Modeling and Simulation of RC, RL, and RLC Circuits as Linear Dynamical Systems

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***Abstract*—This case study presents a systematic approach for modeling and analyzing *RC*, *RL*, and *RLC* circuits using discrete-time linear dynamical systems. The objective is to understand how resistance, capacitance, and inductance influence circuit behavior. Series *RC*, *RL*, and *RLC* circuits were modeled using state-space representations derived from Kirchhoff's Voltage Law and discretized for simulation in MATLAB. A tuning process was developed to design three *RLC* circuits: a resonator, a sensor, and a filter. The resonator produced a stable 440 Hz signal, the sensor successfully isolated an 84 Hz helicopter hum, and the filter reduced unwanted noise while preserving the desired audio range. These results demonstrate the effectiveness of discrete-time linear modeling for predicting circuit responses and provide a practical framework for designing circuits digitally before physical creation.**

I. Introduction

This case study investigates various applications of *RLC* electrical circuits, emphasizing how circuit components influence overall behavior. Successful analysis requires understanding the roles of resistors, capacitors, and inductors, as well as how their interactions determine circuit response under different conditions.

Resistors convert electrical energy into heat, providing resistance R (measured in ohms, Ω) that opposes the flow of current. Capacitors store energy in electric fields, opposing sudden changes in voltage; the degree of voltage opposition is referred to as capacitance C (measured in farads, F). Finally, inductors store energy in magnetic fields, opposing sudden changes in current; the degree of current opposition is referred to as inductance L (measured in henries, H) [2].

Resistors, capacitors, and inductors can be combined with a voltage source to form an *RLC* circuit, a versatile configuration used in filtering, tuning radio receivers, and generating or stabilizing oscillations. The output of an *RLC* circuit depends on the values of R, L, and C, as well as the input voltage V_{in}. These circuits exhibit two phases: the transient state, which is a temporary period immediately following a change in current, and the steady state, which represents long-term stable behavior once transient effects have decayed.

The resonant frequency, denoted f_r, is the input frequency at which the effects of the capacitor and inductor cancel one another, producing resonance as energy alternates between electric and magnetic fields. At resonance, the circuit experiences its maximum output amplitude, while frequencies further from f_r result in reduced output.

The damping ratio ζ quantifies how quickly transient responses decay as the circuit settles into its steady state. In an ideal circuit, resistors are the only elements that dissipate energy and are therefore the primary contributors to damping. A low damping ratio ($\zeta < 1$) produces an underdamped circuit that oscillates before settling, whereas a high damping ratio ($\zeta > 1$) results in an overdamped circuit that settles slowly without oscillation.

The bandwidth BW of an *RLC* circuit determines the range of frequencies, centered around f_r, to be amplified. A small bandwidth indicates a selective circuit, while a large bandwidth allows a broader frequency response.

To explore these concepts, this case study aims to develop and simulate three common *RLC* circuit applications: a resonator, a sensor, and a filter. Section II will derive and apply the equations necessary to tune each circuit; Section III will explore circuit behaviors and discuss the results and limitations of the tuning process.

II. Methods

A. Discrete-Time Derivation for Series RLC Circuit

The first step in simulating any circuit is to derive a mathematical model. This subsection details the derivation of the discrete-time state-space representation for a series *RLC* circuit. The derivation begins with the application of Kirchhoff's Voltage Law and proceeds through discretization of the governing equations, yielding a model suitable for simulation using MATLAB's *ss()*[1] and *lsim()*[2] functions. Notably, though both *RC* and *RL* circuits will be mentioned in this paper, their derivations are provided in the study instructions and will not be repeated here.

1) Discrete-Time Current Equation: Kirchhoff's Voltage Law states that the sum of voltages across the inductor,

[1]Additional information and documentation on the *ss()* command in MATLAB can be found at https://www.mathworks.com/help/releases/R2025b/control/ref/ss.html?overload=ss+false (accessed Nov. 2025).

[2]Additional information and documentation on the *lsim()* command in MATLAB can be found at https://www.mathworks.com/help/releases/R2025b/control/ref/dynamicsystem.lsim.html (accessed Nov. 2025).

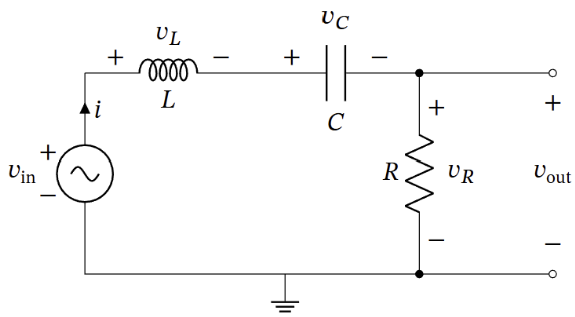

Fig. 1. Diagram of a series*RLC* circuit.

capacitor, and resistor of an *RLC* circuit equals the input voltage:

$$V_{L,k} = V_{\text{in},k} - V_{C,k} - V_{R,k} \tag{1}$$

Where $V_{in,k}$, $V_{L,k}$, $V_{C,k}$, and $V_{R,k}$ represent the input voltage and voltages across the inductor, capacitor, and resistor at time k, respectively.

The voltage–current relationships for the circuit elements are:

$$V_{L,k} = L\frac{i_{k+1} - i_k}{t_{k+1} - t_k}, \qquad \text{(Inductor voltage law)} \tag{2a}$$

$$V_{R,k} = i_k R, \qquad \text{(Ohm's law)} \tag{2b}$$

$$h = t_{k+1} - t_k, \qquad \text{(Discrete time step)} \tag{2c}$$

Substituting (2a)–(2c) into (1) gives:

$$L\frac{i_{k+1} - i_k}{h} = V_{\text{in},k} - V_{C,k} - i_k R \tag{3}$$

Rearranging for i_{k+1} yields:

$$i_{k+1} = \frac{h}{L}V_{\text{in},k} - \frac{h}{L}V_{C,k} + \left(1 - \frac{hR}{L}\right) i_k \tag{4}$$

Equation (4) defines the discrete-time current update, relating the next-step current to the input voltage, capacitor voltage, and previous current.

2) State-Space Representation: Let the state vector contain the capacitor voltage and inductor current:

$$X_k = \begin{bmatrix} V_{C,k} \\ i_k \end{bmatrix}, \qquad \text{(State vector)} \tag{5a}$$

$$u_k = V_{\text{in},k}, \qquad \text{(System input)} \tag{5b}$$

The capacitor voltage update is derived from $i = C\,\frac{dv}{dt}$, discretized as:

$$V_{C,k+1} = V_{C,k} + \frac{h}{C} i_k \tag{6}$$

Combining (5a) with (4) and (6) yields the coupled state equations:

$$X_{k+1} = \begin{bmatrix} V_{C,k} + \frac{h}{C} i_k \\ \frac{h}{L}V_{\text{in},k} - \frac{h}{L}V_{C,k} + \left(1 - \frac{hR}{L}\right) i_k \end{bmatrix} \tag{7}$$

3) Matrix Formulation: These relationships can be expressed compactly in matrix form as:

$$X_{k+1} = \begin{bmatrix} 1 & \frac{h}{C} \\ -\frac{h}{L} & 1 - \frac{hR}{L} \end{bmatrix} X_k + \begin{bmatrix} 0 \\ \frac{h}{L} \end{bmatrix} u_k \tag{8}$$

Equation (8) represents the discrete-time state-space update for the series *RLC* circuit.

4) Final State-Space System: Identifying the system matrices gives:

$$A = \begin{bmatrix} 1 & \frac{h}{C} \\ -\frac{h}{L} & 1 - \frac{hR}{L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{h}{L} \end{bmatrix} \tag{9}$$

Thus, the discrete-time state-space model is expressed as:

$$X_{k+1} = AX_k + Bu_k \tag{10}$$

B. Derivation of Calculations for Inductance, Capacitance, and Resistance

With a functional *RLC* simulation model, the next step in circuit tuning is to derive relationships that allow the calculation of capacitance, inductance, and resistance from more interpretable design parameters. This subsection presents the derivations used to compute these component values for use in the tuning process.

1) Capacitance Given Inductance and Resonant Frequency: The capacitance of an *RLC* circuit can be expressed in terms of its inductance and resonant frequency. The resonant frequency of a series *LC* circuit is given by [1, p. 924]:

$$f_{\text{r}} = \frac{1}{2\pi\sqrt{LC}} \tag{11}$$

Rearranging for capacitance gives:

$$C = \frac{1}{4\pi^2 f_{\text{r}}^2 L} \tag{12}$$

Equation (12) defines the capacitance *C* required to achieve a desired resonant frequency f_{r} for a known inductance *L*.

2) Resistance in Terms of Inductance and Damping Ratio: For an *RLC* circuit with a momentary input rather than a step input, it is often useful to relate the damping ratio to resistance. Using the following relationships [3]:

$$\alpha = \frac{R}{2L} \tag{13}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{14}$$

$$\zeta = \frac{\alpha}{\omega_0} \tag{15}$$

Substituting (13) and (14) into (15) gives:

$$\zeta = \frac{\frac{R}{2L}}{\frac{1}{\sqrt{LC}}} \tag{16}$$

Rearranging for resistance yields:

$$R = 2\zeta\sqrt{\frac{L}{C}} \tag{17}$$

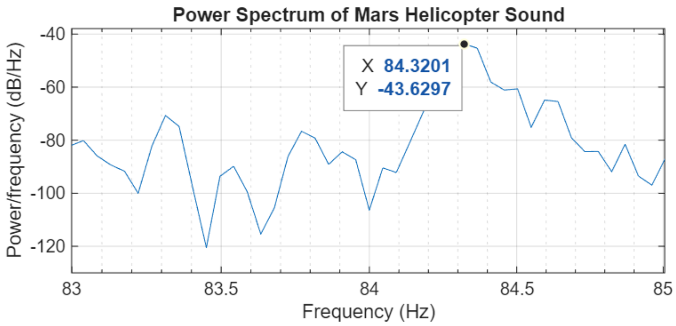

Fig. 2. Power spectrum of the mars helicopter sound, generated using the provided `plotpowerspectrum()` function. The spectrum is zoomed into the 83–85 Hz range, with the frequency corresponding to maximum power marked.

Equation (17) defines the resistance of an *RLC* circuit in terms of its inductance, capacitance, and target damping ratio. This equation is particularly useful in tuning circuits with impulse inputs, such as the resonator.

3) Inductance in Terms of Resistance and Bandwidth: The inductance of a series *RLC* circuit can also be expressed in terms of its resistance and bandwidth. According to [1, p. 931]:

$$L = \frac{R}{2\pi BW} \tag{18}$$

This equation is particularly useful for designing circuits with a continuous voltage input.

C. Final Tuning

Using the derivations above, the capacitance, inductance, and resistance of an *RLC* circuit can be determined based on the desired circuit behavior. This section describes how these relationships were used in the design of each tuned *RLC* circuit.

1) Resonator: The *RLC* resonator was designed to produce an output voltage that oscillates within a specific frequency band when given a momentary voltage input.

The target resonant frequency was 440 Hz, corresponding to the A_4 piano key. Given this target frequency, (12) was used to calculate the capacitance value in terms of inductance and desired bandwidth. To ensure a rapid response after an input pulse, a small inductance of 0.1 H was selected.

To calculate the optimal resistance for the *RLC* resonator, (17) was used to express the resistance in terms of damping ratio, inductance, and previously calculated capacitance. An initial damping ratio of 0.5 was chosen and a binary search process was used to iteratively tune the ratio to its final value of 0.0071875.

2) Sensor: Provided with a NASA sound recording from a lunar helicopter, the goal of the *RLC* sensor was to isolate and amplify the helicopter's 84 Hz noise byproduct, allowing for clear detection of when the helicopter was active. To determine the exact frequency of the helicopter's hum, the provided `plotpowerspectrum()` function was used to identify the frequency with maximum power near 84 Hz, measured as 84.3201 Hz (Fig. 2). With this frequency identified, a target resonant frequency of 84.3201 Hz was used to calculate the capacitance via (12). Then, Equation (18) was applied to compute the inductance based on the desired bandwidth.

To determine the optimal bandwidth, multiple values were tested using a binary search process; the ideal bandwidth would be narrow enough to filter out wind noise and wide enough to capture the helicopter's signal. For each trial, capacitance and inductance were recalculated using (12) and (18). During bandwidth tuning, the `soundsc()`[3] function was used to scale voltage output to its maximum range, effectively disregarding damping effects of an arbitrary resistance value.

Once the optimal bandwidth of 0.34 Hz was determined, multiple resistance values were tested, gradually reducing the resistance until a balance was achieved between output volume and signal clarity. This process revealed that because capacitance and inductance are already defined in terms of *R*, *BW*, and f_r, changing the resistance value has no practical effect on the simulated circuit. Any change in resistance automatically adjusts *L* and *C* to maintain the same *BW* and f_r, canceling any effect on the output amplitude. Therefore, an arbitrary resistance value of $1\ \Omega$ was selected for the final implementation.

3) Filter: The objective of the audio filter was to amplify a broader frequency range while filtering out both a high-frequency hiss and a 60 Hz hum. To achieve this, a larger bandwidth of 400 Hz was chosen to capture the desired vocal range.

Using (12) and (18), capacitance and inductance were calculated based on a midpoint frequency of 440 Hz. The output was then evaluated to identify the most effective resonant frequency, minimizing noise while maintaining clarity. Through this process, a resonant frequency of 600 Hz was selected, providing effective amplification in the target frequency range.

Finally, the bandwidth was refined by testing various values to find one small enough to filter out noise without over-amplifying the 600 Hz component. A final bandwidth of 200 Hz was selected. Changing resistance had negligible impact when *C* and *L* were recalculated to maintain BW and f_r; therefore, an arbitrary resistance value of $1\ \Omega$ was again chosen for the final implementation.

III. Results and Discussion

With the derivation and tuning processes complete, this section presents the results obtained from the developed circuits and provides an analysis of the behavior, findings, and limitations of the *RL*, *RC*, and *RLC* circuits and their derived configurations.

A. Analyzing RC and RL Circuits

Capacitors and inductors exhibit complementary transient and steady-state behaviors in *RC* and *RL* circuits. This arises from their aforementioned physical properties: capacitors store energy in an electric field and oppose voltage changes, while

[3]Additional information and documentation on the *soundsc()* command in MATLAB can be found at https://www.mathworks.com/help/releases/R2025b/matlab/ref/soundsc.html (accessed Nov. 2025).

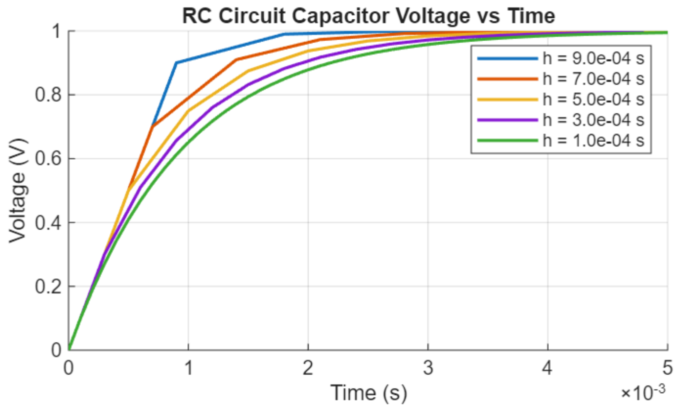

Fig. 3. *RC* voltage across the capacitor over time for various *h* steps.

inductors store energy in a magnetic field and oppose current changes.

1) Sampling Interval: The physical charging behavior of a real capacitor is governed by the function:

$$V_C(t) = V_{\text{in}} \left(1 - e^{-t/RC}\right) \quad \text{(See Appendix B For Derivation)} \tag{19}$$

The sampling interval h is the step size used to simulate the capacitor's charging voltage. As seen in Figure 3, changing this interval affects the precision of the model. The accuracy is inversely related to the sampling interval: as h decreases, the sampling rate increases, and the simulated voltage curve becomes more accurate. Smaller steps capture the steep initial slope of the exponential charge, resulting in higher accuracy. In contrast, larger h values cause a significant deviation from the theoretical curve $V_C(t)$ because large steps miss rapid changes in the differential equation, accumulating error. Because $V_C(t)$ is continuous, the charging of a real capacitor does not depend on the simulation's time step h.

2) RC Time Constant: The RC time constant τ governs the charging speed relative to the theoretical curve. It is defined as the time required for the capacitor voltage to reach $(1 - 1/e) \approx 63.2\%$ of its final steady-state voltage. Therefore, τ measures the circuit's response time: a larger τ indicates a slower response and a longer charge time, while a smaller τ indicates faster charging and response.

3) RC Circuit Response: Figure 3 shows that the voltage across the capacitor in an RC circuit is initially zero and gradually approaches the input voltage V_{in}. The initial current, determined by Ohm's law, is V_{in}/R and decays exponentially according to:

$$i_C(t) = \frac{V_{\text{in}}}{R} e^{-t/RC} \quad \text{(See Appendix B For Derivation)} \tag{20}$$

At $t = 0$, the capacitor acts as a short circuit because it has no stored charge. As $t \to \infty$, the capacitor voltage approaches V_{in}, blocking further current and acting as an open circuit.

4) RL Circuit Response: Figure 4 illustrates that in an *RL* circuit, the voltage across the inductor is initially equal to the input voltage and decays to zero over time. The current $i_L(t)$ is given by:

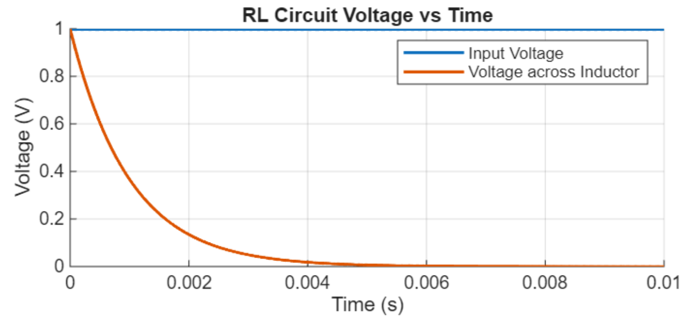

Fig. 4. Voltage across the inductor of an *RL* circuit over time.

$$i_L(t) = \frac{V_{in}}{R} \left(1 - e^{-\frac{R}{L}t}\right) \quad \text{(See Appendix A For Derivation)} \tag{21}$$

The current begins at zero and asymptotically approaches V_{in}/R. At $t = 0$, the inductor resists sudden current changes by generating a counter-voltage, acting as an open circuit. In steady state, the change in current change is zero, causing the inductor to stop generating counter-voltage and instead behave as a short circuit.

B. Analyzing an RLC Circuit with Step Voltage Input

Adjusting the resistance, inductance, and capacitance of series *RLC* circuits with step input reveals how these parameters influence the frequency and amplitude of the output signal, corresponding to pitch, volume, and decay rate of the resulting audio. The decay rate represents the behavior of the amplitude over time: a negative rate indicates decay, while a positive rate indicates growth.

Figure 5 shows that resistance has no measurable effect on output frequency, meaning pitch remains constant across R variations. However, peak amplitude and damping rate both depend on resistance; a higher R reduces peak amplitude and increases decay rate. Capacitance is inversely related to frequency and decay rate, but directly proportional to peak amplitude. Increasing capacitance lowers pitch, raises peak amplitude, and increases the rate of change of the amplitude. Inductance inversely affects both frequency and peak amplitude; increasing L lowers pitch and volume but is largely independent of amplitude decay, confirming that oscillation stability is governed mainly by R and C.

Three common output responses are sustained, decaying, and resonant oscillations. Sustained oscillation occurs under low damping with small resistance, where the circuit rings with nearly constant amplitude. Increasing R produces a decaying oscillation that fades over time, which can also result from increasing C or L. Reducing C sharpens oscillations, increasing frequency and amplitude to create highly resonant, rapidly growing behavior. When heard as sound, these responses correspond respectively to steady tone, fading tone, and a rapidly rising pitch that overwhelms the system.

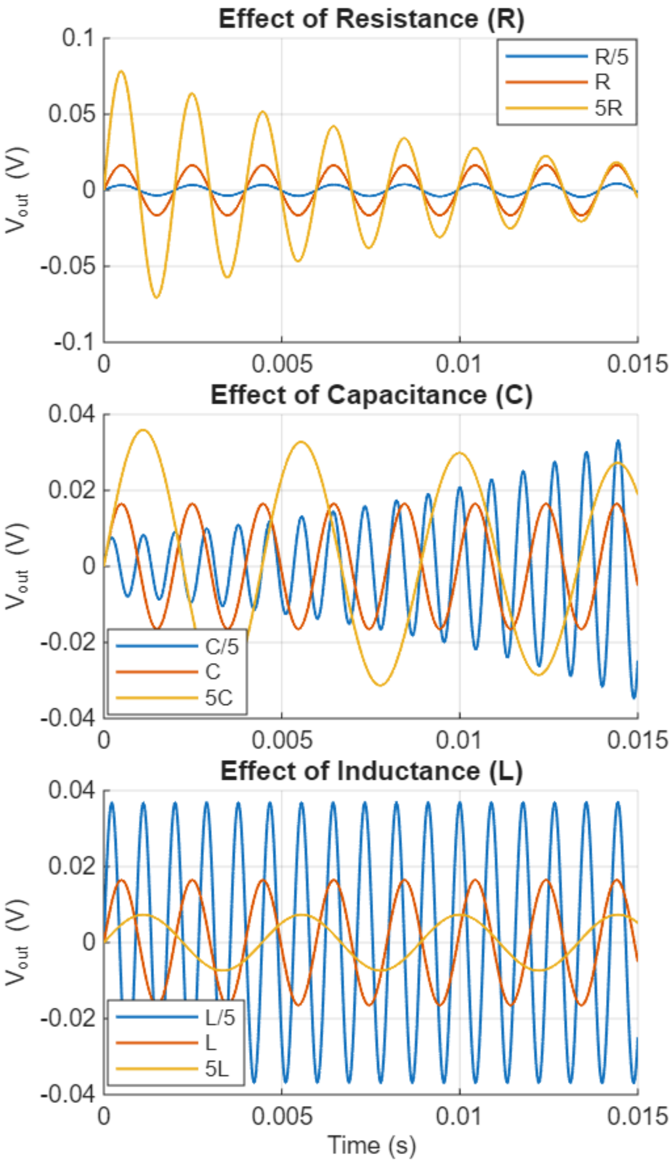

Fig. 5. Variations in R, L, and C for a step-input *RLC* circuit.

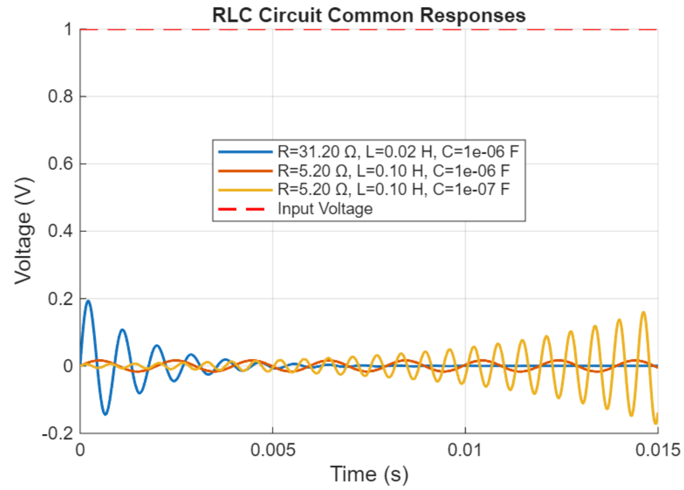

Fig. 6. Responses of an *RLC* circuit to a 1 V step input. Red: steady oscillation. Blue: decaying oscillation. Yellow: growing oscillation.

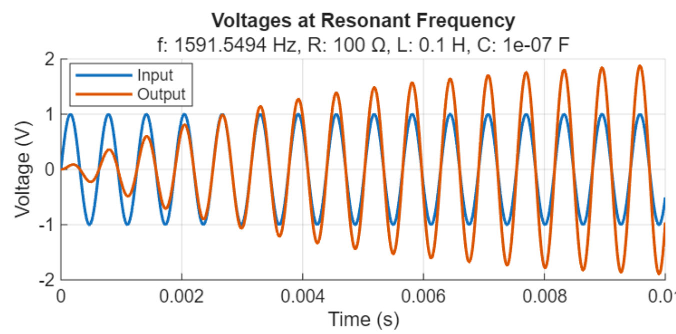

Fig. 7. Voltages at resonant frequency for an oscillating-input *RLC* circuit.

C. Analyzing an RLC Circuit with Oscillating Voltage Input

With an oscillating input, both transient and steady-state responses are observed. The capacitor and inductor must charge during the transient phase, which explains why the rise in peak amplitude. In steady state, the output mirrors the input's sinusoidal frequency and shape. This circuit functions as a band-pass filter, allowing a specific frequency range to pass while attenuating others. Equations (18) and (11) demonstrate how changing component values affect resonant frequency and bandwidth. Gain plots show circuit performance as the ratio of output to input peak voltage across a range of frequencies.

Equation (11) defines that the resonant frequency is inverse related to L and C; this relationship is strongly supported by the gain plots in Figure 8. Figure 7 shows the initial circuit peaking at 1591.5494 Hz, demonstrating high gain. Figure 8 also confirms that increasing L shifts the resonant peak leftward, lowering frequency, and shows the same effect for increasing C. In contrast, changing R affects only amplitude and width, confirming resistance is independent of resonant frequency.

(18) shows bandwidth is directly proportional to R and inversely proportional to L. Increasing R or decreasing L widens the response range. Figure 8 illustrates that higher R decreases resonant gain and flattens the curve, increasing bandwidth, while higher L narrows it. It also confirms that bandwidth is independent of C, as increasing C shifts the resonant frequency lower without changing width.

The frequency and amplitude of the output correspond directly to the pitch and volume of the audio signal. Lower frequencies yield lower pitch, while higher frequencies yield higher pitch. Similarly, smaller amplitudes sound quieter, and larger amplitudes louder. Resistance dictates peak amplitude because it alone dissipates energy. As shown in Figure 8, a smaller R dissipates less energy, producing a higher resonant peak; conversely, larger R dissipates more, damping the circuit and lowering amplitude.

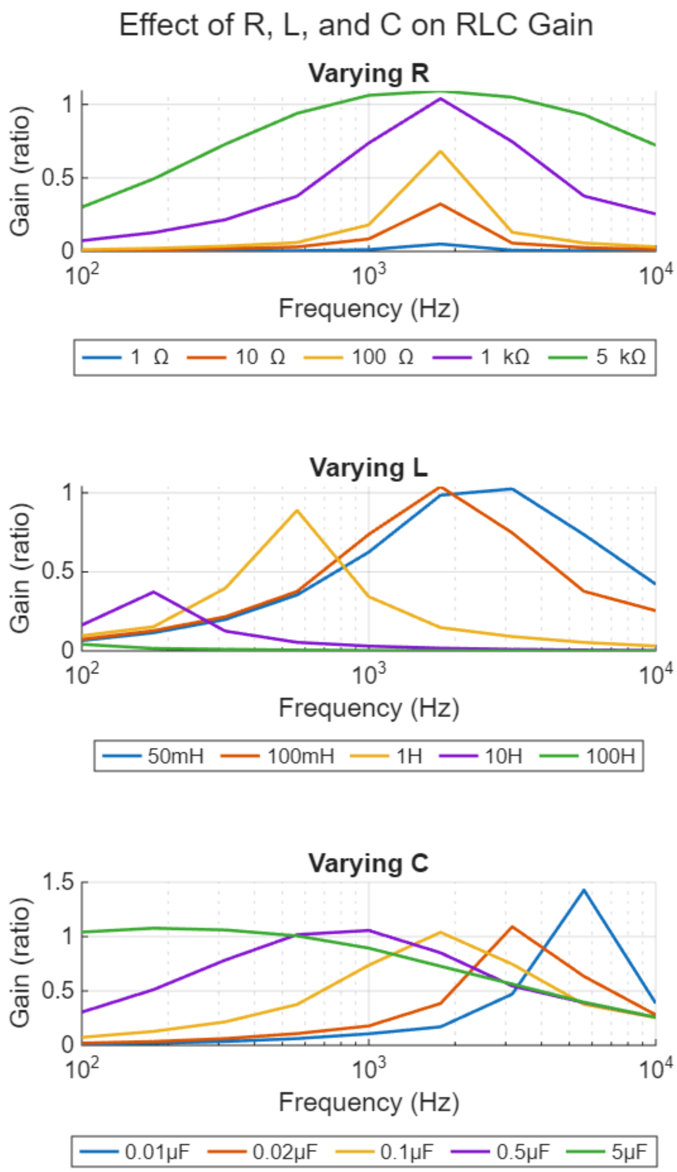

Fig. 8. Gain plots for an oscillating-input *RLC* circuit.

D. Performance, Limitations, and Future Work on the Competition Circuits

1) Resonator: The resonator tuning process described in Section II yielded a circuit that accurately emulated 440 Hz when given a momentary impulse. The resulting waveform demonstrated stable resonance at the target frequency, confirming the effectiveness of the design and tuning procedure.

2) Sensor: The sensor tuning process produced a circuit capable of amplifying the helicopter's hum. The main limitation was output volume: because tuning focused solely on bandwidth and resonant frequency, amplitude adjustment was not feasible without affecting these parameters. As a result, the output was quiet. Despite this, the sensor effectively isolated and amplified the helicopter hum while outputting exponentially less noise in other frequency bands.

3) Filter: The filter tuning process described in Section II yielded a functional *RLC* filter that significantly reduced the broadband hiss and the 60 Hz hum. However, because a single *RLC* circuit had to address two distinct noise sources, compromises were necessary and neither could be fully eliminated. Future improvements could investigate dual-stage filtering—using two *RLC* circuits, each tuned to a specific noise component—for more effective overall performance.

IV. Conclusion

This study explored the process of modeling and tuning *RC*, *RL*, and *RLC* circuits. By deriving and implementing state-space representations, the behavior of these circuits under diverse voltage inputs was successfully simulated and analyzed. The application-specific circuits, including a resonator, sensor, and filter, successfully demonstrated targeted behaviors; the resonator achieved sustained stable oscillation, the sensor isolated the desired signal from helicopter noise, and the filter minimized unwanted frequencies while preserving the relevant audio range.

Despite successfully simulating *RC*, *RL*, and *RLC* circuits, the study faced several limitations. Interdependence of *R*, *L*, and *C* values caused reduced control over amplitude in the sensor and filter; additionally, there were constraints in filter design when attempting to suppress multiple noise components with a single *RLC* stage. Future work may explore multi-stage filtering, adaptive circuit tuning, or the integration of these simulations with physical prototypes for validation.

Overall, this case study confirms that mathematical modeling provides a practical framework for understanding and designing digital *RLC* circuits, offering a foundation for easy circuit prototyping, conceptualization, and design.

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[3] K. Webb, *Section 4: Second-Order Transient Response*. Corvallis, OR, USA: Oregon State University, School of Electrical Engineering and Computer Science. Available: https://web.engr.oregonstate.edu/~webbky/ENGR202_files/SECTION%204%20Second%20Order%20Transient%20Response.pdf [Accessed: 08 Nov. 2025].

V. Notes on the Utilization of AI

Artificial Intelligence tools, such as Underleaf.ai, ChatGPT, and Google Gemini, were utilized during the planning and development of this case study. These tools assisted in tasks including grammar refinement, formatting, reorganization, and logic or consistency checks. All generative responses were thoroughly reviewed, edited, and verified for accuracy prior to use. This paper represents the final product of significant intellectual contribution by Lucas Selvik and Rex Paster and all submitted materials have been carefully checked to ensure accuracy and to authentically represent our original work.

Appendix A
Derivation of Steady State Current in an RL Circuit

The derivation of the current in a series *RL* circuit begins with Kirchhoff's Voltage Law, which states that the sum of voltages around a closed loop equals zero [1]:

$$V_{in} - V_R - V_L = 0 \tag{22}$$

where $V_R = iR$ is the voltage across the resistor and $V_L = L\frac{di}{dt}$ and represents the voltage across the inductor. Substituting these expressions into (22) gives:

$$V_{in} - iR - L\frac{di}{dt} = 0 \tag{23}$$

Rearranging (23) to isolate the derivative term yields:

$$\frac{V_{in}}{R} - i = \frac{L}{R}\frac{di}{dt} \tag{24}$$

This equation is now ready for separation of variables. Moving all terms involving i to one side and t to the other:

$$\int_0^t \frac{R}{L}\,dt = \int_{i_0}^{i} \frac{di}{\frac{V_{in}}{R} - i} \tag{25}$$

where i_0 is the initial current (typically 0 for an unpowered circuit). Integrating both sides of (25) gives:

$$\frac{R}{L}t = -\ln\left|\frac{\frac{V_{in}}{R} - i}{\frac{V_{in}}{R} - i_0}\right| \tag{26}$$

Exponentiating both sides of (26) to solve for i results in:

$$\frac{V_{in}}{R}e^{-\frac{R}{L}t} = \frac{V_{in}}{R} - i \tag{27}$$

Finally, isolating i in (27) gives the current as a function of time:

$$\boxed{i_L(t) = \frac{V_{in}}{R}\left(1 - e^{-\frac{R}{L}t}\right)} \tag{28}$$

In equation (28), V_{in}/R represents the steady-state current. The exponential term shows that the current rises gradually from zero at $t = 0$ to the steady-state value as $t \to \infty$.

Appendix B
Derivation of Charging Behavior in an RC Circuit

The derivation of the voltage and current in a series *RC* circuit begins with Kirchhoff's Voltage Law, which states that the sum of voltages around a closed loop equals the applied voltage [1]:

$$V_{\text{in}} - V_R - V_C = 0 \tag{29}$$

In this equation, $V_R = iR$, where V_R represents the voltage across the resistor, and $V_C = \frac{Q}{C}$, where V_C represents the voltage across the capacitor and Q is the charge on the capacitor. Substituting these expressions into 29 gives:

$$V_{\text{in}} - R\frac{dQ}{dt} - \frac{Q}{C} = 0 \tag{30}$$

Rewriting, we obtain a first-order linear differential equation:

$$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_{\text{in}}}{R} \tag{31}$$

Separating variables and integrating:

$$\int \frac{dQ}{V_{\text{in}}C - Q} = \int \frac{dt}{RC} \tag{32}$$

Solving the integral and applying the initial condition $Q(0) = 0$:

$$Q_C(t) = V_{\text{in}}C\left(1 - e^{-t/RC}\right) \tag{33}$$

The current is the time derivative of charge:

$$\boxed{i_C(t) = \frac{dQ}{dt} = \frac{V_{\text{in}}}{R}e^{-t/RC}} \tag{34}$$

Finally, the voltage across the capacitor is:

$$\boxed{V_C(t) = \frac{Q_C(t)}{C} = V_{\text{in}}\left(1 - e^{-t/RC}\right)} \tag{35}$$

These expressions describe the exponential charging behavior of a capacitor in an *RC* circuit, where *RC* is the time constant governing the rate of voltage change [1].