Problem 6

Consider the problem of the reconstruction of a sparse signal $x \in \mathbb{R}^N$ from given incomplete measurements $y \in \mathbb{R}^M$ and the measurements matrix $K \in \mathbb{R}^{M \times N}$ with $M \ll N$. This problem can be solved by

$$\min_x \|x\|_1 + \frac{\alpha}{2}\|Kx - y\|_2 \,. \tag{1}$$

where $\alpha > 0$ is the regularization parameter.

1. Create a signal $x \in \mathbb{R}^N$ with only $s \ll N$ non-zero entries and a Gaussian random matrix $K^{M \times N}$ with $M \ll N$. Normalize the columns of K and compute $y = Kx$.
2. Implement the proximal gradient method to solve the problem (1) for given y and K.
3. Investigate how does the reconstruction error change as you vary the number of measurements M and the sparsity level s.
4. Does the method still work if we add Gaussian noise to y?
5. Compare the reconstruction results obtained by (1) with the reconstruction results obtained by

$$\min_x \frac{1}{2}\|x\|_2^2 + \frac{\alpha}{2}\|Kx - y\|_2 \,.$$