solutions

2016.9.27

1 L1

1. scalse 1 处于分子尺度，粒子非常稀疏，体微元包含的粒子数的变化dN与体微元有关,也就是$dN = dN(dV)$,因而$\frac{dN}{dV} \neq constant$，也就是密度$\rho$在这个尺度会随体微元的不同而不同.scale 2 处于连续尺度，体元宏观无穷小，而微观无穷大，在此尺度定义密度有：1)定义于一点处，因为体元宏观无穷小，在宏观上就是一点，2)在定义点处连续，因为微观无穷大使体元包含足够多的粒子，这使得附近点之间的粒子数平均不会发生突变，因而密度在空间上是连续分布的.scale 3 处于宏观尺度,这一个尺度上的体元已经感知到了密度的宏观变化

2.

$$\frac{d\alpha}{dt} = \frac{dx/dy}{dt} = \frac{\partial u}{\partial y} \tag{1}$$

$$\frac{d\beta}{dt} = \frac{dy/dx}{dt} = \frac{\partial v}{\partial x} \tag{2}$$

so

$$\frac{d\alpha + d\beta}{dt} = \frac{d\alpha}{dt} + \frac{d\beta}{dt} \tag{3}$$

$$= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{4}$$

3.

$$\tau_x = \frac{\partial u}{\partial y} = -\frac{2U_0 y}{b^2} \tag{5}$$

$$\tau_x\big|_{y=\frac{b}{2}} = -\frac{U_0}{b} \tag{6}$$

4.

$$u = cy \tag{7}$$

$$v = cx \tag{8}$$

$$\tau_x = \mu \frac{\partial \mathbf{v}}{\partial y} = \mu \frac{\partial}{\partial y} \begin{pmatrix} u \\ v \end{pmatrix} = \mu \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{9}$$

$$\tau_y = \mu \frac{\partial \mathbf{v}}{\partial x} = \mu \frac{\partial}{\partial x} \begin{pmatrix} u \\ v \end{pmatrix} = \mu \begin{pmatrix} 0 \\ c \end{pmatrix} \tag{10}$$

(what follows is beyond the scope of the lecture) let $\mathbf{e} = \frac{\mathbf{j}+\mathbf{i}}{\sqrt{\mathbf{2}}}$,

$$\tau_{xy} = \mu \frac{\partial \mathbf{v}}{\partial e} \tag{11}$$

$$= \mu \left(\frac{\partial \mathbf{v}}{\partial x} \cos(\mathbf{e}, \mathbf{i}) + \frac{\partial \mathbf{v}}{\partial y} \cos(\mathbf{e}, \mathbf{j}) \right) \tag{12}$$

$$= \mu \left(\frac{\partial \mathbf{v}}{\partial x} \frac{1}{\sqrt{2}} + \frac{\partial \mathbf{v}}{\partial y} \frac{1}{\sqrt{2}} \right) \tag{13}$$

$$= \frac{\mu}{\sqrt{2}} \left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \begin{pmatrix} c \\ 0 \end{pmatrix} \right) \tag{14}$$

$$= \frac{c\mu}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{15}$$

5.

$$2\sigma l = \rho g h l w \tag{16}$$

$$\Rightarrow h = \frac{2\sigma}{\rho g w} \tag{17}$$

$\sigma = 0.123(1 - 0.00139T)$ so

$h = 9.37mm$

6.

$$\sigma = 0.025N/m \tag{18}$$

$$2 \cdot 2\pi r \sigma = \Delta p \pi r^2 \tag{19}$$

$$\Rightarrow \Delta p = \frac{4\sigma}{r} \tag{20}$$

$$\Rightarrow \Delta p = 50Pa \tag{21}$$

7. $\rho = 996kg/m^3, h = 1mm$

$$\pi\frac{d^2}{4}\rho gh = \sigma\pi d \tag{22}$$
$$\Rightarrow d = \frac{4\sigma}{\rho gh} = 2.989cm \tag{23}$$
$$\tag{24}$$

8.

$$t = \frac{D_c - D_r}{2} = 0.01cm \tag{25}$$

$$\begin{aligned}
F &= \tau A && (26)\\
&= \tau\pi D_r L && (27)\\
&= \mu\frac{v}{t}\pi D_r L && (28)\\
&= \frac{0.85 \times 1000 \times 3.7 \times 10^{-4} \times 0.15 \times 0.3602 \times 3.14 \times \pi}{1 \times 10^{-4}} && (29)\\
&= 1676\text{N} && (30)
\end{aligned}$$

9.

$$G = F \tag{31}$$

$$\begin{aligned}
mg &= \tau A && (32)\\
&= \mu\frac{v}{t}A && (33)\\
&= \mu\frac{v}{t}\pi D_r L && (34)
\end{aligned}$$

10.

$$\begin{aligned}
dM &= rdF && (35)\\
&= r\tau dA && (36)
\end{aligned}$$

$$\tau = \mu\frac{v}{t} = \mu\frac{\omega r}{t} \tag{37}$$
$$dA = 2\pi r \cdot rdx \tag{38}$$
$$r = x\sin\alpha \tag{39}$$

$\Rightarrow$

$$M = \int_0^{D/2} r\mu \frac{r\omega}{h} 2\pi r \frac{dr}{\sin\alpha} \tag{40}$$

$$= \frac{\pi\mu\omega D^4}{32h\sin\alpha} \tag{41}$$

2 L2

1.

$$\begin{cases} \dfrac{dp}{dr} = f = \rho a \\ a = kr \\ kR = g \end{cases} \tag{42}$$

$\Rightarrow$

$$\frac{dp}{dr} = \frac{\rho g r}{R} \tag{43}$$

$\Rightarrow$

$$p = \frac{\rho g r^2}{2R} + p_{atm} \tag{44}$$

so

$$p|_{r=R} = \frac{\rho g R}{2} + p_{atm} \tag{45}$$

$$\approx \frac{\rho g R}{2} \tag{46}$$

$$= 176kMpa \tag{47}$$

2.

$$\rho g \Delta h = \Delta p \tag{48}$$

$$\Rightarrow \Delta h = \frac{\Delta p}{\rho g} \tag{49}$$

$\Rightarrow$

$$\Delta h_{water} = 10.33m \tag{50}$$

$$\Delta h_{sea} = \Delta h_{water}/1.025 = 10.08m \tag{51}$$

$$\Delta h_{Hg} = \Delta h_{water}/13.6 = 0.76m \tag{52}$$

3. let Hg,water,oil denoted by "H, w, o" respectively

$$p_A + \rho_o g h_o + \rho_w g h_w = p_{atm} + \rho_H g h_H \tag{53}$$
$$\Rightarrow p_A - p_{atm} = g(\rho_H h_H - \rho_o h_o - \rho_w h_w) \tag{54}$$

$\Rightarrow$

$$p_A - p_{atm} = 588.6pa \tag{55}$$

4. the resultant accelaration is gravity and inertial

$$\mathbf{r} = \mathbf{g} - \mathbf{a} \tag{56}$$

now the isobars and the direction of the pressure gradient is depict as follow

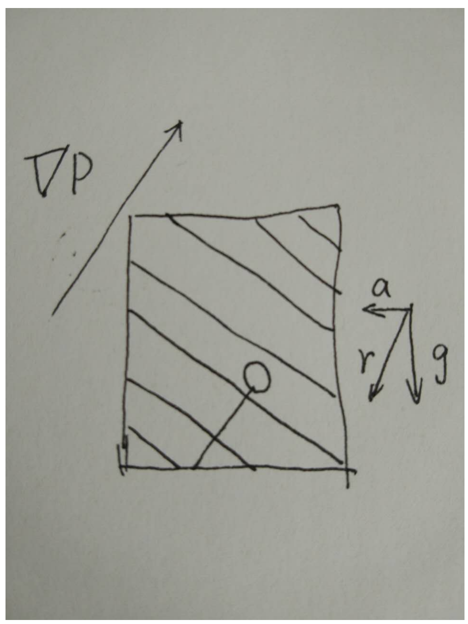


图 1: the balloon

5.

$$dF_y = p_y dA \tag{57}$$

$$\begin{cases} dA = 2\pi r' r d\theta \\ r' = r\sin\theta \\ p_y = p\cos\theta \\ p = \rho g h' \\ h' = h - r\cos\alpha + r\cos\theta \end{cases} \tag{58}$$

$\Rightarrow$

$$dF_y = \rho g(h - r\cos\alpha + r\cos\theta)\cos\theta 2\pi r\sin\theta r d\theta \tag{59}$$
$$= 2\pi\rho g r^2(h - r\cos\alpha + r\cos\theta)\sin\theta\cos\theta d\theta \tag{60}$$

$$F_y = 2\pi\rho g r^2 \int_\alpha^\pi (h - r\cos\alpha + r\cos\theta)\sin\theta\cos\theta\, d\theta \tag{61}$$
$$\tag{62}$$

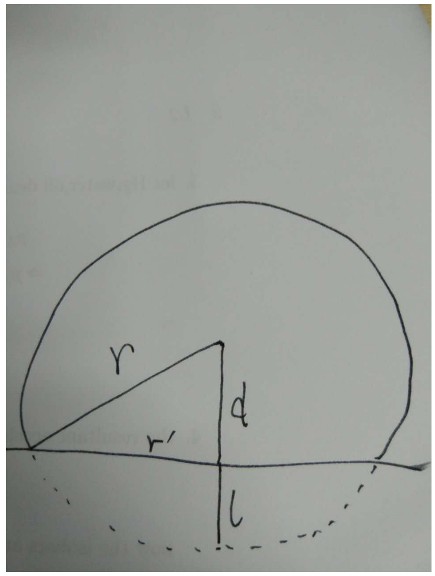
$$h = \frac{2r}{3\sin^2\alpha} + \frac{2r\cos^3\alpha}{3\sin^2\alpha} + r\cos\alpha \tag{63}$$

where $\cos\alpha = \frac{D}{2r}$

altanitive solution:

$$F_y + F_D = \rho g V \tag{64}$$

$$F_D = \rho g S_D h = \rho g \cdot \pi r'^2 \cdot h \tag{65}$$
$$r' = \frac{D}{2} \tag{66}$$
$$V = V_a - V_b \tag{67}$$
$$V_b = \frac{\pi}{3}(3r - l)l^2 \tag{68}$$
$$l = r - d \tag{69}$$
$$d = \sqrt{r^2 - r'^2} \tag{70}$$



let$F_y(h) = 0$, comes the requiring

$$\begin{aligned}
h &= V/S_D && (71)\\
&= \frac{V_a - V_b}{\pi r'^2} && (72)\\
&= \frac{\frac{4}{3}\pi r^3 - \frac{\pi}{3}(3r-l)l^2}{\pi r'^2} && (73)\\
&= \frac{4r^3 - (2r+d)(r-d)^2}{3r'^2} && (74)\\
&= \frac{4}{3}\frac{4r^3 - (2r-d)(r-d)^2}{D^2} && (75)
\end{aligned}$$

where $d = \sqrt{r^2 - (\frac{D}{2})^2}$

6.

$$\begin{cases} F = \rho g h A \\ A = \pi r^2 \end{cases} \tag{76}$$

and the acting point is

$$\Delta y_a = \frac{I_{xx}}{hA} = \frac{\pi r^2}{4hA} \tag{77}$$

the momentum equilibrium

$$F\Delta y_a = Pr \tag{78}$$

$\Rightarrow$

$$P = \frac{F\Delta y_a}{r} \tag{79}$$
$$= \frac{\rho g \pi r}{4} \tag{80}$$
$$= 7.7kN \tag{81}$$

7.

$$F_p = \rho g \bar{h} A \tag{82}$$
$$A = lw \tag{83}$$

$$\Delta y_a = \frac{I_{xx}}{y_c A} \tag{84}$$
$$I_{xx} = \frac{l^3 w}{12} \tag{85}$$

$$\tau w l_c = F_p \Delta y_a \tag{86}$$

$\Rightarrow$

$$\tau = \frac{F_p \Delta y_a}{w l_c} \tag{87}$$
$$= 145.2kN \tag{88}$$

8.

$$F = \rho g \bar{h} A = 10^{10} N \tag{89}$$
$$y_a = \frac{2}{3} h = 85.3m \tag{90}$$

9.

$$F_{up} = F_{down}$$
$$F + \rho_w g x \left(\frac{4}{3}\pi R^3\right) = rg\left(\frac{4}{3}\pi R^3\right)$$
$$x = \frac{rg\left(\frac{4}{3}\pi R^3\right) - F}{\rho_w g\left(\frac{4}{3}\pi R^3\right)}$$

10.

$$\rho_s v_s = \rho_l v_l \tag{91}$$
$$\Rightarrow \quad \frac{v_l}{v_s} = \frac{\rho_s}{\rho_l} \tag{92}$$
$$\tag{93}$$

$$S = v_s - v_l \tag{94}$$
$$\Rightarrow \quad \frac{1}{2}LL\tan\theta = v_s - v_l = 0.1L^2 \tag{95}$$
$$\Rightarrow \quad \tan\theta = 0.2 \tag{96}$$

solution 1:

$$\cos\theta = \frac{1}{\sqrt{1+\tan\theta^2}} = 0.9615 \tag{97}$$

$$h_1 = 0.5 \cdot 1 \cdot \cos\theta = 0.49 \tag{98}$$

$$h_2 = 0.5 \cdot 0.8 \cdot \cos\theta = 0.39 \tag{99}$$

$$h_3 = 0.5 \cdot (1+0.8) \cdot \cos\theta = 0.88 \tag{100}$$

$$\rho = 1000 \tag{101}$$

$$g = 9.8 \tag{102}$$

$$A_1 = 1, A_2 = 0.8, A_3 = 1 \tag{103}$$

$$F_1 = \rho g h_1 A_1 = 4804.8 \tag{104}$$

$$F_2 = \rho g h_2 A_2 = 3075.1 \tag{105}$$

$$F_3 = \rho g h_3 A_3 = 8648.7 \tag{106}$$

$$l_1 = \frac{1}{2} - \frac{1}{3} \tag{107}$$

$$l_2 = \frac{1}{2} - 0.8 \cdot \frac{1}{3} \tag{108}$$

$$I_3 = \frac{1}{12} \tag{109}$$

$$y_3 = \frac{1+0.8}{2\tan\theta} \tag{110}$$

$$l_3 = \frac{I_3}{y_3 A_3} \tag{111}$$

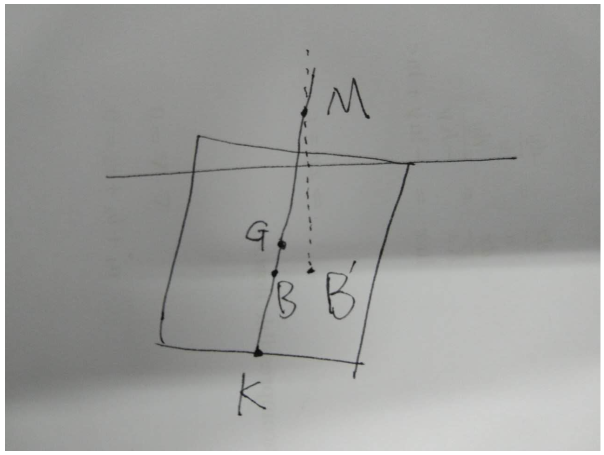
$$M_1 = F_1 l_1 = 800.8 \tag{112}$$

$$M_2 = F_2 l_2 = 717.52 \tag{113}$$

$$M_3 = F_3 l_3 = 160.16 \tag{114}$$

$$M = M_2 + M_3 - M_1 = 76.88 \tag{115}$$

solution 2:



$$KB = 0.9/2 \tag{116}$$
$$KG = 0.5 \tag{117}$$
$$I_{oy} = \frac{1}{12} \tag{118}$$
$$V_s = 0.9 \tag{119}$$
$$BM = \frac{I_{oy}}{V_s} \tag{120}$$
$$GM = KB + BM - KG \tag{121}$$
$$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan\theta^2}} \tag{122}$$
$$l = GM\sin\theta \tag{123}$$
$$\rho_c = 900 \tag{124}$$
$$M = Gl = \rho_c gVl = 73.67 \tag{125}$$

note: the final result of the 2 solution are different ,the reason is still remaind to be solved, you are welcomed to contribute your wisdom to find out the result

3 L3-L4

1. since

$$\mathbf{r} = \mathbf{f}(\mathbf{c}, t) = \mathbf{g}(\mathbf{c})h(t) \tag{126}$$

$\Rightarrow$

$$\mathbf{g}(\mathbf{c}) = \frac{\mathbf{r}}{h(t)} \tag{127}$$

so

$$\begin{aligned} \mathbf{v} &= \frac{\partial \mathbf{r}}{\partial t} &(128)\\ &= \mathbf{g}(\mathbf{c})\dot{h}(t) &(129)\\ &= \frac{\mathbf{r}}{h(t)}\dot{h}(t) &(130) \end{aligned}$$

alternative:

$$\mathbf{c} = \mathbf{g}^{-1}(\frac{\mathbf{r}}{h(t)}) \tag{131}$$

$$\begin{aligned} \mathbf{v} &= \frac{\partial \mathbf{r}}{\partial t} &(132)\\ &= \mathbf{g}(\mathbf{c})\dot{h}(t) &(133)\\ &= \mathbf{g}(\mathbf{g^{-1}}(\frac{\mathbf{r}}{\mathbf{h(t)}}))\dot{h}(t) &(134) \end{aligned}$$

2.

$$\begin{aligned} g(c) &= c &(135)\\ h(t) &= t^2 &(136) \end{aligned}$$

use the result of eq.130

$$\begin{aligned} v &= \frac{x}{h(t)}\dot{h}(t) &(137)\\ &= 2t\frac{x}{t^2} &(138)\\ &= 2\frac{x}{t} &(139)\\ & &(140) \end{aligned}$$

alternative:
and it is easy to find

$$g^{-1}(c) = c \tag{141}$$

$$
\begin{aligned}
v &= g(g^{-1}(\frac{x}{h(t)}))\dot{h}(t) && (142)\\
&= g^{-1}(\frac{x}{h(t)})\dot{h}(t) && (143)\\
&= \frac{x}{h(t)}\dot{h}(t) && (144)\\
&= 2t\frac{x}{t^2} && (145)\\
&= 2\frac{x}{t} && (146)
\end{aligned}
$$

3. at time t the cth element is at $f(c,t)$, so the temperature of the cth element is

$$T = g(f(c,t),t) \tag{147}$$

so the variation rate is

$$\frac{dT}{dt} = \frac{\partial g}{\partial x}\frac{df}{dt} + \frac{\partial g}{\partial t} \tag{148}$$

4.

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{x}{1+t} \\ \dfrac{dy}{dt} = \dfrac{2y}{2+t} \end{cases} \tag{149}$$

$\Rightarrow$

$$\begin{cases} x = c_x(1+t) \\ y = c_y(2+t)^2 \end{cases} \tag{150}$$

with the boundary condition

$$\mathbf{x}(0) = \mathbf{x}_0 \tag{151}$$

the path line through $\mathbf{x}_0$ is

$$\begin{cases} x = x_0(1+t) \\ y = \dfrac{y_0}{4}(2+t)^2 \end{cases} \tag{152}$$

and the streamlines at $t = 0$ can be obtained by let $t = 0$

$$\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = y \end{cases} \tag{153}$$

$\Rightarrow$

$$y = cx \tag{154}$$

5.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{155}$$

since

$$\rho = \rho_0(2 - \cos\omega t) \tag{156}$$

$\Rightarrow$

$$\frac{\partial u}{\partial x} = -\frac{\partial \rho}{\rho \partial t} \tag{157}$$

$$= \frac{\omega \sin\omega t}{\cos\omega t - 2} \tag{158}$$

$$:= f(t) \tag{159}$$

$\Rightarrow$

$$u = \int f(t)dx \tag{160}$$

$$= f(t)x + C \tag{161}$$

apply the boundary condition$u(0,t) = U$

$$u = f(t)x + U \tag{162}$$

6. the face integrals and the volume integrals of divergence is showed below

(a)
$$\int_{A=\partial V} \rho \mathbf{u} \cdot d\mathbf{A} = \int_0^1 dy \int_0^1 dz\, 4x^2 y\Big|_{x=1} - \int_0^1 dy \int_0^1 dz\, 4x^2 y\Big|_{x=0} + \int_0^1 dz \int_0^1 dx\, xyz\Big|_{y=1} - \int_0^1 dz \int_0^1 dx\, xyz\Big|_{y=0} + \int_0^1 dx \int_0^1 dy\, yz^2\Big|_{z=1} - \int_0^1 dx \int_0^1 dy\, yz^2\Big|_{z=0}$$
$$= 2 + 0 + \tfrac{1}{4} + 0 + \tfrac{1}{2} + 0 = \tfrac{11}{4}.$$

(b) $\nabla \cdot \mathbf{u} = \partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 8xy + xz + 2yz$

$$\int_V \nabla \cdot \mathbf{u}\, dV = \int_0^1 dx \int_0^1 dy \int_0^1 dz\, (8xy + xz + 2yz)$$
$$= 8 \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{1}{2} + 2 \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} = \tfrac{11}{4}.$$

7. mass inlet=mass outlet

$$v_1 \cdot 2h = v_2 \cdot 2h - \frac{h \cdot v_2}{2} \tag{163}$$
$$\Rightarrow v_1 = 0.75 v_2 \tag{164}$$

8. mass conservation

$$\frac{\partial(\rho \delta v)}{\partial t} + \frac{\partial(\rho u h)}{\partial x}\delta x = 0 \tag{165}$$
$$\Rightarrow \frac{\partial(\rho h \delta x)}{\partial t} + \frac{\partial(\rho u h)}{\partial x}\delta x = 0 \tag{166}$$
$$\Rightarrow \frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = 0 \tag{167}$$

momentum conservation

$$\frac{\partial(\rho u \delta v)}{\partial t} + \frac{\partial(\rho u^2 h + p)}{\partial x}\delta x = 0 \tag{168}$$
$$\Rightarrow \frac{\partial(\rho u h)}{\partial t} + \frac{\partial(\rho u^2 h + p)}{\partial x} = 0 \tag{169}$$

the total force p due to pressure is

$$p = \int_0^h \rho g y dy = \frac{1}{2}\rho g h^2 \tag{170}$$

so the momentum equation is

$$\frac{\partial(\rho uh)}{\partial t}+\frac{\partial(\rho u^2h+\frac{1}{2}\rho gh)}{\partial x}=0 \tag{171}$$

$$\Rightarrow\frac{\partial(uh)}{\partial t}+\frac{\partial(u^2h+\frac{1}{2}gh)}{\partial x}=0 \tag{172}$$

9. for stream line

$$\begin{aligned}\frac{dx}{u} &= \frac{dy}{v} &(173)\\ \frac{dx}{Kx} &= \frac{dy}{-Ky} &(174)\\ \ln x &= -\ln y+\ln c &(175)\\ & &(176)\end{aligned}$$

$$\Rightarrow xy=c$$

10. it is a incompressible flow,so

$$\nabla\cdot\mathbf{V}=0 \tag{177}$$

$$a_1+b_2+c_3=0 \tag{178}$$

4 L5-L6

1. (a) Steady flow
   Incompressible flow
   Frictionless (inviscid) flow
   Flow along a streamline
   No shaft work
   No heat transfer
   （理想不可压缩流体，定常流动，只有重力作用，沿流线，无其他能量输入输出）
   (b) Venturi effect—measure the flow rate of liquids, disperse perfume or spray paint
   Pour out beer

Arteriosclerosis and vascular flutter

Why do rabbits not suffocate in the burrows ?

Why does a house lose its roof in strong wind ?

...

2. (a) Viscous effect:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 + h_f$$

(b) Pump work:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 - h_s$$

(c) Compressible fluid:

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ \frac{p_1}{\rho_1^\gamma} &= \frac{p_2}{\rho_2^\gamma} \\ \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}v_1^2 + g z_1 &= \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{1}{2}v_2^2 + g z_2 \end{aligned}$$

3. 根据虹吸管速度公式，流出虹吸管水流的速度为

$$\begin{aligned} V &= \sqrt{2gL} \\ &= \sqrt{2 \times 9.81 \times 3.0} \\ &= 7.67 \text{ m/s} \end{aligned}$$

因$Q = AV = \frac{\pi}{4}d^2V$，故

$$\begin{aligned} d &= \sqrt{\frac{4Q}{\pi V}} \\ &= \sqrt{\frac{4 \times 100/3600}{3.142 \times 7.67}} \\ &= 0.0679 \text{ m} \end{aligned}$$

最高截面处压强为

$$\begin{aligned} p_2 &= p_a - \rho g z \\ &= 1.0133 \times 10^5 - 998.2 \times 9.806 \times 6.0 \\ &= 42600 \text{ Pa} \end{aligned}$$

可见最高截面处的压强远大于该温度下的饱和蒸气压，水流不会在最高截面处产生气穴，虹吸管可以正常吸水。

4. (a) For C.V.1

$$\frac{v_1^2}{2}+\frac{p_1}{\rho}+gz_1=\frac{v_2^2}{2}+\frac{p_2}{\rho}+gz_2$$
$$v_1=0 \qquad z_1=z_2$$
$$v_2=\sqrt{\frac{2(p_1-p_2)}{\rho}}=20\ \mathrm{m/s}$$
$$Q=Av_2=\frac{\pi}{4}\times 0.3^2\times 20=1.417\ \mathrm{m^3/s}$$

(b) For C.V.2

$$\frac{v_1^2}{2}+\frac{p_1}{\rho}+gz_1+gh_s=\frac{v_2^2}{2}+\frac{p_2}{\rho}+gz_2$$
$$v_1=0 \qquad p_1=p_2 \qquad z_1=z_2$$
$$gh_s=\frac{v_2^2}{2}$$
$$P_f=\rho gQh_s=\rho Q\frac{v_2^2}{2}=1.22\times 1.417\times\frac{20^2}{2}=346\ \mathrm{W}$$

5.

$$u_1+gz_1=u_2+gz_2$$
$$\Delta u=u_2-u_1=g(z_1-z_2)=gh$$
$$\Delta T=\frac{\Delta u}{c_p}=\frac{gh}{c_p}=\frac{9.8\times 165}{4184}=0.39\ \mathrm{K}$$

5 L7-L8

1. (a)

$$\bar{u}=\frac{1}{h}\int_0^h u dy \tag{179}$$
$$=-\frac{\Delta p h^2}{12\mu L} \tag{180}$$

$$Q = \bar{u}A \tag{181}$$
$$= \bar{u}h \tag{182}$$
$$= -\frac{\Delta p h^3}{12\mu L} \tag{183}$$
$$\tag{184}$$

(b)

$$F_d = \tau \cdot 2L \tag{185}$$
$$\tau = \mu \frac{\partial u}{\partial y}\bigg|_y = -\frac{\Delta p h}{2L} = \frac{12\mu\bar{u}}{h^2} \cdot \frac{h}{2L} = \frac{6\mu\bar{u}}{h} \tag{186}$$
$$C_d = \frac{F_d}{\rho\bar{u}^2 L} = \frac{6\mu\bar{u} \cdot 2L}{h}\frac{1}{\rho\bar{u}^2 L} = \frac{18}{Re} \tag{187}$$

2.

$$\nabla \cdot \mathbf{u} = 0 \tag{188}$$
$$\frac{1}{Re}\nabla^2\mathbf{u} = \nabla p \tag{189}$$

3.

$$\frac{4}{3}\pi r^3 g(\rho' - \rho) = 6\pi\mu r U \tag{190}$$

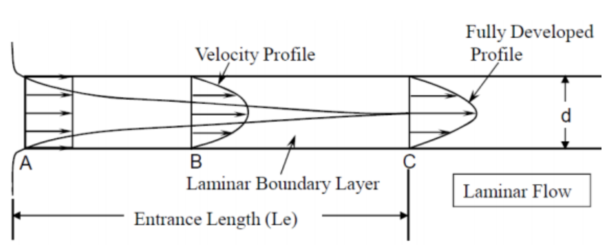
$\Rightarrow$

$$U = \frac{2r^2 g(\rho' - \rho)}{9\mu} \tag{191}$$

4.

$$l = 0.03 d Re \tag{192}$$

where $Re = \frac{\rho U d}{\mu}$



5.

$$Re = \frac{4\rho Q}{\pi\mu d} = 10^3, 10^4, 10^7, 10^8 \tag{193}$$

查看穆迪图(moody diagram)得,阻力系数

$$f = 0.065, 0.032, 0.012, 0.012 \tag{194}$$

又由

$$\bar{u} = \frac{4Q}{\pi d^2} = \frac{Re\pi\mu}{\rho d} \tag{195}$$

所以阻力

$$F = f \cdot \frac{1}{2}\rho\bar{u}^2 L \tag{196}$$

$$= fRe^2\frac{\rho L}{2}(\frac{\pi\mu}{\rho d})^2 \tag{197}$$

$$= (0.065 \times 10^6, 0.032 \times 10^8, 0.012 \times 10^{14}, 0.012 \times 10^{16})\frac{\rho L}{2}(\frac{\pi\mu}{\rho d})^2 \tag{198}$$