Exercise Sheet 4

Exercise 1

Show that regular perturbation fails on the boundary value problem

$$\varepsilon y''(x) + 2y'(x) + y(x) = 0, \quad 0 < x < 1, \quad 0 < \varepsilon \ll 1$$

with $y(0) = 0$, $y(1) = 1$. Find the exact solution and plot it for $\varepsilon = 0.05$ and $\varepsilon = 0.005$. If $x = O(\varepsilon)$, show (on a graph or analytically) that $\varepsilon y''(x)$ is large. If $x = O(1)$, show that $\varepsilon y''(x) = O(1)$. Find an inner and an outer approximation of the exact solution. Find a uniform approximation of the exact solution.

Exercise 2

Use singular perturbation methods to obtain a uniform approximate solutions to the problems

1. $\varepsilon y''(x) + x^{1/3} y'(x) + y(x) = 0$, $y(0) = 0$, $y(1) = e^{-3/2}$,
2. $\varepsilon y''(x) - (2x+1)y'(x) + 2y(x) = 0$, $y(0) = 1$, $y(1) = 0$.

In each case consider $0 < x < 1$ and $0 < \varepsilon \ll 1$.

Exercise 3

Find a uniformly valid approximation of a solution to the problem

$$\varepsilon y''(x) - g(x)y'(x) = f(x) \qquad 0 < x < 1, \quad 0 < \varepsilon \ll 1$$

$$y(0) = 0, \; y(1) = -1\,,$$

where $g(x) > 0$ for $x \in [0,1]$ and g and f are continuous functions on $[0,1]$. Illustrate your general result on the graph for some chosen by you functions g and f.