SSY156 - Modelling and Control of Mechatronic Systems
Peer-2-Peer Homework 04

Question 1 (2 points)

First, we calculate K_m and T_m:

$$K_m = \frac{k_t}{F_m} = \frac{0.5}{0.25} = 2\,\mathrm{rad/(V \cdot s)} \tag{1}$$

$$T_m = \frac{I_m}{F_m} = \frac{1.8}{0.25} = 7.2\,\mathrm{s} \tag{2}$$

Considering that the pole-zero strategy was adopted, the disturbance rejection factor (DRF) is given by:

$$DRF = K_P K_V \tag{3}$$

and Output Recovery Time (ORT) is given by:

$$ORT = max\{Tm, \frac{1}{\zeta\omega_n}\} = max\{Tm, \frac{2}{K_V k_m}\} \tag{4}$$

The tricky part here is that we request the closed-loop system to present ORT=7.2, which is the same value of Tm. In this case, ORT can only assume 7.2 if and only if $\frac{2}{K_V k_m} < T_m$. The second information given in the question is that the Disturbance Rejection Factor (DRF) must be 0.0013, which implies that $K_P K_V = 0.0013$.

Therefore, any K_P and K_V are valid if they respect the following equations:

$$K_P K_V = 0.0013 \tag{5}$$

$$\frac{2}{K_V k_m} < T_m \quad \Rightarrow \quad K_V > \frac{2}{T_m k_m} = 0.1389 \tag{6}$$

Question 2

According to equation (4), the minimum value that ORT can assume is equal T_m, which is equal to 7.2.

This can be confirmed with the following surface plots:

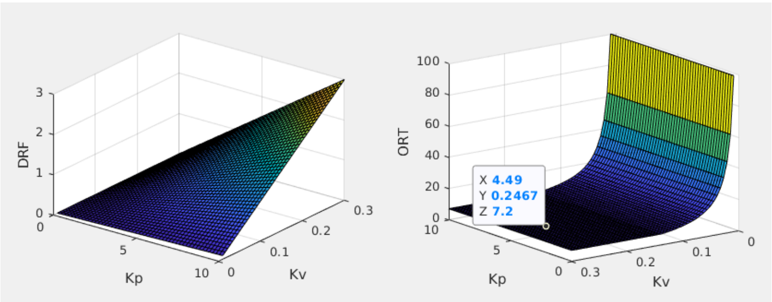

***Figure 1:** DRF and ORT as function of Kp and Kv*

It is important to notice that the output recovery time is limited by T_m, which is the time constant of the drive system.

Question 3

As can be seen in the plots of figure 2, as we increase K_P the disturbance rejection factor will increase and therefore the system will be less affected by the disturbances.

Question 4

If a pole-zero cancellation technique ($T_v = T_m$) is used, the natural frequency and the damping factor of the closed-loop system are given by:

$$K_V = \frac{2\zeta\omega_n}{k_m} \tag{7}$$

$$K_P K_V = \frac{\omega_n^2}{k_m} \tag{8}$$

which implies that:

$$\zeta = \frac{K_V k_m}{2\omega_n} = \frac{K_V k_m}{2\sqrt{K_P K_V k_m}} = \frac{\sqrt{K_V k_m}}{2\sqrt{K_P}} \tag{9}$$

$$\omega_n = \sqrt{K_P K_V k_m} \tag{10}$$

And therefore we can check how the controller parameters influence the closed-loop performance:

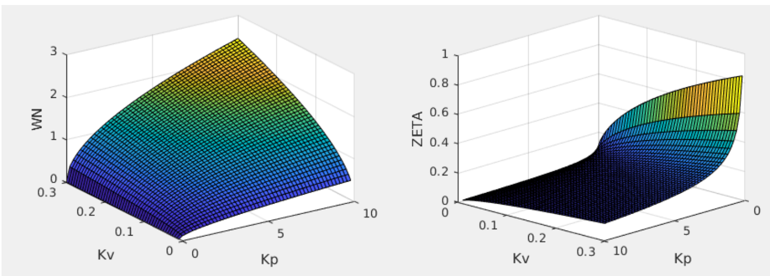

Figure 2: *Close-loop natural frequency and damping factor as function of K_P and K_V*

As can be easily verified, as we increase K_P, the natural frequency will increase and the damping factor will decrease. However, the time constant of the closed-loop system ($\tau = 1/(\zeta\omega_n)$) does not depend on K_P and therefore K_P will not influence the stabilization time of the system. K_P can be used to change, for instance, the damping ratio of the system and consequently the overshoot.