Retirement Policy and Annuity Market Equilibria: Evidence from Chile

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ABSTRACT. Previous research has documented an "annuity puzzle", the empirical regularity that retirees hold too small a fraction of wealth in annuities. To understand the role of pension system design in creating this puzzle, we study Chile's pension system, where the annuitization rate is over 70%. By estimating retiree preferences for retirement assets and the cost of annuitization, we show that Chile's exception to the annuity puzzle is driven by the lack of a mandatory social insurance component and by restrictions on lump sum withdrawals. Reforming these rules would lower annuitization rates and average welfare, but would increase welfare for those at the highest levels of longevity risk. We establish that different coverage levels for mandatory insurance induce a trade-off between providing protection for longevity risk and average welfare, and provide guidance for policy makers seeking to reform the rules governing how retirees can access their retirement savings.

KEYWORDS. pension system design, retirement markets, annuities.

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1. Introduction

Pension systems often combine a mandatory social insurance component with private accounts that give retirees choice over retirement income products. In the United States, for example, retirees receive payments from Social Security and choose how to allocate and spend their accumulated 401k savings. In these settings, economists have documented an "annuity puzzle", the empirical regularity that retirees hold too small a fraction of their private account wealth in an annuity relative to what standard models would predict (Yaari (1965), Friedman and Warshawsky (1990), Mitchell et al. (1999), among others). Since an annuity provides insurance against the risk of outliving ones' assets, under-annuitization increases retirees' risk of financial distress late in life.

Low annuitization rates can arise in equilibrium for several reasons. First, as in other insurance markets, adverse selection can lead to partial or total market unravelling. Second, retirees may value leaving money after their death for others to consume or having liquidity immediately after retirement. And third, public retirement policies may provide retirees with cash flows that contract their demand for private annuitization. For example, retirees in the United States without significant savings are effectively fully annuitized through Social Security payments. These explanations imply that raising annuitization rates may or may not increase welfare. Policymakers intending to maximize retiree welfare must design retirement policy taking into account the spillover effect of public policy onto private market equilibria, as well as selection into private markets based on retiree preferences.

This paper develops a methodology that takes into account these three forces and uses it to study annuitization rates and retiree welfare under different pension system designs. In particular, we focus on reforms widely utilized across the world: varying the coverage of mandatory social insurance and on reforming the assets that are available as an alternative to annuitization. The main challenge in simulating the effects of these reforms is developing a model that incorporates their effects on private market demand for annuities and on the cost of the annuitant pool. To do so, we develop and estimate a discrete-choice model of demand for retirement assets that is founded on a consumption-savings problem. This model allows us to recover a non-parametric distribution of unobserved retiree types. Given this distribution, we re-solve each type's consumption-savings problem under counterfactual pension systems to study how private annuity markets adjust to reforms. Most importantly, this approach allows us to make welfare comparisons across systems and to characterize how different sub-populations are affected.

We estimate the model using data from the Chilean pension system, which stands out as a striking counter-example to the annuity puzzle, as over 70% of retirees in our sample choose to purchase an annuity. The Chilean pension system features two main departures relative to those studied in the annuity puzzle literature. First, it has no mandatory social insurance component covering a fraction of retirement wealth, such as Social Security. Instead, retirees have the ability to allocate wealth between an annuity and an outside option. Second, this outside option is not lump sum withdrawal of savings. Instead, wealth that is not annuitized is drawn down following a government-set schedule.

Through the lens of our model, we find that annuitization rates would fall significantly if the system were reformed to align these two features with other settings. Our results show that Chilean retirees' preferences are not the main driver of the observed exception to the annuity puzzle. Instead, the choice of pension policy levers, including the level of mandatory annuitization and the design of the outside option to the private annuity market, drive the observed private market equilibrium.

Moreover, despite this being a selection market, we find that Chile's fully voluntary annuity market delivers higher average consumer surplus and lower deadweight loss than systems with a mandatory social insurance component. This includes full annuity mandates, as in the UK or Singapore, as well as partial annuity mandates covering different fractions of wealth. Interestingly, this difference is not driven by short lived individuals opting out of annuitization, as has been posited in the previous literature. Instead, it comes from individuals who value liquidity after retirement or leaving funds as bequests. Therefore, while mandatory social insurance eliminates adverse selection for the wealth covered by the mandate, it also creates welfare loss as some of that wealth is inefficiently annuitized.

However, policymakers may want to prioritize insuring those at the highest risk of outliving their savings rather than maximizing average welfare. We find that Chile's system underperforms the reformed system in providing insurance for individuals at the highest risk of living a long life. We simulate private annuity market equilibria under a range of pension systems, covering different amounts of wealth in mandatory social insurance, and establish that there is a trade-off between insuring against longevity risk and maximizing average welfare. In particular, a system that mandates full annuitization of all pension wealth delivers the highest annuity payout and the highest welfare for those at the highest risk of living a long life, but it generates significantly more deadweight loss and less consumer surplus than the fully voluntary system.

Our results provide a roadmap to policymakers who are facing two competing priorities -

insuring the highest risk retirees against the possibility of outliving their savings, and satisfying the heterogenous preferences of retirees over what to do with their accumulated savings. Policymakers interested in maximizing average welfare should allow for unrestricted choice. Despite adverse selection on mortality risk, this setup maximizes the utility derived from the system by allowing for higher consumption immediately after retirement and by allowing for greater bequests. However, policymakers whose primary focus is providing insurance against outliving one's savings should mandate annuitization of around 90% of pension wealth. Allowing a small fraction of wealth to be freely allocated according to the retiree's private preferences increases welfare significantly for populations with low longevity risk while maintaining insurance value for high risk populations.

Related Literature Our work contributes to several strands of the literature that studies insurance markets. Most closely related is recent research on demand and cost estimation in markets with cost-relevant private information, including annuities (Einav et al. (2010)), mortgages (Agarwal et al. (2020) and Allen et al. (2020)), utility contracts (Miravete (2002)), small business lending (Crawford et al. (2018)), consumer credit (Einav et al. (2012), Kawai et al. (2018), Cuesta and Sepúlveda (2019), Nelson (2020)), and health insurance (Cardon and Hendel (2001), Bundorf and Simon (2006), Starc (2014), Handel et al. (2015), Keane and Stavrunova (2016), Finkelstein et al. (2017), Einav et al. (2019), and Tebaldi (2019)). We combine this with models of equilibrium in markets with public and private components, including a literature on long term care insurance (Brown et al. (2007), Brown and Finkelstein (2008)), and emergency care and bankruptcy (Koch (2014), Mahoney (2015), Garthwaite et al. (2018)). Our approach is most closely related to Einav et al. (2010), who study the efficiency of mandating particular guarantee periods in the UK annuity market. In contrast to this paper, we observe a system where retirees can fully select in or out of insurance coverage and study the effects of restricting such choice and providing retirees with different alternatives to annuitization.

A large literature has documented the challenges of insuring retired populations against increasing longevity risk. The private market for longevity insurance in the US is limited by market failures (Brown et al. (2001), Koijen and Yogo (2018), Bhattacharya et al. (2020), Egan et al. (2020)), resulting in low transaction volumes for life annuities (Friedman and Warshawsky (1990), Mitchell et al. (1999), Lockwood (2012), Pashchenko (2013)). We document empirically that in a competitive setting, private annuity take-up rates are largely driven by the design of the pension system and not necessarily indicative of market unravelling. We also contribute to the literature on designing pension policy in light of private preferences (Hosseini (2015), Caliendo et al. (2014), and Horneff et al. (2020)). Instead of relying on calibrations, we directly estimate the distribution of retiree preferences, use these preferences to build demand and cost functions under counterfactual pension systems, and quantify their welfare implications.

This paper is also related to work studying different aspects of the Chilean annuity exchange. Alcalde and Vial (2019) study willingness-to-pay for risk rating and other annuity attributes, while Alcalde and Vial (2018) analyze the role of intermediation. Finally, Fajnzylber et al. (2019) document adverse selection into annuitization, and Aryal et al. (2020) study the role of cognitive costs on annuity product choice and equilibrium payouts.

The rest of the paper proceeds as follows. Section 2 introduces the features of the Chilean pension system that are relevant for this analysis. Section 3 presents evidence of selection on unobserved heterogeneity. Section 4 introduces the consumption-savings model used to value the products offered to retirees and discusses how this model is used to estimate a distribution of unobserved preferences. Section 5 presents the results of the estimation procedure, while Section 6 shows equilibrium outcomes under counterfactual pension systems. Section 7 concludes.

2. Setting and Data

Chileans who are formally employed are required to save 10% of their income in a private retirement account administered by a Pension Fund Administrator (PFA). Upon retirement, those who have saved above a minimum threshold access their savings through an exchange called "SCOMP."[1] This is done either through an intermediary, such as an insurance sales agent or financial adviser, or directly at a PFA. Most women access SCOMP after turning 60, and most men after turning 65.[2]

Over ten firms participate in this exchange at any given time between 2004 and 2013. These firms are simultaneously informed of retirees' pension savings, age, marital status, age and gender of their spouse, number and age of legal beneficiaries, and the annuity contract types the retiree is willing to consider. These types are combinations of a deferral period (number of months without payments), a guarantee period (number of months that the annuity pays out regardless of death), the fraction of total savings that is being annuitized, and whether the annuity includes a transitory rent (a feature that turns it into a front-loaded step function). Retirees do not select a particular contract at this stage, they only give a list of contract types that they'd like to hear offers for. Firms

[1] Retirees who have not saved above the minimum threshold are not elegible for annuitization.

[2] One can enter at any time provided savings are over a minimum threshold, which falls significantly at these ages.

respond with offers, with no restrictions on pricing beyond exceeding a minimum pension, and no requirement to bid on all contracts. All bids must be denominated in an inflation indexed unit of account (Shiller (1998)) called "UF," so all annuities in this setting are measured in real terms.[3]

Wealth that is not allocated to an annuity is placed in a product called programmed withdrawal (PW).[4] PW provides a front-loaded drawdown of pension savings according to a regulated schedule, with two key provisions. First, whatever balance remains upon death is given to heirs. Second, if the retiree is sufficiently poor and lives long enough for payments to fall below a minimum pension, the government will top them up.[5] When an individual chooses PW, their retirement balance remains at a PFA, which invests it in a low risk fund. As a result, PW payments are stochastic, although the variance is small. See Appendix B for more details on PW and the minimum pension guarantee.

Retirees receive annuity offers and information about PW in a packet provided by SCOMP. The document includes a description of PW (Figure A.1) and a ranking of annuity offers by generosity for each contract type (Figure A.2). Retirees are also informed of the risk rating for each company. These ratings are relevant because retirees are only partially insured against the insurance company going bankrupt.[6] After receiving this document, retirees can accept an offer or enter a bargaining stage. In order to bargain, retirees must visit a company's branch location. Firms are not allowed to lower their offers in this stage. On average, these bargained offers represent a modest increase in generosity over offers received within SCOMP, on the order of 2%. Because of this, we do not model bargaining, but we do take into account the final offer generosity from each firm.

Our primary source of data is the individual-level administrative dataset from SCOMP from 2004 to 2013, which includes the retiree's date of birth, gender, geographic location, wealth, and beneficiaries. This data includes contract-level information about prices, contract characteristics and firm identifiers. We observe the contract each retiree chooses, and can compare the characteristics of the chosen contract to the other choices they had. We supplement this data with individual-level administrative death records obtained in mid 2015.

For the remainder of our analysis, we focus on the sample of women who only qualify for single

[3]In December 12, 2017, a UF was worth 40.85 USD.

[4]Fewer than 10% of our sample is able to take a small fraction of their wealth in a lump-sum ("excedente de libre disposición"). When elegible, retirees solicit offers allocating wealth between annuities, PW, and the lump sum. We take this into account throughout the analysis.

[5]The threshold for eligibility is being below the 60% percentile of total wealth.

[6]The government reinsures a minimum pension plus 75% of the difference between the annuity payment and the minimum, up to a cap of 45 UFs. There has been only one bankruptcy since the system's introduction in the 1980s, and that company's annuitants received their full payments for 124 months after bankruptcy.

	N	Mean	10th Pctile	Median	90th Pctile
Panel A: Retiree Characteristics					
Total Wealth (UFs)	34243	2430.57	1067.95	2129.79	4083.08
Age	34243	62.02	60.08	61.17	65.75
Married	34243	.27	0	0	1
Death in 2 Yrs	34243	.0102	0	0	0
Choose Annuity	34243	.714	0	1	1
Panel B: Annuity Characteristics					
Monthly Payment (UFs)	24464	11.083	5.18	9.725	18.4
Guarantee Months	24464	132.49	0	120	240
Deferral Years	24464	.59	0	0	2

Table 1: Average characteristics of our sample and of accepted annuity contracts

life annuities without beneficiaries.[7] We do so in order to avoid modelling a joint survival problem. We think of this sample selection as restricting our analysis to pension systems with single life annuities, because legal beneficiaries and marital status are observable to insurance companies when bidding. Our final dataset consists of 34,243 individuals.

Table 1 presents summary statistics for this sample. Panel A reports statistics for all individuals, while Panel B reports statistics of accepted annuity offers. The annuitization rate for this sample is 71.4%, and the probability of death by two years after retirement is 1.0%. Additionally, there is significant heterogeneity across accepted guarantee periods and deferral periods, with most individuals accepting immediate annuities and a median guarantee period of 10 years. There are two key patterns in the data. First, the fraction of individuals voluntarily choosing annuities is far higher than in other settings (Mitchell et al. (1999)). And second, the market for annuities is unconcentrated (Figure (A.3)), with each of the top ten firms getting a significant share of annuitants.

[7] All women prior to 2008, except those with minor children, and unmarried women starting in 2008 were required by law to purchase single life annuities.

3. Reduced Form Evidence

The primary market failure in voluntary insurance markets is selection on cost-relevant private information. We begin by establishing two facts in the reduced form: first, that there is adverse selection into annuitization and second, that selection into purchasing an annuity is not solely a function of mortality beliefs, but also a function of risk aversion, wealth outside the system, and bequest motives. As a result, the combined effect of selection on cost-relevant and non-cost relevant dimensions of private information is critical in determining market equilibrium.

We investigate the role of selection on private information in the reduced form through the positive correlation test (Chiappori and Salanie (2000)) and the unused observables test (Finkelstein and Poterba (2014)). The former test compares the choice to opt into insurance across populations with differential private information about marginal cost. The latter test instead leverages observable characteristics that correlate with selection into purchasing insurance, but that are not priced on. We observe both preference-relevant unused observables and data on mortality, the cost-relevant dimension of selection in annuity markets.

Table 2 presents results of the positive correlation test, obtained by regressing an indicator for annuity choice on an indicator for death within two years of retirement.[8] Individuals who choose an annuity are 0.46 percentage points less likely to die within 2 years compared to those who choose PW. In comparison, the average death rate is 1%. Column (2) adds controls for all characteristics that are observed by insurance companies at the moment of bidding. Including these controls increases the point estimate to 0.61pp. These results are evidence of economically significant adverse selection into annuitization, consistent with the evidence in Fajnzylber et al. (2019).

We also find evidence of selection across other dimensions of preference. We focus on three unused observables: asset allocations in PFA accounts prior to retirement, the prevalence of intergenerational households in the retiree's municipality of residence, and the fraction of savings that were contributed voluntarily to the system. Beginning with the first, workers invest their pension balances in up to two of five funds prior to retirement. These funds are named A through E, with A being the riskiest and E being the safest. Thus, asset allocation serves as a proxy for risk aversion. As for the second, we calculate each municipality's fraction of households with an elderly member that also have someone under 18 years of age.[9] We think of this fraction as a measure of bequest

[8]We focus on this outcome as we have two-year mortality data for all individuals in our sample.

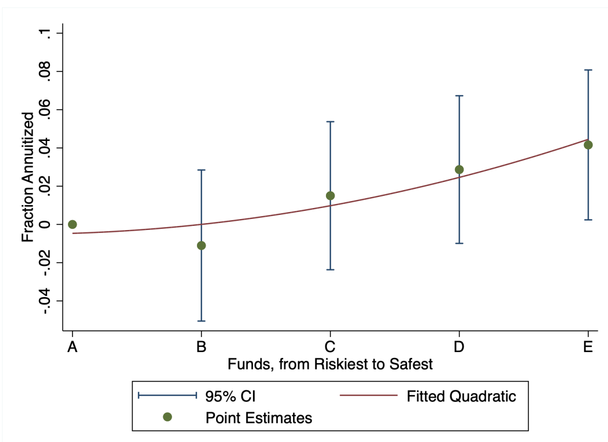
[9]More precisely, Pr[Has member under 18|Has member over 60]. Calculated using the CASEN survey. This typically

	(1)	(2)
	Death in 2 Yrs	Death in 2 Yrs
Choose Annuity	-0.00464***	-0.00610***
	(-3.50)	(-4.22)
Time FEs	Yes	Yes
Age/Wealth Controls	Yes	Yes
Request Controls	No	Yes
Observations	34238	34238

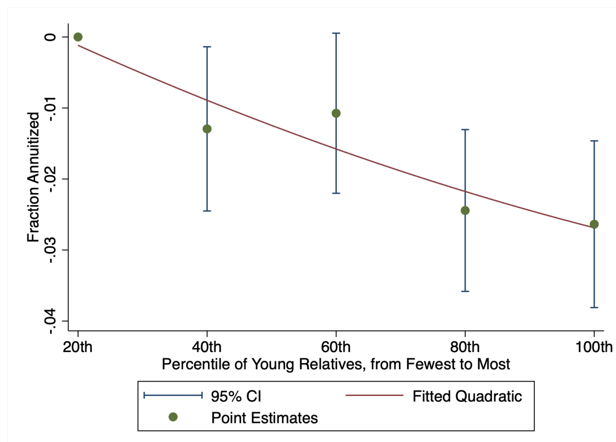
t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

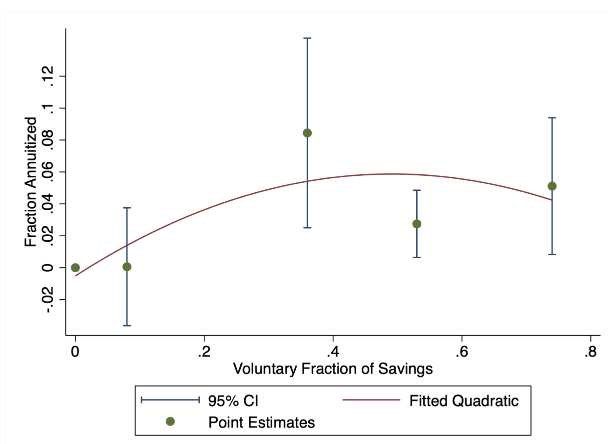
Table 2: Positive correlation test



(a) By Risk Averse Investment



(b) By Bequest Motives



(c) By Voluntary Savings

Figure 1: Variation in Annuitization by Unused Observables

motives. Finally, we observe the fraction of retirement savings that were contributed voluntarily to the system, and think of this as being correlated with having higher wealth outside the pension system.

Figure 1 shows the results of these unused observables tests, computed by regressing an annuitization dummy on each unused observable and controls for the information observed by insurance companies when bidding. Figure 1a shows that individuals with safer investments are also more likely to annuitize, consistent with selection on risk preference. Individuals with some investments in the highest risk A fund are 4 percentage points less likely to annuitize than those with their full investment in the lowest risk E fund. Figure 1b shows that individuals living in communities with more inter-generational households are less likely to annuitize. This is consistent with selection on bequest motives, as high bequest motives make programmed withdrawal more valuable. Finally, Figure 1c shows that individuals with the highest levels of voluntary savings are 4 percentage points more likely to annuitize compared to those with mandatory savings only. This is consistent with higher wealth individuals preferring annuitization.

We interpret these findings as evidence that bequest motives, risk aversion, and outside wealth play an important role in determining willingness to accept annuity offers in this setting. This evidence suggests that Chile's high annuitization rate could be driven in part by a voluntary system co-existing with multiple dimensions of unobserved heterogeneity. It also implies that the effects of counterfactual reforms to the pension system also depend on the relationships between these variables. We now turn to building and estimating a demand system for retirement assets that is able to flexibly account for these relationships and that will allow us to simulate the effects of counterfactual reforms.

4. Model and Estimation

Following the evidence in the previous section, we develop a model to value offers that accounts flexibly for multiple dimensions of private information. In this model, retirees choose the retirement product that maximizes their expected utility. This utility is obtained by solving a finite-horizon consumption-savings problem, taking into account uncertainty about mortality and firm bankruptcy. These assumptions will allow us to value annuity and programmed withdrawal offers, as well as to predict demand when counterfactual assets are introduced or when restrictions on asset allocations

indicates a multi-generational household with grandparents, children, and grandchildren.

are imposed.

Consider valuing a particular annuity offer for a retiree. Let $t = 0$ denote the moment when the individual retires, and let T denote the terminal period. Let ω denote outside wealth (the amount of assets held outside the pension system), γ denote risk aversion, and δ denote the discount factor. Let $d_t = \{0,1\}$ denote whether the individual is alive (0) or dead (1) in period t, and $\{\mu_\tau\}_{\tau=1}^T$ the vector of mortality probabilities. Following the notation in Carroll (2011), let c_t denote consumption in period t, m_t the level of resources available for consumption in t, a_t the remaining assets after t ends, and b_{t+1} the "bank balance" in $t+1$. Let q_t indicate whether the firm is bankrupt (1) or not (0) in period t, and let $\{\psi_{j,\tau}\}_{\tau=1}^T$ be the vector of bankruptcy probabilities for the offering firm. With these objects, we can write the annuity payment in period t conditional on d_t, q_t, the deferral period D and the guarantee period G as $z_t(d_t, q_t, D, G)$.

Assume that the utility derived from consumption is given by the CRRA utility function

$$u(c_t, d_t = 0) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

whereas if the individual dies at the beginning of period t, her terminal utility at t is given by evaluating the CRRA at the expected value of remaining wealth and multiplying it by a bequest motive parameter β:

$$u(d_t = 1) = \beta \cdot \frac{\left(m_t + E[\sum_{\tau=t+1}^{G} \delta^{t-\tau} z_\tau(1, q_\tau, D, G)]\right)^{1-\gamma}}{1-\gamma}$$

and is equal to zero thereafter.[10] The individual's optimal consumption problem is:

$$\max E_0 \left[\sum_{\tau=0}^{T} \delta^\tau u(c_\tau, d_\tau)\right] \quad \text{subject to:}$$

$$a_t = m_t - c_t \quad \forall t \qquad\qquad b_{t+1} = a_t \cdot R \quad \forall t$$

$$m_{t+1} = b_{t+1} + z_{t+1}(d_{t+1}, q_{t+1}, D, G) \quad \forall t \qquad\qquad a_t \geq 0 \quad \forall t$$

Where $R = 1 + r$, and r is the real interest rate, which we assume is deterministic and fixed over time - recall that offers are in real terms. We impose a no borrowing constraint, as this assumption

[10]This assumes that individuals are not risk averse about the remaining uncertainty after death. This assumption is minor, as z_τ is only random if the guarantee period has not expired and the firm is not bankrupt. The risk is bankruptcy prior to guarantee expiry, but this risk is small and most deaths occur after the guarantee period anyway.

greatly simplifies the problem from a computational perspective. In practice, insurance companies can offer loans against their annuity payments, but only do so for five year terms and at interest rates exceeding 20%, so we do not believe this assumption to be restrictive. [11]

To obtain the value of an annuity offer, which is the present discounted value of the expected utility of the optimal state-contingent consumption path, we solve this problem by backward induction using the Endogenous Gridpoint Method (EGM) (Carroll (2006)). Appendix C outlines this procedure in detail.

Valuing a programmed withdrawal offer requires solving a slightly different problem, which we present in Appendix C. The main differences are that there is no deferral or guarantee period, no bankruptcy risk for the asset, and that inheritors receive all remaining balances upon death. As for annuities, we solve this problem numerically.

4.1. Estimation

The previous subsection shows how to obtain the value of any offer given the characteristics of the contract and the individual. This subsection embeds these values into a demand estimation framework to recover the distribution of unobserved characteristics for the population.

Denote the set of individual-offer-firm observables that enter into the annuity valuation problem as X^A_{ioj}, the analogous individual-firm set for a PW offer as X^{PW}_{ij}, and individual i's unobservables - or "type" - as θ_i. We can then denote the value of an annuity offer o that firm j makes to individual i by $V^A(X^A_{ioj}, \theta_i)$, and the value of taking programmed withdrawal from PFA j as $V^{PW}(X^{PW}_{ij}, \theta_i)$.

More precisely, age and gender are observables that affect the utility calculation for both annuities and PW. For annuities, the payment amount, deferral and guarantee periods, and payments upon bankruptcy ρ_{oj} also enter into the problem; we match firm risk ratings to Fitch Ratings' 10 year Average Cumulative Default Rates for Financial Institutions in Emerging Markets for 1990-2011 and use these rates as bankruptcy probability beliefs. As for PW offers, individuals need to take into account the fee. As for types, the unobservables are risk aversion γ_i, outside wealth ω_i, bequest motive β_i, and the mortality probability vector μ_i.

We recover the joint distribution of these unobservables, $F(\theta)$, by applying the estimator

[11] It is also increasingly difficult to accumulate debt as individuals age.

developed by Fox et al. (2011) and Fox et al. (2016). To do so, we discretize the space of types and solve the optimal consumption-savings problem for every individual-offer-type combination. Then, we assume that each individual-type combination selects the highest utility offer available to them, and solve for the joint distribution of types that rationalizes observed choices. Denoting a point in the grid of types by r, we impose that the probability a individual i accepts offer o from firm j if they are of type r when faced with the set of annuity offers $\mathscr{O}_i^A$ and the set of PW offers $\mathscr{O}_i^{PW}$ is

$$s_{iojr} = \begin{cases} 1 \text{ if } V^A(X^A_{ioj}, \theta_r) \geq \max[\max_{o',j' \in \mathscr{O}_i^A} V^A(X^A_{io'j'}, \theta_r), \max_{j' \in \mathscr{O}_i^{PW}} V^{PW}(X^{PW}_{ij'}, \theta_r)] \\ 0 \text{ otherwise} \end{cases}$$

We estimate the type probabilities that minimize the distance between predicted and observed choices by solving:

$$\begin{aligned} &\min_{\pi} \sum_{i,o,j} (y_{ioj} - \sum_r s_{iojr}\pi_r)^2 \quad \text{subject to:} \\ &\pi_r \geq 0 \forall r \quad \text{and} \quad \sum_r \pi_r = 1 \end{aligned} \tag{1}$$

where $y_{ioj} = 1$ if individual i accepts offer o from firm j and 0 otherwise.

The main benefit of this approach is that after estimation one can solve the optimal consumption-savings problem for out of sample assets and predict choice probabilities and selection into assets. The two main concerns are the choice of grid, an issue we will discuss below, and the assumption that each type accepts the offer that maximizes the value obtained from the consumption-savings problem. This implies that, conditional on a contract type, the only source of heterogeneity across firms beyond the offered amount is bankruptcy probability. Thus, the model cannot rationalize a retiree selecting an offer that is not the most generous offer given a risk rating and a contract. While 19% of accepted annuity contracts are dominated, the small monetary amounts lost when accepting a dominated offer - around 1% of the offered amount - leave us unconcerned by this feature of the model.

Another implication of this assumption is that we are also assuming away the existence of non-financial utility terms that can be priced into contracts, such as tastes for firms. Again, since the main variation in contract values comes from the contract terms, and not from variation in offered amounts, we do not think that this assumption is restrictive. Furthermore, our counterfactuals of interest do not require the identification of tastes for firms.

4.2. Implementation

In order to implement the estimator, we need to restrict the mortality probability vector μ, as recovering mortality beliefs for each year between retirement and T is infeasible. To do so, we model μ as the vector from the Chilean Pension Superintendency's retiree mortality tables in place at the time of retirement plus an unobserved shifter that makes retirees effectively younger or older than their retirement age. For example, an individual who retires at 60 with a mortality shifter value of 2 has the mortality rates of a 62 year old. This allows the model to continue to feature selection on mortality without having to separately identify whether this selection comes from a higher death probability in year x or $x+1$.

The resulting type space has four dimensions: risk aversion, outside wealth, bequest motive, and mortality shifter. When solving the optimal consumption-savings problem, we impose $\delta = 0.95$, and $R = 1.03$. For programmed withdrawal, since fees are mostly identical across PFAs during the sample period[12] and we are not interested in modelling competition between them, we solve the optimal consumption-savings problem for one PW offer. Furthermore, we assume that the PW problem is non-stochastic, and set the mean PW return to its empirical counterpart.

The main challenge when constructing a grid over type space is to make it rich enough to span the support of the distribution of unobservables and to distinguish between regions of type space that make different decisions in counterfactuals, but small enough to be implementable. Instead of selecting the grid arbitrarily, we incorporate a model selection step that essentially starts with an extremely large grid and coarsens it as a function of predicted decisions in the counterfactuals of interest and in a subsample of our data. We then use this selected grid in our full sample. See Appendix D for a description of this process and for robustness checks.

Finally, we estimate the model separately for each quartile of pension balances. This allows us to impose as little restrictions as possible on how preferences change with savings.

4.3. Identification

We leverage several sources of variation to identify distributions of unobserved types, all of them conditional on pension balance quartile. The first is selection into contracts. Following the intuition

[12] All firms charge the same fee until July 2010, when an entrant begins offering a 5 bp lower fee.

from the unused observables tests, retirees with different preferences will have different rankings across contract types - even if they all received the same offers. For example, as the number of guarantee periods increases, annuity payouts always decrease. This implies that individuals with no bequest motive will always prefer contracts without guarantee periods, while as bequest motive increases retirees will value contracts with longer guarantee periods more. As another example, contracts with deferral years imply a trade-off between higher annuity payouts until death and an initial period of time without any annuity income. Only individuals who expect to live long enough to recoup this investment and who have sufficient assets outside the system to fund the initial periods will find these offers attractive. Therefore, two distributions that place different mass on different regions of type space will predict different choice probabilities across contract types. This argument is aided by the fact that retirees do not have the same set of contracts in their offer set, either due to the request stage or because of regulatory restrictions.[13] Therefore, cross-individual variation in choice sets helps identify the model.

A second source of variation comes from regulatory changes to the PW drawdown path. Regulators changed the parameters that govern the drawdown path ten times during the sample period. As a result, some retirees face steeper paths than others, which differentially affects relative valuations for the PW contract as a function of unobserved type. A third source of variation arises because firms bid at the individual level, so two individuals with the same observables who retire at different times will receive different offers due to variation in interest rates or in hedging costs.[14]

There are two main concerns with these arguments. The first is the assumption that there is no non-financial utility in the observed offers. If that is not the case, then firms may price on these non-financial terms, creating dependence between the observed offer characteristics and the error term. We believe that the empirical relevance of this concern is minor, as the amount of money lost when an individual accepts a dominated offer is small. Moreover, Castro et al. (2018) document that 96.8% of retirees accept an offer from a company that they do not have a previous relationship with.

The second concern is that the observed variation in offers across individuals is correlated with the distribution of types, by firms screening on observables. We deal with this concern by estimating separate distributions for each pension balance quartile. The remaining observables transmitted to

[13] Offers cannot fall below a minimum pension. As a result, retirees with lower pension balances may not receive bids for contracts that mechanically lower payouts, such as transitory rents, long guarantee periods, or those where a small lump sum withdrawal is taken.

[14] Insurance companies are required to document how their predicted outflows match with the expected payouts from their investments. When they are not aligned, they must deposit funds into a technical reserve account. There is heterogeneity across insurance companies over time in their degree of exposure to this risk.

firms before they formulate their offers are age, which is controlled for in the model, number of legal beneficiaries, for which there is no variation in our subsample, and contracts types requested. This last variable merits further discussion, as one could be concerned about information revelation in the request stage. That is, if individuals with different expected costs request to hear offers for different contract types, then firms could price based on the request phase, creating correlation between the observed offers and the unobserved types.

To check whether this concern is empirically relevant, we take the most commonly requested contract - a “0-0” contract with no guarantee and no deferral period, which is requested by more than 90% of retirees - and study whether offer generosity varies as a function of whether the retiree also requests a contract with a guarantee period or a contract with a deferral period. If there was information revelation in the request stage, then requesting a deferral period would reveal that the individual expects to be long-lived, lowering the generosity of the “0-0” contract. On the converse, requesting a guarantee period contract would reveal that the individual expects to have a non-trivial probability of dying within the guarantee period, and that they care about bequests. Such a person is cheaper to serve and values PW more than the average retiree, so we would expect “0-0” contracts to be more generous. To implement this test, we regress offered amounts on three request dummies - request a deferral period contract, request a guarantee period contract, and the interaction - while controlling flexibly for pension balances and the full interaction of retirement month-year, age and gender fixed effects. Results from this exercise are in Table A.1 in Appendix A. We do not find statistically significant information revelation effects for requesting a guarantee period. We do find significant effects for requesting a deferral period, but they go in the opposite direction to the information revelation story: individuals who request deferral periods receive more generous “0-0” contracts. Overall, we do not find evidence supporting the hypothesis that information revelation in the request stage is a meaningful force in this setting.[15]

5. Estimation Results

To begin, Table 3 presents main features of the estimated type distribution for women in the second quartile of the distribution of pension balances. Results for other quartiles are reported in Appendix A. Additionally, Figure A.5 plots marginal distributions for each dimension of unobserved

[15] If it were a meaningful force, the method would not be invalidated - one could estimate the model conditional on the requested set of contracts.

Panel A: CDF Summary					
Mass Cutoff		1.00E-01	1.00E-02	1.00E-03	1.00E-04
Number of Points with Mass Greater than Cutoff		1	29	36	36
Total Mass for these Points		21.57%	95.92%	100.00%	100.00%
Panel B: Top 10 Mass Points					
Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass	95% CI
1 7.68	0.000	12.575	15	21.57%	(20.13%, 23.01%)
2 44.6	0.000	8.862	-7	5.20%	(-13.60%, 24.00%)
3 0.414	0.000	11.338	15	4.32%	(3.06%, 5.58%)
4 7.89E+03	1.875	16.288	15	4.19%	(2.23%, 6.15%)
5 621	3.125	0.200	-15	3.89%	(2.64%, 5.15%)
6 7.89E+03	5.000	0.200	3	3.71%	(2.43%, 4.98%)
7 7.89E+03	4.375	0.200	-7	3.51%	(2.25%, 4.78%)
8 7.89E+03	5.000	0.200	7	3.42%	(2.17%, 4.66%)
9 621	0.625	20.000	1	3.31%	(1.92%, 4.69%)
10 621	3.750	0.200	-3	3.16%	(1.40%, 4.93%)

Notes: Panel A reports the number of points whose estimated mass is above each cutoff and their total mass. Panel B reports the ten points with the highest estimated masses, their mass estimate, and 95% confidence regions. Confidence regions are obtained by clustering standard errors at the individual level.

Table 3: Descriptive Statistics for Estimated Type Distribution - Second Quartile Women

type. In what follows, we report standard errors that are clustered at the individual level.[16]

The estimated type distribution is disperse, with only one point with mass greater than 10%, 29 points with mass greater than 1%, and 36 points with mass greater than 0.1%. Only considering the 36 points with mass greater than 0.01% results in almost perfect coverage (99.9997%) of the full type distribution. The distribution of the health age shifter is heterogenous, with 53.9% of retirees exhibiting higher death probabilities than those in the Chilean authorities' table. There is significant mass assigned to the health shifter value of 15, which corresponds to a life expectancy of 75 years. As for bequest motives, 10% of retirees behave as if they assign no value to leaving money to their heirs, but there is also significant mass at the largest values of the grid. The distribution of outside wealth has a large mass point at the lowest value (US$ 8,170), consistent with survey evidence (Comisión Asesora Presidencial Sobre el Sistema de Pensiones (2015)) that for many retirees pension savings are their lone asset for funding consumption after retirement. There is also substantial mass at the highest points in the grid. This is reasonable, as this object is meant to capture the value of all assets that can fund consumption and inheritance and our sample is restricted to individuals who can fund an annuity offer above the minimum pension. Finally, the marginal distribution of risk aversion has large mass at $\gamma = 0$, which corresponds to risk neutrality, and most

[16]These standard errors are conservative, as they do not take into account that the true parameter cannot be negative - see Fox et al. (2011).

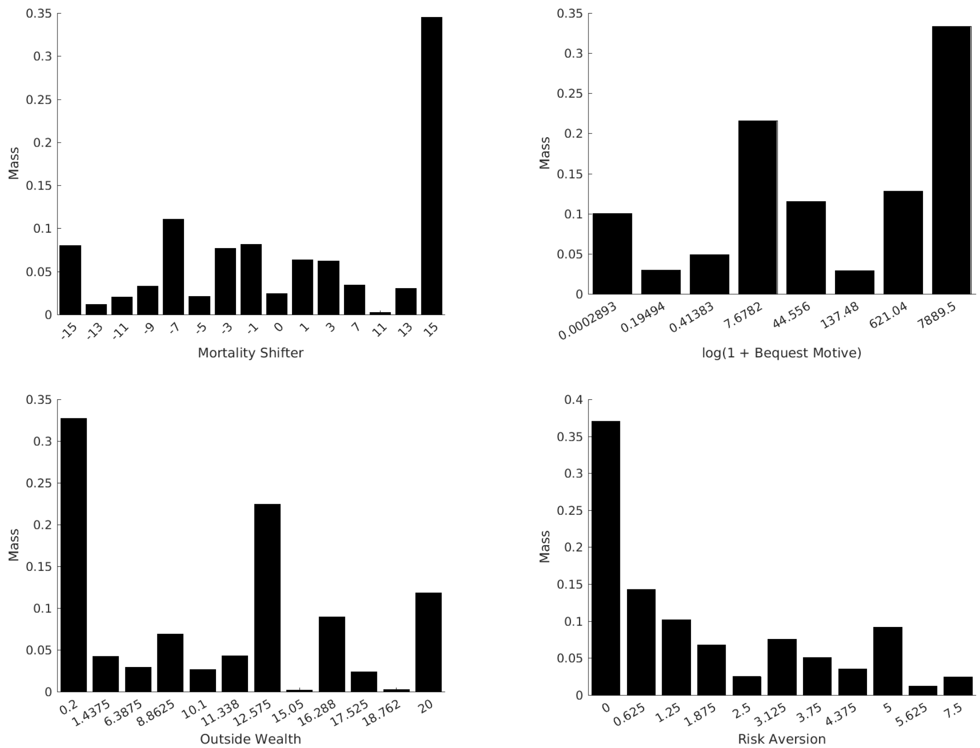


Figure 2: Marginal Distributions - Second Quartile Women

mass below $\gamma = 3$. Note that risk neutral types also have high outside wealth - these are individuals for whom pension balances are a small fraction of their wealth, and for whom the risks embedded into the pension system are small. The mean of the distribution of γ is 1.70.

As argued previously, the relationships across dimensions of unobserved preference can play an important role in shaping selection into annuitization. Figure 3 reports these relationships through heat maps for the joint distribution of mortality shifter and the other dimensions of unobserved type. We observe a wide range of life expectancies for high bequest values, which is particularly interesting because long lived types with high bequest motives may opt out of annuitizing, reducing the cost to serve the annuitant pool. We also find a more standard subgroup of high mortality low bequest types that are likely to always prefer the outside option to annuitization. Turning our attention to risk aversion, there is an interesting subgroup of middle to short lived, high risk aversion types that may annuitize if given a sufficiently attractive offer. These types create regions of advantageous selection when offers are high enough to induce them to purchase insurance. Finally, we find that the group of types with low values of outside wealth mostly spans the possible values of the distribution of mortality. Retirees with low outside wealth and middle to high levels of mortality risk will still value annuitization, as they are exposed to a large risk. Again, the existence of these types may mitigate adverse selection. Finally, these heat maps point to the fact that modelling unobserved preferences using parametric distributions, such as a joint normal, will lead to estimates that average out across these sub-populations, missing important richness.

Table 4 presents measures of in sample and out of sample fit by pension balance quartile.[17] The model does well in fitting the annuitization rate and the probability that an individual is dead by two years after retirement. As we are not using mortality data in estimation, this last result reassures us that the model is recovering reasonable estimates of the distribution of unobserved types. A finer breakdown of in sample fit is presented in Table A.4.

[17] Quartiles defined over the whole sample, not within gender.

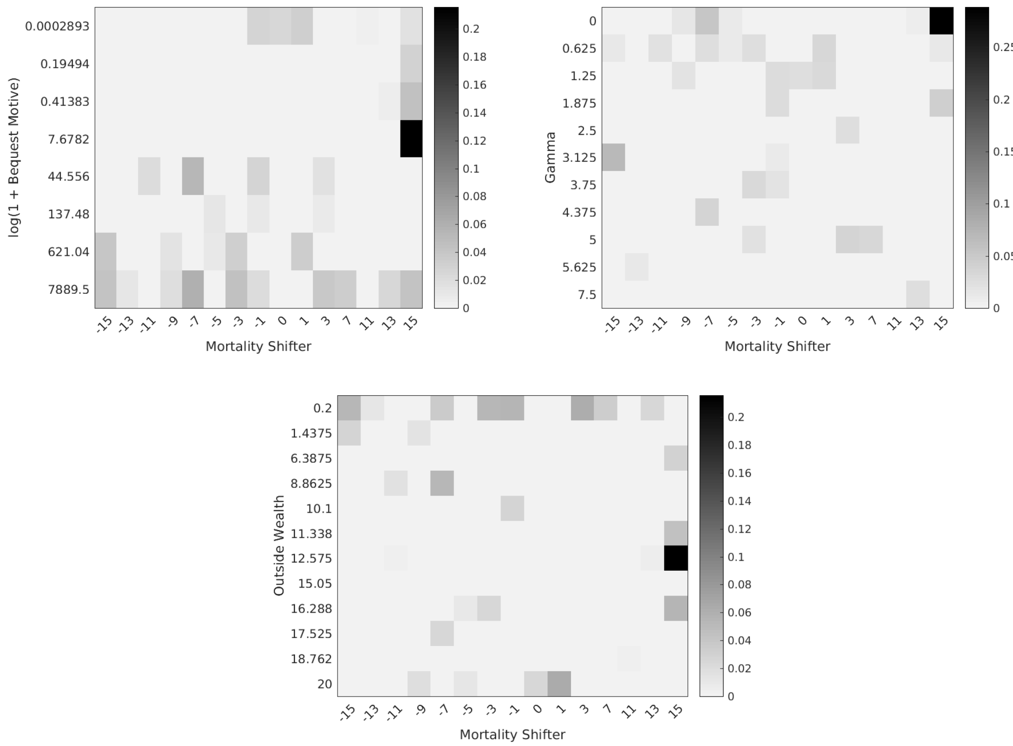


Figure 3: Heat Maps for Joint Distributions

Wealth Quartile	First	Second	Third	Fourth
Fraction Annuitized				
Observed	67.93%	76.51%	75.30%	65.80%
Predicted	58.20%	70.34%	71.34%	64.79%
Two-Year Mortality				
Observed	1.77%	1.70%	1.74%	1.99%
Predicted	1.55%	1.71%	1.39%	1.23%
Number of Offers	263,638	419,381	593,671	428,247
Number of Individuals	9,083	9,180	10,697	6,585
MSE	0.03	0.02	0.02	0.01
R^2	0.32	0.23	0.23	0.26

Table 4: Fit

6. Welfare Effects of Counterfactual Pension Reforms

6.1. Calculating Private Market Annuity Demand, Equilibria, and Welfare

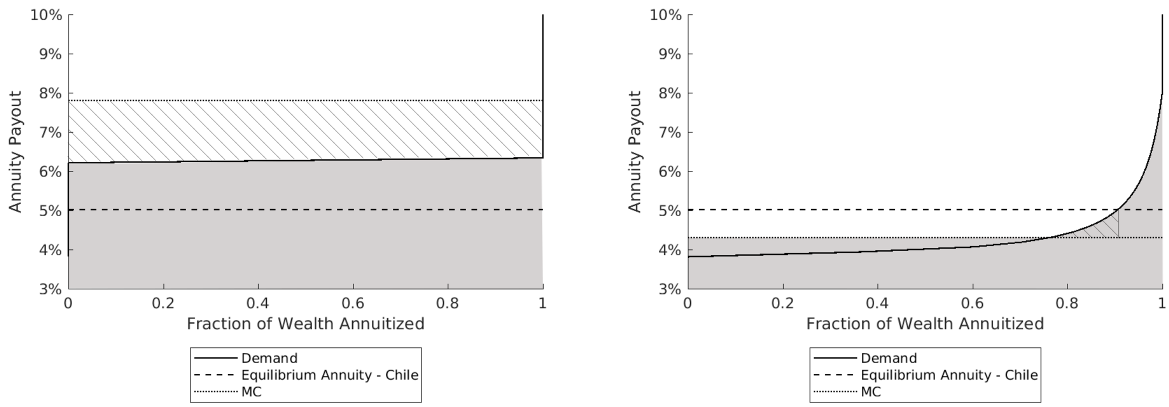
Combining the above estimates with the consumption-savings model introduced in Section 4 allows us to calculate an aggregate annuity demand function, to compute private market annuity equilibria and to calculate welfare under different pension systems. In this subsection, we formalize this procedure.

Denote a pension balance quartile by q and a particular type by i. In order to model annuity demand under different pension systems, let oo denote the alternative to annuitization that is available to retirees, or the outside option, and let $m \in [0,1]$ denote the fraction of pension balances that mandated into annuitization. Let $a_i^q(z,m,oo) \in [0,1-m]$ denote the fraction of wealth type i from quartile q assigns to an annuity of generosity z when a fraction m of their pension savings that are mandated into annuitization and the outside option is oo - this is their demand for annuitization given a system and a generosity.

Computationally, we solve for $a_i^q(z,m,oo)$ by selecting a grid of annuity offers $Z = [\underline{z},...,\bar{z}]$ and, for each $z \in Z$ and type, finding the allocation of wealth between the annuity and the outside option oo that maximizes utility. After calculating these objects, we check whether $a_i^q(\underline{z},m,oo) = 0$ and $a_i^q(\bar{z},m,oo) = 1-m$ for all i, and, if not, expand Z until these conditions hold.

Figure 4 reports two example demand functions, obtained using PW as the outside option.[18] Figure 4a presents demand for type 3 in Table 3, which has low life expectancy and low bequest motives. Demand traces out the amount of wealth that is annuitized over annuity offer generosities - the amount of wealth annuitized that leaves a retiree indifferent between allocating the marginal dollar to the annuity or to the outside option. We also plot the type-specific break even annuity ("MC"), which is the marginal cost per dollar-year of providing an annuity to this type, and the annuity that is offered in equilibrium. This equilibrium annuity corresponds to average cost pricing for the annuitant pool. We present the details on how this equilibrium offer is calculated below. The intersection of demand and MC yields the socially efficient annuitization level, whereas the intersection of demand and the equilibrium annuity corresponds to the actual outcome. This type keeps all wealth in PW, despite the socially efficient outcome being full annuitization. The grey area under the demand curve corresponds to consumer surplus ("CS"), and the striped area between

[18] Without a minimum pension guarantee.



(a) Type 3

(b) Type 5

Figure 4: Demand and welfare for example types

MC and demand corresponds to deadweight loss from adverse selection.

Figure 4b presents the same analysis for type 5. This type annuitizes roughly 90% of their wealth in equilibrium, while the first best is around 75%. Note that for this type the equilibrium annuity payout is much more generous than the type-fair annuity payout. This type receives a transfer from selection into annuitization. In this case, deadweight loss corresponds to the difference between demand and MC for annuitized wealth above the socially efficient level. This type has medium bequest motives, high risk aversion, and low outside wealth. As a result, they have a high valuation for keeping some wealth in programmed withdrawal, as full annuitization implies running the risk of dying early and leaving little money to heirs. We name types that exhibit this behavior "savers", as their bequest motives imply that they keep a balance of liquid assets on hand at all times in order to eliminate the risk of dying without leaving bequests.

Consumer surplus values for each of these types are 6.27% and 5.10% per year, respectively. That is, type 3's equilibrium utility is equivalent to the utility from being fully annuitized at a rate of 6.27% per year. Since lifespans are heterogenous, we multiply these rates by their corresponding type's expected present discounted value of an annuitized dollar. This yields a measure of CS in terms of expected annuity payout per dollar annuitized. In this space, consumer surplus and welfare from receiving the MC annuity is 1 for all types. Consumer surplus from being offered the equilibrium annuity is 0.80 for the left-side type and 1.18 for the right-side. Choice is most valuable for savers, like type 5, who prefer the Chilean equilibrium even above receiving the actuarially fair annuity for their own life expectancy. Type 3, on the other hand, suffers welfare losses because adverse selection leads to the equilibrium annuity falling below the type-fair annuity.

There are also differences in deadweight loss (DWL) across these types. Type 3 has DWL of 19.7%, as they have low MC yet the equilibrium offer precludes annuitization. In contrast, type 5 has DWL of .9%, as the equilibrium outcome is close to the social planner's optimum. Finally, note that consumer surplus and welfare are equal for type 3, but not for type 5. In the former case, there is no discrepancy between private and social values of the selected product, as the type is allocating all their wealth to PW. In the latter, welfare is lower than CS as society values an annuitized dollar at its marginal cost while the type values it at the offered amount.

Having shown how to compute demand and welfare for each type, we turn to calculating equilibria. First, we use the estimated type probabilities to construct aggregate demand from type-specific demand. Second, given an annuity offer rate, demand also allows us to characterize the amount of wealth each type annuitizes. We combine this with expected mortality at the type level to calculate the expected mortality of the annuitant pool. Then, we calculate the the break-even offer for the annuitant pool, or the average cost curve. The intersection of aggregate demand and average cost yields the equilibrium annuitization rate and annuity payout. More precisely, given an offer $z \in Z$, aggregate demand for quartile q is

$$a^q(z,m,oo) = \sum_i a_i^q(z,m,oo) \cdot \hat{\pi}_i^q,$$

where $\hat{\pi}_i^q$ is type i's estimated weight in quartile q. For the quantity $a^q(z,m,oo)$, average cost is

$$c^q(a^q(z),m,oo) = \frac{\sum_i a_i^q(z,m,oo) \cdot \hat{\pi}_i^q \cdot c_i^q}{a^q(z,m,oo)},$$

where c_i^q is the expected cost of giving a one dollar annuity to type i from quartile q - or the type fair annuity for type i. Finally, $c_{af}^q = \sum_i \hat{\pi}_i^q \cdot c_i^q$ is the actuarially fair annuity for the population. Computationally, we calculate quartile-by-quartile equilibria by finding all the $z_c \in Z$ such that $z_c \geq c^q(a^q(z_c),m,oo)$ and $z_{c-1} \leq c^q(a^q(z_{c-1}),m,oo)$. We then use bisection over $[z_{c-1}, z_c]$ to find the $\hat{z}^q(m,oo)$ such that $\hat{z}^q(m,oo) = c^q(a^q(\hat{z}^q),m,oo)$. This an equilibrium annuity rate. Under all scenarios in this paper, this value is unique.

Figure 5 plots demand, average cost, and the actuarially fair annuity for the population of women retiring with a balance amount equal to the second quartile of the distribution of pension balances.

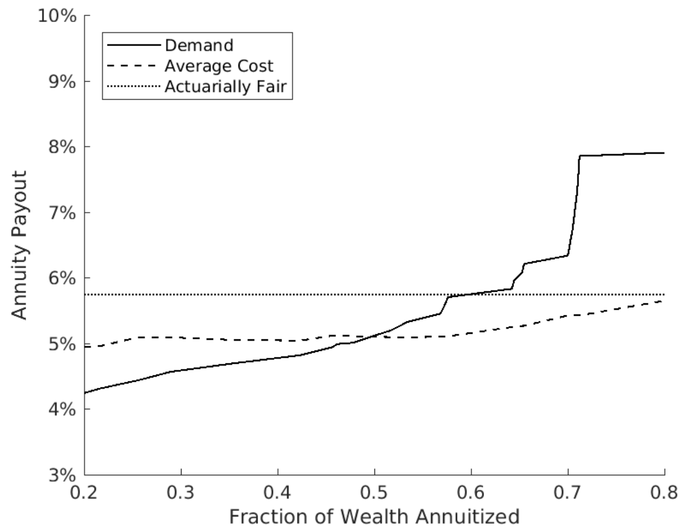


Figure 5: Equilibrium - Second Quartile Women

In this figure, these women are choosing between a simple, immediate annuity and PW. We find a 47.7% annuitization rate at an equilibrium offer of 5% per year. Note also that while our average cost curve is upward sloping, consistent with adverse selection,[19] it is not monotonically increasing. In fact, it is decreasing for several values below the equilibrium. This stems from the existence of individuals with low mortality levels who nonetheless value annuitization.

This equilibrium assumes perfect competition and abstracts away from multiple contract types. We will maintain these assumptions throughout the rest of the paper, to focus on how pension system design affects selection into annuitization.[20] Finally, this equilibrium is representative of a quartile value of pension balances. Equilibria for other quartiles are presented in Appendix A. In what follows, we report average measures of welfare and consumer surplus across pension balance quartiles to reflect system-wide outcomes.

We compute deadweight loss (DWL) in each system and for each quartile by comparing its level of welfare to welfare under the first-best allocation when the outside option is lump sum withdrawal. This allows us to compare systems taking into account welfare loss from both adverse selection and from the drawdown constraints imposed by PW. Welfare under a particular equilibrium for type i in

[19] The break-even offer is lower for low annuitization levels, because longer lived individuals annuitize at lower payouts

[20] We also remove the minimum pension guarantee from PW, as only a fraction of the first quartile women in our sample qualify for it and caveating results by whether they have the MPG or not distracts from the main focus of the paper. This also has the benefit of maintaining a fully funded system both in the baseline and in counterfactuals.

quartile q is

$$W_i^q(m,oo) = \frac{1}{c_i^q}\left((m+a_i^q(\hat{z}^q(m,oo),m,oo))\cdot c_i^q + \int_{m+a_i^q(\hat{z}^q(m,oo),m,oo)}^{1} z_i^q(a,m,oo)\,da\right)$$

where $z_i^q(a,m,oo)$ is the inverse function of $a_i^q(z,m,oo)$ - it outputs the annuity payout that is consistent with a particular fraction of wealth annuitized. The first best allocation is obtained by offering each type choice between lump sum withdrawal and their type-fair annuity, as if types were observable. Denote the fraction of wealth type i annuitizes when given this choice by $a_i^q(c_i^q,0,LS)$. Welfare under the first-best allocation is then

$$W_i^{FB,q}(LS) = \frac{1}{c_i^q}\left(a_i^q(c_i^q,0,LS)\cdot c_i^q + \int_{a_i^q(c_i^q,0,LS)}^{1} z_i^q(a,0,LS)\,da\right).$$

Deadweight loss for a system with mandatory annuitization level m and outside option oo is then $W_i^{FB,q}(LS) - W_i^q(m,oo)$. Thus, deadweight loss arises when a type is over or under annuitized relative to the social optimum, and its magnitude is governed by the difference between the value of the outside option and the social value of annuitization.

We also calculate type i of quartile q's consumer surplus under a system with mandatory annuitization level m and outside option oo as

$$CS_i^q = \frac{1}{c_i^q}\left(m\cdot z_{af}^q + a_i^q(\hat{z}^q(m,oo),m,oo)\cdot \hat{z}^q + \int_{m+a_i^q(\hat{z}^q(m,oo),m,oo)}^{1} z_i^q(a,m,oo)\,da\right).$$

Aggregate DWL and CS are obtained by taking weighted averages across the different types.

These equations highlight that the pension system that generates the highest equilibrium annuitization rate $a_i^q(\hat{z}^q(m,oo),m,oo)$ may not generate the highest aggregate consumer surplus for retirees or social welfare, as the outside option to annuitization also provides value. Types such as savers, for instance, prefer to allocate non-zero fractions of their wealth to both annuities and to alternatives that provide higher liquidity and bequest value.

6.2. Welfare under the Chilean Baseline

Panel A of Table 5 reports equilibrium fractions of wealth annuitized ("Fraction Annuitized") and yearly annuity payouts ("Annuity Rate") under different pension system designs, beginning

		System		
	Baseline	Lump Sum + 50% Mandate	Full Mandate	Lump Sum
	(1)	(2)	(3)	(4)
Panel A: Equilibrium Summary				
Fraction Annuitized	50.72%	18.18%	100.00%	38.78%
Annuity Rate	5.09%	4.99%	5.78%	5.03%
Panel B: Consumer Surplus and Deadweight Loss (per dollar of pension savings)				
Average	1.27	1.11	1.00	1.30
10th Percentile	0.83	0.81	0.73	0.97
Median	1.07	1.08	1.04	1.10
90th Percentile	2.16	1.44	1.27	2.19
Highest Longevity Risk	1.18	1.25	1.34	1.17
Lowest Longevity Risk	0.97	0.86	0.74	1.03
DWL	0.05	0.21	0.32	0.02

Table 5: Outcomes under Alternative Pension Systems

with the Chilean baseline. For the baseline Chilean system shown in Column 1, we find that the average annuitization rate across pension balance quartiles is 50% and the average annuity offer is 5.09%. The Chilean exception to the annuity puzzle persists in this stylized model of the system.

In Column 1 of Panel B in Table 5, we summarize welfare estimates under the Chilean baseline. We find that DWL in the Chilean system is low, 5 cents for every dollar of pension savings. This result is surprising, as voluntary participation in insurance markets is typically thought of as creating inefficiencies due to adverse selection. We find that selection on life expectancy is mitigated by selection on other dimensions of preference, to the point that the Chilean system has nearly zero deadweight loss.

This low level of deadweight loss is driven by subpopulations of retirees whose privately optimal choices attain largely the same welfare as their socially optimal choices. Our estimates show that three such types exist in significant numbers. First are "never annuitants", who do not annuitize at either the first best (marginal cost) generosity or at the equilibrium generosity. Second are types for whom demand is both elastic and at a similar level to their marginal cost. Because their demand is elastic, these types switch rapidly from not placing any wealth in an annuity to placing all their wealth in one. And since the level at which they do so is close to their marginal cost, these types generate little DWL under the voluntary system - even though the fractions of wealth annuitized may be drastically different. Finally, there are "always annuitants", types that fully annuitize both in the private equilibrium and in the first best. Example demand functions for each of these types are plotted in Appendix Figure A.8. These populations outweigh the types who generate deadweight loss due to over- or under-annuitization, such as the types plotted above in Figure 4.

Average consumer surplus in this system is 1.27. Recall that each type would receive CS of 1 if they were given their type-fair annuity. If retirees picked financial instruments solely as a function of their net present value, this could not be the case - one cannot outperform the type-fair annuity by adding an option over another budget-neutral financial product. However, our utility model allows for types such as Type 5 (Panel (b) of Figure 4), who would not fully annuitize even if they were offered their type-fair annuity. Therefore, affording a choice between annuitization and an alternative provides value to some consumers. In fact, the 90th percentile receives more than twice the CS as they would achieve from their type-fair annuity. However, by allowing for adverse selection into annuitization some types are worse off: the 10th percentile of CS is 17 cents per dollar below the value of receiving the type-fair annuity. This dispersion highlights the heterogeneous impact of Chile's voluntary system - some subpopulations gain significantly more welfare than others.

6.3. Equilibrium and Welfare under Alternative Pension Systems

The Chilean pension system features two major departures relative to the pension systems studied in the annuity puzzle literature: the lack of a mandatory social insurance component and the use of PW rather than lump sum withdrawal as the alternative to annuitization. Because our demand system is founded on a consumption-savings problem, we are able to solve for annuity demand, equilibrium, and welfare when these features are modified. First, we show the effect of introducing a mandate into the Chilean system. Then, we show the effect of redesigning the outside option from programmed withdrawal to lump sum withdrawal. Finally, we introduce both a partial mandate and a redesigned outside option. This combination approximates the features of many other pension systems worldwide, including the US Social Security system.

6.3.1. Annuity Mandate

When imposing a full mandate, all pension wealth is annuitized at the actuarially fair amount. This is akin to the UK's former compulsory annuity scheme, Singapore's mandatory annuity policy, and mandatory defined benefit pensions historically offered by public and private employers in the US. Column 2 of Table 5 presents outcomes under this system. Our results show that offered payout generosity increases in this case by 69 bp/year relative to the baseline. This payout reflects that the annuitant population in the Chilean baseline is adversely selected on mortality risk. The mandate gets rid of adverse selection, improving generosity.

Despite this, average CS per dollar is 27% lower than in the baseline, and total DWL is 32 cents per dollar of pension savings, significantly higher than the baseline of 5 cents per dollar. Both metrics show that a fully mandatory pension system causes significant welfare loss to retirees. This loss is largely driven by "never annuitants" and "savers", who highly value the choice afforded by the voluntary baseline system. The full wealth of both of these types would not be annuitized under the social optimum, but is forced into annuitization by the mandate.

Moreover, the distribution of CS shifts downwards, as shown by the changes in the 10th, 50th, and 90th percentile. The vast majority of retirees prefer the baseline system to a full mandate. One population is an exception to this rule - retirees with private information about high longevity risk. These longest lived types gain significantly more welfare from the mandate because the full mandate provides a more generous payout.

6.3.2. Lump Sum Withdrawal

Allowing for lump sum withdrawal requires solving for each type's optimal choice when a counterfactual financial product is introduced as the outside option. Under this reform, each type's annuity demand weakly contracts relative to the baseline, as one can always replicate the PW consumption path under lump sum withdrawal. As a result, both the aggregate demand function and the average cost curve shift, leading to a new equilibrium.

Column 4 of Table 5 shows the new equilibrium and welfare under the redesigned system. The equilibrium fraction of wealth annuitized is now more than 10pp less than under the baseline system, reflecting the value added to retirees when they can access their entire savings rather than a constrained fraction each year. The relatively high value of the outside option also slightly decreases the generosity of the equilibrium annuity. This is due to an increase in adverse selection - retirees willing to turn down a large lump sum in favor of an annuity face higher longevity risk and cost more to insure.

Despite this, there is a small increase in the average welfare of retirees, by about 3 cents per dollar. Behind this improvement are two countervailing forces. Retirees who value annuitization, such as "always annuitants", lose welfare under the reform due to lower annuity generosity. Retirees who value choice, such as "never annuitants" and "savers," gain some welfare from the improved value of the outside option.

Policymakers are often concerned that allowing for unconstrained access to pension funds can lead to some retirees wasting their savings, and then coming back to the government for support late in life - and our consumption-savings model does not include this possibility. We find that the differences in consumer surplus and DWL between a system with lump sum withdrawal and the baseline are small, and may justify the use of PW to minimize government spending to support end of life expenses.

6.3.3. The Chilean Exception to the Annuity Puzzle and Its Welfare Implications

Having introduced the two key differences between the Chilean pension system and pension systems that mix mandatory social insurance with a voluntary component, we now combine them by introducing a partial mandate covering 50% of wealth.[21] That is, we remove half of every types' pension savings and returning it back to them in the form of an actuarially fair annuity. The remainder of wealth is freely allocated between a private market annuity and lump sum withdrawal. To solve for the equilibrium in the private annuity market, we find each type's annuity demand function following the procedure outlined above. Note that this demand takes into account the guaranteed income stream from the mandatory annuity component.

Column 4 in Table 5 presents the equilibrium obtained under this system. We find an equilibrium annuitization rate of 10% and an annuity payout of 4.87% - more than 20bp/year lower than in the baseline. This finding largely replicates the low transaction volumes and low equilibrium annuity generosity documented in the annuity puzzle literature (Mitchell et al. (1999)) solely by reforming the system to incorporate mandatory social insurance and lump sum withdrawal. These results show that the main driver of the Chilean exception to the annuity puzzle is not preferences, but the design of the pension system itself.

Additionally, Column 2 of Panel B of Table 5 reports DWL and CS when both reforms are enacted. We find that DWL is 19 cents per dollar of pension balances. This DWL comes mostly from the mandatory annuitization of wealth that is not annuitized in the social optimum. For example, that of never-annuitants. Aligned with this finding, we also find that average CS falls relative to the baseline. Note that this drop is driven by a decrease in the right tail of the CS distribution - relative to the baseline, 10th percentile and median CS are similar under both systems, while the

[21] We choose 50% following Mitchell et al. (1999). Below, we explore other fractions.

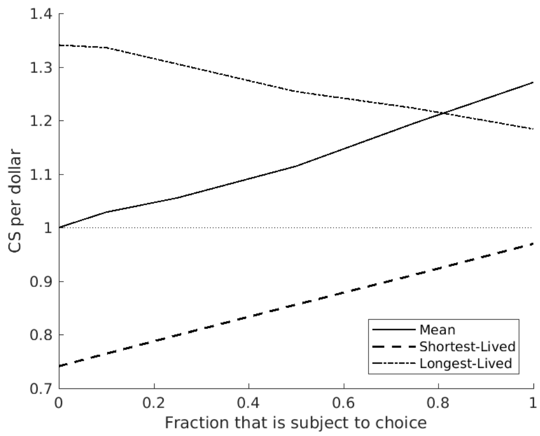
90th percentile is dramatically lower. In the baseline, this right tail is not composed of the types with highest or lowest longevity risk, who have average CS of 1.18 and 0.97, respectively. Instead, it consists of types who have strong preferences for the outside option, including "savers" and "never annuitants". Forcing half their assets to be annuitized makes them significantly worse off. However, the highest longevity risk types are better off under this system - they are likely to annuitize some or all of their wealth, so converting 50% of their assets to the actuarially fair annuity increases their payouts relative to the baseline.

6.4. Implications for Pension System Design

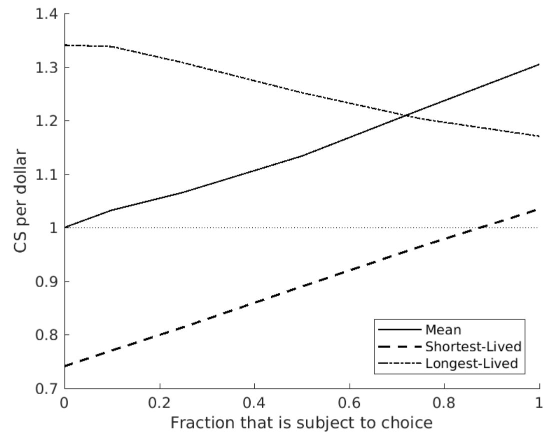
The results in Table 5 show that allowing retirees unrestricted choice maximizes average welfare. However, other systems that restrict choices, particularly a full mandate, provide higher welfare for those with highest longevity risk. Thus, allowing for unrestricted choice may lead to a system with insufficient insurance value for the retirees facing the highest risk of outliving their savings. In this section, we study whether these results generalize as one changes the coverage of mandatory social insurance. This will allow us to determine whether there is a trade-off between average welfare and insurance value.

Figure 6 plots average consumer surplus per dollar of pension savings for the whole population, for those with lowest longevity risk (shortest-lived), and for the highest longevity (longest-lived), as a function of the fraction of wealth that is not placed in mandatory social insurance - the fraction of wealth that is subject to choice. When this fraction is zero, all wealth is placed in an actuarially fair annuity, while when the fraction is one, retirees can freely choose between the outside option and annuitization. We construct the plot by calculating equilibrium annuity offers and annuitization rates when 0%, 25%, 50%, 75%, 90% and 100% of wealth in placed in an actuarially fair annuity. Panel a) reports these consumer surplus when the alternative to annuitization is PW, and Panel b) when it is lump sum withdrawal.

Regardless of the outside option, we find that average CS and CS of the low longevity risk types are monotonically increasing in the fraction of wealth that is subject to choice, while CS for the high longevity risk types is monotonically decreasing. That is, there is a trade-off between these two quantities. Regulators whose only concern is longevity risk should adopt mandatory annuitization of all pension balances, like in the UK's former compulsory annuity scheme, Singapore's mandatory annuity policy, and mandatory defined benefit pensions historically offered by public and private



(a) PW as the outside option



(b) Type 5

Figure 6: CS under partial mandates

employers in the US.

Figure 6 also shows that losses to the high longevity risk types are minimal when allowing for 10% of funds to be subject to choice, while average CS and CS of the shortest lived increases. Therefore, allowing for a small degree of choice increases average welfare without significantly damaging the insurance value of the system.

Allowing for choice beyond 10% of pension balances begins to trade off the welfare of the highest risk types with average welfare. Here, these plots show that given a level of wealth that is freely allocated, long lived individuals are basically indifferent between lump sum and programmed withdrawal, while average CS and CS for the shortest lived are higher under the former. These results suggest that regulators that are interested maximizing the value of the system given a level of protection for the high risk types should combine mandatory annuitization of a fraction of wealth with choice between lump sum withdrawal and private market annuitization for the remainder.

6.5. Discussion

There are three main takeaways from these results. First, voluntary participation in longevity insurance, as implemented in the Chilean system, provides relatively high average welfare to retirees with low deadweight loss, despite this being a selection market. This is because adverse selection is mitigated by selection on non-cost dimensions of preference, such as risk aversion, bequest motives, and outside wealth. Second, one can rationalize the Chilean exception to the annuity puzzle through the differences in institutions between Chile and the developed world. Introducing mandatory social insurance and lump sum withdrawal, two common features in the settings where the annuity puzzle has been documented, leads to low voluntary annuitization levels. And third, designing pension systems requires regulators to trade off provision of longevity insurance with retirees' utility from leaving bequests or increased liquidity at retirement. We quantify this trade-off and show that partial mandate policies can balance the needs of high risk types with the remainder of the population.

These results also have implications for managers of pension systems with unfunded liabilities. To meet the requirement to maintain payout streams despite underfunding, managers have resorted to strategies such as freezing cost of living adjustments (Fitzpatrick and Goda (2020)) and investing in riskier assets (Myers (2020)). Our results lend empirical backing to another common solution: offering retirees choice between the promised payout stream and a lump sum that is worth less than the NPV of the promise. The typical concern with this solution is adverse selection - if only the short lived take out the lump sum, the funding issue will be exacerbated. However, we find that short-lived retirees will not be the only groups to accept the offer - for example, savers are also likely to. As a result, these offers can maintain or reduce liabilities while increasing welfare.

There are three important caveats to our analysis. First, our analysis focuses on decisions made at retirement, while pension balances are the product of decisions made over a lifetime. It is possible that the reforms discussed above could change incentives to save for retirement. While we do not model this margin, we do not believe this to be a major concern, for two reasons. Modifying past contributions is not possible, so this criticism does not apply for individuals who are close to retirement. Additionally, the majority of retirement savings in the Chilean pension system consists of mandatory contributions, and one would have to drop out of employment in the formal sector to reduce them.

Second, as discussed earlier, we do not model the possibility that retirees who do not annuitize spend down all their savings and then rely on the government for support. In such a setting, a fully voluntary system with lump sum withdrawal as the outside option may be particularly risky from a policy perspective. As argued above, making PW the outside option provides some protection against this risk, at minimal welfare cost. Partial mandates would also shield governments against this concern.

Finally, in constructing the average cost curve, we are assuming that the retiree mortality beliefs derived from demand estimation are on average correct. However, recent work (O'Dea and Sturrock

(2020)) highlights the role of pessimistic survival beliefs in contracting annuity demand. Note that our distribution of mortality shifter is in fact centered to the right of zero, suggesting that retirees are on average more pessimistic about their life expectancy than the Chilean Pension Superintendency's mortality tables. If these beliefs are too pessimistic, the average cost curve would be higher than what we simulate, and annuitization would be lower. This bias, as well as any other bias that contracts annuity demand, would bolster the argument for systems with partial or total mandatory annuitization.

7. Conclusion

Chile's voluntary pension system provides a stark counterexample to the annuity puzzle documented across the world's pension systems. We introduce a model of private annuity equilibrium that explicitly models pension policy to study the drivers of these high voluntary annuitization rates. By modeling and estimating the primitive preferences underlying demand, we show that preferences alone do not explain the Chilean exception to the annuity puzzle. Instead, the choice of pension policy levers, including the level of mandatory annuitization and the government's design of the outside option to the private annuity market, drive both private market equilibria and retiree welfare.

Our model also allows us to compare retiree welfare across groups with different private preferences, both under the Chilean baseline pension policy and under alternative policy reforms. Our results show that retirees select into annuities based on private information about mortality, as well as their private preferences over bequests to their heirs, and other dimensions of heterogeneity. As a result, welfare loss due to adverse selection is low in the fully voluntary system under study. Introducing mandatory annuitization harms retirees with strong preferences for bequests, but benefits retirees at the highest risk of outliving their savings. We show that policymakers face a trade-off between the insurance value of the pension system for retirees facing the highest longevity risk and average welfare. A fully voluntary system maximizes average retiree welfare, but a partially mandatory system with a voluntary component provides better insurance value while preserving the benefit of flexibility to satisfy heterogeneous retiree preferences.

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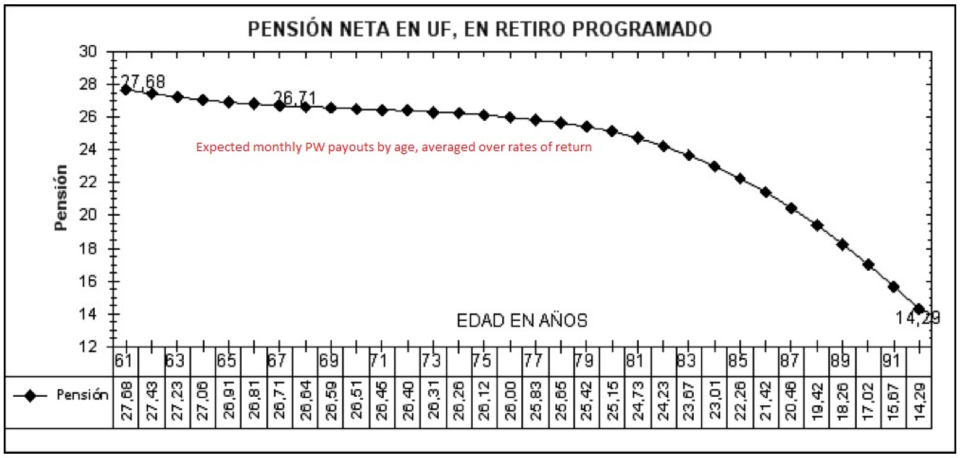
A. Additional Tables and Figures

Figure A.1: Sample printout of programmed withdrawal information conveyed to retiree



Monto Pensión mensual promedio: 24,18 UF

Monto Comisión mensual promedio: 0,30 UF

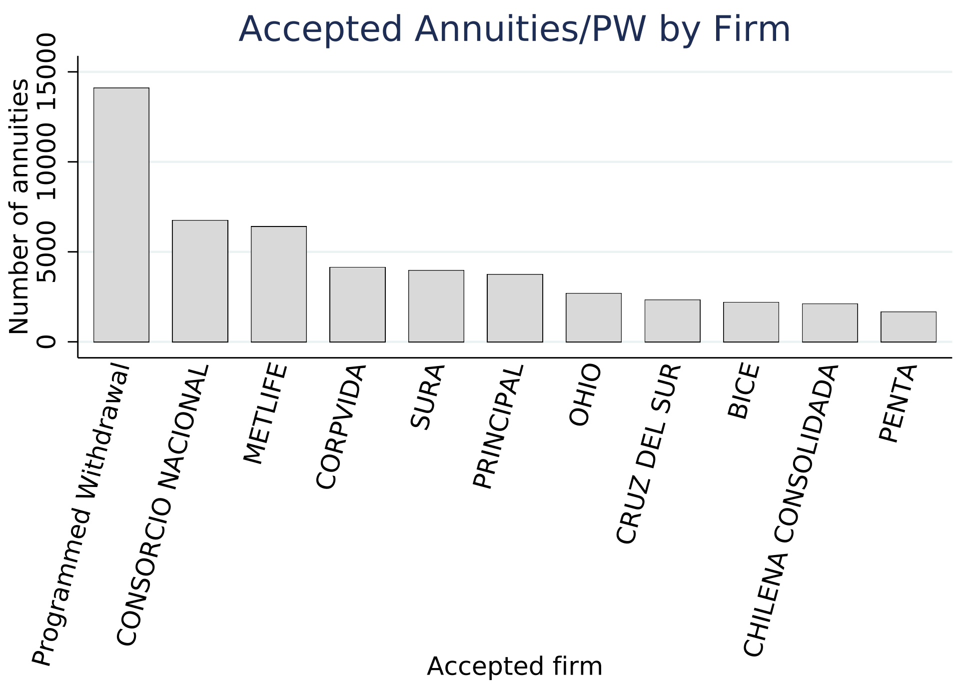
Figure A.2: Sample printout of annuity offers for one contract type

MODALIDAD RENTA VITALICIA INMEDIATA

RENTA VITALICIA INMEDIATA SIMPLE Annuitize full wealth, 0 guarantee, 0 deferral

N° Oferta	Compañía de Seguros de Vida Brand Name	Pensión final Mensual sin Retiro de Excedente UF	Pensión final Mensual en UF Considerando un retiro de excedente de 0,00 UF	Pensión con retiro de Excedente Máximo		Clasificación de riesgo de la Compañía de Seguros (2)
				Pensión final Mensual UF	Excedente UF	
43872093	CRUZ DEL SUR	26,61	<- Monthly payment		Risk rating ->	AA-
43872099	RENTA NACIONAL	26,58				BBB-
43872083	METLIFE	26,52				AA
43872100	CORPSEGUROS	26,34				AA-
43872094	PRINCIPAL	26,28				AA
43872097	CORPVIDA	26,26				AA-
43872084	EUROAMERICA VIDA	26,25				AA-
43872090	PENTA VIDA	26,25				AA-
43872091	OHIO NATIONAL	26,24				AA
43872098	SURA	26,21				AA
43872095	CN LIFE	25,90				AA
43872092	BICE VIDA	25,86				AA+
43872085	CHILENA CONSOLIDADA	25,59				AA
43872086	CONSORCIO VIDA	25,36				AA+

Figure A.3: Participation in market, by insurance firm



	(1)	(2)
	0-0 Offer	log(0-0 Offer)
Request Guarantee	-0.00191	0.000767
	(-0.05)	(0.36)
Request Deferral	0.235*	0.0259***
	(2.50)	(3.61)
Request Both	-0.140	-0.0115
	(-1.47)	(-1.59)
Wealth Spline	Yes	Yes
Age/Month FEs	Yes	Yes
Observations	355092	355092

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.1: Testing for Information Revelation in the Request Stage

Panel A: CDF Summary						
Mass Cutoff		1.00E-01	1.00E-02	1.00E-03	1.00E-04	
Number of Points with Mass Greater than Cutoff		1	23	28	29	
Total Mass for these Points		13.15%	97.94%	99.92%	100.00%	
Panel B: Top 10 Mass Points						
	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass	95% CI
1	7.68	0.000	12.575	15	13.15%	(11.73%, 14.56%)
2	7.89E+03	1.875	16.288	15	9.74%	(7.89%, 11.59%)
3	7.89E+03	5.000	0.200	-3	6.43%	(4.91%, 7.96%)
4	7.89E+03	0.625	18.762	-11	6.12%	(4.41%, 7.83%)
5	7.89E+03	3.750	0.200	7	5.76%	(4.22%, 7.30%)
6	7.89E+03	5.000	0.200	7	4.89%	(3.20%, 6.58%)
7	7.89E+03	1.250	13.812	15	4.73%	(0.59%, 8.86%)
8	7.89E+03	4.375	0.200	-7	4.59%	(3.03%, 6.15%)
9	7.89E+03	5.000	0.200	3	4.33%	(2.59%, 6.07%)
10	137	0.625	20.000	-5	4.27%	(2.82%, 5.72%)

Notes: Panel A reports the number of points whose estimated mass is above each cutoff and their total mass. Panel B reports the ten points with the highest estimated masses, their mass estimate, and 95% confidence regions. Confidence regions are obtained by clustering standard errors at the individual level.

Table A.2: Descriptive Statistics for Estimated Type Distribution - First Quartile Females

Panel A: CDF Summary						
Mass Cutoff		1.00E-01	1.00E-02	1.00E-03	1.00E-04	
Number of Points with Mass Greater than Cutoff		1	29	38	39	
Total Mass for these Points		26.30%	95.49%	99.98%	100.00%	
Panel B: Top 10 Mass Points						
	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass	95% CI
1	7.68	0.000	12.575	15	26.30%	(24.85%, 27.74%)
2	7.89E+03	5.000	0.200	-3	6.04%	(4.90%, 7.17%)
3	137	0.625	20.000	-5	4.77%	(3.55%, 5.98%)
4	0.195	0.000	6.388	15	4.11%	(3.28%, 4.94%)
5	621	3.125	0.200	-15	4.08%	(3.01%, 5.15%)
6	0.414	0.000	11.338	15	3.95%	(2.82%, 5.08%)
7	7.89E+03	5.000	0.200	3	3.92%	(2.80%, 5.03%)
8	44.6	2.500	0.200	3	3.66%	(2.43%, 4.90%)
9	44.6	2.500	0.200	-3	3.13%	(1.87%, 4.39%)
10	7.89E+03	0.625	17.525	-7	3.05%	(2.09%, 4.02%)

Notes: Panel A reports the number of points whose estimated mass is above each cutoff and their total mass. Panel B reports the ten points with the highest estimated masses, their mass estimate, and 95% confidence regions. Confidence regions are obtained by clustering standard errors at the individual level.

Table A.3: Descriptive Statistics for Estimated Type Distribution - Third Quartile Females

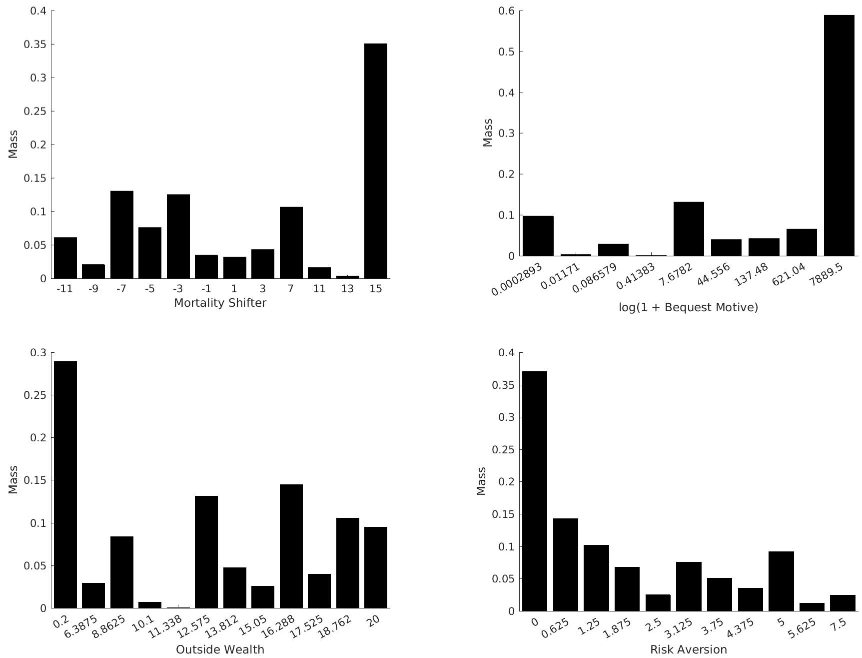


Figure A.4: Marginal Distributions - First Quartile Women

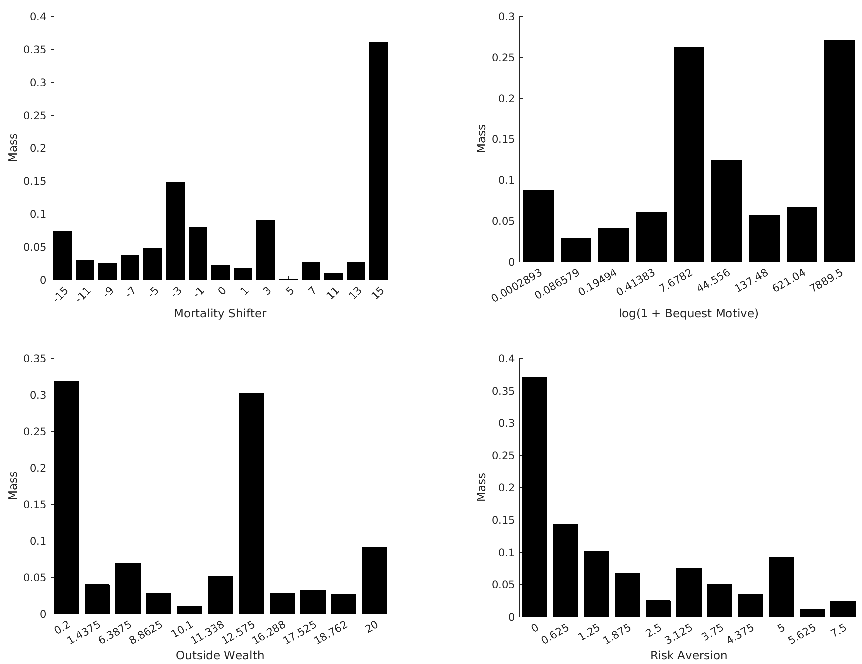


Figure A.5: Marginal Distributions - Third Quartile Women

Wealth Quartile	First	Second	Third	Fourth
Fraction Annuitized				
Observed	67.93%	76.51%	75.30%	65.80%
Predicted	58.20%	70.34%	71.34%	64.79%
Fraction in Mixed Annuities				
Observed	8.99%	11.02%	9.54%	6.26%
Predicted	6.19%	7.45%	6.17%	4.91%
Fraction in Deferred, Non-Guaranteed Annuities				
Observed	3.40%	3.89%	3.77%	3.51%
Predicted	5.58%	8.20%	8.49%	6.73%
Fraction in Deferred & Guaranteed Annuities				
Observed	19.15%	30.40%	33.05%	23.11%
Predicted	5.92%	11.49%	14.46%	10.84%
Fraction in Guaranteed, Non-Deferred Annuities				
Observed	36.07%	34.05%	30.61%	29.79%
Predicted	31.68%	34.36%	33.77%	32.58%
Two-Year Mortality				
Observed	1.77%	1.70%	1.74%	1.99%
Predicted	1.55%	1.71%	1.39%	1.23%
Number of Offers	263,638	419,381	593,671	428,247
Number of Individuals	9,083	9,180	10,697	6,585
MSE	0.03	0.02	0.02	0.01
R^2	0.32	0.23	0.23	0.26

Table A.4: Additional Measures of In-Sample Fit

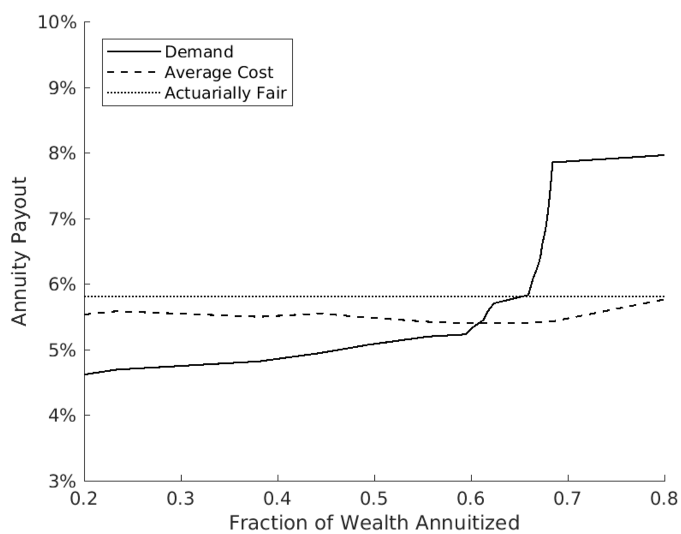


Figure A.6: Equilibrium - First Quartile Women

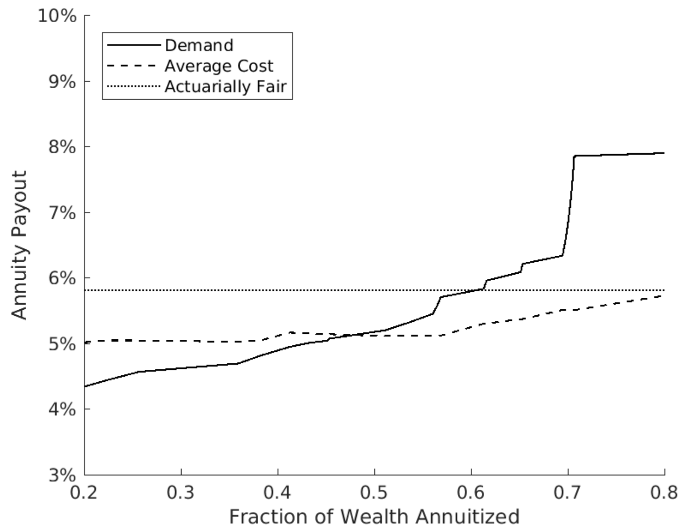
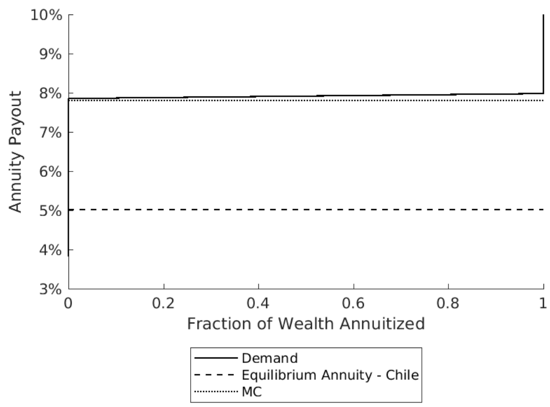
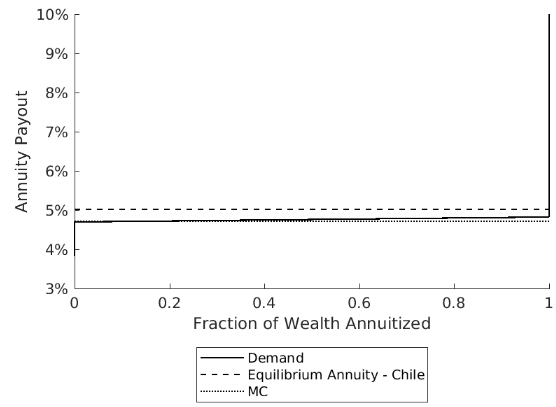


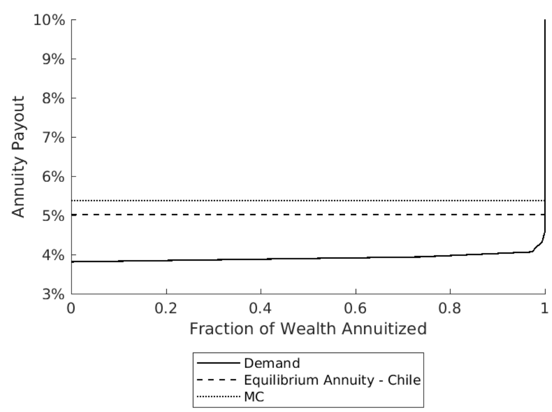
Figure A.7: Equilibrium - Third Quartile Women



(a) Efficient Non-Annuitants



(b) Elastic Demand Close to MC



(c) Efficient Annuitants

Figure A.8: Demand and welfare for example types

B. Online Appendix: Regulatory Details

This appendix presents the formulas for programmed withdrawal and the minimum pension guarantee in detail.

B.1. Programmed Withdrawal

The exposition in this subsection follows Pino (2005). PW payouts in each month of year t for an individual of age x and gender g are

$$PW_t(x,g) = \frac{Balance_t}{CNU_t(x,g) \cdot 12}$$

where balance is the beginning of year account balance in the PFA and CNU is the expected present discounted value of paying out a unit pension. To calculate the CNU, we need to define a few objects. A mortality table issued in year m defines a gender-specific death probability $q^m(x,g)$ for every age x and an adjustment factor $AF^m(x,g)$ - a value meant to correct for increasing longevity expectations for a fixed mortality table.

In year t, the appropriate value for $q^t(x,g)$ is

$$q^t(x,g) = q^m(x,g) \cdot *(1 - AF^m(x,g))^{t-m}$$

Regardless of gender, the tables assume that the probability of being alive at 20 equals 1 and that the probability of being alive at 110 equals 0. For intermediate values, define $l^t(x,g)$, the year t probability of being alive at age x, as

$$l^t(x,g) = l^t(x-1,g) \cdot (1 - q^t(x-1,g)) \text{ for } x \in (20,110]$$

Then $CNU_t(x,g)$ is

$$CNU_t(x,g) = \sum_{j=x}^{110} \frac{l^t(j,g)}{l^t(x,g) \cdot (1+r_{RP})^{j-x}} - \frac{11}{24} \text{ for } x \in (20,110]$$

where $r_{RP} = 0.8 \cdot r_A + 0.2 \cdot \bar{r}$, r_A is the previous year's implicit interest rate for annuities and $\bar{r}$ is the

10 year average return for PW balances. Finally, note that CNU calculations vary for individuals with dependents. We do not report those adjustments, as we work with a no-dependents sample. See Pino (2005) for details. Readers wishing to obtain CNU values will benefit from also reading Vega (2014) and the accompanying Stata module.

B.2. Minimum Pension Guarantee

There are two minimum pension regimes in Chile during our sample period: pre and post 2008. In the first period, any individual with at least 20 years of contributions into the pension system who receives a pension below a minimum guaranteed amount receives a top-up from the government. Since annuity offers cannot fall below this amount, during this period the minimum guaranteed amount is only relevant for valuing programmed withdrawal contracts and for calculating annuity payouts after a default. We value both contracts by taking the UF denominated value of the pension guarantee at the time of retirement and holding it fixed throughout the lifetime of the contract.

Starting in 2008, this guarantee is replaced by an expanded top-up that is available to individuals whose pension falls below a maximum amount. To be precise, the new regime sets a new floor, called the "Pensión Básica Solidaria" or PBS, and a maximum, called the "Pensión Máxima con Aporte Solidario", or PMAS. Annuity offers after this reform cannot fall below the PBS, and individuals funding offers above the PMAS receive no subsidy. For individuals who fund an offer ("Pensión Base", or PB) in between the PBS and the PMAS, the government top up ("Complemento Solidario", or CS) is

$$CS = PBS \cdot (1 - \frac{PB}{PMAS})$$

This amount is added to any annuity offer accepted, regardless of contract type, provided the retiree is 65 or older, has lived in Chile for 20 years after the age of 20, has lived in Chile for 4 of the last 5 years, and is in the 60% percentile or lower in a needs-based poverty index ("Puntaje de Focalización Previsional").

For PW offers, a corrected version of the CS is added to the payout schedule. The correction is meant to ensure that the expected present discounted value of the subsidy is equal under PW and an annuity. See Superintendencia de Pensiones (2018) for details.

C. Online Appendix: Additional Model Details

This appendix section presents a detailed explanation of how the values of annuity and programmed withdrawal offers are calculated. It is divided into four subsections. The first derives the Euler equations for the annuity problem; the second derives the Euler equations for the PW problem; the third presents the computational details of how to solve the annuity problem; and the fourth does the same for the PW problem.

C.1. Derivations for the Annuity Problem

Consider the problem presented in Equation 4. The exogenous variables evolve as follows:

$$d_{t+1} = \begin{cases} 0 \text{ with probability } (1-\mu_{t+1}) \text{ if } d_t = 0 \\ 1 \text{ with probability } \mu_{t+1} \text{ if } d_t = 0 \\ 1 \text{ if } d_t = 1 \end{cases}$$

$$q_{t+1} = \begin{cases} 0 \text{ with probability } (1-\psi_{t+1}) \text{ if } q_t = 0 \\ 1 \text{ with probability } \psi_{t+1} \text{ if } q_t = 0 \\ 1 \text{ if } q_t = 1 \end{cases}$$

$$z_t(d_t,q_t,D,G) = \begin{cases} z \text{ if } q_t = 0 \text{ and } ((d_t = 0 \text{ and } t \geq D) \text{ or } (d_t = 1 \text{ and } D \leq t < G+D)) \\ \rho(z,t)\cdot z \text{ if } q_t = 1 \text{ and } ((d_t = 0 \text{ and } t \geq D) \text{ or } (d_t = 1 \text{ and } D \leq t < G+D)) \\ 0 \text{ otherwise} \end{cases}$$

$$m_0 = \omega,\ d_0 = 0,\ q_0 = 0$$

Where $\rho(z,t)$ is the annuity payment when the firm goes bankrupt:

$$\rho(z,t) = \begin{cases} MPG_t & \text{if } z \leq MPG_t \\ MPG_t + \min((z-MPG)*0.75, 45) & \text{if } z > MPG \end{cases}$$

and MPG is the minimum pension guarantee. For the purposes of this model, we will assume that the MPG is fixed over time.

For expositional clarity, we ignore the no borrowing constraint and derive a solution in an unconstrained setting, and then bring the constraint back in. It is well known that the problems of the previous form can be re-written recursively. In any arbitrary period t, the value of the remaining consumption problem given the current death state d_t, bankruptcy state b_t and liquid assets m_t is $V_t(d_t, q_t, m_t)$, and the Bellman equations are:

$$V_t(d_t, q_t, m_t) = \max_{c_t(d_t, q_t)} \frac{c_t(d_t, q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t(d_t, q_t)' \begin{bmatrix} E_t\left[V_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V_{t+1}(1,1,m_{t+1})\right] \end{bmatrix}$$

where $\Gamma_t(0,0) = \begin{bmatrix} (1-\mu_{t+1})(1-\psi_{t+1}) \\ (1-\mu_{t+1})\psi_{t+1} \\ \mu_{t+1}(1-\psi_{t+1}) \\ \mu_{t+1}\psi_{t+1} \end{bmatrix}$, $\Gamma_t(0,1) = \begin{bmatrix} 0 \\ (1-\mu_{t+1}) \\ 0 \\ \mu_{t+1} \end{bmatrix}$, $\Gamma_t(1,0) = \begin{bmatrix} 0 \\ 0 \\ (1-\psi_{t+1}) \\ \psi_{t+1} \end{bmatrix}$, and $\Gamma_t(1,1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, and each equation is subject to the appropriate dynamic budget constraints and transition rules. We can simplify the previous equation by noting that there is no optimization after death, so for the absorbing state $(d_t = 1, q_t = 1)$ we have that:

$$V_t(1,1,m_t) = \beta \frac{\left[m_t + PDV_t^z(1,1,D,G)\right]^{1-\gamma}}{1-\gamma}$$

$$E_t\left[V_{t+1}(1,1,m_{t+1})\right] = \beta \frac{\left[m_{t+1} + PDV_{t+1}^z(1,1,D,G)\right]^{1-\gamma}}{1-\gamma}$$

where $PDV_t^z(1,1,D,G) = \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot z_\tau(1,1,D,G)$ is the PDV in period t of the payment stream of the guarantee period from $t+1$ to $G+D$.

The expressions are similar in the "dead but not bankrupt" case $(d_t = 1, q_t = 0)$, but take into account that for guaranteed annuities there is uncertainty in the value of future payments:

$$V_t(1,0,m_t) = \beta \frac{[m_t + E[PDV_t^z(1,0,D,G)]]^{1-\gamma}}{1-\gamma}$$

$$E_t\left[V_{t+1}(1,0,m_{t+1})\right] = \beta \frac{\left[m_{t+1} + E[PDV_{t+1}^z(1,0,D,G)]\right]^{1-\gamma}}{1-\gamma}$$

where $E[PDV_t^z(1,0,D,G)]$ is the expected present value in t of the payment stream of the guarantee period from $t+1$ to $G+D$:

$$E[PDV_t^z(1,0,D,G)] = \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot ((1-\Psi_\tau) \cdot z_\tau(1,0,D,G) + \Psi_\tau \cdot z_\tau(1,1,D,G))$$

$$\Psi_\tau = \sum_{\kappa=t+1}^{\tau} \left(\prod_{\tilde{\kappa}=t+1}^{\kappa-1} (1-\psi_{\tilde{\kappa}}) \right) \psi_\kappa$$

and Ψ_τ is the probability that the firm is bankrupt in $\tau > t$, conditional on not being bankrupt in t. As for the remaining states (when the individual is alive), the FOCs from (C.1) are:

$$c_t(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[V'_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] \end{bmatrix}$$

We know that:

$$E_t\left[V'_{t+1}(1,0,m_{t+1})\right] = \beta \cdot [m_{t+1} + \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot ((1-\Psi_\tau) \cdot z_\tau(1,0,D,G) + \Psi_\tau \cdot z_\tau(1,1,D,G))]^{-\gamma}$$

$$E_t\left[V'_{t+1}(1,1,m_{t+1})\right] = \beta \cdot \left[m_{t+1} + \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot z_\tau(1,1,D,G)\right]^{-\gamma}$$

Also, from the Envelope Theorem:

$$V'_t(0,q_t,m_t) = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[V'_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] \end{bmatrix}$$

Combining (C.1) and (C.1), and rolling the equation forward by one year:

$$
\begin{aligned}
c_t(0,q_t)^{-\gamma} &= V_t'(0,q_t,m_t) \\
c_{t+1}(0,q_{t+1})^{-\gamma} &= V_{t+1}'(0,q_{t+1},a_t \cdot R + z_{t+1}(0,q_{t+1},D,G))
\end{aligned}
$$

Substituting back into (C.1) yields the Euler equation:

$$
c_t(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[c_{t+1}(0,0)^{-\gamma}\right] \\ E_t\left[c_{t+1}(0,1)^{-\gamma}\right] \\ E_t\left[V_{t+1}'(1,0,m_{t+1})\right] \\ E_t\left[V_{t+1}'(1,1,m_{t+1})\right] \end{bmatrix}
$$

Following Carroll (2012), note that in equation (C.1) neither m_t nor c_t has any direct effect on V_{t+1}'. Instead, it is their difference, a_t, which enters into the function. This motivates the use of the Endogenous Gridpoint Method to approximate the optimal policy and value functions, as is derived in subsection C.3.

C.2. Derivations for the PW Problem

The individual's optimization problem, which gives the value of accepting a PW offer from firm a, is:

$$
\begin{aligned}
&\max E_0 \left[\sum_{\tau=0}^{T} \delta^\tau u(c_t,d_t) \right] \\
&\quad s.t. \\
&\quad a_t = m_t - c_t \, \forall t \qquad\qquad b_{t+1} = a_t \cdot R_{t+1} \, \forall t \\
&\quad m_{t+1} = b_{t+1} + z_{t+1}(PW_{t+1},d_{t+1},f) \, \forall t \qquad\qquad a_t \geq 0 \, \forall t
\end{aligned}
$$

where $z_t(PW_t,d_t,f)$ denotes the programmed withdrawal payout in period t conditional on pension balance PW_t, death status, and f, the commission rate charged by the firm. The death state and initial conditions are as before, and the remaining exogenous variables evolve as follows:

$$
z_t(PW_t,d_t,a) = \begin{cases} max[z_t(PW_t) \cdot (1-f), MPG] \text{ if } d_t = 0 \\ 0 \text{ if } d_t = 1 \end{cases}
$$

$$PW_{t+1} = (PW_t - z_t(PW_t)) \cdot R_t^{PW}$$

The PW payout function $z_t(PW_t)$ is described in detail in Appendix B. All PFAs are governed by the same PW function, and conditional on the PW balance, will pay out the same amount up before the commission f. As a result, if PFAs provided the same returns over time, the amount of money that is withdrawn every year from the PW account would be the same across PFAs, and only how that money is distributed between the retiree and the PFA would vary across companies. We will assume that in fact PFAs provide the same returns on PW investments, as this simplifies the problem and is not far from reality, where PFA returns vary slightly for the safe investment portfolios where PW balances are invested [22]. Let R_t^{PW} be the return to programmed withdrawal investments. Finally, MPG is the minimum pension guarantee. Every individual who takes PW is guaranteed a payout of at least MPG, and the difference between $z_t(PW_t)$ and MPG (when $z_t(PW_t) < MPG$) is funded by the government. Finally, utility derived from consumption is as before, while upon death utility is:

$$u(d_t = 1) = \beta \cdot \frac{(m_t + PW_t)^{1-\gamma}}{1-\gamma}$$

As before, utility in each state is given by:

$$u(c_t, d_t = 0) = \frac{c_t^{1-\gamma}}{1-\gamma}$$
$$u(d_t = 1) = \beta \cdot \frac{(m_t + PW_t)^{1-\gamma}}{1-\gamma}$$

As in the annuity case, to obtain the value of taking a PW offer we re-write the problem in recursive form. The Bellman equation for the PDV of expected utility under the optimal state-contingent consumption path, for any period t, given the death state, PW account balance, and asset balance, denoted by $V_t(d_t, PW_t, m_t)$, is:

$$V_t(d_t = 0, PW_t, m_t) = \max_{c_t} \frac{c_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}(0, m_{t+1}, PW_t)\right] \\ E_t\left[V_{t+1}(1, m_{t+1}, PW_t)\right] \end{bmatrix}$$

where $\Gamma_t = \begin{bmatrix} 1-\mu_{t+1} \\ \mu_{t+1} \end{bmatrix}$ and, as before, the problem is constrained by dynamic budget constraints and transition rules. Since there is no optimization after death, and inheritors receive the full PW

[22] Illanes (2019) documents this in detail

balance, for the absorbing state $d_t = 1$ we have that:

$$V_t(1, m_t, PW_t) = \beta \frac{[m_t + PW_t]^{1-\gamma}}{1-\gamma}$$

Therefore we can write the expected continuation value for the death state as:

$$E_t[V_{t+1}(1, m_{t+1}, PW_t)] = \frac{\beta}{1-\gamma} \int \left[m_{t+1} + (PW_t - z_t(PW_t)) \cdot R^{PW}\right]^{1-\gamma} dF(R^{PW})$$

For the state where the individual is alive, the expected continuation value is:

$$E_t[V_{t+1}(0, m_{t+1}, PW_t)] = \int V_{t+1}(0, (PW_t - z_t(PW_t)) \cdot R^{PW}, m_{t+1}) dF(R^{PW})$$

With these definitions, the FOCs from (C.2) are:

$$c_t^{-\gamma} = \delta \cdot R \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}'(0, m_{t+1}, PW_t)\right] \\ E_t\left[V_{t+1}'(1, m_{t+1}, PW_t)\right] \end{bmatrix}$$

We know that:

$$E_t\left[V_{t+1}'(1, m_{t+1}, PW_t)\right] = \beta \cdot R \int \left[m_{t+1} + (PW_t - z_t(PW_t)) \cdot R^{PW}\right]^{-\gamma} dF(R^{PW})$$

Also, from the Envelope Theorem:

$$V_t'(0, m_t) = \delta \cdot R \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}'(0, m_{t+1}, PW_t)\right] \\ E_t\left[V_{t+1}'(1, m_{t+1}, PW_t)\right] \end{bmatrix}$$

Combining (C.2) and (C.2), and rolling the equation forward by one year:

$$\begin{aligned} c_t^{-\gamma} &= V_t'(0, m_t) \\ c_{t+1}^{-\gamma} &= V_{t+1}'(0, m_{t+1}, PW_{t+1}) \end{aligned}$$

Substituting back into (C.2) yields the Euler equation:

$$c_t^{-\gamma} = \delta \cdot R \cdot \Gamma_t' \begin{bmatrix} E_t\left[c_{t+1}^{-\gamma}\right] \\ E_t\left[V_{t+1}'(1, m_{t+1}, PW_t)\right] \end{bmatrix}$$

C.3. Computation of the Solution to the Annuity Problem

Having derived the conditions that govern the optimal consumption policy and the value functions for both problems, this subsection presents the details of the numerical procedure used to solve these conditions. Since the problem is solved recursively, we will begin with the solution for period T and work our way backwards. In period T, $\mu_T = 1$ and $T > G+D$, so $m_T = a_{T-1} \cdot R$ and regardless of the bankruptcy state q_T:

$$V_T(0, q_T, m_T) = \beta \cdot \frac{m_T^{1-\gamma}}{1-\gamma}$$

Then in the next-to-last period:

$$V_{T-1}(0, q_{T-1}, m_{T-1}) = \max_{c_{T-1}} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \delta \cdot \beta \cdot \frac{((m_{T-1} - c_{T-1}) \cdot R)^{1-\gamma}}{1-\gamma}$$

Which generates the optimal policy:

$$c_{T-1}^{-\gamma} = \delta \cdot \beta \cdot R^{1-\gamma} \cdot (m_{T-1} - c_{T-1})^{-\gamma}$$
$$c_{T-1}(0, q_{T-1}, m_{T-1}) = \frac{R}{((\delta \cdot \beta \cdot R)^{\frac{1}{\gamma}} + R)} \cdot m_{T-1}$$

And implies that the value function in $T-1$ is:

$$V_{T-1}(0, q_{T-1}, m_{T-1}) = \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma} \right) \left(\frac{R \cdot m_{T-1}}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R} \right)^{1-\gamma}$$

Note that conditional on m_{T-1}, there is no dependence on q_{T-1}. That is, q_{T-1} will shift m_{T-1}, as $m_{T-1} = a_{T-2} \cdot R + z_{T-1}(0, q_{T-1}, D, G)$, but conditional on m_{T-1} it becomes irrelevant. Therefore, given a grid of m_{T-1} one could easily solve for $V_{T-1}(m_{T-1})$, and the value of m_{T-1}'s for other values would be found by interpolation/extrapolation. Note as well that as long as the bequest motive is positive the no-borrowing constraint can be omitted from this stage without loss as the the unconstrained solution always satisfies $c_{T-1} < m_{T-1}$.

Having solved for all the relevant quantities in $T-1$ and T, let us consider the unconstrained problem in $T-2$. From the Euler condition in (C.1) and the optimal policy in (C.3):

$$c_{T-2}(0,q_t)^{-\gamma} = \delta\cdot R\cdot\Gamma_{T-2}(0,q_t)' \begin{bmatrix} E_t\left[c_{T-1}(0,0)^{-\gamma}\right] \\ E_t\left[c_{T-1}(0,1)^{-\gamma}\right] \\ E_t\left[V'_{T-1}(1,0,m_{T-1})\right] \\ E_t\left[V'_{T-1}(1,1,m_{T-1})\right] \end{bmatrix}$$

$$= \delta\cdot R\cdot\Gamma_{T-2}(0,q_t)' \begin{bmatrix} \left(\frac{R}{((\delta\cdot\beta\cdot R)^{1/\gamma}+R)}\right)^{-\gamma}((m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R+z_{T-1}(0,0,D,G))^{-\gamma} \\ \left(\frac{R}{((\delta\cdot\beta\cdot R)^{1/\gamma}+R)}\right)^{-\gamma}((m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R+z_{T-1}(0,1,D,G))^{-\gamma} \\ \beta\cdot\left[(m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R+z_{T-1}(1,0,D,G)+E[PDV^z_{T-1}(1,0,D,G)]\right]^{-\gamma} \\ \beta\cdot\left[(m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R+z_{T-1}(1,1,D,G)+E[PDV^z_{T-1}(1,1,D,G)]\right]^{-\gamma} \end{bmatrix}$$

Unfortunately, this is a non-linear system of equations. To find the value function in $T-2$, one could fix a grid of m_{T-2}, and for each point in the grid solve for optimal consumption and obtain the value function. Interpolation across m's would yield the value function for any m_{T-2}. Note also that the previous derivation is also valid for $0<t<T-2$, so backward induction would allow us to unwind this problem and construct the value function in period 1. The problem in period 0 is slightly different, as the state is $(0,0)$ and wealth is $\omega+z_0(0,0,D,G)+FDA$ with certainty[23], but the same tools apply.

One issue we've abstracted away from up to now is the no-borrowing constraint: $a_{T-1}\geq 0$. Incorporating this constraint implies that when m_{T-1} is sufficiently low, consumption will not be the solution to the aforementioned problem, but rather m_{T-1} itself. This creates a discontinuity in the optimal policy function. Since our approximations to the optimal policy and value functions are constructed by interpolation, it is crucial to incorporate the point where the discontinuity takes place into the grid of points to be evaluated. This ensures that the no-borrowing constraint is properly accounted for in the model. At the point where the no-borrowing constraint binds, $\hat{m}_{T-1}$, the marginal value of consuming m_{T-1} must be equal to the marginal utility of saving 0.

We use the Endogenous Gridpoints Method (Carroll (2006)) to find the solution to the aforementioned problem. At a high level, the strategy is to solve the model for a grid of asset states, and then to interpolate across states to obtain the policy function and the value function. EGM allows us to solve the model efficiently, by re-writing the problem in a way that allows us to back out a solution using an inversion rather than root-finding. The details of the implementation for $T-2$ are presented below:

[23] Recall that FDA is the free disposal amount, another attribute of an annuity offer. In most cases, it is 0.

Numerical Calculation of Policy Function in $T-2$:

1. Select a grid of a_{T-2} with support $[0, \bar{a}_{T-2}]$, where:

$$\bar{a}_{T-2} = R^{T-2}\omega + \sum_{\tau=0}^{T-2} R^{T-2-\tau} z_{\tau}(0,0,D,G)$$

2. Calculate the relevant quantities for the unconstrained problem:

$$m_{T-1}(d_{T-1}, q_{T-1}, D, G) = a_{T-2} \cdot R + z_{T-1}(d_{T-1}, q_{T-1}, D, G))$$

$$\mathfrak{c}_{T-1}(0, q_{T-1}) = \left(\frac{R}{((\delta \cdot \beta \cdot R)^{1/\gamma} + R)} \cdot m_{T-1}(0, q_{T-1}, D, G) \right)^{-\gamma}$$

$$c_{T-2}(0, q_{T-2}) = \left[\delta \cdot R \cdot \Gamma_{T-2}(0, q_{T-2})' \begin{bmatrix} \mathfrak{c}_{T-1}(0,0) \\ \mathfrak{c}_{T-1}(0,1) \\ \beta \cdot [m_{T-1}(1,0,D,G)]^{-\gamma} \\ \beta \cdot [m_{T-1}(1,1,D,G)]]^{-\gamma} \end{bmatrix} \right]^{-\frac{1}{\gamma}}$$

$$\mathfrak{c}_{T-2}(0, q_{T-2}) = c_{T-2}(0, q_{T-2})^{-\gamma}$$

$$m_{T-2}(0, q_{T-2}) = c_{T-2}(0, q_{T-2}) + a_{T-2}$$

$$V_{T-1}(0, q_{T-1}) = \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma} \right) \left(\frac{R \cdot m_{T-1}(0, q_{T-1})}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R} \right)^{1-\gamma}$$

$$V_{T-1}(1, q_{T-1}) = \beta \left(\frac{m_{T-1}(1, q_{T-1})^{1-\gamma}}{1-\gamma} \right)$$

$$V_{T-2}(0, q_{T-2}) = \frac{c_{T-2}(0, q_{T-2})^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_{T-2}(0, q_{T-2}) \begin{bmatrix} V_{T-1}(0,0) \\ V_{T-1}(0,1) \\ V_{T-1}(1,0) \\ V_{T-1}(1,1) \end{bmatrix}$$

3. Denote $\hat{m}_{T-2}(0, q_{T-2})$ the solution to equation (2) when $a_{T-2,i} = 0$. This is the lowest level

of wealth that is unconstrained. Define

$$
\begin{aligned}
\hat{V}_{T-1}(0,q_{T-1}) &= \left(\frac{1+(\delta\cdot\beta\cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right)\left(\frac{R\cdot z_{T-1}(0,q_{T-1},D,G)}{(\delta\cdot\beta\cdot R)^{1/\gamma}+R}\right)^{1-\gamma} \\
\hat{V}_{T-1}(1,q_{T-1}) &= \beta\left(\frac{z_{T-1}(1,q_{T-1},D,G)^{1-\gamma}}{1-\gamma}\right) \\
\hat{\mathfrak{c}}_{T-2,j}(0,q_{T-2}) &= m_{T-2,j}^{-\gamma} \\
\hat{V}_{T-2,j}(0,q_{T-2},m_{T-2}) &= \frac{m_{T-2,j}^{1-\gamma}}{1-\gamma}+\delta\cdot\Gamma_{T-2}(0,q_{T-2})\begin{bmatrix}\hat{V}_{T-1}(0,0)\\ \hat{V}_{T-1}(0,1)\\ \hat{V}_{T-1}(1,0)\\ \hat{V}_{T-1}(1,1)\end{bmatrix}
\end{aligned}
$$

4. Use interpolation to obtain $\grave{\mathfrak{c}}_{T-2}(0,q_{T-2},m_{T-2})$, $\grave{\hat{\mathfrak{c}}}_{T-2,j}(0,q_{T-2})$, $\grave{V}_{T-2,}(0,q_{T-2},m_{T-2})$, and $\grave{\hat{V}}_{T-2,}(0,q_{T-2},m_{T-2})$ for the unconstrained problem.

5. Correct for the no-borrowing constraint by constructing a part exact, part interpolated policy and value function for this period [24]

$$
\begin{aligned}
\grave{\mathfrak{c}}^{*}_{T-2}(0,q_{T-2},m_{T-2}) &= \begin{cases} m_{T-2}^{-\gamma} & \text{if } m_{T-2}<\hat{m}_{T-2}(0,q_{T-2}) \\ \grave{\mathfrak{c}}_{T-2}(0,q_{T-2},m_{T-2}) & \text{otherwise}\end{cases} \\
\grave{V}^{*}_{T-2}(0,q_{T-2},m_{T-2}) &= \begin{cases} \hat{V}_{T-2,}(0,q_{T-2},m_{T-2}) & \text{if } m_{T-2}<\hat{m}_{T-2}(0,q_{T-2}) \\ \grave{V}_{T-2,}(0,q_{T-2},m_{T-2}) & \text{otherwise}\end{cases}
\end{aligned}
$$

There are three issues worth discussing in this procedure: first, we assume that individuals cannot borrow against future annuity payments (the lower bound of a is 0). This is consistent with our knowledge of the Chilean banking system. Second, we set the upper bound of the support of assets as the PDV of initial wealth plus the PDV of the maximum sequence of previous annuity payments. This ensures that the grid of a's spans the optimal asset value in $T-2$, as in the model the agent cannot accumulate more wealth than this value. Third, we interpolate over $\mathfrak{c}(\cdot)$ instead of $c(\cdot)$. This is suggested by Carroll (2011), as the function that enters into the recursion in earlier periods is $\mathfrak{c}(\cdot)$, and not $c(\cdot)$. One could interpolate over $c(\cdot)$, and then raise the interpolated value to the power of $-\frac{1}{\gamma}$, but that is less accurate is simply interpolating over $\grave{\mathfrak{c}}$. With these objects, we can solve the problem for $T-3, T-4, ..., 0$ by recursion.

[24] Note that the solution objects for the $T-2$ problem are exact when the constraint binds.

Numerical Calculation of Policy Function in t:

1. Select a grid of a_t with support $[0, \bar{a}_t]$:

$$\bar{a}_t = R^t \omega + \sum_{\tau=0}^{t} R^{t-\tau} z_\tau(0,0,D,G)$$

2. Calculate the relevant quantities for the unconstrained problem (suppressing the dependence on D and G to simplify notation):

$$m_{t+1}(0,q_{t+1}) = a_t \cdot R + z_{t+1}(0,q_{t+1})$$

$$c_t(0,q_t) = \left[\delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} \grave{\mathfrak{c}}^*_{t+1}(0,0,m_{t+1}(0,0)) \\ \grave{\mathfrak{c}}^*_{t+1}(0,1,m_{t+1}(0,0)) \\ \beta \cdot \left[m_{t+1}(1,0) + E[PDV^z_{t+1}(1,0,D,G)] \right]^{-\gamma} \\ \beta \cdot \left[m_{t+1}(1,1) + E[PDV^z_{t+1}(1,1,D,G)] \right]^{-\gamma} \end{bmatrix} \right]^{-\frac{1}{\gamma}}$$

$$\mathfrak{c}_t(0,q_{T-2}) = c_t(0,q_{T-2})^{-\gamma}$$

$$m_t(0,q_t) = c_t(0,q_t) + a_t$$

$$V_{t+1}(1,q_{t+1}) = \beta \left(\frac{m_{t+1}(1,q_{t+1})^{1-\gamma}}{1-\gamma} \right)$$

$$V_t(0,q_t) = \frac{c_t(0,q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t(0,q_t) \begin{bmatrix} \grave{V}^*_{t+1}(0,0,m_{t+1}) \\ \grave{V}^*_{t+1}(0,1,m_{t+1}) \\ V_{t+1}(1,0) \\ V_{t+1}(1,1) \end{bmatrix}$$

3. Define $\hat{m}_t(0,q_t)$ as the level of wealth obtained at $a_t = 0$ and

$$\hat{V}_{t+1}(1,q_{t+1}) = \beta \left(\frac{E[PDV^z_{t+1}(1,q_{t+1},D,G)]^{1-\gamma}}{1-\gamma} \right)$$

$$\hat{V}_t(0,q_t) = \frac{\hat{m}_t(0,q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_{T-2}(0,q_{T-2}) \begin{bmatrix} \grave{V}^*_{t+1}(0,0,z_{t+1}(0,0)) \\ \grave{V}^*_{t+1}(0,1,z_{t+1}(0,1)) \\ \hat{V}_{t+1}(1,0) \\ \hat{V}_{t+1}(1,1) \end{bmatrix}$$

4. Use interpolation to obtain $\grave{\mathfrak{c}}_t(0,q_t,m_t)$, $\grave{\hat{\mathfrak{c}}}_{t,j}(0,q_t)$, $\grave{V}_{t,}(0,q_t,m_t)$, and $\grave{\hat{V}}_{t,}(0,q_t,m_t)$ for the

unconstrained problem.

5. Correct for the no-borrowing constraint:
$$\grave{\mathfrak{c}}_t^*(0,q_t,m_t) = \begin{cases} m_t^{-\gamma} & \text{if } m_t < \hat{m}_t(0,q_{t+1}) \\ \grave{\mathfrak{c}}_t(0,q_t,m_t) & \text{otherwise} \end{cases}$$
$$\grave{V}_t^*(0,q_t,m_t) = \begin{cases} \hat{V}_t(0,q_t,m_t) & \text{if } m_t < \hat{m}_t(0,q_t) \\ \grave{V}_t(0,q_t,m_t) & \text{otherwise} \end{cases}$$

6. Repeat for $t-1$

Note that again, the constrained segment requires no additional interpolation and hence its implementation is both efficient and precise.We can recover the object of interest (the value of an annuity offer: $V(0,0,\omega_i,D,G)$) after the $t=0$ step in the previous recursion.

C.4. Computation of the Solution to the PW Problem

In period T, $\mu_T = 1$ and $PW_T = 0$, so $m_T = a_{T-1} \cdot R$ and:

$$V_T(0,m_T,PW_T) = \beta \cdot \frac{m_T^{1-\gamma}}{1-\gamma}$$

Then in the next-to-last period:

$$V_{T-1}(0,m_{T-1},PW_{T-1}) = \max_{c_{T-1}} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \frac{\delta \cdot \beta}{1-\gamma}((m_{T-1} - c_{T-1}) \cdot R)^{1-\gamma}$$

The optimal policy and value functions in $T-1$ are then:

$$c_{T-1}(m_{T-1}) = \frac{R}{((\delta \cdot \beta \cdot R)^{\frac{1}{\gamma}} + R)} \cdot m_{T-1}$$
$$V_{T-1}(0,m_{T-1}) = \left(\frac{1+(\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right)\left(\frac{R \cdot m_{T-1}}{(\delta \cdot \beta \cdot R)^{1/\gamma}+R}\right)^{1-\gamma}$$

Note that, conditional on m_{T-1}, there is no dependence on PW_{T-1}. This is because PW_{T-1} will shift m_{T-1}, as $m_{T-1} = a_{T-2} \cdot R + z_{T-1}(PW_{T-1},a)$, but conditional on m_{T-1} it becomes irrelevant.

Additionally, as in the annuity problem, as long as the bequest motive is not negative the unconstrained maximizer satisfies the no-borrowing constraint.

Having solved for all the relevant quantities in $T-1$ and T, we can proceed to solve the problem in $T-2$. There are a few additional objects that need to be introduced before proceeding. First, take K draws from the distribution of R^{PW}. Each draw will be denoted by k, and draws will be held fixed across time periods. Define $\bar{R}_K$ as the largest draw from the distribution of R^{PW}. Second, define the upper bound of the grid of PW, $P\bar{W}$, recursively:

$$
\begin{aligned}
P\bar{W}_1 &= \bar{R}_K \cdot (PW_0 - z_t(PW_0)) \\
P\bar{W}_t &= \bar{R}_K \cdot (P\bar{W}_{t-1} - z_t(P\bar{W}_{t-1}))
\end{aligned}
$$

Finally, define the upper bound of the grid of accumulated assets as:

$$
\bar{a}_t = R^t \omega + \sum_{\tau=0}^{t} R^{t-\tau} z(P\bar{W}_\tau, 0, f)
$$

Numerical Calculation of Policy Function in $T-2$:

1. Select a grid of $(a_{T-2,i}, PW_{T-2,i})$ with support $[0, \bar{a}_{T-2}] \times [0, P\bar{W}_{T-2}]$.

2. Calculate the relevant quantities for the unconstrained problem:

$$
\begin{aligned}
&m_{T-1,k}(0) = a_{T-2} \cdot R + z_{T-1}(R_k^{PW} \cdot (PW_{T-2} - z(PW_{T-2}), 0, a) \\
&m_{T-1,k}(1) = a_{T-2} \cdot R + R_k^{PW} \cdot (PW_{T-2} - z(PW_{T-2})) \\
&E_{T-2}[\mathfrak{c}_{T-1}] = \frac{1}{K} \sum_{k=1}^{K} \left[c_{T-1}(m_{T-1,k}(0))\right]^{-\gamma} \\
&E_{T-2}[V'_{T-1}(1)] = \frac{\beta}{K} \sum_{k=1}^{K} \left[m_{T-1,k}(1)\right]^{-\gamma} \\
&c_{T-2} = \left[\delta \cdot R \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[\mathfrak{c}_{T-1}] \\ E_{T-2}[V'_{T-1}(1)] \end{bmatrix}\right]^{-\frac{1}{\gamma}} \\
&m_{T-2} = c_{T-2} + a_{T-2} \qquad\qquad (2) \\
&\mathfrak{c}_{T-2} = c_{T-2}^{-\gamma} \\
&E_{T-2}[V_{T-1}(0)] = \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right) \cdot \left(\frac{R}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R}\right)^{1-\gamma} \frac{1}{K} \sum_{k=1}^{K} \left[m_{T-1,k}(0)\right]^{1-\gamma} \\
&E_{T-2}[V_{T-1}(1)] = \frac{\beta}{1-\gamma} \cdot \frac{1}{K} \sum_{k=1}^{K} \left[m_{T-1,k}(1)\right]^{1-\gamma} \\
&V_{T-2} = \frac{c_{T-2}^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[V_{T-1}(0)] \\ E_{T-2}[V_{T-1}(1)] \end{bmatrix}
\end{aligned}
$$

3. Denote $\hat{m}_{T-2}(PW_{T-2})$ the solution to (2) when $a_{T-2} = 0$ and the PW balance is PW_{T-2} define

$$
\hat{V}_{T-2}(m_{T-2}, PW_{T-2}) = \frac{m_{T-2}^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[V_{T-1}(0)] \\ E_{T-2}[V_{T-1}(1)] \end{bmatrix}
$$

with the value of V_{T-1} determined by $a_{T-2} = 0$.

4. Use interpolation to obtain $\grave{\mathfrak{c}}_{T-2}(m_{T-2}, PW_{T-2})$ and $\grave{V}_{T-2}(0, m_{T-2}, PW_{T-2})$ for the unconstrained problem. Form the boundary interpolator $\grave{\hat{m}}_{T-2}(PW_{T-2})$ which determines the minimum level of unconstrained wealth for each value of the PW balance.

5. Correct for the no-borrowing constraint by constructing a part exact, part interpolated policy

and value function for this period [25]

$$\grave{\mathfrak{c}}^*_{T-2}(m_{T-2}, PW_{T-2}) = \begin{cases} m_{T-2}^{-\gamma} & \text{if } m_{T-2} < \grave{\hat{m}}_{T-2}(PW_{T-2}) \\ \grave{\mathfrak{c}}_{T-2}(m_{T-2}, PW_{T-2}) & \text{otherwise} \end{cases}$$

$$\grave{V}^*_{T-2}(m_{T-2}, PW_{T-2}) = \begin{cases} \hat{V}_{T-2,}(m_{T-2}, PW_{T-2}) & \text{if } m_{T-2} < \grave{\hat{m}}_{T-2}(PW_{T-2}) \\ \grave{V}_{T-2,}(m_{T-2}, PW_{T-2}) & \text{otherwise} \end{cases}$$

Armed with these objects, we can solve the problem for $T-3, T-4, ..., 0$ by recursion.

<u>Numerical Calculation of Policy Function in t:</u>

1. Select a grid of (a_t, PW_t) with support $[0, \bar{a}_t] \times [0, P\bar{W}_t]$.

2. Calculate the relevant quantities for the unconstrained problem:

$$m_{t+1,k}(0) = a_t \cdot R + z_{t+1}(R_k^{PW} \cdot (PW_t - z_t(PW_t)), 0, a)$$
$$m_{t+1,k}(1) = a_t \cdot R + R_k^{PW} \cdot (PW_t - z_t(PW_t))$$
$$E_t[\mathfrak{c}_{t+1}] = \frac{1}{K} \sum_{k=1}^{K} \grave{\mathfrak{c}}_{t+1}(m_{t+1,k}(0), PW_{t+1,k})$$
$$E_t[V'_{t+1}(1)] = \frac{\beta}{K} \sum_{k=1}^{K} \left[m_{t+1,k}(1)\right]^{-\gamma}$$
$$c_t = \left[\delta \cdot R \cdot \Gamma'_t \begin{bmatrix} E_t[\mathfrak{c}_{t+1}] \\ E_t[V'_{t+1}(1)] \end{bmatrix}\right]^{-\frac{1}{\gamma}}$$
$$m_t = c_t + a_{t,i}$$
$$\mathfrak{c}_t = c_t^{-\gamma}$$
$$E_t[V_{t+1}(0)] = \frac{1}{K} \sum_{k=1}^{K} \grave{V}(0, m_{t+1,k}(0), R_k^{PW} \cdot (PW_t - z(PW_t)))$$
$$E_t[V_{t+1}(1)] = \frac{\beta}{1-\gamma} \cdot \frac{1}{K} \sum_{k=1}^{K} \left[m_{t+1,k}(1)\right]^{1-\gamma}$$
$$V_t = \frac{c_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_t \begin{bmatrix} E_t[V_{t+1}(0)] \\ E_t[V_{t+1}(1)] \end{bmatrix}$$

[25] Note that the solution objects for the $T-2$ problem are exact when the constraint binds.

3. Denote $\hat{m}_t(PW_t)$ the solution when $a_t = 0$ and the PW balance is PW_t define

$$\hat{V}_t(m_t, PW_t) = \frac{m_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t' \begin{bmatrix} E_t[V_{t+1}(0)] \\ E_t[V_{t+1}(1)] \end{bmatrix}$$

   with the value of V_{t+1} determined by $a_t = 0$.

4. Use interpolation to obtain $\grave{\mathfrak{c}}_t(m_t, PW_t)$ and $\grave{V}_t(0, m_t, PW_t)$ for the unconstrained problem. Form the boundary interpolator $\grave{\hat{m}}_t(PW_t)$ which determines the minimum level of unconstrained wealth for each value of the PW balance.

5. Correct for the no-borrowing constraint by constructing a part exact part interpolated policy and value function for this period:

$$\grave{\mathfrak{c}}_t^*(m_t, PW_t) = \begin{cases} m_t^{-\gamma} & \text{if } m_t < \grave{\hat{m}}_t(PW_t) \\ \grave{\mathfrak{c}}_t(m_t, PW_t) & \text{otherwise} \end{cases}$$

$$\grave{V}_t^*(m_t, PW_t) = \begin{cases} \hat{V}_{t,}(m_t, PW_t) & \text{if } m_t < \grave{\hat{m}}_t(PW_t) \\ \grave{V}_{t,}(m_t, PW_t) & \text{otherwise} \end{cases}$$

6. Repeat for $t-1$

We can recover the object of interest (the value of a PW offer: $V_0(0, \omega, PW_0)$) after the $t = 0$ step in the previous recursion.

D. Online Appendix: Grid Selection Details

As mentioned in the main text, we incorporate a grid selection step into the estimation procedure. Through this step we are able to start with a grid that plausibly spans the support of the distribution of types, but that is infeasible to take to the data, and reduce its dimensionality without greatly affecting the outcomes that the model can cover or the predictions that will later be made in the counterfactuals of interest.

We start with an initial grid that has 17 grid points per dimension of the type space, which corresponds to 83,251 total points. For bequest motive, β, the grid is chosen to be uniform over the space of bequeathed wealth for sample retiree. This retiree knows with certainty that he will die the next period and has a risk aversion coefficient of 3. The grid is thus uniform on the wealth

	Bequest Motive	Percentage Consumed
1	7.89e+03	5.00%
2	6.21e+02	10.94%
3	1.37e+02	16.88%
4	4.46e+01	22.81%
5	1.75e+01	28.75%
6	7.68e+00	34.69%
7	3.59e+00	40.62%
8	1.74e+00	46.56%
9	8.52e-01	52.50%
10	4.14e-01	58.44%
11	1.95e-01	64.38%
12	8.66e-02	70.31%
13	3.48e-02	76.25%
14	1.17e-02	82.19%
15	2.81e-03	88.12%
16	2.89e-04	94.06%
17	0	100.00%

Table A.5: Map from bequest motive to fraction of wealth consumed before certain death

consumed in this last period, covering the range of 5% to 100%. Table A.5 presents this mapping for every point in the grid of bequest motives. For risk aversion, the grid spans uniformly from 0 to 10, while for outside wealth the grid spans uniformly from 0.2 to 20 thousand UFs.[26] Finally, the mortality shifter grid spans from -15 to 15, in discrete uniform steps, rounded down. Table A.6 presents the grid points for each dimension.

As it is computationally infeasible to estimate demand on the whole sample with the full grid, we develop a binned-selection procedure. First, we take a random sample of 3% of the population (2019 consumers, with an average of 48.3 offers) and estimate equation (1) using the full grid. This creates a probability distribution over the full grid. We then use the estimates to evaluate four counterfactual choice scenarios of these consumers: first, choice between fully annuitizing their pension balance under actuarially fair rates or programmed withdrawal; and then, between allocating the remaining pension balance to an actuarially fair annuity or to lumpsum withdrawal when 0%, 50% and 90% of pension balances are placed in an actuarially fair annuity. We then bin grid points by comparing the choices made by consumers, such that two grid points belong to the same bin if consumers would make the exact same choices under the last three scenarios, and their

[26]In December 12, 2017, the dollar equivalent range was between 8,170 and 817,000

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
1	7.89e+03	0	0.2	-15
2	6.21e+02	0.62	1.4	-13
3	1.37e+02	1.2	2.7	-11
4	4.46e+01	1.9	3.9	-9
5	1.75e+01	2.5	5.2	-7
6	7.68e+00	3.1	6.4	-5
7	3.59e+00	3.8	7.6	-3
8	1.74e+00	4.4	8.9	-1
9	8.52e-01	5	10	0
10	4.14e-01	5.6	11	1
11	1.95e-01	6.2	13	3
12	8.66e-02	6.9	14	5
13	3.48e-02	7.5	15	7
14	1.17e-02	8.1	16	9
15	2.81e-03	8.8	18	11
16	2.89e-04	9.4	19	13
17	0	10	20	15

Table A.6: Gridpoints by dimension of types for initial grid

choice under the first is equal for at least 98% of them. This produces 4366 bins. For each bin we sample a representative grid point, with a probably proportional to the corresponding estimated from the first step. We estimate demand for the random sample again using the selected 4366 points, using these to further filter out points of low probability. We remove points that are estimated to have a probability below 10^{-6}, leaving 58 points with a cumulative probability of 99.996%. These points correspond to the grid we take to the full sample, concluding our grid construction procedure. The full list of points is available upon request.