Análise de Sistemas de Potência

Aula 04: Operação de Linhas de Transmissão em Regime Permanente

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24 de abril de 2024

Este material é inteiramente baseado no livro "Power Systems Design and Analisys" (6th Edition) dos autores J. Duncan Glover, Thomas J. Overbye e Mulukutlas S. Sarma.

Sumário

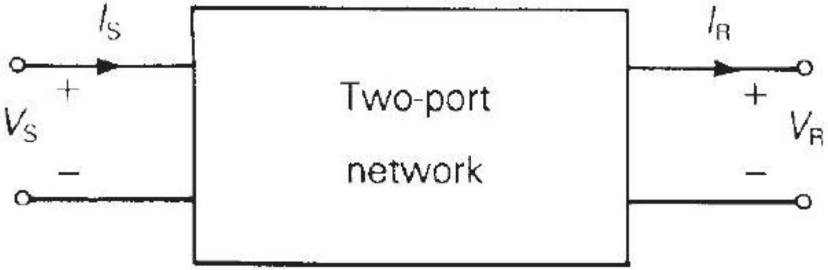
Medium And Short Line Approximations

It is convenient to represent a transmission line by the two-port network shown in Figure below, where V_S and I_S are the sending-end voltage and current, and V_R and I_R are the receiving-end voltage and current.

The relation between the sending-end and receiving-end quantities can be written as:

$$V_S = AV_R + BI_R \quad \textbf{volts}$$

$$I_S = CV_R + DI_R \quad \textbf{A}$$



I_S
I_R
+
V_S
−
Two-port
network
+
V_R
−

In matrix format:

$$\begin{bmatrix} V_{\mathrm{S}} \\ I_{\mathrm{S}} \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_{\mathrm{R}} \\ I_{\mathrm{R}} \end{bmatrix}$$

Where A, B, C**, and** D **are parameters that depend on the transmission-line constants** R, L, C**, and** G**.**

The $ABCD$ **parameters are, in general, complex numbers.**

A **and** D **are dimensionless.** B **has units of ohms, and** C **has units of siemens.**

Network theory texts show that $ABCD$ parameters apply to linear, passive, bilateral two-port networks, with the following general relation:

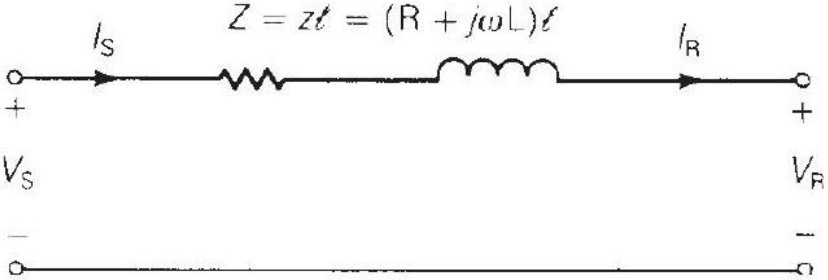
$$AD - BC = 1$$

The circuit in Figure below represents a short transmission line, usually applied to overhead $60 - \mathrm{Hz}$ lines less than $25\ \mathrm{km}$ in length.

Only the series resistance and reactance are included. The shunt admittance is neglected.

The circuit applies to either single-phase or completely transposed three-phase lines operating under balanced conditions.

For a completely transposed three-phase line, Z is the series impedance, V_S and I_R are positive-sequence line-to-neutral voltages, and I_S and I_R are positive sequence line currents.


$Z = z\ell = (R + j\omega L)\ell$
I_S
I_R
+
+
V_S
V_R
−
−

To avoid confusion between total series impedance and series impedance per unit length, use the following notation:

$z = \mathrm{R} + j\omega\mathrm{L}\Omega/\mathrm{m}$, **series impedance per unit length**

$y = \mathrm{G} + j\omega\mathrm{C} \quad \mathrm{S/m}$, **shunt admittance per unit length**

$Z = zl \quad \Omega$, **total series impedance**

$Y = y1 \quad$ **S, total shunt admittance**

$l =$ **line length** m

Recall that shunt conductance G **is usually neglected for overhead transmission.**

The *ABCD* parameters for the short line in Figure are easily obtained by writing a KVL and KCL equation as:

$$V_{\mathrm{S}} = V_{\mathrm{R}} + ZI_{\mathrm{R}}$$
$$I_{\mathrm{S}} = I_{\mathrm{R}}$$

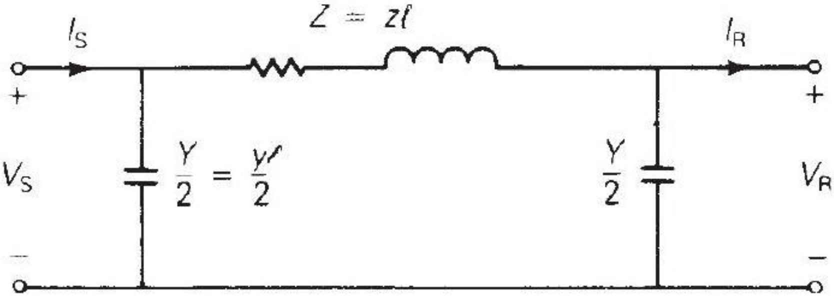
Or, in matrix format:

$$\begin{bmatrix} V_{\mathrm{S}} \\ I_{\mathrm{S}} \end{bmatrix} = \left[\begin{array}{c|c} 1 & Z \\ \hline 0 & 1 \end{array}\right] \begin{bmatrix} V_{\mathrm{R}} \\ I_{\mathrm{R}} \end{bmatrix}$$

Comparing equations, the *ABCD* parameters for a short line are

$$A = D = 1\textbf{per unit}$$
$$B = Z\ \Omega$$
$$C = 0\ \ \mathrm{S}$$

For medium-length lines, typically ranging from 25 to 250 km **at** 60 Hz, **it is common to lump the total shunt capacitance and locate half at each end of the line. Such a circuit, called a nominal π circuit, is shown in Figure below.**



To obtain the $ABCD$ parameters of the nominal π circuit, note first that the current in the series branch in Figure equals $I_{line} = I_{\mathrm{R}} + \frac{V_{\mathrm{R}}Y}{2}$.

Then, writing a KVL equation:

$$
\begin{aligned}
V_{\mathrm{S}} &= V_{\mathrm{R}} + Z\left(I_{\mathrm{R}} + \frac{V_{\mathrm{R}}Y}{2}\right) \\
&= \left(1 + \frac{YZ}{2}\right)V_{\mathrm{R}} + ZI_{\mathrm{R}}
\end{aligned}
$$

Also, writing a KCL equation at the sending end:

$$
I_{\mathrm{S}} = I_{\mathrm{R}} + \frac{V_{\mathrm{R}}Y}{2} + \frac{V_{\mathrm{S}}Y}{2}
$$

Equations above give:

$$I_{\mathrm{S}} = I_{\mathrm{R}} + \frac{V_{\mathrm{R}}Y}{2} + \left[\left(1 + \frac{YZ}{2}\right)V_{\mathrm{R}} + ZI_{\mathrm{R}}\right]\frac{Y}{2}$$
$$= Y\left(1 + \frac{YZ}{4}\right)V_{\mathrm{R}} + \left(1 + \frac{YZ}{2}\right)I_{\mathrm{R}}$$

Writing in matrix format:

$$\begin{bmatrix} V_{\mathrm{S}} \\ I_{\mathrm{S}} \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_{\mathrm{R}} \\ I_{\mathrm{R}} \end{bmatrix}$$

Thus, comparing equations:

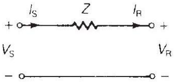
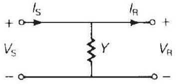
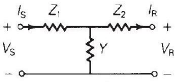
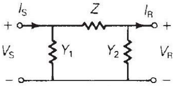
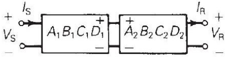
$$A = D = 1 + \frac{YZ}{2} \quad \textbf{per unit}$$

$$B = Z \quad \Omega$$

$$C = Y\left(1 + \frac{YZ}{4}\right) \quad \mathrm{S}$$

Note that for both the short and medium-length lines, the relation $AD - BC = 1$ is verified.

Note also that since the line is the same when viewed from either end, $A = D$.

Circuit	*ABCD* Matrix
 Series impedance	$\left[\begin{array}{c\|c} 1 & Z \\ \hline 0 & 1 \end{array}\right]$
 Shunt admittance	$\left[\begin{array}{c\|c} 1 & 0 \\ \hline Y & 1 \end{array}\right]$
 T circuit	$\left[\begin{array}{c\|c} (1 + YZ_1) & (Z_1 + Z_2 + YZ_1Z_2) \\ \hline Y & (1 + YZ_2) \end{array}\right]$
 π circuit	$\left[\begin{array}{c\|c} (1 + Y_2Z) & Z \\ \hline (Y_1 + Y_2 + Y_1Y_2Z) & (1 + Y_1Z) \end{array}\right]$
 Series networks	$\left[\begin{array}{c\|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right]\left[\begin{array}{c\|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right]=\left[\begin{array}{c\|c} (A_1A_2 + B_1C_2) & (A_1B_2 + B_1D_2) \\ \hline (C_1A_2 + D_1C_2) & (C_1B_2 + D_1D_2) \end{array}\right]$

ABCD parameters can be used to describe the variation of line voltage with line loading.

Voltage regulation is the change in voltage at the receiving end of the line when the load varies from no-load to a specified full load at a specified power factor, while the sending-end voltage is held constant.

Expressed in percent of fullload voltage:

$$\text{percent VR} = \frac{|V_{\mathrm{RNL}}| - |V_{\mathrm{RFL}}|}{|V_{\mathrm{RFL}}|} \times 100$$

Where percent VR is the percent voltage regulation, $|V_{\mathrm{RNL}}|$ is the magnitude of the no-load receiving-end voltage, and $|V_{\mathrm{RFL}}|$ is the magnitude of the full-load receivingend voltage.

The effect of load power factor on voltage regulation is illustrated by the phasor diagrams in Figure below for short lines.

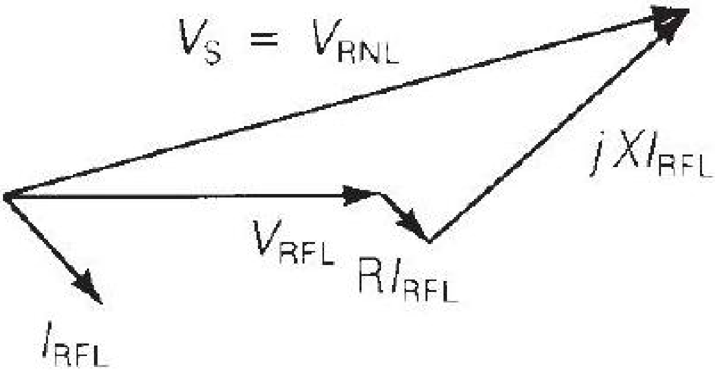
The phasor diagrams are graphical representations of voltage regulation equation for lagging and leading power factor loads. Note that, from voltage regulation equation at no-load, $I_{\mathrm{RNL}} = 0$ and $V_{\mathrm{S}} = V_{\mathrm{RNL}}$ for a short line.

As shown, the higher (worse) voltage regulation occurs for the lagging p.f. load, where V_{RNL} exceeds V_{RFL} by the larger amount.

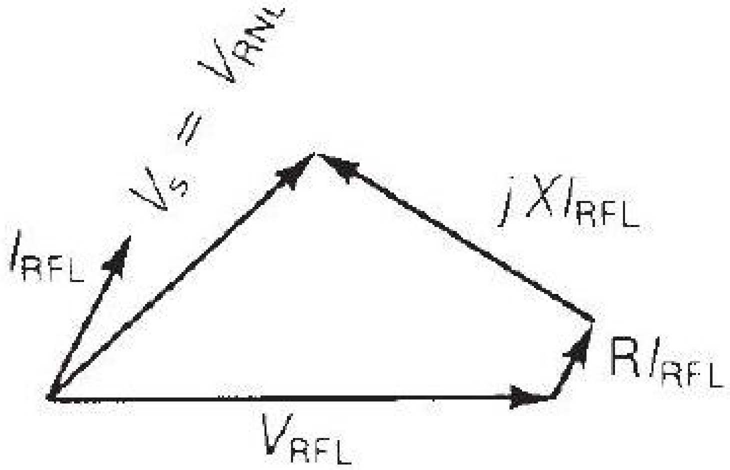
A smaller or even negative voltage regulation occurs for the leading p.f. load. In general, the no-load voltage is (with $I_{\mathrm{RNL}} = 0$):

$$V_{\mathrm{RNL}} = \frac{V_{\mathrm{S}}}{A}$$

which can be used to determine voltage regulation.



(a) Lagging p.f. load



(b) Leading p.f. load

In practice, transmission-line voltages decrease when heavily loaded and increase when lightly loaded.

When voltages on EHV lines are maintained within +5% **of rated voltage, corresponding to about** 10% **voltage regulation, unusual operating problems are not encountered.**

Ten percent voltage regulation for lower voltage lines including transformer-voltage drops is also considered good operating practice.

In addition to voltage regulation, line loadability is an important issue.

Three major line-loading limits are:

(1) the thermal limit, (2) the voltage-drop limit, and (3) the steady-state stability limit.

The maximum temperature of a conductor determines its thermal limit. Conductor temperature affects the conductor sag between towers and the loss of conductor tensile strength due to annealing.

If the temperature is too high, prescribed conductor-to-ground clearances may not be met, or the elastic limit of the conductor may be exceeded such that it cannot shrink to its original length when cooled.

Conductor temperature depends on the current magnitude and its time duration, as well as on ambient temperature, wind velocity, and conductor surface conditions.

For longer line lengths (up to $300 \mathrm{~km}$), line loadability is often determined by the voltage-drop limit.

Although more severe voltage drops may be tolerated in some cases, a heavily loaded line with $V_{\mathrm{R}} / V_{\mathrm{S}} \geq 0.95$ is usually considered safe operating practice.

For line lengths over $300 \mathrm{~km}$, steady-state stability becomes a limiting factor.

Stability, discussed in other section of the book, refers to the ability of synchronous machines on either end of a line to remain in synchronism.

Transmission Line Differential Equations

The line constants R, L**, and** C **are derived another Chapter as per-length values having units of** Ω/m, H/m**, and** F/m**.**

They are not lumped, but rather are uniformly distributed along the length of the line.

In order to account for the distributed nature of transmission-line constants, consider the circuit shown in Figure brlow, which represents a line section of length Δx **.**

$V(x)$ **and** $I(x)$ **denote the voltage and current at position** x**, which is measured in meters from the right, or receiving end of the line.**

Similarly, $V(x+\Delta x)$ **and** $I(x+\Delta x)$ **denote the voltage and current at position** $(x+\Delta x)$**.**

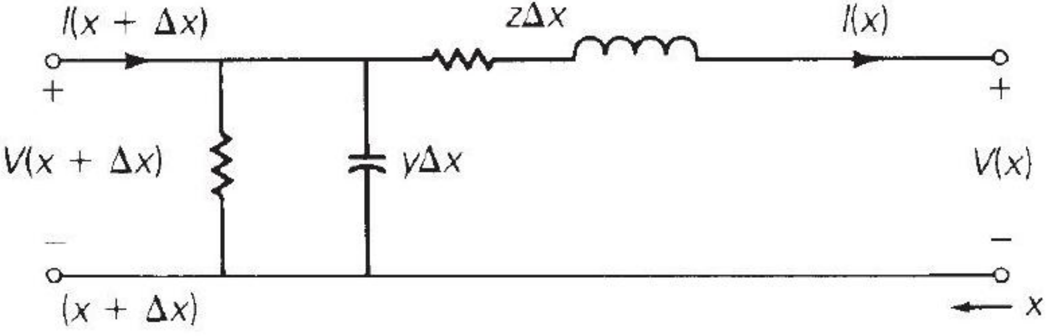
The circuit constants are

$$z = \mathrm{R} + j\omega\mathrm{L} \quad \Omega/\mathrm{m}$$
$$y = \mathrm{G} + j\omega\mathrm{C} \quad \mathrm{S/m}$$

Where G **is usually neglected for overhead** 60 – Hz **lines.**

Writing a KVL equation for the circuit:

$$V(x + \Delta x) = V(x) + (z\Delta x)I(x) \text{ **volts**}$$



Transmission-line section of length Δx.

Rearranging:

$$\frac{V(x + \Delta x) - V(x)}{\Delta x} = zI(x)$$

And taking the limit as Δx approaches zero:

$$\frac{dV(x)}{dx} = zI(x)$$

Similarly, writing a KCL equation for the circuit,

$$I(x+\Delta x) = I(x) + (y\Delta x)V(x+\Delta x) \quad \text{A}$$

Rearranging,

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = yV(x)$$

and taking the limit as Δx approaches zero,

$$\frac{dI(x)}{dx} = yV(x)$$

Equations above are two linear, first-order, homogeneous differential equations with two unknowns, $V(x)$ and $I(x)$.

Eliminate $I(x)$ by differentiating, as follows:

$$\frac{d^2V(x)}{dx^2} = z\frac{dI(x)}{dx} = zyV(x)$$

or

$$\frac{d^2V(x)}{dx^2} - zyV(x) = 0$$

Equation above is a linear, second-order, homogeneous differential equation with one unknown, $V(x)$.

By inspection, its solution is

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \text{ volts}$$

Where A_1 and A_2 are integration constants and

$$\gamma = \sqrt{zy} \quad \mathrm{m}^{-1}$$

γ, whose units are m^{-1}, is called the propagation constant.

Next:

$$\frac{dV(x)}{dx} = \gamma A_1 e^{\gamma x} - \gamma A_2 e^{-\gamma x} = zI(x)$$

Solving for $I(x)$:

$$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{z/\gamma}$$

Using $z/\gamma = z/\sqrt{zy} = \sqrt{z/y}$ this equation becomes:

$$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z_c}$$

Where

$$Z_c = \sqrt{\frac{z}{y}}\Omega$$

Z_c, **whose units are ohms, is called the characteristic impedance.**

Next, the integration constants A_1 and A_2 are evaluated from the boundary conditions. At $x = 0$, which is the receiving end of the line, the receiving-end voltage and current are:

$$V_{\mathrm{R}} = V(0)$$
$$I_{\mathrm{R}} = I(0)$$

Also, at $x = 0$ the equations become:

$$V_{\mathrm{R}} = A_1 + A_2$$
$$I_{\mathrm{R}} = \frac{A_1 - A_2}{Z_c}$$

Solving for A_1 and A_2,

$$A_1 = \frac{V_\mathrm{R} + Z_c I_\mathrm{R}}{2}$$
$$A_2 = \frac{V_\mathrm{R} - Z_c I_\mathrm{R}}{2}$$

Substituting A_1 and A_2 into equations of voltage and current:

$$V(x) = \left(\frac{V_\mathrm{R} + Z_c I_\mathrm{R}}{2}\right) e^{\gamma x} + \left(\frac{V_\mathrm{R} - Z_c I_\mathrm{R}}{2}\right) e^{-\gamma x}$$
$$I(x) = \left(\frac{V_\mathrm{R} + Z_c I_\mathrm{R}}{2Z_c}\right) e^{\gamma x} - \left(\frac{V_\mathrm{R} - Z_c I_\mathrm{R}}{2Z_c}\right) e^{-\gamma x}$$

Rearranging equations above:

$$V(x) = \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) V_{\mathrm{R}} + Z_c \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) I_{\mathrm{R}}$$
$$I(x) = \frac{1}{Z_c}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) V_{\mathrm{R}} + \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) I_{\mathrm{R}}$$

Recognizing the hyperbolic functions cosh and sinh:

$$V(x) = \cosh(\gamma x) V_{\mathrm{R}} + Z_c \sinh(\gamma x) I_{\mathrm{R}}$$
$$I(x) = \frac{1}{Z_c} \sinh(\gamma x) V_{\mathrm{R}} + \cosh(\gamma x) I_{\mathrm{R}}$$

Equations above give the *ABCD* parameters of the distributed line. In matrix format:

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \left[\begin{array}{c|c} A(x) & B(x) \\ \hline C(x) & D(x) \end{array}\right] \begin{bmatrix} V_{\mathrm{R}} \\ I_{\mathrm{R}} \end{bmatrix}$$

where

$$A(x) = D(x) = \cosh(\gamma x) \ \textbf{per unit}$$

$$B(x) = Z_c \sinh(\gamma x) \quad \Omega$$

$$C(x) = \frac{1}{Z_c} \sinh(\gamma x) \quad \mathrm{S}$$

These equations gives the current and voltage at any point x along the line in terms of the receiving-end voltage and current.

For examples, at the sending end, where $x = l$, $V(l) = V_S$ and $I(l) = I_S$. That is:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Where:

$$\begin{aligned} A &= D = \cosh(\gamma l) \ \textbf{per unit} \\ B &= Z_c \sinh(\gamma l) \quad \Omega \\ C &= \frac{1}{Z_c} \sinh(\gamma l) \quad \mathrm{S} \end{aligned}$$

Equations above give the $ABCD$ parameters of the distributed line.

In these equations, the propagation constant γ is a complex quantity with real and imaginary parts denoted α and β. That is:

$$\gamma = \alpha + j\beta \quad \mathrm{m}^{-1}$$

The quantity γl is dimensionless.

Also:

$$e^{\gamma l} = e^{(\alpha l + j\beta l)} = e^{\alpha l} \cdot e^{j\beta l} = e^{\alpha l} \underline{/\beta l}$$

Using this equation, the hyperbolic functions cosh and sinh can be evaluated as follows:

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\angle \beta l + e^{-\alpha l}\angle -\beta l\right)$$

And:

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1}{2}\left(e^{\alpha l}\angle \beta l - e^{-\alpha l}\angle -\beta l\right)$$

Alternatively, the following identities can be used:

$$\cosh(\alpha l + j\beta l) = \cosh(\alpha l)\cos(\beta l) + j\sinh(\alpha l)\sin(\beta l)$$
$$\sinh(\alpha l + j\beta l) = \sinh(\alpha l)\cos(\beta l) + j\cosh(\alpha l)\sin(\beta l)$$

Note that the dimensionless quantity βl is in radians, not degrees.

The $ABCD$ parameters given by equations above are exact parameters valid for any line length.

For accurate calculations, these equations must be used for overhead $60 - \text{Hz}$ lines longer than $250\ \text{km}$.

The $ABCD$ parameters derived in later sections are approximate parameters that are more conveniently used for hand calculations involving short and medium length lines.

Table above summarizes the *ABCD* parameters for short, medium, long, and lossless lines:

Parameter	$A = D$	B	C
Units	**per Unit**	Ω	S
Short line (less than 25 km)	**1**	Z	**0**
Medium line—nominal π **circuit** (25 **to** 250 km)	$1 + \frac{YZ}{2}$	Z	$Y\left(1 + \frac{YZ}{4}\right)$
Long line-equivalent π **circuit (more than** 250 km)	$\cosh(\gamma\ell) = 1 + \frac{Y'Z'}{2}$	$Z_c \sinh(\gamma\ell) = Z'$	$(1/Z_c)\sinh(\gamma\ell)$
Lossless line (R = G = 0)	$\cos(\beta\ell)$	$jZ_c \sin(\beta\ell)$	$\frac{j\sin(\beta\ell)}{Z_c}$

Equivalent π Circuit

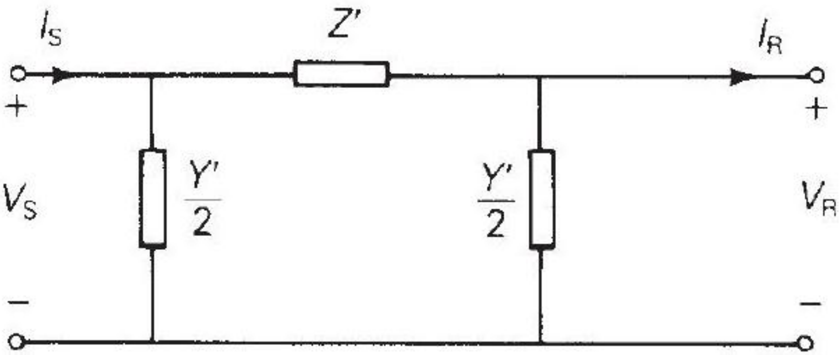
Many computer programs used in power system analysis and design assume circuit representations of components such as transmission lines and transformers.

It is therefore convenient to represent the terminal characteristics of a transmission line by an equivalent circuit instead of its $ABCD$ parameters.

The circuit shown in Figure above is called an equivalent π circuit.

It is identical in structure to the nominal π circuit, except that Z' and Y' are used instead of Z and Y.

The objective is to determine Z' and Y' such that the equivalent π circuit has the same $ABCD$ parameters as those of the distributed line.



I_S
Z'
I_R
+
+
V_S
$\frac{Y'}{2}$
$\frac{Y'}{2}$
V_R
−
−

The $ABCD$ parameters of the equivalent π circuit, which has the same structure as the nominal π, are:

$$A = D = 1 + \frac{Y'Z'}{2} \quad \text{per unit}$$

$$B = Z' \quad \Omega$$

$$C = Y'\left(1 + \frac{Y'Z'}{4}\right) \text{S}$$

Where Z and Y in have been replaced with Z' and Y').

Equating:

$$Z' = Z_c \sinh(\gamma l) = \sqrt{\frac{z}{y}} \sinh(\gamma l)$$

Rewriting in terms of the nominal π circuit impedance $Z = zl$:

$$\begin{aligned} Z' &= zl \left[\sqrt{\frac{z}{y}} \frac{\sinh(\gamma l)}{zl} \right] = zl \left[\frac{\sinh(\gamma l)}{\sqrt{zy}\, l} \right] \\ &= Z F_1 \quad \Omega \end{aligned}$$

Where:

$$F_1 = \frac{\sinh(\gamma l)}{\gamma l} \textbf{ per unit}$$

Similarly, equating:

$$1 + \frac{Y'Z'}{2} = \cosh(\gamma l)$$
$$\frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z'}$$

Using the identity $\tanh\left(\frac{\gamma l}{2}\right) = \frac{\cosh(\gamma l)-1}{\sinh(\gamma l)}$**, becomes:**

$$\frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z_c \sinh(\gamma l)} = \frac{\tanh(\gamma l/2)}{Z_c} = \frac{\tanh(\gamma l/2)}{\sqrt{\frac{z}{y}}}$$

Rewriting in terms of the nominal π circuit admittance $Y = yl$:

$$\frac{Y'}{2} = \frac{yl}{2}\left[\frac{\tanh(\gamma l/2)}{\sqrt{\frac{z}{y}\frac{yl}{2}}}\right] = \frac{yl}{2}\left[\frac{\tanh(\gamma l/2)}{\sqrt{zy}l/2}\right]$$
$$= \frac{Y}{2}F_2 \quad \text{S}$$

Where:

$$F_2 = \frac{\tanh(\gamma l/2)}{\gamma l/2} \text{ per unit}$$

Equations for series impedance and shunt admitance above give the correction factors F_1 and F_2 to convert Z and Y for the nominal π circuit to Z' and Y' for the equivalent π circuit:

$$Z' = Z_c \sinh(\gamma\ell) = ZF_1 = Z\frac{\sinh(\gamma\ell)}{\gamma\ell}$$

$$\frac{Y'}{2} = \frac{\tanh(\gamma\ell/2)}{Z_c} = \frac{Y}{2}F_2 = \frac{Y}{2}\frac{\tanh(\gamma\ell/2)}{(\gamma\ell/2)}$$

Lossless Lines

This section discusses the following concepts for lossless lines:

- **Surge impedance,**
- $ABCD$ **parameters,**
- **Equivalent** π **circuit,**
- **Wavelength,**
- **Surge impedance loading,**
- **Voltage profiles, and**
- **Steady-state stability limit.**

When line losses are neglected, simpler expressions for the line parameters are obtained and the above concepts are more easily understood.

Since transmission and distribution lines for power transfer generally are designed to have low losses, the equations and concepts developed here can be used for quick and reasonably accurate hand calculations leading to initial designs.

More accurate calculations then can be made with computer programs for follow-up analysis and design.

Surge Impedance

For a lossless line, R = G = 0**, and**

$$z = j\omega \mathrm{L} \quad \Omega/\mathrm{m}$$
$$y = j\omega \mathrm{C} \quad \mathrm{S}/\mathrm{m}$$

From equations in last sections:

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega \mathrm{L}}{j\omega \mathrm{C}}} = \sqrt{\frac{\mathrm{L}}{\mathrm{C}}}\,\Omega$$

and

$$\gamma = \sqrt{zy} = \sqrt{(j\omega \mathrm{L})(j\omega \mathrm{C})} = j\omega\sqrt{\mathrm{LC}} = j\beta \quad \mathrm{m}^{-1}$$

Where $\beta = \omega\sqrt{\mathrm{LC}} \quad \mathrm{m}^{-1}$**.**

The characteristic impedance $Z_c = \sqrt{L/C}$, commonly called the surge impedance for a lossless line, is pure real - that is, resistive.

The propagation constant $\gamma = j\beta$ is pure imaginary.

ABCD Parameters

The *ABCD* **parameters are:**

$$A(x) = D(x) = \cosh(\gamma x) = \cosh(j\beta x)$$
$$= \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x) \quad \textbf{per unit}$$

$$\sinh(\gamma x) = \sinh(j\beta x) = \frac{e^{j\beta x} - e^{-j\beta x}}{2} = j\sin(\beta x) \quad \textbf{per unit}$$

$$B(x) = Z_c \sinh(\gamma x) = jZ_c \sin(\beta x) = j\sqrt{\frac{L}{C}}\sin(\beta x) \quad \Omega$$

$$C(x) = \frac{\sinh(\gamma x)}{Z_c} = \frac{j\sin(\beta x)}{\sqrt{\frac{L}{C}}} \text{S}$$

Equivalent π Circuit

For the equivalent π circuit:

$$Z' = jZ_c \sin(\beta l) = jX' \quad \Omega$$

Or:

$$Z' = (j\omega \mathrm{L} l)\left(\frac{\sin(\beta l)}{\beta l}\right) = j\mathrm{X}' \quad \Omega$$

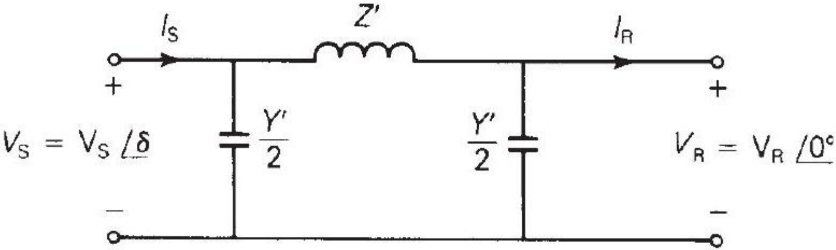
Also:

$$\begin{aligned}\frac{Y'}{2} &= \frac{Y}{2}\frac{\tanh(j\beta l/2)}{j\beta l/2} = \frac{Y}{2}\frac{\sinh(j\beta l/2)}{(j\beta l/2)\cosh(j\beta l/2)} \\ &= \left(\frac{j\omega Cl}{2}\right)\frac{j\sin(\beta l/2)}{(j\beta l/2)\cos(\beta l/2)} = \left(\frac{j\omega Cl}{2}\right)\frac{\tan(\beta l/2)}{\beta l/2} \\ &= \left(\frac{j\omega C' l}{2}\right) \mathrm{S}\end{aligned}$$

Z' and Y' are both pure imaginary.

Also, for βl less than π radians, Z' is pure inductive and Y' is pure capacitive.

Thus the equivalent π circuit for a lossless line, is also lossless.



$$Z' = (j\omega L\ell)\left(\frac{\sin\beta\ell}{\beta\ell}\right) = jX' \quad \Omega$$

$$\frac{Y'}{2} = \left(\frac{j\omega C\ell}{2}\right)\frac{\tan(\beta\ell/2)}{(\beta\ell/2)} = \frac{j\omega C'\ell}{2} \quad S$$

Wavelength

A wavelength is the distance required to change the phase of the voltage or current by 2π radians or $360°$.

For a lossless line, we have:

$$\begin{aligned} V(x) &= A(x)V_R + B(x)I_R \\ &= \cos(\beta x)V_R + jZ_c \sin(\beta x)I_R \end{aligned}$$

And

$$\begin{aligned} I(x) &= C(x)V_R + D(x)I_R \\ &= \frac{j\sin(\beta x)}{Z}V_R + \cos(\beta x)I_R \end{aligned}$$

$V(x)$ **and** $I(x)$ **change phase by** 2π **radians when** $x = 2\pi/\beta$**.**

Denoting wavelength by λ**:**

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mathrm{LC}}} = \frac{1}{f\sqrt{\mathrm{LC}}} \quad \mathrm{m}$$

Or:

$$f\lambda = \frac{1}{\sqrt{\mathrm{LC}}}$$

The term $(1/\sqrt{LC})$ is the velocity of propagation of voltage and current waves along a lossless line.

For overhead lines, $(1/\sqrt{LC}) \approx 3 \times 10^8$ m/s, and for $f = 60$ Hz:

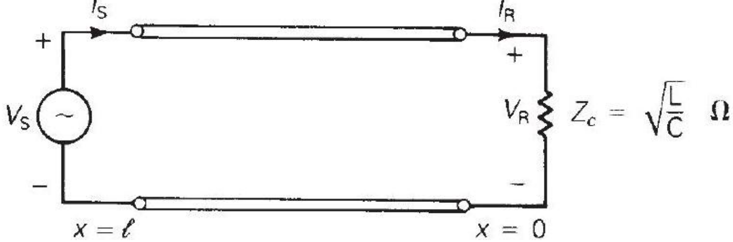
$$\lambda \approx \frac{3 \times 10^8}{60} = 5 \times 10^6 \text{ m} = 5000 \text{ km} = 3100\text{mi}$$

Typical power-line lengths are only a small fraction of the above $60 - \text{Hz}$ wave-length.

Surge Impedance Loading

Surge impedance loading (SIL) is the power delivered by a lossless line to a load resistance equal to the surge impedance $Z_c = \sqrt{L/C}$.

Figure below shows a lossless line terminated by a resistance equal to its surge impedance. This line represents either a single-phase line or one phase-to-neutral of a balanced three-phase line.


I_S
I_R
+
+
V_S
V_R
$Z_c = \sqrt{\frac{L}{C}}\ \Omega$
−
−
$x = \ell$
$x = 0$

At SIL, we have:

$$\begin{aligned}
V(x) &= \cos(\beta x)V_{\mathrm{R}} + jZ_c \sin(\beta x) I_{\mathrm{R}} \\
&= \cos(\beta x)V_{\mathrm{R}} + jZ_c \sin(\beta x)\left(\frac{V_{\mathrm{R}}}{Z_c}\right) \\
&= (\cos\beta x + j\sin\beta x)V_{\mathrm{R}} \\
&= e^{j\beta x}V_{\mathrm{R}} \qquad \textbf{volts} \\
|V(x)| &= |V_{\mathrm{R}}| \qquad \textbf{volts}
\end{aligned}$$

Thus, at SIL, the voltage profile is flat. That is, the voltage magnitude at any point x along a lossless line at SIL is constant.

Also at SIL:

$$
\begin{aligned}
I(x) &= \frac{j \sin(\beta x)}{Z_c} V_{\mathrm{R}} + (\cos \beta x) \frac{V_{\mathrm{R}}}{Z_c} \\
&= (\cos \beta x + j \sin \beta x) \frac{V_{\mathrm{R}}}{Z_c} \\
&= \left(e^{j\beta x} \right) \frac{V_{\mathrm{R}}}{Z_c} \qquad \mathrm{A}
\end{aligned}
$$

Using equations above, the complex power flowing at any point x along the line is:

$$
\begin{aligned}
S(x) = \mathrm{P}(x) + j\mathrm{Q}(x) &= V(x) I^*(x) \\
&= \left(e^{j\beta x} V_{\mathrm{R}}\right)\left(\frac{e^{j\beta x} V_{\mathrm{R}}}{Z_c}\right)^* \\
&= \frac{|V_{\mathrm{R}}|^2}{Z_c}
\end{aligned}
$$

Thus the real power flow along a lossless line at SIL remains constant from the sending end to the receiving end. The reactive power flow is zero.

At rated line voltage, the real power delivered, or SIL, is:

$$\mathrm{SIL} = \frac{V^2_{\mathbf{rated}}}{Z_c}$$

Where rated voltage is used for a single-phase line and rated line-to-line voltage is used for the total real power delivered by a three-phase line.

Table below lists surge impedance and SIL values for typical overhead $60 - \mathrm{Hz}$ **three-phase lines.**

Voltage Profiles

In practice, power lines are not terminated by their surge impedance.

Instead, loadings can vary from a small fraction of SIL during light load conditions up to multiples of SIL, depending on line length and line compensation, during heavy load conditions.

If a line is not terminated by its surge impedance, then the voltage profile is not flat.

Surge impedance and SIL values for typical $60-\mathrm{Hz}$ overhead lines:

$V_{\mathbf{rated}}$ (kV)	$Z_C = \sqrt{L/C}$ (Ω)	$\mathrm{SIL} = \mathrm{V}^2_{\mathbf{rated}}/Z_C$ (MW)
69	366 – 400	12 – 13
138	366 – 405	47 – 52
230	365 – 395	134 – 145
345	280 – 366	325 – 425
500	233 – 294	850 – 1075
765	254 – 266	2200 – 2300

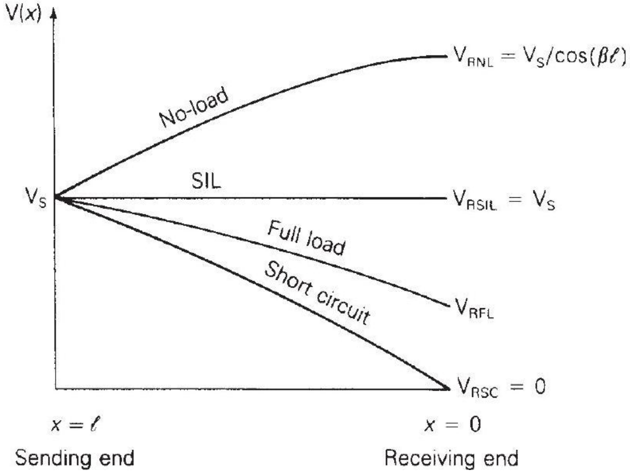
(Source: Electric Power Research Institute (EPRI), EPRI AC Transmission Line Reference Book $200\mathrm{kV}$ and Above (Palo Alto, CA: EPRI, www.epri.com. December 2005); Westinghouse Electric Corporation, Electrical Transmission and Distribution Reference Book, 4th ed. (East Pittsburgh, PA, 1964).)

Figure below shows voltage profiles of lines with a fixed sending-end voltage magnitude V_S for line lengths l up to a quarter wavelength.

This figure shows four loading conditions:

- **(1) no-load;**
- **(2) SIL;**
- **(3) short circuit; and**
- **(4) full load.**

Which are described as follows.



Voltage profiles of an uncompensated lossless line with fixed sending-end voltage for line lengths up to a quarter wavelength.

Figures above summarizes these results, showing a high receiving-end voltage at no-load and a low receiving-end voltage at full load.

This voltage regulation problem becomes more severe as the line length increases. Shunt compensation methods to reduce voltage fluctuations are needed.

1. At no-load, $I_{RNL} = 0$ **yields:**

$$V_{NL}(x) = (\cos \beta x) V_{RNL}$$

The no-load voltage increases from $V_S = (\cos \beta l) V_{RNL}$ **at the sending end to** V_{RNL} **at the receiving end (where** $x = 0$ **).**

2. The voltage profile at SIL is flat.

3. For a short circuit at the load, $V_{RSC} = 0$ **and yields:**

$$V_{SC}(x) = (Z_c \sin \beta x)\, I_{RSC}$$

The voltage decreases from $V_S = (\sin \beta l)\,(Z_c I_{RSC})$ **at the sending end to** $V_{RSC} = 0$ **at the receiving end.**

4. The full-load voltage profile, which depends on the specification of fullload current, lies above the short-circuit voltage profile.

Steady-State Stability Limit

The equivalent π circuit of Figure above can be used to obtain an equation for the real power delivered by a lossless line.

Assume that the voltage magnitudes V_S and V_R at the ends of the line are held constant.

Also, let δ denote the voltage-phase angle at the sending end with respect to the receiving end.

From KVL, the receiving-end current I_{R} is:

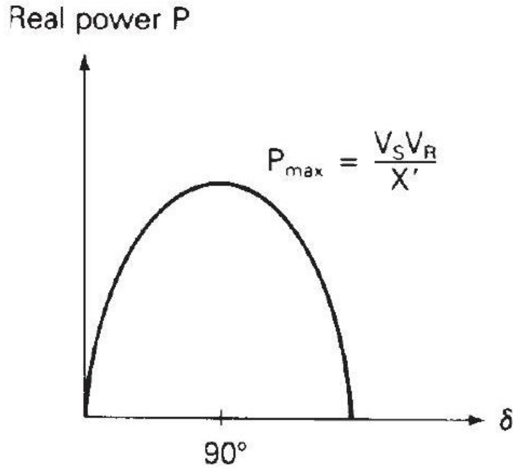
$$
\begin{aligned}
I_{\mathrm{R}} &= \frac{V_{\mathrm{S}} - V_{\mathrm{R}}}{Z'} - \frac{Y'}{2} V_{\mathrm{R}} \\
&= \frac{\mathrm{V}_{\mathrm{S}} e^{j\delta} - \mathrm{V}_{\mathrm{R}}}{j\mathrm{X}'} - \frac{j\omega C' l}{2} \mathrm{V}_{\mathrm{R}}
\end{aligned}
$$

And the complex power S_R delivered to the receiving end is:

$$
\begin{aligned}
S_R = V_R I_R^* &= V_R \left(\frac{V_S e^{j\delta} - V_R}{jX'} \right)^* + \frac{j\omega C' l}{2} V_R^2 \\
&= V_R \left(\frac{V_S e^{-j\delta} - V_R}{-jX'} \right) + \frac{j\omega C l}{2} V_R^2 \\
&= \frac{j\, V_R V_S \cos\delta + V_R V_S \sin\delta - j\, V_R^2}{X'} + \frac{j\omega C l}{2} V_R^2
\end{aligned}
$$

The real power delivered is

$$
P = P_S = P_R = \mathrm{Re}\,(S_R) = \frac{V_R V_S}{X'} \sin\delta \quad \mathrm{W}
$$



Real power delivered by a lossless line versus voltage angle across the line.

Note that since the line is lossless, $P_S = P_R$.

Equation of Active Power in the line is plotted in Figure above. For fixed voltage magnitudes V_S and V_R, the phase angle ∂ increases from $0°$ to $90°$ as the real power delivered increases. The maximum power that the line can deliver, which occurs when $\partial = 90°$, is given by:

$$P_{max} = \frac{V_S V_R}{X'} W$$

P_{max} **represents the theoretical steady-state stability limit of a lossless line.**

If an attempt were made to exceed this steady-state stability limit, then synchronous machines at the sending end would lose synchronism with those at the receiving end.

It is convenient to express the steady-state stability limit in terms of SIL.

$$P = \frac{V_S V_R \sin\delta}{Z_c \sin\beta l} = \left(\frac{V_S V_R}{Z_c}\right)\frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

Expressing V_S **and** V_R **in per-unit of rated line voltage:**

$$P = \left(\frac{V_S}{V_{\mathbf{rated}}}\right)\left(\frac{V_R}{V_{\mathbf{rated}}}\right)\left(\frac{V_{\mathbf{rated}}^2}{Z_c}\right)\frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

$$= V_{\mathbf{Sp.u.}}\, V_{\mathbf{Rp.u.}}\, (\mathrm{SIL})\frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)} \mathrm{W}$$

And for $\delta = 90°$, the theoretical steady-state stability limit is:

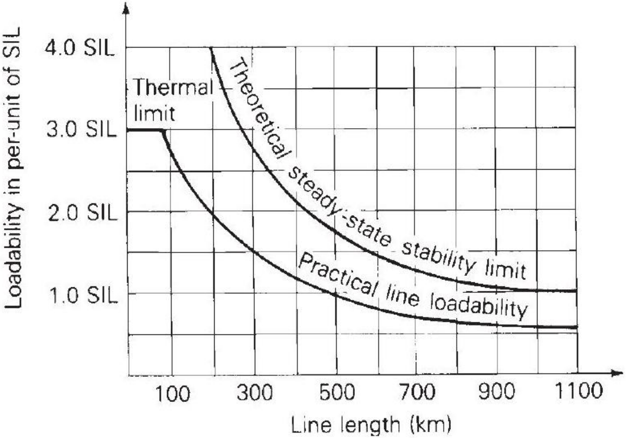
$$P_{max} = \frac{V_{\mathbf{Sp.u.}} V_{\mathbf{Rp.u.}} \ (\mathrm{SIL})}{\sin\left(\frac{2\pi l}{\lambda}\right)} \mathrm{W}$$

This Equations reveal two important factors affecting the steady-state stability limit:

First, active power increases with the square of the line voltage. For example, a doubling of line voltage enables a fourfold increase in maximum power flow.

Second, active power decreases with line length. The Equation is plotted in Figure below for $V_{\mathbf{Sp.u.}} = V_{\mathbf{Rp.u.}} = 1, \lambda = 5000$ km**, and line lengths up to** 1100 km**.**

As shown, the theoretical steady-state stability limit decreases from 4(SIL) for a 200 km **line to about 2 (SIL) for a** 400 km **line.**



Transmission-line loadability curve for $60-\mathrm{Hz}$ **overhead lines-no series or shunt compensation.**

Maximum Power Flow

Maximum power flow, discussed above for lossless lines, is derived here in terms of the *ABCD* parameters for lossy lines. The following notation is used:

$$A = \cosh(\gamma\ell) = \mathrm{A}\angle\theta_{\mathrm{A}}$$

$$B = Z' = Z'\angle\theta z$$

$$V_{\mathrm{S}} = \mathrm{V}_{\mathrm{S}}\angle\delta \qquad V_{\mathrm{R}} = \mathrm{V}_{\mathrm{R}}\angle 0^{\circ}$$

Solving for the receiving-end current:

$$I_{\mathrm{R}} = \frac{V_{\mathrm{S}} - AV_{\mathrm{R}}}{B} = \frac{\mathrm{V}_{\mathrm{S}}e^{j\delta} - \mathrm{A}\mathrm{V}_{\mathrm{R}}e^{j\theta_A}}{Z'e^{j\theta z}}$$

The complex power delivered to the receiving end is

$$S_R = P_R + jQ_R = V_R I_R^* = V_R \left[\frac{V_S e^{j(\delta - \theta_Z)} - A V_R e^{j(\theta_A - \theta_Z)}}{Z'} \right]^*$$

$$= \frac{V_R V_S}{Z'} e^{j(\theta_Z - \delta)} - \frac{A V_R^2}{Z'} e^{j(\theta_Z - \theta_A)}$$

The real and reactive power delivered to the receiving end are thus

$$P_R = \mathrm{Re}\,(S_R) = \frac{V_R V_S}{Z'} \cos(\theta_Z - \delta) - \frac{A V_R^2}{Z'} \cos(\theta_Z - \theta_A)$$

$$Q_R = \mathrm{Im}\,(S_R) = \frac{V_R V_S}{Z'} \sin(\theta_Z - \delta) - \frac{A V_R^2}{Z'} \sin(\theta_Z - \theta_A)$$

Note that for a lossless line, $\theta_A = 0°$, $B = Z' = jX'$, $Z' = X'$, $\theta_Z = 90°$, and (5.5.3) reduces to:

$$
\begin{aligned}
P_R &= \frac{V_R V_S}{X'} \cos(90 - \delta) - \frac{A V_R^2}{X'} \cos(90°) \\
&= \frac{V_R V_S}{X'} \sin \delta
\end{aligned}
$$

Which is the same as the equation found in slides above.

The theoretical maximum real power delivered (or steady-state stability limit) occurs when $\delta = \theta_Z$ in P_R equation:

$$P_{R\,\max} = \frac{V_R V_S}{Z'} - \frac{A V_R^2}{Z'} \cos\left(\theta_Z - \theta_A\right)$$

The second term in equation above, and the fact that Z' is larger than X', reduce $P_{R\,\max}$ to a value somewhat less than that given for a lossless line.

Line Loadability

In practice, power lines are not operated to deliver their theoretical maximum power, which is based on rated terminal voltages and an angular displacement $\delta = 90^\circ$ across the line.

Figure above shows a practical line loadability curve plotted below the theoretical steady-state stability limit.

This curve is based on the voltage-drop limit $V_R/V_S \geq 0.95$ and on a maximum angular displacement of 30 to 35° across the line (or about 45° across the line and equivalent system reactances) in order to maintain stability during transient disturbances.

The curve is valid for typical overhead $60 - \mathrm{Hz}$ lines with no compensation.

Note that for short lines less than $25\ \mathrm{km}$ long, loadability is limited by the thermal rating of the conductors or by terminal equipment ratings, not by voltage drop or stability considerations.