

# transportation problems

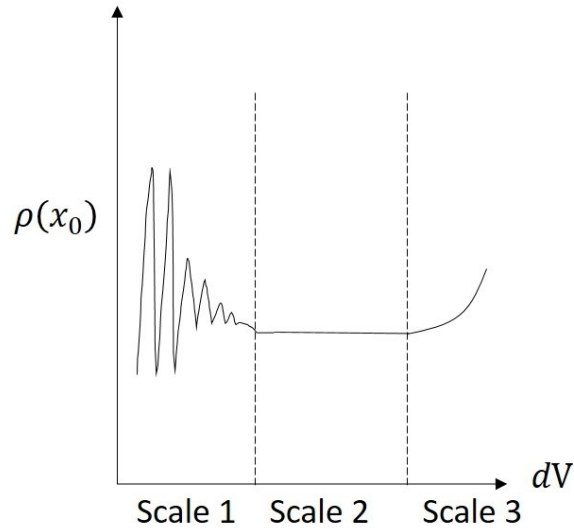
2016.9.27

## 1 L1

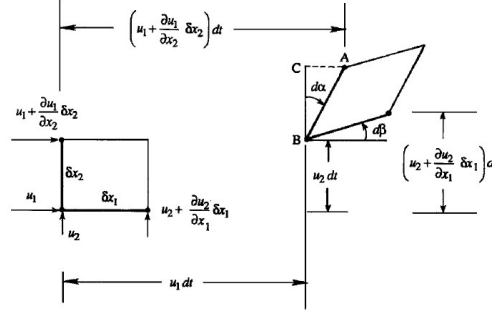
1. In a particle system, the mass density  $\rho$  is defined at some point  $x_0$  as,

$$\rho(x_0) = \frac{m_0 dN}{dV} \quad (1)$$

Where  $m_0$  is the mass of a particle,  $dV$  is the volume of the element that contains  $x_0$ ,  $dN$  is the particle number in the element. The figure below shows how  $\rho(x_0)$  varies at different scales of  $dV$ , explain it respectively.



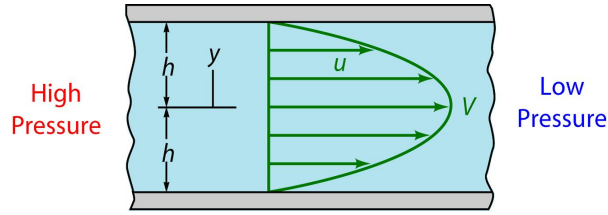
2. Figure below shows the position of an element with sides parallel to the coordinate axes at time  $t$ , and its subsequent position at  $t+dt$ . express the strain rate  $((d\alpha + d\beta)/dt)$  with  $u_1, u_2, x_1, x_2$



3. Consider the viscous flow in a channel of width  $2b$ . The channel is aligned in the  $x$  direction, and the velocity at a distance  $y$  from the centerline is given by the parabolic distribution

$$u(y) = U_0 \left(1 - \frac{y^2}{b^2}\right) \quad (2)$$

In terms of the viscosity  $\mu$ , calculate the shear stress at a distance of  $y = b/2$ .

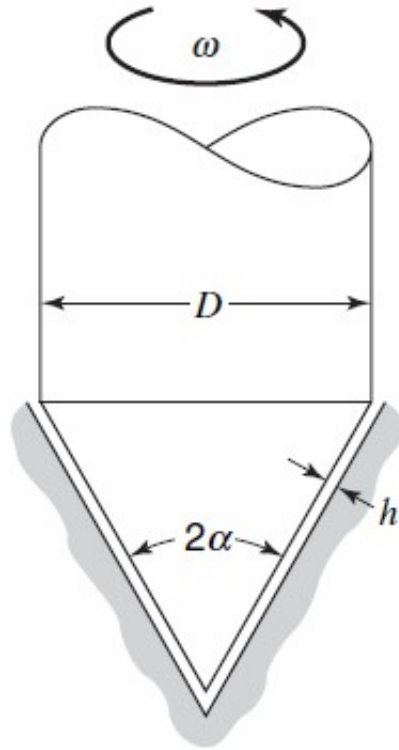


4. Given a velocity field  $v = \begin{pmatrix} u \\ v \end{pmatrix}$  such that

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (3)$$

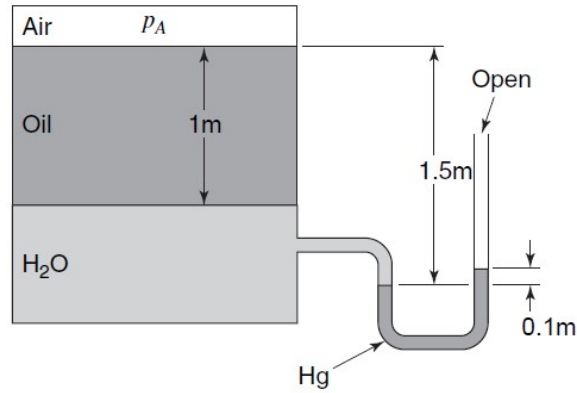
Determine the shear stress  $\tau$  along  $i, j, i + j$  direction (denote them  $\tau_x, \tau_y, \tau_{xy}$  respectively)

5. Two clean and parallel glass plates, separated by a gap of 1.625mm, are dipped in water. If Coefficient of surface tension  $\sigma=0.0735\text{N/m}$ , determine how high the water will rise.
6. Determine the difference in pressure between the inside and outside of a soap film bubble at  $20^\circ\text{C}$  if the diameter of the bubble is 4 mm.
7. Determine the diameter of the glass tube necessary to keep the capillary-height change of water at  $30^\circ\text{C}$  less than 1 mm.
8. An auto lift consists of 36.02-cm-diameter ram that slides in a 36.04-cm-diameter cylinder. The annular region is filled with oil having a kinematic viscosity of  $0.00037\text{m}^2/\text{s}$  and a specific gravity of 0.85. If the rate of travel of the ram is 0.15 m/s, estimate the frictional resistance when 3.14 m of the ram is engaged in the cylinder.
9. 9. If the ram and auto rack in the previous problem together have a mass of 680 kg, estimate the maximum sinking speed of the ram and rack when gravity and viscous friction are the only forces acting. Assume 2.44 m of the ram engaged.
10. The conical pivot shown in the figure has angular velocity  $\omega$  and rests on an oil film of uniform thickness  $h$ . Determine the frictional moment as a function of the angle  $a$ , the viscosity, the angular velocity, the gap distance, and the shaft diameter.

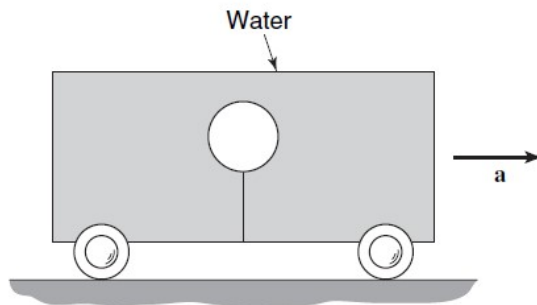


## 2 L2

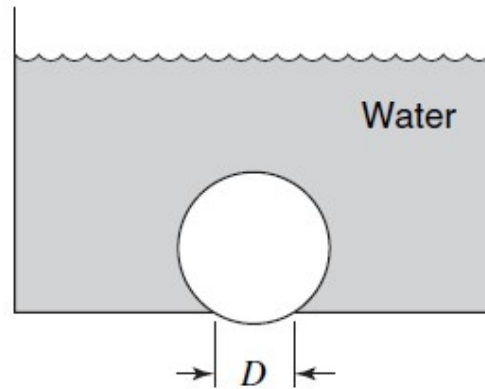
1. Matter is attracted to the center of Earth with a force proportional to the radial distance from the center. Using the known value of  $g$  at the surface where the radius is 6330 km, compute the pressure at Earth's center, assuming the material behaves like a liquid, and that the mean specific gravity is 5.67.
2. Determine the depth change to cause a pressure increase of 1 atm for (a) water, (b) sea water (specific gravity=1.0250), and (c) mercury (SG = 13.6).
3. What is the pressure  $p_A$  in the figure? The specific gravity of the oil is 0.8.



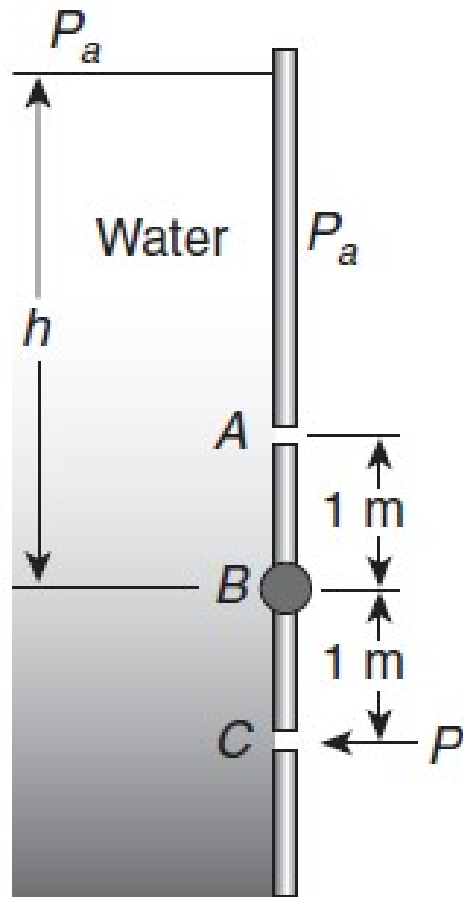
4. The car shown in the figure is accelerated to the right at a uniform rate. What way will the balloon (tied by the string) move relative to the car?



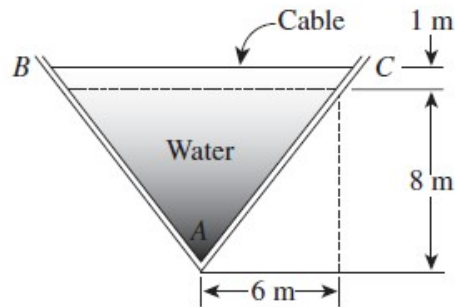
5. It is desired to use a 0.75-m diameter beach ball to stop a drain in a swimming pool. Obtain an expression that relates the drain diameter  $D$  and the minimum water depth  $h$  for which the ball will remain in place.



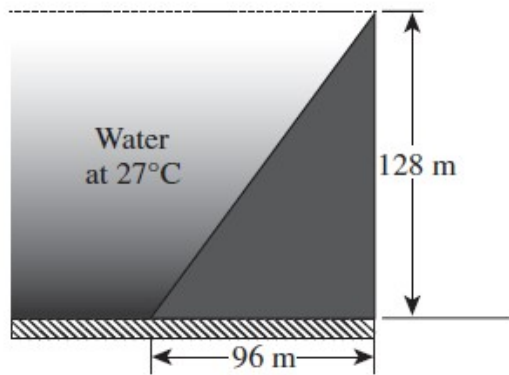
6. The circular gate ABC has a 1m radius and is hinged at B. Neglecting atmospheric pressure, determine the force  $P$  just sufficient to keep the gate from opening when  $h = 12\text{m}$ .



7. The figure below shows an open triangular channel in which the two sides, AB and AC, are held together by cables, spaced 1 m apart, between B and C. Determine the cable tension

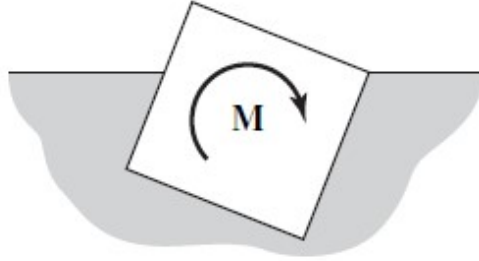


8. The dam shown below is 100 m wide. Determine the magnitude and location of the force on the inclined surface.



9. The float in a toilet tank is a sphere of radius  $R$  and is made of a material with density  $r$ . An upward buoyant force  $F$  is required to shut the ballcock valve. The density of water is designated  $\rho_w$ . Develop an expression for  $x$ , the fraction of the float submerged, in terms of  $R, r, F, g$ , and  $\rho_w$ .
10. A cubical piece of wood with an edge  $L$  in length floats in the water. The specific gravity of the wood is 0.90. What moment  $M$  is required to hold the cube in the position shown? The right-hand edge of the block is flush with the water.





### 3 L3-L4

1. assume the motion is described by

$$\mathbf{r} = \mathbf{f}(\mathbf{c}, t) = \mathbf{g}(\mathbf{c})h(t) \quad (4)$$

find the expression of the velocity field  $\mathbf{v}$ .

2. assume the motion is

$$x = ct^2 \quad (5)$$

use the result of 1, find the expression of the velocity field  $v$ .

3. suppose in a one dimensional flow, the motion of the  $c$ th fluid element is given by

$$x = f(c, t) \quad (6)$$

and the temperature of the flow at time  $t$  is distributed as

$$T = g(x, t) \quad (7)$$

find the temperature variation rate of the  $c$ th element.

4. The velocity components in an unsteady plane flow are given by

$$u = \frac{x}{1+t} \quad (8)$$

$$v = \frac{2y}{2+t} \quad (9)$$

find the path lines and the streamlines equation subjecting to  $\mathbf{x} = \mathbf{x}_0$  at  $t = 0$ .

5. Let a one-dimensional velocity field be  $u = u(x, t)$ . The density varies as  $\rho = \rho_0(2 - \cos \omega t)$ . Find an expression for  $u(x, t)$  if  $u(0, t) = U$ .
6. The components of a mass flow vector  $\rho \mathbf{u}$  are  $\rho u = 4x^2y$ ,  $\rho v = xyz$ ,  $\rho w = yz^2$ . Compute the net outflow through the closed surface formed by the planes  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ . (a) Integrate over the closed surface. (b) Integrate over the volume bounded by that surface.
7. A two-dimensional object is placed in a  $2h$ -wide water tunnel as shown. The upstream velocity,  $v_1$ , is uniform across the cross section. For the downstream velocity profile as shown, find the value of  $v_2$

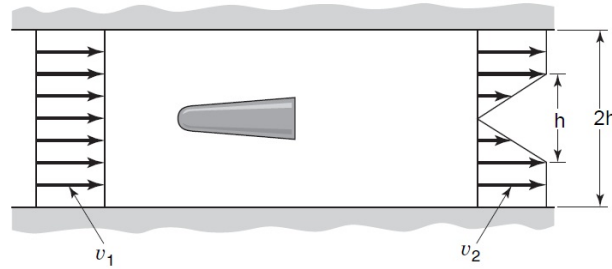


图 1

8. consider fluid in a channel of unit width and that the vertical velocity of the fluid is negligible and the horizontal velocity  $u(x, t)$  is roughly constant throughout any cross section of the channel. assume the fluid is incompressible so the density  $\rho$  is constant, denote the depth of the fluid as  $h(x, t)$  find the mass and momentum conserve equation of the fluid (the gravitational constant is  $g$ ).
9. Given the steady two-dimensional velocity distribution

$$u = Kx, v = -Ky, w = 0 \quad (10)$$

where  $K$  is a positive constant, compute and plot the streamlines of the flow, including directions.

10. Under what conditions does the velocity field

$$V = (a_1x + b_1y + c_1z)\mathbf{i} + (a_2x + b_2y + c_2z)\mathbf{j} + (a_3x + b_3y + c_3z)\mathbf{k} \quad (11)$$

where  $a_1, b_1, \text{ etc.} = \text{const}$ , represent an incompressible flow that conserves mass?