

# transportation problems

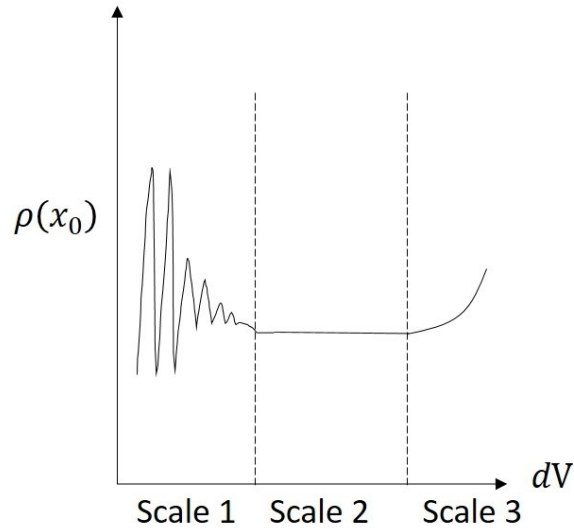
2016.9.27

## 1 L1

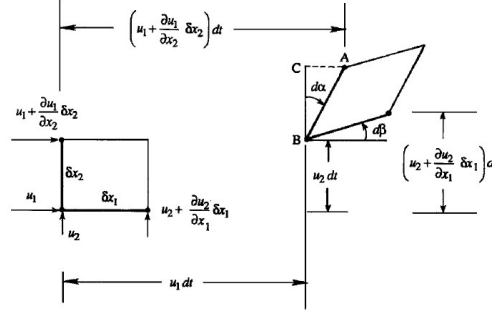
1. In a particle system, the mass density  $\rho$  is defined at some point  $x_0$ ,

$$\rho(x_0) = \frac{m_0 dN}{dV} \quad (1)$$

Where  $m_0$  is the mass of a particle,  $dV$  is the volume of the element that contains  $x_0$ ,  $dN$  is the particle number in the element. The figure below shows how  $\rho(x_0)$  varies at different scales of  $dV$ , explain it respectively.



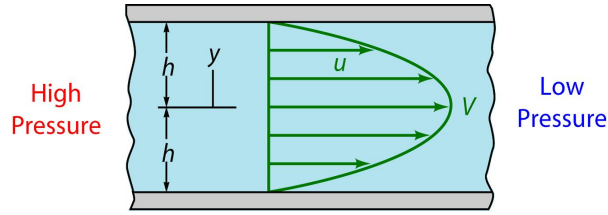
2. Figure below shows the position of an element with sides parallel to the coordinate axes at time  $t$ , and its subsequent position at  $t+dt$ . express the strain rate  $((d\alpha + d\beta)/dt)$  with  $u_1, u_2, x_1, x_2$



3. Consider the viscous flow in a channel of width  $2b$ . The channel is aligned in the  $x$  direction, and the velocity at a distance  $y$  from the centerline is given by the parabolic distribution

$$u(y) = U_0 \left(1 - \frac{y^2}{b^2}\right) \quad (2)$$

In terms of the viscosity  $\mu$ , calculate the shear stress at a distance of  $y = b/2$ .

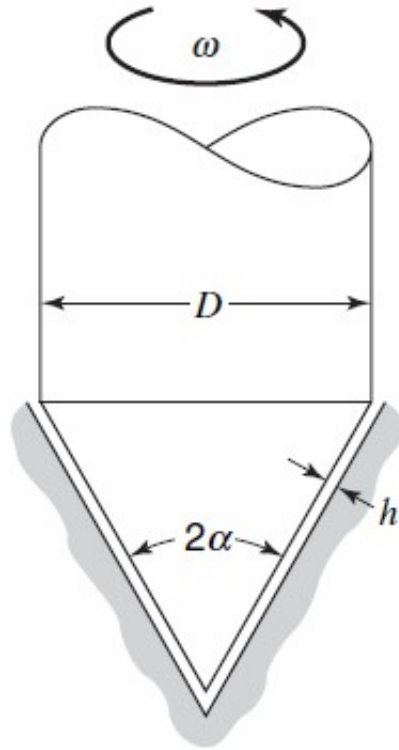


4. Given a velocity field  $v = \begin{pmatrix} u \\ v \end{pmatrix}$  such that

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (3)$$

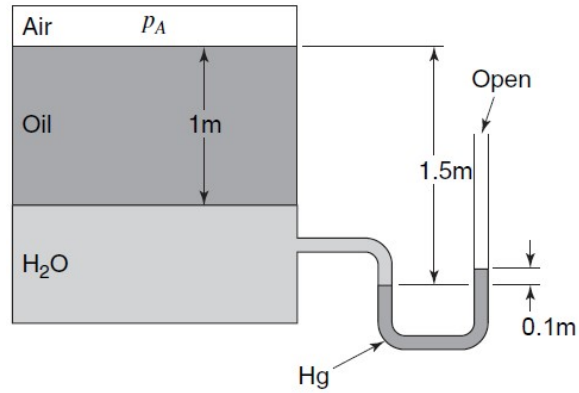
Determine the shear stress  $\tau$  along  $i, j, i + j$  direction (denote them  $\tau_x, \tau_y, \tau_{xy}$  respectively)

5. Two clean and parallel glass plates, separated by a gap of 1.625mm, are dipped in water. If Coefficient of surface tension  $\sigma=0.0735\text{N/m}$ , determine how high the water will rise.
6. Determine the difference in pressure between the inside and outside of a soap film bubble at  $20^\circ\text{C}$  if the diameter of the bubble is 4 mm.
7. Determine the diameter of the glass tube necessary to keep the capillary-height change of water at  $30^\circ\text{C}$  less than 1 mm.
8. An auto lift consists of 36.02-cm-diameter ram that slides in a 36.04-cm-diameter cylinder. The annular region is filled with oil having a kinematic viscosity of  $0.00037\text{m}^2/\text{s}$  and a specific gravity of 0.85. If the rate of travel of the ram is 0.15 m/s, estimate the frictional resistance when 3.14 m of the ram is engaged in the cylinder.
9. If the ram and auto rack in the previous problem together have a mass of 680 kg, estimate the maximum sinking speed of the ram and rack when gravity and viscous friction are the only forces acting. Assume 2.44 m of the ram engaged.
10. The conical pivot shown in the figure has angular velocity  $\omega$  and rests on an oil film of uniform thickness  $h$ . Determine the frictional moment as a function of the angle  $a$ , the viscosity, the angular velocity, the gap distance, and the shaft diameter.

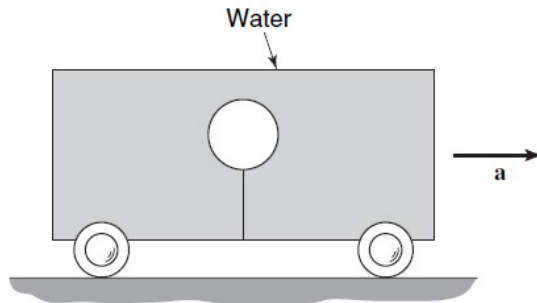


## 2 L2

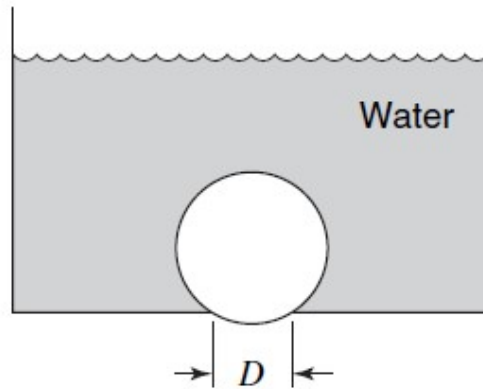
1. Matter is attracted to the center of Earth with a force proportional to the radial distance from the center. Using the known value of  $g$  at the surface where the radius is 6330 km, compute the pressure at Earth's center, assuming the material behaves like a liquid, and that the mean specific gravity is 5.67.
2. Determine the depth change to cause a pressure increase of 1 atm for (a) water, (b) sea water (specific gravity=1.0250), and (c) mercury (SG = 13.6).
3. What is the pressure  $p_A$  in the figure? The specific gravity of the oil is 0.8.



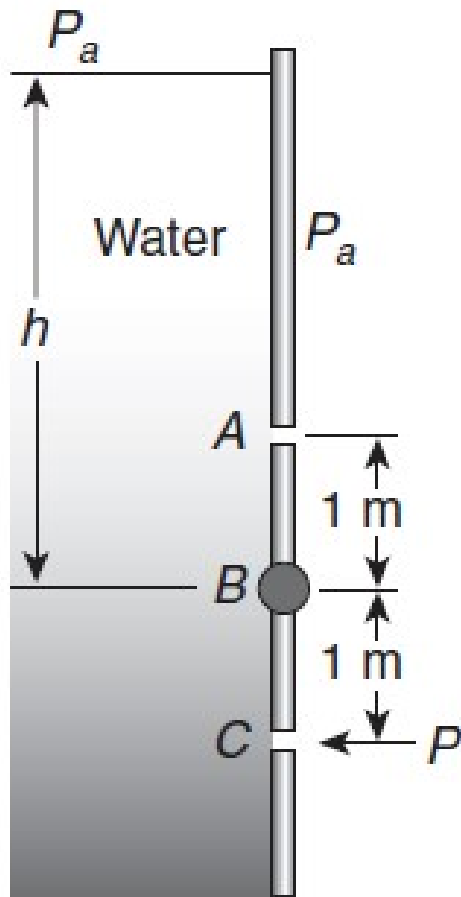
4. The car shown in the figure is accelerated to the right at a uniform rate. What way will the balloon (tied by the string) move relative to the car?



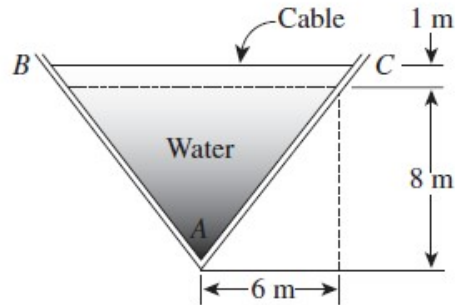
5. It is desired to use a 0.75-m diameter beach ball to stop a drain in a swimming pool. Obtain an expression that relates the drain diameter  $D$  and the minimum water depth  $h$  for which the ball will remain in place.



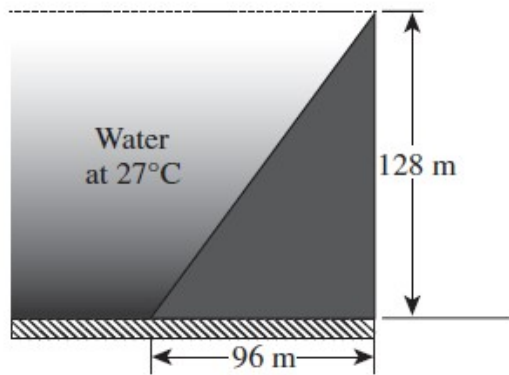
6. The circular gate ABC has a 1m radius and is hinged at B. Neglecting atmospheric pressure, determine the force  $P$  just sufficient to keep the gate from opening when  $h = 12\text{m}$ .



7. The figure below shows an open triangular channel in which the two sides, AB and AC, are held together by cables, spaced 1 m apart, between B and C. Determine the cable tension

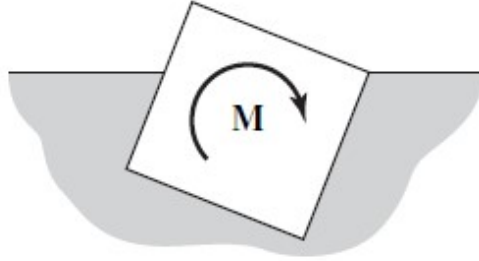


8. The dam shown below is  $100\text{ m}$  wide. Determine the magnitude and location of the force on the inclined surface.



9. The float in a toilet tank is a sphere of radius  $R$  and is made of a material with density  $r$ . An upward buoyant force  $F$  is required to shut the ballcock valve. The density of water is designated  $\rho_w$ . Develop an expression for  $x$ , the fraction of the float submerged, in terms of  $R, r, F, g$ , and  $\rho_w$ .
10. A cubical piece of wood with an edge  $L$  in length floats in the water. The specific gravity of the wood is  $0.90$ . What moment  $M$  is required to hold the cube in the position shown? The right-hand edge of the block is flush with the water.





### 3 L3-L4

1. assume the motion is described by

$$\mathbf{r} = \mathbf{f}(\mathbf{c}, t) = \mathbf{g}(\mathbf{c})h(t) \quad (4)$$

find the expression of the velocity field  $\mathbf{v}$ .

2. assume the motion is

$$x = ct^2 \quad (5)$$

use the result of 1, find the expression of the velocity field  $v$ .

3. suppose in a one dimensional flow, the motion of the  $c$ th fluid element is given by

$$x = f(c, t) \quad (6)$$

and the temperature of the flow at time  $t$  is distributed as

$$T = g(x, t) \quad (7)$$

find the temperature variation rate of the  $c$ th element.

4. The velocity components in an unsteady plane flow are given by

$$u = \frac{x}{1+t} \quad (8)$$

$$v = \frac{2y}{2+t} \quad (9)$$

find the path lines and the streamlines equation subjecting to  $\mathbf{x} = \mathbf{x}_0$  at  $t = 0$ .

5. Let a one-dimensional velocity field be  $u = u(x, t)$ . The density varies as  $\rho = \rho_0(2 - \cos \omega t)$ . Find an expression for  $u(x, t)$  if  $u(0, t) = U$ .
6. The components of a mass flow vector  $\rho \mathbf{u}$  are  $\rho u = 4x^2y$ ,  $\rho v = xyz$ ,  $\rho w = yz^2$ . Compute the net outflow through the closed surface formed by the planes  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ . (a) Integrate over the closed surface. (b) Integrate over the volume bounded by that surface.
7. A two-dimensional object is placed in a  $2h$ -wide water tunnel as shown. The upstream velocity,  $v_1$ , is uniform across the cross section. For the downstream velocity profile as shown, find the value of  $v_2$

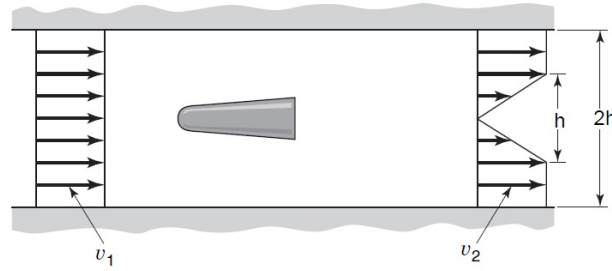


图 1

8. consider fluid in a channel of unit width and that the vertical velocity of the fluid is negligible and the horizontal velocity  $u(x, t)$  is roughly constant throughout any cross section of the channel. assume the fluid is incompressible so the density  $\rho$  is constant, denote the depth of the fluid as  $h(x, t)$  find the mass and momentum conserve equation of the fluid (the gravitational constant is  $g$ ).
9. Given the steady two-dimensional velocity distribution

$$u = Kx, v = -Ky, w = 0 \quad (10)$$

where  $K$  is a positive constant, compute and plot the streamlines of the flow, including directions.

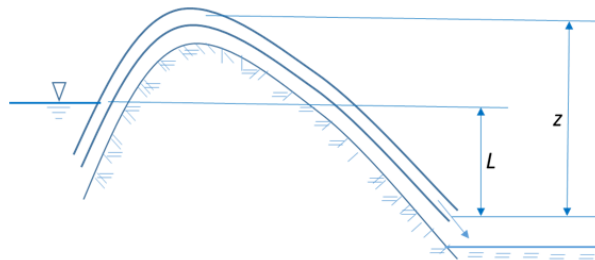
10. Under what conditions does the velocity field

$$\mathbf{V} = (a_1x + b_1y + c_1z)\mathbf{i} + (a_2x + b_2y + c_2z)\mathbf{j} + (a_3x + b_3y + c_3z)\mathbf{k} \quad (11)$$

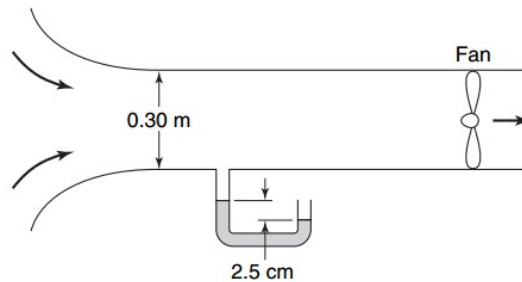
where  $a_1, b_1$ , etc. = const, represent an incompressible flow that conserves mass?

## 4 L3-L4

- Bernoulli Equation may be interpreted physically to mean that the total mechanical energy is conserved for a control volume:
  - What conditions is this relation based upon?
  - Please list a few applications of Bernoulli Equation in life.
- In some cases, the use of Bernoulli Equation is invalid. Then how should we extend Bernoulli Equation for these cases?
- The siphon can transport the water from the reservoir to irrigation canals. Assume the vertical height  $z$  from the highest point to the exit point of the siphon is 6 m and the exit point is 3 m lower than water surface of the reservoir, as shown in figure below. If we need  $100 \text{ m}^3$  of water per hour, try to determine the diameter of the siphon. If the temperature of water is  $20^\circ\text{C}$  (the saturated vapor pressures of water is 2340 Pa accordingly), and the local atmospheric pressure is  $1.0133 \times 10^5 \text{ Pa}$ , determine whether the siphon can transport water normally.



4. A fan draws air from the atmosphere through a 0.30-m-diameter round duct that has a smoothly rounded entrance. A differential manometer connected to an opening in the wall of the duct shows a vacuum pressure of 2.5 cm of water. The density of air is  $1.22 \text{ kg/m}^3$ . Determine the volume rate of air flow in the duct in cubic feet per second. What is the horsepower output of the fan?



5. Multnomah Falls in Oregon has sheer drop of 165 m. Estimate the change in water temperature caused by this drop. [The total energy (the sum of mechanical energy and internal energy) is conserved.]
6. The Stokes number,  $St$ , used in particle-dynamics studies, is a dimensionless combination of five variables: acceleration of gravity  $g$ , viscosity  $\mu$ , density  $\rho$ , particle velocity  $U$ , and particle diameter  $D$ . (a) If  $St$  is proportional to  $\mu$  and inversely proportional to  $g$ , find its form. (b) Show that  $St$  is actually the quotient of two more traditional dimensionless groups.
7. A pendulum has an oscillation period  $T$  which is assumed to depend upon its length  $L$ , bob mass  $m$ , angle of swing  $\theta$ , and the acceleration of gravity. A pendulum 1 m long, with a bob mass of 200g, is tested on earth and found to have a period of 2.04s when swinging at  $20^\circ$ . (a) What is its period when it swings at  $45^\circ$ ? A similarly constructed pendulum, with  $L = 30 \text{ cm}$  and  $m = 100 \text{ g}$ , is to swing on the moon ( $g = 1.62 \text{ m/s}^2$ ) at  $\theta = 20^\circ$ . (b) What will be its period?
8. Use dimensional analysis to determine the energy  $E$  released in an intense point blast if the blastwave propagation distance  $D$  into an

undisturbed atmosphere of density  $\rho$  is known as a function of time  $t$  following the energy release.

9. To good approximation, the thermal conductivity  $k$  of a gas depends only on the density  $\rho$ , mean free path  $l$ , gas constant  $R$ , and absolute temperature  $T$ . For air at  $20^\circ \text{ C}$  and 1 atm,  $k \approx 0.026 \text{ W/m K}$  and  $k \approx 6.5 \text{ E-8 m}$ . Use this information to determine  $k$  for hydrogen at  $20^\circ \text{ C}$  and 1 atm if  $l \approx 1.2 \text{ E-7 m}$ .
10. A student needs to measure the drag on a prototype of characteristic length  $d_p$  moving at velocity  $U_p$  in air at sea-level conditions. He constructs a model of characteristic length  $d_m$ , such that the ratio  $d_p/d_m = a$  factor  $f$ . He then measures the model drag under dynamically similar conditions, in sea-level air. The student claims that the drag force on the prototype will be identical to that of the model. Is this claim correct? Explain.