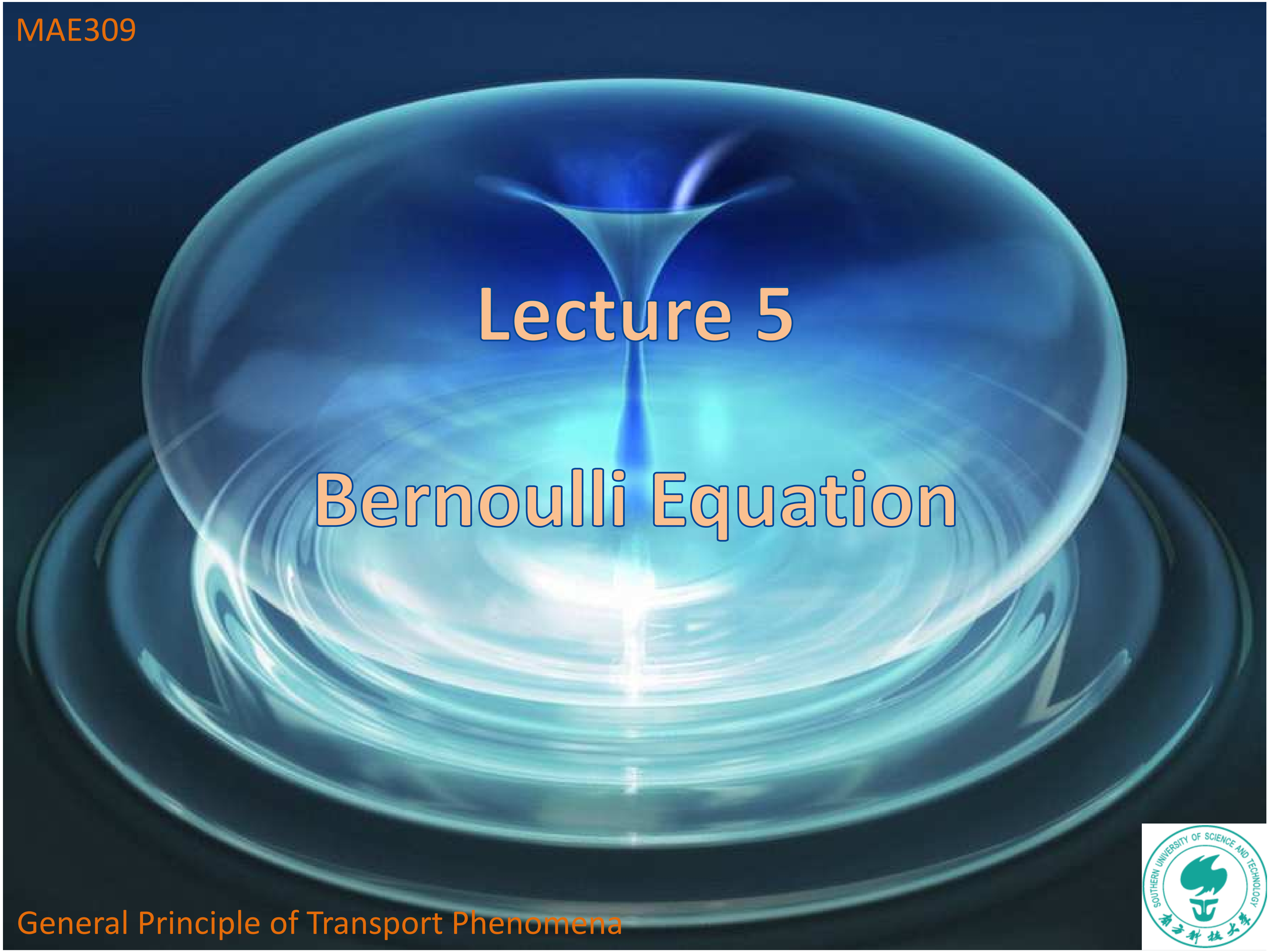


Lecture 5

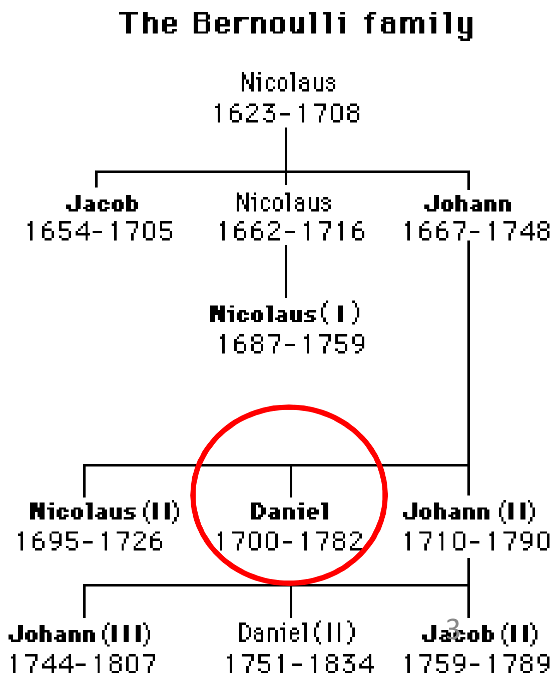
Bernoulli Equation

Learning Objectives

- To understand
  - Bernoulli Equation
  - Derivation of Bernoulli equation
  - Explanation of each Term in Bernoulli equation
  - Venturi Effect
  - How to apply Bernoulli equation to explain many problems in everyday life

Bernoulli Equations

- Bernoulli's Family
  - Daniel Bernoulli (1700-1782) was born in the Netherlands – he wrote the book Hydrodynamica. He hold a Chair at St Petersburg together with his brother Nicolaus II
  - His father Johann wrote the first book on calculus (after Newton & Leibniz)
  - His uncle Jacob was the first to use the term "integral" ("Bernoulli differential equation")
  - His cousin Nicolaus I has contributed to Riccati equations
  - His brother Nicolaus II worked on differential equations and probability
  - His brother Johann II worked in heat & light – he occupied the same chair as his father at Basel University
  - His nephews Johann III, Daniel II and Jacob II are also mathematicians of note in their days



Bernoulli Equations

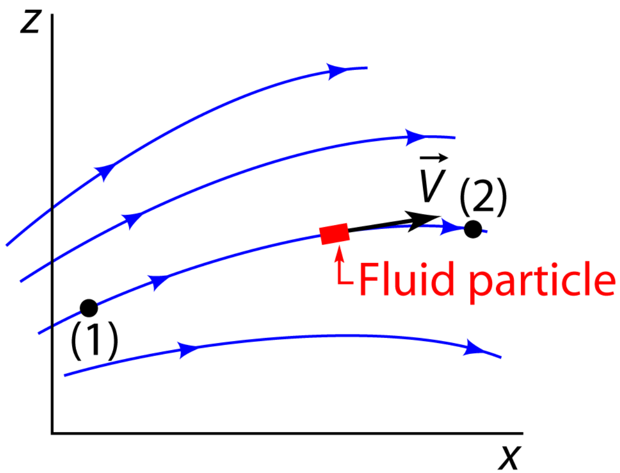
- Bernoulli's Equation for Incompressible Flow

$$\frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{constant}$$

where ρ is density, g is gravity, p is pressure, V is velocity, and z is the elevation of the point above a reference plane, with the positive z-direction pointing upward – so in the direction opposite to the gravitational acceleration

  – The above Bernoulli equation is valid for steady, incompressible flow along a streamline in an "inviscid regions of flow"

  – Constant of integration in general varies from one streamline to another, but remains constant along a particular streamline



Derivation of Bernoulli Equation

- Integral from NS Equation
  - Steady, incompressible, inviscid flow (Euler equation)

$$\cancel{\frac{\partial u}{\partial t}} + \left(\vec{V}\bullet\nabla\right) u = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\cancel{\frac{\partial v}{\partial t}} + \left(\vec{V}\bullet\nabla\right) v = Y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\cancel{\frac{\partial w}{\partial t}} + \left(\vec{V}\bullet\nabla\right) w = Z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

  - Only consider gravity; rewrite in vector form

$$\left(\vec{V}\bullet\nabla\right)\vec{V} = \vec{g} - \frac{1}{\rho}\nabla p$$

Derivation of Bernoulli Equation

- Integral from NS Equation

$\nabla \times \vec{V}$ 旋度

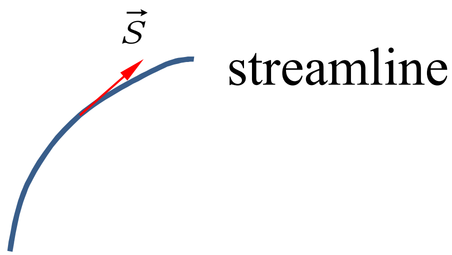
curl or rotation

$$\left(\vec{V}\bullet\nabla\right)\vec{V} = \nabla\frac{\vec{V}^2}{2} - \vec{V}\times\nabla\times\vec{V}$$ Feynman subscript notation

$$\nabla\frac{\vec{V}^2}{2} - \vec{V}\times\nabla\times\vec{V} = \vec{g} - \frac{1}{\rho}\nabla p$$

– Multiple by the unit vector $\vec{s} = \frac{\vec{V}}{\left|\vec{V}\right|}$ along a streamline

$$\vec{s}\bullet\nabla\frac{\vec{V}^2}{2} - \frac{\vec{V}}{\left|\vec{V}\right|}\bullet\vec{V}\times\nabla\times\vec{V} = \vec{s}\bullet\vec{g} - \frac{1}{\rho}\vec{s}\bullet\nabla p$$

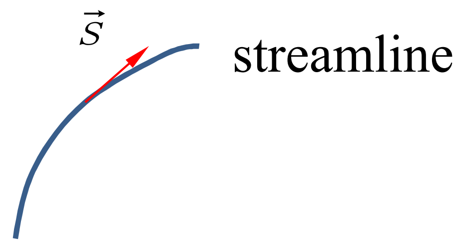


$$\vec{s}\bullet\nabla\frac{\vec{V}^2}{2} = \vec{s}\bullet\vec{g} - \frac{1}{\rho}\vec{s}\bullet\nabla p$$

Derivation of Bernoulli Equation

- Integral from NS Equation

$$\frac{\partial}{\partial s}\left(\frac{\vec{V}^2}{2}+\frac{p}{\rho}\right)+\vec{s}\bullet\vec{g}=0$$



  - Integrate along streamline

$$\frac{\vec{V}^2}{2}+\frac{p}{\rho}+\int\vec{s}\bullet\vec{g}ds=C$$

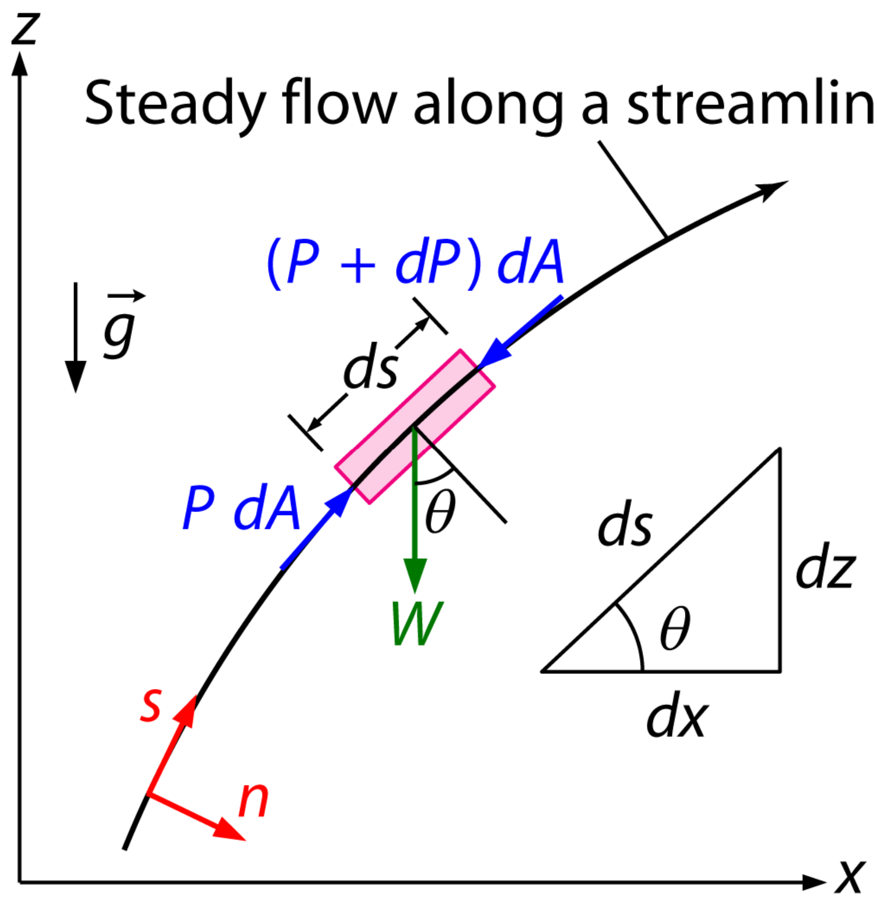
g varies only along z direction

C is constant

$$\frac{\vec{V}^2}{2}+\frac{p}{\rho}+gz=C$$

Derivation of Bernoulli Equation

- Newton's second law
  - Consider motion of fluid particle in a steady flow



Derivation of Bernoulli Equation

- Newton's second law
  - Applying Newton's second law in the s-direction on a particle moving along a streamline

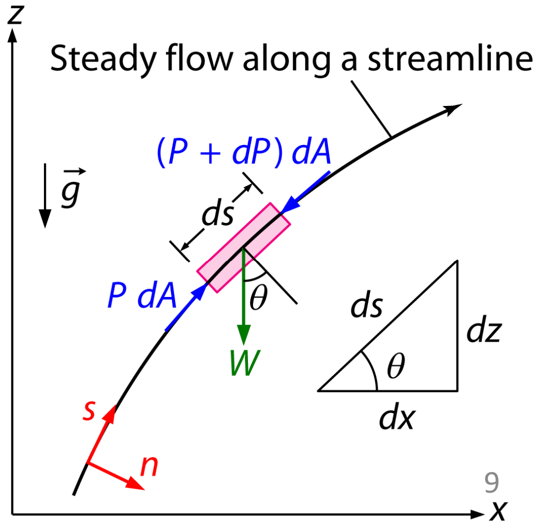
$$\sum F_s = ma_s$$

  - Assuming viscous forces are negligible, only forces acting on fluid particle in s-direction are pressure forces and component of particle's weight

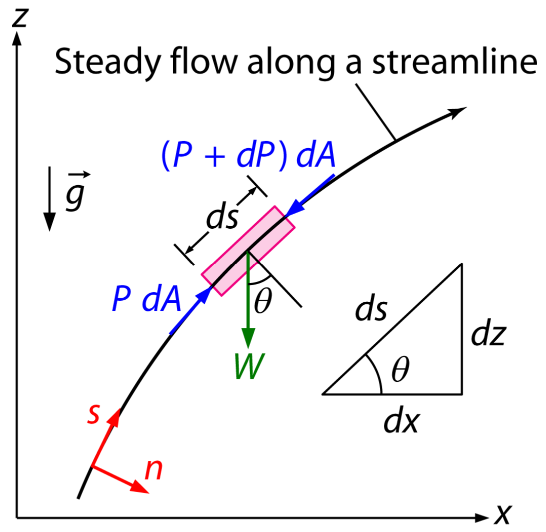
$$PdA - (P + dP)dA - W\sin\theta = ma_s$$

$$-dPdA - W\sin\theta = ma_s$$

θ is the angle between normal to streamline and vertical z-axis



Derivation of Bernoulli Equation



- Newton's second law
  - Velocity along fluid particle

$$V = \frac{ds}{dt}$$

  - Velocity is a function of *s* and *t*

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

  - Steady flow $\partial V/\partial t = 0$

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V$$

Derivation of Bernoulli Equation

- Newton's second law

$$-dPdA - W\sin\theta = m\frac{\partial V}{\partial s}V$$

  – Volume of the fluid particle Ω

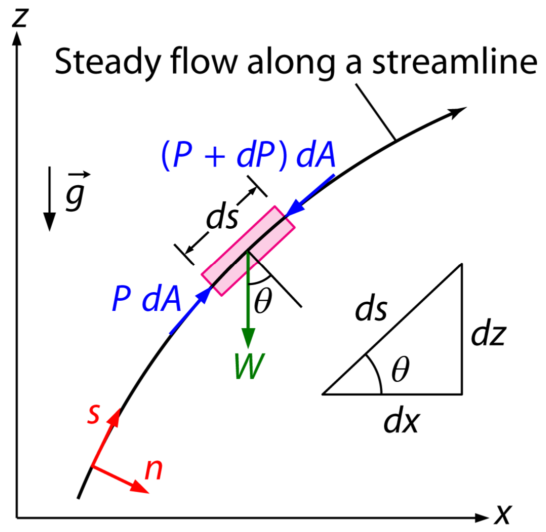
$$m = \rho\Omega = \rho dAds$$

$$W = mg = \rho gdAds$$

$$\sin\theta = \frac{dz}{ds}$$

$$-dPdA - \rho gdAds\frac{dz}{ds} = \rho dAds\frac{\partial V}{\partial s}V$$

$$-dP - \rho gdz = \rho VdV$$



Derivation of Bernoulli Equation

- Newton's second law

$$VdV = \frac{1}{2}d\left(V^2\right)$$

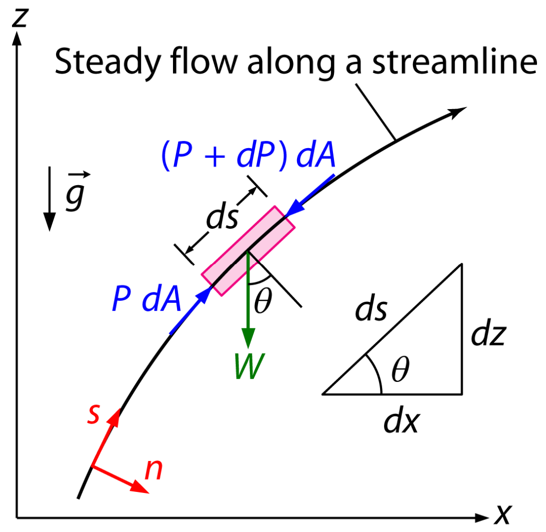
$$-\frac{dP}{\rho} - gdz = \frac{dV^2}{2}$$

  - Integrate the above equation

$$\frac{V^2}{2} + gz + \frac{P}{\rho} = C$$

  - Between any 2 points on the same streamline: steady, incompressible, inviscid flow:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$



Derivation of Bernoulli Equation

- Energy conservation
  - Work done on a fluid particle is equal to the change in its kinetic energy and potential energy

$$\Delta W \quad = \quad \Delta K \quad + \quad \Delta U$$

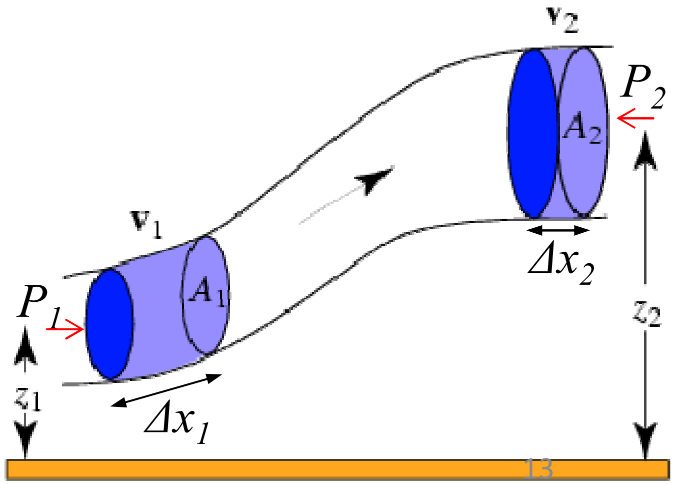
Work Kinetic Energy Potential Energy

✓ Work

$$\begin{aligned} \Delta W &= F_1 \Delta x_1 - F_2 \Delta x_2 \\ &= P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \\ &= P_1 \Omega_1 - P_2 \Omega_2 \end{aligned}$$

Recall continuity equation of incompressible flow, $\Omega_1 = \Omega_2 = \Omega$

$$\Delta W = P_1 \Omega - P_2 \Omega$$

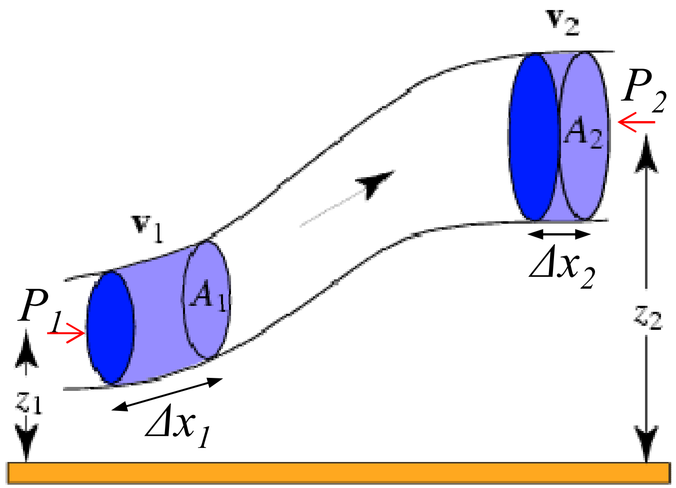


Derivation of Bernoulli Equation

- Energy conservation
  - Work done on a fluid particle is equal to the change in its kinetic energy and potential energy
    - ✓ Change in kinetic energy

$$\Delta E = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

    - ✓ Change in potential energy



$$\Delta U = m_2 g z_2 - m_1 g z_1$$

Recall continuity equation of incompressible flow, $m_1 = m_2 = m$

$$\Delta U = mgz_2 - mgz_1$$

Derivation of Bernoulli Equation

- Energy conservation
  - Work done on a fluid particle is equal to the change in its kinetic energy and potential energy

$$\Delta W \qquad = \qquad \Delta K \qquad + \qquad \Delta U$$

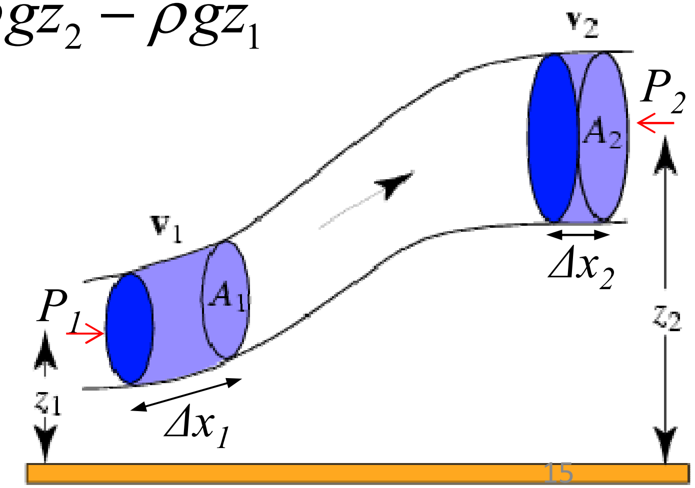
$$P_1\Omega - P_2\Omega \quad = \quad \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \; + \; mgz_2 - mgz_1$$

$$\rho = \frac{m}{\Omega} \;\Rightarrow\; P_1 - P_2 \quad = \quad \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \; + \; \rho g z_2 - \rho g z_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1$$

$$= P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$$

$$= \quad \text{constant}$$



Derivation of Bernoulli Equation

- More explanation

$$\frac{P}{\rho} + \frac{1}{2} v^2 + gz = \text{constant (along a streamline)}$$

  - Each term in the above equation has same units of energy per unit mass
    - ✓ P/ρ: Flow energy
    - ✓ $v^2/2$: Kinetic energy
    - ✓ gz: Potential energy
  - Bernoulli equation can be viewed as a restatement of conservation of mechanical energy
  - The sum of the specific (per unit mass) kinetic, potential, and flow energies of a fluid particle is constant along a streamline in a steady, incompressible and inviscid flow.

Derivation of Bernoulli Equation

- More explanation

$$P + \frac{1}{2}\rho v^2 + \rho g z = \text{constant (along a streamline)}$$

  – Each term in the above equation has same units as pressure

    ✓ P: static pressure → represents the actual thermodynamic pressure of fluid

    ✓ $\rho v^2/2$: dynamic pressure → represents pressure rise when fluid in motion is brought to reset

    ✓ $\rho g z$: hydrostatic pressure → account for the elevation effects

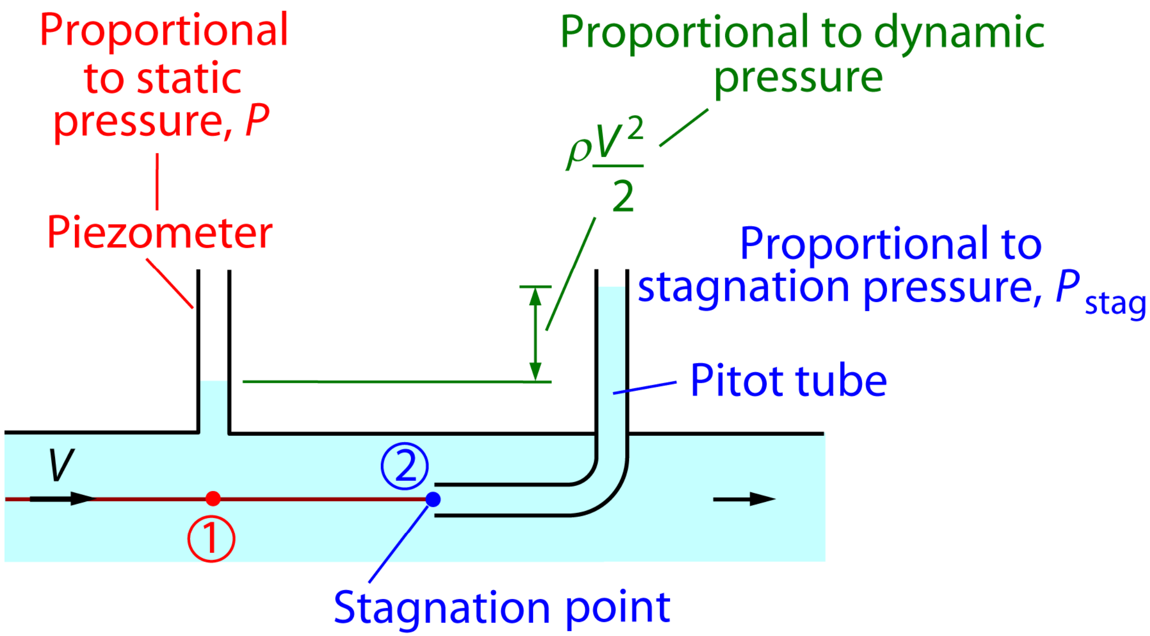
  – Total pressure (P_T): sum of static, dynamic and hydrostatic pressure

  – Bernoulli equation: P_T along streamline is constant

$$P + \frac{1}{2}\rho v^2 + \rho g z = P_T = \text{constant (along a streamline)}$$

Pressure and Velocity Measurement

- Stagnation Pressure
  - Sum of static and dynamic pressure
  - Represents pressure at stagnation point where fluid is brought to rest



$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

Pressure and Velocity Measurement

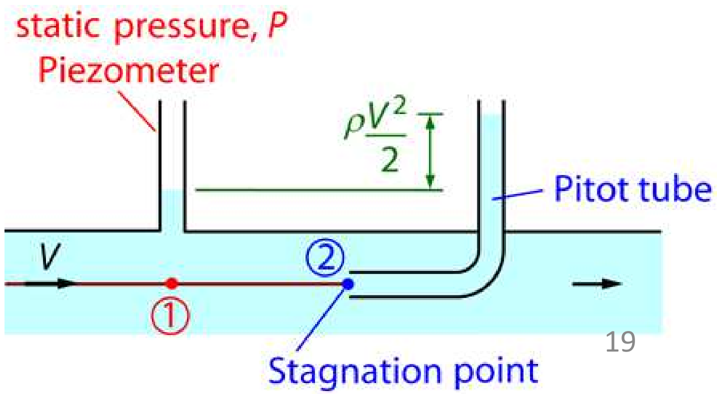
- Velocity Measurement
  - Point 1: $V_1 = V$, $P_1 = P$ (static pressure)
  - Point 1: $V_2 = 0$, $P_2 = P_{stag}$ (static pressure) $z_1 = z_2$
  - Apply Bernoulli equation along streamline between 1 and 2:

$$P_1 + \rho \frac{V_1^2}{2} + \rho g z_1 = P_2 + \rho \frac{V_2^2}{2} + \rho g z_2$$

$$P_{stag} = P + \rho \frac{V^2}{2}$$

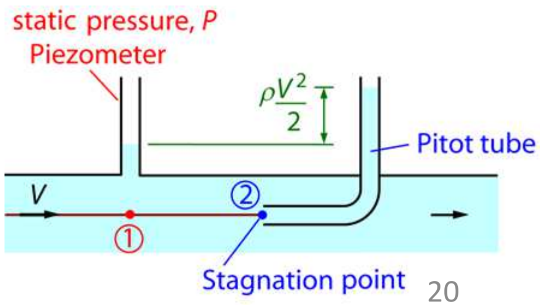
  - Fluid velocity can be deduced from measurement of static and stagnation pressures:

$$V = \sqrt{\frac{2\left(P_{stag} - P\right)}{\rho}}$$



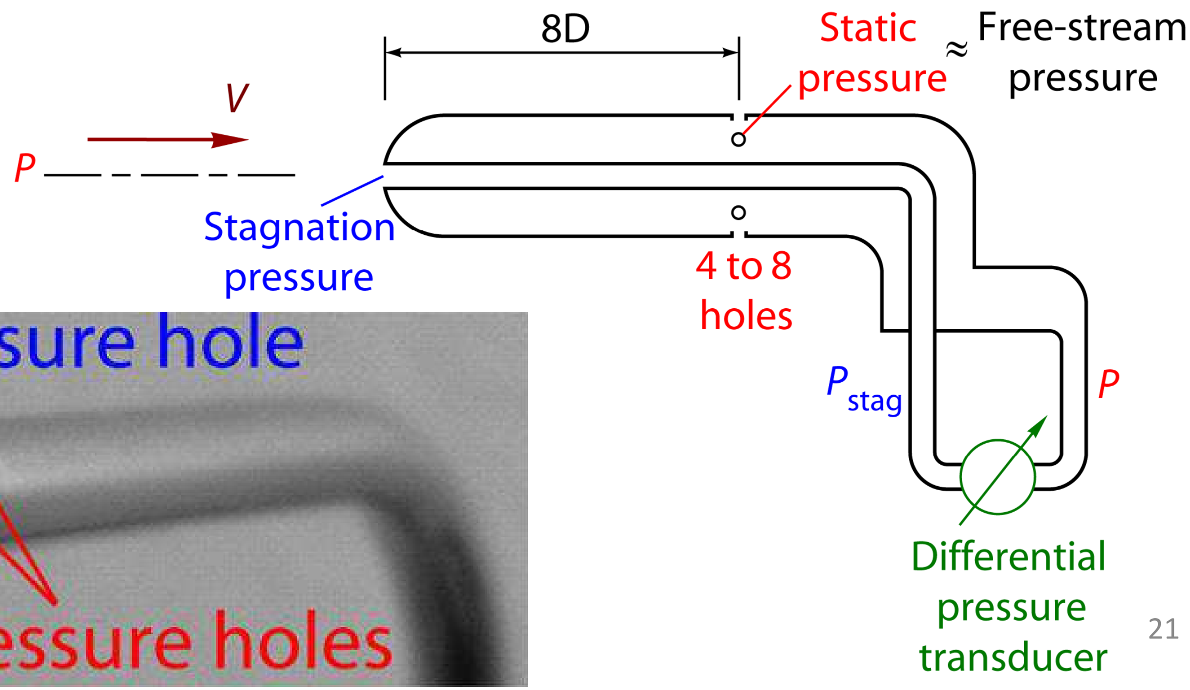
Pressure and Velocity Measurement

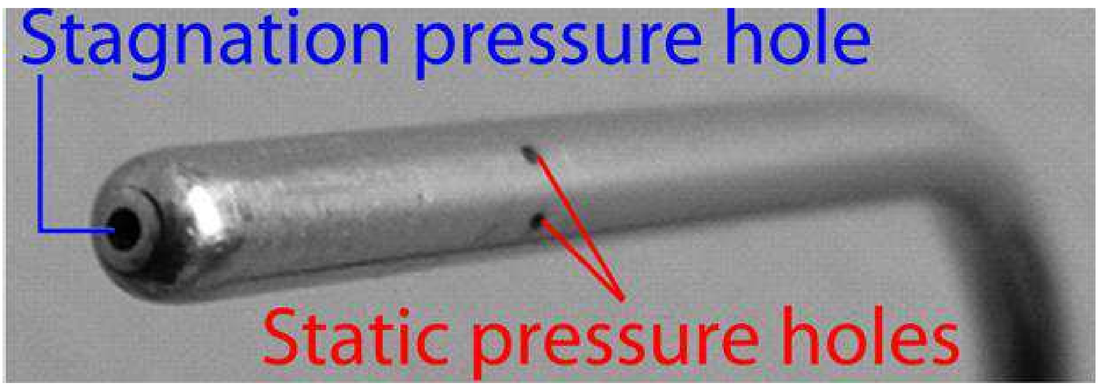
- Devices for pressure measurement
  - Static pressure tap: a small hole drilled into a wall such that plane of hole is parallel to flow direction → measures static pressure
  - Pitot tube: a small tube with its open end aligned into the flow so as to sense full impact pressure of the flowing fluid → measures stagnation pressure
  - Piezometer: vertical transparent tube attached to static pressure tap or Pitot tube: liquid rises in piezometer tube to a column height (head) proportional to pressure being measured



Pressure and Velocity Measurement

- Devices for pressure measurement
  - Pitot-static probe: integrates static pressure holes on a Pitot probe
  - Pitot-static probe connected to pressure transducer or manometer → measures dynamic pressure and hence velocity





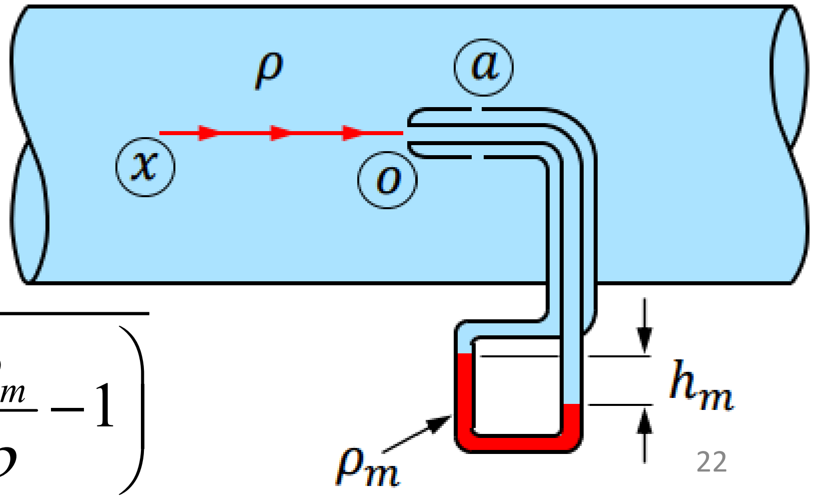
Pressure and Velocity Measurement

- Devices for pressure measurement
  - Static pressure holes (point a) of the outer tube are located such that they measure correct upstream static pressure
  - Two tubes provide the necessary pressure difference measurement using the mercury in it
  - It is possible to use pressure transducers instead of mercury columns to obtain accurate digital readings.

$$P_o = P_x + \rho \frac{V_x^2}{2}$$

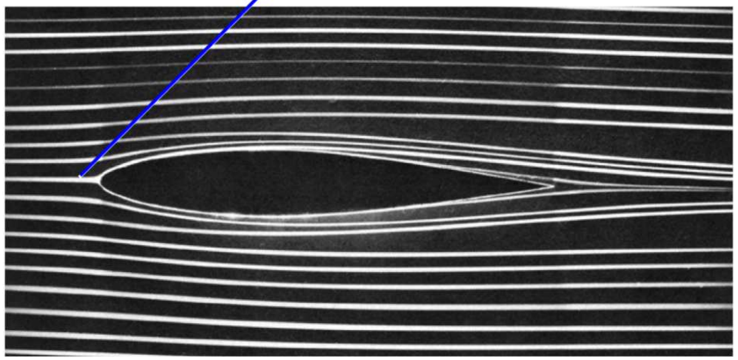
$$P_o - P_x = (\rho_m - \rho) g h_m$$

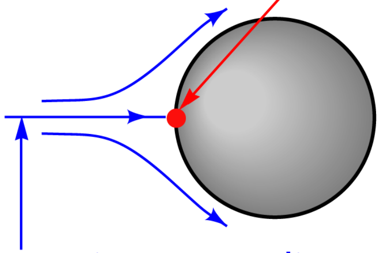
$$V_x = \sqrt{2\left(P_{stag} - P_x\right)/\rho} = \sqrt{2 g h_m \left(\frac{\rho_m}{\rho} - 1\right)}$$

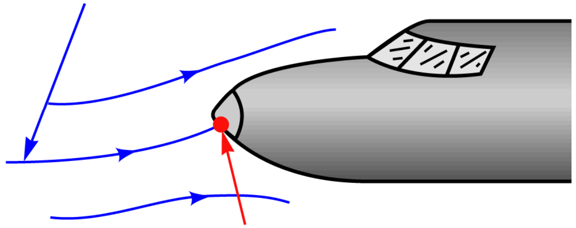


Pressure and Velocity Measurement

- Stagnation point
  - When a stationary body is immersed in a flow, the fluid is brought to rest at nose of body (stagnation point)
  - Stagnation streamline → streamline that extends from far upstream to stagnation point

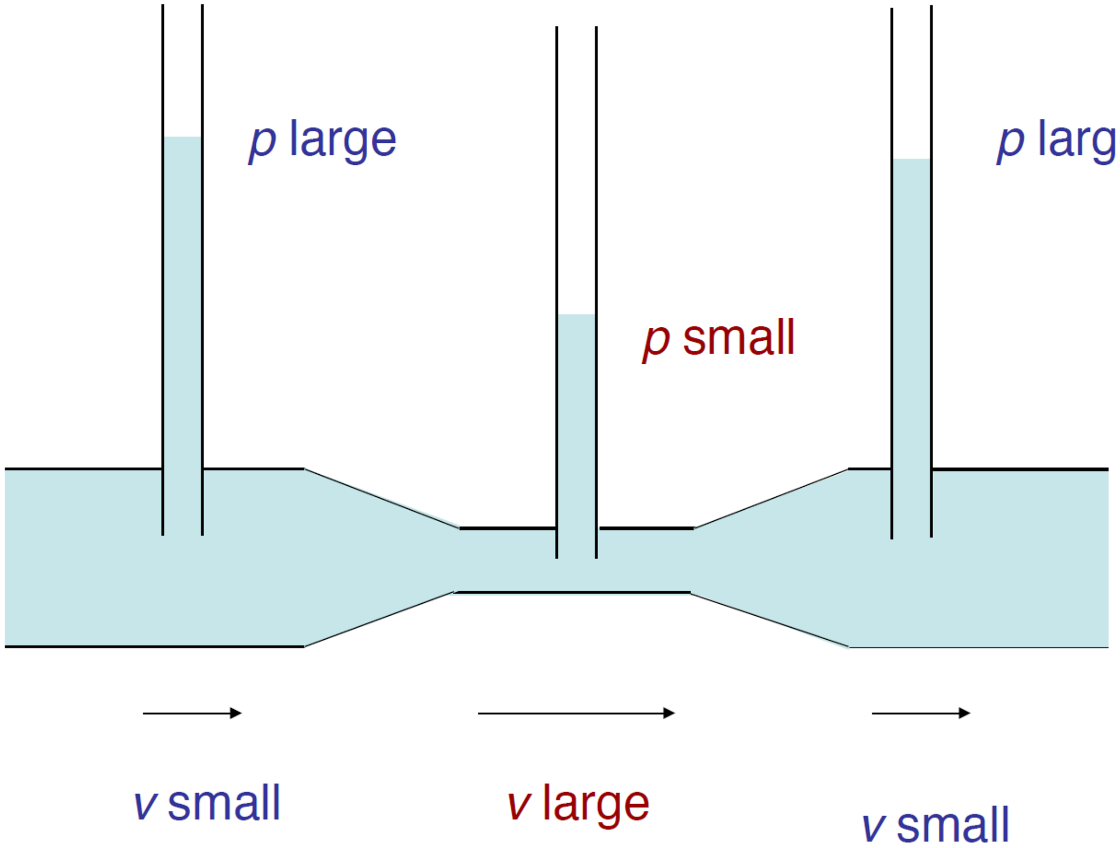






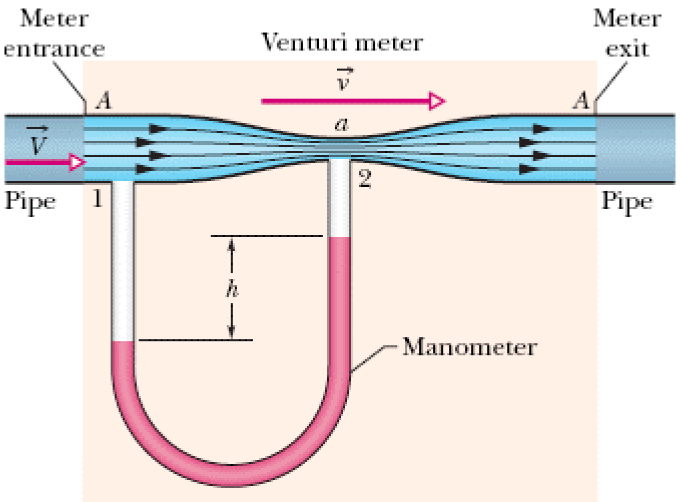
Applications

- Venturi Effect
  - The reduction in fluid pressure that results when a fluid flows through a constricted section (or choke) of a pipe



Applications

- Venturi Effect
  - Devices to measure the flow rate of liquids



$$Q = v_1 A_1 = v_2 A_2$$

$$P_1 + \frac{1}{2}\rho_v v_1^2 + \rho_v g z_1 = P_2 + \frac{1}{2}\rho_v v_2^2 + \rho_v g z_2$$

$$P_1 - P_2 = \frac{1}{2}\rho_v \left(v_2^2 - v_1^2\right)$$

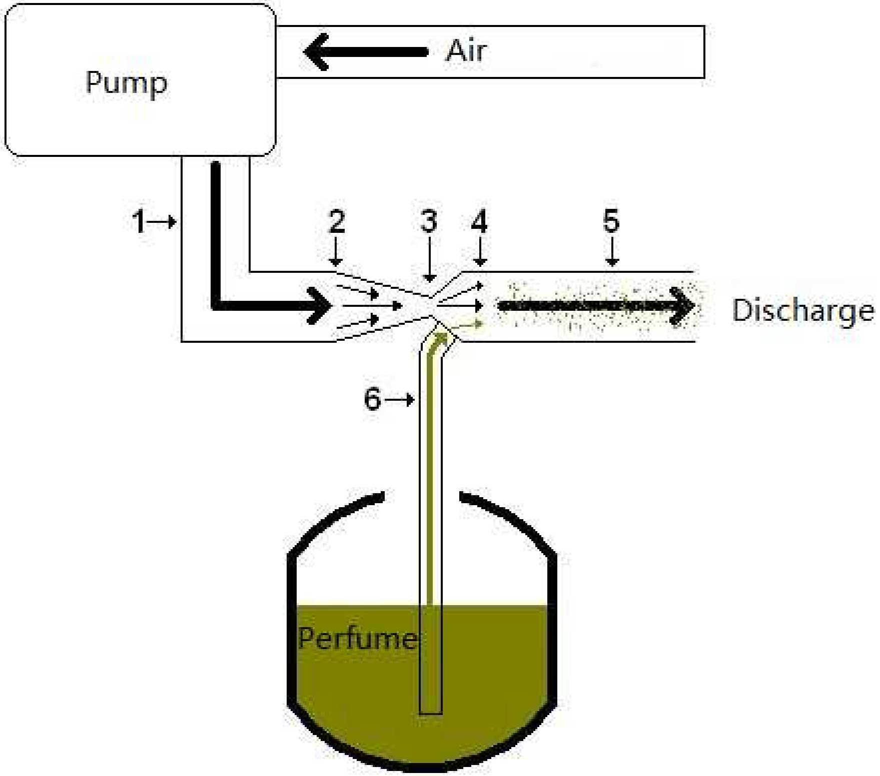
$$Q = A_1 \sqrt{\frac{2}{\rho_v} \bullet \frac{(P_1 - P_2)}{\left(\frac{A_1}{A_2}\right)^2 - 1}} = A_2 \sqrt{\frac{2}{\rho_v} \bullet \frac{(P_1 - P_2)}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

- The densities of fluids in Venturi and Manometer are ρ_v and ρ_m
- The areas of cross sections at point 1 and point 2 are known as A_1 and A_2

$$P_1 - P_2 = \rho_m g h$$

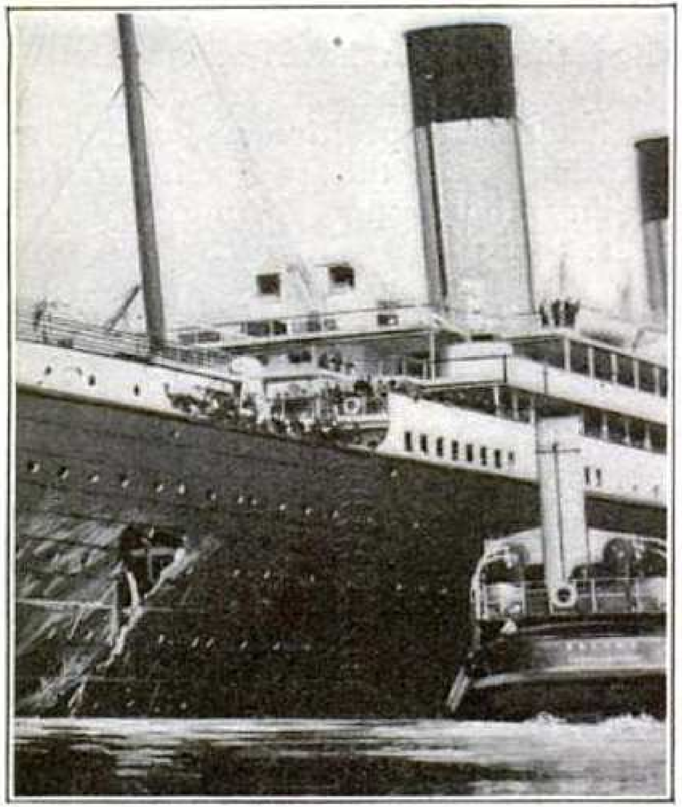
Applications

- Venturi Effect
  - Disperse perfume or spray paint (Atomizers)

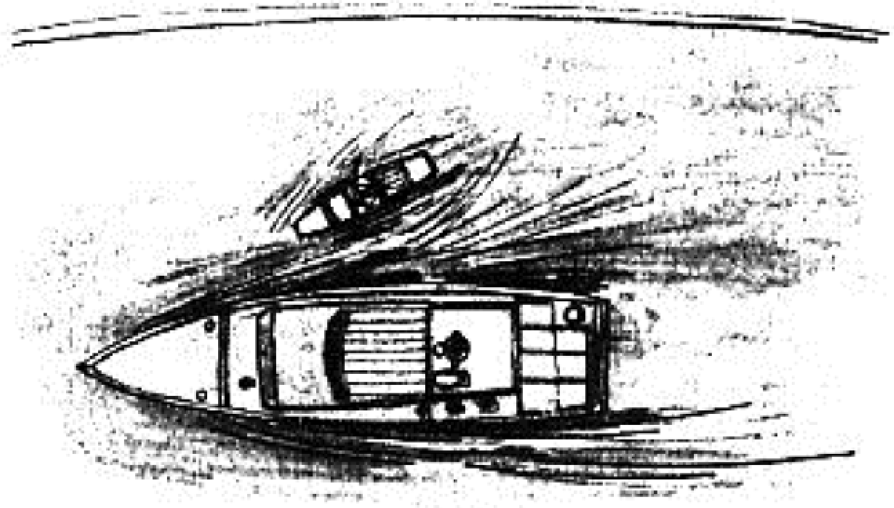


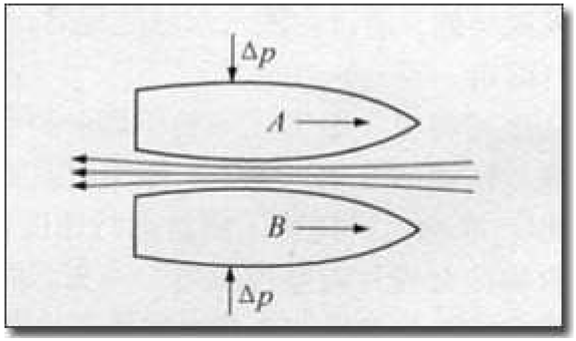
Applications

- Bernoulli's Equation



Olympic collided with *Hawke, 1911*





Applications

- Bernoulli's Equation

A high speed train passing a platform causes a suction effect

Long vehicle and bicycle

Applications

- Pour Out Beer

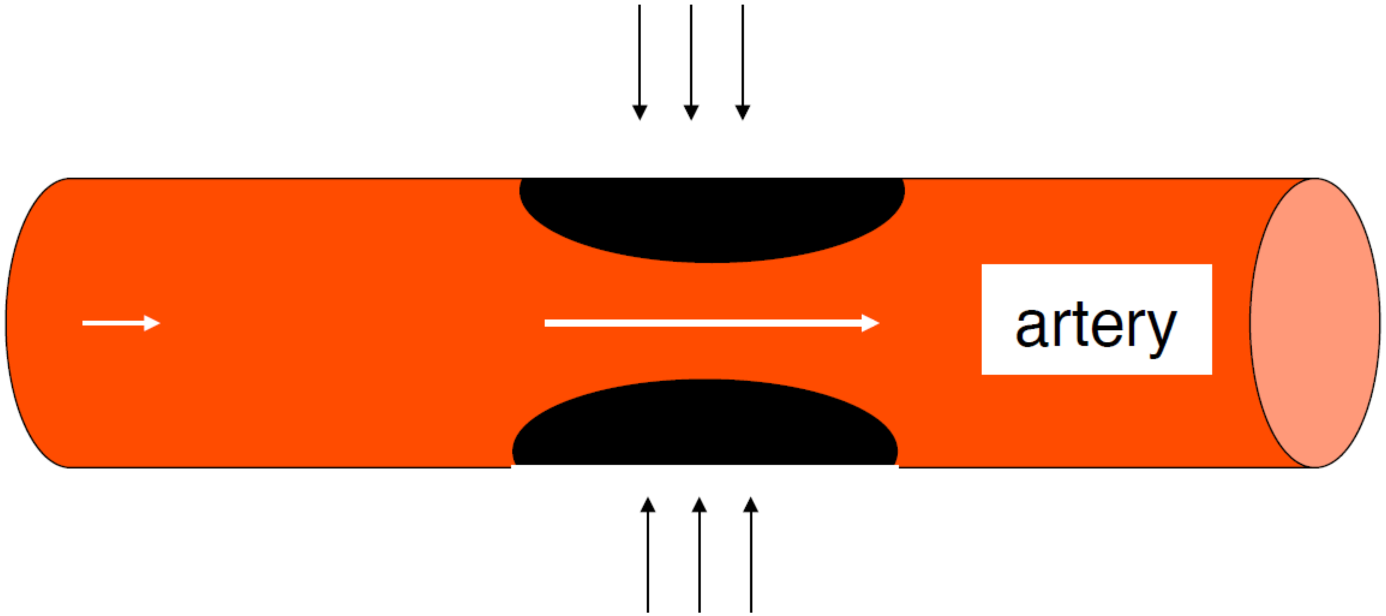
歪门斜倒(邪道)

杯壁(卑鄙)下流

改斜(邪)归正

Applications

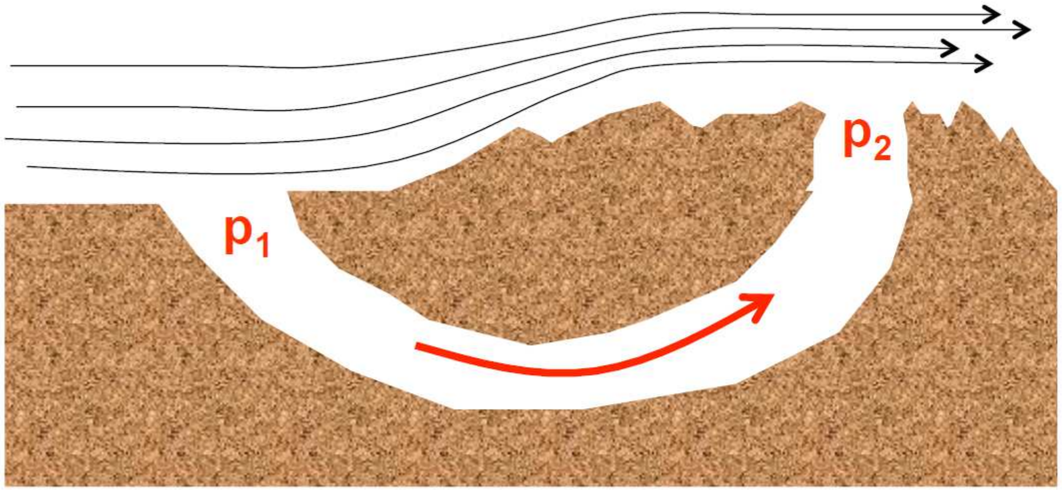
- Arteriosclerosis and Vascular Flutter



- ✓ Flow speeds up at constriction
- ✓ Pressure is lower
- ✓ Internal force acting on artery wall is reduced
- ✓ External forces are unchanged
- ✓ Artery can collapse

Applications

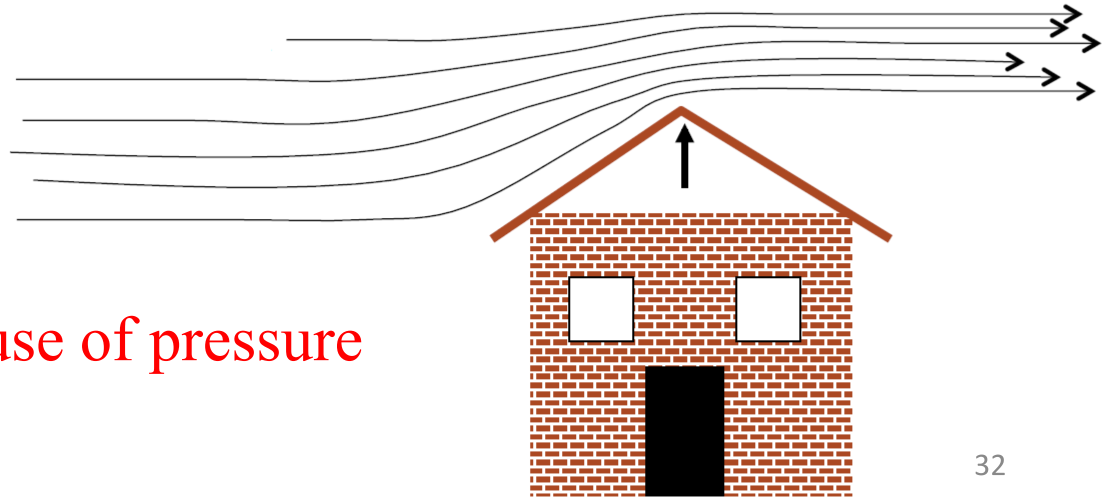
- Why do rabbits not suffocate in the burrows
  - Air must circulate. The burrows must have two entrances.
  - Air flows across the two holes is usually slightly different
  - One hole is usually higher than the other and the a small mound is built around the holes to increase the pressure difference.
    - ✓ Slight pressure difference
    - ✓ Forces flow of air through burrow



$p_2 < p_1$

Applications

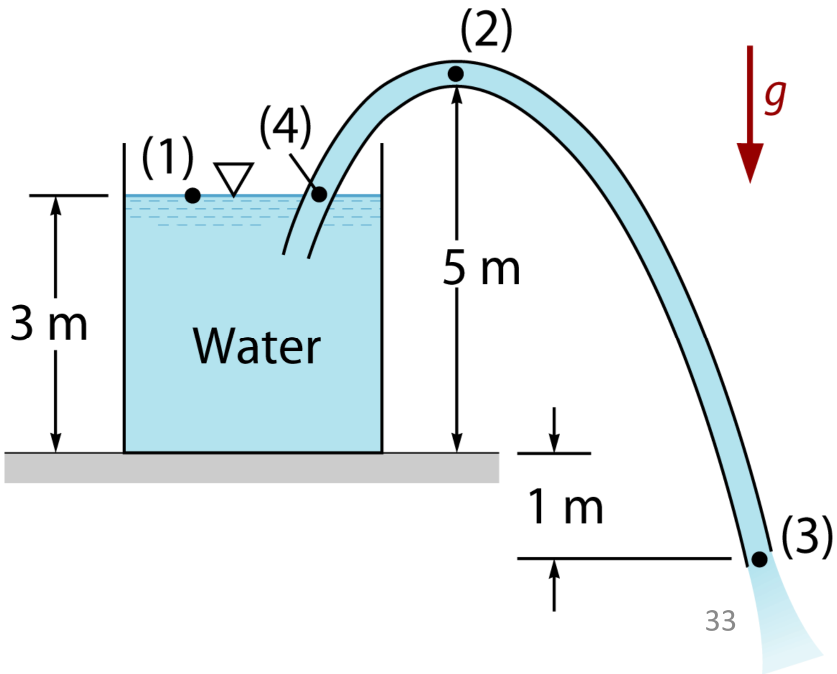
- Why does a house lose its roof in strong wind
  - Air flow is disturbed by the house. The "streamlines" crowd around the top of the roof
  - Faster flow above house
  - Reduced pressure above roof to that inside the house



Roof lifted off because of pressure difference!!!

Examples

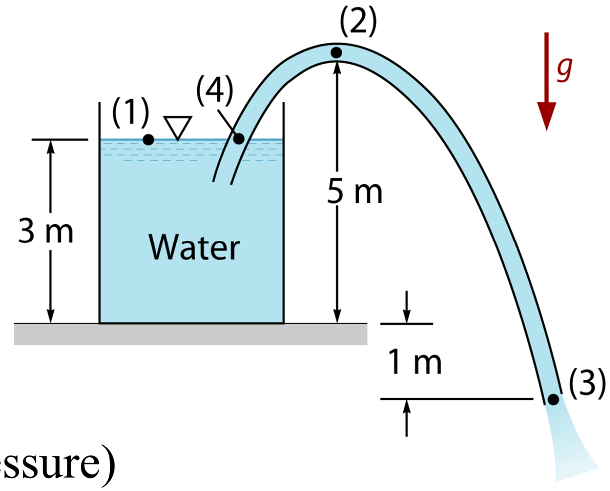
- Siphon Phenomenon
  - Water is siphoned from a large tank through a constant diameter hose
  - Determine:
    a) velocity of water leaving (3) as a free jet
    b) water pressure in tube at (2)
    c) water pressure in tube at (4)



Assume water to be inviscid, incompressible and flow to be steady

Examples

- Siphon Phenomenon
  - Solution:
    - ✓ Part (a): velocity of water leaving (3)



$$z_1 - z_3 = 4 \text{ m}$$

$$P_1 = P_3 = 0 \quad \text{(atmospheric pressure, 0 gage pressure)}$$

$$V_1 \approx 0 \quad \text{(large tank)}$$

Applying Bernoulli equation between (1) and (3)

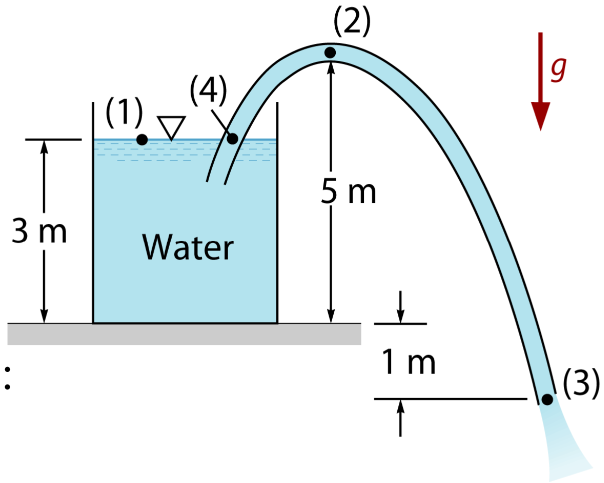
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_3}{\rho} + \frac{V_3^2}{2} + gz_3$$

$$\frac{V_3^2}{2} = g\left(z_1 - z_3\right)$$

$$V_3 = \sqrt{2g\left(z_1 - z_3\right)} = \sqrt{(2)(9.81)(4)} \qquad V_3 = 8.86 \text{ m/s}$$

Examples

- Siphon Phenomenon
  - Solution:
    - ✓ Part (b): water pressure in tube at (2)



Applying continuity equation between (2) and (3):

$$A_2V_2 = A_3V_3$$

Since $A_2 = A_3$, $\quad V_2 = V_3 = 8.86 \text{ m/s}$

Applying Bernoulli equation between (2) and (3)

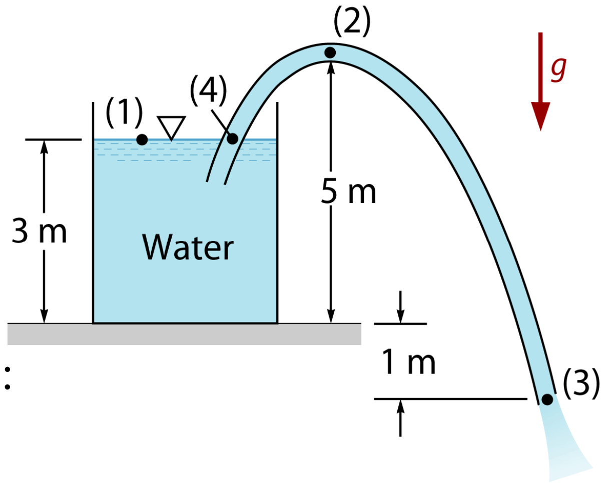
$$\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{P_3}{\rho} + \frac{V_3^2}{2} + gz_3$$

$$\frac{P_2}{\rho} = g\left(z_3 - z_2\right) \qquad z_2 - z_3 = 6 \text{ m}$$

$$P_2 = \rho g\left(z_3 - z_2\right) = (1000)(9.81)(-6) \qquad P_2 = -58.9 \text{ kPa}$$

Examples

- Siphon Phenomenon
  - Solution:
    - ✓ Part (c): water pressure in tube at (4)



Applying continuity equation between (4) and (3):

$$A_4 V_4 = A_3 V_3$$

Since $A_4 = A_3$, $V_4 = V_3 = 8.86 \text{ m/s}$

Applying Bernoulli equation between (4) and (3)

$$\frac{P_4}{\rho} + \frac{V_4^2}{2} + g z_4 = \frac{P_3}{\rho} + \frac{V_3^2}{2} + g z_3$$

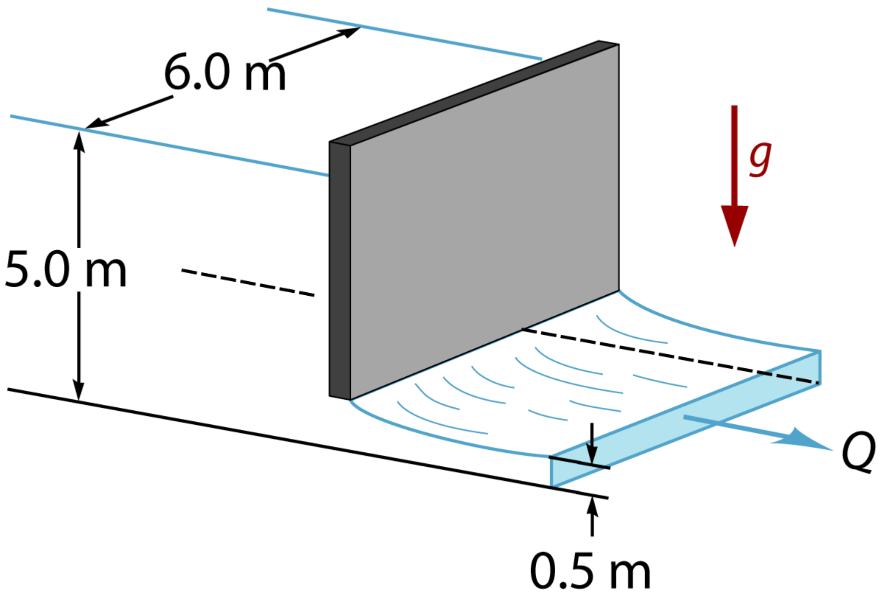
$$\frac{P_4}{\rho} = g\left(z_3 - z_4\right) \qquad z_4 - z_3 = 4 \text{ m}$$

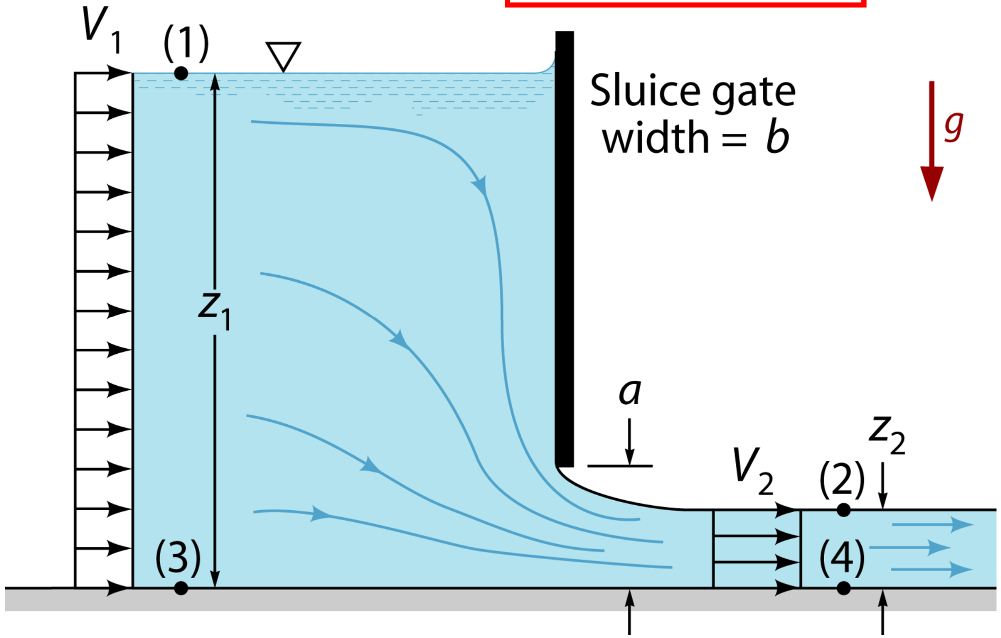
$$P_4 = \rho g\left(z_3 - z_4\right) = (1000)(9.81)(-4) \qquad P_2 = -39.24 \text{ kPa}$$

Examples

- Water flows under sluice gate
  - Determine flow rate Q

$$z_1 = 5 \text{ m}$$
$$z_2 = 0.5 \text{ m}$$
$$b = 6 \text{ m}$$





Assume water to be inviscid, incompressible and flow to be steady

Examples

- Water flows under sluice gate
  - Solution:

Applying continuity equation between (1) and (2):

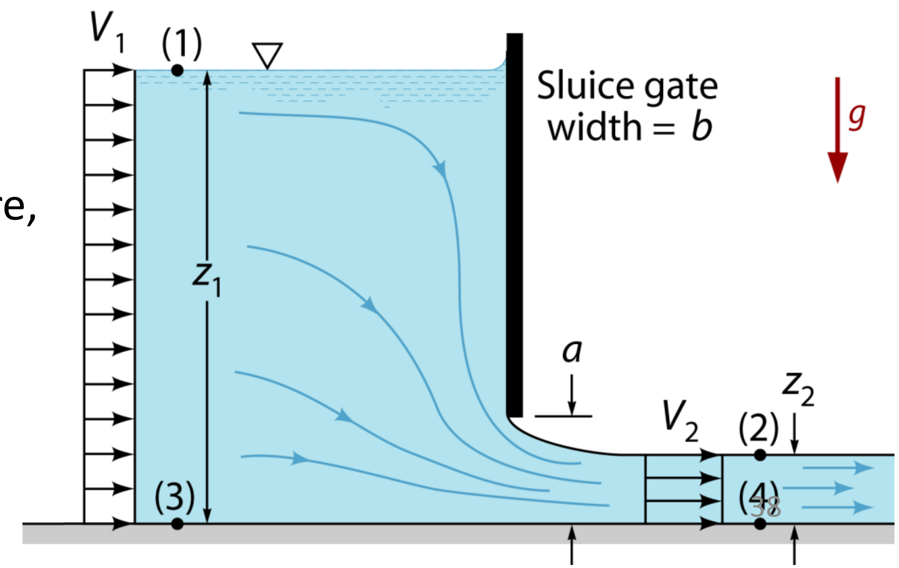
$$Q = A_1V_1 = bz_1V_1 = A_2V_2 = bz_2V_2 \qquad V_1 = V_2\left(\frac{z_2}{z_1}\right)$$

Applying Bernoulli equation between (1) and (2):

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$P_1 = P_2 = 0$$ (atmospheric pressure, 0 gage pressure)

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} = g\left(z_1 - z_2\right)$$



Examples

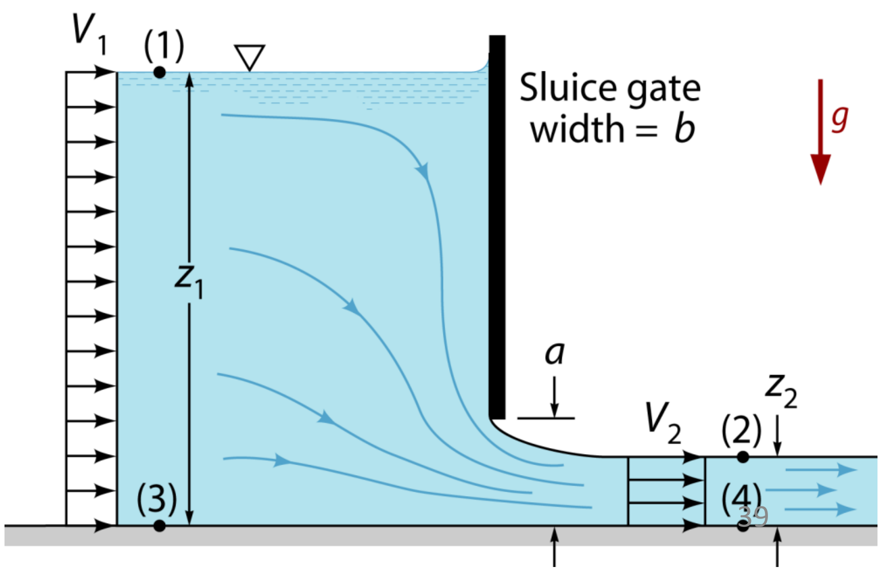
- Water flows under sluice gate
  - Solution:

$$\frac{V_2^2}{2} - \frac{V_2^2}{2}\left(\frac{z_2}{z_1}\right)^2 = g\left(z_1 - z_2\right) \qquad V_2 = \sqrt{\frac{2g\left(z_1 - z_2\right)}{1-\left(z_2/z_1\right)^2}}$$

$$Q = bz_2V_2 = bz_2\sqrt{\frac{2g\left(z_1 - z_2\right)}{1-\left(z_2/z_1\right)^2}}$$

$$Q = (6)(0.5)\sqrt{\frac{(2)(9.81)(5.0-0.5)}{1-(0.5/5.0)^2}}$$

$$Q = 28.33\ \mathrm{m^3/s}$$



Examples

- Water flows under sluice gate
  - Solution:

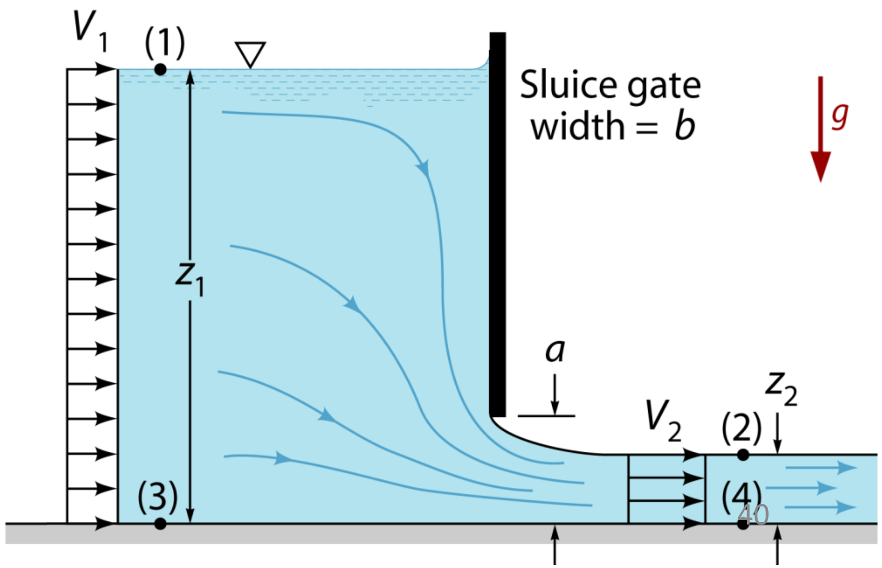
In the limit of $z_1 >> z_2$

$$Q = bz_2V_2 = bz_2\sqrt{\frac{2g\left(z_1 - \cancel{z_2}\right)}{1-\left(\cancel{z_2/z_1}\right)^2}}$$

$$Q = bz_2\sqrt{2gz_1}$$

$$Q = (6)(0.5)\sqrt{(2)(9.81)(5.0)}$$

$$Q = 29.71\ \mathrm{m}^3/\mathrm{s}$$



Examples

- Water flows under sluice gate
  - Solution:

Applying continuity equation between (1) and (2):

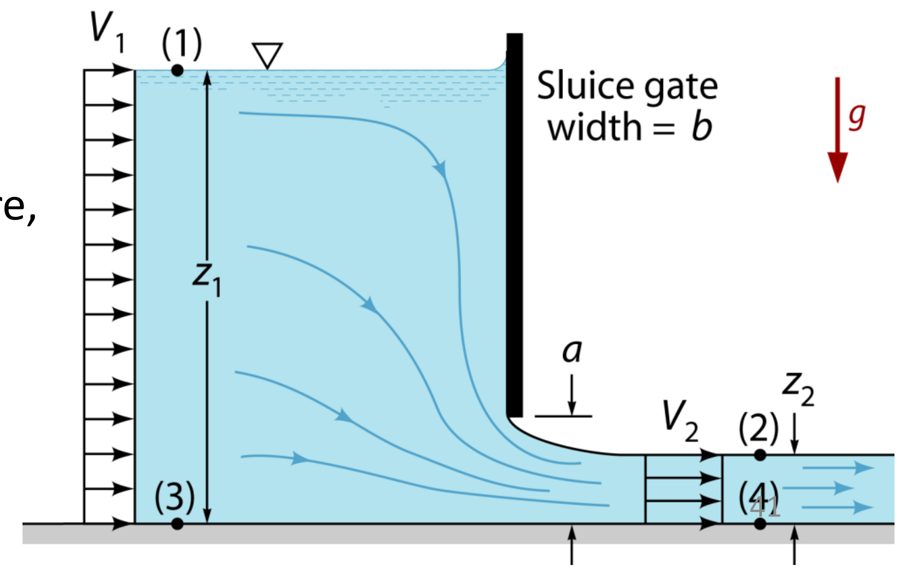
$$Q = A_1 V_1 = b z_1 V_1 = A_2 V_2 = b z_2 V_2 \qquad V_1 = V_2 \left(\frac{z_2}{z_1} \right)$$

Applying Bernoulli equation between (1) and (2):

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$P_1 = P_2 = 0$$ (atmospheric pressure, 0 gage pressure)

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} = g\left(z_1 - z_2\right)$$



Examples

- Water flow through a hole of a tank
  - Determine: the flow velocity V_2 at section 2-2?
  - Solution:

Applying Bernoulli equation between (1-1) and (2-2):

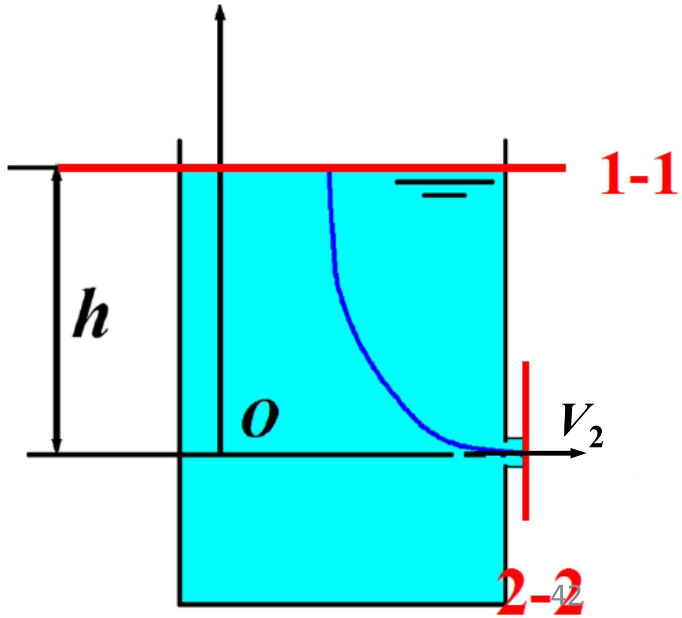
$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{P_2}{\rho}+\frac{V_2^2}{2}+gz_2$$

$$P_1=P_2=P_{atm}$$

$$z_1-z_2=h$$

$$V_1\approx 0$$

$$V_2=\sqrt{2gh}$$

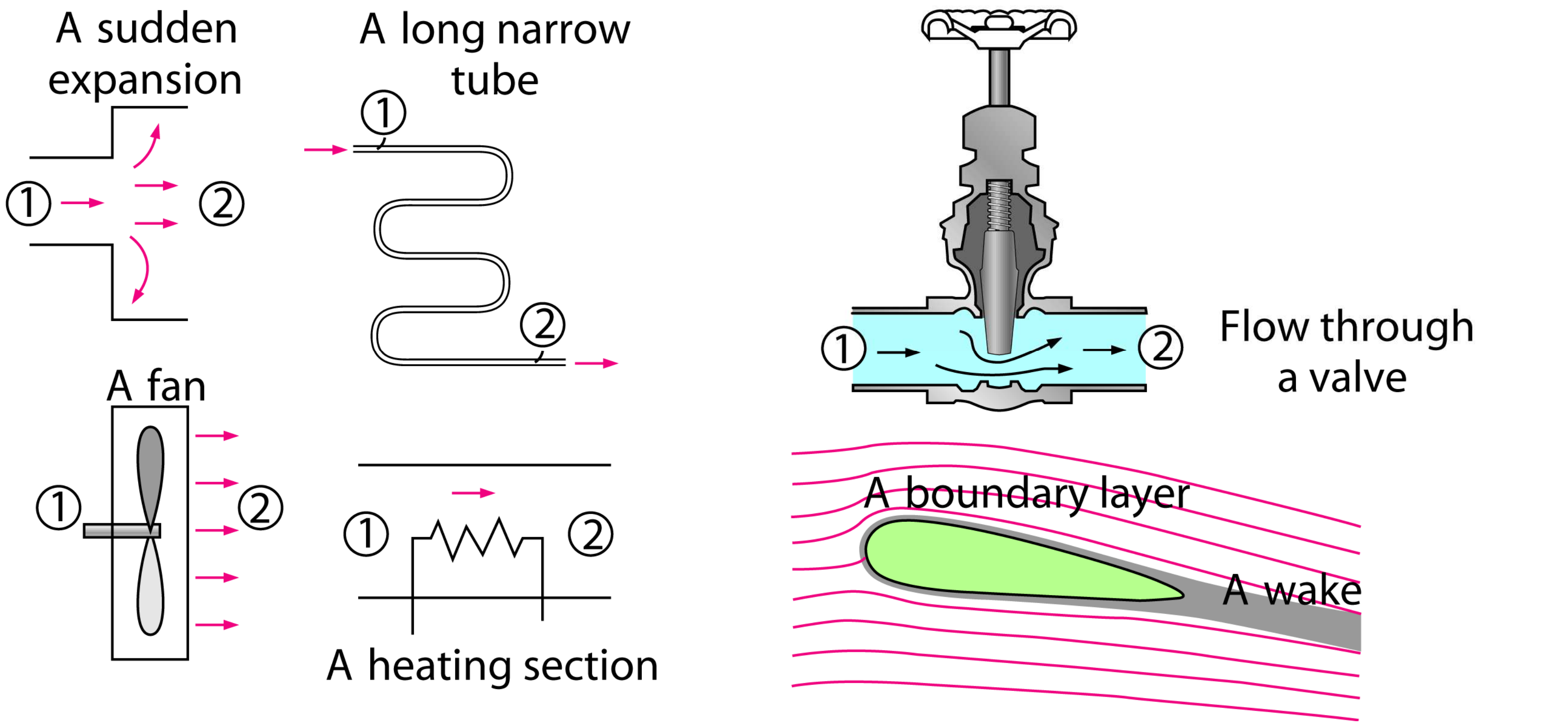


Limitations on Use of Bernoulli Equation

- Assumptions for Bernoulli Equation
  - Steady flow
  - Incompressible flow ⇒ acceptable if flow Mach number is less than 0.3
  - Frictionless (inviscid) flow ⇒ solid walls, wakes downstream of an object, diverging flow sections (diffusers) and flow through long and narrow passages introduce frictional effects
  - Flow along a streamline
  - No shaft work ⇒ pumps, turbines, fans and other fluid machinery carry out energy interactions with fluid particles ⇒ mechanical energy no longer conserved along streamline
  - No heat transfer

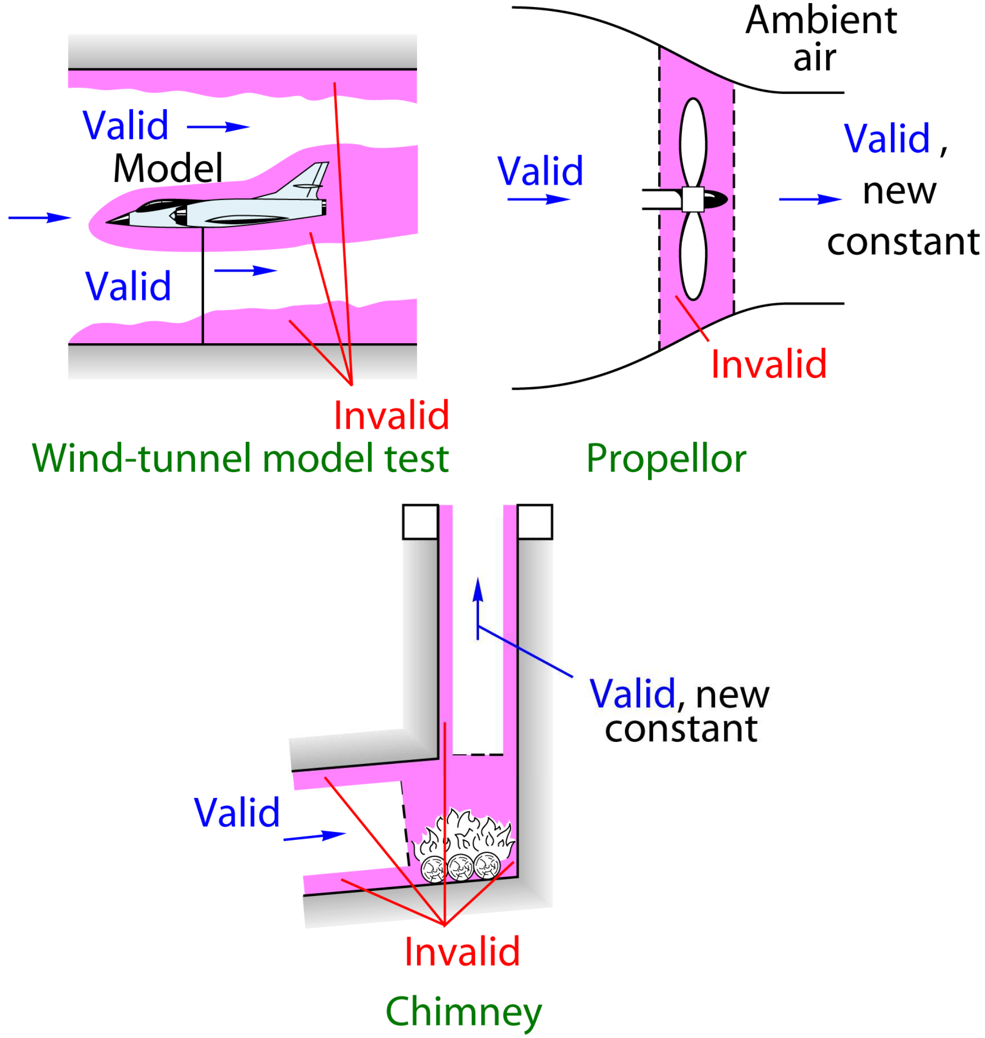
Limitations on Use of Bernoulli Equation

- Examples where use of Bernoulli equation is invalid:



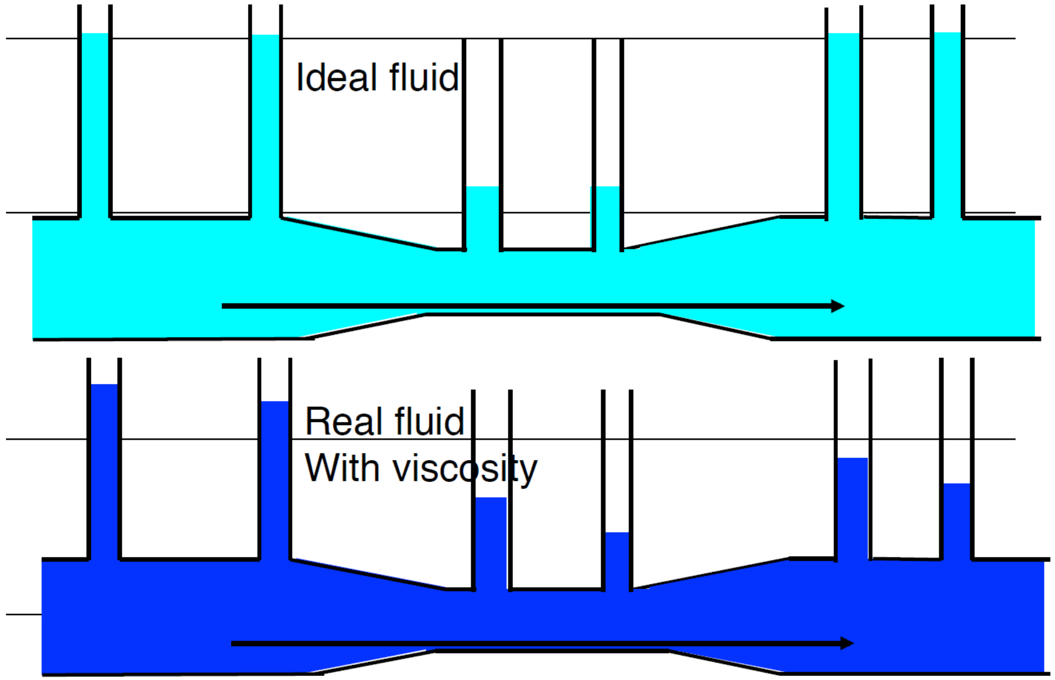
Limitations on Use of Bernoulli Equation

- Examples where use of Bernoulli equation is invalid:



Extended Bernoulli Equation

- Viscous Effect
  - Frictional/viscous force converts mechanical energy into thermal energy
  - It corresponds to a rise in the internal energy of the fluid (heat up the fluid) or to the heat that is lost to the surroundings



Extended Bernoulli Equation

- Viscous Effect
  - Introduce head loss h_f due to viscous force into the original Bernoulli equation
  - h_f is an empirical parameter.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 + h_f$$

Extended Bernoulli Equation

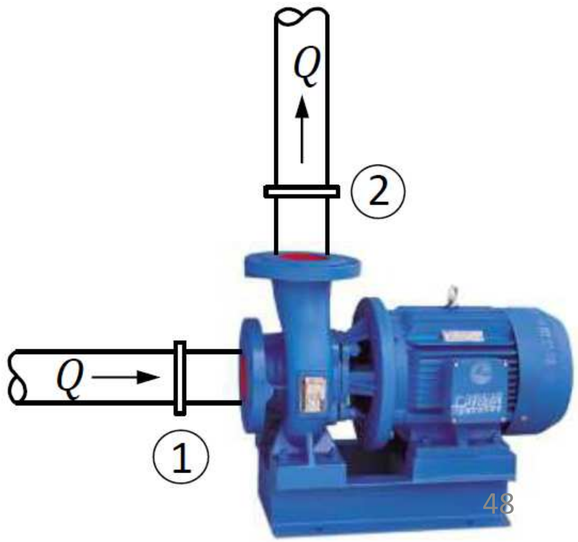
- Pump Work
  - Pump converts mechanical energy into hydraulic energy
  - Pump head h_s can be introduced to Bernoulli equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 - h_s$$

  - Pump head h_s is related to the power delivered to the fluid by the pump (P_f) as follows

$$P_f = \rho g Q h_s$$

where Q is the volumetric flow rate that passes through the pump.



Extended Bernoulli Equation

- Compressible fluid
  - Ideal gas at adiabatic condition

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\frac{P_1}{\rho_1^{\gamma}} = \frac{P_2}{\rho_2^{\gamma}}$$

$$\frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} + \frac{1}{2}v_1^2 + gz_1 = \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2} + \frac{1}{2}v_2^2 + gz_2$$

v is velocity
V is volume

Review

– Bernoulli Equation

$$\frac{p}{\rho} \quad + \quad \frac{1}{2}V^2 \quad + \quad gz = \text{constant}$$

Flow energy Kinetic energy Potential energy

$$P \quad + \quad \frac{1}{2}\rho v^2 \quad + \quad \rho g z = \text{constant}$$

Static pressure Dynamic pressure Hydrostatic pressure

- Bernoulli equation is valid for steady, incompressible flow along a streamline in an "inviscid regions of flow"

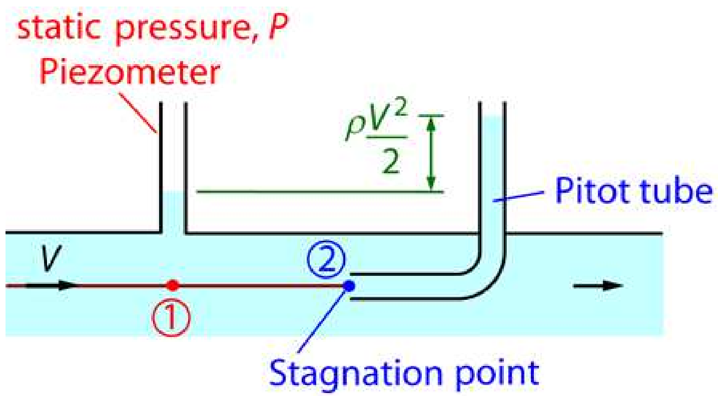
Review

– Stagnation Pressure : sum of static and dynamic pressure

$$P_{stag} = P + \rho \frac{V^2}{2}$$

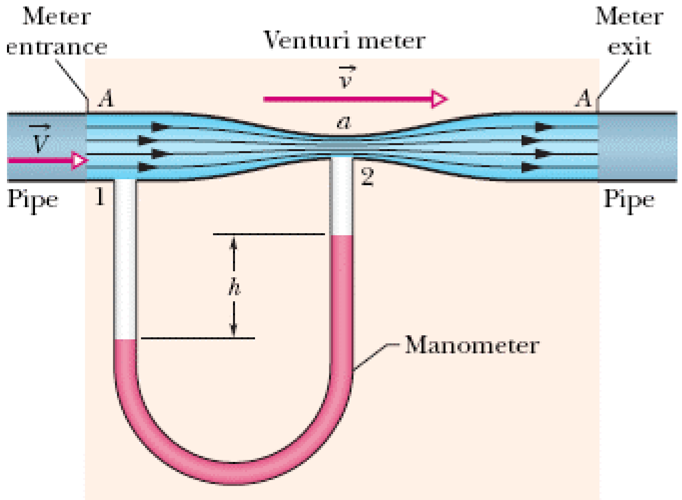
– Velocity Measurement

$$V = \sqrt{\frac{2\left(P_{stag} - P\right)}{\rho}}$$



Review

– Venturi Effect: the reduction in fluid pressure that results when a fluid flows through a constricted section (or choke) of a pipe



– Flow rate measurement:

$$Q = A_1 \sqrt{\frac{2}{\rho_v} \bullet \frac{(P_1 - P_2)}{\left(\frac{A_1}{A_2}\right)^2 - 1}} = A_2 \sqrt{\frac{2}{\rho_v} \bullet \frac{(P_1 - P_2)}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

Review

– Extended Bernoulli Equation

- Viscous Effect

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 + h_f$$

- Pump Work

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 - h_s$$

- Compressible fluid

$$\frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} + \frac{1}{2}v_1^2 + g z_1 = \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2} + \frac{1}{2}v_2^2 + g z_2$$


Thank You for Your Attention!
Any Questions?