SSY156 - Modelling and Control of Mechatronic Systems
HW4 Solution to the Assignment Problem

Considerations

We are considering that when you say that the outer loop should control the "position", you have meant volume of the tank in m^3, since the cross-section area was not given.

Part A

This problem can be easily solved by applying the same procedure of decentralized control learned in class. Further, since the DoF of the system is 1, we do not even need to decouple the system. First, the transfer function can be written in the Laplace domain:

$$M\ddot{q} + B\dot{q} = u \quad \Rightarrow \quad \frac{q}{u} = \frac{B^{-1}}{s\left(1 + \dfrac{M}{B}s\right)} = \frac{k_m}{s\,(1 + T_m s)} \tag{1}$$

where $k_m = B^{-1}$ and $T_m = M/B$ constants were introduced to facilitate the notation.

We then propose the PI velocity feedback and the P position feedback of the following form:

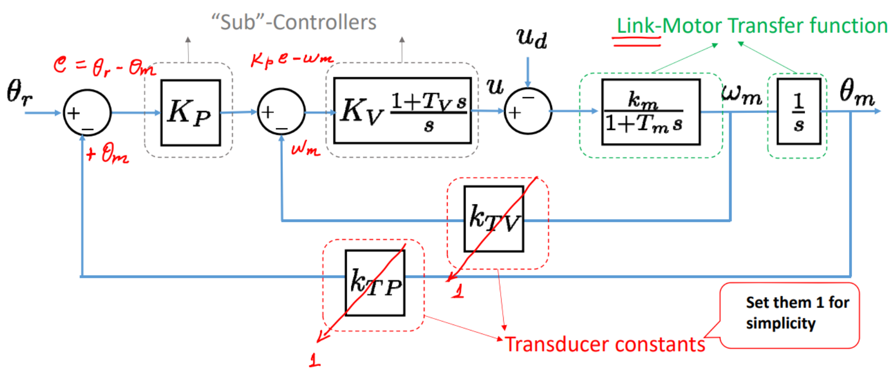

Figure 1: *Control Scheme with position and velocity feedback*

where K_V, K_P and T_V are the controller parameters.

After choosing the controller parameter $T_V = T_m$ (pole-zero cancellation method), we can simplify write the closed-loop transfer function H_{CL} of the system as:

$$H_{CL} = \frac{1}{1 + \dfrac{s}{K_P} + \dfrac{s^2}{k_m K_P K_V}} = \frac{1}{1 + \dfrac{2\zeta s}{\omega_n} + \dfrac{s^2}{w_n^2}} \tag{2}$$

where ω_n and ζ are the desired closed-loop natural frequency and damping coefficient. We then choose $\zeta = 1$ and $\omega_n = 10rad/s$ as good start values for the desired parameters, which will lead to the following controller parameters:

$$T_V = T_m = M/B = 20 \tag{3}$$
$$K_V = 2\zeta\omega_n/k_m = 2\zeta\omega_n B = 10 \tag{4}$$
$$K_P = \omega_n^2/(K_V k_m) = 5 \tag{5}$$

Bellow, follows the step response of the closed-loop system:

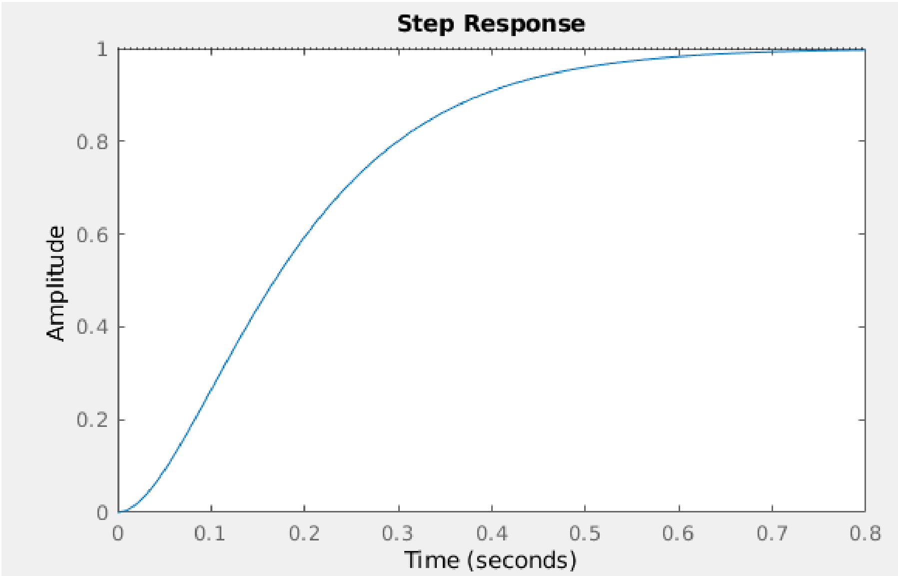

Figure 2: *Step response of the closed-loop system*

Part B

We can start rewriting the closed-loop coefficients in terms of the controller parameters (considering however $T_V = T_m$ constant to allow pole-zero cancellation and simplify the analyze):

$$\zeta = \frac{K_V k_m}{2\omega_n} = \frac{K_V k_m}{2\sqrt{K_P K_V k_m}} = \frac{\sqrt{K_V k_m}}{2\sqrt{K_P}} \tag{6}$$
$$\omega_n = \sqrt{K_P K_V k_m} \tag{7}$$

The time constant of the system can be then written as:

$$\tau = \frac{1}{\zeta\omega_n} = \frac{2}{K_V k_m} \tag{8}$$

Now, it can be easily seen that changing K_P will not change how fast the system responses, but it will change the damping coefficient and natural frequency. This parameter can then be used to adjust the desired overshoot of the system or change the closed-loop behaviour to under-/over-damped.

On the other hand, increasing the parameter K_V will make the system react faster and clearly also change the damping ratio or natural frequency of the system.

Therefore, the controller parameters K_V and K_P can be chosen wisely, so the final system presents the desired performance.

Grading

Grade: 3

The problem presented is very interesting and although it is not directly related to robotics, it requires knowledge of one of the control techniques learned in class.