Análise de Sistemas de Potência

Aula 07: Faltas Assimétricas

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Este material é inteiramente baseado no livro "Power Systems Design and Analisys" (6th Edition) dos autores J. Duncan Glover, Thomas J. Overbye e Mulukutlas S. Sarma.

Sumário

System Representation

A three-phase power system is represented by its sequence networks in this chapter.

The zero-, positive-, and negative-sequence networks of system components generators, motors, transformers, and transmission lines - as developed in last Chapter can be used to construct system zero-, positive-, and negative-sequence networks.

The following assumptions are made:

1. The power system operates under balanced steady-state conditions before the fault occurs. Thus the zero-, positive-, and negative-sequence networks are uncoupled before the fault occurs. **During unsymmetrical faults, they are interconnected only at the fault location**.
2. Prefault load current is neglected. Because of this, the positive-sequence internal voltages of all machines are equal to the prefault voltage V_F. Therefore, the prefault voltage at each bus in the positive-sequence network equals V_F.
3. Transformer winding resistances and shunt admittances are neglected.
4. Transmission-line series resistances and shunt admittances are neglected.
5. Synchronous machine armature resistance, saliency, and saturation are neglected.
6. All nonrotating impedance loads are neglected.
7. Induction motors are either neglected (especially for motors rated 50hp or less) or represented in the same manner as synchronous machines.

Note that these assumptions are made for simplicity in this text, and in practice should not be made for all cases.

For example, in primary and secondary distribution systems, prefault currents may be in some cases comparable to short-circuit currents, and in other cases line resistances may significantly reduce fault currents.

Although fault currents as well as contributions to fault currents on the fault side of $\Delta - Y$ transformers are not affected by $\Delta - Y$ phase shifts, contributions to the fault from the other side of such transformers are affected by $\Delta - Y$ phase shifts for unsymmetrical faults.

Therefore, $\Delta - \mathrm{Y}$ phase-shift effects are included in this chapter.

Consider faults at the general three-phase bus shown in Figure below.

Terminals abc, denoted the fault terminals, are brought out in order to make external connections that represent faults.

Before a fault occurs, the currents I_a, I_b, and I_c are zero.

FIGURE 9.1

General three-phase bus

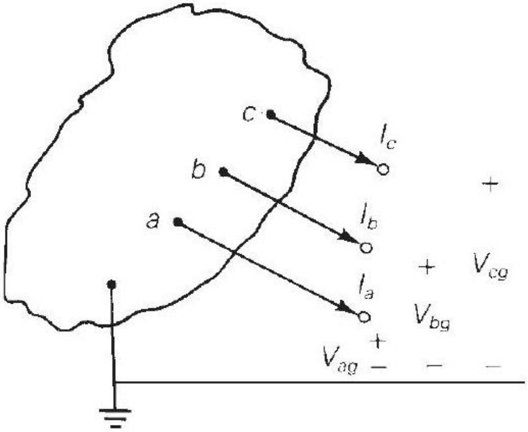


Figure below shows general sequence networks as viewed from the fault terminals.

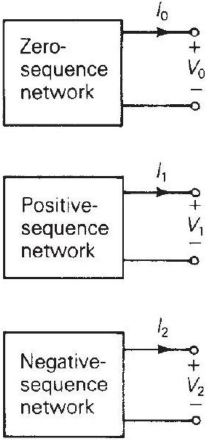
Since the prefault system is balanced, these zero-, positive-, and negative-sequence networks are uncoupled.

Also, the sequence components of the fault currents, I_0, I_1, and I_2, are zero before a fault occurs.

The general sequence networks in Figure are reduced to their Thévenin equivalents as viewed from the fault terminals in Figure.

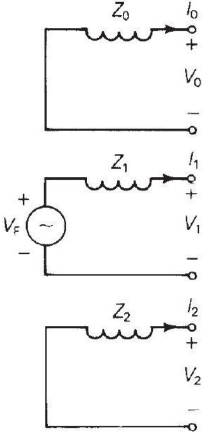
Each sequence network has a Thévenin equivalent impedance. Also, the positive-sequence network has a Thévenin equivalent voltage source, which equals the prefault voltage V_F.

Sequence networks at a general threephase bus in a balanced system:



(a) General sequence networks

Sequence networks at a general threephase bus in a balanced system:



(b) Thévenin equivalents as viewed from fault terminals

The sequence components of the line-to-ground voltages at the fault terminals are:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_F \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

During a bolted three-phase fault, the sequence fault currents are $I_0 = I_2 = 0$ **and** $I_1 = V_F/Z_1$**; therefore, the sequence fault voltages are** $V_0 = V_1 = V_2 = 0$**, which must be true since** $V_{ag} = V_{bg} = V_{cg} = 0$**.**

However, fault voltages need not be zero during unsymmetrical faults, which is considered next.

Single Line-To-Ground Fault

Consider a single line-to-ground fault from phase a to ground at the general threephase bus shown in Figure below.

For generality, a fault impedance Z_{F} is included.

In the case of a bolted fault, $Z_{\mathrm{F}} = 0$, whereas for an arcing fault, Z_{F} is the arc impedance.

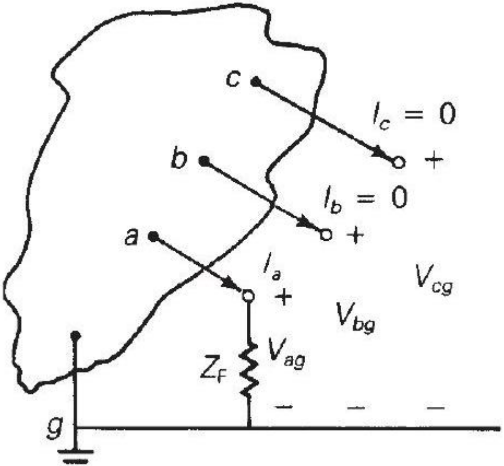
In the case of a transmission-line insulator flashover, Z_{F} includes the total fault impedance between the line and ground, including the impedances of the arc and the transmission tower, as well as the tower footing if there are no neutral wires.

The relations to be derived here apply only to a single line-to-ground fault on phase a.

However, since any of the three phases can be arbitrarily labeled phase a, single line-to-ground faults on other phases are not considered.

Fault conditions in phase domain
Single line-to-ground fault

$$\left.\begin{aligned} I_b &= I_c = 0 \\ V_{ag} &= Z_F I_a \end{aligned}\right\}$$



Now transform equations to the sequence domain:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} I_a \\ I_a \\ I_a \end{bmatrix}$$

Also:

$$(V_0 + V_1 + V_2) = Z_F \, (I_0 + I_1 + I_2)$$

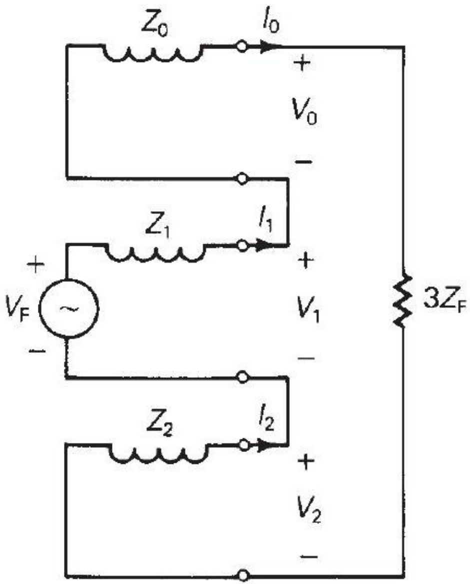
Finally we have the fault conditions in sequence domain:

$$\begin{array}{l} \textbf{Fault conditions in sequence domain} \\ \textbf{Single line-to-ground fault} \end{array} \left\} \begin{array}{l} I_0 = I_1 = I_2 \\ (V_0 + V_1 + V_2) = (3Z_F) \, I_1 \end{array} \right.$$

Equations above can be satisfied by interconnecting the sequence networks in series at the fault terminals through the impedance $(3Z_F)$, as shown in Figure below.

From this figure, the sequence components of the fault currents are:

$$I_0 = I_1 = I_2 = \frac{V_F}{Z_0 + Z_1 + Z_2 + (3Z_F)}$$



Z_0
I_0
+
V_0
−
Z_1
I_1
+
V_F
−
+
V_1
−
$3Z_F$
Z_2
I_2
+
V_2
−

Transforming to the phase domainm, we have:

$$I_a = I_0 + I_1 + I_2 = 3I_1 = \frac{3V_{\mathrm{F}}}{Z_0 + \mathrm{Z}_1 + \mathrm{Z}_2 + (3Z_{\mathrm{F}})}$$

Note also:

$$I_b = \left(I_0 + a^2 I_1 + aI_2\right) = \left(1 + a^2 + a\right) I_1 = 0$$
$$I_c = \left(I_0 + aI_1 + a^2 I_2\right) = \left(1 + a + a^2\right) I_1 = 0$$

These are obvious, since the single line-to-ground fault is on phase a, not phase b or c.

The sequence components of the line-to-ground voltages at the fault are determined from:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_\mathrm{F} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

The line-to-ground voltages at the fault can then be obtained by transforming the sequence voltages to the phase domain.

Line-To-Line Fault

Consider a line-to-line fault from phase b to c, shown in Figure below.

Again, include a fault impedance Z_F for generality.

From Figure,

Fault conditions in phase domain
Line-to-line fault

$$\left.\begin{aligned} I_a &= 0 \\ I_c &= -I_b \\ V_{bg} - V_{cg} &= Z_\mathrm{F} I_b \end{aligned}\right\}$$

Transform equations to the sequence domain, using the first two equations:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{3}\left(a - a^2\right) I_b \\ \frac{1}{3}\left(a^2 - a\right) I_b \end{bmatrix}$$

Using symetrical components equations and third equation, we have:

$$\left(V_0 + a^2 V_1 + a V_2\right) - \left(V_0 + a V_1 + a^2 V_2\right) = Z_{\mathrm{F}} \left(I_0 + a^2 I_1 + a I_2\right)$$

Noting that $I_0 = 0$ and $I_2 = -I_1$, equation simplifies to:

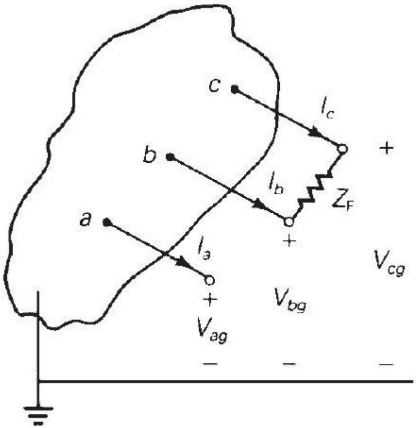
$$\left(a^2 - a\right) V_1 - \left(a^2 - a\right) V_2 = Z_\mathrm{F} \left(a^2 - a\right) I_1$$

or

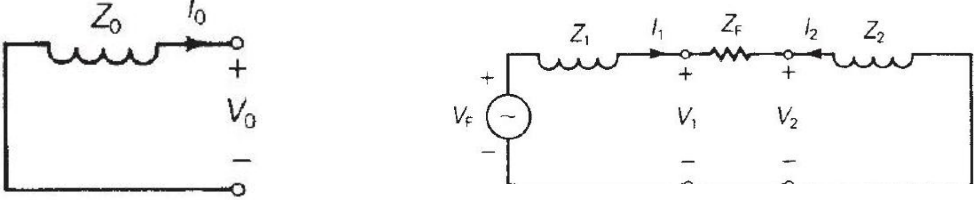
$$V_1 - V_2 = Z_\mathrm{F} I_1$$

Therefore:

$$\left.\begin{array}{l}\textbf{Fault conditions in sequence domain}\\ \textbf{Line-to-line fault}\end{array}\right\} \begin{array}{l} I_0 = 0 \\ I_2 = -I_1 \\ V_1 - V_2 = Z_\mathrm{F} I_1 \end{array}$$



(a) General three-phase bus



(b) Interconnected sequence networks

Equations above are satisfied by connecting the positive- and negative-sequence networks in parallel at the fault terminals through the fault impedance Z_F, as shown in Figure.

From this figure, the fault currents are:

$$I_1 = -I_2 = \frac{V_F}{(Z_1 + Z_2 + Z_F)} \qquad I_0 = 0$$

Transforming to the phase domain and using the identity $\left(a^2 - a\right) = -j\sqrt{3}$, the fault current in phase b is:

$$\begin{aligned} I_b &= I_0 + a^2 I_1 + a I_2 = \left(a^2 - a\right) I_1 \\ &= -j\sqrt{3} I_1 = \frac{-j\sqrt{3} V_F}{(Z_1 + Z_2 + Z_F)} \end{aligned}$$

Note also that:

$$I_a = I_0 + I_1 + I_2 = 0$$

and

$$I_c = I_0 + aI_1 + a^2 I_2 = \left(a - a^2\right) I_1 = -I_b$$

Which verify the fault conditions.

The sequence components of the line-to-ground voltages at the fault are given by:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_\mathrm{F} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Double Line-To-Ground Fault

A double line-to-ground fault from phase b to phase c through fault impedance Z_{F} to ground is shown in Figure below. From this figure,

Fault conditions in the phase domain
Double line-to-ground fault

$$\left.\begin{aligned} I_a &= 0 \\ V_{cg} &= V_{bg} \\ V_{bg} &= Z_{\mathrm{F}}\left(I_b + I_c\right) \end{aligned}\right\}$$

Transforming to the sequence domain:

$$I_0 + I_1 + I_2 = 0$$

Also:

$$\left(V_0 + aV_1 + a^2 V_2\right) = \left(V_0 + a^2 V_1 + aV_2\right)$$

Simplifying,

$$\left(a^2 - a\right) V_2 = \left(a^2 - a\right) V_1$$

Wich give us $V_2 = V_1$. Now:

$$\left(V_0 + a^2 V_1 + a V_2\right) = Z_{\mathrm{F}} \left(I_0 + a^2 I_1 + a I_2 + I_0 + a I_1 + a^2 I_2\right)$$

Using the identity $a^2 + a = -1$ in equation above,

$$(V_0 - V_1) = Z_{\mathrm{F}} \left(2I_0 - I_1 - I_2\right)$$

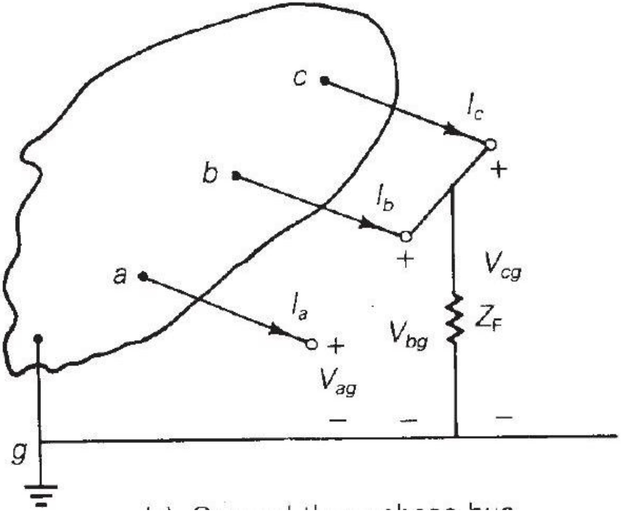
$I_0 = -\left(I_1 + I_2\right)$**; Therefore, equation above becomes:**

$$V_0 - V_1 = \left(3Z_{\mathrm{F}}\right) I_0$$

In summary:

Fault conditions in the sequence domain
Double line-to-ground fault

$$\left.\begin{aligned} & I_0 + I_1 + I_2 = 0 \\ & V_2 = V_1 \\ & V_0 - V_1 = (3Z_\mathrm{F})\, I_0 \end{aligned}\right\}$$



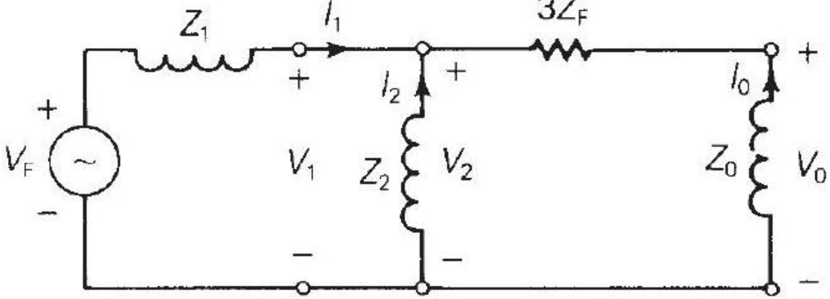
(a) General three-phase bus

Equations above are satisfied by connecting the zero-, positive-, and negative-sequence networks in parallel at the fault terminal;

additionally, $(3Z_F)$ **is included in series with the zero-sequence network.**

This connection is shown in Figure below. From this figure the positive-sequence fault current is:

$$I_1 = \frac{V_F}{Z_1 + [Z_2//(Z_0 + 3Z_F)]} = \frac{V_F}{Z_1 + \left[\frac{Z_2(Z_0+3Z_F)}{Z_2+Z_0+3Z_F}\right]}$$



(b) Interconnected sequence networks

Using current division in Figure above, the negative- and zero-sequence fault currents are:

$$I_2 = (-I_1)\left(\frac{Z_0 + 3Z_F}{Z_0 + 3Z_F + Z_2}\right)$$

$$I_0 = (-I_1)\left(\frac{Z_2}{Z_0 + 3Z_F + Z_2}\right)$$

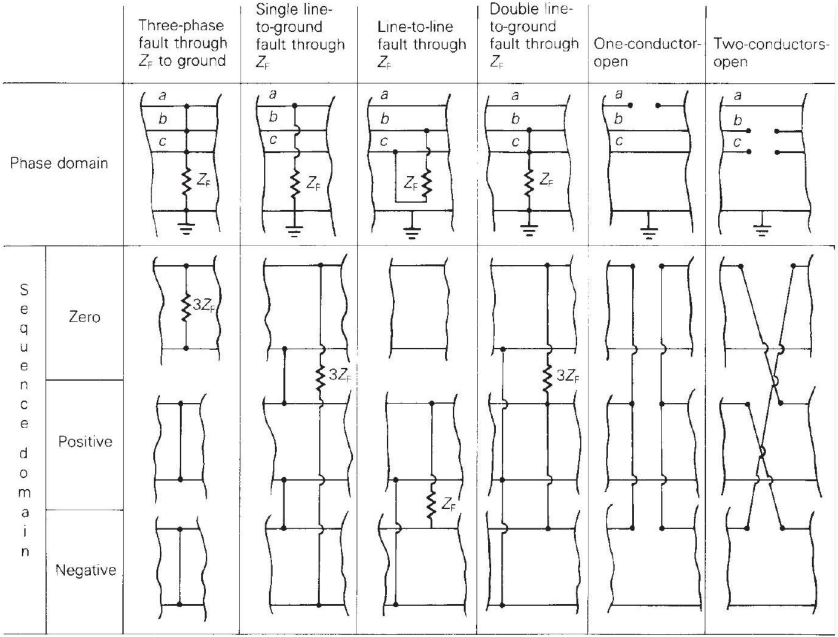
These sequence fault currents can be transformed to the phase domain.

Also, the sequence components of the line-to-ground voltages at the fault are given by:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_F \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Figure below summarizes the sequence network connections for both the balanced three-phase fault and the unsymmetrical faults that have been considered.

Sequence network connections for two additional faults, one-conductor-open and two-conductors-open, are also shown in Figure.


Three-phase fault through Z_F to ground
Single line-to-ground fault through Z_F
Line-to-line fault through Z_F
Double line-to-ground fault through Z_F
One-conductor-open
Two-conductors-open
Phase domain
a
b
c
Z_F
Sequence domain
Zero
Positive
Negative
$3Z_F$
Z_F

Sequence Bus Impedance Matrices

The positive-sequence bus impedance matrix is used for calculating currents and voltages during balanced three-phase faults.

This method is extended here to unsymmetrical faults by representing each sequence network as a bus impedance equivalent circuit (or as a rake equivalent).

A bus impedance matrix can be computed for each sequence network by inverting the corresponding bus admittance network.

For simplicity, resistances, shunt admittances, nonrotating impedance loads, and prefault load currents are neglected.

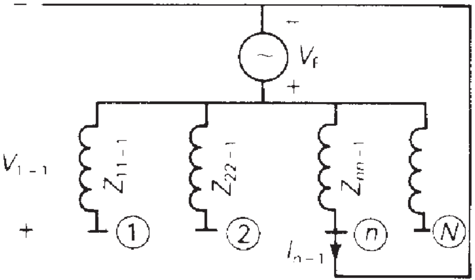
Figure below shows the connection of sequence rake equivalents for both symmetrical and unsymmetrical faults at bus n of an N-bus three-phase power system.

Each bus impedance element has an additional subscript, 0 , 1, or 2, that identifies the sequence rake equivalent in which it is located.

Mutual impedances are not shown in the figure.

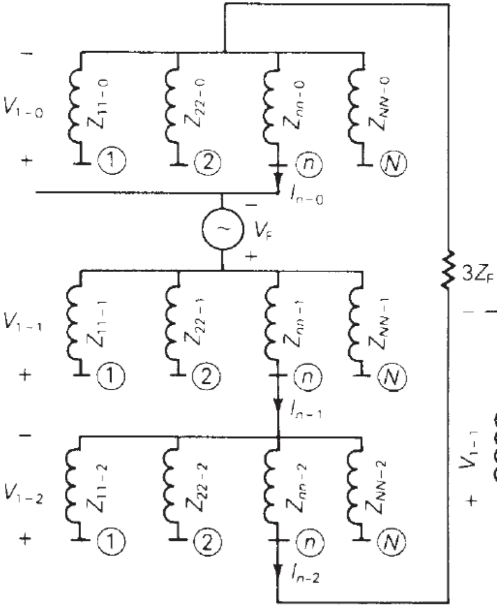
The prefault voltage V_{F} is included in the positive-sequence rake equivalent.

Connection of rake equivalent sequence networks for threephase system faults (mutual impedances not shown)

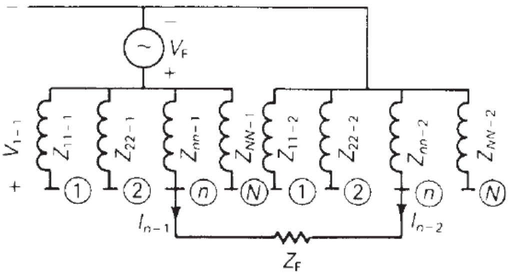


(a) Three-phase fault

(a) Three-phase fault

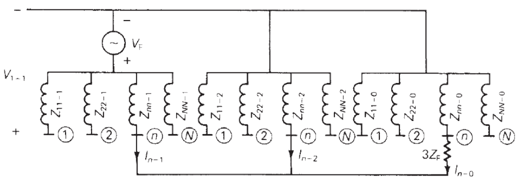


V_{1-0}
Z_{11-0}
Z_{22-0}
Z_{nn-0}
Z_{NN-0}
1
2
n
N
I_{n-0}
V_F
$3Z_F$
V_{1-1}
Z_{11-1}
Z_{22-1}
Z_{nn-1}
Z_{NN-1}
I_{n-1}
V_{1-2}
Z_{11-2}
Z_{22-2}
Z_{nn-2}
Z_{NN-2}
I_{n-2}
V_{1-1}



(c) Line-to-line fault

(c) Line-to-line fault



(d) Double line-to-ground fault

(d) Double line-to-ground fault

From the figure the sequence components of the fault current for each type of fault at bus n are as follows:

BALANCED THREE-PHASE FAULT:

$$I_{n-1} = \frac{V_{\mathrm{F}}}{Z_{nn-1}}$$

$$I_{n-0} = I_{n-2} = 0$$

SINGLE LINE-TO-GROUND FAULT (PHASE a TO GROUND):

$$I_{n-0} = I_{n-1} = I_{n-2} = \frac{V_{\mathrm{F}}}{Z_{nn-0} + Z_{nn-1} + Z_{nn-2} + 3Z_{\mathrm{F}}}$$

LINE-TO-LINE FAULT (PHASE *b* TO *c*):

$$I_{n-1} = -I_{n-2} = \frac{V_{\mathrm{F}}}{Z_{nn-1} + Z_{nn-2} + Z_{\mathrm{F}}}$$

$$I_{n-0} = 0$$

DOUBLE LINE-TO-GROUND FAULT (PHASE *b* TO *c* TO GROUND):

$$I_{n-1} = \frac{V_{\mathrm{F}}}{Z_{nn-1} + \left[\frac{Z_{nn-2}(Z_{nn-0}+3Z_{\mathrm{F}})}{Z_{nn-2}+Z_{nn-0}+3Z_{\mathrm{F}}}\right]}$$

$$I_{n-2} = (-I_{n-1})\left(\frac{Z_{nn-0} + 3Z_{\mathrm{F}}}{Z_{nn-0} + 3Z_{\mathrm{F}} + Z_{nn-2}}\right)$$

$$I_{n-0} = (-I_{n-1})\left(\frac{Z_{nn-2}}{Z_{nn-0} + 3Z_{\mathrm{F}} + Z_{nn-2}}\right)$$

Also from Figure above, the sequence components of the line-to-ground voltages at any bus k during a fault at bus n are:

$$\begin{bmatrix} V_{k-0} \\ V_{k-1} \\ V_{k-2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_{\mathrm{F}} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{kn-0} & 0 & 0 \\ 0 & Z_{kn-1} & 0 \\ 0 & 0 & Z_{kn-2} \end{bmatrix} \begin{bmatrix} I_{n-0} \\ I_{n-1} \\ I_{n-2} \end{bmatrix}$$

If bus k is on the unfaulted side of a $\Delta - Y$ transformer, then the phase angles of V_{k-1} and V_{k-2} are modified to account for $\Delta - Y$ phase shifts.

Also, the above sequence fault currents and sequence voltages can be transformed to the phase domain.

PowerWorld Simulator computes the symmetrical fault current for each of the following faults at any bus in an N-bus power system:

- balanced three-phase fault,
- single line-to-ground fault,
- line-to-line fault, or
- double line-to-ground fault.

For each fault, the Simulator also computes bus voltages and contributions to the fault current from transmission lines and transformers connected to the fault bus.

Input data for the Simulator include machine, transmission-line, and transformer data, as illustrated in Tables below as well as the prefault voltage V_{F} and fault impedance Z_{F}.

When the machine positive-sequence reactance input data consist of direct axis subtransient reactances, the computed symmetrical fault currents are subtransient fault currents.

Alternatively, transient or steady-state fault currents are computed when these input data consist of direct axis transient or synchronous reactances.

Transmission-line positive- and zero-sequence series reactances are those of the equivalent π circuits for long lines or of the nominal π circuit for medium or short lines.

Also, recall that the negative-sequence transmission-line reactance equals the positive-sequence transmission-line reactance.

All machine, line, and transformer reactances are given in per-unit on a common MVA base. Prefault load currents are neglected.

Bus	X_0 **per unit**	$X_1 = X''_d$ **per unit**	X_2 **per unit**	**Neutral Reactance** X_n **per unit**
1	0.0125	0.045	0.045	0
3	0.005	0.0225	0.0225	0.0025

Synchronous machine data for Example 9.8

Bus-to-Bus	X_0 per unit	X_1 per unit
2 – 4	0.3	0.1
2 – 5	0.15	0.05
4 – 5	0.075	0.025

Line data for Example 9.8

Low-Voltage (connection) bus	High-Voltage (connection) bus	Leakage Reactance per unit	Neutral Reactance per unit
$1(\Delta)$	$5(Y)$	0.02	0
$3(\Delta)$	$4(Y)$	0.01	0

Transformer data for Example 9.8

$$S_{\textbf{base}} = 100\text{MVA}$$

$$V_{\textbf{base}} = \begin{cases} 15\text{kV } \textbf{at buses } 1, 3 \\ 345\text{kV } \textbf{at buses } 2, 4, 5 \end{cases}$$

The Simulator computes (but does not show) the zero-, positive-, and negativesequence bus impedance matrices $Z_{\textbf{bus } 0}$, $Z_{\textbf{bus } 1}$, and $Z_{\textbf{bus } 2}$, by inverting the corresponding bus admittance matrices.

After $Z_{\text{bus }0}$, $Z_{\text{bus }1}$, and $Z_{\text{bus }2}$ are computed, are used to compute the sequence fault currents and the sequence voltages at each bus during a fault at bus 1 for the fault type selected by the program user (for example, three-phase fault, or single line-to-ground fault, and so on).

Contributions to the sequence fault currents from each line or transformer branch connected to the fault bus are computed by dividing the sequence voltage across the branch by the branch sequence impedance.

The phase angles of positive- and negative-sequence voltages are also modified to account for $\Delta - Y$ transformer phase shifts.

The sequence currents and sequence voltages are then transformed to the phase domain.

All these computations are then repeated for a fault at bus 2 , then bus 3 , and so on to bus N.

Output data for the fault type and fault impedance selected by the user consist of the fault current in each phase, contributions to the fault current from each branch connected to the fault bus for each phase, and the line-to-ground voltages at each bus-for a fault at bus 1 , then bus 2 , and so on to bus N.