

transportation problems

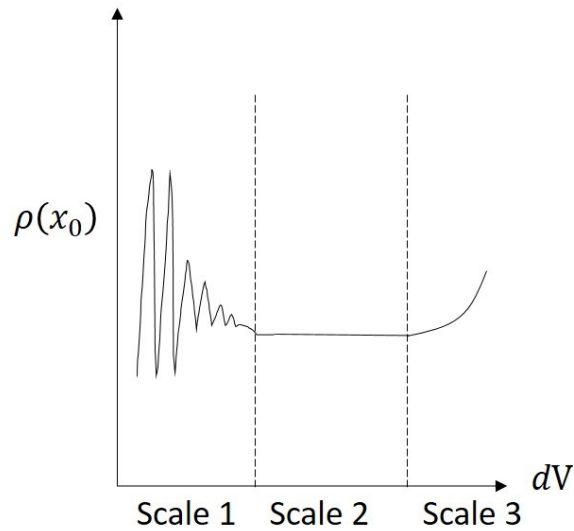
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1 L1

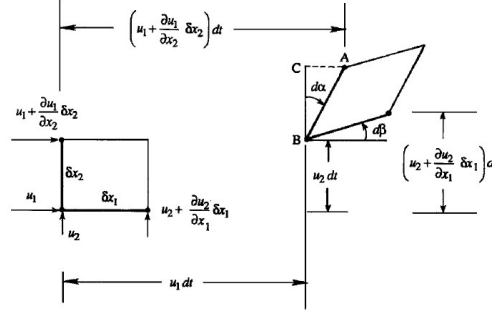
1. In a particle system, the mass density ρ is defined at some point x_0 as,

$$\rho(x_0) = \frac{m_0 dN}{dV} \quad (1)$$

Where m_0 is the mass of a particle, dV is the volume of the element that contains x_0 , dN is the particle number in the element. The figure below shows how $\rho(x_0)$ varies at different scales of dV , explain it respectively.



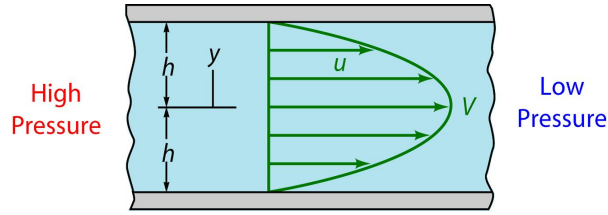
2. Figure below shows the position of an element with sides parallel to the coordinate axes at time t , and its subsequent position at $t+dt$. express the strain rate $((d\alpha + d\beta)/dt)$ with u_1, u_2, x_1, x_2



3. Consider the viscous flow in a channel of width $2b$. The channel is aligned in the x direction, and the velocity at a distance y from the centerline is given by the parabolic distribution

$$u(y) = U_0 \left(1 - \frac{y^2}{b^2}\right) \quad (2)$$

In terms of the viscosity μ , calculate the shear stress at a distance of $y = b/2$.

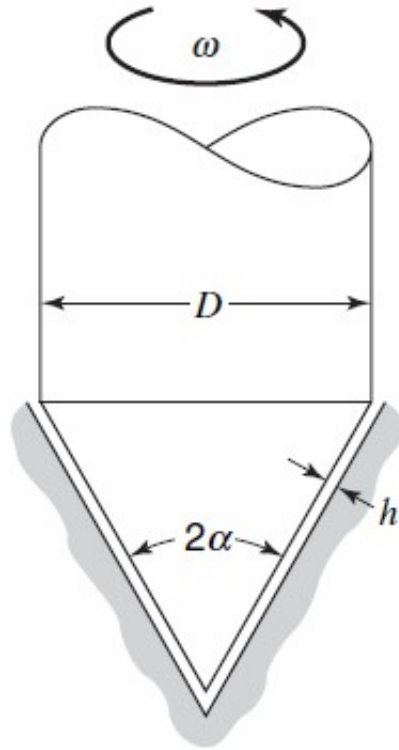


4. Given a velocity field $v = \begin{pmatrix} u \\ v \end{pmatrix}$ such that

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (3)$$

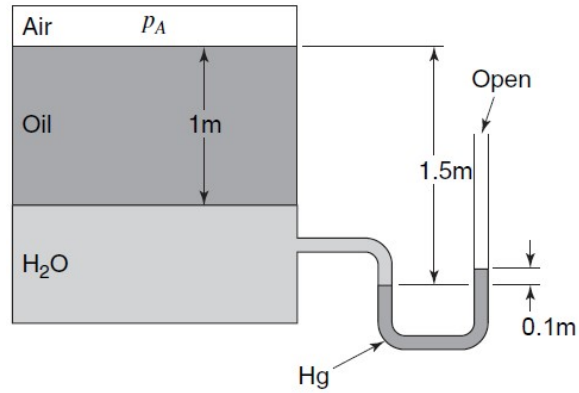
Determine the shear stress τ along $i, j, i + j$ direction (denote them $\tau_x, \tau_y, \tau_{xy}$ respectively)

5. Two clean and parallel glass plates, separated by a gap of 1.625mm, are dipped in water. If Coefficient of surface tension $\sigma=0.0735\text{N/m}$, determine how high the water will rise.
6. Determine the difference in pressure between the inside and outside of a soap film bubble at 20°C if the diameter of the bubble is 4 mm.
7. Determine the diameter of the glass tube necessary to keep the capillary-height change of water at 30°C less than 1 mm.
8. An auto lift consists of 36.02-cm-diameter ram that slides in a 36.04-cm-diameter cylinder. The annular region is filled with oil having a kinematic viscosity of $0.00037\text{m}^2/\text{s}$ and a specific gravity of 0.85. If the rate of travel of the ram is 0.15 m/s, estimate the frictional resistance when 3.14 m of the ram is engaged in the cylinder.
9. 9. If the ram and auto rack in the previous problem together have a mass of 680 kg, estimate the maximum sinking speed of the ram and rack when gravity and viscous friction are the only forces acting. Assume 2.44 m of the ram engaged.
10. The conical pivot shown in the figure has angular velocity ω and rests on an oil film of uniform thickness h . Determine the frictional moment as a function of the angle a , the viscosity, the angular velocity, the gap distance, and the shaft diameter.

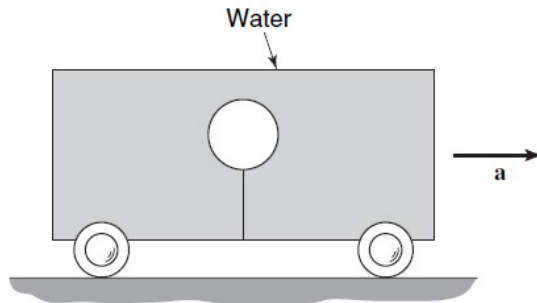


2 L2

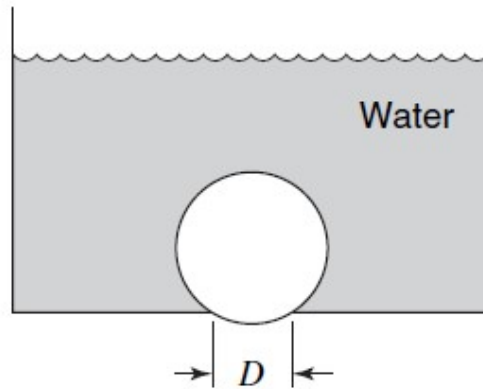
1. Matter is attracted to the center of Earth with a force proportional to the radial distance from the center. Using the known value of g at the surface where the radius is 6330 km, compute the pressure at Earth's center, assuming the material behaves like a liquid, and that the mean specific gravity is 5.67.
2. Determine the depth change to cause a pressure increase of 1 atm for (a) water, (b) sea water (specific gravity=1.0250), and (c) mercury (SG = 13.6).
3. What is the pressure p_A in the figure? The specific gravity of the oil is 0.8.



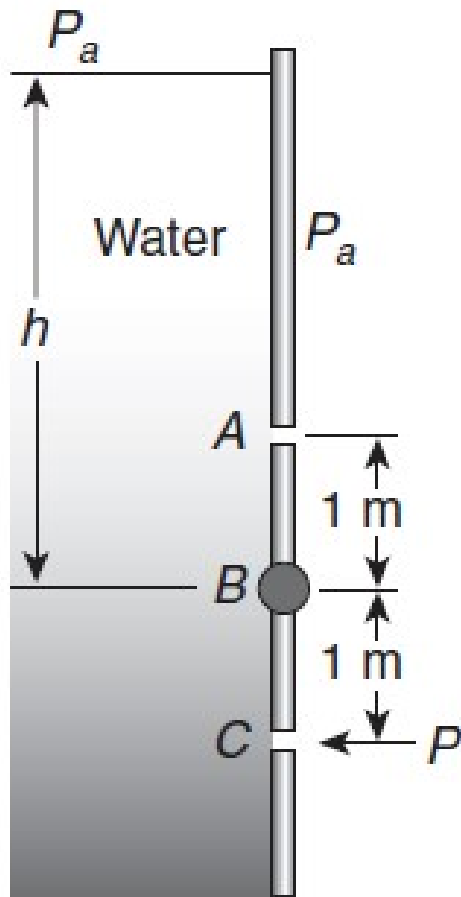
4. The car shown in the figure is accelerated to the right at a uniform rate. What way will the balloon (tied by the string) move relative to the car?



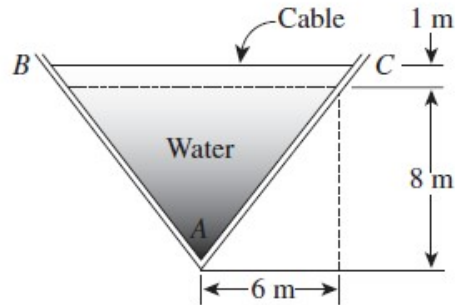
5. It is desired to use a 0.75-m diameter beach ball to stop a drain in a swimming pool. Obtain an expression that relates the drain diameter D and the minimum water depth h for which the ball will remain in place.



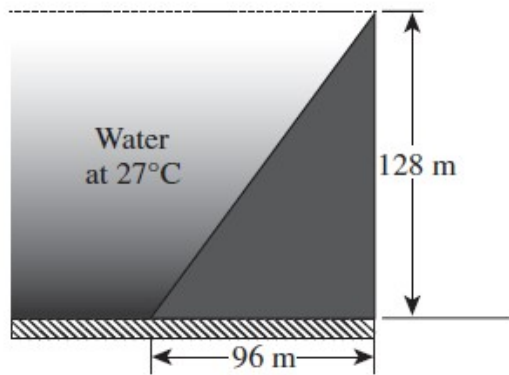
6. The circular gate ABC has a 1m radius and is hinged at B. Neglecting atmospheric pressure, determine the force P just sufficient to keep the gate from opening when $h = 12\text{m}$.



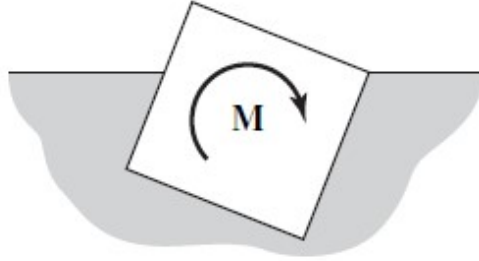
7. The figure below shows an open triangular channel in which the two sides, AB and AC, are held together by cables, spaced 1 m apart, between B and C. Determine the cable tension



8. The dam shown below is 100 m wide. Determine the magnitude and location of the force on the inclined surface.



9. The float in a toilet tank is a sphere of radius R and is made of a material with density r . An upward buoyant force F is required to shut the ballcock valve. The density of water is designated ρ_w . Develop an expression for x , the fraction of the float submerged, in terms of R, r, F, g , and ρ_w .
10. A cubical piece of wood with an edge L in length floats in the water. The specific gravity of the wood is 0.90. What moment M is required to hold the cube in the position shown? The right-hand edge of the block is flush with the water.



3 L3-L4

1. assume the motion is described by

$$\mathbf{r} = \mathbf{f}(\mathbf{c}, t) = \mathbf{g}(\mathbf{c})h(t) \quad (4)$$

find the expression of the velocity field \mathbf{v} .

2. assume the motion is

$$x = ct^2 \quad (5)$$

use the result of 1, find the expression of the velocity field v .

3. suppose in a one dimensional flow, the motion of the c th fluid element is given by

$$x = f(c, t) \quad (6)$$

and the temperature of the flow at time t is distributed as

$$T = g(x, t) \quad (7)$$

find the temperature variation rate of the c th element.

4. The velocity components in an unsteady plane flow are given by

$$u = \frac{x}{1+t} \quad (8)$$

$$v = \frac{2y}{2+t} \quad (9)$$

find the path lines and the streamlines equation subjecting to $\mathbf{x} = \mathbf{x}_0$ at $t = 0$.

5. Let a one-dimensional velocity field be $u = u(x, t)$. The density varies as $\rho = \rho_0(2 - \cos \omega t)$. Find an expression for $u(x, t)$ if $u(0, t) = U$.
6. The components of a mass flow vector $\rho \mathbf{u}$ are $\rho u = 4x^2y$, $\rho v = xyz$, $\rho w = yz^2$. Compute the net outflow through the closed surface formed by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (a) Integrate over the closed surface. (b) Integrate over the volume bounded by that surface.
7. A two-dimensional object is placed in a $2h$ -wide water tunnel as shown. The upstream velocity, v_1 , is uniform across the cross section. For the downstream velocity profile as shown, find the value of v_2

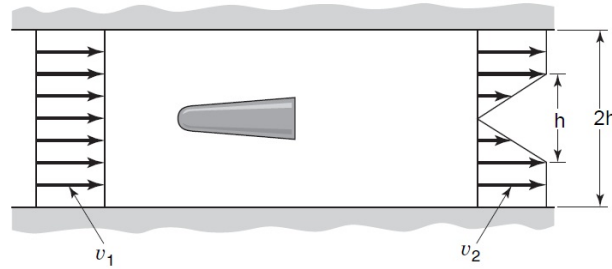


图 1

8. consider fluid in a channel of unit width and that the vertical velocity of the fluid is negligible and the horizontal velocity $u(x, t)$ is roughly constant throughout any cross section of the channel. assume the fluid is incompressible so the density ρ is constant, denote the depth of the fluid as $h(x, t)$ find the mass and momentum conserve equation of the fluid (the gravitational constant is g).
9. Given the steady two-dimensional velocity distribution

$$u = Kx, v = -Ky, w = 0 \quad (10)$$

where K is a positive constant, compute and plot the streamlines of the flow, including directions.

10. Under what conditions does the velocity field

$$V = (a_1x + b_1y + c_1z)\mathbf{i} + (a_2x + b_2y + c_2z)\mathbf{j} + (a_3x + b_3y + c_3z)\mathbf{k} \quad (11)$$

where a_1, b_1 , etc. = const, represent an incompressible flow that conserves mass?