

Important Info for the Lab Assignment evaluation: When you submit answers for each block of questions, they will be graded and you can get from 0 to Total Points. After the evaluation, we will send you back the report, so you can correct the wrong answers. Of course, this correction will not affect the grade that you get in your first submission; it is only intended to give you feedback. In this way you can continue to work on the next parts of the assignment, avoiding that wrong answers in a block will affect the upcoming one.

Assignment Part 2 - Modeling, simulation and identification

Write a report presenting the answers to the following questions. The answers should contain equations and simulation plots where needed. At the end of each block of questions you will find the deadline for submission. Use the section Assignments on PingPong to submit the report (file accepted: pdf).

Question 1 2 points
Write down the generalized coordinates and the generalized forces for the Omni bundle from Lab 1.

Question 2 8 points
Derive the Lagrangian of the system, using the generalized coordinates of the robot. Write the Lagrangian of the system in your report. The kinetic energy and potential energy due to gravity and their derivation (i.e. [link Jacobians, velocity of mass centers and angular velocity of links](#)) must be explained concisely in your report.
Use the coordinate frames suggested for the kinematics, in order to understand positive/negative directions and possible offsets of the variables. The center of mass of each link is located in the center of the link. The inertia matrix of each link should be considered diagonal.

Question 3 5 points
Derive the equations of motion using the Euler-Lagrange method and, in the report, write down the following:
(a) (1 point) Gravity on joint 1
(b) (1 point) The diagonal element (3,3) of inertia matrix
(c) (1 point) Coriolis effect induced on Joint J_2 by velocities of Joints J_2 and J_3 ;
(d) (1 point) Centrifugal effect induced on Joint J_3 by the velocity of Joint J_1 ;
(e) (1 point) Coriolis effect induced on Joint J_3 by velocities of Joints J_1 and J_2 .

Question 4 10 points
(a) (3 points) Using the equations of motion derived in previous question, find a suitable and possibly minimal parameterization, which is linear in the dynamical parameters:

$$\tau = \mathbf{Y}(q_i, \dot{q}_i, \ddot{q}_i)\pi, \quad (1)$$

where π is the parameter vector depending on the dynamical parameters, τ is the input torque applied to the joints and \mathbf{Y} is the regression matrix depending on position, velocity, acceleration of the coordinates and on kinematic parameters. Known parameters and variables as well as unknown parameters must be provided in a table.

- (b) (2 points) Write down the sub-arrays with dimension 3×1 of matrix \mathbf{Y} that contain the gravity acceleration parameter g . Also, write down their corresponding unknown parameters.
- (c) (3 points) B) In lab: Perform an experiment with the robot in the lab. Use the file identification.slx (uploaded on pingpong), which implements a simple controller. The block takes as input some reference trajectories in the joint space and provides as output the actual (measured) values of joint angles, velocities, accelerations, and applied torques. The reference trajectories to implement are:

$$q_1(t) = 0.1333 \left(\sin\left(\frac{\pi}{2}t\right) + \sin(\pi t) + \sin(2\pi t) \right) \quad (2)$$

$$q_2(t) = -0.8 + 0.0667 \left(\sin\left(\frac{\pi}{2}t\right) + \sin(\pi t) + \sin(2\pi t) \right) \quad (3)$$

$$q_3(t) = 2.8 + 0.2 \left(\sin\left(\frac{\pi}{2}t + \frac{\pi}{5}\right) + \sin\left(\pi t + \frac{\pi}{5}\right) + \sin\left(2\pi t + \frac{\pi}{5}\right) \right) \quad (4)$$

Place the robot on point 5 on the board, run the experiment and save the applied torques (τ) and the measured values for position, velocity and acceleration of the joints in order to build the matrix \mathbf{Y} .

(d) (2 points) With this information, compute the vector π using the least squares technique:

$$\pi = (\bar{\mathbf{Y}}^T \bar{\mathbf{Y}})^{-1} \bar{\mathbf{Y}}^T \bar{\tau} \quad (5)$$

where

$$\bar{\mathbf{Y}} = \begin{bmatrix} Y(t_1) \\ \vdots \\ Y(t_n) \end{bmatrix} \quad \bar{\tau} = \begin{bmatrix} \tau(t_1) \\ \vdots \\ \tau(t_n) \end{bmatrix} \quad (6)$$

and N is the number of data from the experiment. In the report, write the parameter vector π and the result of the identification procedure.

Question 5 5 points

Use Matlab/Simulink to simulate the derived model, using the estimated parameters of question 4. Answer the following question (in the report, motivate your answers using simulation plots):

How does the system behave when only gravitational force is applied to it?

Hint: First simulate the manipulator with the initial position, where it is completely hanged. After, try to move it with some small angles, and simulate again.

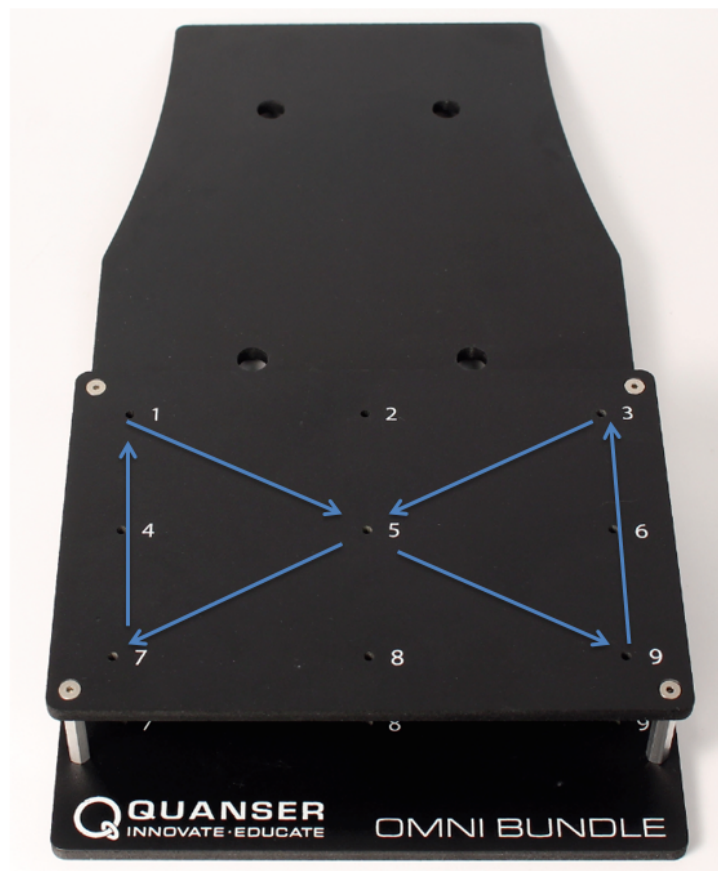


Figure 1: Omni Bundle