SSY156 - Modelling and Control of Mechatronic Systems
Peer-2-Peer Homework 01
Group06_HW06_Solution

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Question 1

First, we define an auxiliary axis AUX with the same orientation of O but centered between R_A and L_A, i.e. in the center of the robot. In this way, the transformation matrix of AUX to O will be easily a translation to the center of the robot and a rotation of θ:

$$A^O_{AUX} = \begin{bmatrix} & & & -L_3 \\ & R_z(\theta) & & L_2 \\ & & & L_6 \\ & \mathbf{0}_3^T & & 1 \end{bmatrix} = \begin{bmatrix} C_\theta & -S_\theta & 0 & -L_3 \\ S_\theta & C_\theta & 0 & L_2 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The arm axes can be then described with help of A^O_{AUX} and a simple rotation and translation:

$$A^O_{L_A} = A^O_{AUX} * A^{AUX}_{L_A} = A^O_{AUX} * \begin{bmatrix} & & & 0 \\ & R_x(-\frac{\pi}{2}) & & \frac{L_5}{2} \\ & & & 0 \\ & \mathbf{0}_3^T & & 1 \end{bmatrix} = \begin{bmatrix} C_\theta & 0 & -S_\theta & -L_3 - S_\theta \cdot \frac{L_5}{2} \\ S_\theta & 0 & C_\theta & L_2 + C_\theta \cdot \frac{L_5}{2} \\ 0 & -1 & 0 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The other arm can be defined similarly:

$$A^O_{R_A} = A^O_{AUX} * A^{AUX}_{R_A} = A^O_{AUX} * \begin{bmatrix} & & & 0 \\ & R_x(\frac{\pi}{2}) & & \frac{-L_5}{2} \\ & & & 0 \\ & \mathbf{0}_3^T & & 1 \end{bmatrix} = \begin{bmatrix} C_\theta & 0 & S_\theta & -L_3 + S_\theta \cdot \frac{L_5}{2} \\ S_\theta & 0 & -C_\theta & L_2 - C_\theta \cdot \frac{L_5}{2} \\ 0 & 1 & 0 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Further, since $A^{L_A}_{L_H}$ and $A^{R_A}_{R_H}$ are given, the hand positions are straight forward to calculate using the previous calculated matrices:

$$A^O_{R_H} = A^O_{R_A} * A^{R_A}_{R_H} \tag{4}$$
$$A^O_{L_H} = A^O_{L_A} * A^{L_A}_{L_H} \tag{5}$$

The head position is then given by one translation and two rotations with respect to AUX:

$$A_C^O = A_{AUX}^O * A_C^{AUX} = A_{AUX}^O * \left[\begin{array}{c|c} R_y(\frac{\pi}{2}) & \begin{matrix} L_4 \\ 0 \\ L_7 \end{matrix} \\ \mathbf{0}_3^T & 1 \end{array}\right] * \left[\begin{array}{c|c} R_z(\frac{\pi}{2}) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \mathbf{0}_3^T & 1 \end{array}\right]$$

$$= A_{AUX}^O * \begin{bmatrix} 0 & 0 & 1 & L_4 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & L_7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_{AUX}^O * \begin{bmatrix} 0 & 0 & 1 & L_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & L_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_C^O = \begin{bmatrix} -S_\theta & 0 & C_\theta & L_4 \cdot C_\theta - L_3 \\ C_\theta & 0 & S_\theta & L_2 + L_4 \cdot S_\theta \\ 0 & 1 & 0 & L_6 + L_7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The last matrix and the easiest one is a pure translation:

$$A_{O_b}^O = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 1 & z_{ob} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Question 2

Given the object coordinate in respect to Ob: $[x_{ob}\ y_{ob}\ z_{ob}]$ in {O}, we can calculate the object coordinate in $\{R_A\}$ with help of the transformation matrix $A_{R_A}^O$ calculated in the last item:

$$\begin{aligned} p_{Ob}^O &= [x_{ob}\ y_{ob}\ z_{ob}] \\ p_{Ob}^{R_A} &= A_O^{R_A} * p_{Ob}^O = \left(A_{R_A}^O\right)^{-1} * p_{Ob}^O \end{aligned} \tag{8}$$

$$A_{R_A}^O = \left[\begin{array}{ccc|c} \overbrace{C_\theta \quad 0 \quad S_\theta}^{R_{R_A}^O} & & & \overbrace{-L_3 + S_\theta \cdot \frac{L_5}{2}}^{p_{R_A}^O} \\ S_\theta & 0 & -C_\theta & L_2 - C_\theta \cdot \frac{L_5}{2} \\ 0 & 1 & 0 & L_6 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{9}$$

$$\left(A_{R_A}^O\right)^{-1} = \left[\begin{array}{c|c} R_O^{R_A} & -R_O^{R_A} * p_{R_A}^O \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} C_\theta & S_\theta & 0 & C_\theta(L_3 - \frac{(L_5 \cdot S_\theta)}{2}) - S_\theta(L_2 - \frac{C_\theta}{2}) \\ 0 & 0 & 1 & -L_6 \\ S_\theta & -C_\theta & 0 & S_\theta(L_3 - \frac{(L_5 \cdot S_\theta)}{2}) + C_\theta(L_2 - \frac{C_\theta}{2}) \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{10}$$

$$p_{Ob}^{R_A} = \left(A_{R_A}^{O}\right)^{-1} * p_{Ob}^{O} = \left(A_{R_A}^{O}\right)^{-1} * \begin{bmatrix} x_{ob} \\ y_{ob} \\ z_{ob} \\ 1 \end{bmatrix} \tag{11}$$

$$p_{Ob}^{R_A} = \begin{bmatrix} C_\theta \cdot x_{ob} - S_\theta \cdot (L_2 - \frac{C_\theta}{2}) + S_\theta \cdot z_{ob} + C_\theta \cdot (L_3 - \frac{(L_5 \cdot S_\theta)}{2}) \\ S_\theta \cdot x_{ob} - C_\theta \cdot z_{ob} - L_6 \\ y_{ob} + C_\theta \cdot (L_2 - \frac{C_\theta}{2}) + S_\theta \cdot (L_3 - \frac{(L_5 \cdot S_\theta)}{2}) \\ 1 \end{bmatrix} \tag{12}$$

Question 3

Assuming $L_1 = L_3 = L_5 = L_6 = 1$ m. and $\theta = 0$, we can write down the orientation of $\{L_A\}$ with respect to $\{Ob\}$ using quaternion with the help of rotation matrix between two frames.

We obtain the rotation matrix by observing how the axes of the two frames are related.

$$R_{LA}^{Ob} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{13}$$

In order to obtain the quaternion parameters, apply the equations of the inverse problem using the data of the rotation matrix. In this case we apply the Matlab function `rotm2quat` to the matrix.

$$\eta = \frac{1}{2}\sqrt{r_{11} + r_{22} + r_{33} + 1} = 0.7071 \tag{14}$$

$$\epsilon = \frac{1}{2} \begin{bmatrix} \operatorname{sgn}(r_{32} - r_{23})\sqrt{r_{11} - r_{22} - r_{33} + 1} \\ \operatorname{sgn}(r_{13} - r_{31})\sqrt{r_{22} - r_{33} - r_{11} + 1} \\ \operatorname{sgn}(r_{21} - r_{12})\sqrt{r_{33} - r_{11} - r_{22} + 1} \end{bmatrix} = \begin{bmatrix} -0.7071 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$