SSY156 - Modelling and Control of Mechatronic Systems
HW2 Solution to the Assignment Problem

February 11, 2019

Question a

The Table-1 shows the DH parameters for the manipulator given in the task.

Table 1: *DH parameters Table*

$Link(i)$	a_i	α_i	d_i	$\theta_i(q_i)$
1	0	$\frac{\pi}{2}$	0	q_1
2	L_2	0	0	q_2
3	L_3	0	0	q_3

The Geometric Jacobian is then given by applying the formulas from the lecture notes:

$$J = \begin{bmatrix} -1.0 \sin(q_1)\,(L_3 \cos(q_2+q_3) + L_2 \cos(q_2)) & -1.0 \cos(q_1)\,(L_3 \sin(q_2+q_3) + L_2 \sin(q_2)) & -1.0\,L_3 \sin(q_2+q_3)\cos(q_1) \\ \cos(q_1)\,(L_3 \cos(q_2+q_3) + L_2 \cos(q_2)) & -1.0 \sin(q_1)\,(L_3 \sin(q_2+q_3) + L_2 \sin(q_2)) & -1.0\,L_3 \sin(q_2+q_3)\sin(q_1) \\ 0 & L_3 \cos(q_2+q_3) + L_2 \cos(q_2) & L_3 \cos(q_2+q_3) \\ 0 & \sin(q_1) & \sin(q_1) \\ 0 & -1.0 \cos(q_1) & -1.0 \cos(q_1) \\ 1.0 & 0 & 0 \end{bmatrix} \tag{1}$$

Question b

In order to find the singularity points of the manipulator, we simply check for which joints angles the linear part of the Jacobian matrix is singular. Considering $L = L_2 = L_3$:

$$\det(J_p) = -\,L^3 * (\cos(q_3) + 1) * (\sin(q_2+q_3) - \sin(q_2)) = 0 \tag{2}$$

Therefore, if we consider $L > 0$, then we conclude that the Jacobian is singular for the following cases:

- $\sin(q_2 + q_3) = \sin(q_2)$
- $cos(q_3) = -1 \quad \Rightarrow \quad q_3 = \pi + 2\pi n \quad \text{where} \quad n = 0, \pm1, \pm2, ...$

Question c

If the end-effector can move in only one direction, the movement is constrained and the Jacobian must be singular. Using a geometrical interpretation, it is clear that the described situation only happens when $q_2 = \pm\pi/2$ and $q_3 = 0$. It can then be verified that these solutions satisfy $det(J_P) = 0$.

Grading

Grade: 3

The exercise covers well the lectures 5 to 7. The exercise is relatively difficult and it is an interesting problem.