Exercise Sheet 2

Exercise 1

Consider the algebraic equation $x^2 + \varepsilon x - 1 = 0$ with $0 < \varepsilon \ll 1$. Find a three-term approximation $x = x_0 + \varepsilon^{\alpha_1} x_1 + \varepsilon^{\alpha_2} x_2 + ...$ for each root. Compare with the exact roots.

Exercise 2

To find approximations to the roots of the cubic equation $x^3 - 4.001x + 0.002 = 0$ why is it easier to examine the equation $x^3 - (4 + \varepsilon)x + 2\varepsilon = 0$? Find a two-term approximation to this equation.

Exercise 3

Find a three-term approximation for the root of $x = 1 + \varepsilon x^2$, $0 < \varepsilon \ll 1$ near $x = 1$. Compare it to the exact solution for $\varepsilon = 0.1$ and $\varepsilon = 0.001$.

Exercise 4

Find a three-term approximation for the quadratic equation $(1 - \varepsilon)x^2 - 2x + 1 = 0$.

Exercise 5

Consider the algebraic equation $g(x, \varepsilon) = 0$, $0 < \varepsilon \ll 1$, where g is a function having derivatives of all order. Assuming $g(x, 0) = 0$ is solvable to obtain x_0, show how to find a three-term perturbation approximation of the form $x = x_0 + x_1\varepsilon + x_2\varepsilon^2$. What condition on g is required to determine x_1 and x_2? Find a three-term approximation to the roots of $e^{\varepsilon x} = x^2 - 1$.