1- The original field eq.

$$\nabla_\mu \nabla^\mu \phi - m^2 \phi = 0$$

$$\nabla_\mu T^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, T^\mu \right)$$

$$\nabla_\mu \nabla^\mu \phi = \nabla_\mu \left(g^{\mu\nu} \nabla_\nu \phi \right) = \nabla_\mu \left(g^{\mu\nu} \partial_\nu \phi \right)$$

Thus

$$\nabla_\mu \nabla^\mu \phi = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \right)$$

And

$$\boxed{\partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \right) - \sqrt{-g}\, m^2 \phi = 0} \quad (1.1)$$

2- Spliting the time derivative

$$\partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \right) = \partial_t \left(\sqrt{-g}\, g^{t\nu} \partial_\nu \phi \right) + \partial_i \left(\sqrt{-g}\, g^{i\nu} \partial_\nu \phi \right)$$

Let

$$\boxed{A^t \equiv \sqrt{-g}\, g^{t\nu} \partial_\nu \phi} \quad (2.1)$$

$$\boxed{A^i = \sqrt{-g}\, g^{i\nu} \partial_\nu \phi} \quad (2.2)$$

$$A^t = \sqrt{-g}\left[g^{tt} \partial_t \phi + g^{ti} \partial_i \phi\right]$$

Using $\boxed{n^\mu = \frac{1}{\alpha}(1, -\beta^i)}$ (2.3) we get

$$A^t = \alpha\sqrt{\gamma}\left[-\frac{1}{\alpha^2} \partial_t \phi + \frac{1}{\alpha^2} \beta^i \partial_i \phi\right]$$

$$= \frac{\cancel{\alpha}\sqrt{\gamma}}{\alpha^{\cancel{2}}}\left[-\partial_t \phi + \beta^i \partial_i \phi\right]$$

$$= -\sqrt{\gamma}\, \frac{1}{\alpha}\left(\partial_t \phi - \beta^i \partial_i \phi\right)$$

$$\Rightarrow \boxed{A^t = -\sqrt{\gamma}\, \mathcal{L}_n \phi} \quad (2.4)$$

Thus $\mathcal{L}_n \phi = \frac{1}{\alpha}\left(\partial_t \phi - \beta^i \partial_i \phi\right)$

$$\Rightarrow \boxed{\partial_t \phi = \alpha\, \mathcal{L}_n \phi + \beta^i \partial_i \phi} \quad (2.5)$$

$$A^i = \sqrt{g}\left[g^{ti}\partial_t \phi + g^{ij}\partial_j \phi\right]$$

$$= \alpha\sqrt{\gamma}\left[\frac{1}{\alpha^2}\beta^i \partial_t \phi + \gamma^{ij}\partial_j \phi - \frac{1}{\alpha^2}\beta^i\beta^j \partial_j \phi\right]$$

$$= \alpha\sqrt{\gamma}\left[\frac{\beta^i}{\alpha^2}\partial_t \phi - \frac{\beta^i\beta^j}{\alpha^2}\partial_j \phi + \gamma^{ij}\partial_j \phi\right]$$

$$= \frac{\alpha\sqrt{\gamma}}{\alpha^2}\left[\beta^i\partial_t \phi - \beta^i\beta^j\partial_j \phi + \alpha^2\gamma^{ij}\partial_j \phi\right]$$

$$= \frac{\sqrt{\gamma}}{\alpha}\left[\beta^i\alpha\mathcal{L}_n \phi + \beta^i\beta^j\partial_j \phi - \beta^i\beta^j\partial_j \phi + \alpha^2\gamma^{ij}\partial_j \phi\right]$$

$$= \frac{\sqrt{\gamma}}{\alpha}\left[\beta^i\alpha\mathcal{L}_n \phi + \alpha^2\gamma^{ij}\partial_j \phi\right]$$

$$\Rightarrow \boxed{A^i = \sqrt{\gamma}\left[\alpha\gamma^{ij}\partial_j \phi + \beta^i\mathcal{L}_n \phi\right]} \quad (2.6)$$

Let

(2.7)

$$\boxed{\Pi \equiv -\sqrt{\gamma}\,\mathcal{L}_n \phi} \quad \Rightarrow \mathcal{L}_n \phi = -\Pi/\sqrt{\gamma}$$

Thus $A^t = \Pi$ and

$$A^i = \sqrt{\gamma}\left[\alpha \gamma^{ij}\partial_j \phi - \frac{\beta^i \Pi}{\sqrt{\gamma}}\right]$$

$$\Rightarrow A^i = \alpha\sqrt{\gamma}\,\gamma^{ij}\partial_j\phi - \beta^i \Pi$$

Let's define ψ_i such that

$$\psi_i \equiv \partial_i \phi$$

$$\boxed{A^i = \alpha\sqrt{\gamma}\,\gamma^{ij}\psi_j - \beta^i \Pi}$$

The original equation becomes

$$\boxed{\partial_t \Pi + \partial_i F^i_{(\Pi)} = S_{(\Pi)}} \qquad (2.7)$$

Where

$$\boxed{F^i_{(\Pi)} = \alpha\sqrt{\gamma}\,\gamma^{ij}\psi_j - \beta^i \Pi} \qquad (2.8)$$

$$\boxed{S_{(\Pi)} = \alpha\sqrt{\gamma}\, m^2 \phi} \qquad (2.9)$$

(2. 5) becomes

$$\boxed{\partial_t \phi = S_{(\phi)}} \qquad (2.10)$$

$$\boxed{S_\phi = \beta^i \psi_i - \frac{\alpha \Pi}{\sqrt{\gamma}}} \qquad (2.11)$$

And finally

$$\partial_t \psi_i = \partial_t(\partial_i \phi) = \partial_i(\partial_t \phi) = \partial_i [S_{(\phi)}]$$

$$= \partial_j [\delta^j{}_i \, S_{(\phi)}] \text{, thus}$$

$$\boxed{\partial_t \psi_i + \partial_j F^j_{(\psi_i)} = S_{(\psi_i)}} \qquad (2.12)$$

Where

$$\boxed{F^j_{(\psi_i)} = -\delta^j{}_i \, S_{(\phi)}} \qquad (2.13)$$

$$\boxed{S_{(\psi_i)} = 0} \qquad (2.14)$$

3- Characteristics

$$\frac{\partial F^i_{(\Pi)}}{\partial \Pi} = -\beta^i \quad \Bigg| \quad \frac{\partial F^i_{(\Pi)}}{\partial \psi_j} = \alpha\sqrt{\gamma}\,\gamma^{ij} \quad \Bigg| \quad \frac{\partial F_{(\Pi)}}{\partial \phi} = 0$$

$$\frac{\partial F^j_{(\psi_i)}}{\partial \Pi} = \frac{\delta^j{}_i \alpha}{\sqrt{\gamma}} \quad \Bigg| \quad \frac{\partial F^j_{(\psi_i)}}{\partial \psi_k} = -\delta^j{}_i \beta^k \quad \Bigg| \quad \frac{\partial F^j_{(\psi_i)}}{\partial \phi} = 0$$

4- Ingoing initial data

Ingoing data satisfies

$$\partial_t \phi - \partial_r \phi = 0$$

As $\dfrac{\partial \phi}{\partial r} = \dfrac{x^i}{r} \partial_i \phi$, we get

$$\partial_t \phi - \frac{x^i}{r} \partial_i \phi = 0$$

Using (2. 11) we get

$$\beta^i \psi_i - \frac{\alpha \Pi}{\sqrt{\gamma}} - \frac{x^i}{r} \psi_i = 0$$

$$\Leftrightarrow \frac{\alpha \Pi}{\sqrt{\gamma}} = \left(\beta^i - \frac{x^i}{r} \right) \psi_i$$

$$\Rightarrow \boxed{\Pi = \frac{\sqrt{\gamma}}{\alpha} \left(\beta^i - \frac{x^i}{r} \right) \psi_i} \quad (4.1)$$

Thus, initial data $f(x, y, z)|_{t=0}$ is

$$\boxed{\begin{cases} \phi = f(x, y, z)|_{t=0} \\ \psi_i = \partial_i f \\ \Pi = (4.1) \end{cases}} \quad (4.2)$$