Tasks (deadline: 17.01.2021)

- **5.1:** Compare European option prices obtained via MC with (i) antithetic variates, (ii) control variates (what will be the control variate?).
- **5.2:** Compare floating strike lookback option prices obtained via MC with (i) antithetic variates, (ii) control variates (what will be the control variate?).

Upload codes with implementation and plots of convergence (with increasing number of simulations) to the Black-Scholes price together with the corresponding confidence intervals for few different parameter sets.

price $= E^{\mathbb{Q}}[e^{-rT} \text{payoff}]$

1 Simulate N trajectories of the underlying S_t on interval $[0,T]$

a) in general use Euler or Milstein scheme

or

b) exact formula for GBM

Define a time grid

0 $\quad \Delta t \quad 2\Delta t \quad \dots \quad T = n\Delta t$

repeat for $i=1,2,\dots,N$

$$S_T^{(i)} = S_0 \exp\Big(\underset{1\le j\le n}{\text{cumsum}}\Big[\big(r - \tfrac{\sigma^2}{2}\big)\Delta t + \sigma\sqrt{\Delta t}\,\underbrace{\varepsilon_j^{(i)}}_{\sim N(0,1)}\Big]\Big)$$

For vanilla option (European) $n = 1$

2. Calculate average payoffs and discount it

$$\text{price} = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}(S_T^{(i)} - K)^+ \text{ or } (K - S_T^{(i)})^+$$

Accuracy

$$CI = \Big[\text{price} - z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{N}},\ \text{price} + z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{N}}\Big]$$

$\sigma = \text{std}(\text{price}^{(i)})$ / $\text{payoff}^{(i)}$

3 Compare with the B-S price for different N

Variance reduction

- antithetic variates

  for part 1 use also $-\varepsilon_j^{(i)}$

  and $\frac{1}{2}\big(\text{payoff}(\varepsilon_j^{(i)}) + \text{payoff}(-\varepsilon_j^{(i)})\big)$ as a new payoff.

- control variate

  Set Z – control variate.
  - $\text{Cov}(Z, \text{payoff}) \neq 0$
  - known $E(Z)$

  1 Simulate sample of N variables Z

  2 Calculate the average of

$$p^{(i)} = \Big[\text{payoff}^{(i)} + c\,[Z^{(i)} - E(Z)]\Big],$$

where $c = -\dfrac{\text{Cov}(\text{payoff}, Z)}{\text{Var}(Z)}$

What should be the control variate?

- S_T, then $E(Z)$ is the expected value of lognormal distrib.
- payoff of the "opposite" option i.e. call for put or put for call, then $E(Z)$ is the B-S price

where $c = -\dfrac{Cov(payoff, Z)}{Var(Z)}$

3. $price = e^{-rT} \frac{1}{N} \sum_{i=1}^{N} p(i)$

$\sigma = e^{-rT} std(\ p(i))$

If $Cov(payoff, Z), Var(Z)$ are not known approximate them using <u>PILOT</u> simulations