Análise de Sistemas de Potência

Aula 06: Teoria das Componentes Simétricas

Prof. Lucas Melo

Universidade Federal do Ceará

12 de agosto de 2024

Este material é inteiramente baseado no livro "Power Systems Design and Analisys" (6th Edition) dos autores J. Duncan Glover, Thomas J. Overbye e Mulukutlas S. Sarma.

Sumário

Definition Of Symmetrical Components

Assume that a set of three-phase voltages designated V_a, V_b, and V_c is given. In accordance with Fortescue, these phase voltages are resolved into the following three sets of sequence components:

1. **Zero-sequence components, consisting of three phasors with equal magnitudes and with zero phase displacement;**
2. **Positive-sequence components, consisting of three phasors with equal magnitudes, $+120^\circ$ phase displacement, and positive sequence;**
3. **Negative-sequence components, consisting of three phasors with equal magnitudes, $+120^\circ$ phase displacement, and negative sequence.**

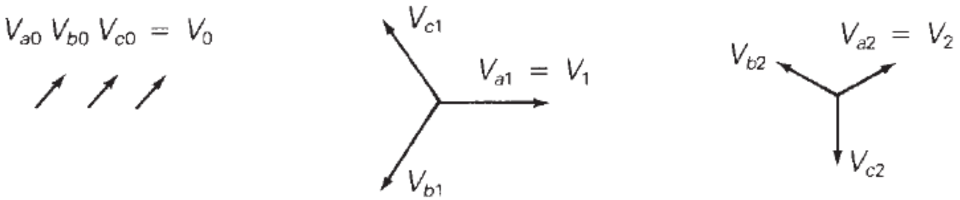
The zero-, positive-, and negative-sequence components of phase a, which are V_{a0}, V_{a1}, and V_{a2}, respectively, are presented in this section.

For simplicity, drop the subscript a and denote these sequence components as V_0, V_1, and V_2. They are defined by the following transformation:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

Where:

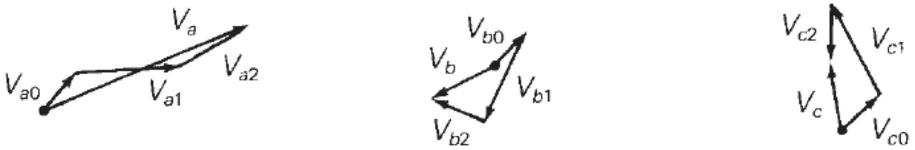
$$a = 1\underline{/120^\circ} = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$$



(a) Zero-sequence components

(b) Positive-sequence components

(c) Negative-sequence components



$$a^4 = a = 1\angle 120^\circ$$

$$a^2 = 1\angle 240^\circ$$

$$a^3 = 1\angle 0^\circ$$

$$1 + a + a^2 = 0$$

$$1 - a = \sqrt{3}\angle 30^\circ$$

$$1 - a^2 = \sqrt{3}\angle 30^\circ$$

$$a^2 - a = \sqrt{3}\angle 270^\circ$$

$$ja = 1\angle 210^\circ$$

$$1 + a = -a^2 = 1\angle 60^\circ$$

$$1 + a^2 = -a = 1\angle 60^\circ$$

$$a + a^2 = -1 = 1\angle 180^\circ$$

Common identities involving $a = 1\angle 120^\circ$

Writing voltage expressions as three separate equations:

$$V_a = V_0 + V_1 + V_2$$
$$V_b = V_0 + a^2 V_1 + a V_2$$
$$V_c = V_0 + a V_1 + a^2 V$$

a **is a complex number with unit magnitude and a** $120°$ **phase angle.**

When any phasor is multiplied by a**, that phasor rotates by** $120°$ **(counterclockwise).**

Similarly, when any phasor is multiplied by $a^2 = (1/120°)\,(1/120°) = 1/240°$**, the phasor rotates by** $240°$**.**

The complex number a **is similar to the well-known complex number** $j = \sqrt{-1} = 1\angle 90°$**, Thus, the only difference between** j **and** a **is that the angle of** j **is** $90°$**, and that of** a **is** $120°$**.**

Equation above can be rewritten more compactly using matrix notation. Define the following vectors $\boldsymbol{V}_{\mathrm{P}}$ and $\boldsymbol{V}_{\mathrm{S}}$, and matrix $\boldsymbol{A}$:

$$\boldsymbol{V}_{\mathrm{P}} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad \boldsymbol{V}_{\mathrm{S}} = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \qquad \boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Where $\boldsymbol{V}_{\mathrm{P}}$ is the column vector of phase voltages, $\boldsymbol{V}_{\mathrm{S}}$ is the column vector of sequence voltages, and $\boldsymbol{A}$ is a 3×3 transformation matrix. Using these definitions:

$$\boldsymbol{V}_{\mathrm{P}} = \boldsymbol{A}\boldsymbol{V}_{\mathrm{S}}$$

The inverse of the A matrix is

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

This Equation can be verified by showing that the product AA^{-1} is the unit matrix. Also, premultiplying by A^{-1} gives:

$$V_S = A^{-1} V_P$$

Using an equation combination, we obtain:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Writing sequence voltage as three separate equations, we have:

$$V_0 = \frac{1}{3}(V_a + V_b + V_c)$$
$$V_1 = \frac{1}{3}(V_a + aV_b + a^2V_c)$$
$$V_2 = \frac{1}{3}(V_a + a^2V_b + aV_c)$$

Equation above shows that there is no zero-sequence voltage in a balanced threephase system because the sum of three balanced phasors is zero.

In an unbalanced three-phase system, line-to-neutral voltages may have a zero-sequence component. However, line-to-line voltages never have a zero-sequence component, since by KVL, their sum is always zero.

The symmetrical component transformation also can be applied to currents, as follows:

$$\boldsymbol{I}_{\mathrm{P}} = \boldsymbol{A}\boldsymbol{I}_{\mathrm{S}}$$

Where $\boldsymbol{I}_{\mathrm{P}}$ is a vector of phase currents:

$$\boldsymbol{I}_{\mathrm{P}} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$\boldsymbol{I}_{\mathrm{S}}$ is a vector of sequence currents,

$$\boldsymbol{I}_{\mathrm{S}} = \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Also,

$$\boldsymbol{I}_{\mathrm{S}} = \boldsymbol{A}^{-1}\boldsymbol{I}_{\mathrm{P}}$$

Sequence currents can be written as separate equations as follows. The phase currents are:

$$I_a = I_0 + I_1 + I_2$$
$$I_b = I_0 + a^2 I_1 + a I_2$$
$$I_c = I_0 + a I_1 + a^2 I_2$$

And the sequence currents are:

$$I_0 = \frac{1}{3}(I_a + I_b + I_c)$$
$$I_1 = \frac{1}{3}(I_a + a I_b + a^2 I_c)$$
$$I_2 = \frac{1}{3}(I_a + a^2 I_b + a I_c)$$

In a three-phase Y-connected system, the neutral current I_n is the sum of the line currents:

$$I_n = I_a + I_b + I_c$$

Comparing equations, give us;

$$I_n = 3I_0$$

The neutral current equals three times the zero-sequence current.

In a balanced Y-connected system, line currents have no zero-sequence component, since the neutral current is zero. Also, in any three-phase system with no neutral path, such as a Δ-connected system or a three-wire Y-connected system with an ungrounded neutral, line currents have no zero-sequence component.

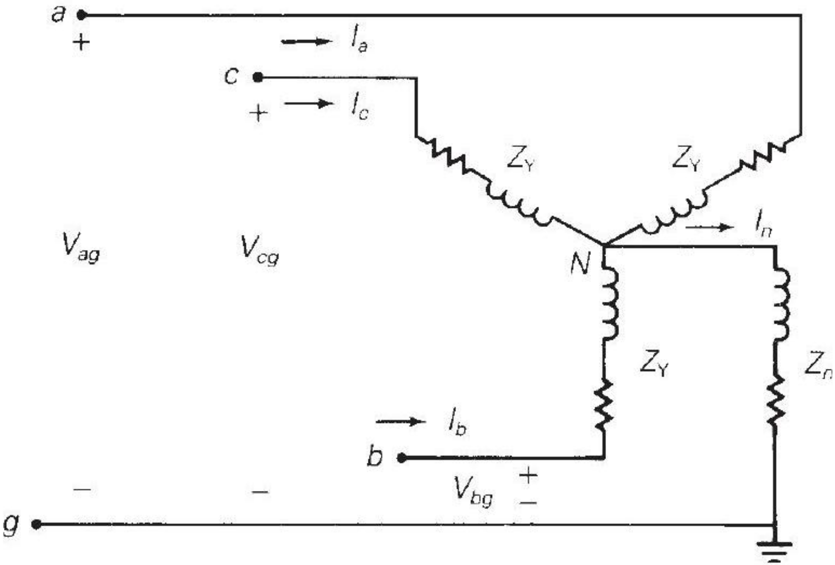
Sequence Networks Of Impedance Loads

Figure below shows a balanced- Y impedance load.

The impedance of each phase is designated Z_{Y}, and a neutral impedance Z_n is connected between the load neutral and ground.

Note from Figure that the line-to-ground voltage V_{ag} is:

$$
\begin{aligned}
V_{ag} &= Z_{\mathrm{Y}} I_a + Z_n I_n \\
&= Z_{\mathrm{Y}} I_a + Z_n \left(I_a + I_b + I_c\right) \\
&= \left(Z_{\mathrm{Y}} + Z_n\right) I_a + Z_n I_b + Z_n I_c
\end{aligned}
$$


a
+
I_a
c
+
I_c
Z_Y
Z_Y
I_n
V_{ag}
V_{cg}
N
Z_Y
Z_n
I_b
b
+
V_{bg}
−
−
−
g

Similar equations can be written for V_{bg} and V_{eg} :

$$
\begin{aligned}
V_{bg} &= Z_n I_a + \left(Z_{\mathrm{Y}} + Z_n\right) I_b + Z_n I_c \\
V_{cg} &= Z_n I_a + Z_n I_b + \left(Z_{\mathrm{Y}} + Z_n\right) I_c
\end{aligned}
$$

This equations can be rewritten in matrix format:

$$
\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} \left(Z_{\mathrm{Y}} + Z_n\right) & Z_n & Z_n \\ Z_n & \left(Z_{\mathrm{Y}} + Z_n\right) & Z_n \\ Z_n & Z_n & \left(Z_{\mathrm{Y}} + Z_n\right) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}
$$

More compactly as:

$$\boldsymbol{V}_{\mathrm{P}} = \boldsymbol{Z}_{\mathrm{P}}\boldsymbol{I}_{\mathrm{P}}$$

Where $\boldsymbol{V}_{\mathrm{p}}$ is the vector of line-to-ground voltages (or phase voltages), $\boldsymbol{I}_{\mathrm{p}}$ is the vector of line currents (or phase currents), and $\boldsymbol{Z}_{\mathrm{P}}$ is the 3×3 phase impedance matrix.

The equations developed in the later section can now be used in to determine the relationship between the sequence voltages and currents, as follows:

$$\boldsymbol{A}\boldsymbol{V}_{\mathrm{S}} = \boldsymbol{Z}_{\mathrm{P}}\boldsymbol{A}\boldsymbol{I}_{\mathrm{S}}$$

Now, premultiplying both sides of $\boldsymbol{A}^{-1}$ gives

$$\boldsymbol{V}_{\mathrm{S}} = (\boldsymbol{A}^{-1}\boldsymbol{Z}_{\mathrm{P}}\boldsymbol{A})\,\boldsymbol{I}_{\mathrm{S}}$$

Or

$$V_{\mathrm{S}} = Z_{\mathrm{S}}I_{\mathrm{S}}$$

Where

$$\boldsymbol{Z}_{\mathrm{S}} = \boldsymbol{A}^{-1}\boldsymbol{Z}_{\mathrm{P}}\boldsymbol{A}$$

The impedance matrix Z_S is called the sequence impedance matrix.

Using the definition of A, its inverse A^{-1}, and Z_P, the sequence impedance matrix Z_S for the balanced-Y load is:

$$Z_S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} (Z_Y + Z_n) & Z_n & Z_n \\ Z_n & (Z_Y + Z_n) & Z_n \\ Z_n & Z_n & (Z_Y + Z_n) \end{bmatrix}$$
$$\times \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Performing the indicated matrix multiplications in last equation, and using the identity $(1 + a + a^2) = 0$:

$$Z_S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} (Z_Y + 3Z_n) & Z_Y & Z_Y \\ (Z_Y + 3Z_n) & a^2 Z_Y & aZ_Y \\ (Z_Y + 3Z_n) & aZ_Y & a^2 Z_Y \end{bmatrix}$$

$$Z_S = \begin{bmatrix} (Z_Y + 3Z_n) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}$$

As shown above, the sequence impedance matrix Z_S for the balanced-Y load is a diagonal matrix.

Since Z_S is diagonal, its definition can be written as three uncoupled equations:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (Z_Y + 3Z_n) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Rewriting as three separate equations:

$$\begin{aligned} V_0 &= (\mathrm{Z}_\mathrm{Y} + 3\mathrm{Z}_n)\, I_0 = \mathrm{Z}_0 I_0 \\ V_1 &= Z_Y I_1 = Z_1 I_1 \\ V_2 &= Z_Y I_2 = Z_2 I_2 \end{aligned}$$

The zero-sequence voltage V_0 depends only on the zerosequence current I_0 and the impedance $(Z_Y + 3Z_n)$. This impedance is called the zero-sequence impedance and is designated Z_0.

The positive-sequence voltage V_1 depends only on the positive-sequence current I_1 and an impedance $Z_1 = Z_Y$ called the positive-sequence impedance.

Similarly, V_2 depends only on I_2 and the negativesequence impedance $Z_2 = Z_Y$.

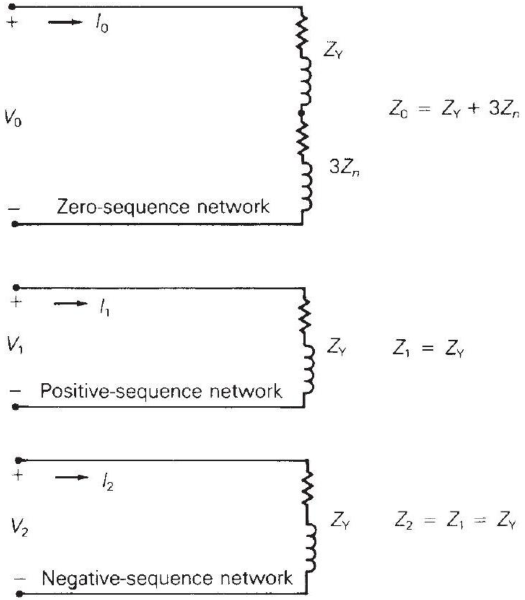
These equations can be represented by the three networks shown in Figure below.

These networks are called the zero-sequence, positive-sequence, and negative-sequence networks.

As shown, each sequence network is separate, uncoupled from the other two.

The separation of these sequence networks is a consequence of the fact that Z_S is a diagonal matrix for a balanced- Y load.

This separation underlies the advantage of symmetrical components.



+
I_0
Z_Y
V_0
$Z_0 = Z_Y + 3Z_n$
$3Z_n$
–
Zero-sequence network
+
I_1
V_1
Z_Y
$Z_1 = Z_Y$
–
Positive-sequence network
+
I_2
V_2
Z_Y
$Z_2 = Z_1 = Z_Y$
–
Negative-sequence network

Note that the neutral impedance does not appear in the positive- and negative-sequence networks of Figure above.

This illustrates the fact that positive- and negative-sequence currents do not flow in neutral impedances.

However, the neutral impedance is multiplied by 3 and placed in the zero-sequence network of the figure.

The voltage $I_0\ (3Z_n)$ across the impedance $3Z_n$ is the voltage drop $(I_n Z_n)$ across the neutral impedance Z_n in Figure above, since $I_n = 3I_0$.

When the neutral of the Y load in Figure above has no return path, then the neutral impedance Z_n is infinite and the term $3Z_n$ in the zero-sequence network becomes an open circuit.

Under this condition of an open neutral, no zero-sequence current exists.

However, when the neutral of the Y load is solidly grounded with a zero-ohm conductor, then the neutral impedance is zero and the term $3Z_n$ in the zero-sequence network becomes a short circuit.

Under this condition of a solidly grounded neutral, zero-sequence current I_o can exist when there is a zero-sequence voltage caused by unbalanced voltages applied to the load.

Figure below shows a balanced- Δ load and its equivalent balanced- Y load.

Since the Δ load has no neutral connection, the equivalent Y load has an open neutral.

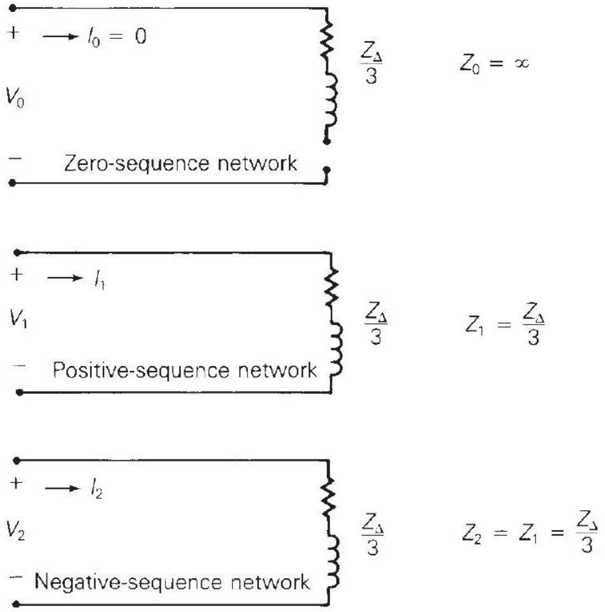
The sequence networks of the equivalent Y load corresponding to a balanced- Δ load are shown in Figure below.

As shown, the equivalent Y impedance $Z_Y = Z_\Delta/3$ appears in each of the sequence networks.

Also, the zero-sequence network has an open circuit, since $Z_n = \infty$ corresponds to an open neutral. No zero-sequence current occurs in the equivalent Y load.

FIGURE 8.5

Sequence networks for an equivalent Y representation of a balanced-Δ load



The sequence networks of Figure above represent the balanced- Δ load as viewed from its terminals, but they do not represent the internal load characteristics.

The currents I_0, I_1, and I_2 in Figure are the sequence components of the line currents feeding the Δ load, not the load currents within the Δ.

The Δ load currents, which are related to the line currents are not shown in Figure.

FIGURE 8.7

General three-phase impedance load (linear, bilateral network, nonrotating equipment)

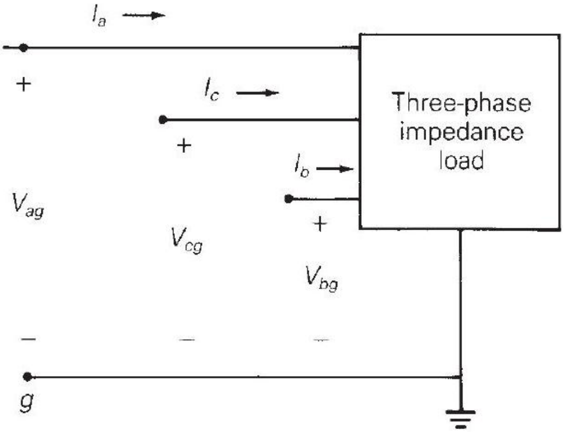


Figure above shows a general three-phase linear impedance load.

The load could represent a balanced load such as the balanced-Y or balanced- Δ load, or an unbalanced impedance load.

The general relationship between the line-to-ground voltages and line currents for this load can be written as:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Or in a compact form:

$$\boldsymbol{V}_{\mathrm{P}} = \boldsymbol{Z}_{\mathrm{P}}\boldsymbol{I}_{\mathrm{P}}$$

Where $\boldsymbol{V}_{\mathrm{P}}$ is the vector of line-to-neutral (or phase) voltages, $\boldsymbol{I}_{\mathrm{P}}$ is the vector of line (or phase) currents, and Z_{P} is a 3×3 phase impedance matrix.

It is assumed here that the load is nonrotating, and that $\boldsymbol{Z}_{\mathrm{P}}$ is a symmetric matrix, which corresponds to a bilateral network.

Since equation above has the same form as Y load phase equations, the relationship between the sequence voltages and currents for the general three-phase load of Figure above is the same as that of Y load sequence equations, which are rewritten here:

$$\boldsymbol{V}_{\mathrm{S}} = \boldsymbol{Z}_{\mathrm{S}}\boldsymbol{I}_{\mathrm{S}}$$

$$\boldsymbol{Z}_{\mathrm{S}} = \boldsymbol{A}^{-1}\boldsymbol{Z}_{\mathrm{P}}\boldsymbol{A}$$

The sequence impedance matrix $\boldsymbol{Z}_{\mathrm{S}}$ given above is a 3×3 matrix with nine sequence impedances, defined as follows:

$$Z_{\mathrm{S}} = \begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix}$$

The diagonal impedances Z_0, Z_1, and Z_2 in this matrix are the self-impedances of the zero-, positive-, and negative-sequence networks.

The off-diagonal impedances are the mutual impedances between sequence networks.

Using the definitions of $\boldsymbol{A}$, $\boldsymbol{A}^{-1}$, $\boldsymbol{Z}_{\mathrm{P}}$, and $\boldsymbol{Z}_{\mathrm{S}}$:

$$\begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{cb} & Z_{cc} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Diagonal Sequence Impedances

Performing the indicated multiplications, and using the identity:

$$(1 + a + a^2) = 0$$

The following separate equations can be obtained.

$$Z_0 = \frac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} + 2Z_{ab} + 2Z_{ac} + 2Z_{bc})$$

$$Z_1 = Z_2 = \frac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} - Z_{ab} - Z_{ac} - Z_{bc})$$

Off-Diagonal Sequence Impedances

$$Z_{01} = Z_{20} = \frac{1}{3}\left(Z_{aa} + a^2 Z_{bb} + a Z_{cc} - a Z_{ab} - a^2 Z_{ac} - Z_{bc}\right)$$

$$Z_{02} = Z_{10} = \frac{1}{3}\left(Z_{aa} + a Z_{bb} + a^2 Z_{cc} - a^2 Z_{ab} - a Z_{ac} - Z_{bc}\right)$$

$$Z_{12} = \frac{1}{3}\left(Z_{aa} + a^2 Z_{bb} + a Z_{cc} + 2a Z_{ab} + 2a^2 Z_{ac} + 2Z_{bc}\right)$$

$$Z_{21} = \frac{1}{3}\left(Z_{aa} + a Z_{bb} + a^2 Z_{cc} + 2a^2 Z_{ab} + 2a Z_{ac} + 2Z_{bc}\right)$$

A symmetrical load is defined as a load whose sequence impedance matrix is diagonal; that is, all the mutual impedances are zero.

Equating these mutual impedances to zero and solving, the following conditions for a symmetrical load are determined.

When both:

$$\left.\begin{aligned} Z_{aa} &= Z_{bb} = Z_{cc} \\ Z_{ab} &= Z_{ac} = Z_{bc} \end{aligned}\right\} \text{ conditions for a symmetrical load}$$

Then

$$\begin{aligned} Z_{01} &= Z_{10} = Z_{02} = Z_{20} = Z_{12} = Z_{21} = 0 \\ Z_0 &= Z_{aa} + 2Z_{ab} \\ Z_1 &= Z_2 = Z_{aa} - Z_{ab} \end{aligned}$$

The conditions for a symmetrical load are that the diagonal phase impedances be equal and that the off-diagonal phase impedances be equal.

These conditions can be verified with the identity $(1 + a + a^2) = 0$ to show that all the mutual sequence impedances are zero.

Note that the positive- and negative-sequence impedances are equal for a symmetrical load, and for a nonsymmetrical load.

This is always true for linear, symmetric impedances that represent nonrotating equipment such as transformers and transmission lines.

However, the positive- and negative sequence impedances of rotating equipment such as generators and motors are generally not equal.

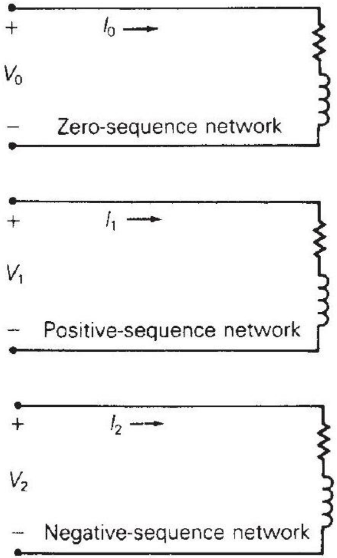
Note also that the zero-sequence impedance Z_0 is not equal to the positive- and negative-sequence impedances of a symmetrical load unless the mutual phase impedances $Z_{ab} = Z_{ac} = Z_{bc}$ are zero.

The sequence networks of a symmetrical impedance load are shown in Figure above.

Since the sequence impedance matrix Z_S is diagonal for a symmetrical load, the sequence networks are separate or uncoupled.

FIGURE 8.8

Sequence networks of a three-phase symmetrical impedance load (linear, bilateral network, nonrotating equipment)



$$Z_0 = Z_{aa} + 2Z_{ab}$$

$$Z_1 = Z_{aa} - Z_{ab}$$

$$Z_2 = Z_1 = Z_{aa} - Z_{ab}$$

Sequence Networks Of Series Impedances

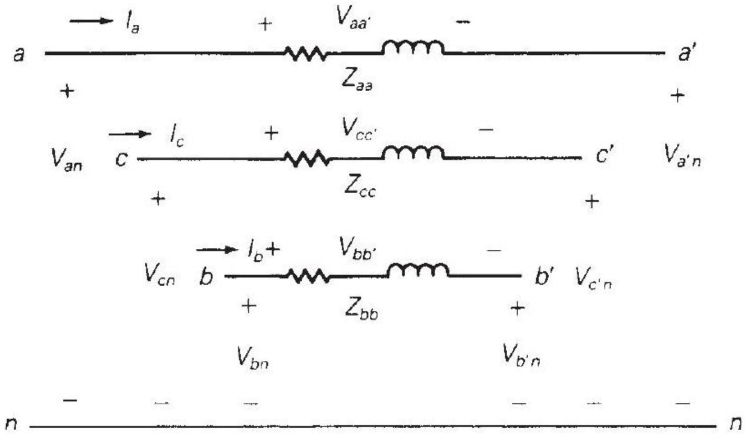
Figure above shows series impedances connected between two three-phase buses denoted abc and $a'b'c'$.

Self-impedances of each phase are denoted Z_{aa}, Z_{bb}, and Z_{cc}.

In general, the series network also may have mutual impedances between phases.

The voltage drops across the series-phase impedances are given by:

$$\begin{bmatrix} V_{an} - V_{a'n} \\ V_{bn} - V_{b'n} \\ V_{cn} - V_{c'n} \end{bmatrix} = \begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$



Three-phase series impedances (linear, bilateral network, nonrotating equipment)

Both self-impedances and mutual impedances are included in equation above.

It is assumed that the impedance matrix is symmetric, which corresponds to a bilateral network.

It is also assumed that these impedances represent nonrotating equipment.

Typical examples are series impedances of transmission lines and of transformers.

In a compact form:

$$\boldsymbol{V}_{\mathrm{P}} - \boldsymbol{V}_{\mathrm{P}'} = \boldsymbol{Z}_{\mathrm{P}} \boldsymbol{I}_{\mathrm{P}}$$

Where $\boldsymbol{V}_{\mathrm{P}}$ is the vector of line-to-neutral voltages at bus abc, $\boldsymbol{V}_{\mathrm{P}'}$ the vector of line-toneutral voltages at bus $a'b'c'$, $\boldsymbol{I}_{\mathrm{P}}$ is the vector of line currents, and $\boldsymbol{Z}_{\mathrm{P}}$ is the 3×3 phase impedance matrix for the series network.

This equation is now transformed to the sequence domain in the same manner that the load-phase impedances were transformed.

Thus,

$$\boldsymbol{V}_{\mathrm{S}} - \boldsymbol{V}_{\mathrm{S'}} = \boldsymbol{Z}_{\mathrm{S}}\boldsymbol{I}_{\mathrm{S}}$$

Where

$$Z_{\mathrm{S}} = A^{-1} Z_{\mathrm{P}} \boldsymbol{A}$$

From the results of Section 8.2, this sequence impedance Z_S matrix is diagonal under the following conditions:

$$\left.\begin{array}{l} Z_{aa} = Z_{bb} = Z_{cc} \\ Z_{ab} = Z_{ac} = Z_{bc} \end{array}\right\} \quad \textbf{conditions for symmetrical series impedances}$$

When the phase impedance matrix Z_P has both equal self-impedances and equal mutual impedances, then:

$$Z_S = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

Where

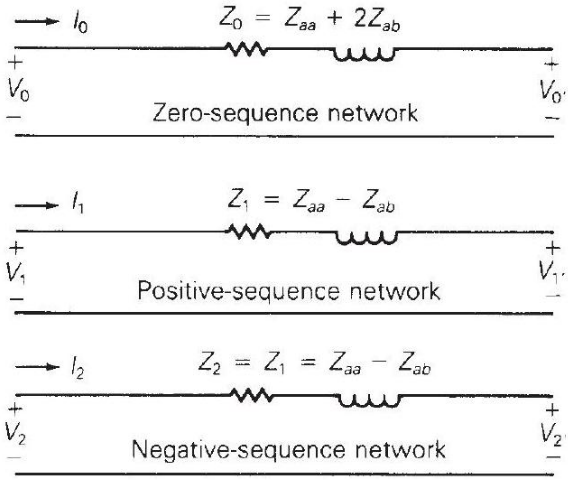
$$Z_0 = Z_{aa} + 2Z_{ab}$$
$$Z_1 = Z_2 = Z_{aa} - Z_{bb}$$

And now we have three uncoupled equations, written as follows:

$$V_0 - V_{0'} = Z_0 I_0$$
$$V_1 - V_{1'} = Z_1 I_1$$
$$V_2 - V_{2'} = Z_2 I_2$$

FIGURE 8.10

Sequence networks of three-phase symmetrical series impedances (linear, bilateral network, nonrotating equipment)



and

$$Z_1 = Z_2 = Z_{aa} - Z_{ab}$$

Equations above are represented by the three uncoupled sequence networks shown in Figure.

From the figure it is apparent that, for symmetrical series impedances, positive-sequence currents produce only positive-sequence voltage drops.

Similarly, negative-sequence currents produce only negative-sequence voltage drops, and zero-sequence currents produce only zero-sequence voltage drops.

However, if the series impedances are not symmetrical, then Z_S is not diagonal, the sequence networks are coupled, and the voltage drop across any one sequence network depends on all three sequence currents.

Sequence Networks Of Three-Phase Lines

We have equations suitable for computer calculation of the series phase impedances, including resistances and inductive reactances, of three-phase overhead transmission lines.

The series phase impedance matrix Z_p for an untransposed line is given by:

$$Z_P = \left[Z_A - Z_B Z_D^{-1} Z_C\right]$$

$$= \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}$$

For a completely transposed line, $\hat{Z}_{\mathrm{P}}$, we have:

$$\hat{Z}_{\mathrm{P}} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ab} \\ Z_{ab} & Z_{aa} & Z_{ab} \\ Z_{ab} & Z_{ab} & Z_{cc} \end{bmatrix}$$

Where:

$$Z_{aa} = \frac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc})$$

$$Z_{ab} = \frac{1}{3}(Z_{ab} + Zbc + Z_{ca})$$

Equation above can be transformed to the sequence domain to obtain:

$$Z_S = \boldsymbol{A}^{-1} Z_P \boldsymbol{A}$$

$\boldsymbol{Z_S}$ is the 3×3 series sequence impedance matrix whose elements are:

$$Z_S = \begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} \Omega/\mathrm{m}$$

In general Z_S is not diagonal. However, if the line is completely transposed:

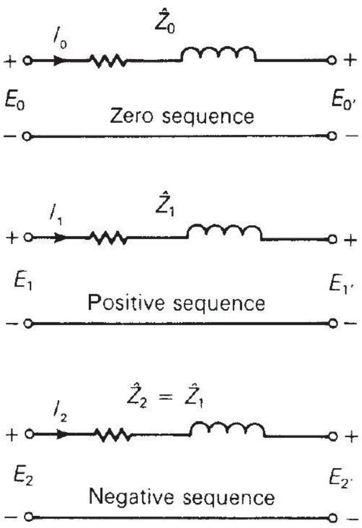
$$\hat{Z}_S = \boldsymbol{A}^{-1}\hat{\boldsymbol{Z}}_P\boldsymbol{A} = \begin{bmatrix} \hat{Z}_0 & 0 & 0 \\ 0 & \hat{Z}_1 & 0 \\ 0 & 0 & \hat{Z}_2 \end{bmatrix}$$

Where:

$$\hat{Z}_0 = \hat{Z}_{\mathbf{aa}} + 2\hat{Z}_{\mathbf{ab}}$$

$$\hat{Z}_1 = \hat{Z}_2 = \hat{Z}_{\mathbf{aa}} - \hat{Z}_{\mathbf{ab}}$$

A circuit representation of the series sequence impedances of a completely transposed three-phase line is shown in Figure below:



Equations suitable for computer calculation of the shunt phase admittances of three-phase overhead transmission lines are common to be developed.

The shunt admittance matrix Y_P for an untransposed line is given by Equation:

$$\mathbf{Y_P} = j\omega\mathbf{C_P} = j(2\pi f)\mathbf{C_P} \;\; S/m \tag{1}$$

Where:

$$\mathbf{C_P} = \begin{bmatrix} C_{aa} & C_{ab} & C_{ac} \\ C_{ba} & C_{bb} & C_{bc} \\ C_{ca} & C_{cb} & C_{cc} \end{bmatrix} \tag{2}$$

And $\hat{Y}_{\mathrm{P}}$ for a completely transposed three-phase line is given by:

$$\hat{\mathbf{Y}}_{\mathbf{P}} = j\omega\hat{\mathbf{C}}_{\mathbf{P}} = j(2\pi f)\hat{\mathbf{C}}_{\mathbf{P}} \quad S/m \tag{3}$$

Where:

$$\hat{\mathbf{C}}_{\mathbf{P}} = \begin{bmatrix} C_{aa} & C_{ab} & C_{ab} \\ C_{ab} & C_{aa} & C_{ab} \\ C_{ab} & C_{ab} & C_{aa} \end{bmatrix} \tag{4}$$

With:

$$\begin{aligned} \hat{C}_{aa} &= \frac{1}{3}[C_{aa} + C_{bb} + C_{cc}] \\ \hat{C}_{ab} &= \frac{1}{3}[C_{ab} + C_{bc} + C_{ca}] \end{aligned} \tag{5}$$

Equations above can be transformed to the sequence domain to obtain:

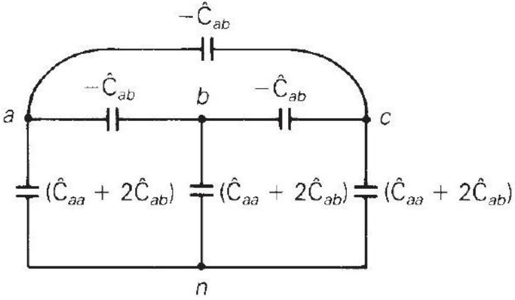
$$\boldsymbol{Y}_{\mathrm{S}} = \boldsymbol{A}^{-1}\boldsymbol{Y}_{\mathrm{P}}\boldsymbol{A}$$

Where:

$$\boldsymbol{Y}_{\mathrm{S}} = \mathbf{G}_{\mathrm{S}} + j(2\pi f)\mathbf{C}_{\mathrm{S}}$$

$$\mathbf{C}_{\mathrm{S}} = \begin{bmatrix} C_0 & C_{01} & C_{02} \\ C_{10} & C_1 & C_{12} \\ C_{20} & C_{21} & C_2 \end{bmatrix} \mathrm{F/m}$$

Circuit representations of the capacitances of a completely transposed three-phase line.



(a) Phase domain

In general, C_S is not diagonal. However, for the completely transposed line:

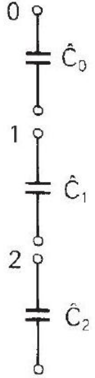
$$\hat{Y}_S = A^{-1}\hat{Y}_P A = \begin{bmatrix} \hat{y}_0 & 0 & 0 \\ 0 & \hat{y}_1 & 0 \\ 0 & 0 & \hat{y}_2 \end{bmatrix} = j(2\pi f)\begin{bmatrix} \hat{C}_0 & 0 & 0 \\ 0 & \hat{C}_1 & 0 \\ 0 & 0 & \hat{C}_2 \end{bmatrix}$$

Where

$$\hat{C}_0 = \hat{C}_{aa} + 2\hat{C}_{ab} \text{ F/m}$$
$$\hat{C}_1 = \hat{C}_2 = \hat{C}_{aa} - \hat{C}_{ab} \text{ F/m}$$

Since $\hat{C}_{ab}$ is negative, the zero-sequence capacitance $\hat{C}_0$ is usually much less than the positive- or negative-sequence capacitance.

Circuit representations of the capacitances of a completely transposed three-phase line.



(b) Sequence domain

Sequence Networks Of Rotating Machines

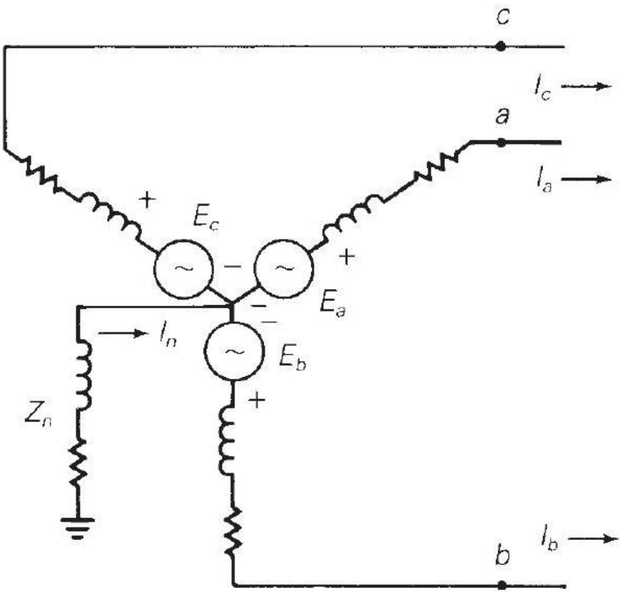
A Y-connected synchronous generator grounded through a neutral impedance Z_n is shown in Figure below.

The internal generator voltages are designated E_a, E_b, and E_c, and the generator line currents are designated I_a, I_b, and I_c.

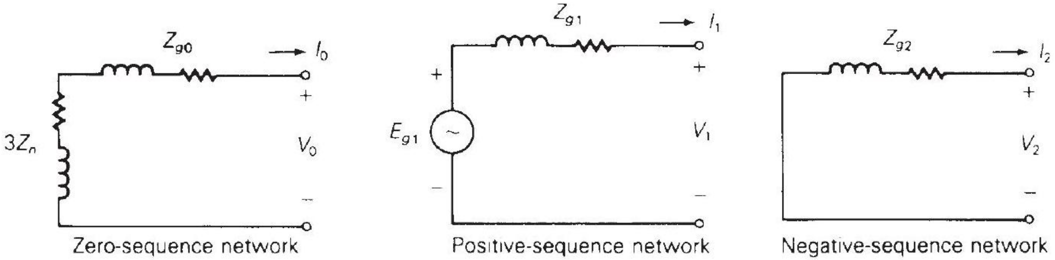
The sequence networks of the generator are shown in next Figure.

Since a three-phase synchronous generator is designed to produce balanced internal phase voltages E_a, E_b, and E_c with only a positive-sequence component, a source voltage E_{g1} is included only in the positive-sequence network.

The sequence components of the line-to-ground voltages at the generator terminals are denoted V_0, V_1, and V_2 in Figure.



c
I_c
a
I_a
E_c
E_a
I_n
E_b
Z_n
I_b
b



Sequence networks of a Y-connected synchronous generator

The voltage drop in the generator neutral impedance is $Z_n I_n$, which can be written as $(3Z_n)\, I_0$, since, the neutral current is three times the zero-sequence current.

Since this voltage drop is due only to zero-sequence current, an impedance $(3Z_n)$ is placed in the zero-sequence network of Figure in series with the generator zero-sequence impedance Z_{g0}.

The sequence impedances of rotating machines are generally not equal.

A detailed analysis of machine-sequence impedances is given in machine theory texts. Here is a brief explanation.

When a synchronous generator stator has balanced three-phase positivesequence currents under steady-state conditions, the net mmf produced by these positive-sequence currents rotates at the synchronous rotor speed in the same direction as that of the rotor.

Under this condition, a high value of magnetic flux penetrates the rotor, and the positive-sequence impedance Z_{g1} has a high value. Under steady-state conditions, the positive-sequence generator impedance is called the synchronous impedance.

When a synchronous generator stator has balanced three-phase negative sequence currents, the net mmf produced by these currents rotates at synchronous speed in the direction opposite to that of the rotor.

With respect to the rotor, the net mmf **is not stationary but rotates at twice synchronous speed.**

Under this condition, currents are induced in the rotor windings that prevent the magnetic flux from penetrating the rotor. As such, the negative-sequence impedance Z_{g2} **is less than the positive-sequence synchronous impedance.**

When a synchronous generator has only zero-sequence currents, which are line (or phase) currents with equal magnitude and phase, then the net mmf produced by these currents is theoretically zero.

The generator zero-sequence impedance Z_{g0} is the smallest sequence impedance and is due to leakage flux, end turns, and harmonic flux from windings that do not produce a perfectly sinusoidal mmf**.**

Typical values of machine-sequence impedances are listed in Table A. 1 in the Appendix.

The positive-sequence machine impedance is synchronous, transient, or subtransient:

- Synchronous impedances are used for steady-state conditions, such as in power-flow studies, which are described in Chapter 6;
- Transient impedances are used for stability studies, which are described in Chapter 13. And;
- Subtransient impedances are used for short-circuit studies, which are described in Chapters 7 and 9.

Unlike the positive-sequence impedances, a machine has only one negative-sequence impedance and only one zero-sequence impedance.

The sequence networks for three-phase synchronous motors and for three-phase induction motors are shown in Figure below.

Synchronous motors have the same sequence networks as synchronous generators, except that the sequence currents for synchronous motors are referenced into rather than out of the sequence networks.

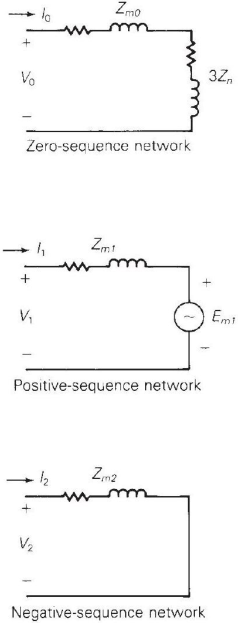
Also, induction motors have the same sequence networks as synchronous motors, except that the positive-sequence voltage source E_{m1} is removed. Induction motors do not have a dc source of magnetic flux in their rotor circuits, and therefore E_{m1} is zero (or a short circuit).

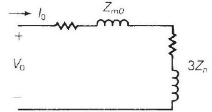
The sequence networks shown in Figures here are simplified networks for rotating machines.

The networks do not take into account such phenomena as machine saliency, saturation effects, and more complicated transient effects. These simplified networks, however, are in many cases accurate enough for power system studies.

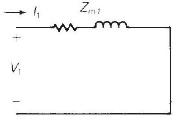
FIGURE 8.15

Sequence networks of three-phase motors

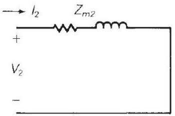




Zero-sequence network



Positive-sequence network

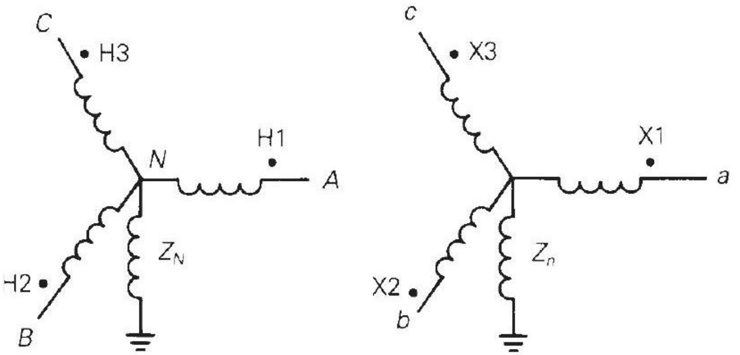


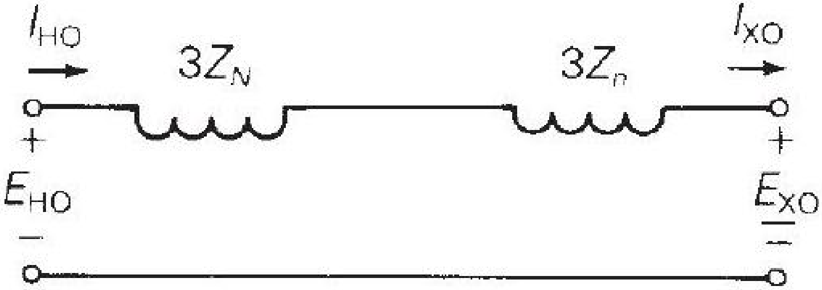
Negative-sequence network

(b) Induction motor

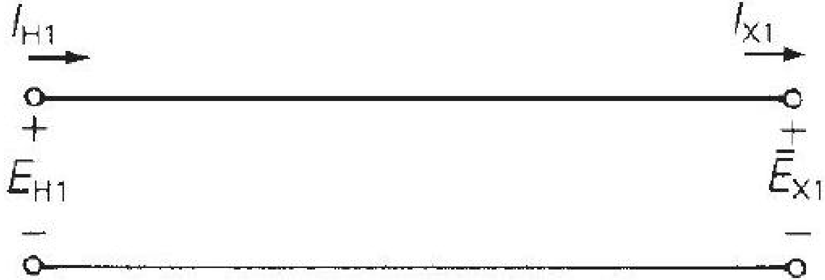
Per-Unit Sequence Models of Three-Phase Two-Winding Transformers

Figure below is a schematic representation of an ideal $\mathrm{Y} - \mathrm{Y}$ transformer grounded through neutral impedances Z_N and Z_n.

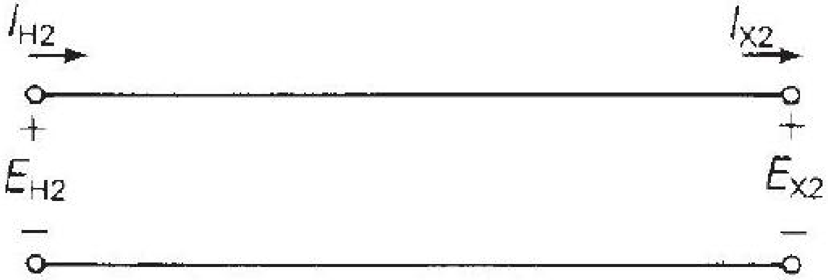




(b) Per-unit zero-sequence network



(c) Per-unit positive-sequence network



(d) Per-unit negative-sequence network

When balanced positive-sequence currents or balanced negative-sequence currents are applied to the transformer, the neutral currents are zero, and there are no voltage drops across the neutral impedances.

Therefore, the per-unit positive- and negative-sequence networks of the ideal $Y - Y$ transformer, are the same as the per-unit single-phase ideal transformer.

Zero-sequence currents have equal magnitudes and equal phase angles. When per-unit sequence currents $I_{\mathrm{Ao}} = I_{\mathrm{Bo}} = I_{\mathrm{Co}} = I_{\mathrm{o}}$ are applied to the high-voltage windings of an ideal Y-Y transformer, the neutral current I_{o} flows through the neutral impedance Z_N, with a voltage drop $(3Z_N)\, I_{\mathrm{o}}$.

Also, per-unit zero-sequence current I_{o} flows in each low-voltage winding, and therefore $3I_{\mathrm{o}}$ flows through neutral impedance Z_n, with a voltage drop $(3I_{\mathrm{o}})\, Z_n$.

Note that if either one of the neutrals of an ideal transformer is ungrounded, then no zero sequence can flow in either the high- or low-voltage windings.

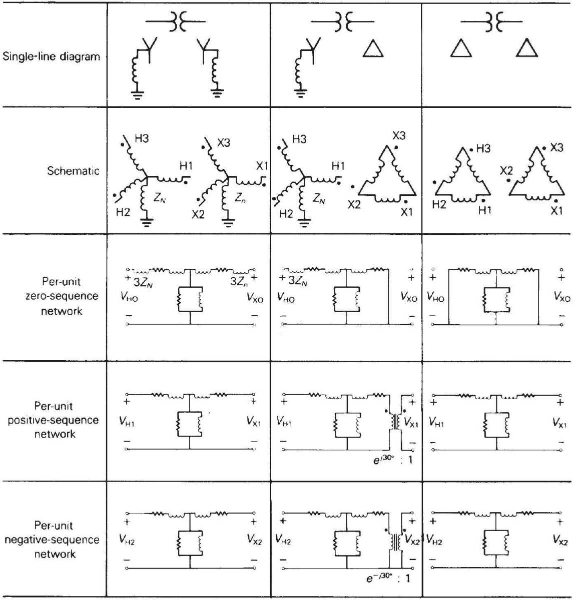
For example, if the high-voltage winding has an open neutral, then $I_N = 3I_0 = 0$, which in turn forces $I_0 = 0$ on the low-voltage side.

This can be shown in the zero-sequence network of Figure below by making $Z_N = \infty$, which corresponds to an open circuit.

The per-unit sequence networks of a practical $\mathrm{Y} - \mathrm{Y}$ transformer are shown in Figure below.

These networks are obtained by adding external impedances to the sequence networks of the ideal transformer, as follows.

The leakage impedances of the high-voltage windings are series impedances with no coupling between phases $(Z_{ab} = 0)$.



Single-line diagram
Schematic
Per-unit zero-sequence network
Per-unit positive-sequence network
Per-unit negative-sequence network
H1
H2
H3
X1
X2
X3
Z_N
Z_n
$3Z_N$
$3Z_n$
V_{HO}
V_{XO}
V_{H1}
V_{X1}
V_{H2}
V_{X2}
$e^{j30°} : 1$
$e^{-j30°} : 1$

If the phase a, b, and c Per-unit sequence networks of practical $Y-Y$, $Y-\Delta$, and $\Delta-\Delta$ transformers windings have equal leakage impedances $Z_{\mathrm{H}} = \mathrm{R}_{\mathrm{H}} + j\mathrm{X}_{\mathrm{H}}$, then the series impedances are symmetrical with sequence networks, where $Z_{\mathrm{H}0} = Z_{\mathrm{H}1} = Z_{\mathrm{H}2} = Z_{\mathrm{H}}$.

Similarly, the leakage impedances of the low-voltage windings are symmetrical series impedances with $Z_{\mathrm{X}0} = Z_{\mathrm{X}1} = Z_{\mathrm{X}2} = Z_{\mathrm{X}}$.

The shunt branches of the practical $\mathrm{Y}-\mathrm{Y}$ transformer, which represent exciting current, are equivalent to the Y load.

Each phase represents a core loss resistor in parallel with a magnetizing inductance.

Assuming these are the same for each phase, then the Y load is symmetrical.

Note that $(3Z_N)$ and $3Z_n)$ have already been included in the zero-sequence network.

The per-unit positive- and negative-sequence transformer impedances of the practical Y-Y transformer in Figure are identical, which is always true for nonrotating equipment.

The per-unit zero-sequence network, however, depends on the neutral impedances Z_N and Z_n.

The per-unit sequence networks of the $\mathrm{Y}-\Delta$ transformer, have the following features:

1. The per-unit impedances do not depend on the winding connections. That is, the per-unit impedances of a transformer that is connected $\mathrm{Y}-\mathrm{Y}$, $\mathrm{Y}-\Delta$, $\Delta-\mathrm{Y}$, or $\Delta-\Delta$ are the same. However, the base voltages do depend on the winding connections.
2. A phase shift is included in the per-unit positive- and negative-sequence networks. For the American standard, the positive-sequence voltages and currents on the high-voltage side of the $\mathrm{Y}-\Delta$ transformer lead the corresponding quantities on the low-voltage side by 30°. For negative sequence, the high-voltage quantities lag by 30°.
3. Zero-sequence currents can flow in the Y winding if there is a neutral connection, and corresponding zero-sequence currents flow within the Δ winding. However, no zero-sequence current enters or leaves the Δ winding.

The phase shifts in the positive- and negative-sequence networks are represented by the phase-shifting transformer.

Also, the zerosequence network provides a path on the Y side for zero-sequence current to flow, but no zero-sequence current can enter or leave the Δ side.

The per-unit sequence networks of the $\Delta - \Delta$ transformer, shown in Figure, have the following features:

1. **The positive- and negative-sequence networks, which are identical, are the same as those for the $Y - Y$ transformer. It is assumed that the windings are labeled so there is no phase shift. Also, the per-unit impedances do not depend on the winding connections, but the base voltages do.**
2. **Zero-sequence currents cannot enter or leave either Δ winding, although they can circulate within the Δ windings.**

Per-Unit Sequence Models Of Three-Phase Three-Winding Transformers

Three identical single-phase three-winding transformers can be connected to form a three-phase bank.

Figure below shows the general per-unit sequence networks of a three-phase three-winding transformer.

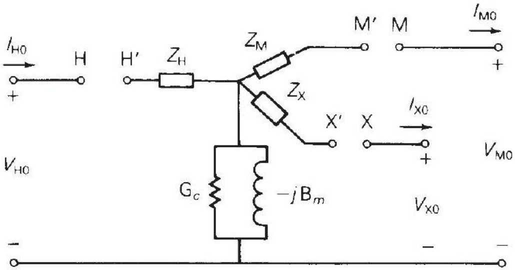
Instead of labeling the windings 1,2 , and 3 , as was done for the single-phase transformer, the letters H, M, and X are used to denote the high-, medium-, and low-voltage windings, respectively.

By convention, a common S_{base} is selected for the H, M, and X terminals, and voltage bases V_{baseH} , $\mathrm{V}_{\text{baseM}}$, and $\mathrm{V}_{\text{basex}}$ are selected in proportion to the rated line-to-line voltages of the transformer.

For the general zero-sequence network, Figure below, the connection between terminals H **and** H' **depends on how the high-voltage windings are connected, as follows:**

1. **Solidly grounded** Y **- Short** H **to** H'**.**
2. **Grounded** Y **through** Z_N**– Connect** $(3Z_N)$ **from** H **to** H'**.**
3. **Ungrounded** Y**-Leave** $\mathrm{H} - \mathrm{H}'$ **open as shown.**
4. Δ**-Short** H' **to the reference bus.**

Per-unit sequence networks of a three-phase three-winding transformer:



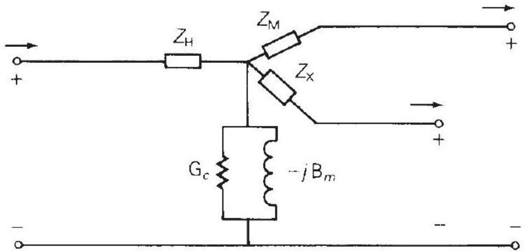
(a) Per-unit zero-sequence network

Terminals $X - X'$ and M – M' are connected in a similar manner.

The impedances of the per-unit negative-sequence network are the same as those of the per-unit positive-sequence network, which is always true for nonrotating equipment.

Phase-shifting transformers, not shown in Figure below, can be included to model phase shift between Δ and Y windings.

Per-unit sequence networks of a three-phase three-winding transformer



(b) Per-unit positive- or negative-sequence network (phase shift not shown)

Power In Sequence Networks

The power delivered to a three-phase network can be determined from the power delivered to the sequence networks.

Let S_p denote the total complex power delivered to the three-phase load, which can be calculated from:

$$S_P = V_{ag} I_a^* + V_{bg} I_b^* + V_{cg} I_c^*$$

Equation above is also valid for the total complex power delivered by the three-phase generator, or for the complex power delivered to any three-phase bus. Rewriting in matrix format:

$$
\begin{aligned}
S_{\mathrm{P}} &= \left[V_{ag} V_{bg} V_{cg}\right] \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} \\
&= \boldsymbol{V}_{\mathrm{P}}^{\mathrm{T}} I_{\mathrm{P}}^*
\end{aligned}
$$

Where T **denotes transpose and * denotes complex conjugate. Now:**

$$
\begin{aligned}
S_{\mathrm{P}} &= \left(\boldsymbol{A}\boldsymbol{V}_s\right)^{\mathrm{T}} \left(\boldsymbol{A}\boldsymbol{I}_s\right)^* \\
&= \boldsymbol{V}_s^{\mathrm{T}} \left[\boldsymbol{A}^{\mathrm{T}} \boldsymbol{A}^*\right] \boldsymbol{I}_s^*
\end{aligned}
$$

Using the definition of A to calculate the term within the brackets, and noting that a and a^2 are conjugates:

$$\begin{aligned}
\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}^{*} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^{*} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \\
&= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\mathbf{U}
\end{aligned}$$

Equation above can now be used to obtain:

$$
\begin{aligned}
S_{\mathrm{P}} &= 3\boldsymbol{V}_{\mathrm{S}}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{S}}^{*} \\
&= 3\left[V_0 + V_1 + V_2\right]\begin{bmatrix} I_0^{*} \\ I_1^{*} \\ I_2^{*} \end{bmatrix} \\
S_{\mathrm{P}} &= 3\left(V_0 I_0^{*} + V_1 I_1^{*} + V_2 I_2^{*}\right) \\
&= 3S_{\mathrm{S}}
\end{aligned}
$$

Thus, the total complex power S_P delivered to a three-phase network equals three times the total complex power S_S delivered to the sequence networks.

The factor of 3 occurs because $A^T A^* = 3\mathbf{U}$.

It is possible to eliminate this factor of 3 by defining a new transformation matrix $A_1 = (1/\sqrt{3})A$ such that $A_1^T A_1^* = \mathbf{U}$, which means that A_1 is a unitary matrix.

Using A_1 instead of A, the total complex power delivered to three-phase networks would equal the total complex power delivered to the sequence networks.

However, standard industry practice for symmetrical components is to use A.