Exercise Sheet 3

Exercise 1

Find the first three terms of the perturbation series solution to the initial value problem

$$\begin{cases} y' = 1 + (1+\varepsilon)y^2 \\ y(0) = 1 \end{cases}$$

for $0 < \varepsilon \ll 1$ and $t > 0$. Find the exact solution and compare the approximation. Is it uniform? Is it enough to consider only the first term of the asymptotic expansion to approximate the exact solution? Are the first two terms enough?

Exercise 2

Consider the initial value problem

$$\begin{cases} y' = -y + \varepsilon y^2 \\ y(0) = 1 \end{cases}$$

for $0 < \varepsilon \ll 1$ and $t > 0$. Find the exact solution and a three-term asymptotic expansion. Compare the exact error of approximation with the theoretical estimate given on the lecture.

Exercise 3

Consider the perturbed harmonic oscillator problem

$$\begin{cases} y'' + (1+\varepsilon)y = 0 \\ y(0) = 1,\, y'(0) = 0 \end{cases}$$

for $0 < \varepsilon \ll 1$ and $t > 0$. Compute a two-term asymptotic expansion of the solution and compare it with the exact solution. What happens for large times, i.e., $t \to \infty$?

Exercise 4

The pendulum problem can be scaled to obtain the initial value problem

$$\begin{cases} y'' + \dfrac{\sin(\varepsilon y)}{\varepsilon} = 0 \\ y(0) = 1,\, y'(0) = 0 \end{cases}$$

for $0 < \varepsilon \ll 1$ and $t > 0$. Apply the regular perturbation method to find a two-term expansion. Show that the correction term is secular term and comment on the validity of the approximation.

Exercise 5

Apply the Poincaré-Lindstedt method to the scaled pendulum problem to obtain a two-term asymptotic expansion.