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Bargaining and Search with Incomplete Information about Outside Options

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This paper considers a model of bargaining in which the seller makes offers and the buyer can search (at a cost) for an outside option; the outside option cannot be credibly communicated, and the seller's offer is recallable by the buyer for one period. There are essentially two equilibrium regimes. For sufficiently high search cost, the game ends immediately; otherwise the search occurs in equilibrium. Compared to the case where the buyer can communicate his outside option, the seller is worse off, and the game results in search for a smaller set of values of the search cost, i.e., less equilibrium delay. *Journal of Economic Literature* Classification Number: C72.

1. INTRODUCTION AND THE MODEL

The existing literature on extensive form models of bargaining has focused on two main strategic factors to explain the outcomes of negotiation, namely, incomplete information and time preference. These two

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categories (which, by the way, have a nonempty intersection) have addressed the issues of disagreement and delay in negotiations between rational players and have sought to draw connections with axiomatically derived solution concepts, like the Nash solution. Examples of the incomplete information work include Chatterjee and Samuelson (1983), Satterthwaite and Williams (1989), Gresik (1991) and the related mechanism design literature of Myerson and Satterthwaite (1983), and others. The time preference aspect is addressed in Rubinstein (1982) and a long line of work proceeding from his paper. Applications tend to use the Rubinstein result, because of its simplicity and its connection with Nash bargaining, even though time preference may not be the real determinant of the allocation of gains from trade in any particular application. This latter point has been made especially forcefully by Hugo Sonnenschein in his presidential address to the Econometric Society in 1989.

A third strategic factor that is generally recognized to be important is a bargainer's ability to generate attractive outside options. (See Raiffa (1982) for a description of the role of "Best Alternatives to the Negotiated Agreement" (BATNA).) This paper and Lee (1994) focus on the analysis of this factor; other recent papers that have done so are Chikte and Deshmukh (1987), Wolinsky (1987), Bester (1988), and Muthoo (1995). There are also related papers that explore the role of fixed, exogenous outside options (Shaked and Sutton, 1984; Binmore, et al., 1989; Shaked, 1987; for example), outside options endogenized through a coalitional bargaining process (Binmore, 1985; Chatterjee and Dutta, 1995, or switching partners (Fudenberg et al., 1987).

The papers by Chikte and Deshmukh (1987), Wolinsky (1987), Bester (1988), and Muthoo (1995) consider bargainers searching for alternatives if the current offer is rejected. Offers cannot be recalled if they are not accepted immediately; as a consequence, search never takes place in equilibrium, since the offerer takes into account a player's search ability in making him or her an offer that cannot be refused.

This finding appears to run counter to the coexistence of search and bargaining in many markets. For example, in the market for senior and middle-level academics, search appears to be a frequently used device to increase the salary or other benefits obtainable from a person's home institution. For those buyers who dislike haggling with car dealers, to give another example, an alternative to the offer/counter-offer process would be to come up with a price quote from either a national pricing service or another dealer.

Both examples above involve recalling offers made by the current negotiating partner as well as outside offers. The scenario at the car dealer's could proceed in the following way, as could the scenarios of other bilateral buyer-seller negotiations. The buyer visits a dealer who first explains the range of cars available at great length. (The delay caused by this is unmodeled in this paper.) The dealer then writes down a price quote for a particular car; the buyer can then make a counter-offer and bargain in the usual Rubinstein (1982) format or leave the dealership to seek an alternative offer. The alternative offer could be from a pricing service (offered, for example, by a well-known credit card company) or a price from another dealer. Either of these has a cost associated with generating it. With the new offer in hand, the buyer can go back and ask for a new offer from the seller. There is some chance that the specific car will have been sold, so that the offer is not recallable. In fact, there is some nonstationarity in the process whereby offers are generated; a second search for a price quote for say, Car X, is more likely to come up blank than the first one, because no more Cars X are available. There is some pressure therefore to conclude agreement. Sometimes there could be a deadline in time, or, say, just two Car X dealers in the vicinity. Search beyond these two then involves a substantially increased cost of travelling to another town, as inhabitants of rural areas will testify.

The car dealer story motivates both the model of this paper and that of Lee (1994). These papers have (i) recall, (ii) one-sided offers, and (iii) a finite (two-period) horizon. Where this paper differs from Lee's companion paper is in the informational assumption about the search outcome—Lee assumes it to be known to both the buyer and the original seller, while this paper considers the case where this is the buyer's private information. Private information about search outcomes could arise if the other dealer refuses to write down a quote. (In the academic market example, this is more common as many schools will not go through the cost of processing an offer without an informal acceptance.) Even if an alternative salary offer is made (in the academic market), this may not reflect the entire offer "package," which would contain subjective elements related to the individual's evaluation of the work and living environment. In the example of the car dealer, the two cars may differ in one or more features, perhaps in color, and an individual's valuation of this difference may be private information. In addition, it is rare that a dealer will put down a written offer in a verifiable format.

The Model

These examples serve to motivate our choice of model. The model is as follows. There are two players, B, the buyer, and S, the seller, with commonly known reservation prices for the item owned by S. These reservation prices are $b = 1$ for the buyer and $s = 0$ for the seller, respectively, so the presence of gains from trade is common knowledge. For convenience, B and S will sometimes be referred to as "he" and "she" in this discussion. S makes an offer p_1 to B in period 1. If B accepts, the payoffs are p_1 to S and $1 - p_1$ to B. B can reject and either search (while still holding on to the offer p_1) or ask for another offer directly without search. (It is immaterial in equilibrium whether the search activity is observed by the seller, since the seller can figure out the equilibrium strategy of the buyer.) However, we assume that search can be observed, even though its outcome cannot.

If B searches at cost c, an offer x_1 is drawn from a commonly known absolutely continuous distribution $F(\cdot)$ with density function $f(\cdot)$. $F(\cdot)$ is assumed to have support on [0, 1]. B can now accept x_1, accept p_1, or ask for another offer. In the event of another offer being solicited by B, the offer p_1 lapses and an offer p_2 is made. Either p_2 or x_1 can now be accepted, although there is a small cost of recall—a small probability that x_1 has disappeared. If neither is accepted, B can search again. An independent draw generates x_2 (again at a cost c). Now the buyer has to accept one of p_2, x_1, x_2 or get a payoff of 0. If the buyer at any time accepts x_i, his payoff is $1 - x_i -$ the search cost, while S gets a payoff of 0. On the other hand, if the buyer at any time accepts p_i during period i, his payoff is $1 - p_i -$ the search costs (if any), while S gets p_i. A schematic representation of the extensive form being used is given in Fig. 1.

Depending on the search cost c, this two-period bargaining and search model has two types of equilibrium—one leads to immediate resolution of bargaining, the other involves both bargaining and search. If c is greater than a cutoff c^*, a search deterring offer p^* will be made by S and be accepted by B immediately. For c below c^*, S will offer a price greater than p^*, which will be rejected by the buyer initially and trigger the subsequent search and bargaining behavior.

The intuition behind the delay is as follows. The seller offers a price that is currently unacceptable but may be acceptable later if the outside option is not sufficiently attractive and the continuation expected payoff is decreasing over time. This is better for the seller than offering the maximum acceptable price (search-deterring offer) when c is very low.

A further point of interest in our model is the comparison of complete and incomplete information settings. First we note that, in the absence of any means of verifying outside options, there is no credible way of communicating the buyer's outside option to the seller, so the incomplete information setting is nonvacuous. (The difference between models where cheap talk plays a role (Farrell and Gibbons, 1989, for example), and our model here is that, although the seller wants the buyer to buy from her rather than to go outside, the buyer does not share this interest and just wants the best price, whether it is from an outside option or from the current seller.)

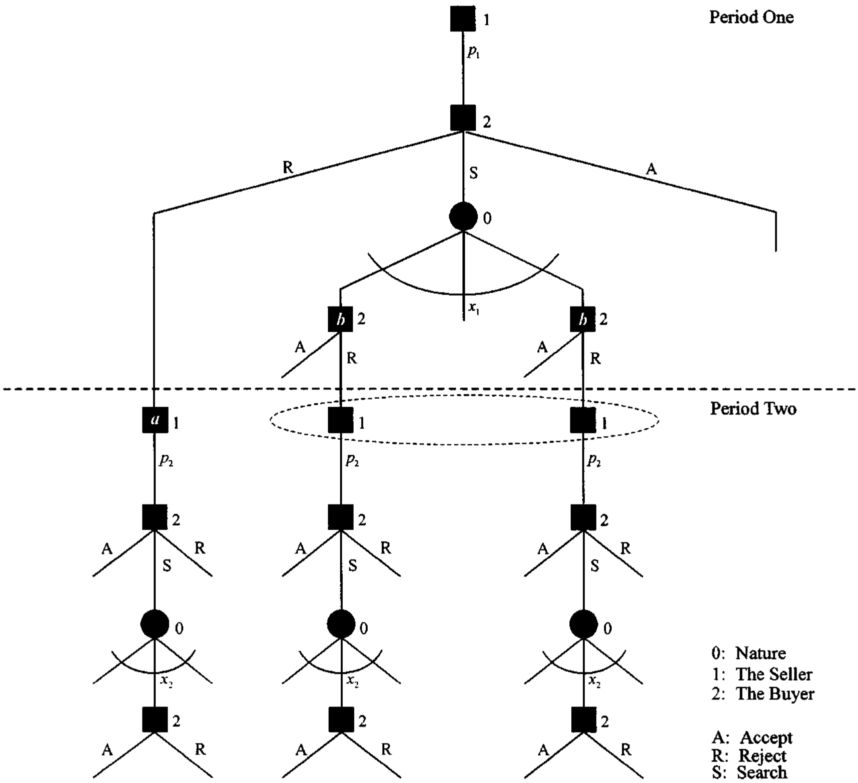

FIG. 1. The extensive form of the search bargaining game.

The complete information model is worked out by Lee (1994). Under complete information as well, there are two types of equilibria, depending on the search cost c. Above a cutoff c^{**}, the price p^* is offered and accepted. Below it, there is search and delay. Interestingly, $c^{**} > c^*$, so that the bargaining ends without delay for a greater range of values of c under incomplete information than under complete information. Furthermore, the seller, on average, is worse off under incomplete information than under complete information. We work out an example in which the buyer is also better off in expected value terms, but we have not been able to prove this in general.

The comparative results might be considered to be somewhat counterintuitive. Why should the buyer be made better off by not being able to communicate his outside option to the seller? The intuition is, however, as follows. In the complete information case, the seller always has the chance to match the buyer's outside option. Here, because of the private information, the seller cannot match the outside option, or the revealed outside option, without creating adverse incentives for the buyer. The chance of ‘‘leakage’’ (a buyer taking his outside option, this being the worst outcome for the seller) makes the seller offer lower prices on average in the low search cost case. (The price p^*, the search deterring price, is the same for both the complete and incomplete information cases.)

The rest of the paper is organized as follow. The next section sets out the equilibria of the two-period model. Section 3 provides discussions on the comparison between complete and incomplete information, along with some comments on extensions to infinite horizon settings. Section 4 concludes. Details of some of the longer proofs are relegated to the appendices.

2. THE ANALYSIS

Before we start the analysis, let us first describe the equilibria of the game. We restrict ourselves to equilibria involving pure strategies on the equilibrium path. The perfect Bayesian equilibria we obtain have the following characteristics on the outcome path. For a cost c above a cutoff c^*, S will offer a price p^* that is immediately accepted by the buyer. The price p^* is the largest price the seller can charge without inducing search by the buyer. The sufficient condition that ensures the above equilibrium is given later in Proposition 6A. Under some conditions on the distribution of outside offers and on what we shall argue to be implausible beliefs, however, a price lower than p^* could be charged (which also leads to immediate resolution of bargaining). This is discussed in Proposition 6B. Thus, for sufficiently high values of c, there could be nonuniqueness of equilibria; however, only one will survive the plausibility restriction.

For a cost c below the cutoff value c^*, we get a unique equilibrium in which the seller finds it optimal to ‘‘take her chances’’ by charging a price p_1 higher than p^* in the beginning. The buyer initially rejects and undertakes a search. If the search option x_1 is in the highest of three regions, the buyer accepts p_1. If it is in the intermediate region, the buyer asks for a second offer. In the lowest region, the buyer accepts the outside option and the game ends. If the second period is ever reached, the seller then charges another price p_2 higher than p^*, (interestingly, $p_2 > p_1$ in this equilibrium), and the buyer searches again. In this equilibrium, search and bargaining both occur, because of both ‘‘recall’’ and the fact that the game ends after two rounds. In addition, only pure strategies are used along the equilibrium path, although mixed strategies are needed off the equilibrium path to support the equilibrium. A formal description of this equilibrium is given in Proposition 7.

For clarity of the presentation, Σ is used to denote the complete game described in the previous section. In addition, Σ^a and Σ^b. are defined as follows:

Σ^a: The subgame starting at node a in Fig. 1 (i.e., the subgame starting at the beginning of the second period, given that the buyer does not search in the first period). Σ^a is essentially a single-period version of the full model.

Σ^b: The subgame starting at node b in Fig. 1 (i.e., the subgame starting at the end of the first period after the buyer has searched and has obtained an x_1).

We start our analysis by examining each player's behavior in Σ^a and Σ^b. Since the buyer's optimal behavior in the second period is the same in both Σ^a and Σ^b, we shall consider this case first.

2.1. *The Buyer's Behavior in the Second Period*

Let $s = \min(x_1, p_2)$. Notice that, for Σ^a, since the buyer does not search in period 1, $s = p_2$. For Σ^b, since the outside option x_1 is not observable by the seller, s is the buyer's private information.

Let $\Phi(s)$ denote the buyer's expected payoff if he searches in the second period. Then, since recall of previous offers is allowed,

$$\Phi(s) = -c + \int_0^s (1 - x) f(x)\, dx + \int_s^1 (1 - s) f(x)\, dx. \tag{1}$$

It is easy to see that $\Phi(s)$ is decreasing in s and takes its maximum value, $1 - c$, at $s = 0$, and its minimum value, $1 - c - \int_0^1 x f(x)\, dx$, at $s = 1$. We assume that $\Phi(s)$ is nonnegative throughout, so c is not "too large."

Now let p^* be the price that makes the buyer indifferent between searching and not searching. Then p^* must satisfy the following condition:

$$1 - p^* = -c + \int_0^{p^*} (1 - x) f(x)\, dx + \int_{p^*}^1 (1 - p^*) f(x)\, dx \equiv \Phi(p^*). \tag{2}$$

Since (2) can be rewritten as $c = \int_0^{p^*} F(x)\, dx$, there exists one and only one p^* satisfying (2). We now have the following lemma, which will be quite useful for our later analysis.

LEMMA 1. *Let* $s \in [0, 1]$. *Then* $1 - s > \Phi(s), \forall s < p^*$ *and* $1 - s < \Phi(s), \forall s > p^*$.

Proof. We can write $(1 - s) - \Phi(s) = c - \int_0^s F(x)\,dx$. Note that the expression is equal to c (> 0) at $s = 0$ and, since $\int_0^s F(x)\,dx$ is increasing in s, that it remains positive until $c - \int_0^s F(x)\,dx = 0$, at $s = p^*$. The same reasoning tells us that it then becomes negative for $s > p^*$. ■

Lemma 1 simply says that a search will take place in the second period if and only if $\min(x_1, p_2) > p^*$. Hence p^* is the cutoff price for a search to occur in the second period. We shall show later that p^* is also the cutoff price for a search to take place in the first period. In fact, Lemma 1 implies that, in any period within a multiperiod model, no price (including both the seller's and outside offers) larger than p^* will be accepted immediately. This is because, if the price is higher than p^*, the buyer can search at least once more, which yields an expected payoff higher than immediate acceptance of the price.

Given the buyer's optimal strategy in the second period, we can characterize the equilibrium of Σ^a and Σ^b. We shall start with Σ^a, the simpler case.

2.2. *The Equilibrium of* Σ^a

Since the buyer does not search in the first period, and since p_1 lapses when p_2 is offered, the seller's strategy in the second period does not depend on the history of Σ in the first period. The equilibrium then depends only on c. In Σ^a, the seller can offer p^*, knowing it will be accepted (by Lemma 1), and get a payoff of p^* (she certainly will not offer anything less than p^*); or she can offer a price p_2 greater than p^*, knowing the buyer will search (again, by Lemma 1) and expect to get $p_2[1 - F(p_2)]$ (since p_2 will only be accepted when $x_2 > p_2$).

Let $\bar{p}$ maximize $p_2[1 - F(p_2)]$. (An assumption is made below to guarantee that $\bar{p}$ is unique.) Then the seller's optimal choice of p_2 in Σ^a will be either p^* or $\bar{p}$, depending on which of p^* and $\bar{p}[1 - F(\bar{p})]$ is larger. In equilibrium, the seller offers p^* if $p^* \geq \bar{p}[1 - F(\bar{p})]$, and otherwise offers $\bar{p}$. In either case, the buyer's expected payoff for going to the second period directly without search in the first period is at most $1 - p^*$. We illustrate this in Fig. 2.

For technical reasons and to facilitate comparison of this model to the corresponding complete information case, the following assumption is made.

Assumption. $p[1 - F(p)]$ is strictly quasi-concave. This assumption guarantees that $p[1 - F(p)]$ is single-peaked and contains no flat portion in the interior of [0, 1]. Among the frequently used probability functions,

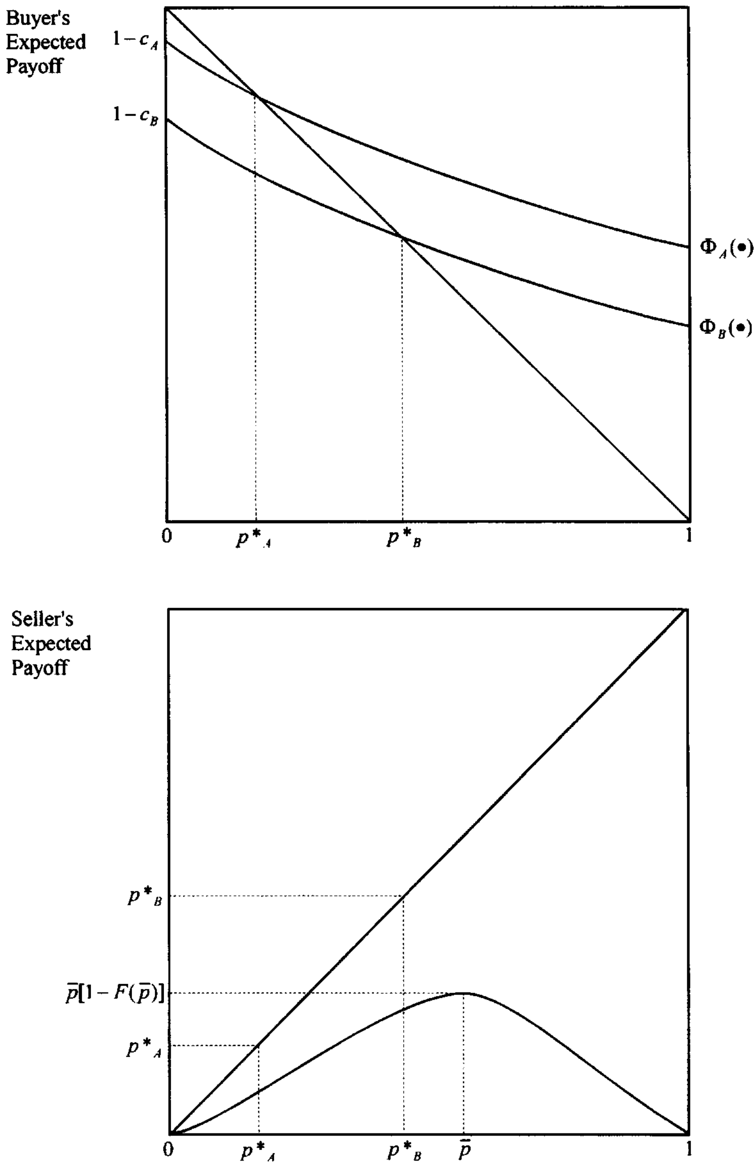

FIG. 2. Illustration of the equilibrium of Σ^a. **Remark:** If $c = c_A$, then the search-deterring price is p_A^*. In this case, since $p_A^* < \bar{p}[1 - F(\bar{p})]$, the seller will offer $\bar{p}$, and the buyer will search. If $c = c_B$, then the search-deterring price is p_B^*. In this case, since $p_B^* > \bar{p}[1 - F(\bar{p})]$, the seller will offer p_B^*, and the buyer will accept immediately.

for example, the family of power functions, $F(x) = x^m$ $(m > 0)$, satisfies this assumption.

2.3. *The Equilibrium of* Σ^b

The equilibrium of Σ^b is more complicated because it depends not only on c but also on p_1 and x_1. In addition, since x_1, is not observable by the seller, the seller must have a belief about x_1, and that belief, in equilibrium, must be consistent with the buyer's behavior in the first period.

There are two types of equilibria for Σ^b—one involves search (in the second period), the other does not. By Lemma 1, for the no-search equilibrium to arise, a price less than or equal to p^* must be offered by the seller in the second period, whereas for the equilibrium to involve a search, a price greater than p^* must be offered. Since p^* is increasing in c, the type of equilibrium that will arise then depends on c. The larger c is, the larger p^* is, and hence the higher is the seller's incentive for offering a search deterring offer. For c below a cutoff value, the seller will strictly prefer offering a price higher than p^* to offering a price below, and vice versa for c above the cutoff. In addition, the equilibrium also depends on p_1. If $p_1 \le p_2 \le p^*$, then, intuitively, the second period should not be reached since no better offer from the seller is expected in the second period. The following proposition considers this case.

PROPOSITION 1. *If $p_1 \le p^*$, the equilibrium strategies of the buyer and the seller in Σ^b are as follows: The buyer accepts p_1 at the end of the first period if $x_1 > p_1$, accepts x_1 if $x_1 \le p_1$. The seller offers $p_2 \ge p_1$ in the second period. The seller's beliefs are such that p_2 is optimal given these beliefs and that $p_2 \le p^*$. The buyer accepts immediately the minimum of x_1 and p_2 in the second period.*

Remark. This proposition is placed here for completeness. It covers two cases, which will be dealt with in detail in Propositions 6A and 6B. In one case, argued to be the only plausible case, $p_1 = p_2 = p^*$, and the seller's belief (given the buyer returns for the second period) is that x_1 is greater than p^*. Another case could potentially arise in which a price p° strictly less than p^* is optimal in the second period. Note that from Lemma 1, a second-period search cannot occur in either of these equilibria.

The remaining discussion on the equilibria of Σ^b will only consider the case when $p_1 < p^*$. Proposition 2 describes a no-search equilibrium of Σ^b under such a condition.

PROPOSITION 2. *Suppose $p_1 > p^*$. Suppose also $p^* \ge \max\ p_2[1 - F(p_2)]^2/[1 - F(p^*)]$. Then, the unique equilibrium of Σ^b is the following: The seller, believing $x_1 \in (p^*, 1]$ whenever the second period is reached,* *offers p^* in the second period. The buyer accepts x_1 at the end of the first period if $x_1 \leq p^*$; otherwise he goes to the second period to accept p^* if $x_1 > p^*$. If the offer p_2 is strictly greater than p^*, as is x_1, the buyer searches and chooses the minimum of x_1, x_2, and p_2.*

Proof. Given that $x_1 \in (p^*, 1]$, the seller has the following two options in the second period. First, she can offer a price $p_2 = p^*$ to get a sure payoff of p^*. Second, she can offer a p_2 greater than p^*, knowing that the buyer will search, and expect to get

$$p_2 \cdot \mathrm{Prob}\big[\, p_2 < x_1 \text{ and } p_2 < x_2 \,|\, x_1 \in (\, p^*, 1]\big]$$
$$= \frac{1 - F(\, p_2)}{1 - F(\, p^*)} p_2 [1 - F(\, p_2)]$$

Clearly, if $p^* \geq \max p_2[1 - F(p_2)]^2/[1 - F(p^*)]$, the seller will offer p^* in the second period.

Since the conjectured second period offer is p^*, the buyer will go to the second period to accept p^* if and only if $x_1 > p^*$. ■

We now turn our attention to the case where there is second-period search by the buyer. Before constructing the equilibrium, let us make the following four observations:

First, if the equilibrium of Σ^b is to involve search, then, as we have argued before (by Lemma 1), the seller's second period offer must be greater than p^*.

Second, for the seller to offer $p_2 > p^*$, it must be the case that the expected payoff from offering such a price is better than the payoff from offering the search-deterring price p^*. This implies that the search cost c is sufficiently low for the seller to take her chances.

Third, since the conjectured second period offer is greater than p^* (according to the first observation), the buyer will go to the second period if and only if $x_1 > p^*$. Therefore, the seller's belief about x_1 must have p^* as the lower bound for second-period search to occur. (Note that a buyer with $x_1 > p^*$ will consider it optimal to go through a second search rather than accepting the outside offer. This follows from the definition of p^*.)

Hence, assuming that the seller believes $x_1 \in (p^*, b]$ $(p^* < b \leq 1)$ whenever the second period is reached (we will verify later that the seller's belief must indeed be in this form in equilibrium), then for the seller to offer $p_2 > p^*$, it must be the case that

$$\max p_2 \cdot \mathrm{Prob}\big[\, p_2 \text{ will be accepted } | x_1 \in (\, p^*, b]\big] > p^*$$
$$\Leftrightarrow \max p_2 \cdot \mathrm{Prob}\big[\, p_2 < x_1 \text{ and } p_2 < x_2 | x_1 \in (\, p^*, b]\big] > p^*$$
$$\Leftrightarrow \max \frac{F(b) - F(\, p_2)}{F(b) - F(\, p^*)} p_2 [1 - F(\, p_2)] > p^*.$$

For simplicity, define $\Psi(p_2, b)$ as follows:

$$\Psi(p_2, b) \equiv \frac{F(b) - F(p_2)}{F(b) - F(p^*)} p_2[1 - F(p_2)]. \tag{3}$$

Furthermore, let $\hat{p}(b)$ be the maximizer and $\Psi(\hat{p}(b), b)$ be the maximum value of $\Psi(p_2, b)$ for a given b.

Notice that $\hat{p}(b)$ and $\Psi(\hat{p}(b), b)$ have the desirable properties described in the following lemma, which we will need in the sequel.

LEMMA 2. *Define* $\mathbf{B} \approx \{b | \Psi(\hat{p}(b), b) \geq p^*$, *and* $p^* < b \leq 1\}$. *Then, for all* $b \in \mathbf{B}$, *both* $\hat{p}(b)$ *and* $\Psi(\hat{p}(b), b)$ *are continuous and monotonically increasing in* b.

Proof. See Appendix A. ■

Finally, our last observation concerns the relationship between p_1 and p_2 in any equilibrium that involves second-period search. Suppose $1 - p_1 > \Phi(p_2)$. Then $\Phi^{-1}(1 - p_1) < p_2$, and all buyers with $x_1 > \Phi^{-1}(1 - p_1)$ will accept p_1. Then the seller's belief about x_1 must have support on $[p^*, \Phi^{-1}(1 - p_1)]$, of which p_2 is not a member. Therefore p_2 is clearly not optimal. (See Fig. 3 for the above discussion.) Therefore, $1 - p_1 \leq \Phi(p_2)$ in any equilibrium with second-period search.

If $1 - p_1 < \Phi(p_2)$, then p_1 will be rejected by all buyers with $x_1 \in (p^*, 1]$, and, therefore, p_2 must maximize $\Psi(p_2, 1)$, where

$$\Psi(p_2, 1) = \frac{1 - F(p_2)}{1 - F(p^*)} p_2[1 - F(p_2)]. \tag{4}$$

To further simplify the notation, let $\hat{p} = \hat{p}(1)$, i.e., $\hat{p}$ maximizes (4). Then, when $1 - p_1 < \Phi(\hat{p})$, the condition $\hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)] > p^*$ (which is exactly the opposite condition of Proposition 2) gives rise to the unique equilibrium of Σ^b, in which the seller offers $p_2 = \hat{p}$ and the buyer searches. Proposition 3 below describes such an equilibrium.

PROPOSITION 3. *Suppose* $1 - p_1 < \Phi(\hat{p})$ *and* $p[1 - F(\hat{p})]^2/[1 - F(p^*)] > p^*$. *Then the following is the unique equilibrium of* Σ^b: *The buyer accepts* x_1 *at the end of the first period if* $x_1 \leq p^*$; *otherwise, expecting* $p_2 = \hat{p}$, *he goes to the second period to search if* $x_1 \in (p^*, 1]$. *The seller, believing* $x_1 \in (p^*, 1]$ *whenever the second period is reached, offers* $p_2 = \hat{p}$.

Proof. See Fig. 4 and previous discussions. ■

The preceding proposition delineates the equilibrium of Σ^b under the conditions that (i) $1 - p_1 < \Phi(\hat{p})$ and (ii) $\hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)] > p^*$.

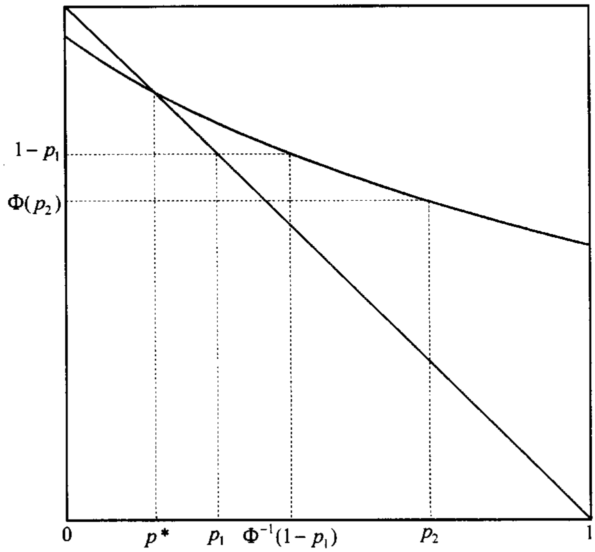

FIG. 3. Illustration of the relationship between equilibrium p_1 and p_2. **Remark:** If $1 - p^* > 1 - p_1 > \Phi(p_2)$, then the buyer will accept x_1 if $x_1 \leq p^*$, will go to the second period to search if $x_1 \in (p^*, \Phi^{-1}(1 - p_1)]$, and will accept p_1 if $x_1 > \Phi^{-1}(1 - p_1)$. Hence, if the second period is reached the seller should believe that $x_1 \in (p^*, \Phi^{-1}(1 - p_1)]$, of which p_2 is not a member.

The first condition implies $p_1 > p^*$. Notice that if the second condition is violated, we will then have the equilibrium described in Proposition 2.

If the first condition is violated, i.e., if $1 - p_1 > \Phi(\hat{p})$ (which, combined with the condition that $p_1 > p^*$, implies $p^* < p_1 < 1 - \Phi(\hat{p})$), then $\hat{p}$ can no longer be the optimal second-period offer. In fact, based on our previous observation that $1 - p_1 \leq \Phi(p_2)$ and the property of $\Phi(\cdot)$, a price smaller than $\hat{p}$ will be offered by the seller in the second period. In this case, there are then two possible second-period offer scenarios. We can have $1 - p_1 = \Phi(p_2)$, in which case those buyer types indifferent between accepting p_1 and waiting for p_2 must accept or reject in the "right" proportion. That is, assuming that the support of the seller's belief is an interval rather than a union of disjoint intervals, a buyer with $x_1 \in (b, 1]$ must accept p_1 and $x_1 \in (p^*, b]$ must wait for p_2. The quantity b, which is unique if it exists, is determined jointly by the following three conditions: (i) p_2 maximizes $\Psi(p_2, b)$, i.e., $p_2 = \hat{p}(b)$; (ii) $\Psi(\hat{p}(b), b) \geq p^*$; and (iii) $1 - p_1 = \Phi(\hat{p}(b))$. The first two conditions guarantee that $\hat{p}(b)$ is the seller's optimal choice of offer, given $x_1 \in (p^*, b]$ (in fact, if (ii) holds for equality, the seller is indifferent between offering p^* and $\hat{p}(b)$, in

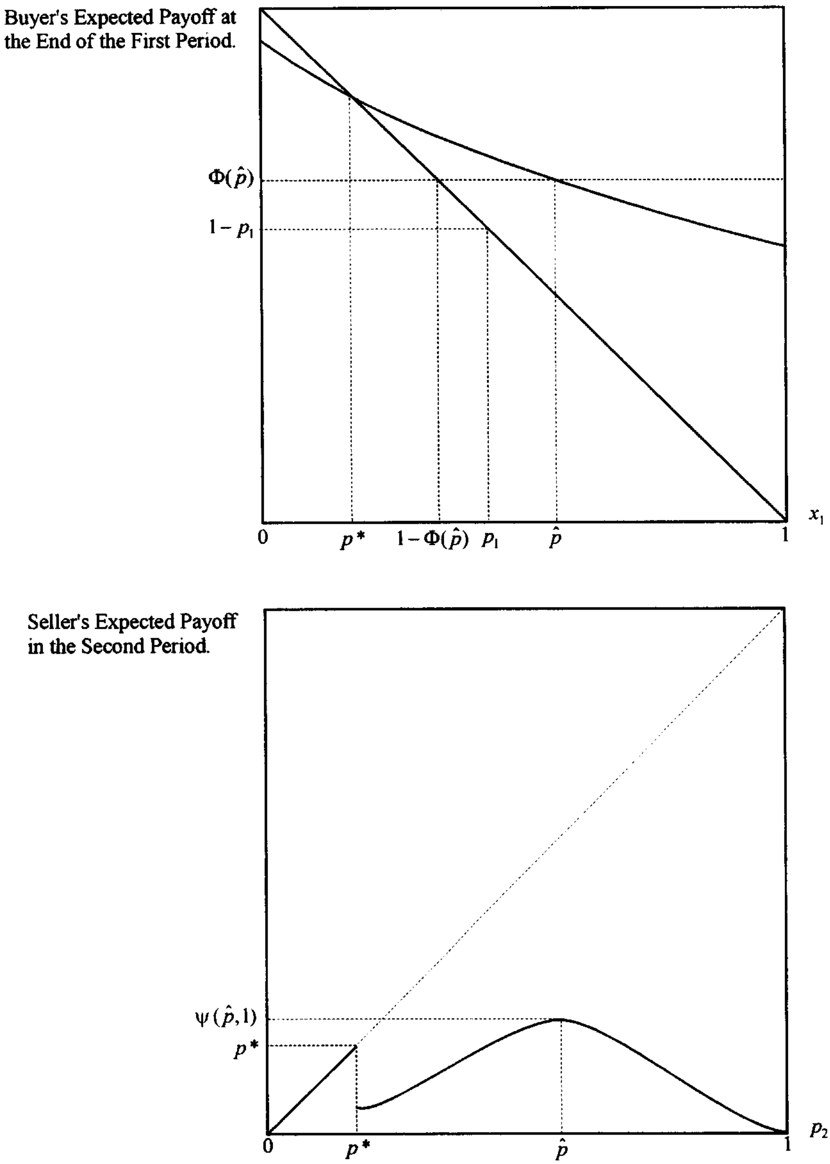

FIG. 4. Illustration of Proposition 3.

which case the equilibrium requires her to offer $\hat{p}(b)$) and the last condition ensures that the buyer will return to the second period if and only if $x_1 \in (p^*, b]$. More specifically, if $x_1 \in (p^*, p_1)$, the buyer strictly prefers going to the second period. If $x_1 \geq p_1$, however, the buyer is indifferent between accepting p_1 at the end of the first period and going to the second period to search again. In this case, the equilibrium requires the buyer to accept p_1 if $x_1 > b$ and to go to the second period if $x_1 < b$. Refer to Fig. 5 for the above discussion.

Notice that, because of the continuity and monotonicity of $\hat{p}(b)$ and $\Psi(\hat{p}(b), b)$ described in Lemma 2, we can always find the "b" that satisfies conditions (i) and (iii) above. However, the $\Psi(\hat{p}(b), b)$ so constructed may not satisfy (ii), and then the $\hat{p}(b)$ so determined cannot be the equilibrium second offer. This situation can arise when p_1 is relatively small, so that $\hat{p}(b)$ and $\Psi(\hat{p}(b), b)$ are also small. In this case, an alternative scenario becomes operative, in which the cutoff b is chosen differently so that (i) $\hat{p}(b)$ maximizes $\Psi(p_2, b)$; (ii) $\Psi(\hat{p}(b), b) = p^*$; and (iii) $1 - p_1 = \alpha(1 - p^*) + (1 - \alpha)\Phi(\hat{p}(b))$. (The existence of $\hat{p}(b)$ and $\Psi(\hat{p}(b), b)$ that satisfy all of the above conditions is guaranteed by Lemma 2.) The first two conditions indicate that the seller is indifferent between offering p^* and $\hat{p}(b)$ in the second period when $x_1 \in (p^*, b]$. The third condition then says that she will offer each price with probabilities α and $1 - \alpha$, respectively. The last condition, for a similar reason explained in the previous scenario, also ensures that the buyer will return to the second period if and only if $x_1 \in (p^*, b]$. Refer to Fig. 6 for the above discussion.

To summarize, when $P_1 \in (p^*, 1 - \Phi(\hat{p})]$, depending on the magnitude of p_1, one of the above two scenarios will arise. The first scenario arises for larger values of p_1 and the second for smaller values of p_1. There exists a cutoff that separates these two situations.

Based on the above discussions, we now have the following propositions. Proofs are omitted.

PROPOSITION 4. *Suppose $p_1 > p^*$ and $1 - p_1 \geq \Phi(\hat{p})$. If there exists a unique quantity "b" such that (i) $\hat{p}(b)$ maximizes $\Psi(p_2, b)$; (ii) $\Psi(\hat{p}(b), b) \geq p^*$; and (iii) $1 - p_1 = \Phi(\hat{p}(b))$; then the unique equilibrium of Σ^b is as follows: The buyer accepts x_1 if $x_1 \leq p^*$, accepts p_1 if $x_1 > b$, and goes to the second period and searches if $p^* < x_1 \leq b$. The seller believing that $x_1 \in (p^*, b]$ whenever the second period is reached, offers $p_2 = \hat{p}(b)$.*

PROPOSITION 5. *Suppose $p_1 > p^*$ and $1 - p_1 \geq \Phi(\hat{p})$. If the unique quantity "b" that satisfies the three conditions of Proposition 4 does not exist, then there must exist a b' such that (i) $p(b')$ maximizes $\Psi(p_2, b')$; (ii) $\Psi(\hat{p}(b'), b') = p^*$; and (iii) $1 - p_1 = \alpha(1 - p^*) + (1 - \alpha)\Phi(\hat{p}(b'))$. The unique equilibrium of Σ^b is as follows: The buyer accepts x_1 if $x_1 \leq p^*$,*

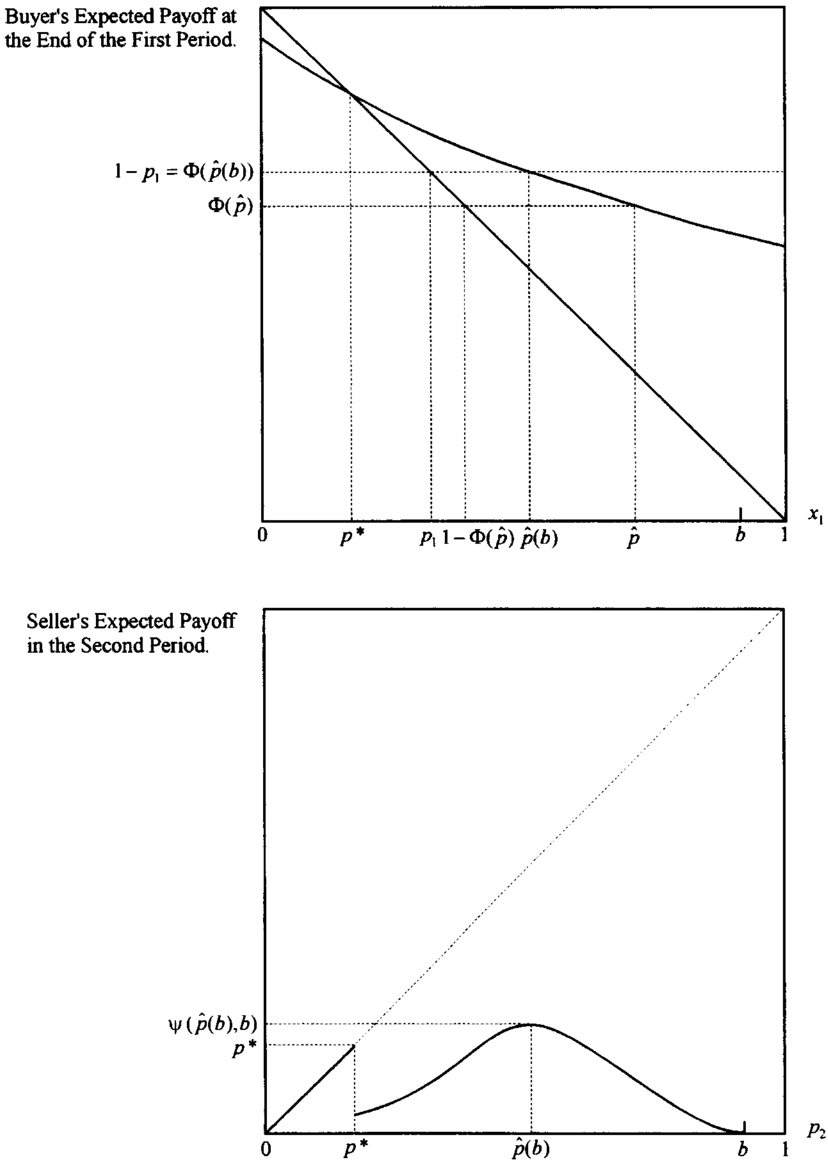

FIG. 5. Illustration of Proposition 4. **Remark:** Given $p_1 < -\Phi(\hat{p})$, the seller offers $\hat{p}(b)$ if $\psi(\hat{p}(b), b) > p^*$, such that $1 - p_1 = \Phi(\hat{p}(b))$. Hence, if $x_1 \leq p^*$, the buyer will accept x_1 at the end of the first period; if $x_1 \in (p^*, \hat{p}(b))$, the buyer strictly prefers returning to the second period; and if $x_1 \geq \hat{p}(b)$, the buyer is indifferent between accepting p_1 at the end of the first period and returning to the second period. In equilibrium, if the buyer is indifferent between accepting p_1 and returning to the second period, he will accept p_1 if his outside offer, x_1, is greater than b

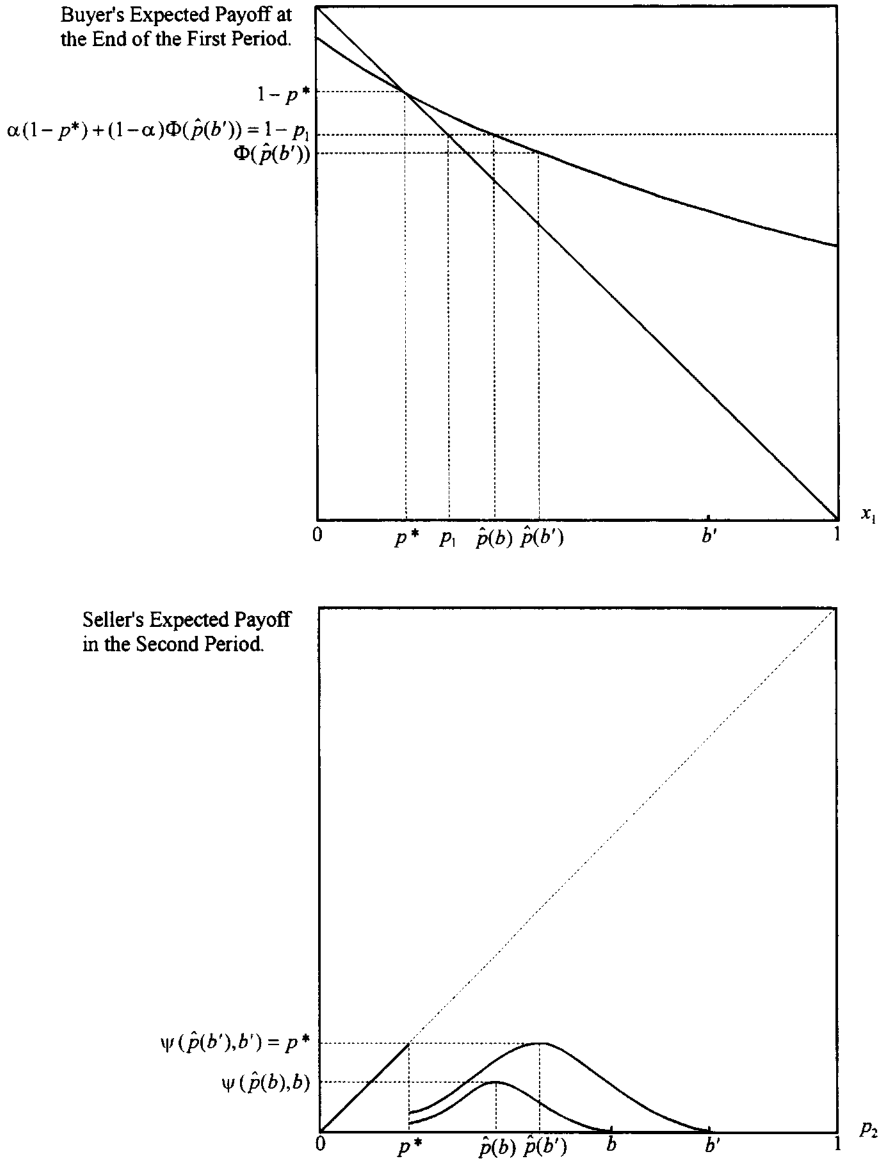

FIG. 6. Illustration of Proposition 5. **Remark:** Here, given p_1, it is not optimal for the seller to offer $\hat{p}(b)$ such that $1-p_1=\Phi(\hat{p}(b))$, since $p^*>\psi(\hat{p}(b),b)$. Hence the seller randomizes in the second period between offering p^* and $\hat{p}(b')$ (with probabilities α and $1-\alpha$, respectively) such that $\alpha(1-p^*)+(1-\alpha)\Phi(\hat{p}(b'))=1-p_1$. If $x_1 \le p^*$, the buyer will accept x_1 immediately; if $x_1 \in (p^*, \Phi^{-1}(1-p_1))$, the buyer strictly prefers returning to the second period; and if $x_1 > \Phi^{-1}(1-p_1)$, the buyer is indifferent between accepting p_1 and returning to the second period, in which case he accepts p_1 if $x_1 > b'$.

accepts p_1 *if* $x_1 > b'$, *and goes to the second period to search if* $p^* < x_1 \leq b'$. *The seller, believing that* $x_1 \in (p^*, b']$ *whenever the second period is reached, offers* $p_2 = \hat{p}(b')$ *with probability* α *and* $p_2 = p^*$ *with probability* $1 - \alpha$.

Propositions 1–5 delineate equilibrium behavior in Σ^b for different values of c and p_1. It can be checked that these propositions have indeed considered all possible values of c and p_1. It is explicitly assumed above that, no matter what the values of c and p_1 are, the buyer finds it optimal to search in the first period (otherwise, the subgame Σ^b will not be reached.) In the following analysis of the equilibrium of the full model, we will examine the consistency of such an assumption. We shall show that only when $\hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)] > p^*$ and $p_1 > p^*$ will the first period involve search. Hence, if these two conditions are met, the scenarios described in Propositions 1 and 2 will appear only off the equilibrium path.

2.4. *The Equilibrium of* Σ

As in both Σ^a and σ^b, there are two types of equilibria for Σ, depending on the search cost—one involves search, the other does not. If a condition similar to that of the optimality of p^* in the second period is satisfied, then our intuition seems to suggest that the seller should offer $p_1 = p^*$ also in the first period (since there is no reason for the seller to delay offering p^*, knowing it will be accepted by the buyer). If this is true, then, search can only occur off the equilibrium path, and the seller's second-period offer will be used to support her equilibrium first-period offer.

We can now check that p^* (possibly along with some other prices less than p^*) will, in fact, be offered in the first period in equilibrium for relatively high values of c. We state this formally in the following proposition.

PROPOSITION 6A. *Suppose* $p^* \geq \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$. *Then the following is an equilibrium of* Σ. *The seller offers* $p_1 = p^*$ *in the first period. If the buyer searches in the first period and returns, the seller, believing* $x_1 \in (p^*, 1]$, *offers* p^* *in the second period. If the buyer does not search in the first period, then the seller either offers* $p_2 = p^*$, *if* $p^* \geq \bar{p}[1 - F(\bar{p})]$, *or offers* $p_2 = \bar{p}$, *if* $\bar{p}[1 - F(\bar{p})] > p^*$. *The buyer accepts any offer less than or equal to* p^* *when offered. In equilibrium, the game ends immediately, with the seller getting* p^* *and the buyer getting* $1 - p^*$, *respectively.*

Proof. See Appendix B. ■

It should be noted that, if $\bar{p}[1 - F(\bar{p})] < p^*$, we could have an equilibrium in which the seller makes a trivial offer $p_1 > p^*$ in the first period and the buyer simply rejects it and goes to the second period directly to accept p^* in the second period. Such an equilibrium would yield exactly the same payoffs for both players as in the equilibrium of Proposition 6A.

Thus, given the conditions of Proposition 6A, it is an equilibrium for the seller to offer p^* in both periods, and for the buyer to accept immediately. However, this is not the only possible equilibrium without search. The next proposition demonstrates that, depending on the probability distribution and with an off-equilibrium belief different from that of Proposition 6A, an equilibrium could exist with the price charged by the seller being below p^*.

PROPOSITION 6B. *Let p° be a price less than p^*. Suppose that* (i) $p^\circ > p_2[1 - F(p_2)]/[1 - F(p^\circ)]$ *for all* $p_2 \in (p^\circ, p^*]$; *and* (ii) $p^\circ > p_2[1 - F(p_2)]^2/[1 - F(p^\circ)]$ *for all* $p_2 \in (p^*, 1]$. *Then the following is an equilibrium of* Σ: *The seller offers* $p_1 = 1 - \Phi(p^\circ)$ *in the first period. If the buyer searches in the first period, the seller, believing* $x_1 \in (p^\circ, 1]$, *offers* p° *in the second period. If the buyer does not search in the first period, then the seller either offers* $p_2 = p^*$, *if* $p^* \geq \bar{p}[1 - F(\bar{p})]$, *or offers* $p_2 = \bar{p}$, *if* $\bar{p}[1 - F(\bar{p})] > p^*$. *The buyer in the first period, accepts any offer less than or equal to* $1 - \Phi(p^\circ)$, *and searches otherwise. If* x_1, *the outcome of first period search, is less than* p°, *the buyer accepts* x_1. *If* $x_1 \geq p^\circ$, *the buyer asks for a second offer from the seller. In the second period, if* $\min(x_1, p_2) > p^*$, *the buyer searches; otherwise, he accepts the minimum of* x_1 *and* p_2 *without search. In equilibrium, the game ends immediately, with the seller getting* $1 - \Phi(p^\circ)$ *and the buyer getting* $\Phi(p^\circ)$, *respectively.*

Proof. See Appendix C. ■

The intuition behind the above equilibrium is simple. If the seller believes that there is a probability that the buyer may enter the second period with an outside offer less than p^*, then it may be optimal for her to offer a price less than p^*.

In the above equilibrium, if the seller believes that the buyer may enter the second period with $x_1 \in (p^\circ, 1]$, then she will do one of the following three things in the second period. First, she can offer $p_2 = p^\circ$ to get a sure payoff of p° (since $p^\circ < p^*$, it will be accepted immediately.) Second, she can offer a price between p° and p^*, i.e., $p_2 \in (p^\circ, p^*]$. In this case, since $p_2 < p^*$, the offer will be accepted by the buyer immediately if it is less than x_1. Hence the seller's maximum expected payoff for making such an offer will be $\max p_2[1 - F(p_2)]/[1 - F(p^\circ)]$, where $[1 - F(p_2)]/[1 - F(p^\circ)]$ is the conditional probability that $x_1 > p_2$ given $x_1 \in (p^\circ, 1]$. Finally, the seller can offer a price greater than p^*. In such a case, the buyer will search and the offer will not be accepted unless both x_1 and x_2 are greater than p_2. Hence the most the seller can get for taking this last option is $\max p_2[1 - F(p_2)]^2/[1 - F(p^\circ)]$, where $[1 - F(p_2)]^2/[1 - F(p^\circ)]$ is the conditional probability that both x_1 and x_2 are greater than p_2, given $x_1 \in (p^\circ, 1]$.

It is certainly possible that, for some forms of $F(\cdot)$ and some values of p°, the first option will yield the highest expected payoff for the seller and, hence, a price smaller than p^* will be offered in the second period. If this is the case, then the seller in the first period will also have to offer a price smaller than p^*. The reason why the seller does not have to offer $p_1 = p^\circ$ and offers $p_1 = 1 - \Phi(p^\circ)$ (which is greater than p°) instead is that the buyer will have to search in the first period to expect p° in the second period. Hence, to deter search, the seller need only offer a price such than $1 - p_1 = \Phi(p^\circ)$.

Notice that offering p^* in both periods is also an equilibrium strategy for the seller, given the conditions of Proposition 6B, if p^* is the optimal second period offer, given that a buyer with x_1 in $(p^*, 1]$ returns for a second period. This equilibrium is more plausible in the following sense. Suppose the equilibrium first period offer is as in Proposition 6B and the seller deviates and offers p^*. The equilibrium can be sustained by the buyer belief that this deviation is a "mistake" and that the second period offer will be p°, in which case the deviating offer will be rejected. However, p^* is the equilibrium offer in an alternative perfect Bayes equilibrium, and the deviation can be interpreted not as a mistake, but as a signal of the seller's intention to offer the same price in the second period. The buyer should believe this signal, because it is the opening choice in a perfect Bayes equilibrium that gives the seller a higher payoff. If the buyer believes this, the seller should certainly deviate; this destroys the candidate (Proposition 6B) equilibrium. Henceforth, we shall disregard the equilibrium outlined in Proposition 6B.

Both types of equilibria described above require high search costs. As the cost of search c becomes smaller, the value of p^* decreases and the seller's expected payoffs in these equilibria also decrease. Eventually, when c becomes small enough, it may no longer be optimal for the seller to resolve bargaining "immediately." In such cases, the seller, as seen in the subgame Σ^b, may then try to take her chances by offering something higher than p^*, knowing that the buyer will search and hoping that her offer will be accepted eventually because of the buyer's unfruitful search. This can happen when the condition for Proposition 6A does not hold, i.e., when $p^* < \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$. We now consider these cases.

When analyzing the equilibria of Σ^b in the previous section, it is explicitly assumed that, for values of p_1 given in the propositions (1–5), the buyer finds it optimal to search in the first period. We now show that, under the condition $p^* < \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$, only when $p_1 > p^*$

will the buyer indeed find it optimal to search. If the seller offers anything lower than p^*, the buyer will accept immediately, and hence the second period will never be reached.

Consider first if the seller offers $p_1 > p^*$ in the first period. Then, as implied by Lemma 1, it is never optimal for the buyer to accept immediately. This is because the buyer can only get $1 - p_1$ if he accepts immediately, whereas if he searches he can expect to get at least $\Phi(p_1)$ (because he can always "recall" p_1 at the end of the first period). Since $p_1 > p^*, \Phi(p_1) > 1 - p_1$. Clearly, accepting p_1 immediately in such a case is not optimal for the buyer.

If the buyer goes to the second period directly without searching in the first period, then the subgame Σ^a will be reached. According to our previous analysis, since $p^* < \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$ (which implies $p^* < \bar{p}[1 - F(\bar{p})]$ by simple algebra), the seller will offer $\bar{p}$ in the second period, and the buyer's expected payoff for taking this action will be $\Phi(\bar{p})$. However, if the buyer does search in the first period, the subgame Σ^b will be reached and one of the situations depicted in Propositions 3 through 5 will arise. This implies that the buyer can expect to get at least $\Phi(\hat{p})$, because in this case the second period seller offer will be no larger than $\hat{p}$. Lemma 3 below shows that $\hat{p} < \bar{p}$. Since $\Phi(\cdot)$ is a decreasing function, it is clear that a search is the best response for the buyer in the first period if $p_1 > p^*$.

LEMMA 3. *Let $\bar{p}$ and $\hat{p}$ be the prices that maximize $p[1 - F(p)]$ and $p[1 - F(p)]^2$, respectively. Then $\bar{p} \geq \hat{p}$.*

Proof. See Appendix D. ■

We next show that if $p^* < \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$ and the seller offers $p_1 \leq p^*$, then the buyer's best response is to accept immediately.

If $p_1 \leq p^*$, the buyer can either (1) accept immediately to get $1 - p_1$, (2) search, in which case the scenario described in Proposition 1 will arise and no offer better than p_1 is expected from the seller in the second period (but the buyer has paid the search cost), or (3) go to the second period immediately without a search, in which case his expected payoff will be $\Phi(\bar{p})$, as discussed before. Clearly, since $p_1 \leq p^*$ and $1 - p_1 > \Phi(p_1) > \Phi(\bar{p})$, immediate acceptance of p_1 is optimal for the buyer.

Hence, given the condition that $p^* < \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$, the buyer's equilibrium strategy in the first period will be to immediately accept any seller offer smaller than or equal to p^* and to search otherwise.

Now we have come to the last part of the analysis—the seller's optimal choice of p_1. The preceding analysis suggests that the seller's optimal choice of p_1 must be either p^*, in which case the seller can ensure a payoff of p^*, or a price greater than p^*, in which case, depending on the size of p_1, one of the three cases described in Propositions 3–5 will arise. Let us define the sets of p_1 that induce the different kinds of subgame behavior described in Propositions 3, 4, and 5 as $\mathbf{P_U}$, $\mathbf{P_M}$, and $\mathbf{P_L}$, respectively. Notice that $\mathbf{P_U}$, $\mathbf{P_M}$, and $\mathbf{P_L}$ together exhaust the interval $(p^*, 1]$ and are mutually exclusive (except in the boundary case). Furthermore, the values of p_1 in $\mathbf{P_U}$ are larger than those in $\mathbf{P_M}$, which, in turn, are larger than those in $\mathbf{P_L}$.

Consider first the cases when $p_1 \in \mathbf{P_U}$. That is, consider the case when the seller's first period offer is such that $p_1 > 1 - \Phi(\hat{p})$. In this case, since $p_1 > p^*$, the buyer will search and the situation depicted in Proposition 3 will arise. Since, according to Proposition 3, the buyer will return to the second period if and only if $x_1 > p^*$ and, once the second period is reached, the seller will offer $p_2 = \hat{p}$ to expect a payoff of $\hat{p}[1 - F(\hat{p})]^2 / [1 - F(p^*)]$, the seller's expected payoff for offering such a p_1 will be

$$\int_{p^*}^{1} \frac{1 - F(\hat{p})}{1 - F(p^*)} \hat{p}[1 - F(\hat{p})] f(x_1)\, dx_1 = \hat{p}[1 - F(\hat{p})]^2, \tag{5}$$

which is independent of p_1. In other words, if $p_1 > 1 - \Phi(\hat{p})$, the value of p_1 becomes irrelevant because p_1 will never be accepted. However, notice that if the seller offers $p_1 = 1 - \Phi(\hat{p})$ (which is the largest member in $\mathbf{P_M}$), she will have exactly the same expected payoff (by Proposition 4). We therefore eliminate from consideration offers p_1 in $\mathbf{P_U}$, as these are payoff-equivalent to the largest offer in $\mathbf{P_M}$.

Consider next the cases when $p_1 \in \mathbf{P_L}$, i.e., those values of p_1 that induce the subgame behavior described in Proposition 5. Then, according to Proposition 5, p_1 will be accepted if and only if $x_1 \in (b', 1]$, and the second period will be reached only with probability $F(b') - F(p^*)$, where b' is chosen so that $\Psi(\hat{p}(b'), b') = p^*$. Furthermore, once the second period is reached, the seller will randomize between offering p^* and $\hat{p}(b')$, and hence her expected payoff in the second period will be p^*. As a result, the seller's expected payoff for offering such a p_1 will be

$$p_1[1 - F(b')] + p^*[F(b') - F(p^*)] \tag{6}$$

Since b' in (6) does not depend on p_1 (see Proposition 5), (6) is increasing in p_1 as long as the value of p_1 stays in $\mathbf{P_L}$. Hence, if the seller is to offer a p_1 in this range, she will offer the largest p_1 possible. Notice that, by condition (ii), which determines the value of b' in Proposition 5, given p^*, b' and $\hat{p}(b')$ are uniquely determined. Hence, $\Phi(\hat{p}(b'))$ in (iii) of Proposition 5 must also be fixed. Thus, when the value of p_1 increases, to maintain the equality in (iii) of Proposition 5, the value of α must decrease because $1 - p^* > 1 - p_1 \geq \Phi(\hat{p}(b'))$ (see Fig. 6). Let p_1' be the largest price in $\mathbf{P_L}$. Then it must be true that $1 - p_1' = \Phi(\hat{p}(b'))$ and $\alpha = 0$. This then means that p^* will be chosen with probability zero in the second period. Hence, in equilibrium, the seller will never play a mixed strategy, even if she may be indifferent between offering p^* and $\hat{p}(b')$ in the second period. The seller's expected payoff for offering $p_1 = p_1'$, according to (6), is

$$p_1'[1 - F(b')] + p^*[F(b') - F(p^*)].$$

However, it can be checked that the p_1' so defined is also a member of $\mathbf{P_M}$ (the smallest one). In fact, p_1' is the price such that condition (ii) of Proposition 4 holds for equality. We therefore can also rule out any $p_1 \in \mathbf{P_L}$ (except p_1') as the possible equilibrium first-period offer.

Hence the first-period equilibrium offer must be either p^* or a price in $\mathbf{P_M}$. According to Proposition 4, if the seller offers $p_1 \in \mathbf{P_M}$, p_1 will be accepted if and only if $x_1 \in (b, 1]$, and the second period will be reached with probability $F(b) - F(p^*)$. Furthermore, since the seller will offer $\hat{p}(b)$ to expect a payoff of $\Psi(\hat{p}(b), b)$ once the second period is reached, the seller's expected payoff for offering such a p_1, denoted as $\mathrm{S}'(p_1)$, will be

$$\mathrm{S}'(p_1) = p_1[1 - F(b)] + [F(b) - F(p^*)]\Psi(\hat{p}(b), b), \tag{7}$$

where b (and hence $\hat{p}(b)$ and $\Psi(\hat{p}(b), b)$) is chosen so that the three conditions of Proposition 4 are satisfied.

Let $\tilde{p}(\in \mathbf{P_M})$ be the maximizer (assumed unique) and $\mathrm{S}'(\tilde{p})$ be the maximum value of (7). Then, in equilibrium, the seller will offer either p^* or $\tilde{p}$ in the first period, depending on which of p^* and $\mathrm{S}'(\tilde{p})$ is larger.

We now summarize the above discussion in the following proposition.

PROPOSITION 7. *If* $p^* < \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$, *then the unique equilibrium of* Σ *is as follows*: *If* $p^* \geq \mathrm{S}'(\tilde{p})$, *the equilibrium calls for the seller to offer* p^* *and the buyer to accept immediately in the first period. If* $p^* < \mathrm{S}'(\tilde{p})$, *the seller offers* $\tilde{p}(> p^*)$ *and the buyer searches in the first period. The subsequent subgame behavior of both players along the equilibrium path is described in Proposition* 4.

The seller's ex ante equilibrium expected payoff, *denoted as* $\mathrm{S}(p)$, *is*

$$\mathrm{S}(p) = \max\{p^*, \mathrm{S}'(\tilde{p})\}.$$

The buyer's ex ante equilibrium expected payoff denoted as B(p), *is*

$$\mathrm{B}(p) = \begin{cases} 1 - p^* & \text{if } p_1 = p^* \\ \mathrm{B}'(\tilde{p}) & \text{if } p_1 = \tilde{p}, \end{cases}$$

where

$$\mathrm{B}'(\tilde{p}) = -c + \int_0^{p^*} (1 - x_1) f(x_1)\, dx_1 + \int_{p^*}^{\hat{p}(b)} \Phi(x_1)\, dx_1 + \int_{\hat{p}(b)}^{1} (1 - \tilde{p}) f(x_1)\, dx_1.$$

Proof. See previous discussion. B$'(\tilde{p})$ can be verified by referring to the buyer's behavior described in Proposition 4. ■

We make some remarks about the uniqueness of the equilibrium. Clearly, if the game has multiple equilibria, it must be because there are different off-equilibrium beliefs that can support different equilibrium second offers. Hence, when checking the uniqueness of the equilibrium, we can thus concentrate only on the seller's belief about x_1 in the second period, and see whether there are beliefs other than those specified previously that can support different equilibria.

If $p^* > \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$, as we pointed out in Propositions 6A and 6B, the equilibrium may not be unique, depending on the form of $F(\cdot)$, although we did argue that the belief used to support the equilibrium in Proposition 6B is not plausible. However, both equilibria call for immediate resolution of bargaining, even if the value of the equilibrium first-period offer is different.

If $p^* < \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$, as required by Proposition 7, then, from our previous discussion, we know that there is no belief that can support $p_2 \le p^*$ in equilibrium, i.e., the equilibrium p_2 must be greater than p^*. Hence, if p^* maximizes S(p), the only possible equilibrium first offer for the seller is p^*, and therefore, the equilibrium is unique. If $\tilde{p}$ maximizes S(p), by assuming that (in the second period) the seller belief about x_1 is an interval, the equilibrium in the subsequent subgame is always unique, regardless of whether the seller follows the equilibrium path (which leads to Proposition 4) or deviates (which leads to Proposition 1, 3, or 5). The equilibrium in this case is also unique.

3. DISCUSSION

The analysis in the preceding section suggests that the incomplete information model has two types of equilibria (namely, ‘‘search'' and ‘‘no search'' equilibria) that are similar qualitatively to those of the complete information model of Lee (1994). A comparison between players' *ex ante* expected payoffs in this model and those in Lee's model shows the following two interesting propositions. To help visualize these results, we construct an example in which $F(\cdot)$ is uniform, and the players' *ex ante* equilibrium expected payoffs in this case are plotted in Fig. 7.

PROPOSITION 8. *The seller's ex ante expected payoff under incomplete information is always less than or equal to her ex ante expected payoff under complete information.*

Although this result is intuitive (the seller is better off to be informed rather than uninformed), its proof is not trivial; it is shown in Appendix E.

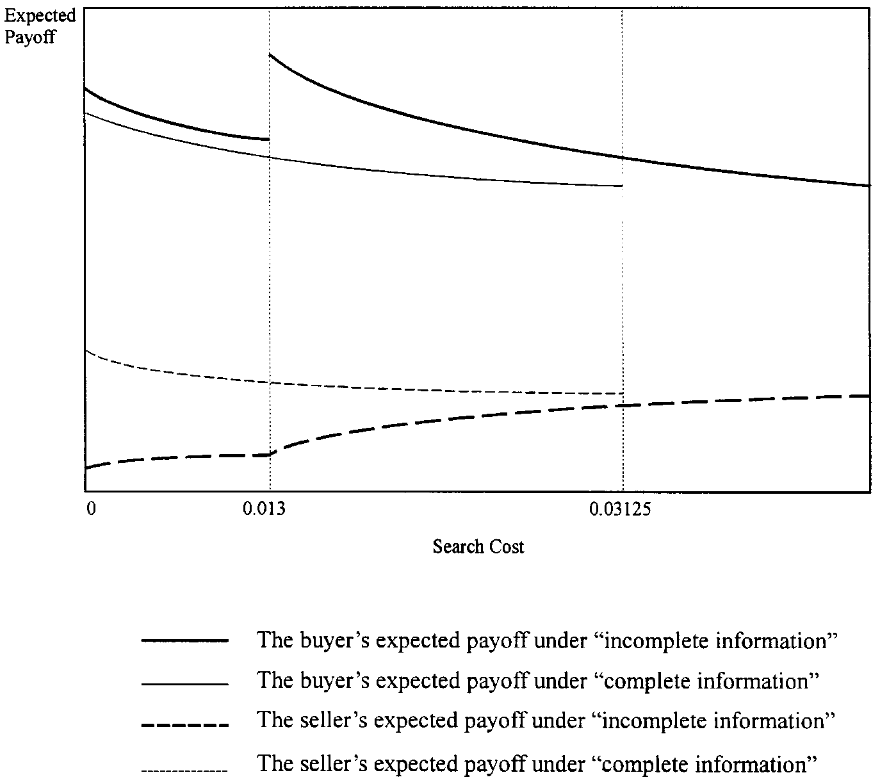

FIG. 7. The seller's and the buyer's *ex ante* equilibrium expected payoffs when $F(\cdot)$ is uniform. When $c > 0.03125$, the expected payoff under incomplete information is the same as that under complete information for both players. This is the case when ''no search'' equilibrium arises—the seller offers p^* and the buyer accepts. The figure is not drawn to scale.

Hence, not being able to observe the buyer's outside option makes the seller worse off. A parallel conclusion about the buyer's *ex ante* expected payoff under each model cannot be drawn. It is not clear whether the buyer is surely better off in one model and worse off in the other. However, as shown in Fig. 7, when $F(\cdot)$ is uniform, the buyer is better off in the incomplete information case.

The reason a definitive comparison of the buyer payoffs is difficult to make in general is that under complete information, the seller in the second period may more than match the buyer's first-period outside offer. For example, if the buyer comes to the second period with $x_1 \in (p^*, p'']$, where $p'' = \inf\{p | p[1 - F(p)] \geq p^*\}$, the seller will offer $p_2 = p^*$, and the buyer will accept. See Lee (1994) and Appendix E for details. This phenomenon cannot occur under incomplete information.

PROPOSITION 9. *The seller will offer the search-deterring price p^* for a larger set of c under incomplete information than under complete information.*

This result follows directly from Proposition 8 and is proved in Appendix E. Readers can also refer to Fig. 7 for this result. In Figure 7, it is shown that the cutoff c that separates "search" equilibrium from "no search" equilibrium in the incomplete information model is 0.013, while the cutoff in the complete information model is 0.03125. Hence, surprisingly, immediate resolution of bargaining is more likely to occur under incomplete information than under complete information. To put it differently, incomplete information bargaining achieves the efficient outcome for a larger domain of values of c.

The intuition behind these results is as follows. Since the buyer is unable to go to the seller with a verifiable outside option and ask for a matching or lower offer, there is more "leakage" from the system than there is with complete information. That is, the buyer accepts a good outside option, since to try to use it to generate a competing seller offer is not credible. The seller compensates for this by generally making lower demands in equilibrium, and by offering the search-deterring price for a greater range of costs than under complete information. The buyer, except in the one case mentioned above, does not lose by not being able to convey outside offers credibly to the seller, because if the seller does match, the buyer is at most only slightly worse off with his outside option than with the seller offer.

A further point of interest for the incomplete information model is that the second-period seller offer is always greater than the first one if the search equilibrium arises. This can be seen from a condition for search equilibrium that $1 - p_1 = \Phi(p_2)$ for equilibrium p_1 and p_2. Since both p_1 and p_2 in this equilibrium must be greater than p^*, this condition ensures that $p_2 \leq p_1$ in equilibrium. This result is intriguing because it is observed quite frequently in real life, but not very often in theoretical models. We often hesitate to buy a product when the price is low and end up having to pay a higher price for the same good.

Our model consists of only two periods. With a finite horizon (beyond two periods), the qualitative features of the above result will remain, although the cutoff values on cost may be different. With an infinite horizon, however, the delay result may or may not hold, depending on the features of the environment. If the process is stationary (i.e., with constant search cost), the infinite horizon stationary equilibrium will not involve search. The price p^* will be offered at every period and accepted in the first period. Thus recall alone is not sufficient to achieve delay in an infinite horizon model; there must be other features of the environment that interact with it to introduce some nonstationarity in the bargaining and search process. For example, we can show that, if the cost of search goes up with the number of searches, then the infinite horizon model may also involve search and delay.[1] Thus recall and some degree of nonstationarity, caused by finiteness or by increasing costs over time, play a major role in determining whether an equilibrium with search exists, as in our model, or one with just bargaining exists, as in the other papers cited in the introduction. Thus the two-period setting used here is not essential to obtain equilibrium search.

4. CONCLUSIONS

In this paper we have analyzed a two-period model of bargaining and search with recall, the result of search being assumed to be the searcher's private information. The equilibria of the model could involve search with

[1] Consider the following example, where $c_1 < c_2 = c_3 = \cdots$. Then there will exist a $p_2^* = p_3^* = \cdots$, such that in every stationary equilibrium, the buyer will reject any price higher than p_2^* from time period 2 onward. Therefore, in period 2, the seller will offer p_2^*, where p_2^* satisfies $1 - p_2^* = \Phi_2(p_2^*)$ as before, and the game will end in two periods. We examine the seller's first-period offer. Clearly, since $c_1 < c_2$, the first-period search-deterring offer p_1^* will be less than p_2^*. It may be optimal for the seller to offer something between p_1^* and p_2^* in period 1. This depends on whether the seller's payoff is maximized by offering p_1^*, with a payoff of p_1^* or $p_1[1 - F(p_1)]$, where $p_1 \in (p_1^*, p_2^*)$. Since p_2^* depends on $c_2, \ldots, c_n$, finding such a p_1 for sufficiently large c_2 is the same as determining the cutoff value c_1^*, such that for $c_1 < c_1^*$, the equilibrium with bargaining and search both occurring will hold. (For further elaboration, see Chatterjee and Lee (1996), an earlier version of this paper.) A referee has pointed out, however, that the best way of modeling these features is not to regard the search process as exogenously specified, but as a process by which a new bargaining round starts if a suitable partner is found. This paper should be considered an initial attempt at addressing these issues rather than the last word on the subject; the referee's comment points to the need for further elaboration in future research.

probability 1 when the search cost is sufficiently small, as well as immediate acceptance of a search-deterring price when the search cost is sufficiently large. The outside option, being private information, without any credible means of communicating this information to the other side, carries with it the initially somewhat surprising implication that the searcher is, in general, better off, and the party without information (the seller) is worse off than in the complete information case.

The recall feature could be used in equilibrium, either just before the end of the game, or just before the end of the first period, when the seller's offer is about to lapse. In the latter case, the recall is not of an outside offer, but that of a price to which the offering party is committed for one period.

In the decision-theoretic search literature, recall is potentially used only in the last period of a finite-horizon game, and never in an infinite-horizon setting, given the independence assumptions we have about outside offers. We argue here that the recall assumption, along with some kind of finiteness of the game, is necessary to achieve equilibria with both bargaining and search.

APPENDIX A: PROOF OF LEMMA 2

We first note that if $b \in \mathbf{B}$, then $\hat{p}(b)$ must be in the interior of $(p^*, b]$. This is easy to check.

The continuity of both $\Psi(\hat{p}(b), b)$ and $\hat{p}(b)$ follows from the "maximum theorem" (see Harris, 1987, for example). The monotonicity of $\Psi(\hat{p}(b), b)$ is demonstrated below by showing $\Psi(\hat{p}(b'), b') > \Psi(\hat{p}(b''), b'')$ for all $b' > b''$:

$$\begin{aligned}
\Psi(\hat{p}(b''), b'') &= \hat{p}(b'')[1 - F(\hat{p}(b''))] \frac{F(b'') - F(\hat{p}(b''))}{F(b'') - F(p^*)} \\
&\leq \hat{p}(b'')[1 - F(\hat{p}(b''))] \frac{F(b'') - F(\hat{p}(b''))}{F(b') - F(p^*)} \\
&< \hat{p}(b')[1 - F(\hat{p}(b'))] \frac{F(b') - F(\hat{p}(b'))}{F(b') - F(p^*)} \\
&= \Psi(\hat{p}(b'), b').
\end{aligned}$$

The second inequality follows from the fact that $\hat{p}(b')$ is the unique maximizer of (3) for $b = b'$, and the first from the fact that $\hat{p}(b'') > p^*$.

We now prove that $\hat{p}(b)$ is increasing in b. The proof is by contradiction. Suppose $\hat{p}(b)$ is not increasing in b; then there must exist b' and b'' such that $b' > b''$ and $\hat{p}(b') < \hat{p}(b'')$. To simplify our notation, we now let $p' = \hat{p}(b')$ and $p'' = \hat{p}(b'')$. Then

$$\begin{aligned}
&\frac{F(b'') - F(p'')}{F(b'') - F(p^*)} p''[1 - F(p'')] > \frac{F(b'') - F(p')}{F(b'') - F(p^*)} p'[1 - F(p')] \\
&\quad \Leftrightarrow F(b'')\{p''[1 - F(p'')] - p'[1 - F(p')]\} \\
&\qquad\qquad > F(p'')p''[1 - F(p'')] \\
&\qquad\qquad\quad - F(p')p'[1 - F(p')] \qquad (i)
\end{aligned}$$

By the same reasoning, we have

$$\begin{aligned}
&F(b')\{p'[1 - F(p')] - p''[1 - F(p'')]\} \\
&\quad > F(p')p'[1 - F(p')] - F(p'')p''[1 - F(p'')] \qquad (ii)
\end{aligned}$$

Adding (ii) to (i), we get

$$[F(b'') - F(b')]\{p''[1 - F(p'')] - p'[1 - F(p')]\} > 0$$

Since $F(b'') - F(b') < 0$, we must have

$$p''[1 - F(p'')] < p'[1 - F(p')]. \qquad (iii)$$

Now, let $\bar{p} = \arg\max_{p \in [0,1]} p[1 - F(p)]$ as before. By the assumption of strict quasi-concavity of $p[1 - F(p)]$, if we can show that both p' and p'' are less than $\bar{p}$, we can conclude that (iii) is a contradiction.

We now show that p' and p'' are both indeed less than $\bar{p}$.

Suppose $p'' > \bar{p}$, and $b'' < 1$, then $F(b'') - F(\bar{p}) > F(b'') - F(p'')$. Hence,

$$\frac{[F(b'') - F(\bar{p})]\bar{p}[1 - F(\bar{p})]}{F(b'') - F(p^*)} > \frac{[F(b'') - F(p'')]p''[1 - F(p'')]}{F(b'') - F(p^*)}.$$

However, this is a contradiction, since p'' is suppose to maximize the right-hand side of the above expression. Hence $p'' < \bar{p}$. A similar argument can be used to show that $p' < \bar{p}$.

By the assumption that $p[1 - F(p)]$ is strictly quasi-concave, $p''[1 - F(p'')] < p'[1 - F(p')] \Leftrightarrow p'' < p'$, which contradicts our original assumption.

Therefore, for $b' > b''$, we must have $p' > p''$, i.e., $\hat{p}(b)$ is increasing in b. ■

APPENDIX B: PROOF OF PROPOSITION 6A

We start our proof by examining the players' behavior in the second period.

The buyer's strategy in the second period follows directly from Lemma 1.

Consider the case when the subgame Σ^a is reached (i.e., the buyer does not search in the first period). Then, by our discussion in Section 2.2, the seller will offer either p^* or $\bar{p}$, depending on which of p^* and $\bar{p}[1 - F(\bar{p})]$ is larger.

Now consider the case when the subgame Σ^b is reached (i.e., the buyer searches in the first period). Then, by Proposition 2, since $p^* > \hat{p}[1 - F(\hat{p})]^2/[1 - F(p^*)]$, the seller will offer p^* in the second period. The buyer, after observing x_1 at the end of the first period, will accept $\min[x_1, p_1]$ if $\min[x_1, p_1] < p^*$; otherwise, he will go to the second period to accept the seller's offer, p^*.

Now, consider the beginning of the first period. It should be clear that the seller will never offer anything strictly less than p^* (by Lemma 1) if we use the off-equilibrium belief described in Proposition 1.

Suppose the seller offers a price $p_1 > p^*$. In this case, the buyer has the following three options. First, he can accept p_1 immediately to get $1 - p_1$. Second, he can reject p_1 and go directly to the second period to get either $1 - p^*$ or $\Phi(\bar{p})$ (see discussion on Section 2.2). Third, he can search in the first period, leading to an expected payoff of $\Phi(p^*) = 1 - p^*$ (since the second-period offer in this case is expected to be p^*). By the decreasing property of $\Phi(\cdot)$ and the definition of p^*, it is clear that the buyer's first alternative is never optimal.

If $\bar{p}[1 - F(\bar{p})] > p^*$, the buyer's expected payoff for taking the second action is $\Phi(\bar{p})$, which is less than his expected payoff for taking the third action $\Phi(p^*)$, because $\bar{p} > p^*$. Hence, if $\bar{p}[1 - F(\bar{p})] > p^*$, the buyer's best response to $p_1 > p^*$ is to search in the first period, leading to a seller offer of $p_2 = p^*$ and an expected payoff of $p^*[1 - F(p^*)]$ for the seller (because the second period will be reached with only probability $1 - F(p^*)$). The seller is therefore better off offering p^* immediately, thus ensuring a payoff of p^*.

If $\bar{p}[1 - F(\bar{p})] < p^*$, the buyer's payoff for taking the second action is $1 - p^*$, which makes him indifferent between the second and third actions. In this case, if the buyer takes his second action, the seller's payoff is p^*, which is the same as her payoff if she offers p^* directly. However, if the buyer assigns some positive probability for choosing his third action, then the seller will get a payoff that is strictly less then p^*. Again, the seller is better off to offer p^* immediately to ensure a payoff of p^*. ■

APPENDIX C: PROOF OF PROPOSITION 6B

The buyer's strategy in the second period follows directly from Lemma 1. The seller's strategy in the second period if the buyer does not search in the first period follows from our discussion in Section 2.2.

If the subgame Σ^b is reached (i.e., the buyer has searched in the first period), then the seller will choose a price p_2 to maximize his second-period expected payoff $S_2(p_2)$, conditional on the belief that $x_1 \in [p^\circ, 1]$, where

$$S_2(p_2) = \begin{cases} p_2 & \text{if } p_2 \in [0, p^\circ] \\ \dfrac{p_2[1 - F(p_2)]}{1 - F(p^\circ)} & \text{if } p_2 \in (p^\circ, p^*] \\ \dfrac{p_2[1 - F(p_2)]^2}{1 - F(p^\circ)} & \text{if } p_2 \in (p^*, 1]. \end{cases}$$

By assumptions (i) and (ii) of the Proposition, it is clear that p^* maximizes $S_2(p_2)$ in the second period.

Now, at the end of the first period, after observing x_1, the buyer will certainly accept $\min[x_1, p_1]$ if $\min[x_1, p_1] < p^\circ$; otherwise, the buyer will return to the second period and accept $p_2 = p^*$.

Now, consider the beginning of the first period. If the seller offers $p_1 > 1 - \Phi(p^\circ)$ (note that $p^\circ < 1 - \Phi(p^*)$ since $p^\circ < p^*$), the buyer's payoff is $1 - p_1$ if he accepts, $\Phi(p^*)$ if he searches (since he expects to get p^* in the second period), and either $1 - p^*$ or $\Phi(\bar{p})$ if he goes to the second period directly without search (depending on whether $p_2 = p^*$ or $p_2 = \bar{p}$). Clearly, the buyer in this case should search. As a result, the seller's expected payoff for offering $p_1 > 1 - \Phi(p^*)$ is $p^\circ[1 - F(p^\circ)]$.

If the seller offers $p_1 \le 1 - \Phi(p^\circ)$, the buyer's payoff is $1 - p_1$ if he accepts, $\Phi(\min(p_1, p^\circ))$ if he searches, and either $1 - p^*$ or $\Phi(\bar{p})$ if he goes to the second period directly without search. Clearly, the buyer's best response is to accept. Given this, the seller can guarantee herself a payoff of $1 - \Phi(p^\circ)$ if she offers $p_1 = 1 - \Phi(p^\circ)$, while her payoff is only $p^\circ[1 - F(p^\circ)]$ if she offers $p_1 > 1 - \Phi(p^\circ)$ (it is straightforward to check that $1 - \Phi(p^\circ) > p^\circ[1 - F(p^\circ)]$). Hence, in equilibrium, the seller offers $p_1 = 1 - \Phi(p^\circ)$, and the buyer accepts immediately. ■

APPENDIX D: PROOF OF LEMMA 3

Suppose $\bar{p} < \hat{p}$. Then $F(\bar{p}) \le F(\hat{p})$, and hence,

$$1 - F(\bar{p}) \ge 1 - F(\hat{p}). \qquad (i)$$

We know that

$$\bar{p}[1 - F(\bar{p})] > \hat{p}[1 - F(\hat{p})]. \qquad (ii)$$

Multiplying (i) and (ii), we get

$$\bar{p}[1 - F(\bar{p})]^2 > \hat{p}[1 - F(\hat{p})]^2.$$

However, this is a contradiction, because $\hat{p}$ maximizes the right-hand side of the above inequality. Hence $\bar{p} \geq \hat{p}$.

APPENDIX E: PROOFS OF PROPOSITION 8 AND PROPOSITION 9

The proof proceeds by direct comparison between the seller's equilibrium expected payoff under incomplete information and the corresponding quantity under complete information; the latter is worked out in Lee (1994). To make the results of our model comparable to those of Lee's model, which assumes discounting over periods, we shall be dealing with the case in which the discount factor in Lee's model, δ, is equal to 1.

The intuition behind the result has already been alluded to in the text, namely that the seller finds it optimal to make lower offers on average because of her inability to verify the buyer's outside option and thus to match it, as she could under complete information. This intuition does not lead to a proof, since the preceding statement has to be verified by deriving both of the prices offered and the probability of acceptance by the buyer under the different search cost regimes, and this effectively leads to the comparison below. Note in particular that the incomplete information equilibrium depends on the endogenously derived cutoff b, which has no counterpart in the complete information model.

Following Lee (1994), when $\delta = 1$, the seller's equilibrium expected payoff under complete information, denoted as $\mathrm{S_c}(p)$, is

$$\mathrm{S_c}(p) = \max \begin{cases} p^* & (i.a) \\ \max_{p_1 \in [1-\Phi(p''),\, 1-\Phi(\bar{p})]} \displaystyle\int_{p^*}^{p''} p^* f(x_1)\, dx_1 & \\ \qquad + \displaystyle\int_{p''}^{x^*(p_1)} x_1 [1 - F(x_1)] f(x_1)\, dx_1 & (i.b) \\ \qquad + \displaystyle\int_{x^*(p_1)}^{1} p_1 f(x_1)\, dx_1, & \end{cases}$$

where $p'' \equiv \inf\{p \mid p[1 - F(p)] \geq p^*\}$, $x^*(p_1)$ satisfies $1 - p_1 = \Phi(x^*(p_1))$, and $\bar{p}$ maximizes $p[1 - F(p)]$.

Let $\tilde{p}_c$ be the maximizer and $\mathrm{S}'_c(\tilde{p}_c)$ be the maximum value of $(i.b)$. Then $\mathrm{S}_c(p)$ can be rewritten as

$$\mathrm{S}_c(p) = \max\{p^*, \mathrm{S}'_c(\tilde{p}_c)\}. \qquad (ii)$$

In equilibrium, the seller offers p^* in the first period if $p^* \geq \mathrm{S}'_c(\tilde{p}_c)$; otherwise, she offers $p_1 = \tilde{p}_c$.

Under incomplete information, the seller's equilibrium expected payoff, denoted as $\mathrm{S}(p)$, is

$$\mathrm{S}(p) = \max\begin{cases} p^* & (iii.a) \\ \max_{p_1 \in \mathbf{P}_M}\left[F(b) - F(\hat{p}(b))\right]\hat{p}(b) & \\ \qquad \times\left[1 - F(\hat{p}(b))\right] + p_1[1 - F(b)], & (iii.b) \end{cases}$$

where $\mathbf{P}_\mathrm{M}$ in $(iii.b)$ is defined as the set of all p_1 satisfying conditions of Proposition 4, and $\hat{p}(b)$ is the maximizer of $\Psi(p_2, b)$ defined in (3).

Now let $\tilde{p}$ maximize $(iii.b)$. Then $\mathrm{S}(p)$ can be rewritten as

$$\mathrm{S}(p) = \max\{p^*, \left[F(b) - F(\hat{p}(b))\right]\hat{p}(b)\left[1 - F(\hat{p}(b))\right] + \tilde{p}\left[1 - F(b)\right]\}. \qquad (iv)$$

In equilibrium, the seller offers $p_1 = p^*$ if p^* is the maximum of (iv); otherwise, she offers $\tilde{p}$.

We want to show that $(i.b)$ is always greater than $(iii.b)$.

First, note that $\tilde{p}_c$, the maximizer of $(i.b)$, is chosen from the interval $[1 - \Phi(p''), 1 - \Phi(\bar{p})]$. We will now show, using the quasi-concavity of $p(1 - F(p))$, that $\tilde{p}$, the maximizer of $(iii.b)$, is also in the same interval. We then use the fact that $\tilde{p}_c$ maximizes $S_c(\cdot)$ within this interval to obtain an inequality.

Since $\tilde{p}_c = 1 - \Phi(x^*(\tilde{p}_c))$, the condition $\tilde{p}_c \in [1 - \Phi(p''), 1 - \Phi(\bar{p})]$ is equivalent to the condition that $x^*(\tilde{p}_c) \in [p'', \bar{p}]$. Furthermore, the equilibrium under the incomplete information model requires that $\tilde{p} = 1 - \Phi(\hat{p}(b))$. Hence, showing $\tilde{p} \in [1 - \Phi(p''), 1 - \Phi(\bar{p})]$ is equivalent to showing that $\hat{p}(b) \in [p'', \bar{p}]$.

Recall that $\bar{p}$ maximizes $p[1 - F(p)]$, and $\hat{p}(b)$ maximizes $\Psi(p_2, b)$. By Lemma 2 and Lemma 3, we know that $\bar{p} \geq \hat{p}(b)$ for all b.

We now show that $\hat{p}(b) \geq p''$.

We know that all $\hat{p}(b)$ satisfying the conditions of Proposition 4 must satisfy

$$\frac{F(b) - F(\hat{p}(b))}{F(b) - F(p^*)}\hat{p}(b)\left[1 - F(\hat{p}(b))\right] \geq p^* = p''\left[1 - F(p'')\right]. \qquad (v)$$

Hence $\hat{p}(b)[1 - F(\hat{p}(b))] \geq p''[1 - F(p'')]$

Now, since $p[1 - F(p)]$ is quasi-concave and $\hat{p}(b)$ and p'' are both less than $\bar{p}$, we know that $\hat{p}(b) > p''$.

Hence we have shown that $\hat{p}(b)$ is in $[p'', \bar{p}]$, which implies that $\tilde{p} \in [1 - \Phi(p''), 1 - \Phi(\bar{p})]$.

Now, by the fact that $\tilde{p}_c$ is the maximizer of $\mathrm{S}'_{\mathrm{c}}(\tilde{p}_c)$, the following must be true:

$$\mathrm{S}'_{\mathrm{c}}(\tilde{p}_c) \tag{vi.a}$$

$$= \int_{p^*}^{p''} p^* f(x_1)\,dx_1 + \int_{p''}^{x^*(\tilde{p}_c)} x_1[1 - F(x_1)] f(x_1)\,dx_1$$

$$+ \int_{x^*(\tilde{p}_c)}^{1} \tilde{p}_c f(x_1)\,dx_1 \tag{vi.b}$$

$$> \int_{p^*}^{p''} p^* f(x_1)\,dx_1 + \int_{p''}^{\hat{p}(b)} x_1[1 - F(x_1)] f(x_1)\,dx_1$$

$$+ \int_{\hat{p}(b)}^{1} \tilde{p} f(x_1)\,dx_1 \tag{vi.c}$$

$$> \tilde{p}[1 - F(\hat{p}(b))] \tag{vi.d}$$

$$= \int_{\hat{p}(b)}^{b} \tilde{p} f(x_1)\,dx_1 + \int_{b}^{1} \tilde{p} f(x_1)\,dx_1 \tag{vi.e}$$

Note that $(iii.b)$ is equal to $\int_{\hat{p}(b)}^{b} \hat{p}(b)[1 - F(\hat{p}(b))] f(x_1)\,dx_1 + \int_{b}^{1} \tilde{p} f(x_1)\,dx_1$.

Since $\tilde{p} = 1 - \Phi(\hat{p}(b)) > \hat{p}(b)[1 - F(\hat{p}(b))]$, it is clear that the value of $(vi.e)$ must be greater that of $(iii.b)$. Hence, $(i.b)$ must be greater than $(iii.b)$. Therefore, $\mathrm{S}_{\mathrm{c}}(p)$, the seller's equilibrium expected payoff under complete information, is greater than or at least equal to $\mathrm{S}(p)$, her equilibrium expected payoff under incomplete information.

Notice that, since $(i.b)$ is greater than $(iii.b)$, whenever it is optimal for the seller to offer the search-deterring price, p^*, in the complete information model (i.e., when $p^* \geq (i.b)$), it must also be optimal for the seller to offer p^* in the incomplete information model. However, the reverse is not true. Hence the seller will offer the search-deterring price for a greater range of c in the incomplete information model than in the complete information model. In other words, under incomplete information, it is easier for players to reach agreement immediately and hence achieve the efficient outcome. ■

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