SSY156 - Modeling and Control of Mechatronic Systems
Winter 2019
Laboratory Assignment 1 - Kinematics
Due date: 06/02/2018 @23:55
Total Points: 20 p
MAX NUMBER OF PAGES: 4 (excluding the appendix with the matlab functions)
Important Info for the Lab Assignment evaluation: When you submit answers for each block of questions, they will be graded and you can get from 0 to Total Points. After the evaluation, we will send you back the report, so you can correct the wrong answers. Of course, this correction will not affect the grade that you get in your first submission; it is only intended to give you feedback. In this way you can continue to work on the next parts of the assignment, avoiding that wrong answers in a block will affect the upcoming one. Hand-written reports are not accepted.

Purpose of the laboratory assignment

The purpose of this lab assignment is to practice with approaches to modeling and control of multi-body mechanical systems.

General outline

This lab assignment is based on the use of simulation software. Matlab Simulink is recommended. The assignment consists of modeling and control tasks. The modeling tasks mainly consist in deriving equations of motion of a mechanical system, which is described next in details, using the Euler-Lagrange method. Then, Simulink models of the system have to be developed for simulation and validation purposes. The motion control problems will be solved based on the developed mathematical models.

The OmniBundle manipulator

The system used in this assignment will be the 3D Systems Geomagic Touch (formerly Sensable Technologies PHANTOM Omni), see Fig.1. The Omni is a robot with six revolute joints. However, only three of the joints are actuated. The three actuated joints J_1, J_2 and J_3 are shown in Fig.1. Also, physical parameters for the system are listed in Table 1.

Figure 1: The OmniBundle manipulator

Parameter	value
L_1	0.132 m
L_2	0.132 m
m_1 (mass joint J_1)	- kg
m_2 (mass link L_1)	- kg
m_3 (mass link L_2)	- kg
r_1 (radius of joint J_1)	0.03 m

Table 1: physical parameters of the OmniBundle
OBS: The masses for the OmniBundle are not specified, but their value is not needed for the first part of the assignment.

Assignment Part 1 - Kinematics

Write a report presenting the answers to the following questions. The answers should contain equations and simulation plots where needed. MATLAB functions should be included in an appendix at the end of the report. Use the section Assignments on PingPong to submit the report (file accepted: pdf).

Question 1 . . . . *3 points*

Use the coordinate frames given in Fig.2 to derive the forward kinematics of the OmniBundle manipulator. The purpose is to determine the position of the end-effector as functions of the joint positions. In your report present also the DH parameters and describe briefly the steps for the derivation. Develop a Matlab function that computes the forward kinematics and compute and report the position of the end-effector in the following cases: $[q_1, q_2, q_3] = [0.67, -0.15, 2.7], [-0.73, 0.25, 1.5]$.

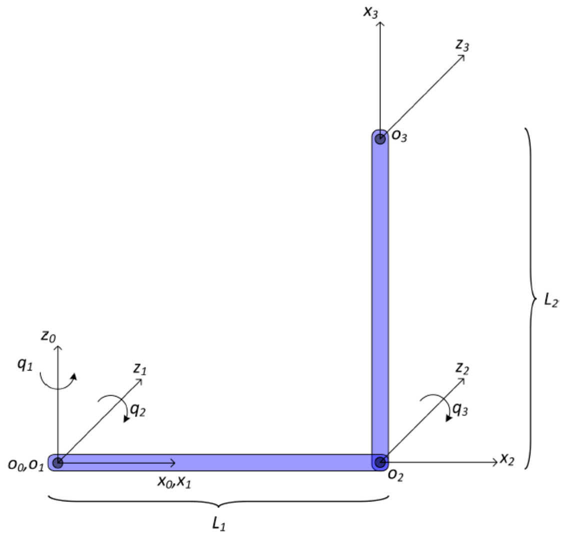

Figure 2: Coordinate frames for the Manipulator

Question 2 . . . . *4 points*

Derive the inverse kinematics of the OmniBundle manipulator. The purpose is to determine the position of each joint given the position of the end-effector. You can use Fig.3 to geometrically derive the inverse kinematics equations. The position of the end-effector is indicated by the point F_3. Note that, in Fig. 3, triangle Zdk is in the vertical $(x-z)$ plane and triangle YXd is in the horizontal plane $(x-y)$ plane. Develop a Matlab function that computes the inverse kinematics and compute and report the value of $[q_1, q_2, q_3]$ corresponding to the following positions in the operational space: $[X, Y, Z] = [0.06, 0.04, 0.02], [0.0847, 0.0123, -0.005]$ m.

Hint: we want to derive q_1, q_2 and q_3 as function of the position in the x_0, y_0, z_0 frame of the point F_3. Use cosine and sine law to find q_3 and q_2. In particular use the L1L2k triangle.

Question 3 . . . . *4 points*

Find the relation between the joint velocities and linear velocity of the end-effector and the joint velocities and the derivatives of the Euler Angles ZYZ, i.e. $\dot{\phi}$, of the end-effector, that are the Jacobian matrices $J_p(q)$ and $J_\phi(q)$ such that:

$$\dot{p}_e = J_p(q)\dot{q} \quad (1)$$

$$\dot{\phi} = J_\phi(q)\dot{q} \quad (2)$$

Describe briefly the steps for your derivations.

Question 4 . . . . *9 points*

A KUKA iiwa 14 manipulator along with an object is modeled using MATLAB/SimMechanics (Fig 4). The D-H parameters are listed in table 2. Also, the frames are shown in Fig. 5.

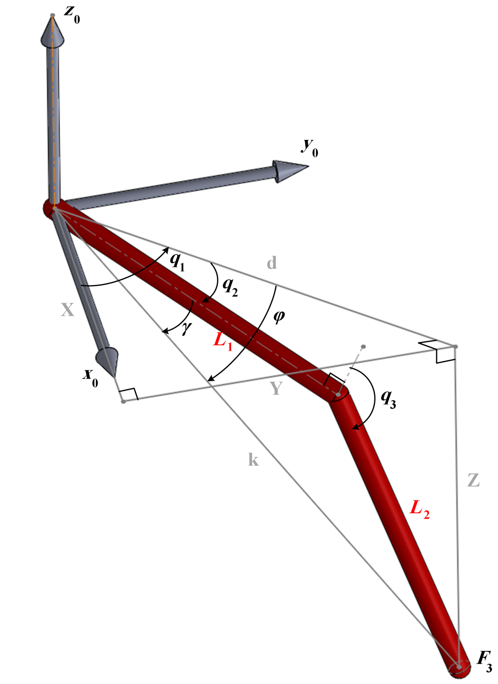

Figure 3: Geometry to derive inverse kinematics

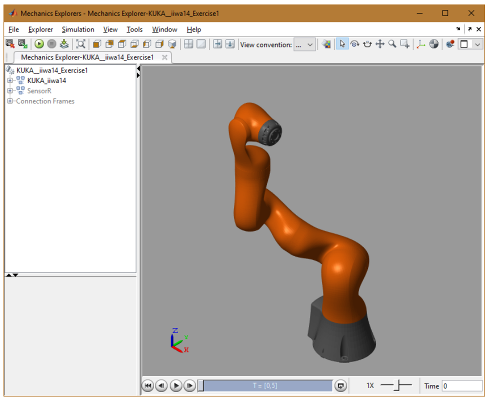

Figure 4: KUKA iiwa 7 modeled in SimMechanics/Simulink

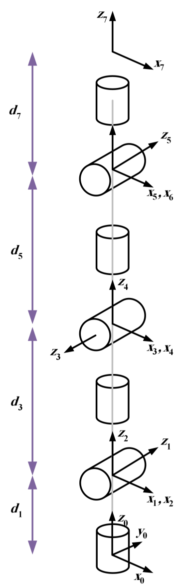

Figure 5: Coordinate frames for KUKA iiwa 7

Link	a_i	α_i	θ_i	d_i
1	0	$\frac{-\pi}{2}$	θ_1	$d_1 = 0.36$
2	0	$\frac{\pi}{2}$	θ_2	0
3	0	$\frac{\pi}{2}$	θ_3	$d_3 = 0.42$
4	0	$\frac{-\pi}{2}$	θ_4	0
5	0	$\frac{-\pi}{2}$	θ_5	$d_5 = 0.4$
6	0	$\frac{\pi}{2}$	θ_6	0
7	0	0	θ_7	$d_7 = 0.131$

Table 2: D-H parameters of KUKA iiwa 7.

(a) (4 points) Using the provided files through PingPong and symbolic toolbox of MATLAB, calculate the Jacobian and plot the required joint angles in order to move the manipulator with constant speed along x-direction. The initial position is ($q = [\pi/6, -\pi/3, 0, -\pi/3, -\pi/6, \pi/2, \pi/2]^T$). There is no need to write the Jacobian matrix in your solution. Just plot the joint angles and provide 3 top-view snapshots of the manipulator in the simulator. Please notice that you have to keep the orientation constant for this question.

(b) (2 points) Assume that the end-effector is at the same initial position of the previous question (a). Using the differential kinematics, derive the required joint angles and velocities for the given linear velocity and position:

$$p = \begin{bmatrix} -0.2 \\ -0.0008t^3 + 0.012t^2 - 0.2 \\ 1 \end{bmatrix} \tag{3}$$

with $t \in [0, 10]$ (sec). There is no need to keep the orientation constant. In the report, show plots of joints angles and velocities.

(c) (3 points) Find two solution for an inverse kinematic of the given manipulator for the position of $[-0.2, 0.2, 1]^T$. Give a brief explanation of your answer.
Hint: Use the result from question 4.b.