B_1(K) \\
&= 0 + B_1(K) \\
&= \{0\}
\end{aligned}$$

We conclude the induced map $(f_1)_*$ is zero.