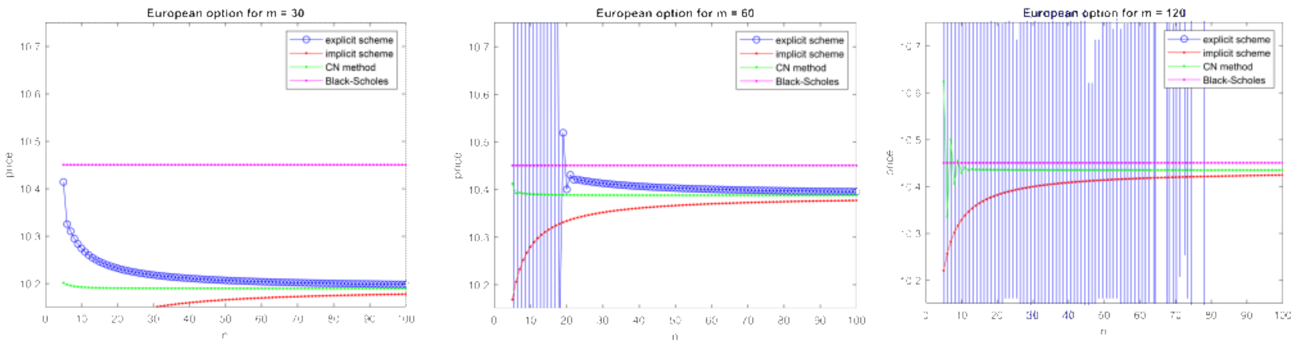
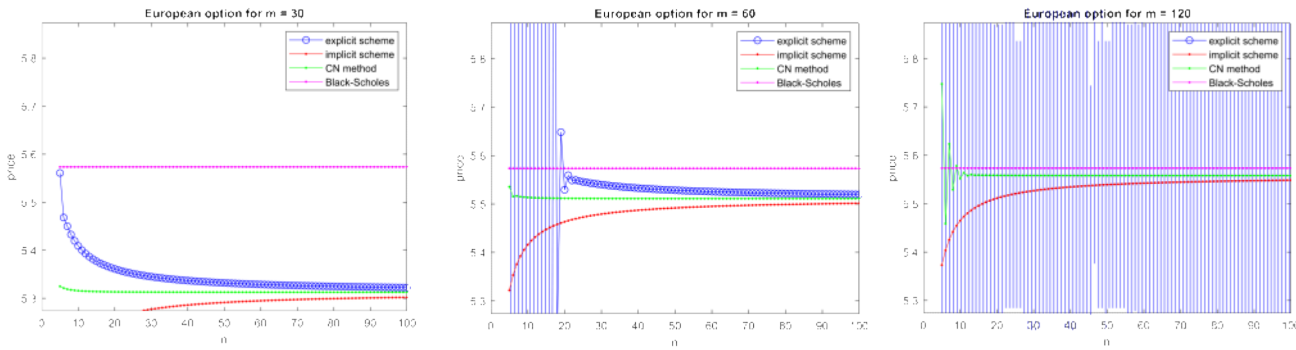
Tasks (deadline: 23.12.2020)

- **4.1:** Evaluate the convergence of explicit finite difference scheme for European vanilla option to the Black-Scholes price for different parameter sets.
- **4.2:** Evaluate the convergence of implicit finite difference scheme for European vanilla option to the Black-Scholes price for different parameter sets. Compare with the results for Task 4.1.

Call option S_0=100, K=100, T=1, r=0.05, \sigma=0.2, S_max=3S_0



Put option S_0=100, K=100, T=1, r=0.05, \sigma=0.2, S_max=3S_0



$$(*)\quad f_{i,j} = a_i f_{i-1,j+1} + b_i f_{i,j+1} + c_i f_{i+1,j+1}$$

$$a_i = \frac{1}{r\Delta t+1}\left(-\tfrac{1}{2} r i\Delta t + \tfrac{1}{2}\sigma^2 i^2 \Delta t\right)$$

$$b_i = \frac{1}{r\Delta t+1}\left(1-\sigma^2 i^2 \Delta t\right)$$

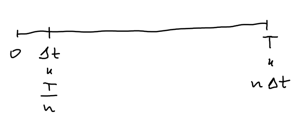
$$c_i = \frac{1}{r\Delta t+1}\left(\tfrac{1}{2} r i\Delta t + \tfrac{1}{2}\sigma^2 i^2 \Delta t\right)$$

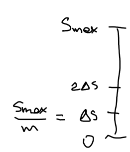
0 Set the parameters

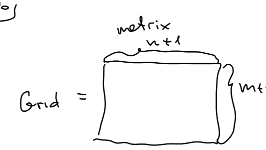
1. Define a grid. Set

- n - number of points on the time grid
- m - number of points on the S grid

set $S_{max} = M \cdot S_0$







2. Calculate the boundary conditions for Grid(1,:), Grid(m+1,0), Grid(:,n+1)

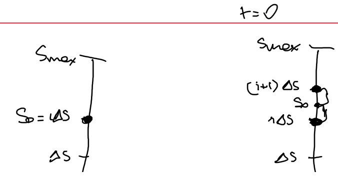
1. For a vanilla call option
   - $f_{0,j}$ = 0, since the call is worthless for S=0
   - $f_{m,j} = S_{max} - Ke^{-r(n-j)\Delta t}$, since for large S the option value asymptotes to $S - Ke^{-r(T-t)}$ (it will always be executed)
   - $f_{i,n} = (i\Delta S - K)^+$
2. For a vanilla put option
   - $f_{0,j} = Ke^{-r(n-j)\Delta t}$
   - $f_{m,j}$ = 0, since the option becomes worthless for large S
   - $f_{i,n} = (K - i\Delta S)^+$

3. for each j (n-1, n-2, ..., 0)
for each i (1, 2, ..., m-1)
calculate $f_{i,j}$ using (*)

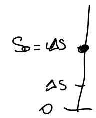
4 The option price is

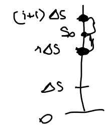
Grid(i, 1), where $i\Delta S \approx S_0$

t=0



use interpolation





use interpolation