Energy Extraction and Quasinormal Modes of Black Hole Binaries: An analytical and numerical study

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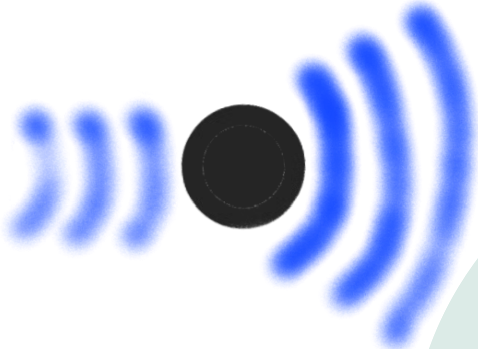
Presentation Outline

1. The relevance of binary systems
2. Modeling binary systems
3. The penrose Process on a Kerr black hole
4. The Penrose Process on static binaries
5. The Penrose Process on non-static binaries
6. Simulating scalar perturbations on GW150914
7. The Assymptotic Iteration Method and QuasinormalModes.jl
8. Revisiting Schwarzschild perturbations with the AIM and QuasinormalModes.jl

The Relevance Of Binary Systems

Accretion Disk Formation



Superradiance

Hawking Radiation

The astrophysical Kerr Black Hole is a two parameter vacumm solution of EFEs.

Even tough they are mathematically very simple, they have very rich interactions with their surroundings

How can we extend these concepts to black holebinaries?

Modeling Binary Systems

Static BBH Model

Two back holes that do not move with respec to to the static observer at infinity.

Exact BBH Model

An exact vacumm solution of Einstein's Field Equations.

Analitic BBH Model

The entire spacetime metric is analytically known at all points in space and time.

Dynamic BBH Model

Two back holes that move with respec to to the static observer at infinity.

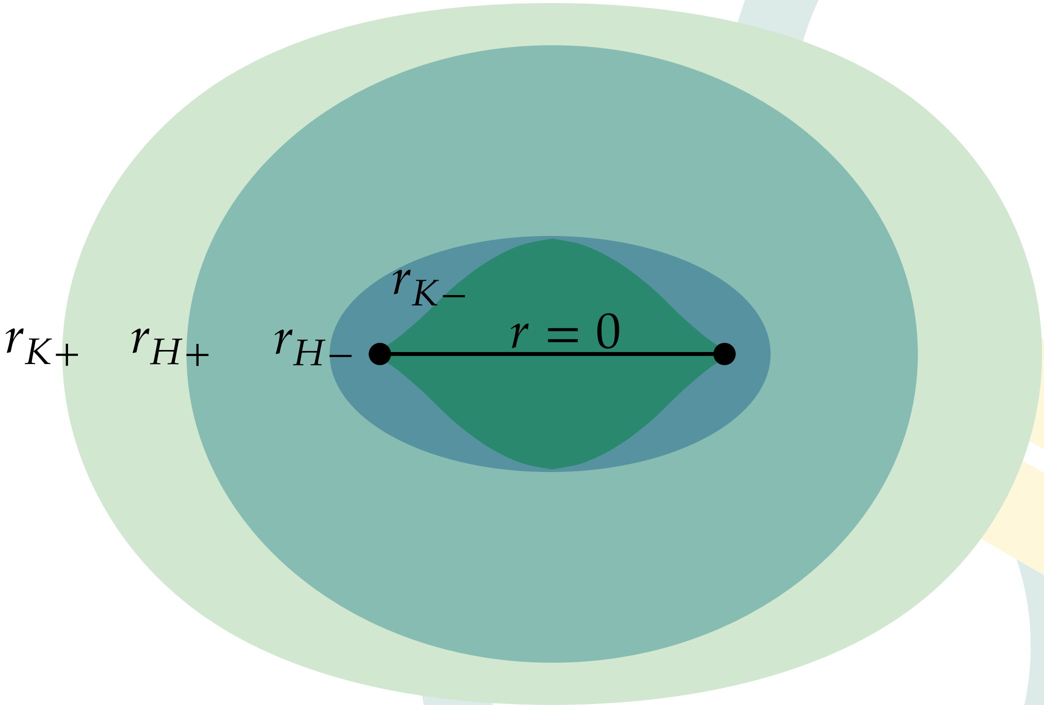
Approximate BBH Model

Non-exact solution of Einstein's Field Equations.
No exotic matter.

Numeric BBH Model

Obtained at a hypersurface by numerically solving the ADM constraint equations.

The Kerr Penrose Process

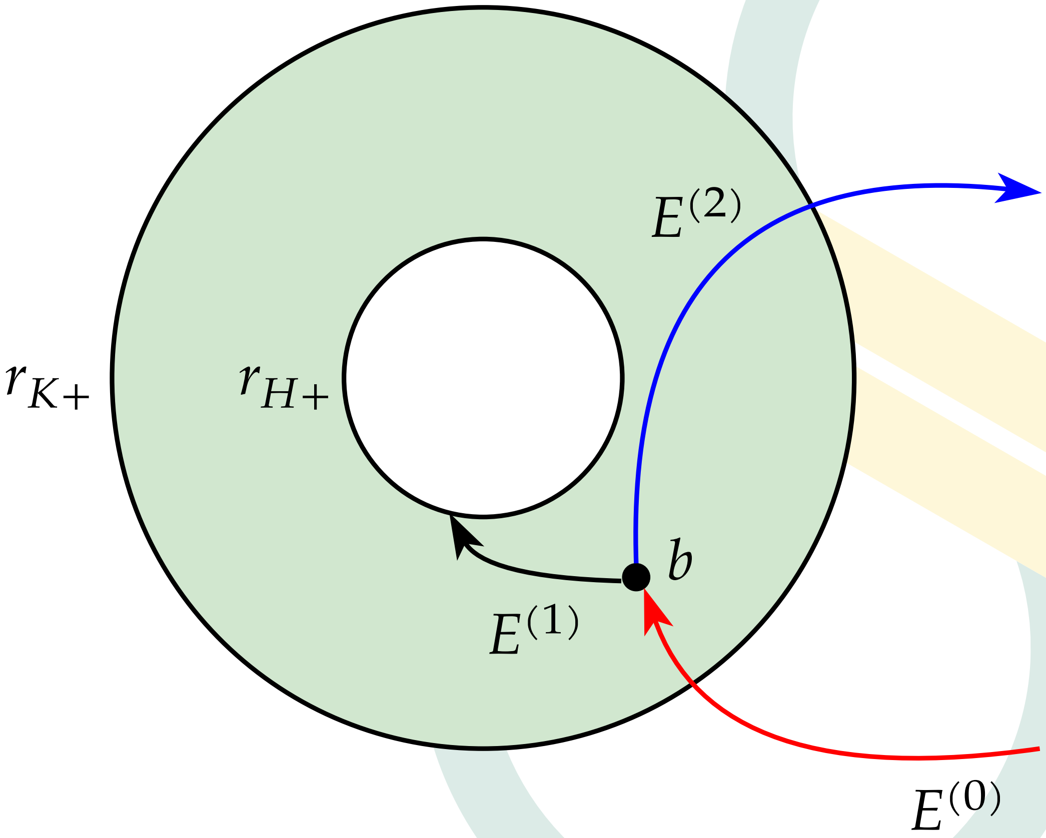


The Kerr spacetime posesses a global time-like Killing vector field.

The outer Killing horizon is located outside the outer event horizon.

The Region between the Killing horizon and outer horizon is know as the Ergosphere

The Kerr Penrose Process



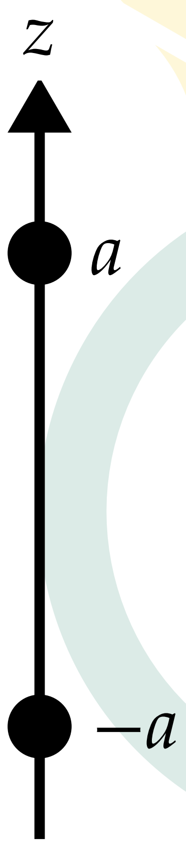
In the ergosphere, a particle may have negative energy according to the static observer at infinity.

One can engineer a particle decay process in such a way that particles leaving the system are more energetic than those entering the system.

In the Kerr metric, this excess energy comes from the black hole's rotation.

The MP Penrose Process

The Majumdar - Papapetrou solution (MP) describes two extremally charged black holes in static equilibrium, thanks to their charge:



$$\mathrm{d}s^2 = -\frac{\mathrm{d}t^2}{U(\rho,z)^2} + U(\rho,z)^2\left[\mathrm{d}\rho^2 + \rho^2\mathrm{d}\phi^2 + \mathrm{d}z^2\right]$$

$$A_\mu = 1 + \frac{M_1}{\sqrt{\rho^2+(z+a)^2}} + \frac{M_2}{\sqrt{\rho^2+(z-a)^2}}$$

$$A_\mu = \left(1 - \frac{1}{U(\rho,z)}\right)\delta_{\mu t}$$

The MP Penrose Process

A charged particle, moving through this binary has energy given by

$$E = \mu\left(1 - \frac{1}{U}\right) + \sqrt{\frac{L^2}{\rho^2 U^4} + \frac{1}{U^2} + \dot{\rho}^2 + \dot{z}^2}$$

At a fixed position, the minimum possible energy is associated with particles at rest, thus

$$E_{\min} = \mu\left(1 - \frac{1}{U}\right) + \frac{1}{U}$$

The minimun energy will be negative if

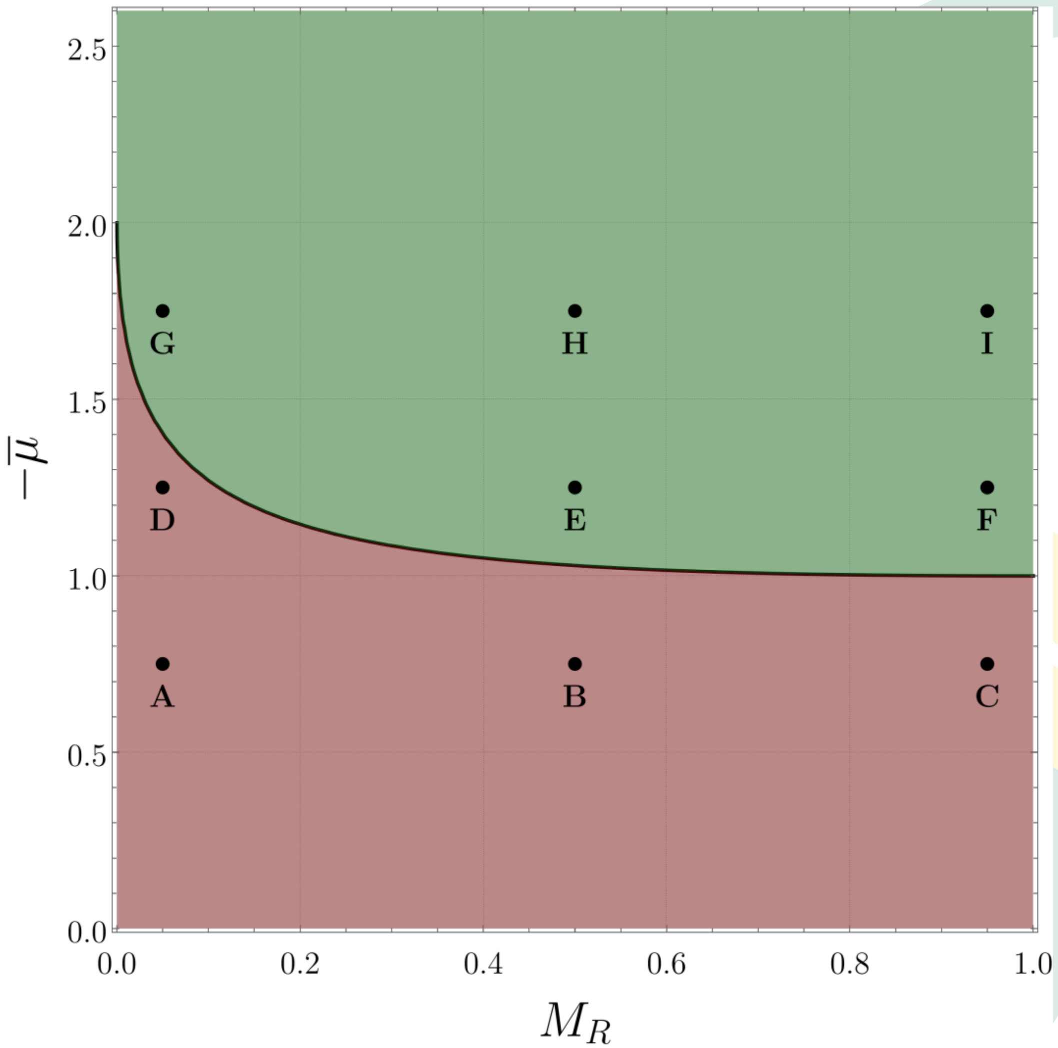
$$\mu < 0$$

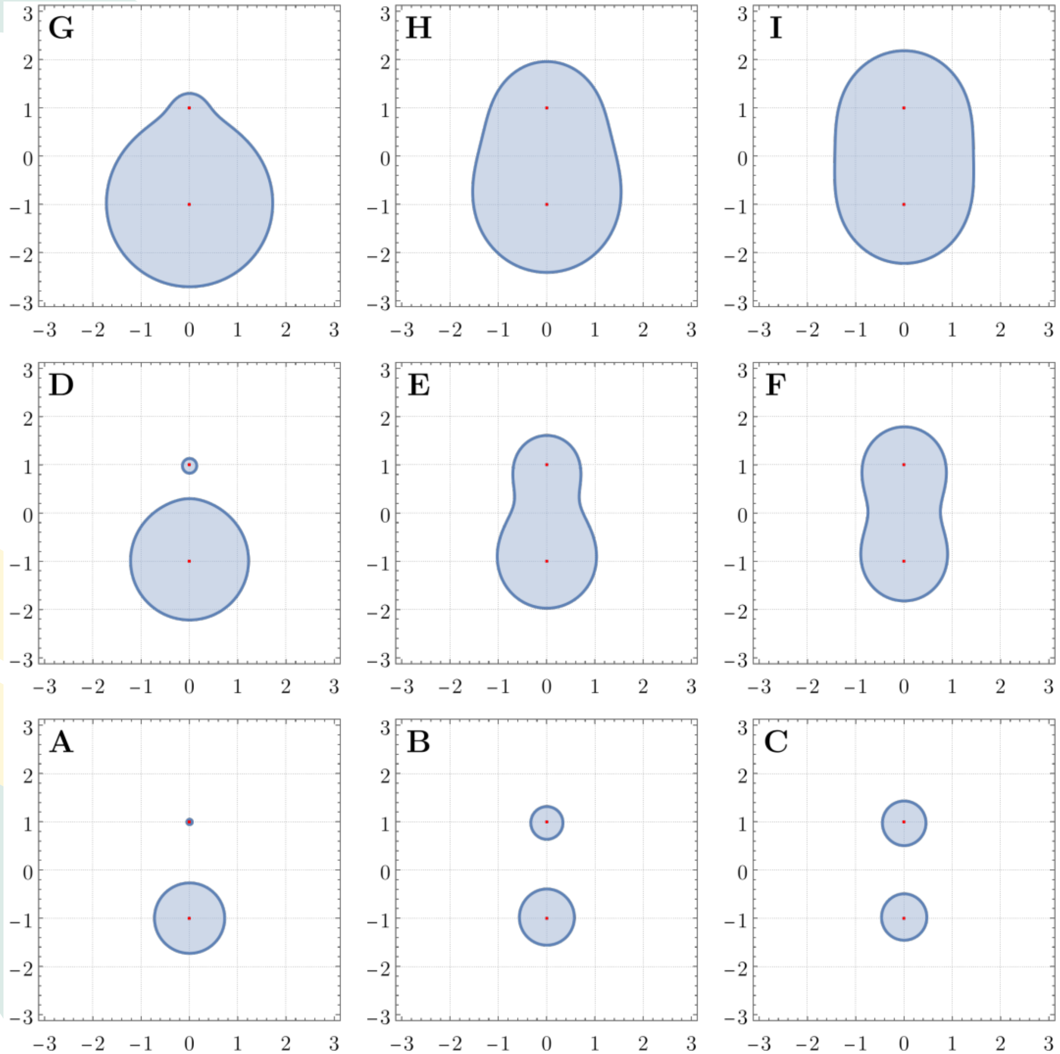
$$\frac{1}{\sqrt{\overline{\rho}^2+(\overline{z}+1)^2}} + \frac{M_R}{\sqrt{\overline{\rho}^2+(\overline{z}-1)^2}} > -\frac{1+M_R}{\overline{\mu}}$$

Where

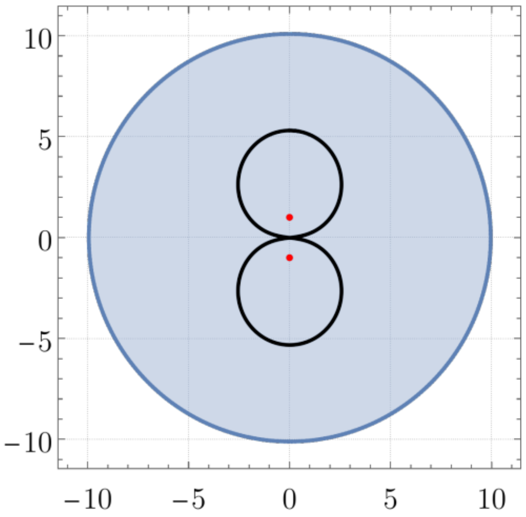
$$M_R = M_2/M_1$$
$$M_T = M_1 + M_2$$
$$\overline{\rho} = \rho/a$$
$$\overline{z} = z/a$$
$$\overline{\mu} = \mu M_T/a$$

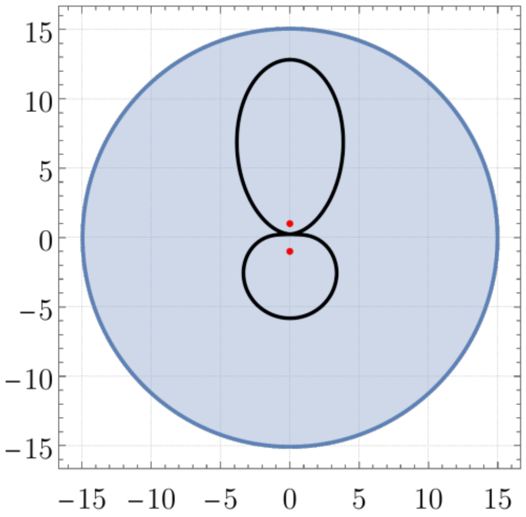
The MP Penrose Process





The MP Penrose Process



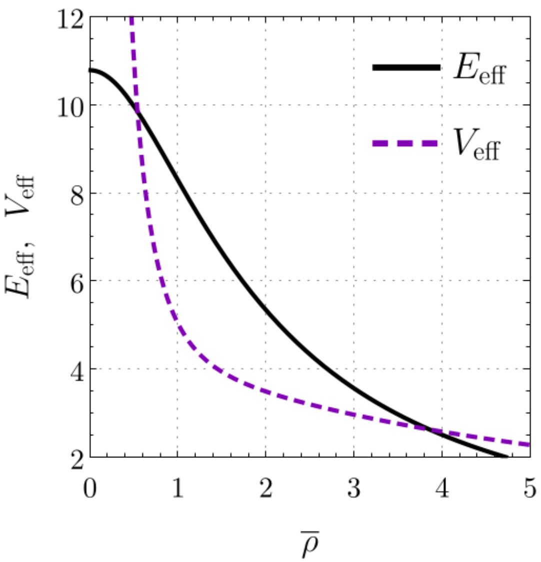


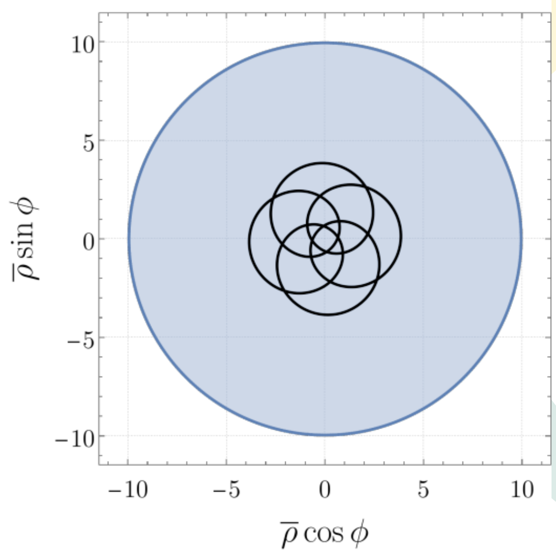
From the particle's energy equation, one can write

$$\dot{\rho}^2 + \dot{z}^2 = E_{\text{eff}}^2(\rho, z) - V_{\text{eff}}(\rho, z)$$

$$E_{\text{eff}}(\rho, z) = E - \mu\left(1 - \frac{1}{U}\right)$$

$$V_{\text{eff}}(\rho, z) = \frac{L^2}{\rho^2 U^4} + \frac{1}{U^2}$$



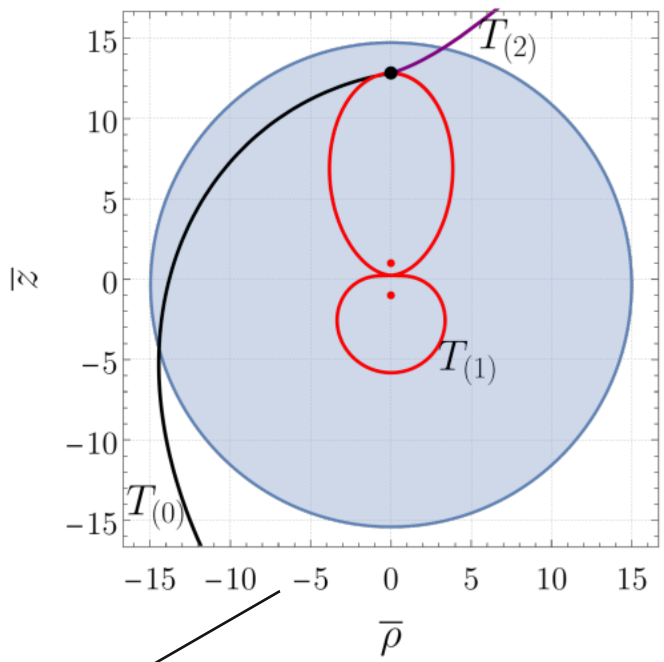


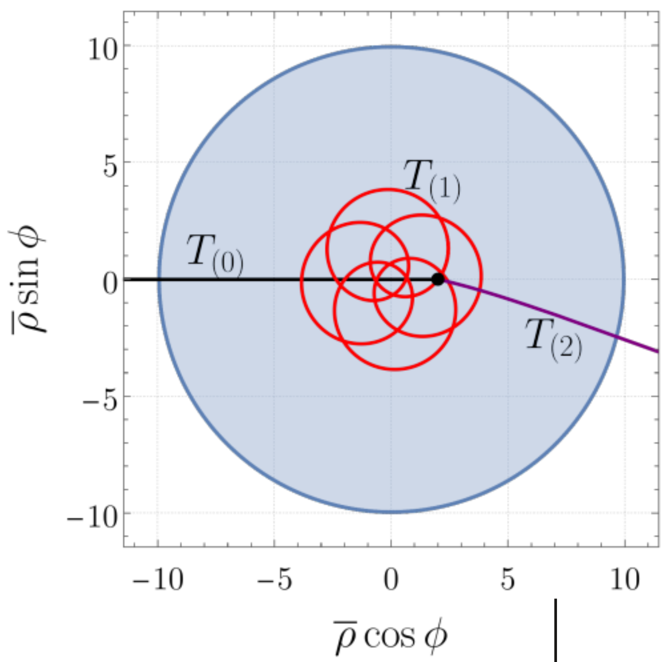
Where

$$E_{\text{eff}}(\rho, z) \geq 0$$

$$E_{\text{eff}}(\rho, z)^2 \geq V_{\text{eff}}(\rho, z)$$

The MP Penrose Process



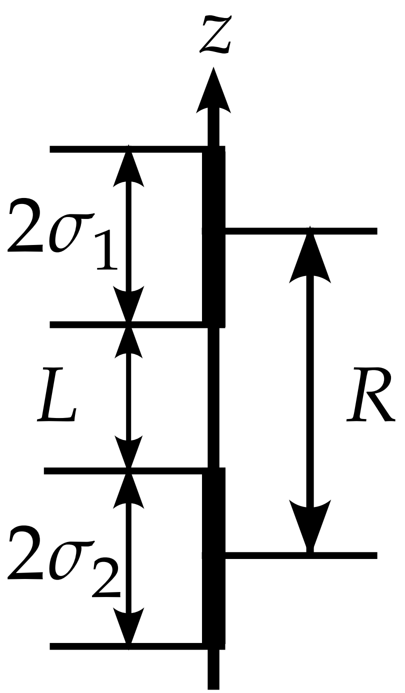


i	$m_{(i)}$	$\mu_{(i)}$	$E_{(i)}$	$L_{(i)}$	$\dot{\rho}_{(i)}$	$\dot{z}_{(i)}$
0	1.00000	-0.08345	1.00000	0	0.60000	0.09699
1	0.70000	-5.00000	-0.02000	0	0.41207	0.00000
2	0.23248	14.69629	4.36171	0	1.34012	0.41719

i	$m_{(i)}$	$\mu_{(i)}$	$E_{(i)}$	$L_{(i)}$	$\dot{\rho}_{(i)}$	$\dot{z}_{(i)}$
0	1.00000	-0.27698	1.00000	0.00000	1.00000	0
1	0.10000	-5.00000	-0.05000	12.85870	1.36059	0
2	0.33342	0.66890	3.01423	-3.85662	2.59116	0

The CMMR Penrose Process

The Cabrera - Munguia, Manko and Ruiz solution (CMMR) describes two Kerr black holes held in static equilibrium by a masless strut (see Ref. [66])



Adapted from Ref. [66]

$$ds^2 = -f(\rho,z)\left[dt - \omega(\rho,z)d\phi\right]^2 + f(\rho,z)^{-1}\left[e^{2\gamma(\rho,z)}\left(d\rho^2 + dz^2\right) + \rho^2 d\phi^2\right]$$

$$a_{1,2} = M_{1,2}/J_{1,2}$$

$$M_T = M_1 + M_2$$

$$J_T = M_1 a_1 + M_2 a_2$$

$$\left(R^2 - M_T^2 + a_*^2\right)\left(a_1 + a_2 - a_*\right) + 2\left(R + M_T\right)\left(J_T - M_T a_*\right) = 0$$

The CMMR Penrose Process

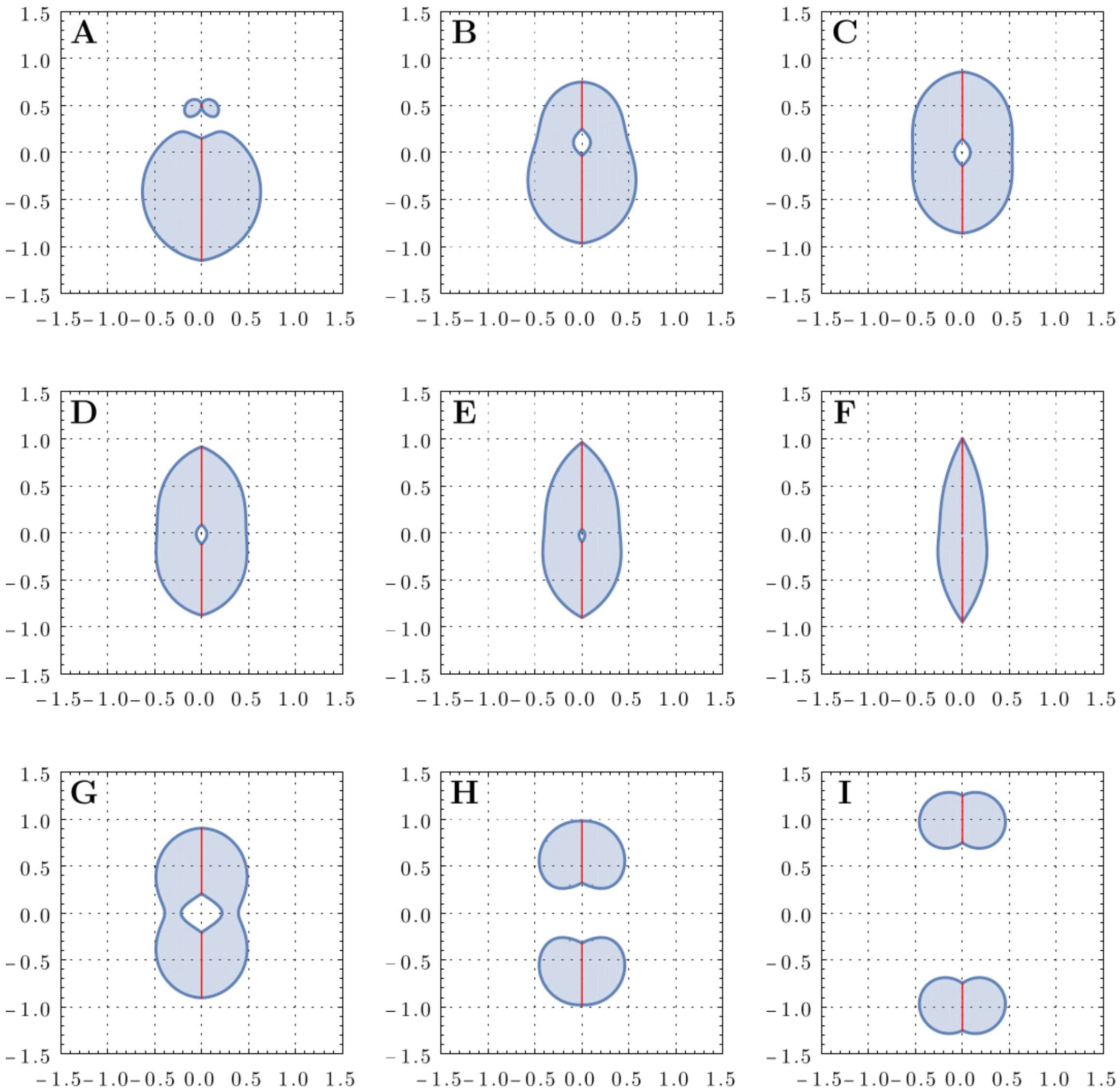
A charged particle, moving through this binary has energy given by

$$E = \frac{-f^2\omega L}{\rho^2-\omega^2 f^2} + \left[\frac{\rho^2 e^{2\gamma}(\dot{\rho}^2+\dot{z}^2)}{\rho^2-\omega^2 f^2} + \left(\frac{\rho f L}{\rho^2-\omega^2 f^2}\right)^2 + \frac{\rho^2 f}{\rho^2-\omega^2 f^2}\right]^{1/2}$$

This metric also posesses a global time-like Killing vector field. The ergosphere is the locus of all points where

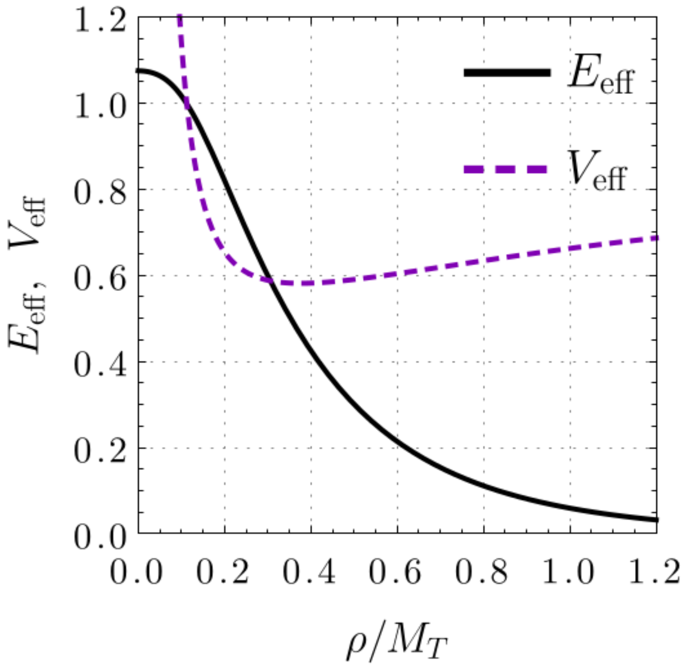
$$f(\rho, z) > 0$$

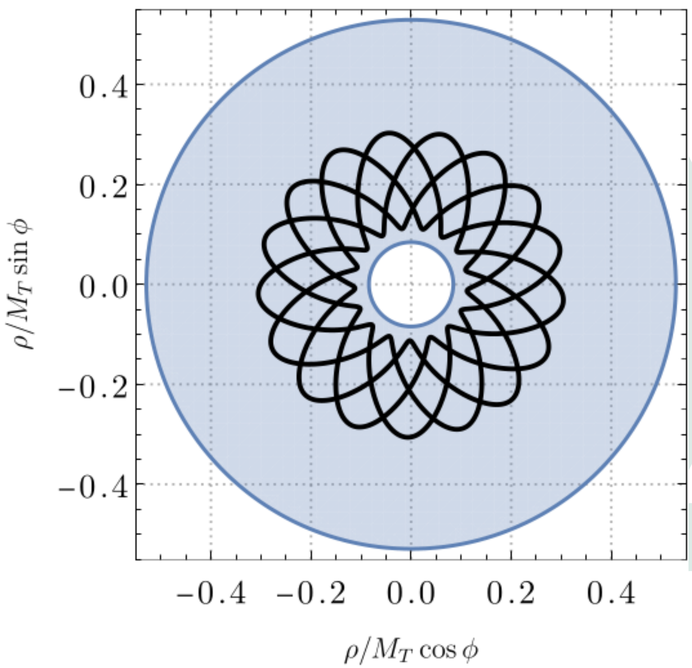
The CMMR Penrose Process

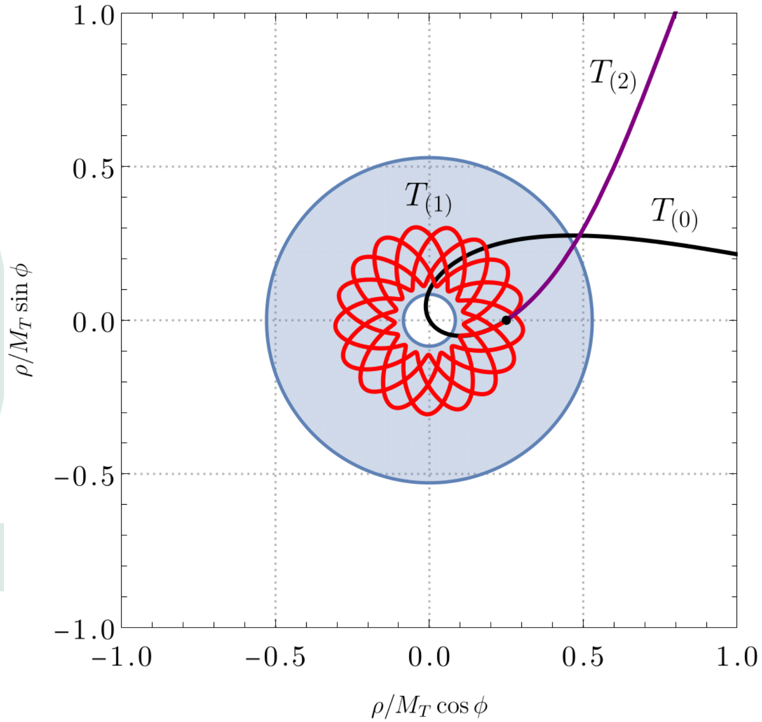


Panel	M_1/M_2	a_1/M_T	a_2/M_T	R/M_T
A	0.16	0.65	0.65	1.00
B	0.58	0.65	0.65	1.00
C	1.00	0.65	0.65	1.00
D	1.00	0.50	0.65	1.00
E	1.00	0.30	0.65	1.00
F	1.00	−0.10	0.65	1.00
G	1.00	0.65	0.65	1.11
H	1.00	0.65	0.65	1.30
I	1.00	0.65	0.65	2.00

The CMMR Penrose Process





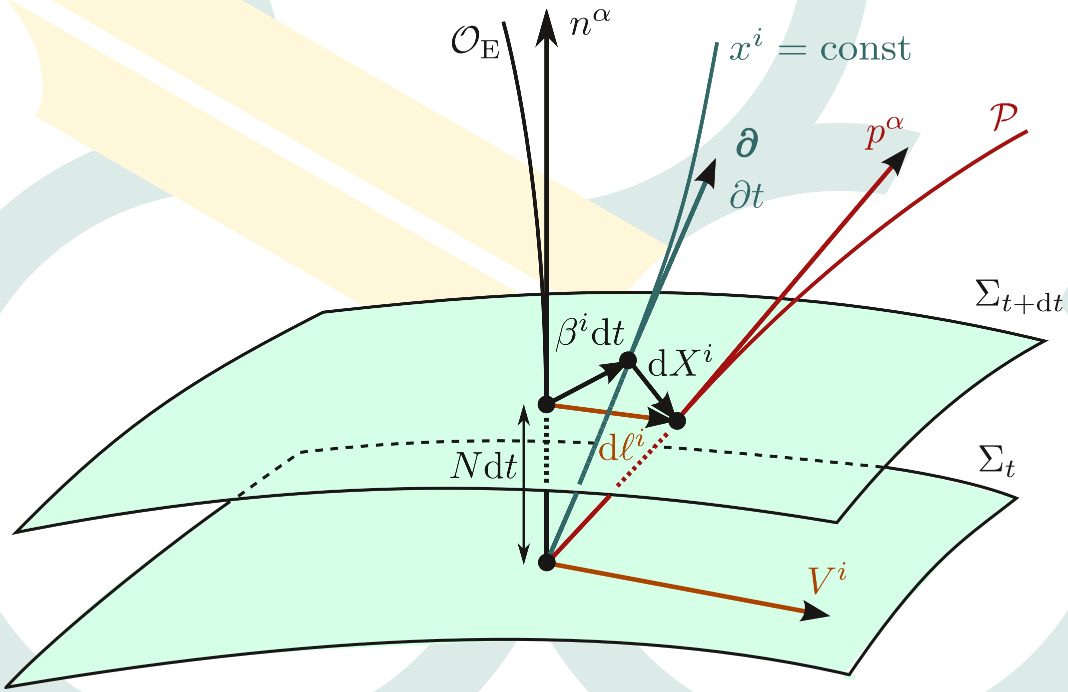


i	$m_{(i)}/m_0$	$E_{(i)}$	$L_{(i)}$	$\dot{\rho}_{(i)}$	$\dot{z}_{(i)}$
0	1.0000000	2.00000	0 .000000	4.343904	0
1	0.0289697	-0.05400	-2.500000	0.313887	0
2	0.3148980	6.35623	0.229993	13.765800	0

Non-Stationary Penrose Process

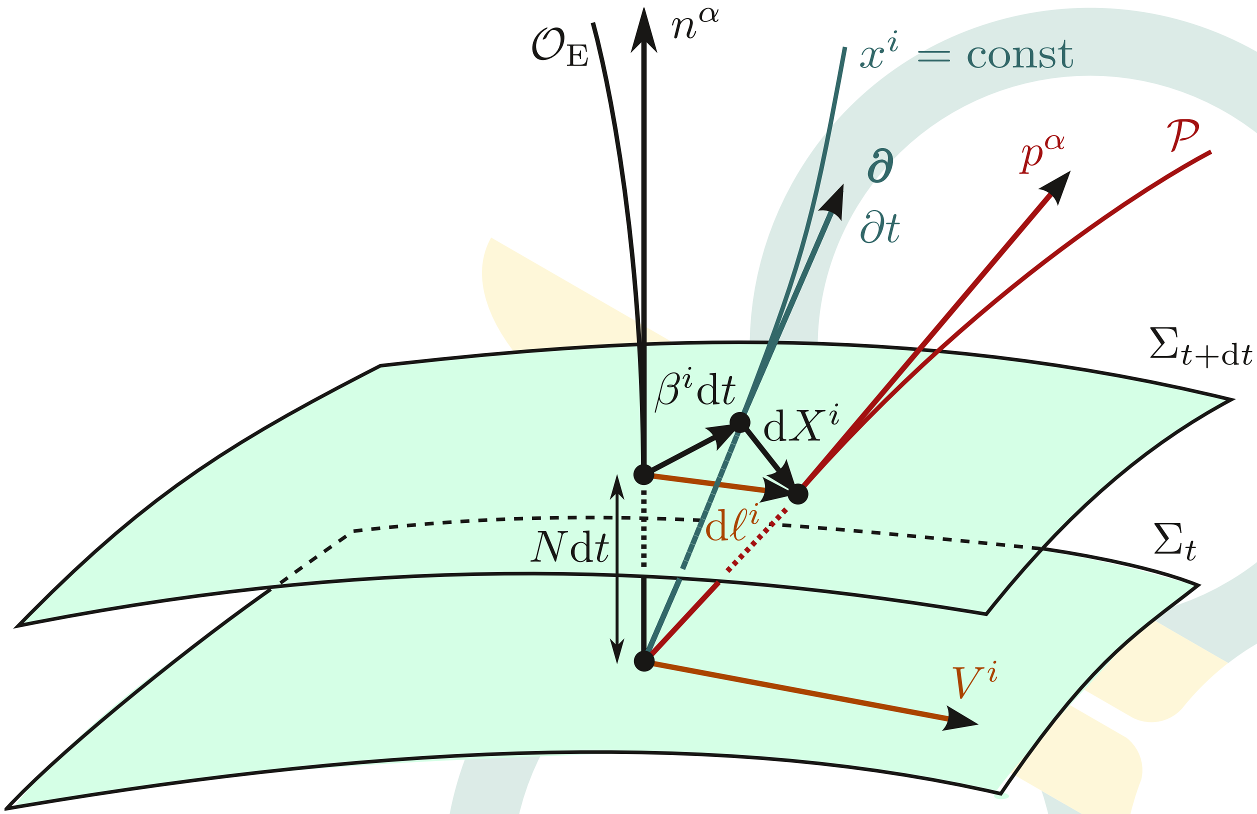
So far, the PP depends on the existence of a conserved negative and "global" energy.

Let us relax this assumption. Our first step is to represent the spacetime of interest in 3+1 form.



$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i\mathrm{d}t)(\mathrm{d}x^j + \beta^j\mathrm{d}t).$$

Non-Stationary Penrose Process



$$p_\mu p^\mu = m^2 \delta$$

$$p^\mu = E(n^\mu + V^\mu)$$

$$E = -n_\mu p^\mu \quad \text{(local)}$$

$$V_\mu V^\mu = V_i V^i = 1 + \delta \left(\frac{m}{E}\right)^2.$$

$$\frac{\mathrm{d}X^i}{\mathrm{d}t} = NV^i - \beta^i$$

$$\frac{\mathrm{d}V^i}{\mathrm{d}t} = NV^j \left[V^i \left(\partial_j \ln N - K_{jk} V^k \right) + 2K^i_{\ j} - {}^3\Gamma^i_{jk} V^k \right] - \gamma^{ij} \partial_j N - V^j \partial_j \beta^i$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = E(N K_{jk} V^j V^k - V^j \partial_j N)$$

Non-Stationary Penrose Process

We define the global energy

$$\varepsilon = -p_\mu \xi^\mu$$

Where

$$\xi^\mu = (\partial_t)^\mu$$

We can bridge the two energy definitions by using the four momentum decomposition and the general 3+1 decomposed metric

$$\varepsilon = (N - \gamma_{ij}\beta^i V^j)E$$

Non-Stationary Penrose Process

$$\varepsilon = (N - \gamma_{ij}\beta^i V^j)E$$

When ε represents a global time-like Killing vector field, ζ^μ can be interpreted as a conserved energy along a particle's trajectory. In general, tough, this is not true.

Note, however, that if the spacetime metric is assymptotically flat, both definitions coincide at spatial infinity. Furthermor, at infinity the global enrgy is again physically meaningful.

This means that even tough we cannot use the global energy to make physical statements close to a gravitational center, we can do so at infinity.

Non-Stationary Penrose Process

$$\varepsilon = (N - \gamma_{ij}\beta^i V^j)E$$