Análise de Sistemas de Potência

Aula 03: Transformadores

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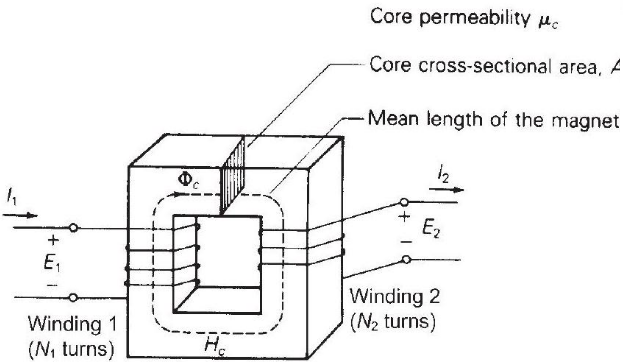
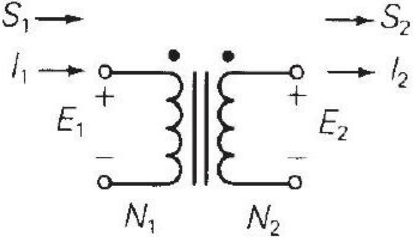
Sumário

The Ideal Transformer

Figure below shows a basic single-phase two-winding transformer, where the two windings are wrapped around a magnetic core [1, 2, 3].

It is assumed here that the transformer is operating under sinusoidal steady-state excitation.


Core permeability μ_c
Core cross-sectional area, A
Mean length of the magnet
Φ_c
I_1
I_2
+
E_1
−
+
E_2
−
Winding 1
(N_1 turns)
Winding 2
(N_2 turns)
H_c


S_1
S_2
I_1
I_2
+
E_1
−
+
E_2
−
N_1
N_2

$$a_t = \frac{N_1}{N_2}$$

Shown in the figure are the phasor voltages E_1 and E_2 across the windings, and the phasor currents I_1 entering winding 1 , which has N_1 turns, and I_2 leaving winding 2, which has N_2 turns.

A phasor flux Φ_c set up in the core and a magnetic field intensity phasor H_c are also shown.

The core has a cross-sectional area denoted A_c, a mean length of the magnetic circuit l_c, and a magnetic permeability μ_c, assumed constant.

For an ideal transformer, the following are assumed:

1. **The windings have zero resistance; therefore, the I^2R losses in the windings are zero.**

2. **The core permeability μ_c is infinite, which corresponds to zero core reluctance.**

3. **There is no leakage flux; that is, the entire flux Φ_c is confined to the core and links both windings.**

4. **There are no core losses.**

A schematic representation of a two-winding transformer is shown in Figure bellow.

Ampere's and Faraday's laws can be used along with the preceding assumptions to derive the ideal transformer relationships.

Ampere's law states that the tangential component of the magnetic field intensity vector integrated along a closed path equals the net current enclosed by that path.

That is:

$$\oint H_{\text{tan}}\, dl = I_{\text{enclosed}}$$

If the core center line shown in Figure is selected as the closed path, and if H_c is constant along the path as well as tangent to the path, then:

$$H_c l_c = N_1 I_1 - N_2 I_2$$

Note that the current I_1 is enclosed N_1 times and I_2 is enclosed N_2 times, one time for each turn of the coils.

Also, using the right-hand rule*, current I_1 contributes to clockwise flux, but current I_2 contributes to counterclockwise flux.

Thus, the net current enclosed is $N_1I_1 - N_2I_2$.

For constant core permeability μ_c, the magnetic flux density B_c within the core, also constant, is:

$$B_c = \mu_c H_c \quad \mathrm{Wb/m^2}$$

And the core flux Φ_c is:

$$\Phi_c = B_c\, A_c \quad \mathrm{Wb}$$

Using the previous equations yields:

$$N_1 I_1 - N_2 I_2 = l_c B_c / \mu_c = \left(\frac{l_c}{\mu_c \, \mathrm{A}_c} \right) \Phi_c$$

Core reluctance R_c is defined as:

$$\mathrm{R}_c = \frac{l_c}{\mu_c \, \mathrm{A}_c}$$

Then we have:

$$N_1 I_1 - N_2 I_2 = \mathrm{R}_c \Phi_c$$

The last Equation can be called Ohm's law for the magnetic circuit, wherein the net magnetomotive force $\mathrm{mmf} = N_1 I_1 - N_2 I_2$ equals the product of the core reluctance R_c and the core flux Φ_c.

Reluctance R_c, which impedes the establishment of flux in a magnetic circuit, is analogous to resistance in an electric circuit.

For an ideal transformer, μ_c is assumed infinite, which means that R_c is 0 , and in this case:

$$N_1 I_1 = N_2 I_2$$

In practice, power transformer windings and cores are contained within enclosures, and the winding directions are not visible.

One way of conveying winding information is to place a dot at one end of each winding such that when current enters a winding at the dot, it produces an mmf **acting in the same direction.**

This dot convention is shown in the schematic of Figure above. The dots are conventionally called polarity marks.

The Equation: $N_1 I_1 = N_2 I_2$ is written for current I_1 entering its dotted terminal and current I_2 leaving its dotted terminal.

As such, I_1 and I_2 are in phase, since $I_1 = (N_2/N_1)\, I_2$.

If the direction chosen for I_2 were reversed, such that both currents entered their dotted terminals, then I_1 would be 180° out of phase with I_2.

Faraday's law states that the voltage $e(t)$ induced across an N-turn winding by a time-varying flux $\phi(t)$ linking the winding is:

$$e(t) = N\frac{d\phi(t)}{dt}$$

Assuming a sinusoidal steady-state flux with constant frequency ω, and representing $e(t)$ and $\phi(t)$ by their phasors E and Φ we have:

$$E = N(j\omega)\Phi$$

For an ideal transformer, the entire flux is assumed to be confined to the core, linking both windings.

From Faraday's law, the induced voltages across the windings of an ideal transformer are:

$$
\begin{aligned}
E_1 &= N_1(j\omega)\Phi_c \\
E_2 &= N_2(j\omega)\Phi_c
\end{aligned}
$$

Dividing equations yields:

$$
\frac{E_1}{E_2} = \frac{N_1\cancel{(j\omega)\Phi_c}}{N_2\cancel{(j\omega)\Phi_c}}
$$

$$
\frac{E_1}{N_1} = \frac{E_2}{N_2}
$$

The dots shown in Figure indicate that the voltages E_1 and E_2, both of which have their + polarities at the dotted terminals, are in phase.

If the polarity chosen for one of the voltages were reversed, then E_1 would be 180° out of phase with E_2.

The turns ratio a_t is defined as follows:

$$a_t = \frac{N_1}{N_2}$$

Using a_t, the basic relations for an ideal single-phase two-winding transformer are:

$$E_1 = \left(\frac{N_1}{N_2}\right) E_2 = a_t E_2$$

$$I_1 = \left(\frac{N_2}{N_1}\right) I_2 = \frac{I_2}{a_t}$$

Two additional relations concerning complex power and impedance can be derived as follows.

The complex power entering winding 1 is:

$$S_1 = E_1 I_1^*$$

And follows that:

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t}\right)^* = E_2 I_2^* = S_2$$

As shown, the complex power S_1 entering winding 1 equals the complex power S_2 leaving winding 2.

That is, an ideal transformer has no real or reactive power loss.

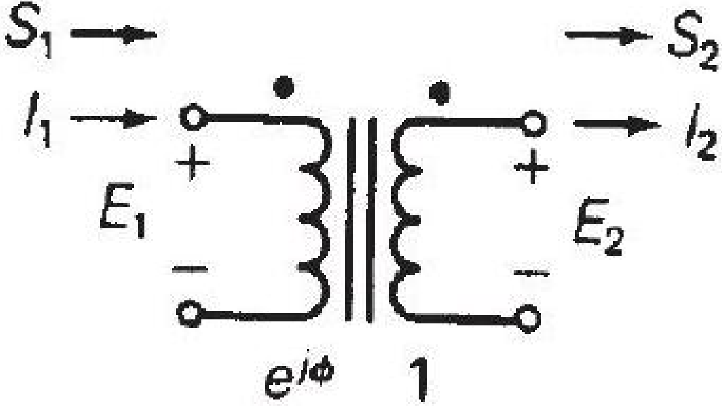
If an impedance Z_2 is connected across winding 2 of the ideal transformer then:

$$Z_2 = \frac{E_2}{I_2}$$

This impedance, when measured from winding 1, is:

$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{I_2/a_t} = a_t^2 Z_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

Thus, the impedance Z_2 connected to winding 2 is referred to winding 1 by multiplying Z_2 by a_t^2, which is the square of the turns ratio.



S_1
S_2
I_1
I_2
+
+
E_1
E_2
−
−
$e^{j\phi}$
1

Figure above shows a schematic of a conceptual single-phase, phase-shifting transformer.

This transformer is not an idealization of an actual transformer since it is physically impossible to obtain a complex turns ratio.

It is used later in this chapter as a mathematical model for representing phase shift of three-phase transformers.

As shown in Figure, the complex turns ratio a_t is defined for the phase-shifting transformer as:

$$a_t = \frac{e^{j\phi}}{1} = e^{j\phi}$$

Where ϕ is the phase-shift angle. The transformer relations are then:

$$E_1 = a_t E_2 = e^{j\phi} E_2$$

$$I_1 = \frac{I_2}{a_t^*} = e^{j\phi} I_2$$

Note that the phase angle of E_1 leads the phase angle of E_2 by ϕ.

Similarly, I_1 leads I_2 by the angle ϕ.

However, the magnitudes are unchanged; that is, $|E_1| = |E_2|$ and $|I_1| = |I_2|$.

From these two relations, the following two additional relations are derived:

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t^*}\right)^* = E_2 I_2^* = S_2$$

$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{\frac{1}{a_t^*} I_2} = |a_t|^2 Z_2 = Z_2$$

Thus, impedance is unchanged when it is referred from one side of an ideal phaseshifting transformer to the other.

Also, the ideal phase-shifting transformer has no real or reactive power losses since $S_1 = S_2$.

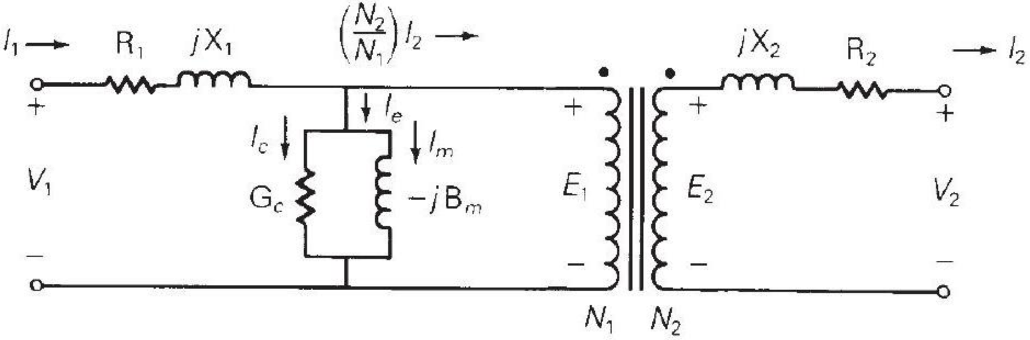
Note that equations for the phase-shifting transformer are the same for the ideal physical transformer except for the complex conjugate $(*)$ in current equations.

The complex conjugate for the phase-shifting transformer is required to make $S_1 = S_2$ (complex power into winding 1 equals complex power out of winding 2).

The Pratical Transformers

The Figure shows an equivalent circuit for a practical single-phase two-winding transformer, which differs from the ideal transformer as follows:

1. **The windings have resistance.**
2. **The core permeability μ_c is finite.**
3. **The magnetic flux is not entirely confined to the core.**
4. **There are real and reactive power losses in the core.**



I_1
R_1
jX_1
$\left(\frac{N_2}{N_1}\right) I_2$
jX_2
R_2
I_2
I_e
I_c
I_m
V_1
G_c
$-jB_m$
E_1
E_2
V_2
N_1
N_2

The resistance R_1 is included in series with winding 1 of the figure to account for I^2R losses in this winding.

A reactance X_1, called the leakage reactance of winding 1 , is also included in series with winding 1 to account for the leakage flux of winding 1.

This leakage flux is the component of the flux that links winding 1 but does not link winding 2 ; it causes a voltage drop $I_1\left(j\mathrm{X}_1\right)$, which is proportional to I_1 and leads I_1 by 90°.

There is also a reactive power loss $I_1^2X_1$ associated with this leakage reactance. Similarly, there is a resistance R_2 and a leakage reactance X_2 in series with winding 2 .

Equation shows that for finite core permeability μ_c, the total mmf **is not zero.**

Dividing equation by N_1 and using the given relation, the result is:

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \frac{\mathrm{R}_c}{N_1}\Phi_c = \frac{\mathrm{R}_c}{N_1}\left(\frac{E_1}{j\omega N_1}\right) = -j\left(\frac{\mathrm{R}_c}{\omega N_1^2}\right) E_1$$

Defining the term on the right-hand side to be I_m, called magnetizing current, it is evident that I_m lags E_1 by $90°$, and can be represented by a shunt inductor with susceptance $\mathrm{B}_m = \left(\frac{R_c}{\omega N_1^2}\right)$ mhos.

However, in reality, there is an additional shunt branch, represented by a resistor with conductance G_c mhos, which carries a current I_c, called the core loss current. I_c is in phase with E_1.

When the core loss current I_c is included becomes:

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = I_c + I_m = \left(\mathrm{G}_c - j\,\mathrm{B}_m\right) E_1$$

The equivalent circuit of transform, which includes the shunt branch with admittance $\left(G_c - j\, B_m\right)$ mhos, satisfies the KCL equation.

Note that when winding 2 is open $(I_2 = 0)$ and when a sinusoidal voltage V_1 is applied to winding 1 , then indicates that the current I_1 will have two components: the core loss current I_c and the magnetizing current I_m.

Associated with I_c is a real power loss $I_c^2/G_c = E_1^2 G_c$ W. This real power loss accounts for both hysteresis and eddy current losses within the core.

Hysteresis loss occurs because a cyclic variation of flux within the core requires energy dissipated as heat.

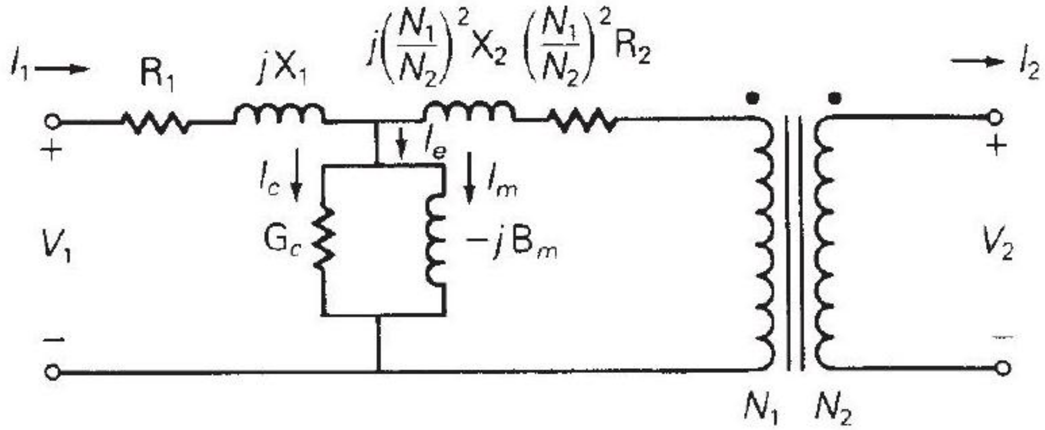
As such, hysteresis loss can be reduced by the use of special high grades of alloy steel as core material.

Eddy current loss occurs because induced currents called eddy currents flow within the magnetic core perpendicular to the flux.

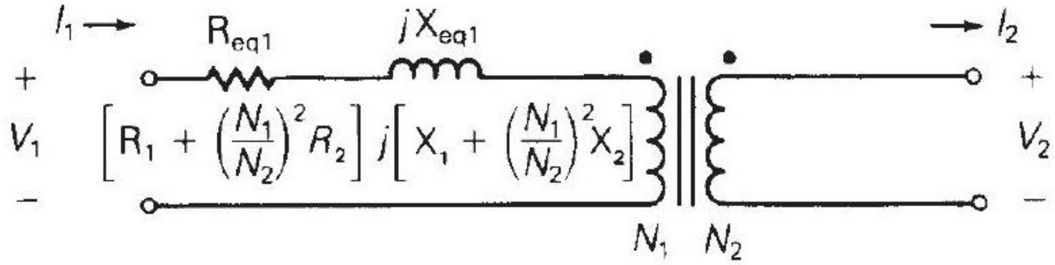
As such, eddy current loss can be reduced by constructing the core with laminated sheets of alloy steel.

Associated with I_m is a reactive power loss $\mathrm{I}_m^2/\mathrm{B}_m = \mathrm{E}_1^2\ \mathrm{B}_m$ var. This reactive power is required to magnetize the core.

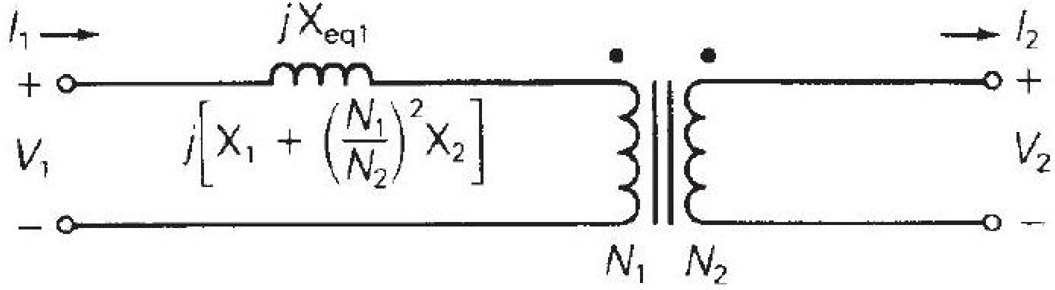
The phasor sum $(I_c + I_m)$ is called the exciting current I_e.



(a) R_2 **and** X_2 **are referred to winding 1**



(b) Neglecting exciting current



(c) Neglecting exciting current and I^2R **winding loss**

Figures above shows three alternative equivalent circuits for a practical single-phase two-winding transformer.

In Figure (a), the resistance R_2 and leakage reactance X_2 of winding 2 are referred to winding 1 via (3.1.21).

In Figure (b), the shunt branch is omitted, which corresponds to neglecting the exciting current. Since the exciting current is usually less than 5% of rated current, neglecting it in power system studies is often valid unless transformer efficiency or exciting current phenomena are of particular concern.

For large power transformers rated more than $500 \mathrm{kVA}$, the winding resistances, which are small compared to the leakage reactances, often can be neglected, as shown in Figure (c).

Thus, a practical transformer operating in sinusoidal steady state is equivalent to an ideal transformer with external impedance and admittance branches, as shown in Figures above.

The external branches can be evaluated from short-circuit and opencircuit tests, as illustrated by the following example.

The following are not represented by the equivalent circuit of Figure 3.5:

1. **Saturation**
2. **Inrush current**
3. **Nonsinusoidal exciting current**
4. **Surge phenomena**

The Per-Unit System

Power-system quantities such as voltage, current, power, and impedance are often expressed in per-unit or percent of specified base values.

For example, if a base voltage of 20kV **is specified, then the voltage** 18kV **is** $(18/20) = 0.9$ **per unit or** 90%**.**

Calculations then can be made with per-unit quantities rather than with the actual quantities.

One advantage of the per-unit system is that by properly specifying base quantities, the transformer equivalent circuit can be simplified.

The ideal transformer winding can be eliminated, such that voltages, currents, and external impedances and admittances expressed in per-unit do not change when they are referred from one side of a transformer to the other.

This can be a significant advantage even in a power system of moderate size, where hundreds of transformers may be encountered.

Another advantage of the per-unit system is that the per-unit impedances of electrical equipment of similar type usually lie within a narrow numerical range when the equipment ratings are used as base values.

Because of this, per-unit impedance data can be checked rapidly for gross errors by someone familiar with per-unit quantities. In addition, manufacturers usually specify the impedances of machines and transformers in per-unit or percent of nameplate rating.

Per-unit quantities are calculated as follows:

$$\text{per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Where:

- Actual quantity is the value of the quantity in the actual units.
- The base value has the same units as the actual quantity, thus making the per-unit quantity dimensionless.
- Also, the base value is always a real number. Therefore, the angle of the per-unit quantity is the same as the angle of the actual quantity.

Two independent base values can be arbitrarily selected at one point in a power system.

Usually the base voltage V_{baseLN} and base complex power S_{base1p} are selected for either a single-phase circuit or for one phase of a three-phase circuit.

Then, in order for electrical laws to be valid in the per-unit system, the following relations must be used for other base values:

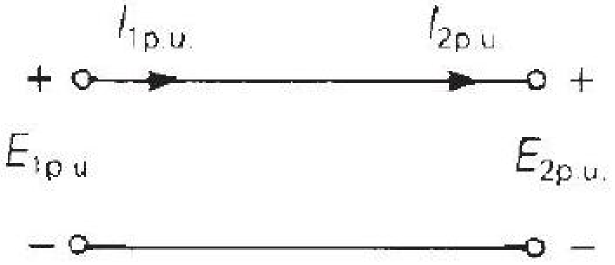
$$
\begin{aligned}
P_{\mathbf{base}1\phi} &= Q_{\mathbf{base}1\phi} = S_{\mathbf{base}1\phi} \\
I_{\mathbf{base}} &= \frac{S_{\mathbf{base}1\phi}}{V_{\mathbf{baseLN}}} \\
Z_{\mathbf{base}} &= R_{\mathbf{base}} = X_{\mathbf{base}} = \frac{V_{\mathbf{baseLN}}}{I_{\mathbf{base}}} = \frac{V^2_{\mathbf{baseLN}}}{S_{\mathbf{base}1\phi}} \\
Y_{\mathbf{base}} &= G_{\mathbf{base}} = B_{\mathbf{base}} = \frac{1}{Z_{\mathbf{base}}}
\end{aligned}
$$

The subscripts LN and 1ϕ denote ”line-to-neutral”and ”per-phase,”respectively, for three-phase circuits. These equations are also valid for single-phase circuits, where subscripts can be omitted.

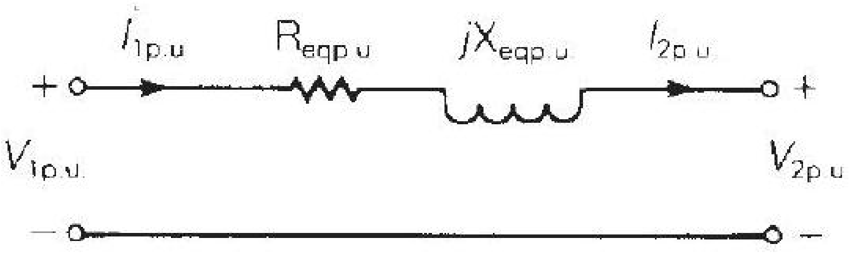
Convention requires adoption of the following two rules for base quantities:

1. The value of $S_{\mathbf{base}1\phi}$ is the same for the entire power system of concern.
2. The ratio of the voltage bases on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings.

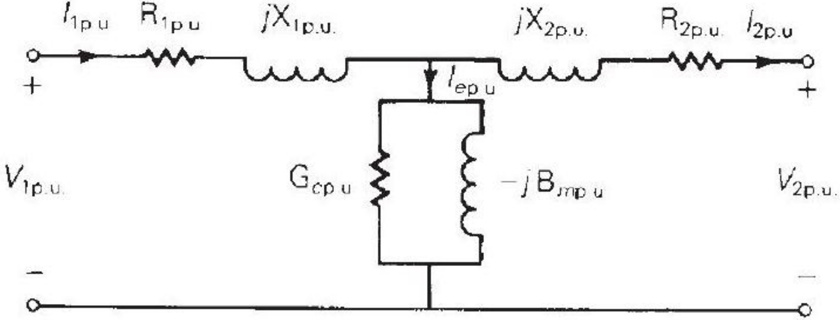
With these two rules, a per-unit impedance remains unchanged when referred from one side of a transformer to the other.



(a) Ideal transformer



(b) Neglecting exciting current



(c) Complete representation

Figures above shows three per-unit circuits of a single-phase two-winding transformer.

The ideal transformer, satisfies the per-unit relations $E_{1\ \mathbf{p.u.}} = E_{2\ \mathbf{p.u.}}$ and $I_{1\ \mathbf{p.u.}} = I_{2\ \mathbf{p...}}$, which can be derived as follows.

First divide voltage equation by $\mathrm{V}_{\mathbf{base\ I}}$:

$$E_{1\ \mathbf{p.u.}} = \frac{E_1}{\mathrm{V}_{\mathbf{base\ I}}} = \frac{N_1}{N_2} \times \frac{E_2}{\mathrm{V}_{\mathbf{base\ I}}}$$

Then, using $\mathrm{V}_{\mathbf{base1}}/\mathrm{V}_{\mathbf{base2}} = \mathrm{V}_{\mathbf{rated}\ 1}/\mathrm{V}_{\mathbf{rated}\ 2} = N_1/N_2$**,**

$$E_{1\ \mathbf{p.u.}} = \frac{N_1}{N_2} \times \frac{E_2}{\left(\frac{N_1}{N_2}\right) \mathrm{V}_{\mathbf{base\ 2}}} = \frac{E_2}{\mathrm{V}_{\mathbf{base\ 2}}} = E_{\mathbf{2\ p.u.}}$$

Similarly, divide current equation by $\mathrm{I}_{\mathbf{base\ 1}}$

$$I_{\mathbf{1\ p.u.}} = \frac{I_1}{\mathrm{I}_{\mathbf{base\ 1}}} = \frac{N_2}{N_1} \times \frac{I_2}{\mathrm{I}_{\mathbf{base\ 1}}}$$

Then, using $\mathrm{I}_{\mathbf{base\ 1}} = \mathrm{S}_{\mathbf{base}}/\mathrm{V}_{\mathbf{base\ 1}} = \mathrm{S}_{\mathbf{base}}/[(N_1/N_2)\ \mathrm{V}_{\mathbf{base\ 2}}] = (N_2/N_1)\ \mathrm{I}_{\mathbf{base\ 2}}$

$$I_{\mathbf{1\ p.u.}} = \frac{N_2}{N_1} \times \frac{I_2}{\left(\frac{N_2}{N_1}\right)_{\mathrm{I}_{\mathbf{base\ 2}}}} = \frac{I_2}{\mathrm{I}_{\mathbf{base\ 2}}} = I_{\mathbf{2\ p.u.}}$$

When only one component, such as a transformer, is considered, the nameplate ratings of that component are usually selected as base values.

When several components are involved, however, the system base values may be different from the nameplate ratings of any particular device.

It is then necessary to convert the per-unit impedance of a device from its nameplate ratings to the system base values.

To convert a per-unit impedance from “old” to “new” base values, use:

$$Z_{\textbf{p.u. new}} = \frac{Z_{\textbf{actual}}}{Z_{\textbf{base-new}}} = \frac{Z_{\textbf{p.u. old}} \cdot Z_{\textbf{base-old}}}{Z_{\textbf{base-new}}}$$

Or, as function of voltage and power:

$$Z_{\textbf{p.u. new}} = Z_{\textbf{p.u. old}} \cdot \left(\frac{V_{\textbf{base-old}}}{V_{\textbf{base-new}}}\right)^2 \cdot \left(\frac{S_{\textbf{base-new}}}{S_{\textbf{base-old}}}\right)$$

3ϕ Per Unit Systems

Balanced three-phase circuits can be solved in per-unit on a per-phase basis after converting Δ-load impedances to equivalent Υ impedances.

Base values can be selected either on a per-phase basis or on a three-phase basis.

Equations 1ϕ systems remain valid for three-phase circuits on a per-phase basis.

Usually $S_{\text{base }3\phi}$ and $V_{\text{base LL}}$ are selected, where the subscripts 3ϕ and LL denote "three-phase" and "line-to-line", respectively.

Then the following relations must be used for other base values:

$$
\begin{aligned}
S_{\mathbf{base}\ 1\phi} &= \frac{S_{\mathbf{base}\ 3\phi}}{3} \\
V_{\mathbf{base\ LN}} &= \frac{V_{\mathbf{base\ LL}}}{\sqrt{3}} \\
S_{\mathbf{base}\ 3\phi} &= P_{\mathbf{base}\ 3\phi} = Q_{\mathbf{base}\ 3\phi} \\
I_{\mathbf{base}} &= \frac{S_{\mathbf{base}\ 1\phi}}{V_{\mathbf{base\ LN}}} = \frac{S_{\mathbf{base}\ 3\phi}}{\sqrt{3}\, V_{\mathbf{base\ LL}}} \\
Z_{\mathbf{base}} &= \frac{V_{\mathbf{base\ LN}}}{I_{\mathbf{base}}} = \frac{V^2_{\mathbf{base\ LN}}}{S_{\mathbf{base}\ 1\phi}} = \frac{V^2_{\mathbf{base\ LL}}}{S_{\mathbf{base}\ 3\phi}} \\
R_{\mathbf{base}} &= X_{\mathbf{base}} = Z_{\mathbf{base}} = \frac{1}{Y_{\mathbf{base}}}
\end{aligned}
$$

Three-Phase Transformer Connections

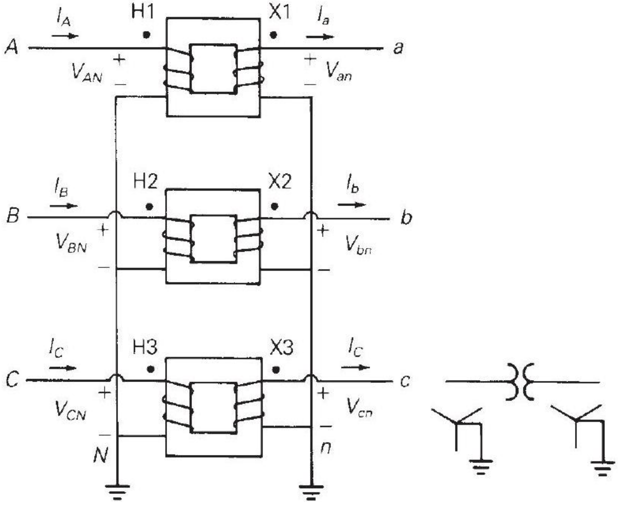
Three identical single-phase two-winding transformers may be connected to form a three-phase bank.

Four ways to connect the windings are:

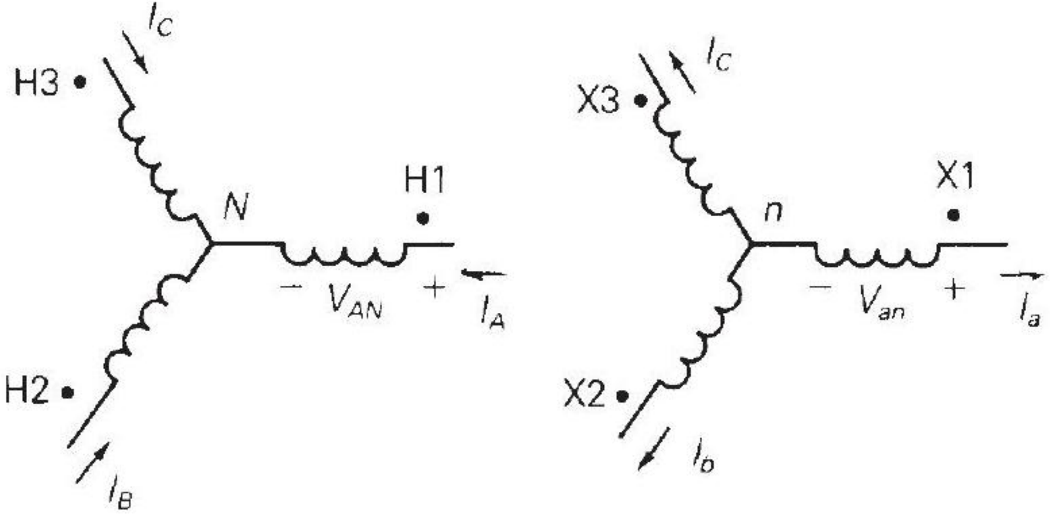
- $Y - Y$,
- $Y - \Delta$,
- $\Delta - Y$,
- $\Delta - \Delta$.

The American standard for marking three-phase transformers substitutes H1, H2**, and** H3 **on the high-voltage terminals and** X1, X2**, and** X3 **on the low-voltage terminals in place of the polarity dots.**

Also, this text uses uppercase letters ABC **to identify phases on the high-voltage side of the transformer and lowercase letters** abc **to identify phases on the low-voltage side of the transformer.**



The transformer high-voltage terminals H1, H2, and H3 are connected to phases *A, B*, and *C*, and the low-voltage terminals X1, X2, and X3 are connected to phases *a, b*, and c, respectively.

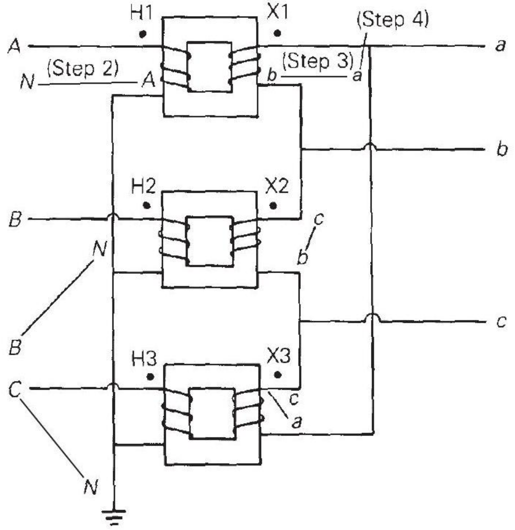


Schematic representation showing phasor relationship for positive sequence operation.

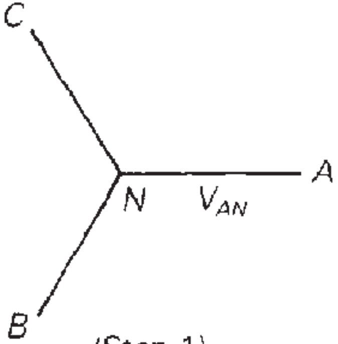
The phases of a $Y - Y$ or a $\Delta - \Delta$ transformer can be labeled so there is no phase shift between corresponding quantities on the low-and high-voltage windings.

However, for $Y - \Delta$ and $\Delta - Y$ transformers, there is always a phase shift.

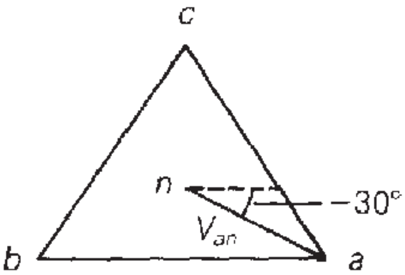
In either a $Y - \Delta$ or $\Delta - Y$ transformer, positive-sequence quantities on the high-voltage side shall lead their corresponding quantities on the low-voltage side by $30°$.



Three-phase two-winding $Y - \Delta$ transformer bank



(Step 1)



(Step 5)

V_{AN} **leads** V_{an} **by** 30°**.**

The $\Delta - Y$ transformer is commonly used as a generator step-up transformer, where the Δ winding is connected to the generator terminals and the Y winding is connected to a transmission line.

One advantage of a high-voltage Y winding is that a neutral point N is provided for grounding on the high-voltage side.

With a permanently grounded neutral, the insulation requirements for the high-voltage transformer windings are reduced. The high-voltage insulation can be graded or tapered from maximum insulation at terminals ABC to minimum insulation at grounded terminal N.

One advantage of the Δ winding is that the undesirable third harmonic magnetizing current, caused by the nonlinear core $B - H$ characteristic, remains trapped inside the Δ winding.

Third harmonic currents are (triple-frequency) zero-sequence currents, which cannot enter or leave a Δ connection, but can flow within the Δ.

The $\mathrm{Y} - \mathrm{Y}$ transformer is seldom used because of difficulties with third harmonic exciting current.

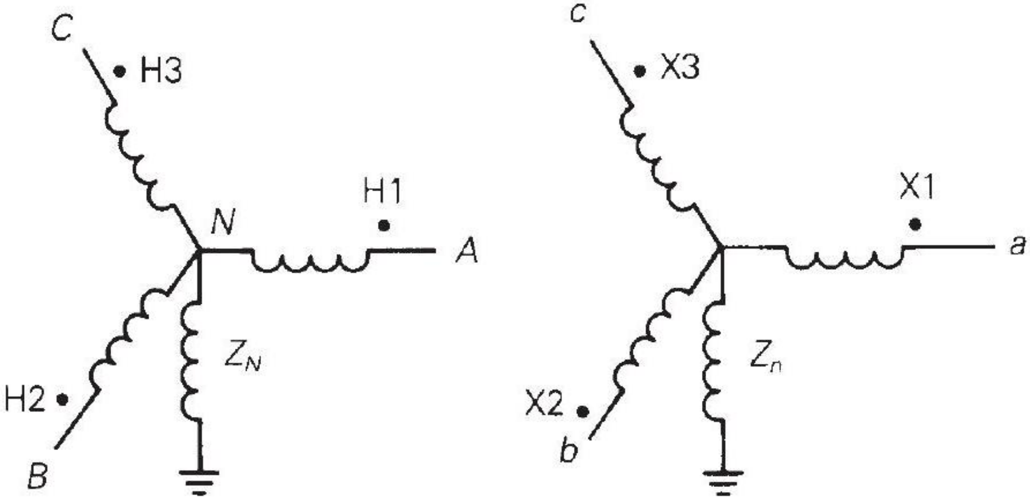
The $\Delta - \Delta$ transformer has the advantage that one phase can be removed for repair or maintenance while the remaining phases continue to operate as a three-phase bank.

This open-Δ connection permits balanced three-phase operation with the kVA rating reduced to 58% **of the original bank.**

Instead of a bank of three single-phase transformers, all six windings may be placed on a common three-phase core to form a three-phase transformer.

The three-phase core contains less iron than the three single-phase units; therefore it costs less, weighs less, requires less floor space, and has a slightly higher efficiency.

However, a winding failure would require replacement of an entire three-phase transformer, compared to replacement of only one phase of a threephase bank.


C
H3
N
H1
A
H2
B
Z_N
c
X3
X1
a
X2
b
Z_n

$I_н$
$I_х$
+
+
$E_н$
$E_х$
–
–

The following are two conventional rules for selecting base quantities:

1. **A common S_{base} is selected for both the H and X terminals.**
2. **The ratio of the voltage bases V_{baseH}/V_{basex} is selected to be equal to the ratio of the rated line-to-line voltages $V_{ratedHLL}/V_{ratedXLL}$.**

When balanced three-phase currents are applied to the transformer, the neutral currents are zero, and there are no voltage drops across the neutral impedances.

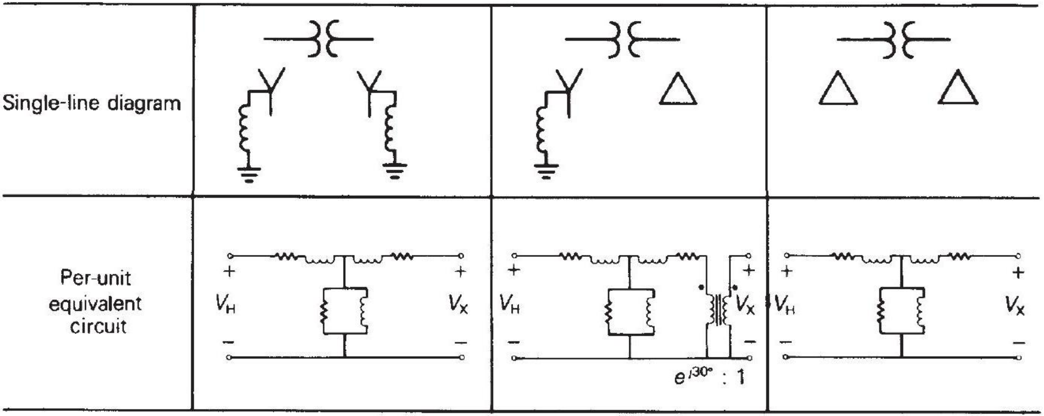
Therefore, the per-unit equivalent circuit of the ideal $\mathrm{Y}-\mathrm{Y}$ **transformer, is the same as the per-unit single-phase ideal transformer.**

The per-unit equivalent circuit of a practical $\mathrm{Y}-\mathrm{Y}$ **transformer is shown in Figure above.**

This network is obtained by adding external impedances to the equivalent circuit of the ideal transformer.

The per-unit equivalent circuit of the $\mathrm{Y}-\Delta$ transformer, shown in Figure, includes a phase shift. For the American standard, the positive-sequence voltages and currents on the high-voltage side of the $\mathrm{Y}-\Delta$ transformer lead the corresponding quantities on the low-voltage side by 30°. The phase shift in the equivalent circuit is represented by the phase-shifting transformer.

The per-unit equivalent circuit of the $\Delta-\Delta$ transformer, shown in Figure, is the same as that of the $\mathrm{Y}-\mathrm{Y}$ transformer. It is assumed that the windings are labeled so there is no phase shift. Also, the per-unit impedances do not depend on the winding connections, but the base voltages do.

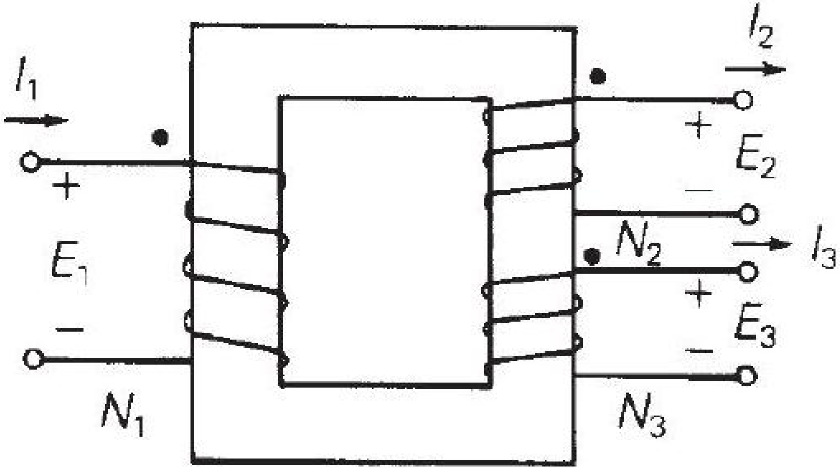


Per-unit equivalent circuits of practical $Y - Y$, $Y - \Delta$, and $\Delta - \Delta$ transformers for balanced three-phase operation.

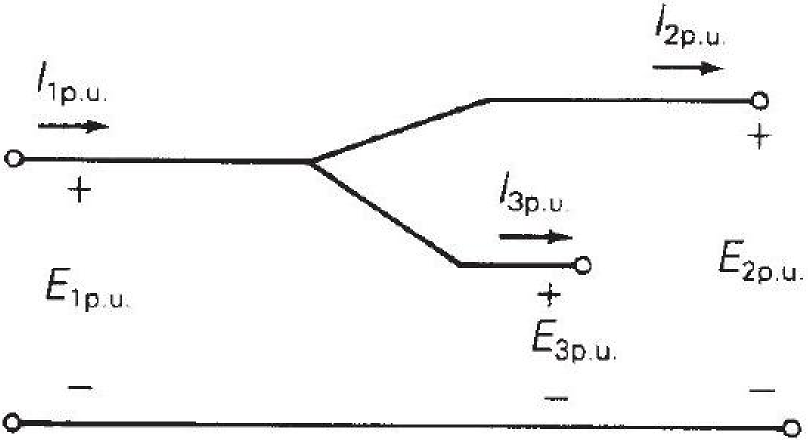
Three-Winding Transformers

Figure above shows a basic single-phase three-winding transformer.

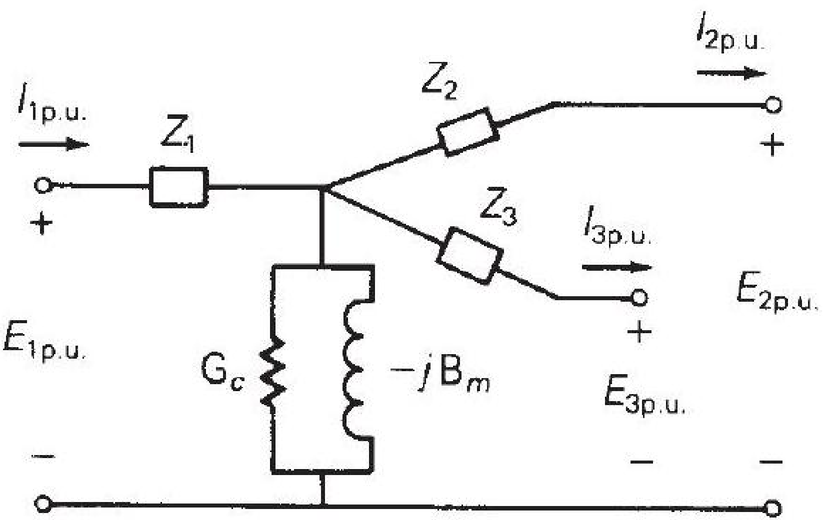
The ideal transformer relations for a two-winding transformer, easily can be extended to obtain corresponding relations for an ideal three-winding transformer.



(a) Basic core and coll configuration



(b) Per-unit equivalent circuit-ideal transformer



(c) Per-unit equivalent circuit - practical transformer

In actual units, these relations are:

$$N_1 I_1 = N_2 I_2 + N_3 I_3$$
$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3}$$

Where I_1 enters the dotted terminal, I_2 and I_3 leave dotted terminals, and E_1, E_2, and E_3 have their + polarities at dotted terminals. In per-unit:

$$I_{1\,\textbf{p.u.}} = I_{2\,\textbf{p.u.}} + I_{3\,\textbf{p.u.}}$$
$$E_{1\,\textbf{p.u.}} = E_{2\,\textbf{p.u.}} = E_{3\,\textbf{p.u.}}$$

In Equations above, a common S_{base} is selected for all three windings, and voltage bases are selected in proportion to the rated voltages of the windings.

The shunt admittance branch, a core loss resistor in parallel with a magnetizing inductor, can be evaluated from an open-circuit test.

Also, when one winding is left open, the three-winding transformer behaves as a two-winding transformer, and standard short-circuit tests can be used to evaluate per-unit leakage impedances,

Per-unit leakage impedances, are:

Z_{12} = **per-unit leakage impedance measured from winding 1 , with winding 2 shorted and winding 3 open.**

Z_{13} = **per-unit leakage impedance measured from winding 1 , with winding 3 shorted and winding 2 open.**

Z_{23} = **per-unit leakage impedance measured from winding 2 , with winding 3 shorted and winding 1 open.**

With winding 2 shorted and winding 3 open, the leakage impedance measured from winding 1 is, neglecting the shunt admittance branch:

$$Z_{12} = Z_1 + Z_2$$

Similarly,

$$Z_{13} = Z_1 + Z_3$$

And

$$Z_{23} = Z_2 + Z_3$$

Solving equations:

$$Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23})$$
$$Z_2 = \frac{1}{2}(Z_{12} + Z_{23} - Z_{13})$$
$$Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12})$$

These equations can be used to evaluate the per-unit series impedances Z_1, Z_2, and Z_3 of the three-winding transformer equivalent circuit from the per-unit leakage impedances Z_{12}, Z_{13}, and Z_{23}, which, in turn, are determined from short-circuit tests.

Note that each of the windings on a three-winding transformer may have a different kVA **rating.**

If the leakage impedances from short-circuit tests are expressed in per-unit based on winding ratings, they must first be converted to per-unit on a common $S_{\mathbf{base}}$ **before they are used in equations.**

Transformers With Off-Nominal Turns Ratios

It has been shown that models of transformers that use per-unit quantities are simpler than those that use actual quantities.

The ideal transformer winding is eliminated when the ratio of the selected voltage bases equals the ratio of the voltage ratings of the windings.

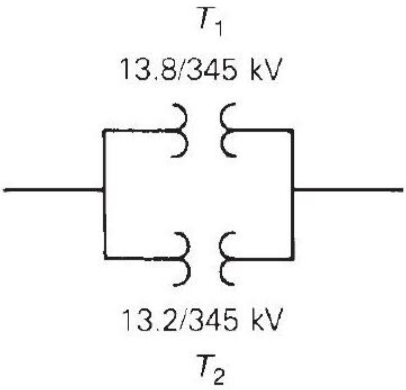
In some cases, however, it is impossible to select voltage bases in this manner.

For example, consider the two transformers connected in parallel in Figure above.

Transformer T_1 **is rated** $13.8/345\mathrm{kV}$ **and** T_2 **is rated** $13.2/345\mathrm{kV}$**.**

If $\mathrm{V}_{\textbf{base H}} = 345\mathrm{kV}$ **is selected, then transformer** T_1 **requires** $\mathrm{V}_{\textbf{base X}} = 13.8\mathrm{kV}$ **and** T_2 **requires** $\mathrm{V}_{\textbf{base X}} = 13.2\mathrm{kV}$**.**

It is clearly impossible to select the appropriate voltage bases for both transformers.



Two transformers connected in parallel

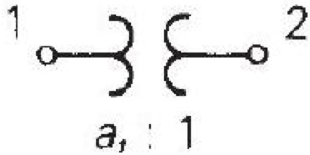
To accommodate this situation, we will develop a per-unit model of a transformer whose voltage ratings are not in proportion to the selected base voltages.

Such a transformer is said to have an off-nominal turns ratio.

Figure above shows a transformer with rated voltages $V_{1\,\text{rated}}$ and $V_{2\,\text{rated}}$, which satisfy:

$$a_t = \frac{V_{1\,\text{rated}}}{V_{2\,\text{rated}}}$$

Where a_t is assumed, in general, to be either real or complex.



(a) Single-line diagram

Suppose the selected voltage bases satisfy:

$$b = \frac{V_{\text{base 1}}}{V_{\text{base 2}}}$$

Defining:

$$c = \frac{a_t}{b}$$

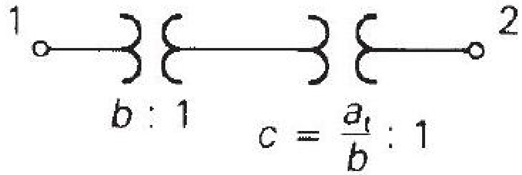
We can be rewritte:

$$V_{\text{1 rated}} = b \cdot \left(\frac{a_t}{b}\right) \cdot V_{\text{2 rated}} = b \cdot c \cdot V_{\text{2 rated}}$$

Equation $V_{1\ \text{rated}} = b \cdot c \cdot V_{2\ \text{rated}}$ can be represented by two transformers in series, as shown in Figure bellow.

The first transformer has the same ratio of rated winding voltages as the ratio of the selected base voltages, b.

Therefore, this transformer has a standard per-unit model.

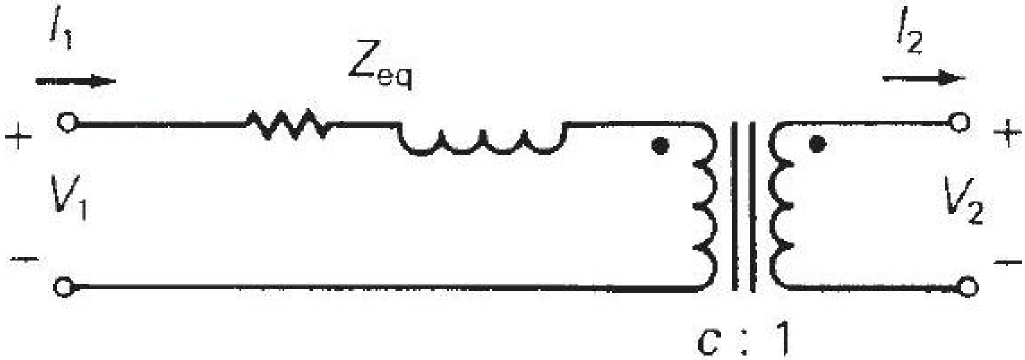


(b) Represented as two transformers in series

Assume that the second transformer is ideal, and all real and reactive losses are associated with the first transformer.

The resulting per-unit model is shown in Figure bellow, where, for simplicity, the shuntexciting branch is neglected.

Note that if $a_t = b$, then the ideal transformer winding shown in this figure can be eliminated, since its turns ratio $c = (a_t/b) = 1$.



(c) Per-unit equivalent circuit (Per-unit impedance is shown)

The per-unit model shown in Figure above is perfectly valid, but it is not suitable for some of the computer programs presented in later chapters because these programs do not accommodate ideal transformer windings.

An alternative representation can be developed, however, by writing nodal equations for this figure as follows:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}$$

both I_1 and $-I_2$ are referenced into their nodes in accordance with the nodal equation method.

Recalling two-port network theory, the admittance parameters are:

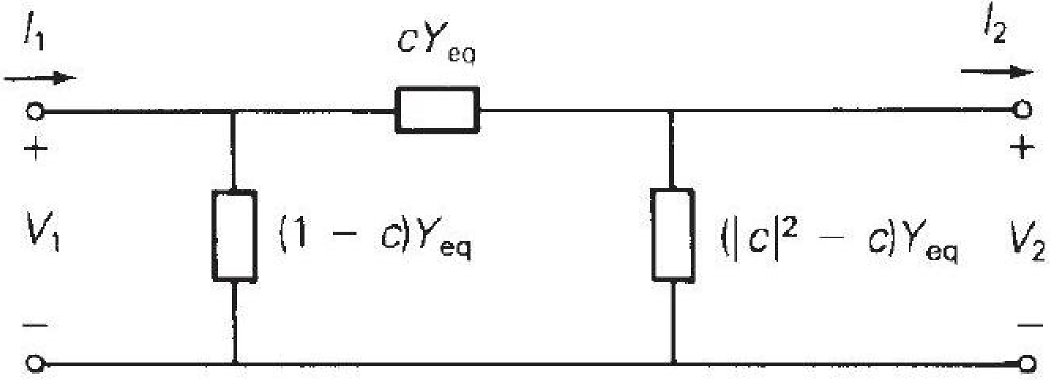
$$
\begin{aligned}
Y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0} &&= \frac{1}{Z_{\text{eq}}} &&= Y_{\text{eq}} \\
Y_{22} &= \left.\frac{-I_2}{V_2}\right|_{V_1=0} &&= \frac{1}{Z_{\text{eq}}/|c|^2} &&= |c|^2 Y_{\text{eq}} \\
Y_{12} &= \left.\frac{I_1}{V_2}\right|_{V_1=0} &&= \frac{-cV_2/Z_{\text{eq}}}{V_2} &&= -cY_{\text{eq}} \\
Y_{21} &= \left.\frac{-I_2}{V_1}\right|_{V_2=0} &&= \frac{-c^* I_1}{V_1} &&= -c^* Y_{\text{eq}}
\end{aligned}
$$

These equations with real or complex c are convenient for representing transformers with off-nominal turns ratios in the computer programs presented later.

Note that when c is complex, Y_{12} is not equal to Y_{21}, and the preceding admittance parameters cannot be synthesized with a passive RLC circuit.

However, the π network shown in Figure bellow, which has the same admittance parameters as equations developed above, can be synthesized for real c.

Note also that when $c = 1$, the shunt branches in this figure become open circuits (zero per unit mhos), and the series branch becomes Y_{eq} per unit mhos (or Z_{eq} per unit ohms).



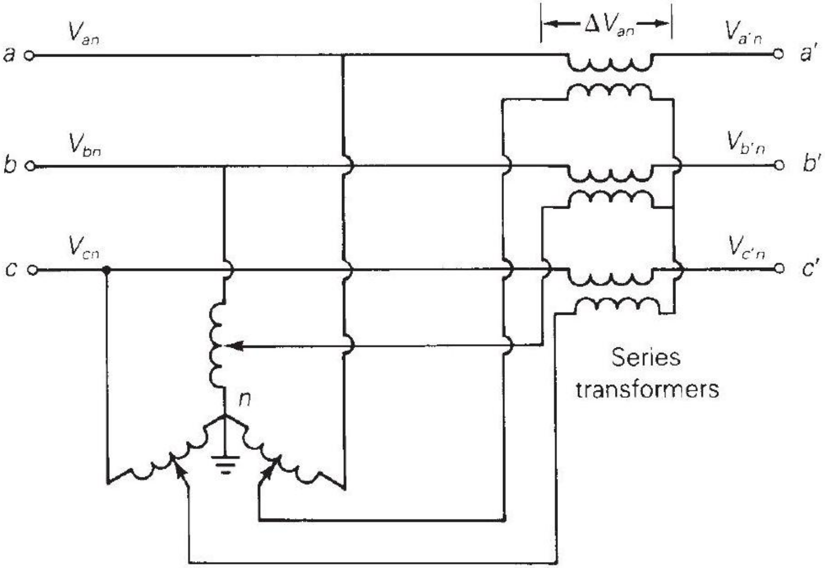
(d) π **circuit representation for real** c

(Per-unit admittances are shown; $Y_{\mathbf{eq}} = \frac{1}{Z_{\mathbf{eq}}}$**)**

The three-phase regulating transformers shown in Figures bellow can be modeled as transformers with off-nominal turns ratios.

For the voltage-magnitude regulating transformer, adjustable voltages ΔV_{an}, ΔV_{bn}, and ΔV_{cn} which have equal magnitudes $\Delta \mathrm{V}$ and which are in phase with the phase voltages V_{an}, V_{bn}, and V_{cn} are placed in the series link between buses $a - a'$, $b - b'$, and $c - c'$.

Modeled as a transformer with an off-nominal turns ratio, $c = (1 + \Delta \mathrm{V})$ for a voltage-magnitude increase toward bus abc, or $c = (1 + \Delta \mathrm{V})^{-1}$ for an increase toward bus $a'b'c'$.

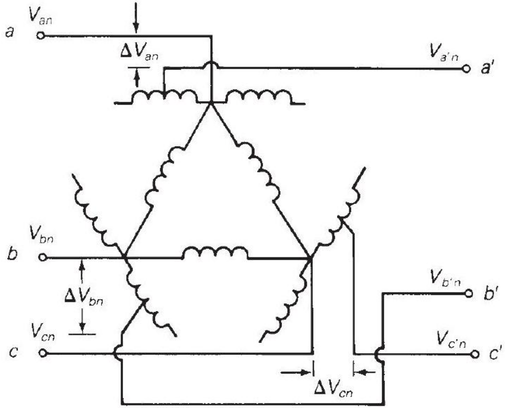


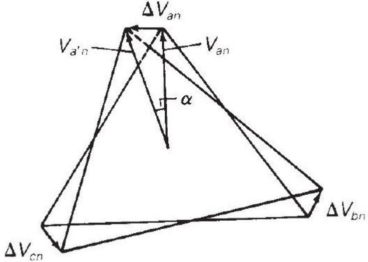
V_{an}
ΔV_{an}
$V_{a'n}$
a
a'
V_{bn}
$V_{b'n}$
b
b'
V_{cn}
$V_{c'n}$
c
c'
n
Series
transformers

For the phase-angle-regulating transformer, the series voltages ΔV_{an}, ΔV_{bn}, and ΔV_{cn}, are $\pm 90°$ out of phase with the phase voltages ΔV_{an}, V_{bn}, and V_{cn}.

The phasor diagram in Figure 3.28 indicates that each of the bus voltages $V_{a'n}$, $V_{b'n}$, and $V_{c'n}$, has a phase shift that is approximately proportional to the magnitude of the added series voltage.

Modeled as a transformer with an off-nominal turns ratio, $c \approx 1\angle\alpha$ for a phase increase toward bus abc or $c \approx 1\angle-\alpha$ for a phase increase toward bus $a'b'c'$.





An example of a phase-angle-regulating transformer. Windings drawn in parallel are on the same core.