Oligopolistic Pricing with Sequential Consumer Search

By Dale O. Stahl II*

N-identical stores compete by choosing prices for a homogeneous good with constant marginal costs. Consumers search sequentially with perfect recall. Some consumers have zero search costs, while all others have a positive search cost, c. There is a unique symmetric Nash Equilibrium price distribution that changes smoothly from "marginal cost pricing" to "monopoly pricing" as search cost and population parameters change. Remarkably, as the number of stores increases, the NE becomes more monopolistic.

This paper bridges the gap between two starkly contrasting results of Nash Equilibrium (*NE*) price-setting for a homogeneous good with identical costs and no capacity constraints. If consumers can search costlessly (i.e., they are fully informed of the price set by each store), then the unique *NE* is the Walrasian price (Joseph Bertrand, 1883). On the other hand, if search costs are bounded above zero, then the unique *NE* is the monopoly price (Peter Diamond, 1971). Results for other cases do not provide a smooth transition from the Bertrand result to the Diamond result.

There is a rich but spotty literature on consumer search. Most models with a finite number of stores (for example, Avishay Braverman, 1980; Hal Varian, 1980; Steven Salop and Joseph Stiglitz, 1977; and Stiglitz, 1979) do not consider sequential search; instead, a consumer is either fully informed or totally ignorant of store prices. Kenneth Burdett and Kenneth Judd (1983) consider noisy sequential search and optimal nonsequential search. Louis Wilde and Allan Schwartz (1979) consider *ad hoc* nonsequential search rules. John Carlson and Preston McAfee (1983) consider sequential consumer search from a finite number of stores, but the search rule is based on a conjectured distribution of prices that is unrelated to the equilibrium distribution. Stiglitz (1987) considers optimal consumer search from a finite number of stores with no recall. Jennifer Reinganum (1979) and Raphael Rob (1985) consider optimal sequential search for a model with a continuum of stores and consumers. Search costs are bounded above zero for Reinganum; nonetheless, there is price dispersion via nonidentical store costs. The distribution of search costs is not bounded above zero for Rob. However, Rob (like Stiglitz) makes the dubious assumption that consumers can "see" deviations by stores before they actually search. This departure from the notion of an *NE* is crucial to their results. It is more reasonable to assume that consumers know only the *NE* distribution of prices upon which they base their search rules; information about actual prices is obtained only via search. The literature does not include a proper game-theoretic study of optimal sequential search from a finite number of identical stores with recall.[1]

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[1]Burdett and Judd (1983), Richard Manning and Peter Morgan (1982), and Morgan and Manning (1985) consider optimal search in a broader context that includes multiple samples at one time. Newspapers and trade magazines are often cited as a justification for considering "parallel" search, but the parallel search models assume that the consumer can choose the sample size, which is not the case in the examples cited. Even after obtaining published prices (usually subject to change without notice), an individual consumer often must search sequentially to verify the published price, availability, quality, style, etc. Therefore, sequential search is a practical constraint in most situations.

The motivation for most of this literature seems to have been the desire for a model in which price differences among stores could be explained as an equilibrium result. Accordingly, these models produce mixed-strategy NE which are interpreted as price dispersion. No attention was given to the gap between the Bertrand and Diamond results.

Rob's and Stiglitz's models also necessarily exclude a mass of consumers who search costlessly, so no consumer is ever fully informed. Casual empiricism suggests that there is a non-negligible measure of consumers who seem to derive enjoyment from shopping itself. To capture the impact of these consumer types, our models need a mass of costless searchers. Any model that cannot accommodate costless searchers as well as costly searchers is seriously deficient.

This paper considers a two-stage model in which there are $N \geq 2$ stores who each fix a price for a homogeneous good in the first stage. Stores have identical constant marginal costs of supply which, without loss of generality, we take as zero. Let $F(p)$ denote the NE distribution of prices. As is commonly assumed in the literature, we assume the first sample is free, but thereafter each sample costs c^i. In stage two, consumer i adopts an optimal sequential search strategy given search costs c^i and perfect recall. When a consumer stops searching, she purchases an amount $D(p)$ from the lowest-priced store in her sample. The static nature of store-pricing strategies is justified when the cost of revising prices is sufficiently high as to make it suboptimal to do so in the time it takes consumers to search.

We assume that the revenue function, $R(p) \equiv pD(p)$, is continuous and has a unique maximum at $\hat{p}$, and that for all $p < \hat{p}$, $R(p)$ is continuously differentiable and strictly increasing. Consumers are effectively identical except for search costs about which stores have no individual information.

We will focus on the case in which there are two types of consumers. A proportion $\mu \in [0,1]$ of consumers have zero-search costs; we will call these consumers "shoppers." The remaining $1-\mu$ proportion of consumers have a common search cost $c > 0$.

The Bertrand result and the Diamond result obtain in our model when $\mu = 1$ and $\mu = 0$, respectively. When $\mu \in (0,1)$, there is no pure-strategy NE, but there is a unique symmetric NE in mixed strategies; thus, our model produces price dispersion. Moreover, the NE distribution $F(p)$ changes continuously from the degenerate distribution at the monopoly price (Diamond) to the degenerate distribution at zero price (Bertrand) as μ goes from 0 to 1. Further, as the search cost c tends to zero (given $\mu < 1$), the NE converges to the degenerate distribution at zero price. These two properties fulfill the objective of closing the gap between the contrasting Bertrand and Diamond results.

Remarkably, the NE does not converge to the degenerate distribution at zero price as the number of stores increases. To the contrary, the NE approaches the degenerate distribution at the monopoly price. The reason for this result is that as N increases, the probability of being the lowest-priced store decreases exponentially, thus undermining the incentive for lowering price. In this model, entry has a noncompetitive outcome. With few exceptions, the literature has focused on "long-run" (free-entry) equilibria. Robert Rosenthal (1980) shows that in Varian's model with (exogenously) informed and uniformed consumers, the NE becomes more monopolistic as the number of stores increases. It is interesting that we obtain the same result with endogenously informed and uninformed consumers. Stiglitz (1987), notwithstanding the above reservations, shows that prices can become less competitive as the number of firms increases.

As pricing becomes more monopolistic with an increase in the number of stores, the consumers with positive search costs are obviously worse off. It turns out that the costless shoppers are better off, so the effects on net welfare depend on the proportion of shoppers, μ. As a robust example demonstrates, it is possible that total surplus declines monotonically in N.

Section I of this paper develops the optimal consumer search strategy. Section II derives the NE of prices. Section III presents asymptotic results, and Section IV presents local comparative statics. Section V develops welfare effects. Section VI concludes with a further discussion. The Appendix contains the formal proofs.

I. Optimal Consumer Search

For consumers with search cost c, continued search is profitable if the expected benefits from continued search exceed the costs. The benefits are measured by consumer surplus. Given an observed price z, the benefit of finding a price $p < z$ is

$$(1)\qquad CS(p;z) \equiv \int_p^z D(x)\,dx.$$

We assume that $CS(0;z) < \infty$ for all $z \geq 0$. Recalling that p is the excess of price above marginal cost, this is an innocuous assumption. To find the *ex ante* expected benefits, we integrate $CS(p;z)$ with respect to the price distribution $F(p)$:

$$(2)\qquad ECS(z) \equiv \int_b^z CS(p;z)\,dF(p),$$

where b is the lower bound of the support of $F(p)$.[2] Let P denote the upper bound of the support of $F(p)$. Then, given $z \leq P$, and integrating by parts:

$$(3)\qquad ECS(z) \equiv \int_b^z D(p)F(p)\,dp.$$

Sampling the last store is profitable if $ECS(z) > c$. Clearly, $ECS(z)$ is strictly increasing for all $z \in [b, P)$, as long as $D(z) > 0$, which we will see holds for NE distributions F since $P \leq \hat{p}$. Therefore, if there exists an $r_F < P$ such that $ECS(r_F) = c$, then $ECS(z) \lesseqgtr c$ as $z \lesseqgtr r_F$; that is, r_F is unique. Define r_F to be equal to this unique root of $[ESC(z) - c]$, if it exists, and otherwise, let $r_F = +\infty$. Then, the optimal search rule can be stated as: search the last store if the lowest observed price, z, is greater than "reservation price" r_F; stop and purchase from a store with price z if $z < r_F$; the indifferent case $(z = r_F)$ can be specified either way.

In a symmetric NE, a consumer samples from the same price distribution, $F(p)$, each time she visits a new store. In these circumstances, it is well-known (for example, Meir Kohn and Steven Shavell, 1974), that when there are $K \geq 1$ stores left to search, the optimal stopping rule is independent of K. In other words, there exists a unique reservation price r_F, as defined above, such that it is optimal to continue search if the lowest observed price is greater than r_F, and optimal to stop search if the lowest observed price is less than r_F. In the event that all of the first $N-1$ stores visited have prices exceeding r_F, then the consumer visits the last store and picks the store with the lowest price among the N stores. If more than one store has the lowest price, we assume that the consumers are equally distributed among the lowest-priced stores.[3]

For price-setting behavior, it does not matter how we specify the indifferent case for the consumers with $c > 0$; however, it does matter how we specify the indifferent case for the consumers with $c = 0$. Indeed, if shoppers (consumers with $c = 0$) stop at the first store with price $p \leq r_F$ (which equals b for shoppers), then the monopoly price is an NE because all consumers will visit one store only! This is not a reasonable NE in the presence of shoppers, because at zero cost the shoppers could visit one more store, and this behavior will induce beneficial price competition among the stores. Moreover, if shoppers were prone to err slightly in their assessment of the NE-price distribution, then additional searches would occur and induce price undercutting. Therefore, in the spirit of "trembling hand perfection" (Reinhard Selten, 1975), we assume that the shoppers do not stop before they sample at least two

[2] The "support" of a probability distribution is the smallest closed set with probability one.

[3] As a matter of methodology, optimal search is an important benchmark. On the other hand, the results will be qualitatively unchanged if, instead, the high-search-cost consumers had a reservation price fixed at a constant, or equal to a monotonic transformation of the optimal reservation price.

prices equal to b. We will see in the next section that the event $p = b$ has zero probability in equilibrium. Therefore, the shoppers (who have zero-search costs) will, with probability one, sample the prices of all N stores and choose the lowest price.

II. Store Price-Setting

We focus on symmetric NE; from here on we suppress the qualifier "symmetric."[4] Initially, we will assume an exogenous consumer reservation price r, and determine an NE in price-setting conditional on r: let $F(p; r)$ denote the cumulative probability distribution of prices adopted by each of the N stores in such a conditional NE. This distribution will produce a reservation price r_F. Obviously, we want a consistent reservation price r^* that produces an NE distribution $F(p; r^*)$, which in turn produces $r_F = r^*$. Finding such a consistent reservation price will be the last step in this section.

We take the number of stores, N, as exogenously fixed. It is important to characterize equilibrium for arbitrary N rather than focusing exclusive attention on the long-run free-entry case. After all, the dynamic story behind free entry refers to profits with N above and below the long-run equilibrium number, N^*.

LEMMA 1: *Given $\mu \in (0,1)$, if $F(p; r)$ is an NE-distribution conditional on reservation price r, then it is atomless.*

The intuition for this fact is that profits could be discretely increased by undercutting atoms, so price distributions with atoms cannot be optimal.

Let P_r be the maximal element of the support of $F(p; r)$, and let $b(r)$ be the minimal element of the support.

[4] In an asymmetric NE, the consumer's posterior distribution on prices will change as observations are made. This nonstationarity makes it much more difficult to solve for asymmetric NE, and I have not attempted to do so.

LEMMA 2: *If $F(p; r)$ is an NE-distribution conditional on reservation price r, then $P_r = \min\{r, \hat{p}\}$.*

In other words, the upper limit of the support of $F(p; r)$ is the lesser of the monopoly price $\hat{p}$ and the consumer reservation price r. Intuitively, if $p_j > b(r)$, then only the $c > 0$ type consumers will buy at $p_j = P_r < r$; but then P_r yields less revenue than $p_j = r$. It should not be surprising that pricing above the monopoly price $\hat{p}$ is never optimal. Given a conditional NE-distribution $F(p; r)$, consumers with positive search cost always observe $p_j \le r$ and hence stop with probability one at the first store they visit. Only the shoppers engage in real search activity.

Given Lemmas 1 and 2, it is easy to derive the profits a store (say j) can expect when it sets price $p_j \le \hat{p}$ while all other stores play $F(p; r)$.

$$(4) \quad E\pi_j(p_j, F) \equiv \left\{\mu\left[1 - F(p_j; r)\right]^{N-1} + (1-\mu)/N\right\} \cdot R(p_j).$$

The first component is from the shoppers and involves the probability that p_j is lower than the other $N - 1$ prices. The second component is the share of captured consumers with positive search costs.

Definition: A distribution $F(p; r)$ is a conditional NE iff $E\pi(p, F)$ is equal to a constant (say π) for all p in the support of F and not greater than π for any p. $F(p; r)$ is a consistent NE iff F is a conditional NE and r is a consistent reservation price.

Then conditional NE-expected profits

$$\pi = E\pi(P_r, F) = R(P_r) \cdot (1-\mu)/N.$$

Solving $E\pi[p, F(p; r)] = \pi$ for $F(p; r)$ *yields*:

$$(5) \quad F(p; r) = 1 - \left[\left(\frac{1-\mu}{N\mu}\right)\left(\frac{R(P_r)}{R(p)} - 1\right)\right]^{\frac{1}{N-1}}.$$

The density function $f(p;r)$ is the derivative of $F(p;r)$ with respect to p:

$$(6)\quad f(p;r)=\left(\frac{1-\mu}{N(N-1)\mu}\right)\times\left[\left(\frac{1-\mu}{N\mu}\right)\left(\frac{R(P_r)}{R(p)}-1\right)\right]^{-\frac{N-2}{N-1}}\times\frac{R(P_r)}{R(p)}\,\frac{R'(p)}{R(p)},$$

which is nonnegative on $[0, P_r]$ since $R(p)$ is strictly increasing for all $p<\hat{p}$. To complete the characterization of the $F(p;r)$, we need to derive the lower bound of the support, $b(r)$. At the lower bound, $F[b(r);r]=0$; so from equation (5) and the fact that $R(p)$ is strictly increasing below $\hat{p}$, $b(r)$ is the unique solution to

$$(7)\quad R[b(r)]=\left[\frac{1-\mu}{1+(N-1)\mu}\right]\cdot R(P_r).$$

Given the consumer reservation price r, which defines P_r, and the lower bound defined by equation (7), the distribution $F(p;r)$ defined by equation (5) constitutes an NE in price-setting conditional on reservation price r. This fact can be demonstrated with three observations. First, note that $E\pi(p,F)=R(p)\cdot(1-\mu)/N<\pi$ if $P_r<p\le r$, or $E\pi(p,F)=0$ if $r<p$. Second, for all $p<b(r)$, the store captures $(\mu+(1-\mu)/N)$ of the customers, so $E\pi(p,F)<R(b)[\mu+(1-\mu)/N]=\pi$. Third, for all $p\in[b(r),P_r]$, by construction of $F(p;r)$, $E\pi(p,F)=\pi$, and $F(p;r)$ is a proper probability distribution.

However, before we declare our problem solved, we must establish that the consumer reservation price r is consistent with $F(p;r)$ as defined by equations (5) and (7), or that there exists at least one such consistent r. Recall that r_F was defined as the unique solution to $ECS(z)=c$, if one exists, or ∞ otherwise. Thus, a consistent reservation price $r^*\le\hat{p}$ must satisfy:

$$(8)\quad H(r^*;\mu,N,c)\equiv\int_{b(r^*)}^{r^*}D(p)F(p;r^*,\mu,N)\,dp-c=0,$$

where $b(r^*)$ is defined by equation (7); the parameters (μ, N) are listed among the arguments of H to highlight their effect via equation (5). Note that due to Lemma 2 reservation prices at or above $\hat{p}$ might as well be infinite from the point of view of the stores; thus, without loss of generality, we can focus attention exclusively on solutions in the $[0,\hat{p})$ interval. Observe that if $H(\cdot;\mu,N,c)$ is strictly increasing on $[0,\hat{p})$, then there is either a *unique* root or none at all. To ensure this uniqueness, we can impose a further restriction on the revenue function.

ASSUMPTION C: *$pR'(p)/R^2(p)$ is decreasing for all $p\in[0,\hat{p})$.*

This restriction can be rewritten as: $D''(p)\le R'(p)/p^2+2[D'(p)]^2/D(p)$; note that the right-hand side is positive for all $p\in[0,\hat{p})$. It is also satisfied for convex demand functions of the form $D(p)=(1-p)^\beta$ for $\beta\in(0,1)$. C is clearly satisfied for all concave (and linear) demand functions.

LEMMA 3: *Given C, $\partial H(r,c)/\partial r>0$ for all $r<\hat{p}$.*

Hence, C is sufficient to ensure the existence of a unique root of $H(r,c)$ in the $[0,\hat{p})$ interval or none at all.

Now we formally define a consistent reservation price

$$(9)\quad \rho(\mu,N,c)\equiv\begin{cases} r^*, & \text{if } H(r^*;\mu,N,c)=0 \\ & \text{and } r^*\in[0,\hat{p}) \\ +\infty, & \text{otherwise.}\end{cases}$$

Observe that this ρ defines a unique distribution function $F(p;\rho)$, which in turn reproduces the same ρ as the optimal reservation price for $F(p;\rho)$. Thus, we have established our first major result.

PROPOSITION 1: *$F(p;\rho)$, defined by equations* (5), (7), *and* (9) *specify a consistent*

NE; further, given C, there is no other symmetric NE.

The structure of this *NE* is very similar to that of Rosenthal (1980) and Varian (1980). The essential difference is that their uninformed consumers essentially have an exogenous reservation price greater than or equal to the monopoly price, so the upper bound of the *NE* distribution is always the monopoly price $\hat{p}$. In contrast, the model of this paper has the consumer reservation price endogenously determined, and consequently the upper bound on the *NE* distribution depends on the parameters (μ, N, c). This dependency is explored in the next sections.

Before closing this section, we should note that the Bertrand and Diamond results are degenerate special cases. When $\mu = 0$ (i.e., no shoppers), from equation (4) $E\pi(p, F) = R(p)/N$, so clearly the unique *NE* is for all stores to choose the monopoly price $\hat{p}$. When $\mu = 1$, equation (4) becomes $E\pi(p, F) = \{[1 - F(p; r)]^{N-1} + T\}R(p)$, where T stands for a complicated expression representing the probability that p is tied for the low price. Now observe that there can be no atoms above $b(r)$ since a p slightly below such an atom would avoid ties and earn discretely more profits. But then since F must be atomless above $b(r)$, $F(P_r; r) = 1$, so $E\pi(P_r, F) = 0$, but $E\pi(p, F) > 0$ for some $p \in (b(r), P_r)$, thus contradicting the optimality of P_r. Thus, F must be concentrated at $b(r)$, and the usual undercutting argument establishes that the unique *NE* is for all stores to price at marginal cost (i.e., zero).

III. Asymptotic Results

We now want to explore how the parameters (μ, N, c) affect the consistent reservation price, the resulting *NE* distribution of prices, the lower and upper bound of the support of this distribution, and *NE*-expected profits. For convenience of notation, let

$$\beta(\mu, N, c) \equiv b[\rho(\mu, N, c), \mu, N], \tag{10}$$

$$P_r(\mu, N, c) \equiv \min\{\rho(\mu, N, c), \hat{p}\},$$

$$\Phi(p; \mu, N, c) \equiv F[p; \rho(\mu, N, c), \mu, N],$$

and

$$\pi(\mu, N, c) \equiv R[P_r(\mu, N, c)] \cdot (1 - \mu)/N.$$

In this section, we will consider the asymptotic results as μ approaches 0 or 1, as c approaches 0 or increases without bound, and as N increases without bound. To aid in the statement of convergence results, let $\delta(p)$ denote the degenerate probability distribution with unit mass at p.

Our first asymptotic result concerns the effect of the proportion of shoppers, μ, on the *NE*.

PROPOSITION 2: *(a) As $\mu \to 0$, $\beta(\mu, N, c) \to P_r(\mu, N, c) \to \hat{p}$, and $\Phi(\cdot; \mu, N, c) \to \delta(\hat{p})$; hence $\pi(\mu, N, c) \to R(\hat{p})/N$. (b) Conversely, as $\mu \to 1$, $\rho(\mu, N, c) \to r \in (0, \hat{p})$, $\beta(\mu, N, c) \to 0$ and $\Phi(\cdot; \mu, N, c)$ converges weakly*[5] *to $\delta(0)$; hence $\pi(\mu, N, c) \to 0$.*

In other words, as the proportion of shoppers becomes vanishingly small, the entire *NE* distribution converges to the monoply price $\hat{p}$ (the Diamond result). Intuitively, most consumers will accept a price of $\hat{p}$, so it is not optimal to price much lower to attract the small number of shoppers. The proof establishes further that for sufficiently small μ, the upper bound $P_r = \hat{p}$. As the proportion of shoppers approaches one, the lower bound of the support approaches zero because more shoppers induce more cutthroat competition. On the other hand the upper bound does not approach zero because the costly searchers will purchase at prices significantly above zero. Nonetheless, the probability mass shifts toward zero, in the sense that for any store and positive price p, the probability that the store charges more than p coverages to zero.

PROPOSITION 3: *(a) As $c \to 0$, $P_r(\mu, N, c) \to \beta(\mu, N, c) \to 0$, and $\Phi(\cdot; \mu, N, c) \to \delta(0)$, and $\pi(\mu, N, c) \to 0$. (b) There is some $\bar{c} > 0$ such that for all $c > \bar{c}$, $P_r(\mu, N, c) = \hat{p}$, and*

[5] That is, for all $p > 0$, $\Phi(p; \mu, N, c) \to 1$: see Patrick Billingsley (1968).

$\beta(\mu, N, c)$, $\Phi(\cdot\,; \mu, N, c)$, and $\pi(\mu, N, c)$ are constant in c.

In other words, as the search cost decreases to zero, the entire *NE* distribution converges to the degenerate distribution at zero (the Bertrand result). Intuitively, less costly search lowers the reservation price and hence these consumers behave more like shoppers. Conversely, for $c > \int D(p)\,dp$, which is greater than $\int D(p)F(p)\,dp$, it follows from equation (8) that there does not exist a root of $H(r; \mu, N, c)$ in $[0, \hat{p})$, so the upper bound of the support of the *NE* distribution is the monopoly price. From then on, the *NE* distribution is independent of c.

Next, we explore the asymptotic effects of N.

PROPOSITION 4: *(a) There is an N' such that for all $N > N'$, $P_r(\mu, N, c) = \hat{p}$. (b) As $N \to \infty$, $\beta(\mu, N, c) \to 0$, $\Phi(\cdot\,; \mu, N, c)$ converges weakly to $\delta(\hat{p})$, and $\pi(\mu, N, c) \to 0$.*

The proof shows that all the probability mass becomes more and more concentrated at P_r. The reason for this seemingly paradoxical result is that as N increases, the likelihood of being the lowest-priced store decreases exponentially, thus undermining the incentive to lower prices. Given this result and equation (8), it follows that as the number of stores increases, eventually there is no root of $H(r; \mu, N, c)$ in $[0, \hat{p})$, so the upper bound on the distribution becomes the monopoly price $\hat{p}$. Expected profits are $R(\hat{p})(1-\mu)/N$, which clearly goes to zero, and similarly, using equation (7), the lower bound must also approach zero.

We now address the *long-run free-entry* case. Suppose there is a fixed cost κ to entry. Then, the long-run equilibrium number of stores is the largest integer, $N^*(\kappa)$, such that $\pi[\mu, N^*(\kappa), c] \geq \kappa$. (By proposition 4, $N^*(\kappa)$ is well-defined for $\kappa > 0$.)

PROPOSITION 5: *$N^*(\kappa)$ is a decreasing step function, and $N^*(\kappa) \to \infty$ as $\kappa \to 0$. Consequently, as $\kappa \to 0$, $\Phi(\cdot\,; \mu, N^*(\kappa), c)$ converges weakly to $\delta(\hat{p})$.*

In other words, as entry costs decrease, more stores will enter, and then Proposition 4 tells us that pricing eventually becomes more monopolistic!

IV. Local Comparative Statics

When $H(r; \mu, N, c)$ has a root in $(0, \hat{p})$, we use the implicit function theorem to determine the local behavior of r, β, Φ, and π. For this purpose, we derive the signs of the partial derivatives of H with respect to (r, μ, c). It is immediate that $\partial H(r; \mu, N, c)/\partial c = -1$. Not quite immediate, but still straightforward, we have $\partial H(r; \mu, N, c)/\partial \mu > 0$. Lemma 3 established that under assumption C, $\partial H(r; \mu, N, c)/\partial r > 0$.

Our first comparative statics result concerns the effect of the proportion of shoppers, μ, on the *NE*.

PROPOSITION 6: *Given C, (a) $\partial \rho(\mu, N, c)/\partial \mu \leq 0$ (strict inequality iff $\rho(\mu, N, c) < \hat{p}$), (b) $\partial \beta(\mu, N, c)/\partial \mu < 0$, (c) $\partial \Phi(p; \mu, N, c)/\partial \mu > 0$ for all $p \in (b, P_r)$, and (d) $\partial \pi(\mu, N, c)/\partial \mu < 0$.*

In other words, the *NE* moves continuously and monotonically from the Diamond result to the Bertrand results as the proportion of consumers with zero-search costs goes from 0 to 1. As the proportion of shoppers increases, the entire *NE* distribution shifts to lower prices: the lower and upper bounds of the support shift down; moreover, probability mass shifts down everywhere (i.e., the new distribution is first-order stochastically dominated).

PROPOSITION 7: *Given C, (a) $\partial \rho(\mu, N, c)/\partial c \geq 0$, and $\partial \beta(\mu, N, c)/\partial c \geq 0$; (b) $\partial \Phi(p; \mu, N, c)/\partial c \leq 0$ for all $p \in (b, P_r)$; and (c) $\partial \pi(\mu, N, c)/\partial c \geq 0$ (all strict inequalities iff $\rho(\mu, N, c) < \hat{p}$).*

In other words, as the search cost decreases to zero, the *NE* moves continuously and monotonically toward the Bertrand result. Conversely, as search cost increases, the upper bound of the support and the probability mass move monotonically toward the monopoly price $\hat{p}$. Since search costs enter only through the consumer reservation price, if $\rho(\mu, N, c) \nless \hat{p}$, then the *NE* is unaffected by c.

TABLE 1—EXAMPLE OF A CONSISTENT NE: $D(p)=1-p$, $\mu=0.5$, AND $c=0.05$

$N=$	2	3	4	10	20	30	100	1000
$P_r=$	0.1139	0.1305	0.1471	0.2379	0.3764	0.4879	0.5000	0.5000
$b=$	0.0349	0.0292	0.0257	0.0168	0.0113	0.0081	0.0025	0.0003
$\pi=$	0.0252	0.0189	0.0157	0.0091	0.0059	0.0042	0.0013	0.0001
$\bar{p}=$	0.0594	0.0754	0.0914	0.1790	0.3112	0.4163	0.4702	0.4952
$L=$	0.0525	0.0520	0.0512	0.0464	0.0392	0.0315	0.0126	0.0021
$TCS=$	0.3218	0.3148	0.3079	0.2733	0.2285	0.2005	0.1918	0.1881
$TS=$	0.3723	0.3715	0.3707	0.3640	0.3459	0.3269	0.3168	0.3131

We next explore the local effects of N. Treating N as a real number instead of an integer, we can differentiate equation (5) with respect to N.

$$\text{(11)}\quad \frac{\partial F(p;r,\mu,N)}{\partial N} = \frac{1-F}{N(N-1)}(N\ln(1-F)+1).$$

Since $F(b;r)=0$, the partial derivative of H with respect to N, $\partial H(r;\mu,N,c)/\partial N$, is just the integral with respect to $p\in[b,P_r]$ of $D(p)\cdot\partial F(p;r,\mu,N)/\partial N$. From equation (11), note that $\partial F/\partial N=0$ at $p=r$, and $\partial F/\partial N=1/[N(N-1)]>0$ at $p=b$; $\partial F/\partial N$ is positive up to the p', where $F(p')=1-\exp(-1/N)$; and $\partial F/\partial N$ is negative between this p' and r. If the interval (b,p') is not too large relative to the (p',r) interval, then a reasonable conjecture is

Hypothesis HN.

$$\partial H(r;\mu,N,c)/\partial N<0.$$

So far, a proof of this hypothesis for general demand functions has eluded me. However, I do have a proof for constant demand (available from the author upon request), and I have numerically verified this hypothesis for linear demand functions and a variety of other demand functions. On the other hand, for any admissible demand function, HN will be satisfied when there are sufficiently many stores.

LEMMA 4: *There is an N' such that for all $N>N'$, HN is satisfied.*

PROPOSITION 8: (*a*) *Under hypothesis HN and C, $\partial\rho(\mu,N,c)/\partial N>0$ (whenever $\rho(\mu,N,c)<\hat{p}$), and (b) under hypothesis HN and C, and/or whenever $\rho(\mu,N,c)=\infty$, $\partial\Phi(p;\mu,N,c)/\partial N<0$ for all p between p' and $P_r(\mu,N,c)$.*

Thus, under hypothesis HN and assumption C, $\rho(\mu,N,c)$ will increase monotonically with N until there ceases to be a root of $H(r;\mu,N,c)$ in the range $[0,\hat{p})$ at which time $\rho(\mu,N,c)$ is defined as $+\infty$ and $P_r(\mu,N,c)=\hat{p}$. Moreover, since $F(P_r)=1$, the probability density function at the upper bound $P_r(\mu,N,c)$ must also increase indefinitely as N increases. This local effect is consistent with the asymptotic result (Proposition 4) that for large N, $P_r(\mu,N,c)=\hat{p}$ and the distribution converges to $\delta(\hat{p})$. It also follows that the mean price, $\bar{p}\equiv\int pdF$, will also increase with the number of stores.

Table 1 shows results for the canonical linear demand function $D(p)=1-p$. Figure 1 displays the density function plotted on $[b,P_r]$ for selected values of N; it shows how probability mass is shifted toward P_r as N increases.

To investigate the total effect of N on the lower bound, $\beta(\mu,N,c)$, we apply the chain rule to equation (7) to derive that

$$\text{(12)}\quad \operatorname{sign}\left(\frac{\partial\beta(\mu,N,c)}{\partial N}\right) = -\operatorname{sign}\left(\frac{\mu}{1+(N-1)\mu}-\frac{R'(P_r)}{R(P_r)}\times\frac{\partial\rho(\mu,N,c)}{\partial N}\right).$$

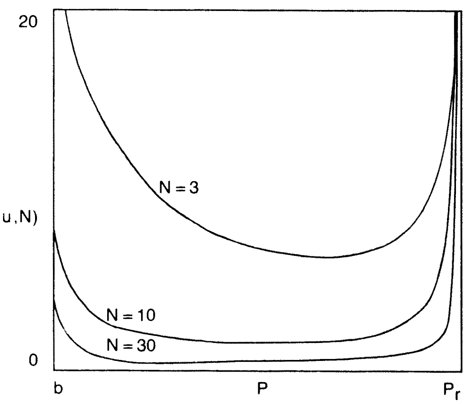

FIGURE 1. DENSITY FUNCTION FOR: $D(p)=1-p$, $\mu=0.5$, AND $c=0.05$

Observe that when either P_r equals the monopoly price, or marginal revenue is arbitrarily close to zero, the second term is negligible, so $\partial\beta(\mu, N, c)/\partial N<0$. Thus, recalling Proposition 4, for sufficiently large N, $\partial\beta(\mu, N, c)/\partial N<0$.[6] For constant demand, linear demand, and a variety of other cases, I have numerically verified that the sequence of $\beta(\mu, N, c)$ generated by the integers $N\geq 2$ is strictly decreasing, given any $\mu\in(0,1)$ and $c>0$ (for example, see Table 1). Thus, for a wide class of demand specifications, as the number of stores increases, competition induces stores to consider low prices (so β decreases), but the probability of being the lowest-priced store decreases thereby discouraging the stores from putting much probability mass on these low prices; instead, they put more probability mass at the higher prices. Unfortunately, a proof that $\partial\beta(\mu, N, c)/\partial N<0$ in general has eluded me.

Similarly, one would expect the total effect of N on expected profits to be negative. For constant demand, linear demand, and a variety of other cases, I have numerically verified that the sequence of $\pi(\mu, N, c)$ generated by the integers $N\geq 2$ is indeed decreasing, given any $\mu\in(0,1)$, and $c>0$. While a general proof that $\partial\pi(\mu, N, c)/\partial N<0$ has eluded me, the behavior of profits and the lower bound are closely linked via equation (7).

LEMMA 5: *If* $\partial\beta(\mu, N, c)/\partial N<0$, *then* $\partial\pi(\mu, N, c)/\partial N<0$.

It would be desirable to have a more definitive general result on the local effect of the number of stores on expected profits and the lower bound of the *NE*-price distribution. Nonetheless, given any specific demand function, the *NE* can be explicitly computed via equations (5), (7), and (9), and these effects will be manifest.

V. Welfare Analysis

We have characterized the *NE* distribution of prices and how it depends on the parameters, but what is the ultimate impact on social welfare? In cases where pricing becomes more monopolistic, clearly the positive-search-cost consumers suffer. On the other hand, if the lower bound of the support decreases, then perhaps the shoppers are benefiting. Given consumer surplus as our measure of welfare, we can compute the expected consumer surplus, the expected producer surplus, and the expected total surplus.

First, we characterize the price that the shoppers expect to receive, having gone to the trouble of sampling all N stores. The cumulative distribution function of the lowest price of N samples from the price distribution $F(p;\rho)$ is

$$(13)\quad G(p)\equiv 1-[1-F(p;\rho)]^N$$

$$=1-\left[\frac{R(b)\,[R(P_r)-R(p)]}{R(p)\,[R(P_r)-R(b)]}\right]^{\frac{N}{N-1}}.$$

The second expression follows by substitution from equations (5) and (7); clearly, $G(P_r)=1$ and $G(b)=0$. Let $L(\mu, N, c)$ de-

[6] Note that HN is not required; indeed, if HN does not hold, then $\partial\rho/\partial N\leq 0$, so $\partial\beta/\partial N<0$. Also recall from Propositions 2 and 3 that for small μ and/or large c, P_r is close to $\hat{p}$, so we would have $\partial\beta/\partial N<0$.

note the expectation of the lowest price. Then,

$$(14)\quad L(\mu, N, c) = b + \int_b^{P_r}[1 - G(p)]\,dp.$$

Recalling the asymptotic and comparative statics results of Sections IV and V, as the proportion of shoppers, μ, goes from 0 to 1, the NE goes from having all mass at the monopoly price $\hat{p}$ to having all mass at zero; hence, $L(\mu, N, c)$ goes continuously from $\hat{p}$ to 0. Similarly, as the search cost approaches zero, $L(\mu, N, c) \to 0$.

It is also straightforward to verify that as the number of stores grows without bound, $L(\mu, N, c) \to 0$ also. Thus, while consumers with high search costs suffer because the mean price they face is close to the monopoly price, the shoppers benefit from more stores because the mean price they face is close to zero. The total effect would appear to depend on the proportion of shoppers.

Using consumer surplus as our welfare measure and taking the monopoly price as our benchmark, the consumer surplus of the positive-search-cost consumers is $\int CS(p;\ \hat{p})\,dF(p)$, and for the shoppers it is $\int CS(p;\hat{p})\,dG(p)$, where the integration is over $[b, P_r]$. It is convenient to *define* $\hat{G}(p) \equiv \mu G(p) + (1-\mu)F(p)$. Then, using equation (1) and integrating by parts, total expected consumer surplus is

$$(15)\quad TCS \equiv CS(P_r;\hat{p}) + \int_b^{P_r} D(p)\hat{G}(p)\,dp.$$

Producer surplus from the positive-search-cost consumers is just $\int R(p)\,dF(p)$, and from the shoppers it is $\int R(p)\,dG(p)$. Then total expected producer surplus is

$$(16)\quad TPS \equiv R(P_r) - \int_b^{P_r} R'(p)\hat{G}(p)\,dp,$$

which is also equal to $N\pi = (1-\mu)R(P_r)$.

Finally, total expected surplus is just the sum of expected consumer surplus and producer surplus:

$$(17)\quad TS \equiv CS(P_r;\hat{P}) + R(P_r) - \int_b^{P_r} pD'(p)\hat{G}(p)\,dp.$$

Of course, the efficient outcome (Bertrand) would have $TPS = 0$ and $TS = TCS = CS(0;\hat{p})$, the maximum possible. Observing that $\partial\hat{G}/\partial P_r < 0$, and $\partial\hat{G}/\partial b < 0$, it follows that anything that decreases P_r (or b) will increase both TCS and TS and decrease TPS. Thus, not surprisingly, increasing the proportion of shoppers or lowering search costs, will unambiguously increase social welfare. The effect of the number of stores is less transparent.

The behavior of TCS as a function of N depends on $\partial\rho/\partial N$ (Proposition 8) and $\partial\hat{G}/\partial N$, which in turn depends on $\partial F/\partial N$ equation (11). TPS is increasing in N until $P_r = \hat{p}$, after which TPS is constant. TS depends on the integral of $pD'(p)\partial\hat{G}/\partial N$, which is difficult to analyze. While a general result has not been derived, given any specific demand function and parameters (μ, N, c), the NE and the above surplus measures can be computed. Table 1 shows these results for linear demand when $\mu = .5$, and $c = 0.05$ (10 percent of $\hat{p}$). Despite the relatively large proportion of shoppers, TCS and TS decline monotonically in N. (Indeed, given $\mu \leq 0.7$, TCS and TS decline monotonically in N). It appears that with $N = 2$, shoppers can already expect to find a very low price, so there is not much more to gain from further entry, but the positive-search-cost consumers suffer from entry. While this remarkable result is not a general theorem, the point is that it is easy to construct reasonable, robust examples in which social welfare declines with entry!

VI. Discussion

This paper has provided a bridge between the contrasting Bertrand and Diamond results. As the proportion of shoppers goes from 1 to 0, the NE changes continuously from the Bertrand NE to the Diamond NE.[7] As the cost of search goes to zero, the NE converges to the Bertrand NE. Remarkably, as the number of stores increases, the NE moves toward the Diamond NE. The reason for the latter result is that with more stores, the probability of being the lowest-priced store decreases exponentially, thus lowering the expected gain from low prices relative to the profits from the captive consumers with high search costs. Consequently, entry discourages more competitive pricing, and instead leads to more monopolistic pricing. On the other hand, the shoppers benefit from having more stores. The effects on total social welfare depend on demand specifications, but for a robust class of reasonable examples total surplus declines monotonically with entry.

The uniqueness of the NE as well as the local comparative statics results rested on Assumption C. Since C is a sufficient but not necessary condition, these results hold for a wider (yet to be characterized) class of demand functions. The local effect of the number of stores also depends on the demand function, but a characterization of this effect in terms of the properties of the demand function has not been derived (other than for linear and constant demand functions). Another shortcoming is that the effect of N on the lower bound of prices and expected profits has so far eluded analytic determination. Nonetheless, for any specific demand function, the NE is computable, and all the effects will be manifest.

[7]This smooth transition between the Bertrand and Diamond results depends crucially on the shoppers having zero-search costs. If, instead, there are two types of consumers with search costs $c_h > c_1 > 0$ respectively, the Diamond result would prevail. It would be desirable to have a smooth transition as $c_1 \to 0$. One way to obtain such a result is to consider ε-perfect NE. Specifically, suppose consumers compute their reservation price with some error. Given ε, for c_1 sufficiently small, there is positive probability that the c_1 types have a reservation price below b, which induces price-cutting by the stores. The resulting ε-perfect NE will be similar to the one derived in this paper and, in addition, will depend continuously on c_1.

It would be desirable to extend this model in several ways. First, the model should be extended to permit more general distributions of search costs. Second, the model could be modified to permit a positive cost to second visits; consumers would have perfect recall but would incur some cost to return to a store. This modification will alter the reservation prices of consumers. Third, it would be interesting to know if there are any asymmetric NE.

In terms of the store specifications, the model could be modified to permit increasing marginal costs, fixed costs, capacity constraints, and heterogeneous marginal costs. These modifications would permit the study of the robustness of various results in the literature to the Bertrand assumption (For example, David Kreps and Jose Scheinkman, 1983, and Stahl, 1988).

Last, but not least, we have allowed consumers to act over a time, but we have required stores to adopt a static strategy. The defense of this assumption was that the cost of revising prices makes it suboptimal to do so in the time it takes consumers to search. On the other hand, it may be interesting to investigate a dynamic model in which stores could choose time-dependent pricing strategies.

APPENDIX

PROOF OF LEMMA 1:

See Varian (1980), Proposition 3. (Varian's proof assumes the shoppers have sampled all N stores. Nevertheless, his argument is valid if the shoppers have sampled at least two stores. Specifically, to show there can be no atom at p', it is enough to observe that if shoppers sample at least two stores with $p = p'$, then by undercutting an atom at p', a store will discretely increase its share of shoppers.)

PROOF OF LEMMA 2:

Lemma 1 implies that $b(r) < P_r$, and no shoppers stop at any $p > b$. Then, if $P_r < \min(r, \hat{p})$, by charging p slightly above P_r, the store will lose no non-shoppers and gain revenue. Therefore, this p dominates P_r, a contradiction; hence, $P_r \geq \min(r, \hat{p})$. Suppose $r \leq \hat{p}$ and $r < P_r$. For $p = P_r$, the store "captures" no consumers—that is, all consumers leave and search again, *and* no consumer will return because $1 - F(P_r) = 0$. On the other hand, $p = r$ will at least capture $1/N$ of the non-shoppers, which gives positive profits, a contradiction. The remaining case has $\hat{p} \leq P_r \leq r$. Since $\hat{p}$ is the unique maximum of $R(p)$, $E\pi(p, F)$ is strictly decreasing for $p > \hat{p}$, implying that $P_r > \hat{p}$ is suboptimal. Therefore, $P_r = \min\{r, \hat{p}\}$. □

PROOF OF LEMMA 3:

Given $r < \hat{p}$, from equation (8), $\partial H(r; \mu, N, c)/\partial r = D(r) + \int_b^r D(x) \cdot (\partial F/\partial r)\, dx$. Now by differentiating equation (5) with respect to r, and using equation (6), we find that

$$\frac{D(x)\,\partial F/\partial r}{D(r)\,\partial F/\partial x} = -\frac{rR'(r)/R^2(r)}{xR'(x)/R^2(x)},$$

which, given C, lies in the interval $(-1,0)$ for all $x < r$. Therefore, $\partial H/\partial r > D(r)[1 - \int_b^r(\partial F/\partial x)\, dx] = 0$.

PROOF OF PROPOSITION 2:

(a) Since $(1-\mu/[1+(N-1)\mu] \to 1$ as $\mu \to 0$, from equation (7), $\beta(\mu, N, c) \to P_r$. Then $H(r; \mu, N, c) \to -c < 0$. Therefore, for sufficiently small μ, $\rho(\mu, N, c)$ does not have a root in $[0, \hat{p})$, implying that $P_r(\mu, N, c) = \hat{p}$. Clearly, $\pi(\mu, N, c) \to R(\hat{p})/N$. (b) From equation (7), as $\mu \to 1$, $\beta(\mu, N, c) \to 0$. Note that $\rho(\mu, N, c)$ is continuous in μ (for example, see the proof of Proposition 5(a)). Suppose $\rho(\mu, N, c) \to 0$; but then $H(r) \to -c < 0$, a contradiction. Therefore, $\rho(\mu, N, c) \to r > 0$. Let $\mu = 1 - 1/t$. Then for any $p \in (0, r]$, $[1 - F(p; r, 1-1/t, N)]^{N-1} = [R(r)/R(p) - 1]/N(t-1) \to 0$ as $t \to \infty$, implying that $\Phi(p; \mu, N, c) \to 1$. Clearly, $\pi(\mu, N, c) \to 0$. □

PROOF OF PROPOSITION 3:

(a) Suppose to the contrary that as $c \to 0$, $P_r(\mu, N, c) \to P^* > 0$; but then $H(r; \mu, N, c) > 0$, since by equation (7) $\beta(\mu, N, c) < P_r(\mu, N, c)$, which is a contradiction. Clearly, $\pi(\mu, N, c) \to 0$. (b) Suppose to the contrary that for all $c > 0$, $\rho(\mu, N, c) < \hat{p}$; but $H(r; \mu, N, c)$ is bounded above (by say K), so for $c > K$, we have a contradiction. □

PROOF OF PROPOSITION 4:

(a) Suppose to the contrary that for all N, $\rho(\mu, N, c) < \hat{p}$; that is, $H(r; \mu, N, c)$ has a root in $[0, \hat{p})$ for all N. Let r be a limit point of $\rho(\mu, N, c)$ as $N \to \infty$. For $\alpha \in (0, r]$, choose $p \le r - \alpha$. Note that $F(p; r, \mu, N)$ has the form $1 - (w_N/N)^{1/(N-1)}$, where w_N is determined by (p, P_{rN}, μ), and is bounded above zero. It follows that then $F(p; r, \mu, N) \to 0$ as $N \to \infty$. Therefore, given α and any $\varepsilon > 0$, there exists an $N_{\alpha\varepsilon}$ such that for all $N > N_{\alpha\varepsilon}$, $F(p; r, \mu, N) < \varepsilon$ for all $p \le r - \alpha$. Thus, $H(r; \mu, N, c) \le$

$$\int_0^r D(x) F(x; r, \mu, N)\, dx - c$$

$$< \int_{b(r)}^{r-\alpha} D(x)\varepsilon dx + \int_{r-\alpha}^{r} D(x)\, dx - c$$

$$< \varepsilon CS(r) + \alpha D(r-\alpha) - c.$$

First, observe that if $r = 0$, so $\alpha = 0$ is our only choice, then the above inequalities still hold, so by choice of ε we would have $H(r; \mu, N, c) < 0$, a contradiction; hence $r > 0$. Second, given $r > 0$, for sufficiently large N, we would again have $H(r; \mu, N, c) < 0$, a contradiction. Therefore, for sufficiently large N, $H(r; \mu, N, c)$ ceases to have a root in $[0, \hat{p})$, implying that $P_r = \hat{p}$. (b) In addition, taking any $\alpha \in (0, r]$, we have shown that for all $p < P_r$, $\Phi(p; \mu, N, c) \to 0$. Next, given N sufficiently large so $P_r = \hat{p}$, $\pi(\mu, N, c) = (1-\mu)R(\hat{p})/N \to 0$, and from equation (7) $R[\beta(\mu, N, c)] = \{N/[1+(N-1)\mu]\} \cdot \pi(\mu, N, c) \to 0$; hence, we must have $\beta(\mu, N, c) \to 0$ as well. □

PROOF OF PROPOSITION 5:

Define $\eta(\kappa) \equiv \{x \in \mathscr{R} \mid x \ge x'$ for all x' such that $\pi(\mu, x', c) \ge \kappa\}$; that is, the optimal industry size when not confined to be an integer. Now observe that we must have $\partial\pi/\partial x \le 0$ at η. If $\partial\pi/\partial x < 0$, then $d\eta/d\kappa < 0$. Suppose instead that $\partial\pi/\partial x = 0$, and consider a decrease in κ; it must be that η increases. Therefore, $d\eta/d\kappa = -\infty$. In any event, $d\eta/d\kappa < 0$; in other words, $\eta(\kappa)$ is a strictly decreasing function. But then $N^*(\kappa)$ is the unique decreasing step function characterized by the largest integer not exceeding $\eta(\kappa)$. Next, consider a monotonic sequence of positive $\kappa \to 0$. If for all $\kappa > 0$, $\eta(\kappa) < N'$ for some $N' < \infty$, then for κ sufficiently small, $\pi[\mu, \eta(\kappa), c] > \pi(\mu, N', c)/2 > 0$. However, by Proposition 4, $\lim_{N\to\infty}\pi(\mu, N, c) = 0$, so for κ sufficiently small, there is some $N'' > N' > \eta(\kappa)$ that is feasible, which contradicts the definition of $\eta(\cdot)$. Therefore, $\eta(\kappa) \to \infty$ as $\kappa \to 0$; consequently, $N^*(\kappa) \to \infty$ as $\kappa \to 0$. Now weak convergence of $\Phi[\cdot, \mu, N^*(\kappa), c]$ follows directly from Proposition 4. □

PROOF OF PROPOSITION 6:

(a) From equations (5) and (8), $\partial H(r; \mu, N, c)/\partial\mu > 0$. Thus, from Lemma 3 and the implicit function theorem, $\partial\rho(\mu, N, c)/\partial\mu < 0$. $\rho\pi(\mu, N, c)/\partial\mu < 0$. (b) From equation (7), $\partial b(r, \mu, N)/\partial r > 0 > \partial b(r, \mu, N)/\partial\mu$. Using the chain rule: $\partial\beta(\mu, N, c)/\partial\mu = (\partial b(r, \mu, N)/\partial r) \cdot (\partial\rho(\mu, N, c)/\partial\mu) + \partial b(r, \mu, N)/\partial\mu < 0$. (c) In the proof of Lemma 3, we saw that $\partial F(p; r, \mu, N)/\partial r < 0$. From equation (5), $\partial F(p; r, \mu, N)/\partial\mu > 0$. Then using the chain rule: $\partial\Phi(p; \mu, N, c)/\partial\mu = [\partial F(p; r, \mu, N)/\partial r] \cdot [\partial\rho(\mu, N, c)/\partial\mu] + \partial F(p; r, \mu, N)/\partial\mu < 0$. (d) From equation (10), $\partial\pi(\mu, N, c)/\partial\mu = -R(P_r)/N + R'(P_r) \cdot (\partial\rho/\partial\mu) \cdot (1-\bar{\mu})/N < 0$. □

PROOF OF PROPOSITION 7:

Clearly, if $\rho \ge \hat{p}$, then c has no effect; so suppose $\rho < \hat{p}$. (a) From equation (8), $\partial H(r; \mu, N, c)/\partial c = -1$; thus, using Lemma 3 and the implicit function theorem, $\partial\rho(\mu, N, c)/\partial c > 0$. (b) From equation (7), $\partial b(r, \mu, N)/\partial r > 0 = \partial b(r, \mu, N)/\partial c$; thus, $\partial\beta(\mu, N, c)/\partial c = (\partial b/\partial r) \cdot (\partial\rho/\partial c) > 0$. (c) From equation (5), $\partial F(p; r, \mu, N)/\partial r < 0 = \partial F(p; r, \mu, N)/\partial c$; thus, $\partial\Phi(p; \mu, N, c)/\partial c = (\partial F/\partial r) \cdot (\partial\rho/\partial c) > 0$. (d) From equation (10), $\partial\pi(\mu, N, c)/\partial c = R'(P_r) \cdot (\partial\pi/\partial c) \cdot (1-\mu)/N > 0$. □

PROOF OF LEMMA 4:

Let $Y(p, N) \equiv F(p, N) + \exp(-1/N) - 1$, which equals 0 at p'. Note that $\partial Y/\partial p = \partial F/\partial p > 0$, and $\partial Y/\partial N = (1/N^2)\exp(-1/N) - \partial F/\partial N$, which is positive at p' since $\partial F/\partial N = 0$ at p' (by definition of p'). Hence, by the implicit function theorem, $dp'/dN < 0$. But suppose $p' \to \bar{p}' > b$. For large N there is a constant w such that, $1 - F(\bar{p}') \approx \exp\{\ln(w/N) \cdot [1/(N-1)]\}$ which is strictly less than $\exp(-1/N)$, contradicting the definition of p'. Hence, $p' \to b$, so $\lim_{N \to \infty}(\partial H/\partial N) = -\alpha < 0$. Therefore, there is an N' such that for all $N > N'$, $\partial H/\partial N < -\alpha/2 < 0$. □

PROOF OF PROPOSITION 8:

(a) Using HN, Lemma 3, and the implicit function theorem, $\partial \rho(\mu, N, c)/\partial N > 0$. (b) Using the chain rule: $\partial \Phi(p; \mu, N, c)/\partial N = (\partial F/\partial r) \cdot (\partial \rho/\partial N) + \partial F/\partial N$. Given (a), the first term is negative, and given $p \in (p', P')$, where $F(p') = 1 - \exp(-1/N)$, the second term is negative also. □

PROOF OF LEMMA 5:

From equation (10),

$$\partial \pi(\mu, N, c)/\partial N = -R(b)(1-\mu)/N^2 + (\partial \beta/\partial N) \cdot R'(b) \cdot [1 + (N-1)\mu]/N.$$

The first term is negative, and by the premise of this Lemma the second term is negative also. □

REFERENCES

Bertrand, Joseph, "Théorie mathématique de la richesse sociale," *Journal des Savants*, 1883, 499–508.

Billingsley, Patrick, *Convergence in Probability Measures*, Wiley & Sons, 1968.

Braverman, Avishay, "Consumer Search and Alternative Market Equilibria," *Review of Economic Studies*, April 1980, *47*, 487–502.

Burdett, Kenneth and Judd, Kenneth L., "Equilibrium Price Dispersion," *Econometrica*, July 1983, *51*, 955–70.

Carlson, John and McAfee, R. Preston, "Discrete Equilibrium Price Dispersion," *Journal of Political Economy*, June 1983, *91*, 480–93.

Diamond, Peter, "A Model of Price Adjustment," *Journal of Economic Theory*, June 1971, *3*, 156–68.

Kohn, Meir G. and Shavell, Steven, "The Theory of Search," *Journal of Economic Theory*, October 1974, *9*, 93–123.

Kreps, David M. and Scheinkman, Jose A., "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes, *Bell Journal of Economics*, Autumn 1983 *14*, 326–37.

Manning, Richard and Morgan, Peter, "Search and Consumer Theory," *Review of Economic Studies*, April 1982, *49*, 203–16.

Morgan, Peter and Manning, Richard, "Optimal Search," *Econometrica*, July 1985, *53*, 923–44.

Reinganum, Jennifer F., "A Simple Model of Equilibrium Price Dispersion," *Journal of Political Economy*, August 1979, *87*, 851–58.

Rob, Raphael, "Equilibrium Price Distributions," *Review of Economic Studies* July 1985, *52*, 457–504.

Rosenthal, Robert W., "A Model in Which an Increase in the Number of Sellers Leads to a Higher Price," *Econometrica*, September 1980, *48*, 1575–79.

Salop, Steven C. and Stiglitz, Joseph E., "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," *Review of Economic Studies*, October 1977, *44*, 493–510.

Selten, Reinhard, "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 1975, *4*, 25–55.

Stahl, Dale O., "Bertrand Competition for Inputs and Walrasian Outcomes," *American Economic Review*, March 1988, *78*, 189–201.

Stiglitz, Joseph E., "Equilibrium in Product Markets with Imperfect Information," *American Economic Review*, May 1979, *69*, 339–45.

______, "Competition and the Number of Firms in a Market: Are Duopolies More Competitive than Atomistic Markets?," *Journal of Political Economy*, October 1987, *95*, 1041–61.

Varian, Hal R., "A Model of Sales," *American Economic Review*, September 1980, *70*, 651–59.

Wilde, Louis L. and Schwartz, Allan, "Equilibrium Comparison Shopping," *Review of Economic Studies*, July 1979, *46*, 543–53.