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Author(s): Jennifer F. Reinganum
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Jennifer F. Reinganum

Northwestern University

This paper demonstrates that price dispersion can exist even within the context of a very simple model. Identical buyers with elastic demand curves sample sequentially from a known price distribution, at a fixed cost per observation. Firms are assumed to be perfectly informed of buyers' reservation prices and demand functions. Given the firms' distribution of marginal costs, firms' behavior as monopolistic competitors results in their offering a distribution of prices which is consistent with expected utility maximization by buyers and with expected profit maximization by sellers.

I. Introduction

Since the publication of Stigler's seminal article "The Economics of Information" (Stigler 1961), many authors have examined optimal search strategies for agents facing stochastic prices, wages, or demand conditions (McCall 1970; Mirman and Porter 1974; Rothschild 1974*a*, 1974*b*; and Lippman and McCall 1976). While the models developed were insightful analyses of the problems of individual agents, many search models have suffered from the fact that the price dispersion upon which they are predicated disappears when the strategies are implemented in a simple market setting (Diamond 1971; Rothschild 1973; Ioannides 1975; Butters 1977*b*). It will be shown that the reasons for these failures lie in part with the strategies themselves and in part with the market settings in which they are typically placed. Specifically, we are interested in demonstrating that price dispersion may exist even within the framework of a very simple model. Fur-

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thermore, the persistence and degree of price dispersion in this particular model will be shown to depend crucially upon two concepts that are conspicuously absent from most search models: the existence of differing marginal costs among firms and the nonzero elasticity of consumers' demand curves.

As Butters (1977*a*) correctly noted, some deviation from the deadly "simplest model"—wherein identical consumers with unitary demand search sequentially, at a fixed cost per observation, over identical monopolistically competitive firms—is required in order to obtain a nondegenerate price distribution. That is, imperfect information *alone* is insufficient to support price dispersion. Several recent papers have presented models which include such deviations (e.g., Green 1976; Butters 1977*a*; Salop 1977; and Wilde 1977) with interesting results.

This paper focuses on and demonstrates the existence of price dispersion for a model which is remarkably similar to the "simplest" one. Specifically, the model which follows differs from the "simplest model" in but two respects: (1) the peculiar assumption of unitary demand is abandoned, and buyers are assumed to purchase goods according to their (elastic) demand functions; (2) firms are not identical; each has constant marginal costs, and each value of marginal costs occurs with some frequency among the firms in the industry.

These assumptions, along with the remaining elements of the "simplest model," will be shown to be sufficient to support a nondegenerate distribution of product prices in equilibrium.

II. Buyers' Behavior

Assume that there is a continuum of identical buyers, each possessed of a strictly quasi-concave, twice continuously differentiable utility function over commodities. Further assume that buyers' demand functions for all commodities are defined, downward sloping in their own prices, and continuously differentiable. The utility function can then be rewritten in terms of prices and wealth as $U(W,p_0,p)$ where W denotes wealth, p_0 is the vector of fixed prices for all other goods, and p is the price paid by the buyer for the "product," to be defined below. We will make the further simplifying assumption that the indirect utility function $U(W,p_0,p)$ can be written as $V(p_0, p) + W$, where V is strictly decreasing in its second argument. The assumption of additive separability in wealth is not strictly necessary but it facilitates comparison with the familiar formulae of Rothschild (1973, 1974*b*) and De Groot (1970) (compare eqq. [1] and [1′] below).

Buyers are assumed to have perfect information in the markets for all commodities except one, hereafter referred to as the "product" market. The product market is characterized by a distribution of prices. Each consumer, in attempting to maximize the expected utility of his limited wealth, engages in search behavior in the product market. Insofar as this activity increases expected utility, buyers attempt to ascertain a minimum price for the product.

Rothschild (1974*b*) has discussed the case in which buyers do not know the distribution of prices from which they are sampling. If buyers are allowed sampling with recall (that is, they are allowed to choose any one among the prices they have been quoted), then the optimal search procedure is not significantly different from the one obtained when buyers know the distribution of product prices. Therefore it will be assumed that buyers sample with recall from a known price distribution. Denote this distribution by $F(p)$ where $F(\cdot)$ is continuously differentiable almost everywhere and has positive density $dF(p)$ on the closed interval $[\underline{p},\bar{p}]$ for some $\underline{p}$ and $\bar{p} > \underline{p}$ in $R+$.

Since nonsequential search is optimal only if there are economies of scale in sampling, it will be assumed that each buyer follows a sequential search strategy, soliciting price quotations so long as the expected increase in utility from doing so is positive. More precisely, if the fixed sampling cost per observation is k, and if the lowest price encountered on the first n searches is S, then the expected gain in utility from searching once more is

$$\begin{aligned} h(S) = &\int_{\underline{p}}^{S} [V(p_0,p) + W - (n+1)k]dF(p) \\ &+ \int_{S}^{\bar{p}} [V(p_0,S) + W - (n+1)k]dF(p) \\ &- [V(p_0,S) + W - nk], \end{aligned}$$

which simplifies to

$$h(S) = \int_{\underline{p}}^{S} [V(p_0,p) - V(p_0,S)]dF(p) - k. \tag{1}$$

Since buyers must search at least once, we require that $h(\bar{p}) \geqslant 0$. Then for W and $V(p_0,\underline{p})$ finite, it can be shown that the optimal stopping rule for the sequential search procedure exists and is myopic (see De Groot [1970] or Lippman and McCall [1976]). That is, there exists a unique reservation price p_r such that $h(p_r) = 0$ and the optimal strategy is to buy if the sampled price p is less than or equal to p_r, and to continue searching if p is greater than p_r. Since all buyers are identical, they have a common reservation price p_r and common demand curves for all goods. The quantity demanded (wealth and all other prices held constant) by the buyer when quoted price p will be

$$d(p) = \begin{cases} q(p) & \text{for } p \leqslant p_r \\ 0 & \text{for } p > p_r, \end{cases}$$

where $q(p)$ is continuously differentiable. Since we wish to demonstrate that price dispersion may exist even within the framework of an extremely simple model (and is not due entirely to some eccentricities in consumer demand), we assume that the demand curve has constant elasticity $e < -1$.

The usual assumption that

$$d(p) = \begin{cases} 1 & \text{for } p \leqslant p_r \\ 0 & \text{for } p > p_r, \end{cases}$$

along with a utility function of the form $U(W,p_0,p) = W - g(p_0) - p$, generates the familiar "searching for the lowest price" rule: search until a price m is encountered such that

$$\int_{\underline{p}}^{m} (m - p)dF(p) - k \leqslant 0. \tag{1'}$$

This assumption of a completely inelastic demand curve seems somewhat strange given marginalist demand theory. Accordingly, we have eliminated it, thus allowing for the interplay of various substitution effects among all commodities as well as the income effects of obtaining a lower product price. This elasticity will prove critical to the market's ability to support a distribution of prices for the homogeneous product.

III. Sellers' Behavior

Assume that there is a continuum of firms which offers a homogeneous product for sale. Let the continuum of firms be the set J, a finite closed interval of the real line. Each firm j in J has a constant marginal cost c_j at which it can instantaneously produce the product, where $c_j \in [\underline{c}, \bar{c}]$ for some $\underline{c}$ and $\bar{c} > \underline{c}$ in R^+. Suppose that $\bar{c} \leqslant \underline{c}e/(1 + e)$ and that the costs c_j occur with distribution $G(c)$ = relative frequency of firms j in J with $c_j \leqslant c$, where G is continuously differentiable with positive density $dG(c)$.

Sellers are assumed to be perfectly informed of buyers' reservation prices and demand curves, and they exploit this knowledge in their price-setting behavior. Each seller j in J is assumed to choose his product price p_j so as to maximize his expected profits Π_j.

$$E[\Pi_j] = \begin{cases} (p_j - c_j)q(p_j)E[N_j] & \text{for } p_j \leqslant p_r \\ 0 & \text{for } p_j > p_r, \end{cases} \tag{2}$$

where N_j is defined to be the number of buyers who sample firm j. While both the "number" of buyers and the "number" of sellers in the market are infinite, the proper interpretation is that we are actually considering very large finite numbers of both: say, n sellers and m buyers. Then forming the ratio $\lambda = m/n$ and letting m and n approach infinity, holding λ constant, gives a measure of the average number of buyers per seller.

IV. Equilibrium Conditions

The notion of equilibrium to be used here is the Nash equilibrium concept. That is, in equilibrium, there should be no incentive for any seller to alter his quoted price nor for any buyer to alter his reservation price.

DEFINITION: An *equilibrium* is a reservation price p_r and a frequency distribution $F_{p_r}(p)$ over a price range $[\underline{p}^*,\bar{p}^*]$ such that:

1. $F_{p_r}(\cdot)$ is C^1 almost everywhere and has positive density $dF_{p_r}(\cdot)$ over $[\underline{p}^*,\bar{p}^*]$ for $\bar{p}^* \geqslant \underline{p}^*$;

2. Given $F_{p_r}(\cdot)$ over $[\underline{p}^*,\bar{p}^*]$, buyers choose optimal stopping rule (reservation price) p_r; and

3. Given p_r, firms collectively generate $F_{p_r}(\cdot)$ over $[\underline{p}^*,\bar{p}^*]$ by individually choosing p_j so as to maximize expected profits.

Notice that if $\bar{p}^* = \underline{p}^*$, then the equilibrium involves a single market price. If $\bar{p}^* > \underline{p}^*$, then the equilibrium is characterized by price dispersion.

PROPOSITION: There exists an equilibrium such that $\bar{p}^* > \underline{p}^*$. That is, there exists an equilibrium with price dispersion.

PROOF: Given the distribution $F(p) = G[p(1 + e)/e]$ on $[\underline{p},\bar{p}] = [\underline{c}e/(1 + e),\bar{c}e/(1 + e)]$, there exists a unique reservation price p_r (see, e.g., De Groot [1970]). Furthermore, $\bar{p} \geqslant p_r > \underline{p}$. To see this, recall that

$$h(S) = \int_{\underline{p}}^{S} [V(p_0,p) - V(p_0,S)]dF(p) - k.$$

Let us examine $h(S)$ as S changes. Since $h(S)$ is differentiable, we can apply Leibniz's rule:

$$h'(S) = -V_2(p_0,S)F(S) > 0.$$

The inequality follows because V is strictly decreasing in its second argument. Thus $h(S)$ is strictly increasing. Since $h(\underline{p}) = -k$ and $h(p_r) = 0$, it follows that $p_r > \underline{p}$. Similarly, since $h(\bar{p}) \geqslant 0$ and $h(p_r) = 0$, it follows that $\bar{p} \geqslant p_r$.

Now, given a reservation price p_r and the demand function $d(p)$, and recalling that firms act so as to maximize expected profits, from equation (2) we see that in equilibrium, p_j will be at or below the reservation price p_r for all sellers in the market. Since $p_j \leqslant p_r$, all buyers who search j on the first search will buy from j. Because buyers have no prior knowledge of who quotes which price, it is reasonable to require that sellers be treated symmetrically in the sense that, on average, each seller is approached by the same number of buyers on the first search. Therefore, $E[N_j] = \lambda$ for all j in J, and $E[\Pi_j] = (p_j - c_j)q(p_j)\lambda$. Maximizing with respect to p_j yields the familiar condition that $p_j^*(1 + 1/e) = c_j$, where $e < -1$ is the elasticity of demand. Thus $\underline{p}^* = \underline{p} = \underline{c}e/(1 + e)$ and $\bar{p}^* = p_r \leqslant \bar{c}e/(1 + e)$.

If $p_j^* \leqslant p_r$, then a cumulative distribution function $F_{p_r}(p)$ is induced upon prices by the distribution $G(c)$ over costs: $F_{p_r}(p) = G[p(1+e)/e]$. And if $p_j^* > p_r$, then seller j's maximal expected profits are obtained by setting the price $p_j = p_r$. Since $p_r > \underline{p} \geq \bar{c}$, all firms are making positive expected profits, so there is no incentive for exit. Thus the cumulative distribution function $F_{p_r}(p)$ is the one induced by $G(c)$ except at the upper endpoint p_r, which is a mass point. It remains to be shown that the substitution of this distribution $F_{p_r}(p)$ for $F(p)$ leaves the reservation price unchanged. That is, we must show that

$$\int_{\underline{p}}^{p_r} [V(p_0,p) - V(p_0,p_r)]dF_{p_r}(p) - k = 0. \tag{3}$$

Recall that the reservation price p_r satisfies

$$h(p_r) = \int_{\underline{p}}^{p_r} [V(p_0,p) - V(p_0,p_r)]dF(p) - k = 0$$

by definition. Now $F_{p_r}(p) = F(p)$ except at $p = p_r$, where $F_{p_r}(p)$ has a mass point. But the function $[V(p_0,p) - V(p_0,p_r)]$ is zero at $p = p_r$, so equation (3) follows.

Thus we have established that, given the reservation price p_r, sellers will offer a nondegenerate range of prices $[\underline{p}^*,\bar{p}^*] = [\underline{c}e/(1+e),p_r]$ with frequency distribution

$$F_{p_r}(p) = \begin{cases} G[p(1+e)/e] & \text{for } p < p_r \\ 1 & \text{for } p = p_r \end{cases}.$$

Furthermore, given this distribution and price range, buyers still choose reservation price p_r. Thus p_r, $F_{p_r}(p)$, and $[\underline{c}e/(1+e),p_r]$ constitute an equilibrium with price dispersion.

V. Conclusion

Assuming the optimal behavior of buyers and sellers and given a distribution of marginal costs, we have demonstrated that there exists an (induced) equilibrium distribution of prices which is nondegenerate. This result is *not* equivalent to the statement that "for any finite positive level of sampling costs k and for an infinite number of prices, it will never pay to search until the minimum price is encountered"—this much is obvious. The set of sellers who offer the product at the minimum price has measure zero. One key to the existence of price dispersion in this model is that buyers do not purchase the same amount of the product for all prices at or below the reservation price. For if they did, then the expected profit-maximizing price would be the reservation price p_r for *all* firms, since in that case $E[\Pi_j] = (p_j - c_j)Q\lambda$, where Q is the fixed quantity demanded per buyer. Then expected profits are clearly maximized when p_j is set as high as the market will bear—p_r. In other words, in this model, not only is the product price not driven down to the minimum price (this due to the existence of search costs) but the price is also prevented from jumping up to the reservation price p_r by the substitution effects of other goods—that is, by the more acceptable proposition that buyers consult demand functions rather than purchase the same quantity regardless of its price.

Of course, eliminating the assumption of different marginal costs will also reverse the result. Both a cost dispersion and elastic demand are essential to provide firms with an incentive to price below the reservation price.

VI. Discussion

Many simplifying assumptions have been made above. The first, that buyers are identical and have simple demand and utility functions, is not objectionable. Although it is patently unrealistic, one can only expect that differences among buyers would reinforce the result rather than reverse it.

The source of the sampling cost k has been left unspecified. It may be interpreted as the cost of making a trip to the store, the cost of a telephone call, or the cost of crossing the street to ask a neighbor. In any case, it is assumed that buyers are unable to trade information at a cost of less than k per observation.

While labor-market job search models typically assume that one either accepts a job offer or rejects it—that is, 0–1 demand—if one considers the number of hours supplied as an elastic function of the wage rate, then the above-developed model is clearly applicable.

A plausible story for the existence of different costs is not difficult to imagine for the short run—anything from different wage contracts to varying age of equipment to different locations with respect to the factory may be invoked. Given this short-run cost dispersion, the existence of imperfect information allows relatively inefficient firms (with $c_j > \underline{c}$) to remain in operation. Seller j's maximized expected profits increase by $-dE[\Pi_j^*]/dc_j = \lambda q[c_j e/(1+e)]$ with a decrease in marginal costs c_j, so all firms would benefit from cost-reducing investment. However, the lower c_j is, the greater is the marginal benefit from c_j reduction. Since it is likely that the marginal cost of c_j reduction also increases as c_j decreases, it is not clear whether the optimal rate of c_j reduction is increasing, constant, or decreasing in c_j. If it is constant, then the cost dispersion $[\underline{c}, \bar{c}]$ will persist in the long run. If the optimal rate of c_j reduction is increasing (decreasing) in c_j, then the cost dispersion may be expected to diminish (increase) in the long run.

Although this model provides a rationale for the existence of equilibrium price dispersion, it does *not* explain observed equilibrium search behavior. That is, in equilibrium, only a single search is required. But since we observe that consumers often sample several stores before buying, some modification of the model is required to capture this aspect of search behavior. Perhaps the simplest modification is to consider consumer heterogeneity. If there are several classes of buyers with different search costs (and hence different reservation prices), it is conjectured that equilibrium will be characterized by both price dispersion and consumer search.

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