Outside Options and the Failure of the Coase Conjecture[†]

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A buyer wishes to purchase a good from a seller who chooses a sequence of prices over time. Each period the buyer can also exercise an outside option, abandoning their search or moving on to another seller. We show there is a unique equilibrium in which the seller charges a constant price in every period equal to the monopoly price, contravening the Coase conjecture. We then embed the single-seller model into a search framework and show the result provides a foundation for the usual "no haggling" assumption. (*JEL* C78, D42, D43, L12, L13)

The Coase conjecture is a cornerstone of modern microeconomic theory, informing monopoly theory and providing a canonical example of the problem of commitment. The idea is that, for any given price, high value buyers are more likely to purchase than low value buyers, leading to negative selection in the demand pool. Accordingly, the seller cuts its price over time, causing high value buyers to delay their purchases. The seller's inability to commit thus leads its later selves to exert a negative externality on its former selves, reducing its overall profit (Gul, Sonnenschein, and Wilson 1986). This idea of negative selection is robust: versions of it hold when costs are nonlinear (Kahn 1986), when goods depreciate over time (Bond and Samuelson 1984), when there is entry of new buyers (Sobel 1991) and when the buyers face future competition (Fuchs and Skrzypacz 2010).

In this article, we show that the Coase conjecture fails in a natural environment where buyers have an outside option they can choose to exercise. We consider a seller who faces a buyer (or a continuum of buyers) with unknown value for the good and unknown outside option. Each period, the seller chooses price p_t; the buyer then chooses to buy, wait, or exercise his option and terminate the game. The outside option may come from the possibility of buying another product. For example, waiting for the price of the iPhone to fall, a customer could instead buy a comparable phone from a competitor. Similarly, when bargaining over the price of a car, a buyer may abandon negotiations and move on to the next dealer. The outside option may also come from pursuing other objectives once the negotiation has terminated. For example, after he finishes bargaining at a bazaar, a tourist could start seeing local attractions.

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In Section I, we take the outside option as exogenous and show that there is unique equilibrium in which the seller charges a constant price equal to the monopoly price against its demand. Intuitively, if the buyer expects the price to stay high, then those with low values exercise their outside option and exit the market. As a result, there is no negative selection in the demand pool and no price cut is forthcoming.

To understand why the monopoly pricing equilibrium is unique, suppose prices do fall over time, and let $\underline{u}$ be the lowest net value of types who delay. In the continuation game, the seller will never lower its price below $\underline{u}$, since this would leave rents on the table. Type $\underline{u}$ will thus receive zero utility and should take his outside option in period $t = 1$, contradicting the assumption that he delays. The overall idea is that the outside option results in low value types exiting and positive selection in the demand pool. This is very natural: when shopping in a bazaar, a high price quote from a seller is more likely to lead a low value buyer to move on to the next stall rather than wait for a price reduction.

High-priced equilibria have been produced in other durable goods models. Ausubel and Deneckere (1989) show that if the buyer expects an out-of-equilibrium price cut to be followed by Coasian pricing, then one can sustain prices that are arbitrarily high. Fudenberg, Levine, and Tirole (1987) suppose the seller has the option to consume the good and show that if the buyer expects the seller to consume early, he is less willing to delay purchasing, causing the seller to become more pessimistic over time and justifying the belief that the seller consumes early. In both of these cases there are multiple equilibria, including those with Coasian dynamics; in our paper, the monopoly equilibrium is unique.

Analogs to the unraveling logic in our article have been seen before. In Diamond (1971), N firms choose prices $\{p_1, \ldots, p_N\}$, and buyers pay a positive cost to search one more firm; in equilibrium, all firms choose the monopoly price. Both papers feature a holdup problem—a buyer must be rewarded some utility for paying a search cost (or forgoing an outside option) but the seller has the incentive to extract the utility after the buyer has made his decision. Hence, the buyer with the lowest utility regrets searching (or forgoing the outside option), leading this behavior to unravel from the bottom. However, the models have different implications, since Diamond's buyers are held up when they leave a seller, whereas our buyers are held up when they stay. If a firm could make multiple price offers, Diamond's sellers would thus suffer from the Coase problem whereas our seller charges the monopoly price. Similar logic is at work in Perry (1986), in which two parties bargain via alternating offers under two-sided uncertainty. He proves that if there is a known fixed cost of making an offer and no discounting, then the game terminates in a single round. However, this result breaks down when discounting is introduced (Cramton 1991) or when offer costs are private information (Rubinstein 1985; Bikhchandani 1992), with buyers signaling their types by waiting. In comparison, our article considers a Coasian framework, discounting payoffs and allowing both the buyer's valuation and outside option to be private information.[1]

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[1] There are also other related papers. In Compte and Jehiel (2002), two parties bargain via alternating offers, where one is tempted to mimic a commitment type but is restrained by the outside option of the opponent. Deb (2010) shows that there is a constant price equilibrium in a Coasian model when buyers' values are stochastic, where the constant renewal of high types prevents the seller from walking down the demand curve.

We end Section I by analyzing the robustness of our monopoly pricing result. In Section IA we show that the result continues to hold even if taking the outside option is reversible. For example, when bargaining at a bazaar, a buyer may return to negotiate after some time has passed. In Section IB we then examine the form of the equilibrium if some types have no outside options and show that the optimal first-period price converges to the monopoly price as the number of such zero-option types shrinks.

In Section II, we endogenize the outside option by considering a model of sequential search. Each period, a buyer chooses whether to stay with the current seller or move on to an alternative seller where he receives a new value draw; the chosen seller then quotes a sequence of prices over time. In this case, the value from future search opportunities constitutes a buyer's outside option and allows us to apply our monopoly pricing result. In equilibrium, a seller faces buyers who arrive over time, receive one offer, buy if their value is sufficiently high, and otherwise move on.

Our model thus shows that even if the Coase conjecture applies to a single seller, it may fail when there are competing sellers who generate an outside option for a buyer. This result complements Ausubel and Deneckere (1987) and Gul (1987) who consider direct competition between price-setting sellers. These models have multiple equilibria ranging from traditional Bertrand pricing to monopolistic pricing that is sustained through Bertrand punishments. In contrast, our model has a unique equilibrium and does not rely on non-Markovian punishments that might be hard to coordinate. Our monopoly pricing result significantly simplifies the analysis: once we have proved that sellers choose constant prices, the equilibrium is similar to Wolinsky (1986) or Anderson and Renault (1999), albeit with proportional discounting. One can thus view our results as providing a foundation for Wolinsky's assumption that sellers do not haggle.

I. The Failure of the Coase Conjecture

We start by looking at how the presence of the buyer's outside option changes equilibrium play in an otherwise standard Coase conjecture setting. A monopoly seller tries to sell a durable good to a single buyer in periods $t \in \{1, 2, \dots\}$. The seller's cost of producing the good is $c = 0$ and is commonly known. The buyer privately knows his value for the good $v \in V \subset [0, \bar{v}]$ and the value of his outside option $w \in W \subset [\underline{w}, \overline{w}]$, where $\underline{w} > 0$ and $\bar{v}$, $\overline{w}$ are finite. The values (v, w) are drawn from a distribution with support contained in $V \times W$; the distribution is commonly known.[2]

The timing is as follows: at the start of any period t, the seller chooses price $p_t \geq 0$. The buyer then chooses whether to buy the good, exercise his outside option, or wait. All actions are publicly observable, and we allow for mixed strategies. The game continues only if the buyer chooses to wait. Waiting is costly as the buyer and seller discount their utility with a common discount factor $\delta \in (0, 1)$. If the buyer buys the good in period t, he obtains utility $\delta^t(v - p_t)$, and the seller obtains profit

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[2] The distribution over (v, w) is not restricted and may have gaps and atoms; the assumption of bounded support is made to ensure a monopoly price exists. We do require that outside options are bounded below; in Section IB we show that our results are robust to including a small number of zero-option types.

$\delta^t p_t$. If the buyer exercises the outside option in period t, he obtains utility $\delta^t w$, and the seller obtains profit 0. If the buyer waits forever, both the seller and buyer obtain 0. A *public history* of the game—or, for short, *history*—is any finite sequence of the seller's and buyer's consecutive actions, starting with the first price decision of the seller.[3]

A *Perfect Bayesian Equilibrium* (PBE) is a history-contingent sequence of the seller's offers p_t, the buyer's acceptance and exercise decisions, and of updated beliefs about the buyer's values (v, w) such that: actions are optimal given beliefs; beliefs are derived from actions from Bayes' rule whenever possible, including off the equilibrium path; and the seller's actions, even zero-probability actions, do not change its belief about the buyer's type. An equilibrium is *essentially unique* if all equilibria lead to the same payoffs. Notably, essential uniqueness does not pin down prices off the equilibrium path.

Define $u := v - w$ as the *net value* of the buyer. Let $F(u)$ be the probability the buyer's net value is strictly below u (which coincides with the cdf when there is no atom) and assume the monopoly price $p^m \in \text{argmax } p(1 - F(p))$ is unique.[4] The buyer and seller follow *monopoly strategies* if in every period

   (i) The seller charges p^m;

   (ii) The buyer buys the good if $u \geq p$ and otherwise exercises his outside option.

We first show that in any PBE, prices cannot fall below the lowest net value $\underline{u}$ at any history. If this did happen then the seller would give type $\underline{u}$ rents in some periods. When rents are maximal the buyer purchases immediately in any PBE; raising the price a little would not induce any delay, because of discounting, and is therefore a profitable deviation.

LEMMA 1: *In any PBE, if the seller believes that the buyer's net value is above $\underline{u}$ at some history, then it charges prices above $\underline{u}$ at that history.*

PROOF:
Fix a PBE. Let $\underline{u}(h)$ be the minimum of the support of the seller's belief about the net values of the buyer at history h, and let $\underline{p}(h)$ be the minimum of the support of prices the seller charges at h. We wish to show that $\underline{p}(h) > \underline{u}(h)$ for all histories h. By way of contradiction, assume that $\underline{p}(h) < \underline{u}(h)$ for some histories h, and let $\Delta = \sup_h(\underline{u}(h) - \underline{p}(h)) > 0$ be the least upper bound on undiscounted rents the lowest net-value buyer types can receive. Since prices are nonnegative, Δ is finite; for any $\epsilon \in \left(0, \frac{1-\delta}{2}\right)$, we can then pick a history h_t at time t at which $\underline{u}(h_t) - \underline{p}(h_t) > (1 - \epsilon)\Delta$.

We claim that at history h_t the seller puts probability 1 on the buyer immediately buying at any price $p \leq \underline{p}(h_t) + \epsilon \Delta$. At time t, each type prefers to buy now rather than take the outside option because he gets positive rents from buying, $u - p \geq \underline{u}(h_t) - \underline{p}(h_t) - \epsilon \Delta > 0$. It follows that $v - p > w > \delta^{s-t} w$, so each type prefers to buy at t rather that exit in period $s > t$. Finally, we wish to show that each type prefers to buy at t rather than buy in a future period. To see this, suppose some positive measure set of types did decide to wait and purchase at time $s > t$, where h_s is the resulting continuation of history h_t. By Bayes' rule $\underline{u}(h_s) \geq \underline{u}(h_t)$, so the definitions of p and of h_t imply that

$$\underline{u}(h_s) - p \geq \underline{u}(h_t) - \underline{p}(h_t) - \epsilon \Delta > (1 - 2\epsilon)\Delta > \delta^{s-t}(\underline{u}(h_s) - \underline{p}(h_s)),$$

where the last inequality is self-evident if $\underline{u}(h_s) - \underline{p}(h_s) \leq 0$ and otherwise follows from the definition of Δ and the assumption that $\epsilon < (1 - \delta)/2$. Because the types remaining at history h_s have values $v = u + w \geq \underline{u}(h_s)$, the above inequality yields $v - p > \delta^{s-t}(v - \underline{p}(h_s))$ so all these types prefer to buy at time t rather than delay until time s, contradicting the assumption that some types delay.

Any price $p \leq \underline{p}(h_t) + \epsilon \Delta$ thus leads to an immediate sale. Hence, prices $p \in [\underline{p}(h_t), \underline{p}(h_t) + \epsilon \Delta)$ are not a best response, and $\underline{p}(h_t)$ is not the minimum of the support of prices the seller charges at h_t, contradicting the supposition that $\Delta > 0$.

PROPOSITION 1: *There is a PBE in which the buyer and seller use monopoly strategies. This PBE is essentially unique.*

PROOF:

To check that the monopoly strategies form a PBE is straightforward: if the buyer exits or buys in period $t = 1$, then charging the monopoly price forever is a best response; if the seller charges a constant price, then the buyer will exit or buy in period $t = 1$.

To demonstrate the payoff-equivalence of all PBE, we claim that in period 1 the buyer either buys or exits, irrespective of the price posted by the seller. Suppose, by contradiction, that there is a PBE and a seller's price p_1 (either on path or off path) such that some positive measure subset of types decide to delay, and let $\underline{u}(h_2)$ be the minimum of the support of their net values. Exiting in period $t \geq 2$ cannot be better than exiting at $t = 1$, so a positive measure of types with $u < \underline{u}(h_2) + \epsilon$ and $\epsilon \in \left(0, \frac{1-\delta}{\delta} w\right)$ buy in some period $t \geq 2$ with positive probability. Lemma 1 and the consistency of the seller's beliefs implies that $\underline{p}(h_t) \geq \underline{u}(h_t) \geq \underline{u}(h_2)$, so for $u < \underline{u}(h_2) + \epsilon$, we have

$$\delta^{t-1}(v - \underline{p}(h_t)) \leq \delta^{t-1}(v - \underline{u}(h_2)) < \delta^{t-1}(v - u + \epsilon) = \delta^{t-1}(w + \epsilon) \leq w,$$

where the last inequality uses $\epsilon \leq \frac{1-\delta}{\delta} w$. Hence, these types prefer to exit at $t = 1$ rather than wait, contradicting the assumption that a positive mass delays.

Thus, for any price p_1 posted by the seller, the buyer either buys or exercises his outside option at time 1 in any PBE. Incentive compatibility for the buyer then implies that the buyer buys if $v - p_1 > w$ and exercises his outside option if $v - p_1 < w$. An indifferent buyer, $v - p_1 = w$, can do either if there is measure zero of

such types; if there is a positive mass of indifferent types, incentive compatibility for the seller requires that these indifferent types choose to buy with probability 1. Given the buyer's behavior, the seller maximizes $p_1(1 - F(p_1))$, which has a unique optimum.

Lemma 1 states that the seller will never give any rents to the type with the lowest net valuation $\underline{u}$. Proposition 1 uses this to show that type $\underline{u}$ should take the outside option rather than delay, causing the set of waiting buyers to unravel from the bottom. Since there is no delay for any initial price p_1, the seller's problem collapses to the static problem.

Proposition 1 has interesting implications for the theory of monopoly pricing. It suggests that sellers may offer a range of heterogeneous products in order to help them commit to high prices. If a monopolist sells a single product, it might be tempted to cut its price over time via the Coase conjecture. If it also has a second product that appeals to low-value customers, this will provide an outside option, clearing out the low end of the market and helping it to commit to the high price. In addition, the result shows that the seller makes the same profits as if it committed to prices at time $t = 1$. Hence, with outside options, the sequentially optimal mechanism can be implemented in prices, as in Skreta (2006), and coincides with the optimal commitment mechanism.[5]

In the model, we suppose there is a single buyer, but one can obtain the same result when the seller sells a continuum of goods to a continuum of buyers with different values and outside options. As elucidated by Gul, Sonnenschein, and Wilson (1986), this alternative formulation requires that seller's and buyers' actions are the same for any two histories that differ only in acceptance/exit choices of a measure-zero set of buyers. While their model does not allow for an exit option, the analysis of the equivalence of the single-buyer and continuum-of-buyers formulations is unchanged.

We now consider two extensions of the basic model.

A. *Reversible Exit*

In the baseline model we assumed that if a buyer chooses to exercise the outside option, the game ends and the buyer never returns. This is the right model if the buyer purchases a substitute; however, if the buyer simply abandons negotiations then he might return at a later date (e.g., the tourist at the bazaar who wishes to see local sights). Assuming that the buyer receives flow payoff of $(1 - \delta)w$ when outside the market and that the seller can see whether the buyer is present, Proposition 1 applies, and the seller chooses a single monopoly price. The idea is the same as before: the lowest type $\underline{u}$ expects to receive no surplus, so there is no point ever returning to the negotiations. To prove the result formally, one can restrict the claim of Lemma 1 to histories at which the buyer is present.

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[5]The result also suggests that the introduction of outside options has applications outside the simple Coase conjecture setting: for example, when selling one good over time to multiple buyers (e.g., McAfee and Vincent 1997), the Myerson auction should be the unique sequentially optimal mechanism.

B. *Types with No Outside Option*

The baseline model also assumed that the buyer's outside option was bounded below by $\underline{w} > 0$. To the contrary, if all types' outside options were equal to zero, there is a standard Coasian equilibirum, with prices falling over time. One might be concerned that if there were only a few such zero-option types, then it is no longer sequentially optimal for the seller to charge the monopoly price in all periods since we would be in a standard Coasian setting once all the high-option types have exited. However, we show below that the seller's profits change continuously when we add zero-option types.

To gain some intuition, consider the special case where the buyer has $w = 0$ with probability α and a constant $w > 0$ with probability $1 - \alpha$, and suppose $v \in [\underline{v}, \overline{v}]$ with $\underline{v} > 0$, so we can use backward induction to construct a pure strategy PBE.[6] For any initial price p_1, types with values above some cutoff x_1 purchase immediately, and high-option types with values below some cutoff z exit, leaving $[z, x_1]$ high-option types and $[\underline{v}, x_1]$ zero-option types in the market. Given subsequent prices $\{p_2, p_3, \dots\}$ and cutoffs $\{x_2, x_3, \dots\}$, buyer z is determined so that he is indifferent between exiting at time $t = 1$ and buying at time $\tau := \min\{t : x_t \leq z\}$, i.e., $w = \delta^{\tau-1}(z - p_\tau)$. Since z is indifferent between staying and exiting the market at $t = 1$, types $[z, x_1]$ strictly prefer to buy or remain in the market in later periods in any equilibrium, allowing us to ignore the exit constraints in periods $t \geq 2$. For any given z we can thus characterize prices and cutoffs via backwards induction, as in Gul, Sonnenschein, and Wilson (1986, Theorem 1).

As $\alpha \to 0$, the mass of delaying types converges to zero, and profits converge to those in the static problem. To see why, suppose this were not the case and the set of delaying high-option types converged to $[z, x_1]$ with $x_1 > z$. As $\alpha \to 0$, the seller would work through these types increasingly slowly as the incentive to drop the price below z and sell to the zero-option types decreases. Specifically, since selling to type z would cause a discrete fall in the price, the seller would prefer to subdivide any set $[z, x_t]$ if α is sufficiently small, postponing the price drop and raising revenue from (most of) the remaining high-option types. As a result, $\tau \to \infty$ and type z would prefer to exit in period $t = 1$, contradicting the assumption that a positive mass of high-option types delays.

We now formally show that the equilibrium payoffs are continuous in the distribution of types. We derive this continuity result in a discretized version of the model in order to avoid measurability issues arising in games with a continuum of types and actions.[7] As before, the seller chooses prices over time, and the buyer chooses whether to buy, wait, or take the outside option. The buyer privately knows his value for the good $v \in V \subset [0, \overline{v}]$ and the outside option $w \in W \subset [0, \overline{w}]$, where $\overline{v}, \overline{w}$ are finite. What changes is that (i) we allow the outside option to take value zero, and (ii) we discretize the type and action spaces: we restrict the sets V and W to be finite, and assume that there is a lowest monetary unit p_0 and the seller is restricted to post prices $p \in \{kp_0 \mid k = 0, 1, 2, \dots\}$.

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[6] If $\underline{v} = 0$, then there may be non-Markovian equilibria with higher profits, as in Ausubel and Deneckere (1989).

[7] We would like to thank one of the referees for suggesting the discretized approach to the continuity result.

Consider a sequence of cdf distributions G^α of (v, w) such that $G^\alpha \to G^0$ in distribution as $\alpha \to 0$. Assume that the limit distribution G^0 has support contained in $[0, \bar{v}] \times [\underline{w}, \bar{w}]$ for some $\underline{w} > 0$ and has a unique static monopoly price.[8] When defining a PBE, we assume that the seller puts positive probability on buyer's types only in the support of G^α; as a result, the model under G^0 is a special case of the baseline model. Such an equilibrium exists for any α by Fudenberg and Levine (1983, Theorem 6.1). In addition, under the limit distribution G^0, an analog to Proposition 1 holds when the price grid is sufficiently fine, implying that there is an essentially unique equilibrium. We then have:

PROPOSITION 2: *Assume p_0 is sufficiently small. As $\alpha \to 0$ the PBE payoffs of the buyer and seller converge pointwise to the unique PBE payoff profile at $\alpha = 0$.*

The proof can be found in the Appendix.

Proposition 2 asserts that the PBE payoffs are upper hemi-continuous as $\alpha \to 0$. While true in our model, this is not a general property of PBE; for example, in Spence's signaling model, payoffs are discontinuous in a separating equilibrium as the mass of low types shrinks to zero. The general problem is that while the limit of equilibrium strategies is optimal given limit beliefs, the PBE beliefs may fail upper hemi-continuity. In our setting, this arises because the limit beliefs at some histories may put positive probability on $w < \underline{w}$ as $\alpha \to 0$, which is inconsistent with beliefs at $\alpha = 0$. We thus prove Proposition 2 by showing that Proposition 1, and its discretized analog, rely only on histories where beliefs are consistent in the limit as $\alpha \to 0$. Roughly, we suppose that high-option types delay with positive probability, prove the seller's beliefs are consistent on these histories, and show that the high-option type with the lowest net value would then prefer to exit.

Proposition 2 considers what happens as $\alpha \to 0$, for a fixed δ. However, the order of limits matters. As $\delta \to 1$ for fixed α, monopoly pricing remains an equilibrium, but there is another equilibrium with Coasian dynamics. To understand why, observe that, in a pure strategy equilibrium, buyers either exit in period $t = 1$ or stay until they buy. If any buyers choose to remain, the $t \geq 2$ subgame is analogous to a standard Coase model, and prices will converge to the lowest net value as $\delta \to 1$, as in Gul, Sonnenschein, and Wilson (1986, Theorems 3–4).[9]

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[8] In order for there to be a unique PBE under G^0, there must be a unique equilibrium to the static game (in which a buyer must either buy the good or take the outside option). With a continuous price space we need only assume there is a unique monopoly price; if there is an atom of buyer types they must all buy, else the seller will lower its price. With discrete prices and types, the same is true if the price grid is sufficiently fine. Let p^* maximize $p(1 - F^0(p))$ on the grid of prices, where $F^0(p)$ is the measure of net values $v - w$ strictly below p in the limit distribution G^0. If there were an atom at p^* of size m, and $p_0 \leq mp^*$, then there would be at least two monopoly prices, p^* and $p^* - p_0$, depending on how we break the tie at the atom. Hence, the assumption of a unique monopoly price implies that p^* does not lie on the grid of net values.

[9] There is a subtlety here that is not present in the standard Coase model since the period $t \geq 2$ distribution is endogenous, determined by the buyers who delay in $t = 1$. In particular, the increase in δ raises buyers' continuation values in the $t \geq 2$ continuation game, and thus more buyers delay for a fixed p_1. While this may affect the rate prices fall as $\delta \to 1$, the Cosian dynamics can be supported in the $t \geq 2$ subgame even in the worst-case scenario where all high-value buyers delay, i.e., $x_1 = \bar{v}$, while the introduction of more low-value buyers speeds up convergence.

II. Search

In Section I we examined a single seller facing a single buyer with outside option w. We now endogenize the outside option by considering a model of sequential search and show that there is a unique equilibrium where sellers charge the monopoly price against their residual demand.

The market consists of mass one of ex ante identical sellers, each with zero marginal cost, and mass one of buyers. A buyer has private value at seller i, denoted v_i; these values are constant across time and i.i.d. across sellers with continuous density $f(\cdot)$ and distribution $F(\cdot)$ on $[0, \bar{v}]$.

Each period proceeds as follows:

  (i) A buyer chooses to stay with his current seller or picks a new one at random;

 (ii) If a buyer arrives at a new seller, he observes his value v_i at that seller;

(iii) The seller quotes a price to the buyer; we can either assume that seller i observes the entire history of the market or just the history at seller i. Denote the price sequence charged by seller i by $\{p_1^i, p_2^i, \ldots\}$.

Since $\delta < 1$, seller j has some market power and can guarantee itself a strictly positive profit. That is, even if the prices of all sellers (including itself) are zero in future, the seller adds value $v - \delta \max\{v, w\}$ when $v > \delta w$ and can thus make positive profits, where $w < \bar{v}$ is the value of moving onto another seller. Since the seller earns positive profits and the density of values is continuous, seller j will thus charge $p_1^j < \bar{v}$ in any equilibrium, and the buyer will earn strictly positive utility in expectation. Hence, when facing seller i, the buyer has a strictly positive outside option, $w > 0$, allowing us to apply Proposition 1 and conclude that in any PBE each seller i charges a constant price p_i. It is then straightforward to solve for the optimal prices:

PROPOSITION 3: *Suppose the hazard rate $f(v)/[1 - F(v)]$ is increasing in v. There is an essentially unique equilibrium in which each seller charges a constant price p satisfying*

$$(1) \qquad\qquad\qquad p\left[\frac{f(v^*)}{1 - F(v^*)}\right] = 1.$$

A buyer purchases if $v \geq v^*$ and otherwise moves on to another seller. The cutoff v^* satisfies

$$(2) \qquad\qquad\qquad v^* - p = \delta E_v[\max\{v - p, v^* - p\}].$$

PROOF:

Suppose seller i chooses p_i and other sellers charge some distribution of prices inducing continuation value w. We can define a cutoff v_i^* where a buyer is indifferent

between buying from seller i and continuing his search, i.e., $v_i^* - p_i = \delta w$. Seller i's profit per buyer is thus proportional to

$$\Pi = p_i[1 - F(v_i^*)] = p_i[1 - F(\delta w + p_i)].$$

Profits are weakly negative at the boundaries where $v_i^* \in \{0, \bar{v}\}$, so the optimal cutoff v_i^* must be interior. Differentiating then yields the necessary first-order condition

$$p_i f(\delta w + p_i) - (1 - F(\delta w + p_i)) = 0,$$

which is sufficient because the objective is log-concave. Since $f(\cdot)$ has increasing hazard, this equation has a unique solution, implying that any equilibrium must be symmetric, $p_i = p$ for all i. In such a symmetric equilibrium the f.o.c. yields (1) as required. The equilibrium cutoff itself satisfies $\delta w = v^* - p$, or (2). The equilibrium is unique: an increase in p raises the cutoff and, since $f(\cdot)$ has increasing hazard rate, strictly raises the left-hand side of (1).

  With competition via sequential search, the Coase conjecture fails, and sellers set a constant price; in equilibrium the sellers all choose the same price. The pricing formula (1) says that when seller i lowers its price by ϵ, it loses ϵ on all its current customers (measure 1) but gains sales from the marginal customers (measure $f(v^*)$) of all those who visit it $\left(\text{measure } 1/(1 - F(v^*))\right)$.
  An increase in δ raises the competition between sellers, lowering prices p. Observe that equation (2) implies that $v^*(p; \delta)$ increases in $(p; \delta)$ pointwise. Hence, an increase in δ implies that p must fall in order for (1) to be satisfied with equality. In the limit, as $\delta \to 1$ we obtain perfect competition: $v^*(p; \delta) \to \bar{v}$ uniformly across $p \geq 0$, implying that $f(v^*(p; \delta))/\left(1 - F(v^*(p; \delta))\right) \to \infty$ and the equilibrium price converges to 0.[10] From a buyer's perspective, limited competition reduces his utility vis-a-vis no competition since the unwillingness of fellow buyers to wait helps sellers commit to high prices; however, buyers still benefit from lots of competition.
  One can extend this result to allow for correlation of buyers' values across sellers. Assume buyers have one of finitely many private types $\theta \in \Theta$ with probability mass function $g(\theta)$ and values that are conditionally i.i.d., $v_i \sim f(\cdot \mid \theta)$ with full support on $[0, \bar{v}]$. Over time, buyers with higher types tend to leave the market sooner; however, if seller i's prices depend only on the history at seller i (which is analogous to the sellers not knowing in which order the buyers visit them), then Proposition 3 extends, and the unique equilibrium price is given by

$$p \sum_{\theta \in \Theta} \left[\frac{f(v^*(\theta) \mid \theta)}{1 - F(v^*(\theta) \mid \theta)} \right] g(\theta) = 1,$$

where the cutoff for type θ is determined by $v^*(\theta) - p = \delta E_{v \mid \theta}[\max\{v - p, v^*(\theta) - p\}]$.

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[10]The fact that $f(v)/[1 - F(v)] \to \infty$ follows from the assumption that the hazard rate is increasing, so a limit exits, and that v has finite support. With infinite support, this condition may not hold (e.g., with an exponential distribution, where the hazard rate is constant).

III. Conclusion

This article considered a classic Coase pricing game, where buyers have an outside option that they may exercise each period, either abandoning their search or moving on to another seller. The outside option leads low-value buyers to exit the market rather than delay consumption, countering the negative selection that drives the Coase conjecture. This has a stark effect—there is a unique equilibrium in which the seller charges the monopoly price in every period, and buyers either immediately purchase or exit. By embedding the single-seller model into a search framework and endogenizing the outside option, this result also simplifies pricing in a search model, providing a foundation for the traditional price-posting assumption.

The article has practical implications when studying durable-goods firms in contexts where buyers have outside options. For example, in monopolization cases, our model might make one more skeptical of firms using Coasian logic to argue that they face competition from their past selves (e.g., *United States v. Alcoa*, 148 F.2d 416 (2d Cir. 1945)). And in merger cases, the result illustrates how a requirement that merging firms licence their product to a competitor (e.g., the Borland and Ashton-Tate 1991 merger) may affect buyers' outside options and the merged firm's commitment power.[11]

There are a number of interesting ways to extend the model. First, outside options may arrive as the game proceeds, and this arrival may or may not be observable to the seller. For example, Apple first released the iPhone in June 2007 and had an effective monopoly on smartphones until the Android launched in October 2008. Second, buyers may enter the market over time, mitigating the unravelling force analogous to Section IB. Third, in the search model, there might be Diamond-style switching costs when moving between sellers, meaning that while some buyers switch, others will wait for a price drop.

APPENDIX: PROOF OF PROPOSITION 2

A. *Preliminaries*

Consider a sequence of distributions G^α of (v, w) such that $G^\alpha \to G^0$ (in distribution) as $\alpha \to 0$. Assume that the limit distribution G^0 has support contained in $[0, \bar{v}] \times [\underline{w}, \overline{w}]$ for some $\underline{w} > 0$ and has a unique monopoly price. We then wish to show that PBE payoffs under G^α converge to the unique PBE payoffs under G^0 as $\alpha \to 0$.

The proof of Proposition 2 is based on the observation that the set of payoff profiles and the set of strategy-belief profiles is compact. The compactness implies that it is enough to show that all convergent sequences of strategy-belief profiles have the same limit payoffs. If we knew that the limit strategy-belief profile is a PBE, we could then invoke a discretized analog of Proposition 1 to show the uniqueness of limit payoffs. The problem is that a limit of PBEs may not be a PBE itself: in the limit, buyers' actions are optimal given beliefs, but the limit beliefs may fail to

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[11] See Orbach (2004) for discussion of the Alcoa case, and Katz and Shapiro (1999) for Borland and Ashton-Tate.

satisfy the PBE consistency condition. That is, at some histories h, the limit belief may put positive probability on buyer types outside the support of G^0 (i.e., with $w < \underline{w}$). We overcome this problem by showing that an analog of Proposition 1 obtains when the limit beliefs are consistent on a large set of histories we call H, and then proving that the beliefs are indeed consistent on H.

Let us first introduce some notation and define the set H. At public history h, denote by $\sigma_s(h)$ the strategy of the seller, by $\sigma_{v,w}(h)$ the strategy of the buyer of type (v, w), and by $\lambda(h)$ the belief of the seller over buyer types. In particular, $\sigma_{v,w}(h)(a)$ is the probability that buyer of type (v, w) takes action a at history h, and $\lambda(h)(v, w)$ is the probability buyer has type (v, w) at history h. We denote the profile of buyers' strategies by $\sigma = (\sigma_s, (\sigma_{v,w})_{(v,w) \in V \times W})$. For any profile of strategies σ and beliefs λ, we define $H = H(\sigma, \lambda)$ recursively as the smallest set of histories such that:

(i) The empty history $\oslash$ belongs to H.

(ii) If $h \in H$ and a is the seller's action at h, then $(h, a) \in H$.

(iii) If $h \in H$ and a is the buyer's action at h, then $(h, a) \in H$ if and only if the belief $\lambda(h)$ puts strictly positive probability on buyer types (v, w) that take action a with positive probability according to the strategy $\sigma_{v,w}$, that is

$$\sum_{(v,w) \in V \times W} \lambda(h)(v, w) \cdot \sigma_{v,w}(h)(a) > 0.$$

Intuitively, set H allows for deviations by the seller but does not allow buyers to take actions outside the support of $\sigma_{v,w}$.

We then say that a profile of strategies σ and beliefs λ is a *quasi-PBE* if (i) the buyers' strategies are optimal given beliefs at any history, and (ii) the seller updates her beliefs according to the Bayes' rule at histories from set H. Importantly, at histories outside H the quasi-PBE allows the seller to put positive probability on any type in $V \times W$, including types not in the support of $\lambda(\oslash)$. Of course, any PBE is a quasi-PBE, but a quasi-PBE does not insist on applying Bayes' rule after the buyer has taken a zero-probability action. In a one-period game, the two concepts coincide.

Next we formulate and prove the analog of Proposition 1 for quasi-PBE in the discretized model.

LEMMA 1*: *Suppose p_0 is sufficiently small. In any quasi-PBE, if the seller believes that the buyer's net value is above $\underline{u}$ at some history from H, then she charges prices above $\underline{u} - p_0$ at that history.*

The proof of Lemma 1* is similar to the proof of Lemma 1, except we wish to show that $\underline{p}(h) \geq \underline{u}(h) - p_0$ for all $h \in H$, rather than $\underline{p}(h) \geq \underline{u}(h)$ for all h, defining $\Delta = \sup_{h \in H}(\underline{u}(h) - \underline{p}(h) - p_0)$ accordingly. To do this we suppose, by contradiction, that $\Delta > 0$. For any h_t where Δ is nearly attained, we claim that any price $p \leq \underline{p}(h_t) + p_0 + \epsilon \Delta$ leads the buyer to purchase at time t, which guarantees that there exists a price on the grid to which the seller can defect. To prove the buyer

prefers to buy at time t rather than time $s > t$, we suppose, by contradiction, that some mass of buyers does delay after the deviation. Since the seller deviates, while buyers are playing equilibrium strategies, the history h_s is contained in H, allowing us to use Bayes' rule. The algebra is then similar to Lemma 1, with the additional assumption that $p_0 \leq \frac{1-\delta}{\delta} \underline{w}$.

PROPOSITION 1*: *Consider distribution G^0 and suppose p_0 is sufficiently small. There is a quasi-PBE (and PBE) in which the buyer and seller use monopoly strategies. This quasi-PBE (and PBE) is essentially unique.*

The essential uniqueness of quasi-PBE is a stronger property than the essential uniqueness of PBE. For instance, if the probability of the low type is zero in the Spence signaling model, then there is a unique PBE but multiple quasi-PBEs. For this reason, the PBE payoffs in Spence's model are not upper hemi-continuous even though the quasi-PBEs payoffs are upper hemi-continuous. Indeed, our proof below applies to a broad class of games and may have other applications beyond the current model.

The proof of Proposition 1* is largely the same as Proposition 1. For any initial price p_1, if a positive mass of types delay then the resulting history $h_2 \in H$ leads to a lowest type $\underline{u}(h_2)$ that delays. Furthermore, by Bayes' rule $\underline{u}(h_2)$ is a lower bound on net values at all continuations of history h_2 that belong to H, and the buyers put probability 1 on continuation histories from H. Thus Lemma 1* implies that type $\underline{u}(h_2)$ expects to obtain at most $\delta(v - \underline{u}(h_2) + p_0)$ and, if $p_0 < \frac{1-\delta}{\delta} \underline{w}$, strictly prefers to exit and obtain $\underline{w}$ instead. Intuitively, if the lowest net value buyer delays he obtains rents at most δp_0, but he gives up the rental value of the outside option $(1 - \delta)w$. Hence, all buyers purchase immediately or exit for any initial p_1 in any quasi-PBE. Since we assumed that there is a unique monopoly price and the price grid is sufficiently fine, there is an essentially unique static PBE (see footnote 8), and an essentially unique quasi-PBE to the entire game.

B. *Proof of Proposition 2*

We endow the set of profiles of strategies and beliefs (σ, λ) with the standard product topology. Since all buyer types' net values are bounded above by $\bar{v}$ we can restrict attention to strategies in which all prices come from the compact subgrid $\Gamma = \left\{ kp_0 \mid k = 0, 1, 2, \ldots \lfloor \frac{\bar{v}}{p_0} \rfloor + 1 \right\}$, where $\lfloor x \rfloor$ denotes the largest integer such that $\lfloor x \rfloor \leq x$. For each history h, the seller's action $\sigma_s(h)$ takes value in the compact set of lotteries $\Delta(\Gamma) \subset [0, 1]^{\Gamma}$; any of the finite number of types $(v, w) \in V \times W$ takes actions $\sigma_{v,w}(h)$ from the compact set of lotteries $\Delta(\{\text{exit,buy,wait}\}) \subset [0, 1]^{\{\text{exit,buy,wait}\}}$; and the seller's belief lies in $\Delta(V \times W) \subset [0, 1]^{V \times W}$. Thus, for a fixed history h the set of strategy-belief profiles is compact in the standard topology on $[0, 1]^{\Gamma} \times [0, 1]^{\{\text{exit,buy,wait}\} \times V \times W} \times [0, 1]^{V \times W}$. By Tychonoff's Theorem, the set of strategy-belief profiles is compact in the product topology over histories h.

Given a strategy-belief profile (σ, λ), let $\Pi = (\Pi_s, (\Pi_{v,w})_{(v,w) \in V \times W})$ be the profile of expected payoffs at the null history $\oslash$. Observe that payoffs Π are continuous in

(σ, λ). This follows because discounting and the boundedness of per-period payoffs imply that periods in the far future have a negligible impact on initial payoffs. Thus product-topology convergence of strategy-belief profiles on any finitely long set of histories implies the convergence of payoffs obtained at these histories.

Let $\Pi^* \in \mathbb{R} \times \mathbb{R}^{V \times W}$ be the unique profile of expected payoffs that, by Proposition 1*, obtains in all quasi-PBE under G^0. We wish to show that any sequence of strategy-belief profiles $(\sigma^\alpha, \lambda^\alpha)$ that form a PBE for initial belief G^α induces payoffs Π^α that converge to Π^* in the standard topology on $\mathbb{R} \times \mathbb{R}^{V \times W}$ as $\alpha \to 0$. Let us initially consider the case when the sequence of strategy-belief profiles is convergent.

Claim: If the sequence of strategy-belief profiles $(\sigma^\alpha, \lambda^\alpha)$ converges as $\alpha \to 0$, then the sequence of payoff profiles Π^α converges to Π^* in the standard topology on $\mathbb{R} \times \mathbb{R}^{V \times W}$.

To prove the claim denote by (σ^0, λ^0) the limit of $(\sigma^\alpha, \lambda^\alpha)$ as $\alpha \to 0$. Let us show that (σ^0, λ^0) is a quasi-PBE. First, as the payoffs are continuous in the strategy-belief profile, the Maximum Theorem implies that the best responses are upper hemi-continuous, i.e., if seller's strategy σ_s^α is a best response to $((\sigma_{v,w}^\alpha)_{(v,w) \in V \times W}, \lambda^\alpha)$ for all α, then σ_s^0 is a best-response to $((\sigma_{v,w}^0)_{(v,w) \in V \times W}, \lambda^0)$, and similarly for buyer's strategies. Hence, given λ^0, the strategies σ^0 are mutual best responses. Second, to show that the limit profile satisfies the consistency conditions of a quasi-PBE, we need to check the seller updates his beliefs according to Bayes' rule at histories from the set $H = H(\sigma^0, \lambda^0)$:

(i) At the empty history the seller's belief is $\lambda^0(\oslash) = \lim_{\alpha \to 0} \lambda^\alpha(\oslash)$ $= \lim_{\alpha \to 0} G^\alpha = G^0$ and, hence, agrees with the distribution of buyer types G^0.

(ii) If $h \in H$ and a is the seller's action at h, then beliefs λ^α are unaffected by the choice of a, and so $\lambda^0 = \lim_{\alpha \to 0} \lambda^\alpha$ are also unaffected.

(iii) If $h \in H$ and a is the buyer's action at h such that

$$\sum_{(v,w) \in V \times W} \lambda^0(h)(v, w) \cdot \sigma_{v,w}^0(h)(a) > 0,$$

then $\sum_{(v,w) \in V \times W} \lambda^\alpha(h)(v, w) \, \sigma_{v,w}^\alpha(h)(a) > 0$ for α close to 0. Since the denominator is strictly positive, the continuity of Bayes' rule implies

$$\lambda^0(\{h, a\})(v, w) = \lim_{\alpha \to 0} \lambda^\alpha(\{h, a\})(v, w)$$

$$= \lim_{\alpha \to 0} \frac{\lambda^\alpha(h)(v, w) \cdot \sigma_{v,w}^\alpha(h)(a)}{\sum_{v,w} \lambda^\alpha(h)(v, w) \cdot \sigma_{v,w}^\alpha(h)(a)}$$

$$= \frac{\lambda^0(h)(v, w) \cdot \sigma_{v,w}^0(h)(a)}{\sum_{v,w} \lambda^0(h)(v, w) \cdot \sigma_{v,w}^0(h)(a)}.$$

The limit beliefs λ^0 are thus consistent with the limit strategies σ^0, and the limit strategy-belief profile (σ^0, λ^0) is a quasi-PBE. By Proposition 1*, the limit strategy-belief profile induces payoffs Π^*. As the payoffs are continuous in the strategy-belief profile, so the sequence of payoffs Π^α converges to Π^*. This proves the claim.

Now, consider the general case when the sequence of strategy-belief profiles $(\sigma^\alpha, \lambda^\alpha)$ need not be convergent. By way of contradiction, assume that Π^α does not converge to Π^*. Then there is an open neighborhood $U \subseteq \mathbb{R} \times \mathbb{R}^{V \times W}$ of Π^* and a subsequence of $(\sigma^\alpha, \lambda^\alpha)$ such that the payoff profiles along the subsequence never belong to U. Since the space of strategy-belief profiles is compact, this last subsequence has a convergent subsubsequence $(\sigma^{\alpha_n}, \lambda^{\alpha_n})$. Yet the above claim implies that since $(\sigma^{\alpha_n}, \lambda^{\alpha_n})$ converges, so the corresponding payoffs Π^{α_n} converge to Π^*, yielding a contradiction. Hence, the payoffs of any sequence of PBEs converge to the unique quasi-PBE payoffs Π^*, which is also the unique PBE payoff.

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