# Análise de Sistemas de Potência

## Aula 05: Curto Circuito Simétrico

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**Este material é inteiramente baseado no livro "Power Systems Design and Analisys" (6th Edition) dos autores J. Duncan Glover, Thomas J. Overbye e Mulukutlas S. Sarma.**

# Sumário

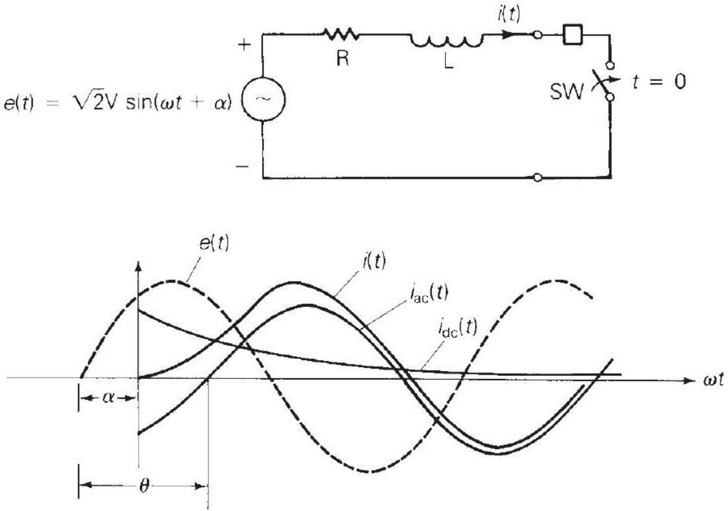
# Series R-L Circuit Transients

Consider the series $\mathrm{R} - \mathrm{L}$ circuit shown in Figure below.

The closing of switch $\mathrm{SW}$ at $t = 0$ represents to a first approximation a three-phase short circuit at the terminals of an unloaded synchronous machine.

For simplicity, assume zero fault impedance; that is, the short circuit is a solid or "bolted" fault.

The current is assumed to be zero before SW closes, and the source angle $\alpha$ determines the source voltage at $t = 0$.




**Current in a series** R – L **circuit with ac voltage source**

**Writing a KVL equation for the circuit,**

$$\frac{\mathrm{L}di(t)}{dt} + \mathrm{R}i(t) = \sqrt{2}V\sin(\omega t + \alpha) \quad t \geq 0$$

**The solution for this equation is:**

$$\begin{aligned} i(t) &= i_{\mathrm{ac}}(t) + i_{\mathrm{dc}}(t) \\ &= \frac{\sqrt{2}\,\mathrm{V}}{\mathrm{Z}}\left[\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta)e^{-t/\mathrm{T}}\right] \quad \mathrm{A} \end{aligned}$$

**Where:**

$$i_{\mathrm{ac}}(t) = \frac{\sqrt{2}\,\mathrm{V}}{\mathrm{Z}}\sin(\omega t + \alpha - \theta) \quad \mathrm{A}$$

$$i_{\mathrm{dc}}(t) = -\frac{\sqrt{2}\,\mathrm{V}}{\mathrm{Z}}\sin(\alpha - \theta)e^{-t/T} \quad \mathrm{A}$$

$$Z = \sqrt{\mathrm{R}^2 + (\omega\mathrm{L})^2} = \sqrt{\mathrm{R}^2 + \mathrm{X}^2} \quad \Omega$$

$$\theta = \tan^{-1}\frac{\omega\mathrm{L}}{\mathrm{R}} = \tan^{-1}\frac{\mathrm{X}}{\mathrm{R}}$$

$$\mathrm{T} = \frac{\mathrm{L}}{\mathrm{R}} = \frac{\mathrm{X}}{\omega\mathrm{R}} = \frac{\mathrm{X}}{2\pi f\mathrm{R}} \quad \mathrm{s}$$

**The total fault current, called the asymmetrical fault current, is plotted in Figure above along with its two components.**

**The ac fault current (also called symmetrical or steady-state fault current), is a sinusoid.**

**The *dc* offset current, decays exponentially with time constant $T = L/R$.**

**The rms ac fault current is $I_{ac} = V/Z$.**

**The magnitude of the dc offset, which depends on $\alpha$, varies from 0 when $\alpha = \theta$ to $\sqrt{2}I_{ac}$ when $\alpha = (\theta \pm \pi/2)$.**

**Note that a short circuit may occur at any instant during a cycle of the ac source; that is, $\alpha$ can have any value. To find the largest fault current, choose $\alpha = (\theta - \pi/2)$:**

$$i(t) = \sqrt{2} I_{ac} \left[ \sin(\omega t - \pi/2) + e^{-t/T} \right] \text{ A}$$

**Where:**

$$I_{ac} = \frac{V}{Z} \quad \text{A}$$

**The rms value of $i(t)$ is of interest.**

**Since $i(t)$ is not strictly periodic, its rms value is not strictly defined.**

**However, to calculate the rms asymmetrical fault current with maximum dc offset, treat the exponential term as a constant, stretching the rms concept as follows:**

$$
\begin{aligned}
I_{rms}(t) &= \sqrt{[I_{ac}]^2 + [I_{dc}(t)]^2} \\
&= \sqrt{[I_{ac}]^2 + \left[\sqrt{2} I_{ac} e^{-t/T}\right]^2} \\
&= I_{ac}\sqrt{1 + 2e^{-2t/T}} \quad \text{A}
\end{aligned}
$$

**It is convenient to use** $T = X/(2\pi f R)$ **and** $t = \tau/f$**, where** $\tau$ **is time in cycles:**

$$I_{rms}(\tau) = K(\tau) I_{ac} \quad A$$

**Where**

$$K(\tau) = \sqrt{1 + 2e^{-4\pi\tau/(X/R)}} \text{ per unit}$$

**From this equations, the rms asymmetrical fault current equals the rms ac fault current times an "asymmetry factor"** $K(\tau)$**.**

$I_{rms}(\tau)$ **decreases from** $\sqrt{3} I_{ac}$ **when** $\tau = 0$ **to** $I_{ac}$ **when** $\tau$ **is large.**

**Also, higher** $X$ **to** $R$ **ratios (** $X/R$ **) give higher values of** $I_{rms}(\tau)$**.**

**The above series $R - L$ short-circuit currents are summarized in Table above.**

| Component | Instantaneous Current (A) | rms Current (A) |
|---|---|---|
| **Symmetrical (ac)** | $i_{ac}(t) = \frac{\sqrt{2}\,V}{Z}\sin(\omega t + \alpha - \theta)$ | $I_{ac} = \frac{V}{Z}$ |
| **dc offset** | $i_{dc}(t) = \frac{-\sqrt{2}\,V}{Z}\sin(\alpha - \theta)e^{-t/T}$ | |
| **Asymmetrical (total)** | $i(t) = i_{ac}(t) + i_{dc}(t)$ | $I_{rms}(t) = \sqrt{I_{ac}^2 + i_{dc}(t)^2}$ **with maximum dc offset:** $I_{rms}(\tau) = K(\tau)I_{ac}$ |

**Short-circuit current—series R-L circuit.**

# Three-Phase Short Circuit—Unloaded Synchronous Machine

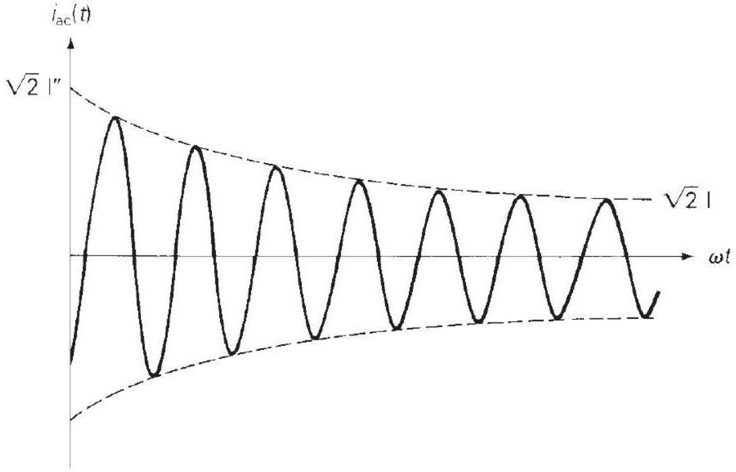
One way to investigate a three-phase short circuit at the terminals of a synchronous machine is to perform a test on an actual machine. Figure below shows an oscillogram of the ac fault current in one phase of an unloaded synchronous machine during such a test.

The dc offset has been removed from the oscillogram. As shown, the amplitude of the sinusoidal waveform decreases from a high initial value to a lower steady-state value.

A physical explanation for this phenomenon is that the magnetic flux caused by the short-circuit armature currents (or by the resultant armature MMF) is initially forced to flow through high-reluctance paths that do not link the field winding or damper circuits of the machine.

This is a result of the theorem of constant flux linkages that states that the flux linking a closed winding cannot change instantaneously.

The armature inductance, which is inversely proportional to reluctance, is therefore initially low. As the flux then moves toward the lower reluctance paths, the armature inductance increases.




**The ac fault current in one phase of an unloaded synchronous machine during a three-phase short circuit (the dc offset current is removed)**

**The ac fault current in a synchronous machine can be modeled by the series $R - L$ circuit of the initial slides if a time-varying inductance $L(t)$ or reactance $X(t) = \omega L(t)$ is employed.**

**In standard machine theory texts, the following reactances are defined:**

$$X''_d = \text{ direct axis subtransient reactance}$$
$$X'_d = \text{ direct axis transient reactance}$$
$$X_d = \text{ direct axis synchronous reactance}$$

**Where $X''_d < X'_d < X_d$.**

**The subscript $d$ refers to the direct axis.**

**There are similar quadrature axis reactances $X''_q$, $X'_q$, and $X_q$.**

**However, if the armature resistance is small, the quadrature axis reactances do not significantly affect the short-circuit current.**

**Using the above direct axis reactances, the instantaneous ac fault current can be written as:**

$$i_{ac}(t) = \sqrt{2}E_g \left[ \left( \frac{1}{X''_d} - \frac{1}{X'_d} \right) e^{-t/T''_d} + \left( \frac{1}{X'_d} - \frac{1}{X_d} \right) e^{-t/T'_d} + \frac{1}{X_d} \right] \sin\left( \omega t + \alpha - \frac{\pi}{2} \right)$$

**Where $E_g$ is the rms line-to-neutral prefault terminal voltage of the unloaded synchronous machine. The armature resistance is neglected.**

**Note that at $t = 0$, when the fault occurs, the rms value of $i_{\text{ac}}(t)$ is:**

$$I_{\text{ac}}(0) = \frac{E_g}{X''_d} = I''$$

**Which is called the rms subtransient fault current, $I''$.**

**The duration of $I''$ is determined by the time constant $T''_d$, called the direct axis short-circuit sub-transient time constant.**

**At a later time, when $t$ is large compared to $T''_d$ but small compared to the direct axis short-circuit transient time constant $T'_d$, the first exponential term has decayed almost to zero, but the second exponential has not decayed significantly.**

**The rms ac fault current then equals the rms transient fault current, given by:**

$$I' = \frac{E_g}{X'_d}$$

**When $t$ is much larger than $T'_d$, the rms ac fault current approaches its steady-state value, given by:**

$$I_{ac}(\infty) = \frac{E_g}{X_d} = I$$

**Since the three-phase no-load voltages are displaced** $120^\circ$ **from each other, the three-phase ac fault currents are also displaced** $120^\circ$ **from each other.**

**In addition to the ac fault current, each phase has a different dc offset. The maximum dc offset in any one phase, which occurs when** $\alpha = 0$ **is:**

$$i_{\mathrm{dcmax}}(t) = \frac{\sqrt{2}\mathrm{E}_g}{\mathrm{X}''_d} e^{-t/\mathrm{T_A}} = \sqrt{2}\mathrm{I}'' e^{-t/\mathrm{T_A}}$$

**Where** $\mathrm{T_A}$ **is called the armature time constant.**

**Note that the magnitude of the maximum dc offset depends only on the rms subtransient fault current** $\mathrm{I}''$**.**

**The above synchronous machine short-circuit currents are summarized in Table below.**

| Component | Instantaneous Current (A) | rms Current (A) |
|---|---|---|
| Symmetrical (ac) | (7.2.1) | $I_{ac}(t) = E_g\left[\left(\frac{1}{X''_d} - \frac{1}{X'_d}\right)e^{-t/T''_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)e^{-t/T'_d} + \frac{1}{X_d}\right]$ |
| Subtransient | | $I'' = E_g/X''_d$ |
| Transient | | $I' = E_g/X'_d$ |
| Steady-state | | $I = E_g/X_d$ |
| Maximum dc offset | $i_{dc}(t) = \sqrt{2}I''e^{-t/T_A}$ | |
| Asymmetrical (total) | $i(t) = i_{ac}(t) + i_{dc}(t)$ | $I_{rms}(t) = \sqrt{I_{ac}(t)^2 + i_{dc}(t)^2}$ with maximum dc offset: $I_{rms}(t) = \sqrt{I_{ac}(t)^2 + [\sqrt{2}I''e^{-t/T_A}]^2}$ |

**Short-circuit current—unloaded synchronous machine.**

**Machine reactances $X''_d$, $X'_d$, and $X_d$ as well as time constants $T''_d$, $T'_d$ and $T_A$ are usually provided by synchronous machine manufacturers.**

**They also can be obtained from a three-phase short-circuit test by analyzing an oscillogram such as that in Figure.**

**Typical values of synchronous machine reactances and time constants are given in Appendix Table A.1.**

# Power System Three-Phase Short Circuits

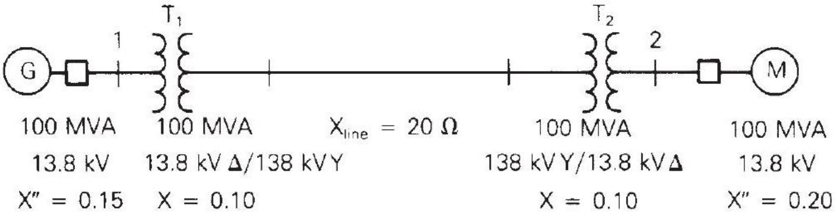
In order to calculate the subtransient fault current for a three-phase short circuit in a power system, make the following assumptions:

1. Transformers are represented by their leakage reactances. Winding resistances, shunt admittances, and $\Delta - \mathrm{Y}$ phase shifts are neglected.
2. Transmission lines are represented by their equivalent series reactances. Neglect series resistances and shunt admittances.
3. Synchronous machines are represented by constant-voltage sources behind subtransient reactances. Neglect armature resistance, saliency, and saturation.
4. Neglect all nonrotating impedance loads.
5. Especially for small motors rated less than 50hp, either neglect induction motors or represent them in the same manner as synchronous machines.

These assumptions are made for simplicity in this text, and in practice they should not be made for all cases.

For example, in distribution systems, resistances of primary and secondary distribution lines may in some cases significantly reduce fault current magnitudes.

**Figure below shows a single-line diagram consisting of a synchronous generator feeding a synchronous motor through two transformers and a transmission line.**

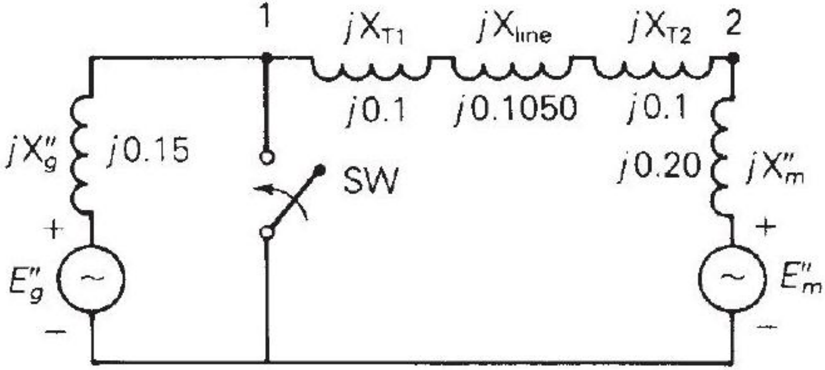



**Single-line diagram of a synchronous generator feeding a synchronous motor.**

**Consider a three-phase short circuit at bus 1.**

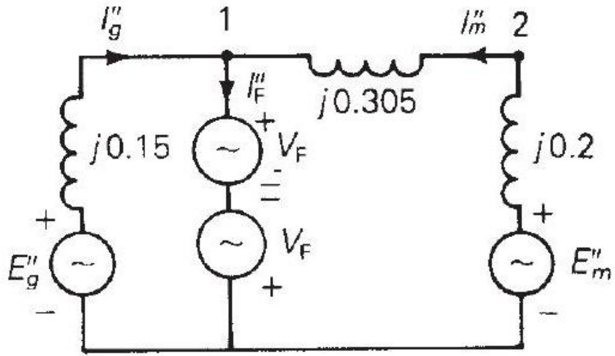
**The positive-sequence equivalent circuit is shown in Figure below, where the voltages $E_g''$ and $E_m''$ are the prefault internal voltages behind the subtransient reactances of the machines, and the closing of switch SW represents the fault.**

**For purposes of calculating the subtransient fault current, $E_g''$ and $E_m''$ are assumed to be constant-voltage sources.**




**(a) Three-phase short circuit**

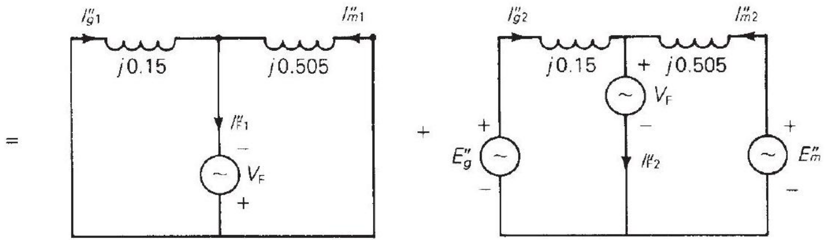
**In Figure below the fault is represented by two opposing voltage sources with equal phasor values $V_F$.**




**(b) Short circuit represented by two opposing voltage sources**

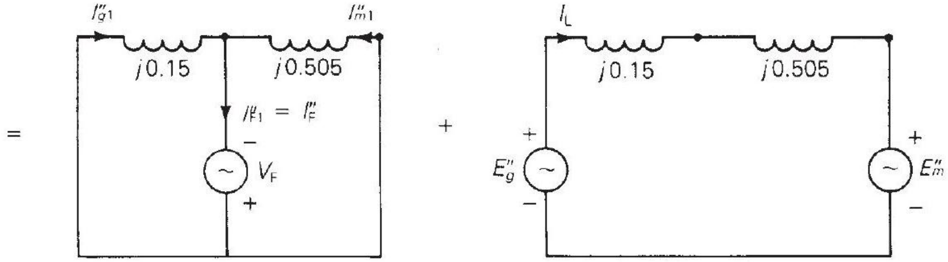
Using superposition, the fault current then can be calculated from the two circuits shown in Figure below.

However, if $V_F$ equals the prefault voltage at the fault, then the second circuit in Figure below represents the system before the fault occurs.




(c) Application of superposition

As such, $I''_{F_2} = 0$ and $V_F$, which has no effect, can be removed from the second circuit, as shown in Figure below.




(d) $V_F$ set equal to prefault voltage at fault

**The subtransient fault current is then determined from the first circuit in Figure above,** $I''_{F} = I''_{F1}$**.**

**The contribution to the fault from the generator is** $I''_{g} = I''_{g1} + I''_{g2} = I''_{g1} + I_{L}$**, where** $I_{L}$ **is the prefault generator current.**

**Similarly,** $I''_{m} = I''_{m1} - I_{L}$**.**

# Bus Impedance Matrix

Now to extend the results of the previous section to calculate subtransient fault currents for three-phase faults in an $N$-bus power system, the system is modeled by its positive-sequence network, where lines and transformers are represented by series reactances and synchronous machines are represented by constant-voltage sources behind subtransient reactances.

As before, all resistances, shunt admittances, and nonrotating impedance loads, and also for simplicity prefault load currents, are neglected.

Consider a three-phase short circuit at any bus $n$. To analyze two separate circuits, use the superposition method described above.

In the first circuit, all machine-voltage sources are short-circuited, and the only source is due to the prefault voltage at the fault. Writing nodal equations for the first circuit:

$$Y_{\text{bus}} E^{(1)} = I^{(1)}$$

Where $Y_{\text{bus}}$ is the positive-sequence bus admittance matrix, $E^{(1)}$ is the vector of bus voltages, and $I^{(1)}$ is the vector of current sources. The superscript (1) denotes the first circuit.

**Solving:**

$$\boldsymbol{Z}_{\mathbf{bus}}\,\boldsymbol{I}^{(1)} = \boldsymbol{E}^{(1)}$$

**Where**

$$\boldsymbol{Z}_{\mathbf{bus}} = \boldsymbol{Y}_{\mathbf{bus}}^{-1}$$

**$\boldsymbol{Z}_{\mathbf{bus}}$, the inverse of $\boldsymbol{Y}_{\mathbf{bus}}$, is called the positive-sequence bus impedance matrix. Both $\boldsymbol{Z}_{\mathbf{bus}}$ and $\boldsymbol{Y}_{\mathbf{bus}}$ are symmetric matrices.**

**Since the first circuit contains only one source, located at faulted bus $n$, the current source vector contains only one nonzero component, $I_n^{(1)} = -I''_{Fnn}$.**

**Also, the voltage at faulted bus $n$ in the first circuit is $E_n^{(1)} = -V_F$.**

**Rewriting:**

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & \cdots & Z_{2N} \\ \vdots & & & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & \cdots & Z_{nN} \\ \vdots & & & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nn} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I''_{Fn} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} E_1^{(1)} \\ E_2^{(1)} \\ \vdots \\ -V_F \\ \vdots \\ E_N^{(1)} \end{bmatrix}$$

**The minus sign associated with the current source indicates that the current injected into bus $n$ is the negative of $I''_{Fn}$, since $I''_{Fn}$ flows away from bus $n$ to the neutral. The subtransient fault current is:**

$$I''_{\mathrm{F}n} = \frac{V_{\mathrm{F}}}{Z_{nn}}$$

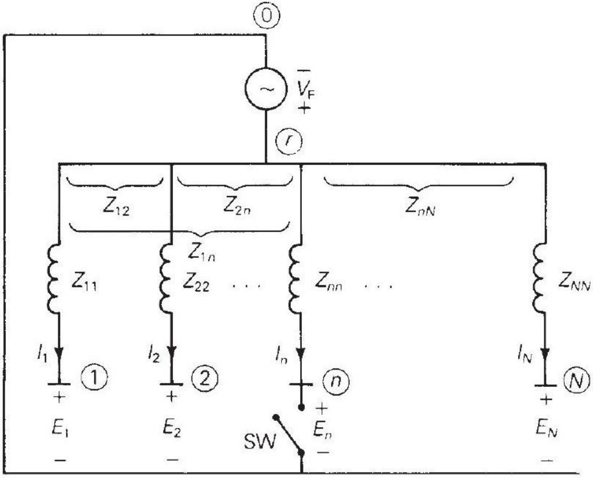
**Also, the voltage at any bus $k$ in the first circuit is:**

$$E_k^{(1)} = Z_{kn}\left(-I''_{\mathrm{F}n}\right) = \frac{-Z_{kn}}{Z_{nn}} V_{\mathrm{F}}$$

**The second circuit represents the prefault conditions. Neglecting prefault load current, all voltages throughout the second circuit are equal to the prefault voltage; that is, $E^{(2)} = V_{\mathrm{F}}$ for each bus $k$. Applying superposition:**

$$
\begin{aligned}
E_k &= E_k^{(1)} + E_k^{(2)} = \frac{-Z_{kn}}{Z_{nn}} V_{\mathrm{F}} + V_{\mathrm{F}} \\
&= \left(1 - \frac{Z_{kn}}{Z_{nn}}\right) V_{\mathrm{F}} \quad k = 1, 2, \ldots, N
\end{aligned}
$$

**Figure below shows a bus impedance equivalent circuit that illustrates the short-circuit currents in an $N$-bus system.**

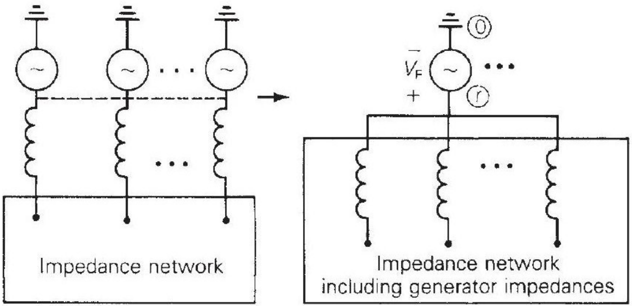


**This circuit is given the name rake equivalent in Neuenswander due to its shape, which is similar to a garden rake.**

**The diagonal elements $Z_{11}, Z_{22}, \ldots, Z_{NN}$ of the bus impedance matrix, which are the self-impedances, are shown in Figure above.**

**The off-diagonal elements, or the mutual impedances, are indicated by the brackets in the figure.**

Neglecting prefault load currents, the internal voltage sources of all synchronous machines are equal both in magnitude and phase. As such, they can be connected, as shown in Figure below, and replaced by one equivalent source $V_F$ from neutral bus o to a references bus, denoted $r$. This equivalent source is also shown in the rake equivalent of Figure above.



**Using $Z_{\text{bus}}$, the fault currents are given by:**

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & \cdots & Z_{2N} \\ \vdots & & & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & \cdots & Z_{nN} \\ \vdots & & & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nn} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_F - E_1 \\ V_F - E_2 \\ \vdots \\ V_F - E_n \\ \vdots \\ V_F - E_N \end{bmatrix}$$

**Where $I_1, I_2, \ldots$ are the branch currents and $(V_F - E_1), (V_F - E_2), \ldots$ are the voltages across the branches.**

**If switch SW in Figure above is open, all currents are zero and the voltage at each bus with respect to the neutral equals $V_{\mathrm{F}}$. This corresponds to prefault conditions, neglecting prefault load currents.**

**If switch** SW **is closed, corresponding to a short circuit at bus** $n$, $E_n = 0$ **and all currents except** $I_n$ **remain zero.**

**The fault current is affect the overall learning experience.**

$I''_{\mathrm{F}n} = I_n = V_{\mathrm{F}}/Z_{nn}$.

**This fault current also induces a voltage drop** $Z_{kn}I_n = (Z_{kn}/Z_{nn})\, V_{\mathrm{F}}$ **across each branch** $k$**.**

**The voltage at bus** $k$ **with respect to the neutral then equals** $V_{\mathrm{F}}$ **minus this voltage drop.**

**As shown by Figure above, subtransient fault currents throughout an $N$-bus system can be determined from the bus impedance matrix and the prefault voltage.**

**$Z_{\text{bus}}$ can be computed by first constructing $Y_{\text{bus}}$, via nodal equations, and then inverting $Y_{\text{bus}}$. Once $Z_{\text{bus}}$ has been obtained, these fault currents are easily computed.**