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Optimal sales mechanism with outside options ☆

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Abstract

This paper studies the optimal design of sales mechanisms when a buyer can quit a negotiation for an outside option at any time. The main results show that the profit-maximizing mechanism induces a set of buyer types to delay purchasing a good if the value of the outside option is highly dispersed among buyer types. Moreover, to prevent a buyer from quitting a negotiation, the profit-maximizing mechanism also features an upfront payment, which is compensated later by a price discount. The seller can implement the profit-maximizing mechanism by offering a declining price path or a menu of European options.

JEL classification: C78; D82; D86

Keywords: Delay; Option contract; Outside option; Upfront payment; Quitting right

1. Introduction

In sales negotiations, parties often face a choice between haggling or promptly quitting a negotiation. There is clearly a trade-off: the parties can haggle in search of a delayed agreement, but it is sometimes more beneficial for both parties to immediately quit the negotiation and opt

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for outside options. This trade-off is resolved in various manners. For example, a delay is normal in business-to-business negotiations, whereas retailers typically offer a fixed-price with no room for haggling.

Our aim is to address the following questions: what leads negotiating parties to choose a delay rather than an immediate breakdown, or vice versa? How does the effort to avoid a breakdown or delay affect the division of the surplus and other features of the negotiation process and outcome?

Suppose that a buyer (she) and a seller (he) negotiate the terms of trade for a single indivisible good. Prior to the negotiation, the buyer (informed party) privately learns her type, which simultaneously determines the values of both the seller's good and her outside option. The seller (uninformed party) can choose and commit to any mechanism, thus allowing him to choose a stochastic time-dependent price path, arrange auxiliary transfers between two parties that may depend on the buyer's reports, break off the negotiation at any time, etc.[1]

Sales negotiations with the seller's full commitment power have long been studied in the literature. Our model has two distinctive features. First, the buyer's outside option depends on her type (type-dependent outside option). Second, the buyer can break off and quit the negotiation at any time, both on and off the equilibrium path, in which case she can still enjoy the outside option (quitting rights).[2] Implicitly, signing any contract that abrogates the buyer's outside option or quitting right is impossible or prohibitively costly.

The main results of the paper show when and how a delay occurs in profit-maximizing (hereafter, optimal) mechanisms rather than an immediate outcome.[3] The seller *strictly* prefers delaying to immediately breaking off the negotiation if (i) the value of the outside option is positively related to the gain from trading and (ii) the outside options are highly dispersed among buyer types.[4] Otherwise, it is optimal for the seller to commit to a fixed-price mechanism, such that the buyer either purchases the seller's good or quits the negotiation immediately.

The occurrence of a delay in optimal mechanisms shows that the no-haggling result (Stokey, 1979; Riley and Zeckhauser, 1983; Samuelson, 1984) does not hold if the buyer has the quitting right and type-dependent outside options. The comparative statics with respect to outside options also explains why delays remain the norm in markets for automobiles, inputs for manufacturing, and so forth. Buyers in these markets have heterogeneous outside options owing to differences in their financial status and/or information, and thus the seller can make more profit by delaying. By contrast, a fixed-price mechanism is optimal for retail transactions in which buyers have similar outside options.

To see why a delay can be strictly beneficial for the seller, observe first that the outside option remains available to the buyer even after the negotiation breaks down. Additionally, because the value of the outside option is type-dependent and unknown to the seller, any possibility of breakdown concedes information rent to buyer types with a valuable outside option. Thus, all else being equal, the seller can extract more profit by delaying the negotiation instead of breaking it off immediately.

1 Board and Pycia (2014), Hwang and Li (2017), Hwang (2018), and Chang and Lee (2021) study the same problem but without the seller's commitment power.

2 Quitting rights are not equivalent to ex post participation constraints which require the agent's payoff to be higher than her outside option only on the equilibrium path (Compte and Jehiel, 2009, Section III).

3 We treat the two parties breaking off (quitting) the negotiation as another manner of resolving bargaining. Hence, we assume that no delay occurs if the two parties either trade the good or break off at time zero.

4 The dispersion is measured by the elasticity of the value of the outside option with respect to the gain from trading across buyer types.

This intuition can be better understood through a two-type example in which the buyer may have either a high (high type) or a low (low type) gain from trading. For simple exposition, suppose that the value of the high type's outside option is positive, while the low type's outside option is worth zero, which means that the ratio of the high type's outside option to the low type's is *infinite*. In other words, between the two buyer types, the dispersion of outside options prevails over the dispersion of values of the seller's good. Furthermore, suppose that the outside option is always available to both types during the negotiation.

Observe first that a fixed-price mechanism is optimal among all *one-shot mechanisms* such that the negotiation situation is resolved at time zero for certain. By definition, if the two parties fail to achieve an immediate agreement in a one-shot mechanism, the negotiation breaks down and the buyer exercises her outside option. Hence, the problem of solving for the optimal one-shot mechanism is mathematically equivalent to the static screening problem with no outside options, except that now the difference between the value of the good and the value of the outside option (net-value) constitutes the reservation price of the buyer. The optimality of a fixed-price mechanism among one-shot mechanisms then follows from the no-haggling result.

To see how a delay generates more profit, suppose that the seller offers the following two options to the buyer. First, the buyer can trade immediately at a high price, but suppose that this price is acceptable to the high type only. Second, the buyer can trade at a lower price, but this lower price is available only after a delay. Faced with these two options, if the delay is sufficiently long, then the high type would never opt for the second option as her outside option generates a higher payoff. By contrast, the low type would always choose the second option regardless of the length of the delay. By offering an option to delay, the seller can therefore extract more profit without conceding any further rent.

Notably, the delay can never be replaced by any other screening scheme. In particular, the seller would ultimately earn a strictly lower profit if the delay were replaced by a lottery that dictates either trade or breakdown at time zero. A mechanism with such a lottery necessarily allows a higher information rent for the high type because the high type can still enjoy the outside option even if the lottery dictates a negotiation breakdown. In turn, a higher information rent causes a lower expected profit for the seller. Note that the quitting right and the outside option may not be exercised on the equilibrium path, but these possibilities still affect the incentive cost on the equilibrium path.

In addition to a delay in the transaction, if the arrival time of the outside option is stochastic, then the optimal mechanisms also feature an upfront payment from the buyer. The buyer would be compensated for this payment by a corresponding price discount when she purchases the good in the future. However, if she withdraws from the negotiation without trading, she would lose the money paid as the upfront payment. Hence, the upfront payment prevents the buyer from quitting during the negotiation, which results in less information rent to the buyer.

The upfront payment and delay make the optimal mechanism resemble option contracts. Indeed, the seller can implement the optimal mechanism by offering a menu of European options. The optimality of option-like contracts stands in contrast to the case without outside options, in which a posting-price mechanism (with a time-varying price path) is optimal with or without the seller's commitment power (Skreta, 2006). The result also contrasts with the revenue management problem, in which committing to a posting-price mechanism is often sufficient to achieve the optimal profit level (Hörner and Samuelson, 2011; Board and Skrzypacz, 2016).

Related literature This paper considers the mechanism design problem in which the buyer has a type-dependent outside option and quitting rights. Relatedly, Lewis and Sappington (1989), Jullien (2000), and Rochet and Stole (2002), among others, discuss type-dependent participation constraints in nonlinear pricing. Different from the current article, these papers focus on the case without quitting rights, meaning that the buyer cannot opt for an outside option once she has participated in a mechanism.

The implications of quitting rights (veto constraints) in mechanism design have been studied by Compte and Jehiel (2007, 2009), who show that the full-extraction theorem of Crémer and McLean (1988) does not hold when agents have quitting rights. Similar considerations can be found in Matthews and Postlewaite (1989) and Forges (1999) for games with pre-play communications.

While this paper focuses on the case in which two parties trade a single good, other authors, for example, Thanassoulis (2004), Manelli and Vincent (2006, 2007), Pavlov (2011), and Rochet and Thanassoulis (2019) have shown that the no-haggling result may fail when parties negotiate the terms of trade for multiple goods simultaneously. The no-haggling result may also not hold due to budget constraints (Che and Gale, 2000) or risk attitudes (Laffont and Martimort, 2002).

The bargaining literature has also studied various ways that a delay may arise in negotiations. Different from our model, this literature focuses on the case in which the uninformed party has no commitment power, and hence revises the price every period. Among the various sources of delay identified in the bargaining literature, Fuchs and Skrzypacz (2010) and Hwang (2018) show that the interplay between outside options and asymmetric information can also result in a delay.

2. Model

Environment A buyer and a seller negotiate the terms of trade for an indivisible good at discrete times $t = n\Delta = 0, \Delta, 2\Delta, \ldots$, where $\Delta > 0$ denotes the time interval between consecutive *periods*. The buyer's value of the good θ is drawn from $\Theta := [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_+$. The realization of θ is the buyer's private information, and the seller has a prior over Θ with a distribution F and differentiable density f. The seller's reserve price is normalized to 0. The buyer and the seller have the common discount rate $r > 0$.

The buyer has an outside option for which she can always quit the negotiation. The value of the outside option also depends on θ and is denoted by $w(\theta) \geq 0$. We assume that $w(\theta)$ is constant over time. The outside option and the seller's good are mutually exclusive; therefore, the outside option loses its value to the buyer once the two parties trade, and vice versa. The availability of the outside option $Z_n \in \{0, 1\}$ in period n is also the buyer's private information, where $Z_n = 1$ iff the outside option is available. The realizations of θ and $(Z)_{n=0}^{\infty}$ are independent. For most parts of the paper, except Section 6.2, we focus on the case in which $(Z_n)_{n\geq 0}$ follows the binomial process with arrival rate $\lambda \in (0, \infty)$. Formally, suppose the following two assumptions.

(Z1) $Z_0 = 0$ with probability 1.[5]
(Z2) $\mathbb{P}\{Z_n = 1 | Z_{n-1} = 0\} = 1 - e^{-\lambda\Delta}$ and $\mathbb{P}\{Z_n = 1 | Z_{n-1} = 1\} = 1$ for all $n \geq 1$.

Equivalently, the period in which the outside option arrives, which we denote by $n_{arrival}$, is a geometric random variable with density $p(n) = e^{-\lambda(n-1)\Delta}(1 - e^{-\lambda\Delta})$ for any $n \in \mathbb{N}$. We also

[5] We assume (Z1) to avoid issues related to the mechanism design with multidimensional private information (θ and Z_0). Alternatively, we could assume $\mathbb{P}\{Z_0 = 1\} = 1 - e^{-\lambda\Delta}$. We discuss in Section 5.2 that our main results remain valid under this alternative assumption if Δ is sufficiently small.

consider the case $Z_n = 1$ for all $n \geq 0$ (thus, $n_{arrival} = 0$ almost surely), in which case λ is said to be infinite. Independent of the negotiation process, the buyer can guarantee herself the *autarky payoff*

$$\phi(\theta) := \mathbb{E}\left[e^{-rn_{arrival}\Delta}\right] w(\theta) = \begin{cases} \frac{e^{-r\Delta}(1-e^{-\lambda\Delta})}{1-e^{-(\lambda+r)\Delta}} w(\theta) & \text{if } \lambda < \infty \\ w(\theta) & \text{if } \lambda = \infty \end{cases}$$

by exercising her outside option immediately in period $n_{arrival}$. Define $u(\theta) := \theta - \phi(\theta)$ as the *net-value* (net gain from trading), and let $\underline{u} := \min_{\theta \in \Theta} u(\theta)$ and $\overline{u} := \max_{\theta \in \Theta} u(\theta)$ respectively denote the minimum and maximum values of $u(\theta)$ among buyer types.

As discussed in the introduction, the dispersion of $\phi(\theta)$ among buyer types plays a key role in the analysis. Formally, we measure the local dispersion of $\phi(\theta)$ by its elasticity with respect to θ[6]:

$$\epsilon_\phi(\theta) := \begin{cases} \theta\phi'(\theta^-)/\phi(\theta) & \text{if } \phi(\theta) > 0 \\ 1 & \text{if } \phi(\theta) = 0 \end{cases} \tag{1}$$

where $\phi'(\theta^-)$ is the left derivative of ϕ. Similarly, the elasticity of $u(\theta)$ with respect to θ is

$$\epsilon_u(\theta) := \theta u'(\theta^-)/u(\theta).$$

The negotiation is mediated by a mechanism that we introduce shortly. Here, we first describe the possible outcomes and the final payoffs in each case. There are two ways for the negotiation to conclude. First, suppose that a trade occurs in period $n \geq 0$. In this case, the buyer obtains

$$e^{-rn\Delta}\theta - \sum_{k=0}^{\infty} e^{-rk\Delta} p_k$$

as her final payoff, where p_k is the payment to the seller in period k. The buyer can also quit at any $n \geq 0$ without trading. The buyer still retains the outside option; hence, her final payoff is

$$e^{-rn\Delta} w(\theta) - \sum_{k=0}^{n} e^{-rk\Delta} p_k \quad \text{if } Z_n = 1, \quad \text{and} \quad e^{-rn\Delta}\phi(\theta) - \sum_{k=0}^{n} e^{-rk\Delta} p_k \quad \text{if } Z_n = 0.$$

If the negotiation continues forever, the buyer's final payoff equals the total discounted net payments. In all cases, the seller's final payoff equals the total discounted net payments from the buyer.

Assumption 1. $u(\theta) = \theta - \phi(\theta) > 0$ for a.e. (almost every) $\theta \in \Theta$.

Assumption 2. $f(\theta) > 0$ for all θ, and both $\frac{f(\theta)}{1-F(\theta)}$ and $\theta f(\theta) + F(\theta)$ strictly increase in θ.

Assumption 3. $w(\theta)$ and $\phi(\theta)$ are convex, continuous, and piecewise twice differentiable.[7]

[6] We assume that ϕ is convex (Assumption 3). Under this assumption, $\{\theta : \phi(\theta) = 0\}$ is always an (possibly empty) interval. In addition, $\epsilon_\phi(\theta) \in (-\infty, 0] \cup [1, \infty)$ outside of this interval. We may assign any value between 0 and 1 for $\epsilon_\phi(\theta)$ such that $\phi(\theta) = 0$; none of the results in this paper depend on which value to choose.

[7] Note that we allow $w(\theta)$ to be non-monotonic. Additionally, $\phi(\theta)$ is proportional to $w(\theta)$, hence, $\phi(\theta)$ satisfies Assumption 3 whenever $w(\theta)$ satisfies it, and vice versa.

Assumption 1 can be made without loss because the optimal mechanism would exclude any θ such that $u(\theta) \leq 0$. Assumption 2 simplifies the analysis by avoiding bunching. We assume that $w(\theta)$ is convex for two reasons. First, convexity arises naturally in many contexts. For example, suppose that the buyer can switch to an outside seller, which serves as her outside option. In addition, suppose that each θ assigns a value $\alpha\theta \geq 0$ to the outside seller's good. Then, $w(\theta)$ is convex whenever the outside seller employs an incentive-compatible mechanism.[8] Second, the convexity simplifies the analysis. Without any restriction, the graph of $u(\theta)$ can be arbitrary; thus, the exclusion of low net-value types can occur at any point in Θ. The convexity of $w(\theta)$ guarantees $u(\theta)$ to be single-peaked, and hence, exclusion occurs only at either end of Θ. Additionally, we show in Section 4 that the optimal allocation for each θ hinges on whether $\theta \in \{\theta' : \epsilon_\phi(\theta')/\epsilon_u(\theta') > 1\}$. This set is always an interval if $w(\theta)$ is convex, which also simplifies the analysis.[9]

Assumption 4. $-u'(\theta^-)/u(\theta) > \frac{1}{2} f'(\theta)/f(\theta)$ for a.e. $\theta \in \Theta$ such that $\phi'(\theta^-) > 1$.[10]

Note that Assumption 4 vacuously holds true if $\{\theta : \phi'(\theta^-) > 1\} = \emptyset$.[11] In addition, $u(\theta)$ deceases, and therefore, $-u'(\theta^-)/u(\theta) > 0$ over the interval $\{\theta : \phi'(\theta^-) > 1\}$. Hence, Assumption 4 also holds if $f(\theta)$ (weakly) decreases in θ over this interval. Assumption 4 helps to obtain a complete characterization of the optimal mechanism, particularly with regard to the allocation for buyer types such that $\phi'(\theta^-) > 1$. In short, Assumption 4 guarantees that a version of local incentive-compatibility is sufficient for the global incentive-compatibility. This is generally not the case for our setup because two distant types may have the same net-value (the same gain from trading).

Mechanism The negotiation is mediated by a mechanism. According to the revelation principle, we may focus on incentive-compatible direct mechanisms (Myerson, 1986; Pavan, 2015). The seller can commit to any direct mechanism, and as is standard, he can also bind himself to always obeying its recommendation. A direct mechanism is characterized as a stochastic process $\mu = (A, P) \equiv (A_n, P_n)_{n=0}^{\infty}$. The realization $(A_n, P_n) \in \{-1, 0, 1\} \times \mathbb{R}$ represents the mechanism's recommendation in period n: $A_n = 1$ if parties are recommended to trade in period n, $A_n = -1$ if the buyer is recommended to quit the negotiation, and $A_n = 0$ if it is recommended to proceed to the next period with the null action. P_n specifies the recommended transfer from the buyer to the seller.

The timeline is as follows. In each period $n \geq 0$, the buyer sends a message to the mechanism, and then the recommendation (A_n, P_n) is realized. Denote by m_n the buyer's message in period n. Here, $m_0 \in M_0 \equiv \Theta$ reveals the buyer's type, and $m_n \in M_n \equiv \{0, 1\}$, $n \geq 1$, reveals the realization of Z_n. The distribution of (A_n, P_n) depends on the history of the messages $m^n = (m_k)_{k=0}^n$

[8] Suppose that each type θ pays $p(\theta)$ and obtains a good with probability $x(\theta)$ in the outside seller's mechanism. By the standard payoff equivalence argument (Börgers, 2015, Proposition 2.2), $w(\theta) = (\alpha\theta)x(\theta) - p(\theta) = \alpha \int_{\underline{\theta}}^{\theta} x(s)\mathrm{d}s + w(\underline{\theta})$. The incentive-compatibility implies that $x(\theta)$ is monotone, and hence, $w(\theta)$ is convex.

[9] While the convexity of $w(\theta)$ is essential for the complete characterization of the optimal mechanism in Section 4, the main takeaways of this paper remain valid under weaker conditions. See footnote 21 in Section 4.2.

[10] In fact, it suffices to impose the following slightly weaker assumption:

$$\left[\frac{u(\theta)}{u'(\theta^-)} - \frac{1 - F(\theta)}{f(\theta)}\right] f(\theta) \text{ strictly increases in } \theta \text{ over the interval } \{\theta : \phi'(\theta^-) > 1\}.$$

[11] This interval is indeed empty if we interpret the outside option as the buyer's payoff to switch to another seller who offers the identical product. See footnote 8.

and realized recommendations $(A_k, P_k)_{k=0}^{n-1}$ thus far. The realized (A_n, P_n) is implemented if the buyer approves it, and then the negotiation proceeds to the next period. If the buyer vetoes, then neither trade nor transfer occurs in period n, and the two parties quit the negotiation indefinitely. The buyer can still enjoy her outside option; hence, the buyer obtains $w(\theta)$ if $Z_n = 1$ or $\phi(\theta)$ if $Z_n = 0$ as her continuation payoff. None of the previous actions and transfers is undone after the buyer quits the mechanism. In particular, no previous transfers are refunded.[12]

The buyer chooses which message to send (communication strategy) and whether to veto the realized (A_n, P_n) (veto strategy) in each period n. A communication strategy is called *truthful* if it always chooses $m_0 = \theta$ and $m_n = Z_n$ for $n \geq 1$ almost surely. A veto strategy is *obedient* if it never vetoes. $\mu = (A, P)$ is called *incentive-compatible* if and only if the buyer's payoff maximization problem in μ admits the truthful and obedient strategy as an optimal solution.[13] Let $\mathcal{M}^{IC}$ be the set of all incentive-compatible mechanisms. For any $\mu = (A, P) \in \mathcal{M}^{IC}$ and $\theta \in \Theta$, $(\widehat{A}(\theta), \widehat{P}(\theta)) = (\widehat{A}_n(\theta), \widehat{P}_n(\theta))_{n=0}^{\infty}$ denotes the process of recommendations conditional on (i) the buyer's type being θ and (ii) the buyer adopting the truthful and obedient strategy. Define

$$\Pi_S(\mu) := \int_{\Theta} \mathbb{E}\left[\sum_{n=0}^{\infty} e^{-rn\Delta} \widehat{P}_n(\theta)\right] \mathrm{d}F(\theta)$$

as the seller's expected profit. Note that the negotiation is concluded in period n such that $\widehat{A}_k(\theta) \neq 1$ and $\widehat{P}_k(\theta) = 0$ for all $k > n$ because neither trade nor transfer is induced any longer. Thus, we define the *delay of transaction (or the total duration of negotiation)*, when faced with θ, by

$$\delta(\theta; \mu) := \Delta \cdot \inf\{n : \widehat{A}_k(\theta) \neq 1 \text{ and } \widehat{P}_k(\theta) = 0 \text{ for any } k > n\}.$$

We say the transaction is delayed (or the negotiation is delayed) for θ iff $\delta(\theta; \mu) > 0$. Note that we consider the transaction to be delayed if, for example, the buyer trades at time zero but continues to make post-trade transfers for a while afterward. The *expected delay* is then defined by $\delta(\mu) := \mathbb{E}\left[\int_{\Theta} \delta(\theta; \mu)\mathrm{d}F(\theta)\right]$. $\mu \in \mathcal{M}^{IC}$ is called a *one-shot* mechanism if all transactions occur in period 0 (i.e., $\delta(\mu) = 0$). μ is the *optimal one-shot mechanism* if μ generates the highest profit among all one-shot mechanisms. $\mu \in \mathcal{M}^{IC}$ is called *(globally) optimal* if $\Pi_S(\mu) = \overline{\Pi}_S := \sup_{\tilde{\mu} \in \mathcal{M}^{IC}} \Pi_S(\tilde{\mu})$.

Finally, we will restrict our attention to incentive-compatible mechanisms such that the buyer is never recommended to quit before period $n_{arrival}$. This restriction is inconsequential because a mechanism can postpone any break-up with the buyer in period $n < n_{arrival}$ by recommending $A_k = P_k = 0$ for any $n \leq k < n_{arrival}$, without changing the negotiation outcome at all.

3. Optimal one-shot mechanism

We first characterize the optimal one-shot mechanism for the benchmark. Any one-shot mechanism is identified by $x(\theta)$ and $q(\theta)$, which are the probability that θ trades in period $n = 0$ and her expected payment, respectively. The buyer obtains the autarky payoff $\phi(\theta)$ if no trade occurs,

[12] The negotiation may continue even after the buyer trades with the seller; for example, the seller may commit to making a transfer to the buyer after a trade occurs.

[13] The existence of an optimal solution is a part of the requirement for μ being incentive-compatible.

which is the case with probability $1-x(\theta)$. Hence, a one-shot mechanism is incentive-compatible iff

$$\begin{aligned} & x(\theta)\theta + (1-x(\theta))\phi(\theta) - q(\theta) \geq \max\{\phi(\theta), x(\theta')\theta + (1-x(\theta'))\phi(\theta) - q(\theta')\} \\ \iff & x(\theta)(\theta-\phi(\theta)) - q(\theta) \geq \max\{0, x(\theta')(\theta-\phi(\theta)) - q(\theta')\} \end{aligned}$$

for all $\theta, \theta' \in \Theta$. The above is equivalent to the incentive-compatibility constraint for the static screening problem, except that θ is replaced by $u(\theta) = \theta - \phi(\theta)$ on both sides. Hence, according to the no-haggling result (Stokey, 1979), among all one-shot mechanisms, it is optimal to commit to a single fixed price

$$p^{\text{one-shot}} \in \arg\max_p \int_{\theta: u(\theta)\geq p} p \, \mathrm{d}F(\theta). \tag{2}$$

The price $p^{\text{one-shot}}$ is accepted at $t=0$ iff $u(\theta) \geq p^{\text{one-shot}}$; otherwise, the buyer instantly quits and then takes the outside option in period $n = n_{arrival}$. Herein, we denote this *optimal one-shot mechanism* by $\mu^*_{one-shot}$. Note that $\mu^*_{one-shot}$ features the familiar bang-bang allocation in which the seller trades with the highest net-value types (no distortion on the top) while other buyer types are completely rationed (complete distortion on the bottom).[14]

4. Main results

4.1. Relaxed problem

To characterize the (globally) optimal mechanism, we begin by constructing a relaxed problem that provides an upper bound for $\overline{\Pi}_S$. For any $\mu = (A, P) \in \mathcal{M}^{IC}$ let $\tau(\theta;\mu) := \inf\{n \in \mathbb{N}_0 : \widehat{A}_n(\theta) = 1\}$ and $\sigma(\theta;\mu) := \inf\{n : \widehat{A}_n(\theta) = -1\}$ be the periods in which μ recommends each nonnull action. Then, the expected discounted trading volume and the total discounted transfer are

$$x(\theta;\mu) := \mathbb{E}\left[e^{-r\tau(\theta;\mu)\Delta}\right] \in [0,1] \quad \text{and} \quad q(\theta;\mu) := \mathbb{E}\left[\sum_{n=0}^{\infty} e^{-rn\Delta}\widehat{P}_n(\theta)\right] \in \mathbb{R},$$

with the convention $\inf(\emptyset) = \infty$ and $e^{-\infty} = 0$. Similarly,

$$y(\theta;\mu) := \frac{\mathbb{E}[e^{-r\sigma(\theta;\mu)\Delta}]}{B}$$

where $B := \mathbb{E}[e^{-rn_{arrival}\Delta}]$ and thus $y(\theta;\mu) \in [0,1]$.[15] The buyer's expected payoff in μ is

$$\begin{aligned} V^{\mu}(\theta) &= \mathbb{E}\left[e^{-r\tau(\theta;\mu)\Delta}\theta + e^{-r\sigma(\theta;\mu)\Delta}w(\theta) - \sum_{n=0}^{\infty} e^{-rn\Delta}\widehat{P}_n(\theta)\right] \\ &= x(\theta;\mu)\theta + y(\theta;\mu)\phi(\theta) - q(\theta;\mu). \end{aligned}$$

[14] There might be multiple price levels that maximize the seller's static profit (2) and hence multiple optimal one-shot mechanisms. However, all of these optimal one-shot mechanisms result in the same profit level.

[15] $1/B$ is multiplied merely for normalization, which ensures that $y(\theta;\mu)$ lies in $[0,1]$ for all θ, λ, and Δ.

Lemma 1. *For any $\mu \in \mathcal{M}^{IC}$ and $\theta, \tilde{\theta} \in \Theta$,*

$$x(\theta;\mu)\theta + y(\theta;\mu)\phi(\theta) - q(\theta;\mu) \geq x(\tilde{\theta};\mu)\theta + y(\tilde{\theta};\mu)\phi(\theta) - q(\tilde{\theta};\mu), \tag{3}$$

$$x(\theta;\mu)\theta + y(\theta;\mu)\phi(\theta) - q(\theta;\mu) \geq \phi(\theta), \tag{4}$$

$$x(\theta;\mu),\ y(\theta;\mu) \in [0,1], \text{ and } x(\theta;\mu) + y(\theta;\mu) \in [0,1]. \tag{5}$$

In addition, $V^\mu(\theta)$ is convex, and

$$V^\mu(\theta) = V^\mu(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} [x(s;\mu) + \phi'(s)y(s;\mu)]ds \qquad \forall \theta \in \Theta.$$

Sketch of proof. First, (3) and (4) are required for the buyer's truthful report of her type and individual-rationality at time zero, both of which are necessary for the incentive-compatibility. (5) is also straightforward if $\lambda = \infty$. Thus, it suffices to prove the following under the assumption $\lambda < \infty$:

$$\max_{\mu \in \mathcal{M}^{IC}} x(\theta;\mu) + y(\theta;\mu) = \max_{\mu \in \mathcal{M}^{IC}} \mathbb{E}\left[e^{-r\tau(\theta;\mu)\Delta} + \frac{1}{B}e^{-r\sigma(\theta;\mu)\Delta}\right] \leq 1 \quad \forall \theta \in \Theta.$$

Because $B \in [0,1]$, $x(\theta;\mu) + y(\theta;\mu)$ is maximized only if the buyer exercises the outside option immediately upon its arrival in period $n_{arrival}$, unless the parties had already traded earlier. Hence, we may restrict our attention to mechanisms with the recommendation rule below with some n^*:

- if the outside option arrives in or before period n^*, the buyer exercises the outside option immediately;
- if no outside option arrives by period n^*, the two parties trade in period n^*.

Given that the arrival rate of the outside option is constant over time, $x(\theta;\mu) + y(\theta;\mu)$ is in fact independent of n^* (see Section A.1 for details). In particular, we may choose $n^* = 0$; thus,

$$\max_{\mu \in \mathcal{M}^{IC}} x(\theta;\mu) + y(\theta;\mu) = e^{-r\cdot 0\cdot\Delta} + \frac{1}{B}e^{-r\cdot\infty\cdot\Delta} = 1.$$

Finally, $V^\mu(\theta) = \max_{\theta' \in \Theta} x(\theta';\mu)\theta + y(\theta';\mu)\phi(\theta) - q(\theta';\mu)$ is the maximum of convex mappings, and hence $V^\mu(\theta)$ is also convex in θ. The integral representation of $V^\mu(\theta)$ follows the envelope theorem. □

Lemma 1 leads to the following upper bound for $\overline{\Pi}_S = \sup_{\mu \in \mathcal{M}^{IC}} \Pi_S(\mu)$:

$$\max_{(x(\theta),y(\theta),q(\theta))_{\theta\in\Theta}} \int_{\Theta} q(\theta)\mathrm{d}F(\theta) \quad \text{subject to} \quad (3), (4), \text{ and } (5). \tag{6}$$

(6) is mathematically equivalent to the *static* screening problem with two-dimensional allocation $(x(\theta), y(\theta))$ and type-dependent individual-rationality constraints.[16] Applying the envelope theorem to (3) and then following the standard revenue-equivalence argument, we can transform (6) into the following equivalent form:

[16] Jullien (2000) considers a nonlinear pricing problem that is closely related to (6). Unfortunately, (6) fails two assumptions, Homogeneity and CVU (Jullien, 2000, p. 10 and p. 23), which are required for his analysis.

$$\Pi_R := \max_{x,y,\underline{\pi}} \int_{\Theta} \left\{ \left[\theta - \frac{1-F(\theta)}{f(\theta)} \right] x(\theta) + \left[\phi(\theta) - \phi'(\theta)\frac{1-F(\theta)}{f(\theta)} \right] y(\theta) \right\} \mathrm{d}F(\theta) - \underline{\pi} \quad \text{(R)}$$

subject to

$$(\theta'-\theta)x(\theta') + (\phi(\theta')-\phi(\theta))y(\theta') + \int_{\theta'}^{\theta} \left[x(s)+\phi'(s^-)y(s)\right] \mathrm{d}s \geq 0 \qquad \forall \theta, \theta' \in \Theta$$

$$\underline{\pi} + \int_{\underline{\theta}}^{\theta} \left[x(s)+\phi'(s^-)y(s)\right] \mathrm{d}s \geq \phi(\theta) \quad \text{and} \quad 0 \leq y(\theta) \leq 1 - x(\theta) \leq 1 \qquad \forall \theta \in \Theta$$

Here, the first and second constraints are equivalent to (3) and (4), respectively, where $\underline{\pi}$ denotes the expected payoff of the lowest buyer type $\underline{\theta}$. We will refer to (R) as the *relaxed problem* and its optimized value as Π_R.

We need to introduce further notations to present the solution of the relaxed problem (R). The convexity of ϕ guarantees the existence of $\theta^\dagger$ such that[17]

$$\epsilon_\phi(\theta) > 1 \quad \iff \quad \theta > \theta^\dagger. \tag{7}$$

Additionally, for any $p \in [\underline{u}, \overline{u}]$,

$$\begin{aligned} &\underline{\theta}(p) := \inf\{\theta \in \Theta : u(\theta) \geq p\}, && \overline{\theta}(p) := \sup\{\theta \in \Theta : u(\theta) \geq p\}, \\ &\mathcal{U}(p) := [\underline{\theta}(p), \overline{\theta}(p)], && \mathcal{T}(p) := \{\theta \in \Theta \backslash \mathcal{U}(p) : \epsilon_\phi(\theta)/\epsilon_u(\theta) > 1\}. \end{aligned} \tag{8}$$

$\mathcal{U}(p)$ is the set of the types that prefer to trade at price p rather than $\phi(\theta)$. $\mathcal{T}(p)$ is the set of the types outside of $\mathcal{U}(p)$ such that $\epsilon_\phi(\theta)/\epsilon_u(\theta) > 1$, or equivalently, (i) $u(\theta)$ and $\phi(\theta)$ are positively related, and (ii) $\phi(\theta)$ is locally more dispersed than $u(\theta)$. Note that $\epsilon_\phi(\theta)/\epsilon_u(\theta) > 1$ if and only if both $u'(\theta^-) = 1 - \phi'(\theta^-) > 0$[18] and $\epsilon_\phi(\theta) - \epsilon_u(\theta) = \theta(\epsilon_\phi(\theta) - 1)/u(\theta) > 0$ hold simultaneously. Thus,

$$\epsilon_\phi(\theta)/\epsilon_u(\theta) > 1 \quad \iff \quad \epsilon_\phi(\theta) > 1 \quad \text{and} \quad \phi'(\theta^-) < 1. \tag{9}$$

Additionally, $u'(\theta^-) = 1 - \phi'(\theta^-) < 0 < \phi'(\theta^-)$ at any $\theta > \overline{\theta}(p)$; therefore, any buyer types higher than $\overline{\theta}(p)$ are excluded from $\mathcal{T}(p)$. Consequently,

$$\begin{aligned} \mathcal{T}(p) &= \{\theta \in \Theta \backslash \mathcal{U}(p) : \epsilon_\phi(\theta) > 1 \text{ and } \phi'(\theta^-) < 1\} \\ &= (\theta^\dagger, \overline{\theta}] \cap (\underline{\theta}, \underline{\theta}(p)) = (\theta^\dagger, \underline{\theta}(p)) \quad \forall p \in [\underline{u}, \overline{u}] \end{aligned} \tag{10}$$

and $\mathcal{T}(p) \neq \emptyset$ iff $\theta^\dagger < \underline{\theta}(p)$. $\mathcal{U}(p)$ and $\mathcal{T}(p)$ are illustrated in Fig. 1 for the case of

$$\phi(\theta) = \begin{cases} a\theta & \text{if } \theta \in [0,a], \\ \theta^2 & \text{if } \theta \in (a,1], \end{cases} \quad \text{and hence,} \quad \epsilon_\phi(\theta) = \begin{cases} 1 & \text{if } \theta \in [0,a], \\ 2 & \text{if } \theta \in (a,1]. \end{cases} \tag{11}$$

We now present the solution for (R). For any fixed $\underline{\pi}$, by temporarily ignoring the constraint (3), (R) can be analyzed as a special case of an SCLP (separated continuous linear program). In

17 $\epsilon_\phi(\theta) > 1$ iff $\theta\phi'(\theta^-) - \phi(\theta) > 0$, and $\theta\phi'(\theta^-) - \phi(\theta)$ is increasing in θ because ϕ is convex.

18 $\epsilon_\phi(\theta) > 0$ whenever $u'(\theta^-) = 1 - \phi'(\theta^-) \leq 0$.

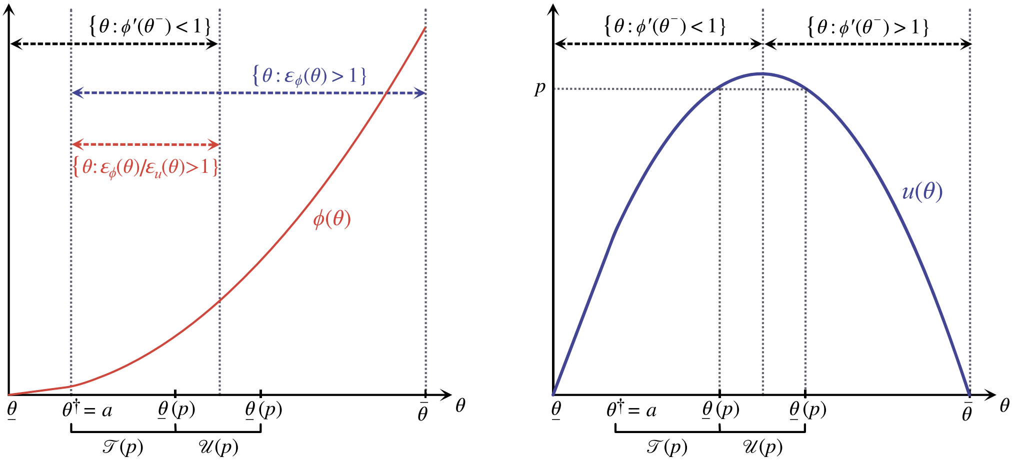

Fig. 1. $\mathcal{U}(p)$ and $\mathcal{T}(p)$.

the online appendix, we characterize the solution of this SCLP based on the mathematical properties of SCLPs as studied by Anderson et al. (1983), Shapiro (2001) and others. This solution also satisfies the constraint (3); hence, it also solves (R).

Lemma 2. *Suppose Assumptions 1-4. A solution for* (R) *exists, and*

$$\Pi_R = \max_{p\in[\underline{u},\overline{u}]} \left\{ \int_{\mathcal{U}(p)} p \, \mathrm{d}F(\theta) + \int_{\mathcal{T}(p)} (\epsilon_\phi(\theta) - 1)\phi(\theta) \, \mathrm{d}F(\theta) \right\}. \tag{R*}$$

The detailed proof can be found in the online appendix. In what follows, let $\mathcal{P}_{R^*}$ denote the set of solutions (arg maxima) for the optimization on the right-hand side of (R*). The term $\int_{\mathcal{U}(p)} p \, \mathrm{d}F(\theta)$ in (R*) is the bound for the seller's profit from a trade with no delay. For concreteness, suppose that the seller aims to trade with $\Theta^\star \subset \Theta$ at $t = 0$. By the same intuition that the optimal one-shot mechanism posts a single price, it is optimal to trade with all buyer types in $\Theta^\star$ at the identical price, e.g., p', so that the expected profit from a trade at $t = 0$ is $\int_{\Theta^\star} p' \, \mathrm{d}F(\theta)$. Note that all other buyer types outside of $\mathcal{U}(p')$ find the outside option more attractive than p' because $u(\theta) < p'$; thus, we necessarily have $\Theta^\star \subset \mathcal{U}(p')$ and $\int_{\Theta^\star} p' \, \mathrm{d}F(\theta) \leq \int_{\mathcal{U}(p')} p' \, \mathrm{d}F(\theta)$.

The next term is the bound for the seller's profit from arranging a delayed trade for the buyer types outside of $\mathcal{U}(p)$. Recall that the integration range $\mathcal{T}(p)$ includes only buyer types for which $\phi(\theta)$ is (locally) more dispersed than $u(\theta)$. The connection between the seller's profit and $\epsilon_\phi(\theta)/\epsilon_u(\theta)$ can be understood similarly to the two-type example in the introduction; the more $\phi(\theta)$ is dispersed among buyer types, the less incentive cost is incurred to arrange a delayed trade. This point is discussed in more detail later after we present the optimal mechanism in the next subsection.

4.2. *Optimal mechanism when $\lambda = \infty$*

Optimal mechanism Fix $\lambda = \infty$ and $\Delta > 0$. For each $p \in [\underline{u}, \overline{u}]$, define a mechanism $\mu^*_{\infty,\Delta}(p)$ as follows. For each message $m_0 \in \Theta$ that the buyer sends at the beginning (in period 0):

- If $m_0 \in \mathcal{U}(p) = [\underline{\theta}(p), \overline{\theta}(p)]$, the two parties are recommended to trade in period 0 at price p and then terminate the negotiation after the trade.
- If $m_0 \in \mathcal{T}(p) = (\theta^\dagger, \underline{\theta}(p))$, $\mu^*_{\infty,\Delta}(p)$ recommends that the parties wait until period $n(m_0)$ with neither any nonnull action nor transfer. Then, in any period $n \geq n(m_0)$, $\mu^*_{\infty,\Delta}(p)$ randomizes over the two recommendations: (i) With probability $1-\beta(m_0)$, $\mu^*_{\infty,\Delta}(p)$ recommends trading in period n at price $\rho(m_0)$. The negotiation ends after the trade. (ii) With probability $\beta(m_0)$, $\mu^*_{\infty,\Delta}(p)$ waits one more period.
- If $m_0 \notin \mathcal{U}(p) \cup \mathcal{T}(p)$, $\mu^*_{\infty,\Delta}(p)$ recommends the parties to break off the negotiation instantly.

To complete the description of $\mu^*_{\infty,\Delta}(p)$, we still need to specify $\beta(m_0) \in [0,1]$, $\rho(m_0) \in \mathbb{R}$, and $n(m_0) \in \mathbb{N}_0$ for each $m_0 \in \mathcal{T}(p)$. Define $n(m_0)$ and $\beta(m_0)$ by

$$n(m_0) = \max\{n \in \mathbb{N}_0 : e^{-rn\Delta} \geq \phi'(m_0^-)\} \quad \text{and} \quad \beta(m_0) = \frac{e^{-rn(m_0)\Delta} - \phi'(m_0^-)}{e^{-rn(m_0)\Delta} - e^{-r\Delta}\phi'(m_0^-)} \tag{12}$$

so that the expected trading volume $x(m_0; \mu^*_{\infty,\Delta}(p)) = \mathbb{E}[e^{-r\tau(m_0;\mu^*_{\infty,\Delta}(p))\Delta}]$ equals

$$\begin{aligned} x(m_0; \mu^*_{\infty,\Delta}(p)) &= \sum_{k=0}^{\infty} e^{-r(k+n(m_0))\Delta} \beta(m_0)^k (1-\beta(m_0)) = \frac{e^{-rn(m_0)\Delta}(1-\beta(m_0))}{1-\beta(m_0)e^{-r\Delta}} \\ &= \phi'(m_0^-). \end{aligned} \tag{13}$$

In addition, choose

$$\rho(m_0) = \frac{(\epsilon_\phi(m_0)-1)\phi(m_0)}{\phi'(m_0^-)} \qquad \Longrightarrow \qquad q(m_0; \mu^*_{\infty,\Delta}(p)) = (\epsilon_\phi(m_0)-1)\phi(m_0). \tag{14}$$

The next proposition shows that (i) $\mu^*_{\infty,\Delta}(p)$ is incentive-compatible for any p and (ii) $\mu^*_{\infty,\Delta}(p)$ achieves $\overline{\Pi}_S$ if we choose p from $\mathcal{P}_{R^*}$, the set of arguments of maxima of (R*).

Proposition 1. *Suppose $\lambda = \infty$. (i) $\mu^*_{\infty,\Delta}(p)$ is incentive-compatible for any $p \in [\underline{u}, \overline{u}]$. (ii) For any $p^* \in \mathcal{P}_{R^*}$, $\Pi_S(\mu^*_{\infty,\Delta}(p^*)) = \Pi_R = \overline{\Pi}_S$; thus, $\mu^*_{\infty,\Delta}(p^*)$ is optimal.*

Proof. We first show that $\Pi_S(\mu^*_{\infty,\Delta}(p)) = \Pi_R$ if $p \in \mathcal{P}_{R^*}$, under the assumption that $\mu^*_{\infty,\Delta}(p)$ is incentive-compatible. The seller earns p if $\theta \in \mathcal{U}(p)$ and 0 if $\theta \notin \mathcal{T}(p) \cup \mathcal{U}(p)$. If $\theta \in \mathcal{T}(p)$, then by (14), the buyer's expected discounted payment is $q(\theta; \mu^*_{\infty,\Delta}(p)) = \phi(\theta)(\epsilon_\phi(\theta)-1)$. Therefore, the discounted expected payment of each buyer type is

$$q(\theta; \mu^*_{\infty,\Delta}(p)) = \begin{cases} p & \text{if } \theta \in \mathcal{U}(p), \\ \phi(\theta)(\epsilon_\phi(\theta)-1) & \text{if } \theta \in \mathcal{T}(p), \end{cases}$$

and $q(\theta; \mu^*_{\infty,\Delta}(p)) = 0$ for all other buyer types. Consequently,

$$\Pi_S(\mu^*_{\infty,\Delta}(p)) = \int_{\mathcal{U}(p)} p \, \mathrm{d}F(\theta) + \int_{\mathcal{T}(p)} \phi(\theta)(\epsilon_\phi(\theta)-1)\mathrm{d}F(\theta) = \Pi_R \quad \forall p \in \mathcal{P}_{R^*}.$$

Next, we prove that $\mu^*_{\infty,\Delta}(p)$ is incentive-compatible for any $p \geq 0$. Let $V^*(m_0|\theta)$ denote the buyer type θ's expected payoff from reporting m_0 (and then behaving optimally in the continuation games) in $\mu^*_{\infty,\Delta}(p)$. Also, define $V^*(\theta) := V^*(\theta|\theta)$. Note first that, once the buyer chooses

m_0, the mechanism's recommendations in period $n \geq 1$ no longer depend on the buyer's messages at $n \geq 1$. Thus, it is trivially optimal for the buyer to be truthful in any period $n \geq 1$. For $m_0 \in \mathcal{U}(p)$, $V^*(m_0|\theta) = \max\{\theta - p, \phi(\theta)\}$ where $\theta - p$ is the payoff from accepting the mechanism's recommendation after reporting m_0, and $\phi(\theta)$ is the payoff from vetoing. In addition, $\theta - p \geq \phi(\theta)$ if and only if $\theta \in \mathcal{U}(p)$; thus,

$$V^*(m_0|\theta) = \begin{cases} \theta - p \geq \phi(\theta) & \text{if } \theta \in \mathcal{U}(p) \\ \phi(\theta) & \text{if } \theta \notin \mathcal{U}(p) \end{cases} \quad \forall \theta \in \Theta \quad \text{and} \quad \forall m_0 \in \mathcal{U}(p). \tag{15}$$

This shows that no buyer type $\theta \in (\mathcal{U}(p))^c$ has an incentive to send $m_0 \in \mathcal{U}(p)$, because it never yields more than $\phi(\theta)$. (15) also reveals that each $\theta \in \mathcal{U}(p)$ finds it optimal to be obedient (i.e., it is optimal for the buyer to accept to trade at p) conditional on the truthful report of $m_0 = \theta$.

Now consider the case in which the buyer chooses $m_0 \in \mathcal{T}(p)$. To analyze the buyer's optimal strategy in the ensuing periods, note first that quitting in period 0 (which yields $w(\theta)$ as the final payoff) dominates quitting in any period $0 < n < n(m_0)$ (which yields $e^{-rn\Delta} w(\theta)$). Moreover, if the buyer finds it optimal to be obedient to the recommendation at $n = n(m_0) - 1$, then it is also optimal to be obedient in all following periods, as $\mu^*_{\infty,\Delta}(p)$ employs the same randomization rule at any $n \geq n(m_0)$. Thus, after reporting $m_0 \in \mathcal{T}(p)$, the buyer's best response is either (A) to remain obedient in all periods $n \geq 0$ or (B) to veto in period 0. The buyer's expected payoff is

$$\underbrace{\phi'(m_0^-)}_{x(m_0;\mu^*_{\infty,\Delta}(p))} \theta - \underbrace{(\epsilon_\phi(m_0) - 1)\phi(m_0)}_{q(m_0;\mu^*_{\infty,\Delta}(p))} = \phi'(m_0^-)(\theta - m_0) + \phi(m_0),$$

if she chooses (A). On the other hand, the final payoff is $\phi(\theta)$ if she chooses (B). Hence,

$$V^*(m_0|\theta) = \max\{\phi'(m_0^-)(\theta - m_0) + \phi(m_0), \phi(\theta)\} = \phi(\theta) \leq V^*(\theta|\theta) \tag{16}$$

where the second equality holds because ϕ is convex and the last inequality holds as equality if $m_0 = \theta$.[19] In conclusion, any $\theta \in (\mathcal{T}(p))^c$ has no incentive to send $m_0 \in \mathcal{T}(p)$; moreover, each $\theta \in \mathcal{T}(p)$ finds it optimal to be obedient after she sends m_0 truthfully. $\square$

Example 1. Suppose that θ is drawn from the uniform distribution over $\Theta = [0, 1]$ and that ϕ and ϵ_ϕ are given by (11) with $a \in [0, 0.2]$. Thus, $\theta^\dagger = a$. First, the unique element in $\mathcal{P}_{R^*}$ can be obtained from the first-order condition.

$$p^* = \frac{10 - \sqrt{2}}{49} = \arg\max_{p'} \left\{ \int_{\mathcal{U}(p')} p' \, \mathrm{d}F(\theta) + \int_{\mathcal{T}(p')} (\epsilon_\phi(\theta) - 1)\phi(\theta) \mathrm{d}F(\theta) \right\}.$$

One can also calculate $\mathcal{U}(p^*) = [\underline{\theta}(p^*), \overline{\theta}(p^*)] = [(3 - \sqrt{2})/7, (4 + \sqrt{2})/7]$ and $\mathcal{T}(p^*) = [\theta^\dagger, \underline{\theta}(p^*)] = [a, (3 - \sqrt{2})/7]$. Now, consider the mechanism $\mu^*_{\infty,\Delta}(p^*)$, which is optimal according to the last proposition. All buyer types outside of $\mathcal{U}(p^*) \cup \mathcal{T}(p^*)$ are excluded from $\mu^*_{\infty,\Delta}(p^*)$ (the gray area in Fig. 2). For each $\theta \in \mathcal{U}(p^*) \cup \mathcal{T}(p^*)$, the total discounted trading volume and transfer are

$$x(\theta; \mu^*_{\infty,\Delta}(p^*)) = \begin{cases} 1 & \text{if } \theta \in \mathcal{U}(p^*) \\ 2\theta & \text{if } \theta \in \mathcal{T}(p^*) \end{cases} \quad \text{and} \quad q(\theta; \mu^*_{\infty,\Delta}(p^*)) = \begin{cases} p^* & \text{if } \theta \in \mathcal{U}(p^*) \\ \theta^2 & \text{if } \theta \in \mathcal{T}(p^*). \end{cases}$$

[19] $(\theta - m_0)\phi'(m_0^-) + \phi(m_0)$ is the tangent to the graph of $\phi(\theta)$ at $\theta = m_0$. As ϕ is convex, this tangent lies below ϕ.

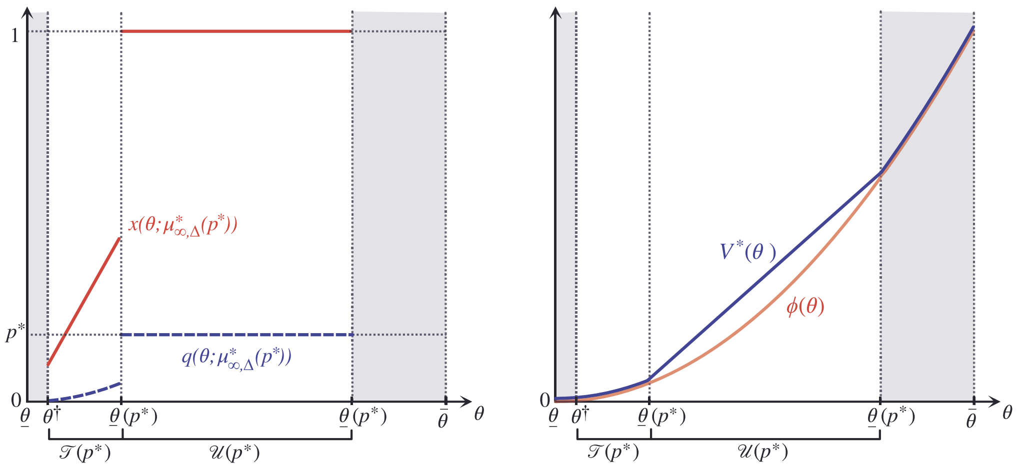

Fig. 2. Graph for Example 1 ($a = 0.05$).

Each buyer type's expected payoff in $\mu^*_{\infty,\Delta}(p^*)$ is

$$V^*(\theta) = V^{\mu^*_{\infty,\Delta}(p^*)}(\theta) = \begin{cases} \theta - p^* > \phi(\theta) & \text{if } \theta \in \mathcal{U}(p^*), \\ \phi(\theta) & \text{if } \theta \in \mathcal{T}(p^*), \end{cases}$$

and $V^*(\theta) = \phi(\theta)$ for all excluded types. All buyer types in $\mathcal{T}(p^*) = [a, (3-\sqrt{2})/7]$ trade after a delay, and these buyer types obtain zero rent in the sense that $V^*(\theta) = \phi(\theta)$.

In general, $\mathcal{P}_{R^*}$ includes multiple elements. $\mu^*_{\infty,\Delta}(p^*)$ is still well-defined for each $p^* \in \mathcal{P}_{R^*}$, and all of these mechanisms generate the optimal profit level $\overline{\Pi}_S$. Note that the expected delay

$$\delta(\mu^*_{\infty,\Delta}(p^*)) = \int_{\mathcal{T}(p^*)} \frac{1}{r} \log \frac{1}{x(\theta; \mu^*_{\infty,\Delta}(p^*))} \mathrm{d}F(\theta) + O(\Delta) \tag{17}$$

is positive if $\mathcal{T}(p^*) \neq \emptyset$. Here, $O(\Delta)$ stands for a term that vanishes as $\Delta \to 0$. If $\mathcal{T}(p^*) = \emptyset$ for some $p^* \in \mathcal{P}_{R^*}$, then $\mu^*_{\infty,\Delta}(p^*)$ degenerates to a one-shot mechanism, which means that a delay is not essential to achieve the optimal profit level. In the remaining part of this subsection, we focus on the mechanism $\mu^*_{\infty,\Delta}(p^*)$ such that $p^* \in \mathcal{P}_{R^*}$ and $\mathcal{T}(p^*) \neq \emptyset$; hence $\mu^*_{\infty,\Delta}(p^*)$ is optimal and $\delta(\mu^*_{\infty,\Delta}(p^*)) > 0$.

Benefit of delay The mechanism $\mu^*_{\infty,\Delta}(p^*)$ features the familiar "no-distortion-at-the-top" and "complete-rationing-at-the-bottom" properties. Buyer types in $\mathcal{U}(p^*)$ have large gains from trading (net-value); thus, $\mu^*_{\infty,\Delta}(p^*)$ trades with them at the highest price p^* with no delay. On the other hand, buyer types at either end of Θ (the gray area in Fig. 2) are completely excluded because they have low net-values relative to the high incentive cost of inducing them to be truthful and obedient throughout the negotiation.

More interestingly, $\mu^*_{\infty,\Delta}(p^*)$ rations buyer types in $\mathcal{T}(p^*)$ only partially and not completely. This stands in contrast to the bang-bang allocation in the optimal mechanism with *type-independent* outside options (Stokey, 1979); the bang-bang allocation also arises in $\mu^*_{one-shot}$ in Section 3. For both cases, the bang-bang allocation arises because the incentive constraints (ICs) should bind downward to maximize profit. As a result, any attempt to trade with an extra type, say θ_0, leads to an additional incentive cost for a nearby higher net-value type, say θ_1; otherwise, the IC for θ_1 (which was already binding beforehand) is violated. Due to the linearity of the mechanism's optimization problem, this trade-off always pushes the trading volume of θ_0 toward either 0 or 1.

To understand why the bang-bang allocation may not be optimal with type-dependent outside options, it helps to consider a two-type case first. Suppose two buyer types θ_h and θ_l such that $u(\theta_h) > u(\theta_l) > 0$. In addition, suppose that $\mathbb{P}\{\theta = \theta_h\}$ is large enough; hence, the seller can benefit from discriminating θ_h and θ_l. Moreover, suppose that

$$\frac{u(\theta_h) - u(\theta_l)}{u(\theta_l)} < \frac{\phi(\theta_h) - \phi(\theta_l)}{\phi(\theta_l)} \iff \bar{t}(\theta_h) := \frac{1}{r}\log\frac{\theta_h}{\phi(\theta_h)} < \bar{t}(\theta_l) := \frac{1}{r}\log\frac{\theta_l}{\phi(\theta_l)}. \quad \text{(D)}$$

Note that (D) combines two assumptions. First, $\phi(\theta)$ and $u(\theta)$ are positively related. Second, the dispersion of $\phi(\theta)$ is larger than the dispersion of $u(\theta)$ between θ_h and θ_l. Broadly, the IR constraints are more (less) likely to bind than IC constraints when $\phi(\theta)$ is more (less) dispersed. Thus, under (D), the downward IC for θ_h does not necessarily bind any longer, and the bang-bang allocation may not be optimal. More specifically, consider the following mechanism.

- If $m_0 = \theta_h$, then the seller trades with the buyer immediately at price $u(\theta_h) = \theta_h - \phi(\theta_h)$.
- If $m_0 = \theta_l$, then the two parties are recommended to wait until time $\hat{t} \in (\bar{t}(\theta_h), \bar{t}(\theta_l))$ (equivalently, $\phi(\theta_l)/\theta_l < e^{-r\hat{t}} < \phi(\theta_h)/\theta_h$) and then trade at price $\hat{p} = \epsilon > 0$.

The buyer type θ_h rejects any trade arranged at time t such that $t > \bar{t}(\theta_h)$ (equivalently, $e^{-rt} < e^{-r\bar{t}(\theta_h)}$) because her outside option already yields a higher payoff:

$$e^{-rt}(\theta_h - p) < e^{-r\bar{t}(\theta_h)}\theta_h \le \phi(\theta_h) \quad \forall t > \bar{t}(\theta_h) \quad \text{and} \quad \forall p \ge 0. \tag{18}$$

Consequently, θ_h chooses to trade at price equal to $u(\theta_h)$ by sending $m_0 = \theta_h$, and the mechanism extracts the entire net-value from θ_h. On the other hand, θ_l chooses $m_0 = \theta_l$ as long as $\hat{p} = \epsilon$ is sufficiently low; thus, the mechanism can generate additional profit from θ_l on top of the full extraction from θ_h. In particular, a complete exclusion of θ_l is never optimal.[20]

Condition (D) is crucial. The buyer type θ_l also never trades at $t > \bar{t}(\theta_l)$ for a reason similar to (18). Thus, if $\bar{t}(\theta_l) < \bar{t}(\theta_h)$, then it is impossible to trade with θ_l at some time after $\bar{t}(\theta_h)$ in the first place; hence, any attempt to trade with θ_l necessarily interferes with the IC for θ_h as in the case with the type-independent outside option. In general, we may interpret $\bar{t}(\theta) := \frac{1}{r}\log\frac{\theta}{\phi(\theta)}$ as the type-dependent *deadline* in bargaining, before which the negotiation must be resolved (otherwise, the IR for θ is violated). (D) guarantees that the order between $\bar{t}(\theta)$ coincides with the order between $u(\theta)$ so that the mechanism can invite extra types without affecting the IC for higher net-value types.

Returning to the case of $\Theta = [\underline{\theta}, \overline{\theta}]$, note that the continuous-type correspondence of (D) is $\epsilon_\phi(\theta)/\epsilon_u(\theta) > 1$. By definition of $\mathcal{T}(p^*)$, a buyer type $\theta \in (\mathcal{U}(p^*))^c$ satisfies this condition iff $\theta \in \mathcal{T}(p^*)$, and for these buyer types

$$\frac{\mathrm{d}\bar{t}}{\mathrm{d}u} = \frac{\mathrm{d}\bar{t}(\theta)}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}u} = \frac{\phi(\theta) - \theta\phi'(\theta^-)}{r\theta\phi(\theta)}\frac{1}{1-\phi'(\theta^-)} = \frac{1-\epsilon_\phi(\theta)}{r\theta(1-\phi'(\theta^-))} < 0.$$

[20] A similar phenomenon arises in the nonlinear pricing problem, where more participation is induced under type-dependent participation constraints (Lewis and Sappington, 1989; Julien, 2000; Rochet and Stole, 2002).

Therefore, based on reasoning similar to that for the two-type case, $\mu^*_{\infty,\Delta}(p^*)$ can generate additional profit from lower net-value types in $\mathcal{T}(p^*)$ by arranging a delayed trade, without any interference with its profit from the top; see (16) in the proof of Proposition 1.

The remaining and important question is whether the seller can generate the same profit by rationing buyer types in $\mathcal{T}(p^*)$ in alternative ways. For example, what if the seller rations these buyer types with a *one-shot lottery* (which randomly dictates to trade or break off at $t = 0$) rather than a delay? In fact, such a one-shot lottery can never generate the same profit as $\mu^*_{\infty,\Delta}(p^*)$ if and only if $\Pi_S(\mu^*_{\infty,\Delta}(p^*)) > \Pi_S(\mu^*_{one-shot})$. Formally, this conclusion immediately follows the construction of $\mu^*_{one-shot}$ in Section 3. Recall that $\mu^*_{one-shot}$ was allowed to employ any such one-shot lotteries but chose not to. Thus, the suboptimality of $\mu^*_{one-shot}$ immediately proves that the delay in $\mu^*_{\infty,\Delta}(p^*)$ can never be replaced with a one-shot lottery.

Intuitively, if a one-shot lottery is resolved with the outcome "breakdown," then the buyer can still exercise her outside option as if nothing happened with the seller. Therefore, any mechanism based on a one-shot lottery inevitably induces the buyer to exercise the outside option with a positive probability. Then, given that the value of the outside option is type-dependent and the buyer's private information, the mechanism has to yield more information rent to buyer types with better outside options, which results in a lower profit. On the other hand, the optimal mechanism can avoid a costly breakdown of the negotiation by inducing a delay and hence can generate more profit.[21] Indeed, note that a breakdown of the negotiation occurs with zero probability in $\mu^*_{\infty,\Delta}(p^*)$ once the buyer types outside of $\mathcal{U}(p^*) \cup \mathcal{T}(p^*)$ are excluded at the very beginning of the negotiation.

Recall that the expected delay in $\mu^*_{\infty,\Delta}(p^*)$ is positive iff $\mathcal{T}(p^*)$ is non-empty. $\underline{\theta}(p)$ increases in p, and hence $\mathcal{T}(p) = (\theta^\dagger, \underline{\theta}(p))$ is also monotone in the sense that $\mathcal{T}(p_1) \subset \mathcal{T}(p_2)$ whenever $p_1 \leq p_2$. Therefore, all the mechanisms in $\{\mu^*_{\infty,\Delta}(p^*) : p^* \in \mathcal{P}_{R^*}\}$ induce a positive delay iff

$$\mathcal{T}(p^*) \neq \emptyset \quad \forall p^* \in \mathcal{P}_{R^*} \quad \Longleftrightarrow \quad \mathcal{T}(\underline{p}^*) \neq \emptyset \quad \text{where} \quad \underline{p}^* := \min \mathcal{P}_{R^*}.$$

The next proposition shows that all optimal mechanisms, not only those in $\{\mu^*_{\infty,\Delta}(p^*) : p^* \in \mathcal{P}_{R^*}\}$, need to delay transactions with some buyer types to achieve $\overline{\Pi}_S$ under the same condition.

Proposition 2. *Suppose $\lambda = \infty$. If $\mathcal{T}(\underline{p}^*) \neq \emptyset$, then all optimal mechanisms induce a delay of transaction. If $\mathcal{T}(\underline{p}^*) = \emptyset$, then the optimal mechanism $\mu^*_{\infty,\Delta}(\underline{p}^*)$ degenerates to a one-shot mechanism, and hence the seller can achieve $\overline{\Pi}_S$ without a delay.*

Proof. The second part is straightforward: if $\mathcal{T}(\underline{p}^*) = \emptyset$, then all buyer types trade with the seller or exercise the outside option at time zero in $\mu^*_{\infty,\Delta}(\underline{p}^*)$; hence, $\mu^*_{\infty,\Delta}(\underline{p}^*)$ is a one-shot mechanism. For the first part, suppose that the optimal one-shot mechanism $\mu^*_{one-shot}$ achieves $\overline{\Pi}_S$. Then,

$$\int\limits_{\mathcal{U}(p^{one-shot})} p^{one-shot}\, \mathrm{d}F(\theta) \geq \max_p \left[\int\limits_{\mathcal{U}(p)} p\, \mathrm{d}F(\theta) + \int\limits_{\mathcal{T}(p)} \phi(\theta)(\epsilon_\phi(\theta) - 1)\mathrm{d}F(\theta) \right] = \overline{\Pi}_S$$

[21] This argument does not exploit the convexity of ϕ, indicating that a delay may arise with non-convex ϕ. Indeed, suppose ϕ is not convex and $\mathcal{T}(p^{one-shot}) \neq \emptyset$. $\mu^*_{one-shot}$ is suboptimal in this case because, by the same argument discussed here, we can arrange a delayed trade for $\theta \in \mathcal{T}(p^{one-shot})$ without lowering the profit from $\mathcal{U}(p^{one-shot})$.

where the last equality holds due to Proposition 1. Because $\phi(\theta)(\epsilon_\phi(\theta)-1)>0$ for all $\theta \in \mathcal{T}(p)$, this inequality holds only if $\mathcal{T}(p^{one-shot})=\emptyset$ and $p^{one-shot} \in \mathcal{P}_{R^*}$ (i.e., $p^{one-shot}$ maximizes the right-hand side). Then, $\mathcal{T}(\underline{p}^*) \subset \mathcal{T}(p^{one-shot})=\emptyset$ by the monotonicity of $\mathcal{T}(p)$. □

4.3. Optimal mechanism when $\lambda < \infty$

Finally, we discuss the optimal mechanisms for the case $\lambda < \infty$. The autarky payoff is now

$$\phi(\theta)=\mathbb{E}\left[e^{-rn_{arrival}\Delta}\right]w(\theta)=\frac{e^{-r\Delta}(1-e^{-\lambda\Delta})}{1-e^{-(\lambda+r)\Delta}}w(\theta)=\frac{\lambda}{\lambda+r}w(\theta)+O(\Delta) \tag{19}$$

where $O(\Delta)$ stands for a term that vanishes as $\Delta \to 0$. With the autarky payoff being substituted with (19), there is no change in either (R) or (R*). Thus, any mechanism that induces the same allocation as $\mu^*_{\infty,\Delta}(p^*)$ for some $p^* \in \mathcal{P}_{R^*}$ (as defined in Section 4.2), if any, should be optimal.

Our first observation is that, however, $\mu^*_{\infty,\Delta}(p^*)$ itself may not be incentive-compatible any longer. Let us illustrate this issue for the case that Δ is small, although the same problem generally occurs. Consider $\theta \in \mathcal{T}(p^*)$. Recall that $\mu^*_{\infty,\Delta}(p^*)$ recommends this buyer type to delay by time $\frac{1}{r}\log(1/\phi'(\theta^-))+O(\Delta)$ and then trade at price $(\epsilon_\phi(\theta)-1)\phi(\theta)/\phi'(\theta^-)$ with probability close to 1. If the buyer obeys this recommendation, then her expected payoff is approximately

$$e^{-r\left(\frac{1}{r}\log\frac{1}{\phi'(\theta^-)}\right)}\left[\theta-\frac{(\epsilon_\phi(\theta)-1)\phi(\theta)}{\phi'(\theta^-)}\right]=\phi'(\theta^-)\left[\theta-\theta+\frac{\phi(\theta)}{\phi'(\theta^-)}\right]=\phi(\theta). \tag{20}$$

However, suppose that the outside option arrives at time $t=\epsilon \approx 0$. If the buyer disobeys the mechanism and exercises this outside option, then her final discounted payoff is $e^{-r\epsilon}w(\theta) \approx w(\theta)$, which is larger than (20). Thus, $\mu^*_{\infty,\Delta}(p^*)$ is no longer incentive-compatible.

This problem can be remedied by requiring each buyer type in $\mathcal{T}(p^*)$ to pay a large upfront fee at $t=0$ and promising that this upfront fee is compensated by a price discount if the buyer remains obedient until the end of the negotiation. More specifically, for each $p \in [\underline{u}, \overline{u}]$, consider the following mechanism, which we denote by $\mu^*_{\lambda,\Delta}(p)$.

- If $m_0 \in \mathcal{U}(p)$, the parties are recommended to trade in period 0 at price p.
- If $m_0 \in \mathcal{T}(p)$, the buyer is asked to pay the seller the upfront fee $\tilde{\nu}(m_0)=w(m_0)-\phi(m_0)$ at $t=0$. Then, the mechanism recommends waiting until period $\tilde{n}(m_0)$ with neither any nonnull action nor transfer. In any period $n \geq \tilde{n}(m_0)$, the mechanism randomizes over the two recommendations: (i) With probability $1-\tilde{\beta}(m_0)$, it is recommended to trade in period n at price $\tilde{p}(m_0)$. (ii) With probability $\tilde{\beta}(m_0)$, it is recommended to wait one more period.
- If $m_0 \notin \mathcal{U}(p) \cup \mathcal{T}(p)$, then the mechanism recommends breaking off the negotiation at $t=0$.

For each $m_0 \in \mathcal{T}(p)$, similar to the case of $\lambda=\infty$, choose

$$\tilde{n}(m_0)=\max\{n \in \mathbb{N}_0 : e^{-rn\Delta} \geq \phi'(m_0^-)\} \quad \text{and} \quad \tilde{\beta}(m_0)=\frac{e^{-r\tilde{n}(m_0)\Delta}-\phi'(m_0^-)}{e^{-r\tilde{n}(m_0)\Delta}-e^{-r\Delta}\phi'(m_0^-)}$$

$$\Longrightarrow \quad x(m_0;\mu^*_{\lambda,\Delta}(p))=\sum_{k=0}^{\infty}e^{-r(k+\tilde{n}(m_0))\Delta}\tilde{\beta}(m_0)^k(1-\tilde{\beta}(m_0))=\phi'(m_0^-),$$

and

$$\tilde{p}(m_0)=m_0-\frac{w(m_0)}{\phi'(m_0^-)} \quad \Longrightarrow \quad \tilde{\nu}(m_0)+x(m_0;\mu^*_{\lambda,\Delta}(p))\tilde{p}(m_0)=(\epsilon_\phi(m_0)-1)\phi(m_0).$$

With the upfront fee, the expected payoff of each buyer type $\theta \in \mathcal{T}(p)$ from being truthful and obedient to $\mu^*_{\lambda,\Delta}(p)$ still equals her autarky payoff:

$$-\tilde{\nu}(\theta)+\sum_{k=0}^{\infty}(1-\tilde{\beta}(\theta))\tilde{\beta}(\theta)^k e^{-r(k+\tilde{n}(\theta))\Delta}[\theta-\tilde{\rho}(\theta)] = -\tilde{\nu}(\theta)+x(\theta;\mu^*_{\lambda,\Delta}(p))[\theta-\tilde{\rho}(\theta)] = \phi(\theta).$$

However, immediately after the buyer pays the upfront fee, her payoff from remaining obedient is

$$\sum_{k=0}^{\infty}(1-\tilde{\beta}(\theta))(\tilde{\beta}(\theta))^k e^{-r(k+\tilde{n}(\theta))\Delta}[\theta-\tilde{\rho}(\theta)] = \phi(\theta)+\tilde{\nu}(\theta) \geq w(\theta).$$

Hence, the buyer has no incentive to exercise the outside option even if it arrives at $t=\epsilon \approx 0$. Consequently, $\mu^*_{\lambda,\Delta}(p)$ is now incentive-compatible and generates the exactly optimal profit level $\overline{\Pi}_S = \Pi_R$ for the seller if we choose p from $\mathcal{P}_{R^*}$.

Proposition 3. *Suppose $\lambda < \infty$. (i) $\mu^*_{\lambda,\Delta}(p)$ is incentive-compatible for any $p \in [\underline{u}, \overline{u}]$. (ii) For any $p^* \in \mathcal{P}_{R^*}$, $\Pi_S(\mu^*_{\lambda,\Delta}(p^*)) = \Pi_R = \overline{\Pi}_S$ hence, $\mu^*_{\lambda,\Delta}(p^*)$ is optimal.*

Similar to the delay in transaction, the upfront fee is indispensable to implementing the optimal mechanism. For any $\mu = (A, P)$, define each buyer type's upfront fee by the total discounted payment before μ recommends the first nonnull action in period $n = \min\{\tau(\theta;\mu), \sigma(\theta;\mu)\}$.

$$\nu(\theta;\mu) := \mathbb{E}\left[\sum_{0\leq k<\min\{\tau(\theta;\mu),\sigma(\theta;\mu)\}} e^{-rk\Delta}\widehat{P}_k(\theta)\right] \quad \forall\theta\in\Theta.$$

We say that μ induces a positive upfront fee if $\int_\Theta \nu(\theta;\mu)\mathrm{d}F(\theta) > 0$. The following proposition states the necessary and sufficient condition for all optimal mechanisms inducing positive upfront fees and delays. Recall that $\underline{p}^*$ stands for the smallest element in $\mathcal{P}_{R^*}$.

Proposition 4. *Suppose $\lambda < \infty$ and $\Delta > 0$. If $\mathcal{T}(\underline{p}^*) \neq \emptyset$, then all optimal mechanisms induce positive delay of transaction and upfront fees. If $\mathcal{T}(\underline{p}^*) = \emptyset$, $\mu^*_{\lambda,\Delta}(\underline{p}^*)$ degenerates to a one-shot mechanism.*

The proof is in Section A.3. The key step in the proof is to show that, in any optimal mechanism (not only the one constructed above), there is a buyer type θ such that (i) the seller induces a delayed trade with θ and (ii) the buyer's payoff from participating in the optimal mechanism is exactly $\phi(\theta)$. Then, similar to the discussion above, an upfront fee should be arranged to ensure that this buyer never walks away before the trade.

5. Optimal mechanism in the limiting cases

We investigate in this section how the optimal mechanisms behave in the limiting case $\Delta \to 0$ and/or $\lambda \to \infty$. We need to develop some notations first. The objective function in (R*) and its maximizers $\mathcal{P}_{R^*}$ generally depend on λ and Δ. In this section, we denote by $\mathcal{P}^{\lambda,\Delta}_{R^*}$ these maximizers to emphasize this dependence on λ and Δ. Note that $\underline{\theta}(p)$ and $\overline{\theta}(p)$ also depend on

λ and Δ, which we will denote by $\overline{\theta}^{\lambda,\Delta}(p)$ and $\underline{\theta}^{\lambda,\Delta}(p)$ in this section. In addition, $\underline{\theta}^{\lambda,0}(p) := \lim_{\Delta\to 0}\underline{\theta}^{\lambda,\Delta}(p)$ and $\overline{\theta}^{\lambda,0}(p) := \lim_{\Delta\to 0}\overline{\theta}^{\lambda,\Delta}(p)$ represent their continuous-time limits. For any $p \in [\underline{u}, \overline{u}]$,

$$\mathcal{U}^{\lambda,\Delta}(p) := [\underline{\theta}^{\lambda,\Delta}(p), \overline{\theta}^{\lambda,\Delta}(p)], \quad \mathcal{T}^{\lambda,\Delta}(p) := (\theta^{\dagger}, \underline{\theta}^{\lambda,\Delta}(p))$$
$$\mathcal{U}^{\lambda,0}(p) := [\underline{\theta}^{\lambda,0}(p), \overline{\theta}^{\lambda,0}(p)], \quad \mathcal{T}^{\lambda,0}(p) := (\theta^{\dagger}, \underline{\theta}^{\lambda,0}(p)).$$

Finally, note that $\mathcal{P}^{\lambda,\Delta}_{R^*}$, $\mathcal{U}^{\lambda,\Delta}(p)$, and $\mathcal{T}^{\lambda,\Delta}(p)$ are independent of Δ when $\lambda = \infty$; thus, we'll often use $\mathcal{P}^{\infty}_{R^*}$, $\mathcal{U}^{\infty}(p)$, and $\mathcal{T}^{\infty}(p)$ as shorthand for them in this case.

5.1. Continuous-time limit: $\Delta \to 0$

In this subsection, we fix $\lambda \in (0, \infty]$ and investigate the continuous-time limit of each buyer type's allocation in $\mu^*_{\lambda,\Delta}(p^{\lambda,\Delta})$, where $p^{\lambda,\Delta}$ is optimally chosen from $\mathcal{P}^{\lambda,\Delta}_{R^*}$. The buyer's allocation takes the simple form in the limit, allowing simple implementation with a conventional declining price path or a menu of option contracts.

Note that each buyer type's allocation in $\mu^*_{\lambda,\Delta}(p^{\lambda,\Delta})$ is characterized by the (i) stochastic trading time, (ii) upfront fee at $t = 0$, and (iii) (follow-up) payment upon trading, which we denote by $\tau^{\lambda,\Delta}(\theta; p^{\lambda,\Delta})$, $\nu^{\lambda,\Delta}(\theta; p^{\lambda,\Delta})$, and $\rho^{\lambda,\Delta}(\theta; p^{\lambda,\Delta})$, respectively, in this section. In what follows, suppose that the sequence $p^{\lambda,\Delta}$ is convergent as $\Delta \to 0$.

First, suppose $\lambda = \infty$. $\mathcal{P}^{\infty,\Delta}_{R^*}$ is independent of Δ in this case, and pick $p^{\infty} \in \mathcal{P}^{\infty,\Delta}_{R^*}$. From the definition of $\mu^*_{\infty,\Delta}(p^{\infty})$ in Section 4.2, the upfront fee and the (follow-up) payment upon trading are given by

$$\nu^{\infty,\Delta}(\theta; p^{\infty}) = 0 \quad \forall\theta, \quad \text{and} \quad \rho^{\infty,\Delta}(\theta; p^{\infty}) = \begin{cases} p^{\infty} & \text{if } \theta \in \mathcal{U}^{\infty}(p^{\infty}) \\ \theta - \frac{w(\theta)}{w'(\theta^-)} & \text{if } \theta \in \mathcal{T}^{\infty}(p^{\infty}) \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

Note that both $\nu^{\infty,\Delta}(\theta; p^{\infty})$ and $\rho^{\infty,\Delta}(\theta; p^{\infty})$ are independent of Δ. The next proposition examines the convergence of $\tau^{\infty,\Delta}(\theta; p^{\infty})$. The proof is in Appendix A.4.

Proposition 5. *Suppose $\lambda = \infty$ and $p^{\infty} \in \mathcal{P}^{\infty}_{R^*}$. (i) For all θ, $\nu^{\infty,\Delta}(\theta; p^{\infty})$ and $\rho^{\infty,\Delta}(\theta; p^{\infty})$ are given as (21), and $\tau^{\infty,\Delta}(\theta; p^{\infty})$ converges in probability to the following as $\Delta \to 0$:*

$$\operatorname*{plim}_{\Delta\to 0} \tau^{\infty,\Delta}(\theta; p^{\infty}) = \begin{cases} 0 & \text{if } \theta \in \mathcal{U}^{\infty}(p^{\infty}) \\ \frac{1}{r}\log\frac{1}{w'(\theta^-)} & \text{if } \theta \in \mathcal{T}^{\infty}(p^{\infty}) \\ \infty & \text{otherwise.} \end{cases}$$

Alternatively, each buyer type's allocation in the limit can be described as follows:

- If $\theta \in \mathcal{U}^{\infty}(p^{\infty})$, the two parties trade at price p^{∞} at $t = 0$.
- If $\theta \in \mathcal{T}^{\infty}(p^{\infty})$, the two parties trade at deterministic time $\frac{1}{r}\log\frac{1}{w'(\theta^-)}$ and price $\theta - \frac{w(\theta)}{w'(\theta^-)}$.

For all other cases, the two parties break off instantly at $t = 0$. Note that both $\rho^{\infty,\Delta}(\theta; p^{\infty})$ and $\operatorname{plim}_{\Delta\to 0}\tau^{\infty,\Delta}(\theta; p^{\infty})$ are monotone in θ; hence, we can implement the optimal mechanism with a declining price path $\langle p^*(t)\rangle_{t\geq 0}$ in the continuous-time limit. This optimal price path is

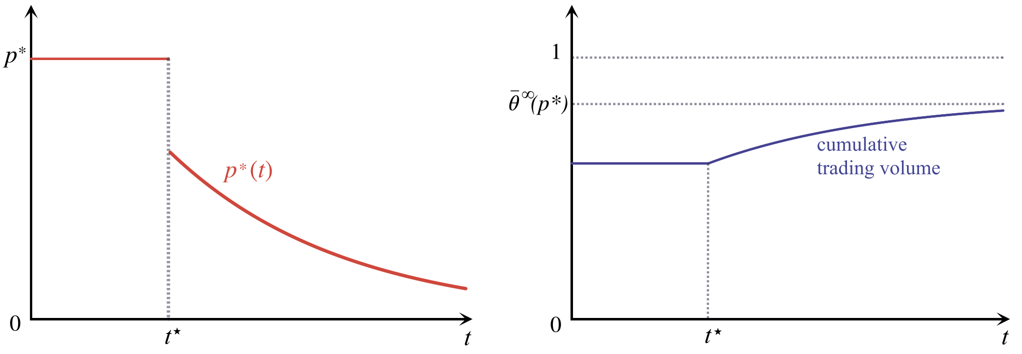

Fig. 3. Optimal price path for Example 1 ($a = 0$ and $r = 0.1$).

illustrated in Fig. 3 for Example 1, where $p^* = p^\infty$. A bulk of trade occurs at $t = 0$ with buyer types in $\mathcal{U}^\infty(p^\infty)$, and then, buyer types in $\mathcal{T}^\infty(p^\infty)$ are gradually skimmed over time.

Next, consider the case $\lambda \in (0, \infty)$. Suppose that $p^{\lambda,\Delta} \in \mathcal{P}^{\lambda,\Delta}_{R^*}$ is convergent as $\Delta \to 0$, and let $p^{\lambda,0} := \lim_{\Delta\to 0} p^{\lambda,\Delta}$ denote this limit. Similar to the case of $\lambda = \infty$, each buyer type's allocation in $\mu^*_{\lambda,\Delta}(p^{\lambda,\Delta})$ converges in probability as in the next proposition, whose proof is almost identical to that of Proposition 5 and is hence omitted.

Proposition 6. *Suppose $0 < \lambda < \infty$, and $p^{\lambda,\Delta} \in \mathcal{P}^{\lambda,\Delta}_{R^*}$ is convergent to $p^{\lambda,0}$ as $\Delta \to 0$. The allocations of each buyer type θ in $\mu^*_{\lambda,\Delta}(p^{\lambda,\Delta})$ converge to the following:*

$$\lim_{\Delta\to 0} \nu^{\lambda,\Delta}(\theta; p^{\lambda,\Delta}) = \begin{cases} 0 & \text{if } \theta \in \mathcal{U}^{\lambda,0}(p^{\lambda,0}) \\ \frac{r}{\lambda+r} w(\theta) & \text{if } \theta \in \mathcal{T}^{\lambda,0}(p^{\lambda,0}) \\ 0 & \text{otherwise}, \end{cases}$$

$$\lim_{\Delta\to 0} \rho^{\lambda,\Delta}(\theta; p^{\lambda,\Delta}) = \begin{cases} p^{\lambda,0} & \text{if } \theta \in \mathcal{U}^{\lambda,0}(p^{\lambda,0}) \\ \theta - \frac{(\lambda+r)w(\theta)}{\lambda w'(\theta^-)} & \text{if } \theta \in \mathcal{T}^{\lambda,0}(p^{\lambda,0}) \\ 0 & \text{otherwise}, \end{cases}$$

$$\operatorname*{plim}_{\Delta\to 0} \tau^{\lambda,\Delta}(\theta; p^{\lambda,\Delta}) = \begin{cases} 0 & \text{if } \theta \in \mathcal{U}^{\lambda,0}(p^{\lambda,0}) \\ \frac{1}{r}\log\frac{\lambda+r}{\lambda w'(\theta^-)} & \text{if } \theta \in \mathcal{T}^{\lambda,0}(p^{\lambda,0}) \\ \infty & \text{otherwise}. \end{cases} \tag{22}$$

Alternatively, each buyer type's allocation in the limit can be described as follows:

- If $\theta \in \mathcal{U}^{\lambda,0}(p^{\lambda,0})$, the two parties trade at price $p^{\lambda,0}$ instantly at $t = 0$.
- If $\theta \in \mathcal{T}^{\lambda,0}(p^{\lambda,0})$, the buyer pays the upfront fee $\frac{r}{\lambda+r} w(\theta)$ at $t = 0$. Then, the buyer trades with the seller at time $\frac{1}{r}\log\frac{\lambda+r}{\lambda w'(\theta^-)}$ by additionally paying the seller $\theta - \frac{\lambda+r}{\lambda}\frac{w(\theta)}{w'(\theta^-)}$.

For all other cases, the two parties break off instantly at $t = 0$. Due to the necessity of upfront fees, the optimal mechanism cannot be implemented simply with a price path. With a delay and upfront payment, the optimal mechanism in fact features an option-contract-like structure. Indeed, the seller can implement the optimal outcome with a menu of call options. Suppose that, based on the buyer's message $\theta \in \Theta$, the seller offers a European call option with premium $\lim_{\Delta\to 0}\nu^{\lambda,\Delta}(\theta;p^{\lambda,\Delta})$, maturity time $\operatorname{plim}_{\Delta\to 0}\tau^{\lambda,\Delta}(\theta;p^{\lambda,\Delta})$, and strike price $\lim_{\Delta\to 0}\rho^{\lambda,\Delta}(\theta;p^{\lambda,\Delta})$ as in the last proposition. Each type finds it optimal to report her θ truthfully; hence, this menu of options achieves $\overline{\Pi}_S$.

5.2. *Frictionless limit: $\lambda\to\infty$*

Next, we examine the case in which the friction in the outside option's arrival process vanishes to zero or, equivalently, the limit $\lambda\to\infty$. Before taking this limit, however, we first take $\Delta\to 0$ and then investigate how the continuous-time limit of the optimal mechanisms changes as $\lambda\to\infty$. To see that this exercise is sensible, note that the outside option's arrival time for the case $\lambda<\infty$ converges in probability to the arrival time for the case $\lambda=\infty$ only if we take $\Delta\to 0$ first and then $\lambda\to\infty$.

Formally, let $n^{\lambda}_{arrival}$ denote the *period* in which the outside option arrives. Again, the superscript λ is added to $n_{arrival}$ only in this section to emphasize its dependence on λ. Recall that

$$n^{\lambda}_{arrival}=\begin{cases}\text{geometric random variable with the support } \mathbb{N}=\{1,2,\ldots\} & \text{if } \lambda<\infty,\\ \text{0 with probability 1} & \text{if } \lambda=\infty.\end{cases}$$

Importantly, by Assumption (Z1) in Section 2, $n^{\lambda}_{arrival}=0$ with zero probability when $\lambda<\infty$. Note that the arrival *time* of the outside option is $n^{\lambda}_{arrival}\Delta$. Then, $\lim_{\lambda\to\infty}\operatorname{plim}_{\Delta\to 0} n^{\lambda,\Delta}_{arrival}\Delta = 0 = n^{\infty}_{arrival}\Delta$ while $\operatorname{plim}_{\lambda\to\infty} n^{\lambda}_{arrival}\Delta \neq 0 = n^{\infty}_{arrival}$ if we fix $\Delta>0$ and only take the limit $\lambda\to\infty$.

A comparison between the last two propositions shows that the continuous-time limit of $\mu^*_{\lambda,\Delta}(p^{\lambda,\Delta})$ indeed converges to the continuous-time limit of $\mu^*_{\infty,\Delta}(p^{\infty})$ as $\lambda\to\infty$. More precisely, choose $p^{\lambda,\Delta}\in\mathcal{P}^{\lambda,\Delta}_{R*}$ for each λ and Δ such that the double-limit $\lim_{\lambda\to\infty}\lim_{\Delta\to 0}p^{\lambda,\Delta}$ is well-defined. One can show that this double limit must belong to $\mathcal{P}^{\infty}_{R*}$[22]; hence, there is $\tilde{p}^{\infty}\in\mathcal{P}^{\infty}_{R*}$ such that (i) $\mu^*_{\infty,\Delta}(\tilde{p}^{\infty})$ is exactly optimal whenever $\lambda=\infty$ and $\Delta>0$ and (ii) each buyer type's continuous-time-limit allocation in $\mu^*_{\lambda,\Delta}(p^{\lambda,\Delta})$ converges to her continuous-time-limit allocation in $\mu^*_{\infty,\Delta}(\tilde{p}^{\infty})$ in the following sense:

$$\begin{aligned}&\lim_{\substack{\lambda\to\infty\\ \lambda<\infty}}\operatorname*{plim}_{\Delta\to 0}(\nu^{\lambda,\Delta}(\theta;p^{\lambda,\Delta}),\tau^{\lambda,\Delta}(\theta;p^{\lambda,\Delta}),\rho^{\lambda,\Delta}(\theta;p^{\lambda,\Delta}))\\ &=\operatorname*{plim}_{\Delta\to 0}\left(\nu^{\infty,\Delta}(\theta;\tilde{p}^{\infty}),\tau^{\infty,\Delta}(\theta;\tilde{p}^{\infty}),\rho^{\infty,\Delta}(\theta;\tilde{p}^{\infty})\right).\end{aligned}$$

Finally, we close this section by discussing how the observations in this section are affected if we replace (Z1) with the following alternative assumption.

(Z1′) $\mathbb{P}\{Z_0=1\}=\mathbb{P}\{Z_n=1|Z_{n-1}=0\}=1-e^{-\lambda\Delta}$ for all $n\geq 1$.

Under (Z1′), the buyer has two-dimensional private information upon her participation in the negotiation. Hence, well-known technical issues for multidimensional mechanism design problems

[22] Denote the objective function in (R*) by $H^{\lambda,\Delta}(p)$. $H^{\lambda,\Delta}(p)$, $\lim_{\Delta\to 0}H^{\lambda,\Delta}(p)$, and $\lim_{\lambda\to\infty}\lim_{\Delta\to 0}H^{\lambda,\Delta}(p)$ are all well-defined and continuous in p. In addition both integrals in (R*) are monotone in Δ and λ. Hence, by Dini's theorem, both $\lim_{\Delta\to 0}H^{\lambda,\Delta}(p)$ and $\lim_{\lambda\to\infty}\lim_{\Delta\to 0}H^{\lambda,\Delta}(p)$ hold uniformly with respect to p and $\lim_{\lambda\to\infty}\lim_{\Delta\to 0}p^{\lambda,\Delta}\in\mathcal{P}^{\infty}_{R*}$.

(Rochet and Stole, 2003) arise, making it difficult to characterize the optimal mechanism. However, such technical issues disappear in limiting cases. First, note that $\mathbb{P}\{Z_0 = 0|\lambda < \infty\} = e^{-\lambda\Delta}$ converges to zero as $\lambda \to \infty$ *for any fixed* $\Delta > 0$. Hence, it is always optimal for the seller to ignore this negligibly possible event $Z_0 = 0$ whenever λ is sufficiently large, in which case the seller's optimal mechanism should never condition the buyer's allocation on Z_0. As a result, the optimal mechanism for case $\lambda = \infty$ is also (approximately) optimal for case $0 \ll \lambda < \infty$. What if $\lambda < \infty$ is small? Note that the probability $\mathbb{P}\{Z_0 = 1|\lambda < \infty\} = 1 - e^{-\lambda\Delta}$ converges to 0 as $\Delta \to 0$. Thus, similar to the other limiting case $\lambda \to \infty$, it is always optimal for the seller to disregard the negligible possibility that $Z_0 = 1$. As a result, the mechanism $\mu^*_{\lambda,\Delta}(p^{\lambda,\Delta})$, which was optimal under assumption (Z1), remains (approximately) optimal under (Z1$'$) provided that Δ is small.

6. Discussions

6.1. Comparative statics

Propositions 2 and 4 show that all optimal mechanisms deviate from the fixed-price mechanism iff $\mathcal{T}(\underline{p}^*) \neq \emptyset$. In this subsection, we show that this condition is more likely to hold for environments such that ϕ is more dispersed in terms of $\epsilon_\phi(\theta)/\epsilon_u(\theta)$. Identify each environment by F and ϕ, and define $\mathcal{E}$ as the set of environments that satisfy Assumptions 1–4. For simplicity, we also assume ϕ is differentiable in all environments in $\mathcal{E}$. Let

$$Q_E := \int\limits_{\Theta} \mathbb{1}_{\{\epsilon_\phi(\theta)/\epsilon_u(\theta)>1\}} \mathrm{d}F(\theta)$$

be the fraction of types at which $\phi(\theta)$ is locally elastic with respect to $u(\theta)$. Additionally, let μ^*_E be the optimal mechanism (as characterized in the last section) for E.

Now, consider $E_1 = (F_1, \phi_1)$ and $E_2 = (F_2, \phi_2) \in \mathcal{E}$ such that $Q_{E_1} < Q_{E_2}$. Additionally, suppose that $\delta(\mu^*_{E_1}) > 0$. Seemingly contrary to the discussion in Section 4, however, we cannot generally conclude that a delay also occurs in $\mu^*_{E_2}$. The reason is that a change in the environment from E_1 to E_2 has two opposite effects on the delay. All things being equal, the change increases Q_E and therefore encourages the seller to arrange more delay, which is consistent with the intuition discussed in the last sections. On the other hand, the change may also affect the distribution of $u(\theta)$ in a way that the seller's overall gain from any price discrimination decreases; in this case, a delay may disappear in the optimal mechanism even if the outside option is more dispersed in E_2.

To isolate the effect of a change in Q_E from a change in net-values, we restrict our attention to environments such that the net-values are identically distributed. For any $E = (F, \phi) \in \mathcal{E}$ let $\theta_E(q) := \inf\{\theta \in \Theta : q \leq F(\theta)\}$ and $u_E(q) := \theta_E(q) - \phi(\theta_E(q))$ be a buyer type at quantile q of F and her net-value, respectively. Fix $E_0 = (F_0, \phi_0) \in \mathcal{E}$, and define $\mathcal{E}(E_0) := \{E = (F, \phi) \in \mathcal{E} : u_E(q) = u_{E_0}(q)\ \forall q \in [0, 1]\}$ as the set of all environments with the same net-value at each quantile q. Restricting attention to $\mathcal{E}(E_0)$, we can obtain the next proposition.

Proposition 7. *Fix $E_0 \in \mathcal{E}$ and suppose there is $E' \in \mathcal{E}(E_0)$ with $\delta(\mu^*_{E'}) > 0$. There are cutoffs $\overline{Q}$ and $\underline{Q} \in [0, 1]$ such that, for all $E \in \mathcal{E}(E_0)$, $\delta(\mu^*_E) > 0$ if $Q_E \geq \overline{Q}$, and $\delta(\mu^*_E) = 0$ if $Q_E \leq \underline{Q}$.*

The proof is in Appendix A.5. Here, we assume that there is $E' \in \mathcal{E}_E$ such that $\delta(\mu^*_{E'}) > 0$ to exclude the trivial case that the optimal mechanism induces no delay in all environments in $\mathcal{E}(E_0)$.

6.2. General arrival processes

In this section, we show that the main results of the paper hold for a larger class of arrival processes beyond the binomial process. Let g be the probability mass function of $n_{arrival}$ (the period in which the outside option arrives). In addition, let $\text{supp}(n_{arrival}) \subset \mathbb{N} \cup \{\infty\}$ denote the support of $n_{arrival}$, where $n_{arrival} = \infty$ stands for the case that the outside option remains unavailable indefinitely (i.e., $Z_n = 0$ for all $n < \infty$).[23]

Whether the paper's main results hold in more general environments hinges on whether Lemma 1 is valid, from which all other results follow. The next lemma shows that Lemma 1 continues to hold under the following two conditions.

(G_1) If n and $n' \in \text{supp}(n_{arrival})$, $\{k \in \mathbb{N} \cup \{\infty\} : n \leq k \leq n'\} \subset \text{supp}(n_{arrival})$.[24]
(G_2) $g(n)/\sum_{k \geq n} g(k)$ increases in n.

The condition (G_1) requires that $\text{supp}(n_{arrival})$ has an interval-like structure. The condition (G_2) assumes the monotone hazard-rate.

Lemma 3. *Suppose Assumptions 1-4, (Z1), (G_1), and (G_2) hold. Then, Lemma 1 holds true.*

The proof is in Appendix A.2. All other main results in the previous sections also continue to hold. The formulation of the relaxed problem remains intact, and thus Lemma 2 continues to be valid without any change. The mechanism discussed in the previous sections is still incentive-compatible and optimal.

7. Concluding remarks

This paper considered sales negotiations in which the buyer can opt out for a type-dependent outside option during the negotiation. The outside option has a substantial effect on the negotiation process and outcome. Most notably, delays in the transaction and upfront payment schemes arise in all optimal mechanisms to avoid negotiation breakdown.

We finish by noting the implications of this paper's findings for bargaining theory. In a recent paper, Board and Pycia (2014) consider the bargaining problem without the seller's commitment power but otherwise identical to our model with $\lambda = \infty$. They show that the seller insists on a fixed price indefinitely, and hence, no delay occurs in equilibrium. Our finding shows that the seller can earn more profit by offering a declining price path and inducing a delay, but, according to Board and Pycia, this strategy violates sequential rationality.

23 Note that $0 \notin \text{supp}(n_{arrival})$. Thus, we maintain the assumption (Z1).

24 Define $n < \infty$ for any $n \in \mathbb{N}$. The condition (G_1) is crucial. For example, suppose $\mathbb{P}\{\Delta n_{arrival} = \epsilon\} = \mathbb{P}\{\Delta n_{arrival} = \infty\} = 1/2$, where $n_{arrival} = \infty$ iff the outside option never arrives. In addition, suppose that the seller commits to trading at $n = \epsilon/\Delta$ if $Z_n = 0$ and breaking off the negotiation at $n = \epsilon/\Delta$ otherwise. Then, Lemma 3 fails because $x(\theta; \mu) + y(\theta; \mu) = \frac{1}{2}e^{-r\epsilon} + 1 > 1$. The author thanks an anonymous reviewer for this example.

This observation contrasts with the prediction of the Coase conjecture. Since Coase (1972), the bargaining literature has emphasized the adverse effect on profit of the seller's inability to commit to a fixed price. However, our results demonstrate that the Coase conjecture is reversed when buyers have type-dependent outside options and quitting rights; it is the seller's inability to change his price that may undermine the seller's profit.

Appendix A. Proofs

A.1. *Proof of Lemma 1*

Let $\{\mathfrak{F}_n^Z\}_{n\geq 0}$ denote the filtration generated by Z, and let $\mathscr{T}$ be the set of all $(\{\mathfrak{F}_n^Z\}_{n\geq 0})$-stopping times. Note that $B = \mathbb{E}[e^{-rn_{arrival}\Delta}] < 1$ in the definition of y. Thus, to derive the upper bound for $x(\theta)+y(\theta)$, we may restrict our attention to μ such that

$$\tau(\theta;\mu)\begin{cases} <\infty & \text{if } \tau(\theta;\mu) < n_{arrival} \\ =\infty & \text{if } \tau(\theta;\mu) \geq n_{arrival} \end{cases} \quad \text{and}$$

$$\sigma(\theta;\mu) = \begin{cases} \infty & \text{if } \tau(\theta;\mu) < n_{arrival} \\ n_{arrival} & \text{if } \tau(\theta;\mu) \geq n_{arrival} \end{cases} \quad \text{a.s.}$$

Therefore,

$$x(\theta)+y(\theta) \leq \sup_{\tau\in\mathscr{T}} \mathbb{E}\left[e^{-r\tau\Delta}\mathbb{1}_{\{\tau<n_{arrival}\}} + \frac{e^{-rn_{arrival}\Delta}}{B}\mathbb{1}_{\{\tau\geq n_{arrival}\}}\right]. \tag{A.1}$$

By a direct calculation, for any $\tau\in\mathscr{T}$ and its possible realization n,

$$\mathbb{E}\left[e^{-r\tau\Delta}\mathbb{1}_{\{n<n_{arrival}\}} + \frac{e^{-rn_{arrival}\Delta}}{B}\mathbb{1}_{\{n\geq n_{arrival}\}}\middle|\tau=n\right] = 1$$

Thus, the right-hand side of (A.1) is 1. Consequently, $x(\theta)+y(\theta)\leq 1$.

A.2. *Proof of Lemma 3*

It suffices to show that the right-hand side of (A.1) in the proof of Lemma 1 is less than 1. Let $\underline{n}$ and $\overline{n}$ denote the smallest and highest elements in the support, respectively. In addition,

$$\alpha(n) := \mathbb{E}\left[e^{-rn\Delta}\mathbb{1}_{\{n<n_{arrival}\}} + \frac{e^{-rn_{arrival}\Delta}}{B}\mathbb{1}_{\{n_{arrival}\leq n\}}\right] \quad \forall n\geq 0.$$

$$\Longrightarrow \quad \alpha(n) = e^{-rn\Delta}\ \forall n<\underline{n} \quad \text{and}$$

$$\alpha(n) = \sum_{k>n} g(k)e^{-rn\Delta} + \sum_{k\leq n} e^{-rk\Delta}g(k)/B \quad \forall n\geq\underline{n}.$$

In particular, $\alpha(\infty)=1$. Then, $\alpha(n-1)\geq\alpha(n)$ for all $0\leq n<\underline{n}$. In addition, for $\underline{n}\leq n<\infty$,

$$\alpha(n)\geq\alpha(n+1) \quad \Longleftrightarrow \quad \frac{(1-e^{-r\Delta})B}{e^{-r\Delta}(1-B)} \geq \frac{g(n+1)}{\sum_{k\geq n+1} g(k)}$$

Note that $\frac{(1-e^{-r\Delta})B}{e^{-r\Delta}(1-B)} > 0$ is constant, while the hazard-rate $\frac{g(n+1)}{\sum_{k\geq n+1} g(k)}$ increases in n. Thus, there is $n^\star\geq\underline{n}$ such that $\alpha(n)$ initially decreases over $[\underline{n}, n^\star]$ and then increases over $[n^\star, \overline{n}]$.

Now, suppose $g(\underline{n}) \geq \frac{(1-e^{-r\Delta})B}{e^{-r\Delta}(1-B)}$. Then, $\alpha(n)$ decreases over $[0, \underline{n})$ and then increases over $[\underline{n}, \overline{n}]$. Hence, $\alpha(n) \leq \max\{\alpha(0), \lim_{n \to \bar{n}} \alpha(n), 1\} \leq 1$ for any n. Next, suppose $g(\underline{n}) < \frac{(1-e^{-r\Delta})B}{e^{-r\Delta}(1-B)}$. Then,

$$\alpha(\underline{n}) = [1 - g(\underline{n})]e^{-r\underline{n}\Delta} + \frac{g(\underline{n})}{B} e^{-r\underline{n}\Delta} = e^{-r\underline{n}\Delta}\left[1 + \frac{1-B}{B} g(\underline{n})\right] < \frac{e^{-r\underline{n}\Delta}}{e^{-r\Delta}} < 1 = \alpha(0).$$

Hence, $\alpha(0) \geq \alpha(n)$ for all $n \leq \underline{n}$. Recall that $\alpha(n)$ decreases over $[\underline{n}, n^\star]$ and then increases over $[n^\star, \overline{n}]$. Therefore, $\alpha(n) \leq \max\{\alpha(0), \lim_{n \to \bar{n}} \alpha(n), 1\} \leq 1$ for any $n \leq \overline{n}$.

A.3. Proof of Proposition 4

For any $\nu \in \mathcal{M}^{IC}$, let $V^\nu(\theta)$ denote the payoff of a buyer type θ from participating in ν. Suppose $\mathcal{T}(\underline{p}^*) \neq \emptyset$ and choose any optimal $\mu \in \mathcal{M}^{IC}$. $\mathcal{T}(\underline{p}^*) = \emptyset$ if $\phi'(\theta^-) \geq 1$ for all θ; thus, suppose $\{\theta : \phi'(\theta^-) < 1\} \neq \emptyset$ without loss. By Proposition 3, $\Pi_S(\mu) = \Pi_R$. Hence, $(x, y, \underline{\pi}) = (x(\cdot; \mu), y(\cdot; \mu), \phi(\underline{\theta}))$ must solve (R), and by Lemma A.8 in the online appendix, there are θ^* and θ^{**} such that $\theta^\dagger < \theta^* \leq \theta^{**}$, $(x(\theta; \mu), y(\theta; \mu)) = (\phi'(\theta^-), 0)$ for $\theta \in (\theta^\dagger, \theta^*)$, and

$$V^\mu(\theta) = \phi(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} [x(s; \mu) + \phi'(s) y(s; \mu)] \mathrm{d}s = \phi(\theta) \quad \theta \in (\theta^\dagger, \theta^*).$$

Note that $x(\theta; \mu) < 1$ and $y(\theta; \mu) = 0$ for $\theta \in (\theta^\dagger, \theta^*)$; that is, these buyer types trade after a delay. On the other hand, $V^\mu(\theta) = \phi(\theta) < w(\theta)$ for these buyer types; hence, by the same argument used for (20) in the main text, μ is incentive-compatible only if μ arranges an upfront fee for $\theta \in (\theta^\dagger, \theta^*)$.

A.4. Proof of Proposition 5

All buyer types in $\Theta \backslash \mathcal{T}^\infty(p^\infty)$ trade or exercise the outside option at $t = 0$; hence, the convergence of the allocation for each $\theta \in \Theta \backslash \mathcal{T}^\infty(p^\infty)$ is straightforward. We focus on $\theta \in \mathcal{T}^\infty(p^\infty)$ in this proof. Recall from (12) that $\exp\{-r[\Delta + \tau^{\infty,\Delta}(\theta; p^\infty)]\} \leq \exp\{-r\Delta[1 + n(\theta)]\} \leq \phi'(\theta^-) = w'(\theta^-) = \mathbb{E}[e^{-r\tau^{\infty,\Delta}(\theta; p^\infty)}]$ with probability 1, where $n(\theta) = \max\{n \in \mathbb{N}_0 : e^{-rn\Delta} \geq \phi'(\theta^-)\}$. Thus, for any $\epsilon > 0$, the following inequality holds for all sufficiently small $\Delta > 0$:

$$\begin{aligned}
&\mathbb{P}\left\{|w'(\theta^-) - e^{-r\tau^{\infty,\Delta}(\theta; p^\infty)}| \geq \epsilon\right\} \\
\leq\ &\mathbb{P}\left\{|w'(\theta^-) - e^{-r\left(\Delta + \tau^{\infty,\Delta}(\theta; p^\infty)\right)}| \geq \frac{\epsilon}{2}\right\} \\
\leq\ &\frac{2}{\epsilon}\mathbb{E}\left[w'(\theta^-) - e^{-r(\Delta + \tau^{\infty,\Delta}(\theta; p^\infty))}\right] \qquad (\because \text{Markov's inequality})
\end{aligned}$$

From (13), $\mathbb{E}[w'(\theta^-) - e^{-r(\Delta + \tau^{\infty,\Delta}(\theta; p^\infty))}] = w'(\theta^-) - e^{-r\Delta}\mathbb{E}[e^{-r\tau^{\infty,\Delta}(\theta; p^\infty))}] = (1 - e^{-r\Delta})w'(\theta^-)$ vanishes to zero as $\Delta \to 0$. Hence, $\operatorname{plim}_{\Delta \to 0} e^{-r\tau^{\infty,\Delta}(\theta; p^\infty)} = w'(\theta^-)$ and $\operatorname{plim}_{\Delta \to 0} \tau^{\infty,\Delta}(\theta; p^\infty) = \frac{1}{r} \log \frac{1}{w'(\theta^-)}$. To calculate the limit of $\rho^{\infty,\Delta}(\theta; p^\infty)$, recall from (14) that

$$\rho^{\infty,\Delta}(\theta; p^\infty) = \rho(\theta) = \frac{(\epsilon_w(\theta) - 1)w(\theta)}{w'(\theta^-)} = \theta - \frac{w(\theta)}{w'(\theta)} \quad \forall \theta \in \mathcal{T}^{\infty,\Delta}(p^\infty).$$

A.5. Proof of Proposition 7

For any quantile q, $u'_E(q) = [1 - \phi'(\theta_E(q))]\theta'_E(q)$ has the same sign for all $E \in \mathcal{E}(E_0)$. Let $\hat{q}$ be the quantile such that $\phi'(\theta_E(q)) < 1$ iff $q < \hat{q}$ for all $E \in \mathcal{E}(E_0)$. Additionally, for each environment E, let $q_E^\dagger$ be the quantile such that $\theta_E(q_E^\dagger) = \theta^\dagger$. Then, by (9) in Section 4.1,

$$Q_E = \int_\Theta \mathbb{1}_{\{\epsilon_\phi(\theta)/\epsilon_u(\theta)>1\}} \mathrm{d}F(\theta) = \int_{\theta_E(q_E^\dagger)<\theta<\theta_E(\hat{q})} \mathrm{d}F(\theta) = (\hat{q} - q_E^\dagger)\mathbb{1}_{\{\hat{q}>q_E^\dagger\}}.$$

Denote by $\mu^*_{one-shot}$ the optimal one-shot mechanism that offers a single price $p^{one-shot}$. The profit in the optimal one-shot mechanism depends only on the distribution of net-values. Therefore, $\mu^*_{one-shot}$ posts the same price and generates the same profit in all $E \in \mathcal{E}(E_0)$. Define $\underline{q} := \min\{q \in [0,1] : u_{E_0}(q) \geq p^{one-shot}\}$ and $\overline{Q} := \hat{q} - \underline{q}$. Now, suppose that $Q_E > \overline{Q}$ (equivalently, $\theta_E(q_E^\dagger) < \theta_E(\underline{q})$) and consider the mechanism $\mu^*_{\lambda,\Delta}(p^{one-shot})$ as defined in Section 4.3. Then,

$$\Pi_S(\mu^*_{\lambda,\Delta}(p^{one-shot})) - \Pi_S(\mu^*_{one-shot}) = \int_{\theta_E(q_E^\dagger)}^{\theta_E(\underline{q})} (\epsilon_\phi(\theta) - 1)\phi(\theta)\mathrm{d}F(\theta) > 0.$$

Hence, $\Pi_S(\mu^*_E) > \Pi_S(\mu^*_{one-shot})$ and $\delta(\mu^*_E) > 0$. The proof for the case of $Q_E \leq \underline{Q}$ is trivial; the proposition holds trivially with $\underline{Q} = 0$ (where the optimal mechanism degenerates into a one-shot mechanism) and thus is also true with sufficiently small $\underline{Q} > 0$.

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2021.105279.

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