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Homework exercise 1: Solution sheet

Please use this sheet to fill in your results and hand it in together with the rest of your solution, i.e. your commented written calculations. Notice that giving only the results will not be credited.

a)

Discretized cost functional:

$$J \approx h \cdot \left[\sum_{k=0}^{N-1} x_k^T \cdot Q \cdot x_k + u_k^T \cdot R \cdot u_k \right]$$

b) No results have to be filled in.

c)

Discretized differential equation:

$$x_{k+1} = (I + hA)\, x_k + (B \cdot h) u_k$$

d)

$$H = \begin{bmatrix} Q & & & & 0 \\ & Q & & & \\ & & \ddots & & \\ & & & R & \\ 0 & & & & R \end{bmatrix} \cdot 2h$$

$$f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad d = 0$$

$$
A_{\mathrm{eq}} = \left[\begin{array}{c|cccccc}
 & 0 & 0 & 0 & \cdots & 0 & 0 \\
 & -B_D & 0 & 0 & \cdots & 0 & 0 \\
I_{n.N} & -A_D.B_D & -B_D & 0 & \cdots & 0 & 0 \\
 & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 & -A_D^{N-2}.B_D & -A^{N-3}.B_D & & \cdots & -B_d & 0
\end{array}\right]
$$

$$
b_{\mathrm{eq}} = \begin{bmatrix} I \\ A_D \\ A_D^2 \\ \vdots \\ A_D^{N-1} \end{bmatrix} . X_0
$$

$$
A_{\mathrm{ineq}} = \begin{bmatrix} 0, & 0, & \ldots, & 0 \end{bmatrix}
$$

$$
b_{\mathrm{ineq}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

e)

Optimal solution:

$$y^* = -H^{-1} \cdot \left(A_{eq}^T \cdot \pi^* + f \right)$$

where: $\pi^* = -(A_{eq} \cdot H^{-1} \cdot A_{eq}^T)^{-1} (A_{eq} \cdot H^{-1} \cdot f + b_{eq})$

f) No results have to be filled in.

g) No results have to be filled in.

h) No results have to be filled in.