Energia conservada de uma métrica ADM

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \quad \text{onde} \quad \dot{x}^\mu = \frac{dx^\mu}{d\lambda}$$

$$\mathcal{L} = \frac{1}{2} \left\{ -N^2 \dot{t}^2 + \gamma_{ij} \left[\dot{x}^i \dot{x}^j + \beta^j \dot{x}^i \dot{t} + \beta^i \dot{t} \dot{x}^j + \beta^i \beta^j \dot{t}^2 \right] \right\}$$

$$\Rightarrow \mathcal{L} = \frac{1}{2} \left\{ \left[-N^2 + \gamma_{ij} \beta^i \beta^j \right] \dot{t}^2 + \left[\gamma_{ij} \beta^j \dot{x}^i + \gamma_{ij} \beta^i \dot{x}^j \right] \dot{t} + \gamma_{ij} \dot{x}^i \dot{x}^j \right\}$$

$$\Rightarrow \mathcal{L} = \frac{1}{2} \left\{ \left[-N^2 + \gamma_{ij} \beta^i \beta^j \right] \dot{t}^2 + 2 \gamma_{ij} \beta^i \dot{x}^j \dot{t} + \gamma_{ij} \dot{x}^i \dot{x}^j \right\}$$

$$\Rightarrow \boxed{\mathcal{L} = \frac{1}{2} \left[-N^2 + \gamma_{ij} \beta^i \beta^j \right] \dot{t}^2 + \gamma_{ij} \beta^i \dot{x}^j \dot{t} + \frac{1}{2} \gamma_{ij} \dot{x}^i \dot{x}^j} \qquad (1)$$

$$-\frac{\partial \mathcal{L}}{\partial \dot{t}} = -\left[-N^2 + \gamma_{ij} \beta^i \beta^j \right] \dot{t} - \gamma_{ij} \beta^i \dot{x}^j$$

$$\Rightarrow \boxed{E_G = \left[N^2 - \gamma_{ij} \beta^i \beta^j \right] \dot{t} - \gamma_{ij} \beta^i \dot{x}^j} \qquad (2)$$

Temos também que $\dot{t} = E_L / N$, logo

$$E_G = \left[N^2 - \gamma_{ij} \beta^i \beta^j \right] \frac{E_L}{N} - \gamma_{ij} \beta^i \dot{x}^j$$

Adicionalmente temos

$$V^i = \frac{1}{N}\left(\frac{dx^i}{dt} + \beta^i\right) \Rightarrow \frac{dx^i}{dt} = NV^i - \beta^i$$

$$\frac{dx^i}{d\lambda} = \frac{dx^i}{dt}\frac{dt}{d\lambda} = \left(NV^i - \beta^i\right)\frac{E_L}{N}$$, logo

$$E_G = \left[N^2 - \partial_{ij}\beta^i\beta^j\right]\frac{E_L}{N} - \partial_{ij}\beta^i\left(NV^j - \beta^j\right)\frac{E_L}{N}$$

$$= \left[N^2 \cancel{- \partial_{ij}\beta^i\beta^j} - N\partial_{ij}\beta^i V^j \cancel{+ \partial_{ij}\beta^i\beta^j}\right]\frac{E_L}{N}$$

$$\Rightarrow \boxed{E_G = \left(N - \partial_{ij}\beta^i V^j\right)E_L} \quad (3)$$