Available online at www.sciencedirect.com
ScienceDirect
Games and Economic Behavior 62 (2008) 417–435
GAMES and Economic Behavior
www.elsevier.com/locate/geb

Bargaining, search, and outside options

Anita Gantner *

Department of Economics, University of Innsbruck, Universitätsstrasse 15, 6020 Innsbruck, Austria

Received 12 December 2005
Available online 21 September 2007

Abstract

This paper studies a two-sided incomplete information bargaining model between a seller and a buyer. The buyer has an outside option, which is modeled as a sequential search process during which he can choose to return to bargaining at any time. Two cases are considered: In Regime I, both agents have symmetric information about the search parameters. We find that, in contrast to bargaining with complete information, the option to return to bargaining is not redundant in equilibrium. However, the no-delay result still holds. In Regime II, where agents have asymmetric information about the outside option, delay is possible. The solution characterizes the parameters for renegotiation and those for search with no return to the bargaining table.

JEL classification: C78; D83; C61

Keywords: Bargaining; Two-sided incomplete information; Outside option; Search

1. Introduction

Consider a negotiation between a seller and a buyer over the price of a house. The buyer is looking for a house with particular features, one that might not be easily substituted by another house offered on the real estate market. Each agent knows his own valuation of the house, but is uncertain about the other's valuation. While the seller currently has no other potential bargaining partner, the buyer can quit negotiations at any time and look around for better alternatives, but he can also return to renegotiate with the seller.

* Fax: +43 512 507 2980.
E-mail address: anita.gantner@uibk.ac.at.

0899-8256/$ – see front matter
doi:10.1016/j.geb.2007.05.006

This situation captures three crucial strategic factors in any bargaining setting: information, time preference and outside options. While private information may lead agents to conceal the true size of their potential gains from trade, time preference helps to uncover the true valuation of the agent who is less willing to delay an agreement. Following the time preference aspect addressed in Rubinstein (1982), examples of incomplete information work include Fudenberg and Tirole (1983), a two-period model with one-sided and two-sided incomplete information; Rubinstein (1985), a one-sided incomplete information model with infinite time horizon; Chatterjee and Samuelson (1987, 1988), where two-sided incomplete information models are analyzed with an infinite time horizon; and Cramton (1992), where the time between offers is not fixed and it is endogenously determined which agent makes the first offer.

The third strategic factor, the outside option, has been explored in different ways. While Shaked and Sutton (1984) as well as Binmore (1985) look at fixed, exogenous outside options in complete information settings, Bester (1988) considers a model with quality uncertainty where buyers can quit the bargaining partner and search for another seller to bargain with. Chikte and Deshmukh (1987) analyze how search ability affects a model where two bargainers who must search for alternatives if the current offer is rejected.

This paper analyzes an infinite horizon model of alternating-offers bargaining, where one agent (the buyer) can search for alternative offers which are non-negotiable. The buyer can choose to return to bargaining at any time, as long as the seller has not quit. The structure of the model follows Muthoo's (1995) approach of interlacing a bargaining process and a search process. Adding the option to return to bargaining is not trivial: It is not *a priori* evident whether equilibrium payoffs should be higher for the agent with a return option, since he can move between the bargaining and search process, or whether payoffs should be higher in a game without return option, since it might be strategically advantageous if the threat to opt out forever is automatically credible (Schelling, 1965). Muthoo shows that with *complete information* bargaining, the option to return is redundant, i.e. in equilibrium, a game with the option to return to bargaining and a game without such option are identical. The present paper introduces two-sided incomplete information bargaining and varying information about the outside option in Muthoo's model, which has important implications for the results.

Earlier papers that incorporate bargaining in a search model such as Mortensen (1982) (following the game theoretic approaches of Diamond and Maskin, 1979 and Mortensen, 1978) assume that agreements are instantaneous and the available surplus is divided equally according to Nash's axiomatic bargaining solution. This is also the approach by Baucells and Lippman (2004), who focus on the impact of the buyer's availability on agents' payoffs. Arnold and Lippman (1998) analyze a seller's choice between posting a price and bargaining in a model with incomplete information about buyers' valuations and their bargaining abilities.

As Rubinstein and Wolinsky (1985) remark, analyzing the bargaining problem with a strategic approach "constitutes an attempt to look into the bargaining black-box." In this sense, this paper is complementing the bargaining and search literature by characterizing a sequential equilibrium for an alternating-offers bargaining model with two-sided incomplete information, where the buyer has an outside option that allows him to search for alternative non-negotiable offers. Two regimes concerning the information about the outside option are considered: symmetric information, where both bargaining partners know the distribution parameters of the outside option, and asymmetric information, where the seller has only a probability distribution over the buyer's possible outside options. Search outcomes here are verifiable, in contrast to Chatterjee and Lee's (1998) two-period bargaining model with outside option for the buyer, where the search outcome is private information, but the reservation prices in the bargaining process are common knowledge.

The present bargaining model follows the two-sided incomplete information model by Chatterjee and Samuelson (1987) with two possible types for each player and offers that are restricted to "high price" and "low price." The allocation of the gains from trade is therefore reduced to the question of *who* gets the high surplus. As Chatterjee and Samuelson (1987) show, this game has a unique sequential equilibrium and bargaining proceeds only for a finite but endogenously determined number of periods. The restriction of the offers might seem quite strong in its implications, however, in a later paper, Chatterjee and Samuelson (1988) examine the model without restrictions. There exist multiple equilibria, but only one of them has plausible assumptions about updating beliefs. This equilibrium shares the features of the restricted-offers case. The authors emphasize that the multitude of equilibria does not alter the model's qualitative results and implications for bargaining. In view of these findings, the restricted-offers case shall be considered in this paper in order to simplify the analysis while retaining the important aspects of the model. The results presented here are thus also a direct extension of the Chatterjee and Samuelson analysis by introducing an outside option into their model.

The paper is organized as follows. In Section 2, the model is set up. Section 3 analyzes the bargaining equilibrium with a fixed outside option. In Section 4, the outside option is explicitly modeled as a sequential search process, and the equilibrium is characterized in two regimes: In Section 4.1 the case where both players have symmetric information about the outside option is considered. In Section 4.2 only the agent with an outside option knows the search parameters while his bargaining partner's information contains only a probability distribution over the possible qualities of the outside option. Section 5 concludes.

2. The bargaining-search model

Seller S and buyer B bargain over the price of an indivisible good. They are imperfectly informed about each other's valuation for the good. Each agent can be one of two possible types: B can have a high valuation v^h, or a low valuation v^l, and S can have a high cost c^h or a low cost c^l. Let $c^l \leqslant v^l < c^h \leqslant v^h$. At time 0, the buyer's prior belief that he faces a low-cost seller is π_S^0, and the seller's prior belief that he faces a high-valuation buyer is π_B^0. The priors π_S^0 and π_B^0 are assumed to be given exogenously and are common knowledge. Agents update their beliefs according to Bayes' rule.

Bargaining between B and S proceeds as follows: In each round, S can demand a price p_S or quit bargaining. In the same round, B responds with one of three choices: he can offer $p_B \geqslant p_S$, or offer $p_B < p_S$, or opt out. The outside option is to buy via search. Search is sequential and modeled in the following way: non-negotiable offers y arrive according to a Poisson process with arrival rate $\lambda > 0$ and $E[y] < \infty$. The magnitudes of the outside options located by B are common knowledge. The time interval between successive arrivals of offers is distributed exponentially. Notice that while time is discrete during bargaining, a continuous-time model is used to describe search.

The structure of the bargaining-search game is illustrated in Fig. 1. Let G ("game") denote the move-structure of a subgame starting in the bargaining phase and N ("nature") denote the move-structure of a subgame starting in the search phase.[1] The game begins at time 0 where S

[1] The term "subgame" shall be used in a non-standard way as in Cramton (1992). It would be a proper subgame if it were not for the incomplete information about agents' valuations.

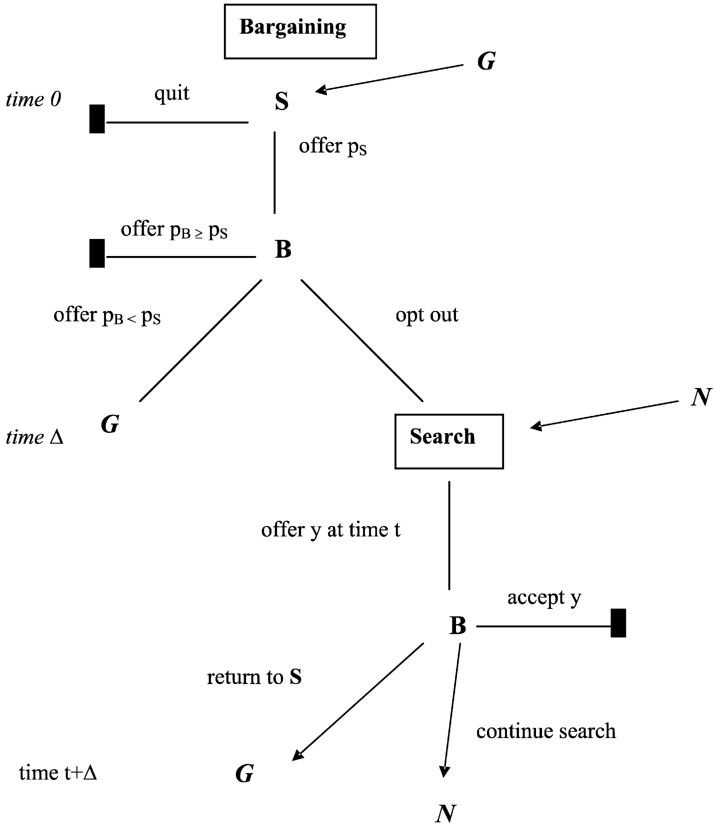

Fig. 1. The bargaining-search game.

makes an offer to B. In any round of G, B immediately responds to S's demand p_S. If B offers $p_B \geqslant p_S$, the good is traded for p_B and the game is over.[2] If B offers $p_B < p_S$, bargaining proceeds to the next round, which takes Δ time units and another subgame with identical move-structure G starts. If B opts out, the game proceeds to the subgame with move-structure N, in which B searches until he locates an outside offer. During this time, there are no decisions to be made by S. When B locates an outside offer y at time t, he has to choose between one of three options: he can accept y and the game is over, or he can continue search and another (identical) subgame N starts, or he can return to bargaining with S, which takes Δ time units and brings him back to G.

The payoffs to the players are now described. Let $r > 0$ be the common rate of time preference. For notational convenience, let the discount factor $\mathrm{e}^{-r\Delta}$ be denoted by δ. If the game is terminated by an agreement between B and S over a price p at time $\Delta T + t$, B receives $(v^i - p)\delta^T \mathrm{e}^{-rt}$ and S receives $(p - c^i)\delta^T \mathrm{e}^{-rt}$, depending on the type $i = h, l$. If the game is terminated by B accepting an outside offer y at time $\Delta T + t$, a buyer of type i receives a payoff

[2] Note that even though B is given an unrestricted set of possibilities in his response to S's offer, we can ignore the case where $p_B > p_S$ since a rational B would never offer a price that is higher than the offer he received.

of $(v^i - y)\delta^T e^{-rt}$, and S receives zero. In case B and S perpetually disagree or if B searches forever, the payoffs are zero for each player.

Two tie-breaking assumptions shall be made:

(A1) In case of indifference between certain payoffs, agents prefer agreement to disagreement.

(A2) In case of indifference between two payoffs, where one gives M with certainty while the other gives an expected payoff of M, players prefer the former choice.

(A1) and (A2) will be used to focus on an equilibrium in which B does not randomize between accepting the bargaining offer and taking the outside option when these actions yield equal payoffs, but strictly prefers the former. Note that (A1) and (A2) will apply for a set of games of measure zero. The assumptions are related to the fact that only restricted offers are considered in this model. Consider the model without assumptions (A1) and (A2). Suppose the buyer is indifferent between taking the outside option and accepting S's high price offer. If offers were not restricted, a low-cost S could lower his offer by some ϵ, thus removing indifference on B's side. Then the two seller types would separate, since lowering the offer is not individually rational for a high-cost S. In a subgame with the latter, B would then randomize between accepting S's offer and taking the outside option. Note that the high-cost S is bound to offer c^h, thus the type of the seller is known and there is no room for strategic actions. In the subgame with a low-cost S, B might now speculate on even lower offers from S's side, and the game will have multiple equilibria, including those with and without commitment (see, e.g. Sobel and Takahashi, 1983). Since the outside option does not yield additional insights into these cases, and they are discussed already in Chatterjee and Samuelson (1987, 1988), we use (A1) and (A2) to remain within the framework that focuses on novel strategic implications of the outside option.

3. The bargaining equilibrium with a fixed outside option

To simplify the analysis, this section will focus on pure bargaining behavior with a fixed outside option. The latter shall be viewed as a guaranteed and non-negotiable price p at which a buyer can purchase the good at any time. Note that the surplus for the two buyer types from accepting a given outside option differs. If $p < v^l$, both buyer types will always take the outside option, therefore we will restrict our attention to outside options with $p \geqslant v^l$. Both agents shall have the option to quit the bargaining process with a zero payoff. Offers are restricted to the set $\{v^l, c^h\}$, that is, there are only two possible choices for each agent: a *low price offer* v^l and a *high price offer* c^h.[3]

To simplify notation, let n_S denote the first round where the seller demands the low price v^l (i.e. a flexible S reveals his type), and let n_B denote the first round where the buyer offers the high price c^h or takes the outside option. Then a pure strategy for the flexible seller consists of a decision to demand the high price for $n_S - 1$ rounds and to demand the low price in round n_S. Similarly, a pure strategy for the flexible buyer consists of a decision to offer the low price for $n_B - 1$ rounds and to offer the high price or take the outside option in round n_B. As Chatterjee and Samuelson (1987) note, avoiding to specify the actions a flexible seller takes after n_S is not

[3] Note that this is equivalent to a process of one-sided offers, where S makes an offer and B just responds by accepting, rejecting or taking the outside option.

problematic, since the game will not proceed past n_S. n_S and n_B thus give information about the endpoint of bargaining.

The fixed outside option in this section is known to both agents. Since the high-valuation buyer and the low-cost seller can reach a positive payoff by agreeing on either price, i.e., they can trade with either type of opponent, they shall be called flexible agents. The best a low-valuation buyer and a high-cost seller can do is offer their true valuation, they shall be called inflexible agents. When two inflexible types are matched, mutually beneficial trade is not possible. If the outside option is $p = v^l$, the inflexible buyer is indifferent between quitting the game and taking the outside option. If $p > v^l$, he will quit the game as soon as he concludes that his opponent is also inflexible.

Furthermore, let $\{q_S^i\}$ be a seller's mixed strategy, such that q_S^i is the probability that he plays a pure strategy with $n_S = i$. Let $\{q_B^i\}$ be a buyer's mixed strategy, such that q_B^i is the probability that he plays a pure strategy with $n_B = i$, where $\sum_{i=1}^{\infty} q_S^i = 1$ and $\sum_{i=1}^{\infty} q_B^i = 1$. Since the flexible buyer can choose to take the outside option, he can ensure himself $v^h - v^l$ in each round if he has been demanded the low price and $\max\{v^h - p, v^h - c^h\}$ if he has been demanded the high price. Let $\mathcal{E}_B^T$ denote the expected payoff to a flexible buyer from playing a pure strategy $n_B = T$:

$$\mathcal{E}_B^T = \sum_{i=1}^{T} \pi_S q_S^i (v^h - v^l) \delta^{i-1} + \left[\pi_S \left(1 - \sum_{i=1}^{T} q_S^i\right) + (1 - \pi_S)\right] \max\{v^h - p, v^h - c^h\} \delta^{T-1}. \tag{1}$$

Let $\mathcal{V}_B^T = \sum_{i=T}^{\infty} q_B^i \mathcal{E}_B^i$ be the expected payoff to a buyer from the remainder of the game, given round T has been reached and no flexible agent has revealed his type so far. Then $\mathcal{V}_B^1 = \sum_{i=1}^{\infty} q_B^i \mathcal{E}_B^i$ is the flexible buyer's present value in round 1 if he plays mixed strategy $\{q_B^i\}$. The seller's $\mathcal{E}_S^T$ and $\mathcal{V}_S^T$ are defined analogously.[4]

Then a sequential equilibrium in the bargaining game with outside options consists of a pair of mixed strategies $\{q_S^{i*}\}$ and $\{q_B^{i*}\}$ such that

$$\mathcal{V}_S^T(\{q_S^{i*}\}, \{q_B^{i*}\}) \geqslant \mathcal{V}_S^T(\{q_S^i\}, \{q_B^{i*}\})\ \forall \{q_S^i\}, \tag{2}$$

$$\mathcal{V}_B^T(\{q_S^{i*}\}, \{q_B^{i*}\}) \geqslant \mathcal{V}_B^T(\{q_S^{i*}\}, \{q_B^i\})\ \forall \{q_B^i\} \tag{3}$$

for all T and for consistent beliefs π_B^T, π_S^T, which are updated according to Bayes' rule.

3.1. Pure strategies

If pure strategies are played, either $n_S > n_B$ or $n_S \leqslant n_B$.

Proposition 1. *In the bargaining game with a fixed outside option, a Nash equilibrium with $n_S = 1$ and $n_B = 1$ exists if $p < c^h$, or if the seller has a sufficiently high prior belief that the buyer is inflexible ($\pi_B^0 \leqslant \bar{\pi}_B$). A Nash equilibrium with $n_S = 2$ and $n_B = 1$ exists if $p \geqslant c^h$ and $\pi_B^0 > \bar{\pi}_B$ and $\pi_S^0 \leqslant \bar{\pi}_S$.*

[4] The obvious dependency of $\mathcal{E}_B^i$, $\mathcal{E}_S^i$, $\mathcal{V}_B^i$ and $\mathcal{V}_S^i$ on q_B^i and q_S^i, respectively, is omitted for simplicity.

Proof. Clearly, if $p < c^h$, a flexible buyer would rather take the outside option than accept the high price c^h at any time during bargaining. Thus, $n_S \leqslant n_B$, and following the arguments of Chatterjee and Samuelson (1987), we have $n_S = 1$, since delay is costly. Both types of buyer accept $p_S = v^l$ in round 1. On the other hand, if $p \geqslant c^h$, the outside option does not provide any additional gains for either type of buyer, and the bargaining equilibrium is as described in Chatterjee and Samuelson (1987): $n_S = 1$ is a best response to $n_B = 1$ if demanding the low price v^l and receiving $v^l - c^l$ in round 1 is better for S than demanding the high price c^h, when B will offer c^h in round 1 with probability π_B^0, and with probability $1 - \pi_B^0$ the game continues to the next round:

$$\pi_B^0\big(c^h - c^l\big) + \delta\big(1 - \pi_B^0\big)\big(v^l - c^l\big) \leqslant v^l - c^l \tag{4}$$

which gives a boundary value for π_B^0:

$$\pi_B^0 \leqslant \frac{(v^l - c^l)(1-\delta)}{c^h - c^l - \delta(v^l - c^l)} \equiv \bar{\pi}_B. \tag{5}$$

Given $n_S = 1$, a buyer will infer from an offer of c^h that he faces an inflexible seller. Then a flexible buyer accepts the high price. An inflexible buyer quits.

An equilibrium with $n_S > 1$ can thus only exist if $p \geqslant c^h$ and the seller's belief that the buyer is flexible is sufficiently high ($\pi_B^0 > \bar{\pi}_B$). Given $n_S > 1$, B will reveal his type before S if

$$\delta\pi_S^0\big(v^h - v^l\big) + \delta\big(1 - \pi_S^0\big)\big(v^h - c^h\big) \leqslant v^h - c^h, \tag{6}$$

that is, if B's prior that S is flexible is sufficiently low:

$$\pi_S \leqslant \frac{(v^h - c^h)(1-\delta)}{\delta(c^h - v^l)} \equiv \bar{\pi}_S. \tag{7}$$

Since delay is costly, this occurs in round 1, i.e. $n_B = 1$. Then $n_S = 2$ is a best response, since S must conclude that he faces an inflexible buyer if he is offered the low price in round 1. Note that assumption (A1) is used to establish that if $p = c^h$, B will choose the agreement on the high price rather than take the outside option. □

3.2. Mixed strategy equilibrium

In the Chatterjee and Samuelson model without outside option, there exists a mixed strategy equilibrium if $\pi_B^0 > \bar{\pi}_B$ and $\pi_S^0 > \bar{\pi}_S$. Because a flexible agent will never continue the game indefinitely, there exists some round T beyond which a flexible agent will not proceed. He will immediately make the offer that gives him the low surplus if he concludes that the opponent is inflexible, which happens at the latest in round T. This implies that the potentially infinite-horizon game must have a finite horizon if at least one of the players is flexible. Each randomization is determined such that the previous agent is indifferent between the revealing and concealing offer, which supports the previous agent's randomization. Thus, we get a sequence of probabilities until the first round T where the probabilities exceed one. This identifies the round T by which a flexible agent has stopped in equilibrium, where T is determined endogenously.

The question in the present model is how the outside option changes this reasoning.

Proposition 2. *A mixed strategy equilibrium exists if and only if $p > c^h$, $\pi_B > \bar{\pi}_B$ and $\pi_S > \bar{\pi}_S$, and is constructed as in the game without outside options.*

Proof. Note first that in a mixed strategy equilibrium with $q_B^i > 0$ we have $\mathcal{E}_B^i = \mathcal{E}_B^{i-1}$, otherwise a flexible buyer would not be indifferent between revealing his type and continuing the randomization. Thus

$$\begin{aligned}\mathcal{V}_B^1 &= \sum_{i=1}^{\infty} q_B^i \mathcal{E}_B^i = \mathcal{E}_B^1 \sum_{i=1}^{\infty} q_B^i = \mathcal{E}_B^1 \\ &= \pi_S q_S^1 \big(v^h - v^l\big) + \big(1 - \pi_S q_S^1\big) \max\big\{v^h - p, v^h - c^h\big\}. \end{aligned} \tag{8}$$

To show why a mixed strategy equilibrium with $p < c^h$ cannot exist, suppose first that $q_S^1 > 0$, i.e. S starts randomizing in round 1. B's expected payoff from playing $n_B = 1$ is $\mathcal{E}_B^1 = \pi_S q_S^1(v^h - v^l) + (1 - \pi_S q_S^1)(v^h - p)$. Clearly, the flexible buyer would be better off entering the randomization than taking the outside option, since $\forall q_S^1 > 0$ we have $\mathcal{V}_B^1 = \mathcal{E}_B^1 > v^h - p$.[5] If a mixed strategy equilibrium exists, there must be a q_B^1 such that $\mathcal{E}_S^1 \leqslant \mathcal{E}_S^2$, otherwise it would be better for S to reveal his type in round 1. Since S gets zero with probability $\pi_B q_B^1$ (when B takes the outside option), we have $\mathcal{E}_S^1 = v^l - c^l$ and $\mathcal{E}_S^2 = [\pi_B(1 - q_B^1) + 1 - \pi_B](v^l - c^l)\delta$. For any $q_B^1 \geqslant 0$ we have $\mathcal{E}_S^1 > \mathcal{E}_S^2$, therefore the flexible seller prefers to reveal his type in round 1, and a mixed strategy equilibrium with $q_S^1 > 0$ does not exist.

Now suppose $q_S^1 = 0$, and the buyer starts randomizing first. Then the flexible buyer would have to be indifferent in each round between choosing the outside option and continuing the randomization, with an expected payoff of $v^h - p$, i.e. $\mathcal{V}_B^1 = \sum_{t=1}^{\infty} q_B^T \mathcal{E}_B^T = \mathcal{E}_B^1 = v^h - p$. The seller, on the other hand, can get at most $v^l - c^l$ by revealing his type, since otherwise the buyer takes the outside option, leaving him with a payoff of zero. Again, the seller would prefer to get $v^l - c^l$ as early as possible, since delay is costly. Therefore, $q_S^T = 0$ cannot be part of a flexible seller's mixed equilibrium strategy for any T.

When $p = c^h$, the result relies on assumption (A1) that in case of equality of payoffs, B prefers an agreement to disagreement, and the case is treated like $p > c^h$. With $p > c^h$, the outside option is not relevant for bargaining behavior and the mixed strategy equilibrium is thus constructed as in the Chatterjee and Samuelson (1987) model. □

With $p < c^h$, a flexible buyer will never accept the high price. The only offer he will accept from the seller is v^l and since delay is costly, the flexible seller can do no better than reveal his type in round 1 and demand the low price. The good outside option makes it possible for the buyer to receive the full gains from trade in the first round, independent of π_B and π_S. This equilibrium is a sequential equilibrium and unique, which follows from the arguments in Chatterjee and Samuelson (1987). We have another no-delay result:

Corollary 1. *In equilibrium, the outside option is never taken after round* 1.

Proof. This follows directly from Propositions 1 and 2. With pure strategies, the game with at least one flexible agent ends in round 1. An inflexible buyer would only take the outside option if $p < v^l$, but then so would the flexible buyer, and the game ends in round 1. If $p > v^l$, the inflexible buyer prefers quitting to taking the outside option (which happens in round 2). By Proposition 2, a mixed-strategy equilibrium exists only if $p > c^h$. If T_B is reached and no agent

[5] The inflexible buyer would also prefer to remain in the bargaining process as long as there is a chance that he faces a flexible seller.

has revealed his type yet, the buyer will prefer to offer the high price in T_B to taking the outside option, since $\delta^T(v^h - p^h) < \delta^T(v^h - c^h)$. The outside option thus causes no delay. □

4. The bargaining-search equilibrium

In this section, the outside option is modeled as a sequential search process, as described in the setup of the bargaining-search model in Section 2. Assuming that outside offers are not always available, these non-negotiable price offers y shall be modeled as a Poisson process with a given arrival rate $\lambda > 0$ and a cumulative distribution function $F(y)$, whose support is on the open interval $(0, \infty)$. Payoffs are discounted at the continuous time rate $r > 0$, reflecting the cost of search. At any point during search, the buyer can choose to return to bargaining. The last offer y he found during search remains available when he returns to bargaining (i.e. it is verifiable). Attention shall be restricted to pure strategies in this part of the paper.

Following the standard approach for a sequential search model without an option to leave the search process (e.g. Lippman and McCall, 1976, 1981), a buyer is said to follow a reservation price policy with reservation price p, if he accepts any offer which is at most p and rejects all offers above p. To find the optimal reservation prices for the two types of buyers, let the expected return from search to the buyer of type $i = h, l$ associated with the reservation price p be $R^i(p)$. Since offers higher than p arrive with probability $1 - F(p)$, the expected return from search for type $i = h, l$ can be written as

$$\begin{aligned} R^i(p) &= \frac{\lambda}{r+\lambda}\left[R^i(p)\big(1 - F(p)\big) + \int_0^p (v^i - y)\,\mathrm{d}F(y)\right] \\ &= \frac{\lambda}{r+\lambda F(p)} \int_0^p \big(v^i - y\big)\,\mathrm{d}F(y). \end{aligned} \tag{9}$$

Then the gain $R^i(p) - (v^i - p)$ for a buyer of type i from following the reservation price policy vis-à-vis having an offer of p and accepting it is given by

$$R^i(p) - \big(v^i - p\big) = \frac{\lambda}{r+\lambda F(p)} \int_0^p (p - y)\,\mathrm{d}F(y) - \frac{r(v^i - p)}{r + \lambda F(p)}. \tag{10}$$

Denoting $H = \int_0^p (p - y)\,\mathrm{d}F(y)$, we have

$$R^i(p) - \big(v^i - p\big) = \frac{\lambda H - r(v^i - p)}{r + \lambda F(p)}. \tag{11}$$

The optimal reservation price p^{*i} is then the unique solution to $\lambda H - r(v^i - p) = 0$ and $R^i(p) = v^i - p^{*i}$ (see e.g. Arnold and Lippman, 1998).

Since the reservation price p^{*i} is increasing in the buyer's valuation,[6] if a flexible buyer has an outside option with $p^{*h} < v^l$, then the inflexible buyer's reservation price must be characterized by $p^{*l} < p^{*h} < v^l$, thus search would be the best option for both types, independent of the

[6] Burdett (1981) showed that a right translation of a wage offer distribution increases the reservation wage by less than the amount of the translation.

seller's demand. In order to restrict our attention to two-sided incomplete information, it shall be assumed that the inflexible buyer's return from search $v^l - p^{*l}$ is at most zero.[7] This implies that $p^{*h} > v^l$, i.e. the flexible buyer would prefer to buy the good from S for the low price to opting out. We thus exclude a pure search process with expected return $v^h - p^{*h}$ in which the seller's demand does not influence the flexible buyer's decision to opt out. Since the results are driven by the flexible buyer's outside option, we can just consider $p^* = p^{*h}$ in what follows.

The buyer's option to quit the game is now replaced by the possibility to search forever. Search incurs no fixed cost, thus the payoff from searching forever is zero. Varying information about the outside option will be considered in the following two sections.

4.1. Regime I: symmetric information about the outside option

By symmetric information about the outside option we mean that both the seller and the buyer know the parameters of the distribution of the outside offers. Then the buyer's expected return from starting search is a credible outside option. This expected payoff also includes the option to return to bargaining (RTB), and is thus different from a fixed outside option as well as from a standard search problem.

In the bargaining-search game as described in Fig. 1, let $M_N^{\Delta T+t}$ be the maximum expected payoff to a flexible buyer with belief π_S^T when he chooses to search (start subgame N) after having bargained for T rounds and having spent time t for search, and when both players behave optimally thereafter. Let $M_G^{\Delta T+t}$ be the maximum expected payoff to a flexible buyer when he chooses to RTB (start subgame G) at time $\Delta T + t$ and both players behave optimally thereafter. $M_N^{\Delta T+t}$ and $M_G^{\Delta T+t}$ are given by

$$M_N^{\Delta T+t} = \int_0^\infty \left[\mathrm{e}^{-rt} \int_0^\infty \max\left\{\delta M_G^{\Delta T+t}, M_N^{\Delta T+t}, v^h - y\right\} \mathrm{d}F(y) \right] \lambda \mathrm{e}^{-\lambda t} \,\mathrm{d}t, \tag{12}$$

$$M_G^{\Delta T+t} = \pi_S^T \left(v^h - v^l\right) + \left(1 - \pi_S^T\right) \max\left\{M_N^{\Delta T+t}, v^h - y\right\}. \tag{13}$$

Note that during search, B does not update his belief about the type of S, thus the variables depend on B's belief π_S^T after T rounds of bargaining, independent of how much time was spend during search. The payoff from returning to bargaining, $M_G^{\Delta T+t}$, is delayed by Δ units of time.

B switches from the bargaining to the search process by opting out, he does so if search offers a higher expected payoff than the current payoff from bargaining. If S offers the high price c^h, any price from search $y < c^h$ would be better. Once search is started, the optimal search policy from a standard sequential search problem prescribes that any $y \leqslant p^*$ should be accepted and any $y > p^*$ rejected, where p^* is the optimal reservation price. However, since return to bargaining is possible, the buyer could also just search in order to find a sufficiently low price not too far

[7] Gantner (2003) analyzed a fixed outside option game where the two buyer types have access to different outside options. When $p^{*l} < v^l < c^h < p^{*h}$, the game is then one of one-sided incomplete information, since only a flexible buyer remains in the bargaining process. It is shown that with pure strategies, any $n_S \geqslant N$, $n_B = 1$ are equilibrium strategies, where N is the smallest integer satisfying condition $\pi_S < \frac{(v^h - c^h)(1-\delta^{N-1})}{\delta^{N-1}(c^h - v^l)}$. A higher initial belief π_S pushes N to later rounds, but allows for multiple equilibrium strategies for the seller. The equilibrium *outcome*, however, is unique; the game ends in the first round with S getting the high surplus from playing with a flexible buyer. This allocation occurs also in a mixed-strategy equilibrium, where, in expectation, B (whose type is revealed), concedes before S in the spirit of Ordover and Rubinstein (1986).

into the future, so that a return to S is worthwhile. In the latter case B may violate the optimal search policy with reservation price p^*, since his intention is just to search for a credible threat.

In the following, the optimal strategies for both types of subgames N and G are described before the solution to the entire problem is characterized.

Lemma 1. *A flexible buyer's equilibrium strategy prescribes to return to bargaining at most once to an unknown type of seller, and only if he located offer $y < c^h$ from search. Upon RTB, B accepts if offered the low price from S, otherwise B follows the static optimal search policy, i.e. he accepts any $y \leqslant p^*$ and continues search if $y > p^*$. B updates his belief to $\pi_S^{T+1} = 1$ when he faces the low price upon RTB, and $\pi_S^{T+1} = 0$ when he faces the high price upon RTB at time $\Delta T + t$. Upon RTB, a flexible seller's equilibrium strategy prescribes to offer the low price if and only if $y < c^h$.*

Proof. If B considers to choose RTB, he must have opted out and hence, by Proposition 1, $\pi_B^0 > \bar{\pi}_B$. Suppose the number of times B chooses RTB is infinite, i.e. we have a cycle of switching between the bargaining and search process. But then B must eventually find an offer y from search at time $\Delta T + t$ with $v^h - y > \delta M_G^{\Delta T+t}$, and $v^h - y > M_N^{\Delta T+t}$, so that he would prefer to accept y rather than continuing the game. Thus the number of RTB must be finite.

Now suppose B chooses RTB n times, with $1 \leqslant n < \infty$. Then the nth time B chooses RTB at time $\Delta T + t$, it must be that $\delta M_G^{\Delta T+t} > M_N^{\Delta T+t}$, i.e RTB is better than continuing search. Upon the nth RTB, B chooses between accepting S's offer, taking the outside offer y, or returning to the search process, i.e. $M_G^{\Delta(T+1)+t} = \max\{v^h - p_S, v^h - y, M_N^{\Delta(T+1)+t}\}$. Since returning to the search process now does not include RTB anymore, its expected payoff is just $M_N^{\Delta(T+1)+t} = v^h - p^*$. The expected payoff of RTB at time $\Delta(T+1)+t$ thus depends on S's offer p_S, the optimal reservation price p^* and the offer y located from search. Suppose $p^* < c^h$. Because B would never accept the high price c^h from S, but rather follow the static optimal stopping rule from search, a flexible S should always offer the low price upon RTB and any time he is called to play (in particular in round 1). Now suppose $p^* \geqslant c^h$. Then using Proposition 1, a necessary and sufficient condition for the flexible seller to offer the low price is that the buyer returns with an offer $y < c^h$. The buyer, on the other hand, has no reason to return a second time once the low price has been offered, since all bargaining and search parameters are known after his return and delay is costly. Thus the number of RTB is at most one. Note that assumption (A2) is used to establish that if $c^h = p^*$, B will choose the agreement with S on the high price rather than continuing search. Assumption (A1) is used to establish that if $y = c^h$, B will choose the agreement with S over taking the outside offer y.

Then B must update his beliefs in the following way: if he chooses RTB at time $\Delta T + t$ and is offered the low price v^l, then $\pi_B^{T+1} = 1$, i.e. the low price offer must come from a flexible seller. If B is offered the high price c^h upon return, then $\pi_B^{T+1} = 0$. Upon RTB, B follows the static optimal search policy in the following sense: he chooses the smaller offer between y and the seller's offer and accepts if this is less or equal to p^*, and he continues search if both the seller's offer and y are greater than p^*. □

Lemma 2. *During the search phase, a flexible buyer's equilibrium strategy prescribes to choose RTB to an unknown type of seller if and only if $y < c^h$ and $\min\{y, p^*\} > \frac{v^h(1-\delta)+\delta\pi_S^T v^l}{1-\delta(1-\pi_S^T)} \equiv \bar{y}^T$. Otherwise, he follows the static optimal search policy, i.e. he accepts y if $y \leqslant p^*$ and continues search if $y > p^*$.*

Proof. Consider the search phase of a game where the seller has always offered the high price so far. B's expected return from the search phase is given by (12) and (13). Suppose B located offer y at time $\Delta T + t$. For RTB to be optimal, we must have (i) $\delta M_G^{\Delta T+t} > M_N^{\Delta T+t}$ and (ii) $\delta M_G^{\Delta T+t} > v^h - y$. From Lemma 1, we know that $y < c^h$ is a necessary condition to reveal the seller's type, but (i) and (ii) require that the offer y from search is also sufficiently high, so that B has no incentive to accept it without choosing RTB first. To see this, consider the expected payoff for a flexible buyer from RTB after he found an offer $y < c^h$ during search at time $\Delta T + t$ as given in (13). Suppose, first, that in (13) we have $M_N^{\Delta T+t} > v^h - y$. Since by Lemma 1 RTB is chosen at most once, the expected return from search is just the return from following the static optimal reservation price policy: $M_N^{\Delta T+t} = v^h - p^*$. Then using the assumption $M_N^{\Delta T+t} > v^h - y$ and the necessary condition for RTB, $y < c^h$, we have $p^* < y < c^h$. Thus, if the buyer returns to bargaining and faces the high price c^h again, it would be more profitable for him to continue search following the static reservation price policy rather than accepting the current outside offer y. Given that in this case a flexible seller would offer the low price, the condition that the expected value of RTB is higher than the expected value of search is then

$$\delta M_G^{\Delta T+t} = \delta\big[\pi_S^T\big(v^h - v^l\big) + \big(1-\pi_S^T\big)\big(v^h - p^*\big)\big] > v^h - p^* \tag{14}$$

or

$$p^* > \frac{v^h(1-\delta) + \delta\pi_S^T v^l}{1-\delta(1-\pi_S^T)} \equiv \bar{y}^T. \tag{15}$$

This condition can also be expressed in beliefs: $\pi_S^T > \frac{(v^h-p^*)(1-\delta)}{\delta(p^*-v^l)} \equiv \bar{\pi}_S(y)$. It can easily be seen that the latter condition is synonymous to the derivation $\bar{\pi}_S$ from Proposition 1 in the game without search, just that here the relevant "offer at hand" is p^* rather than c^h.

Now suppose that in (13) we have $M_N^{\Delta T+t} \leqslant v^h - y$, or $p^* \geqslant y$. The expected value of RTB here is

$$\delta M_G^{\Delta T+t} = \delta\big[\pi_S^T\big(v^h - v^l\big) + \big(1-\pi_S^T\big)\big(v^h - y\big)\big] > v^h - y \tag{16}$$

for the case that RTB is optimal and $y \leqslant p^*$ was found. Also here, the outside offer y must not be too low: From (16) we get

$$y > \frac{v^h(1-\delta) + \delta\pi_S^T v^l}{1-\delta(1-\pi_S^T)} \equiv \bar{y}^T \tag{17}$$

or, expressed in beliefs, $\pi_S^T > \frac{(v^h-y)(1-\delta)}{\delta(y-v^l)} > \bar{\pi}_S \equiv \bar{\pi}_S(p^*)$. Together, (15) and (17) give the result stated in the lemma: RTB is optimal if and only if $y < c^h$ and $\bar{y}^T < \min\{p^*, y\}$. Notice that $\bar{\pi}_S(y) > \bar{\pi}_S$ and also $\bar{\pi}_S(p^*) > \bar{\pi}_S$, i.e. the critical value of the buyer's belief that makes it worthwhile returning to S is higher in the game with search, or the set of possible parameters for which bargaining is continued is smaller. Thus B proceeds in the following way during search: After locating an offer $y < c^h$ at time $\Delta T + t$, this offer has to be compared to p^*. For all $y > p^*$ RTB is optimal if $p^* > \bar{y}^T$ and continuing search is optimal if $p^* \leqslant \bar{y}^T$. For all $y \leqslant p^*$, RTB is optimal if $y > \bar{y}^T$ and accepting y is optimal if $y \leqslant \bar{y}^T$. □

Lemma 3. *During the bargaining phase, a flexible buyer's equilibrium strategy prescribes to opt out at time ΔT if S demanded the high price and at least one of the following conditions hold*:

(i) $p^* < c^h$,
(ii) $\pi_S^T > \bar{\pi}_S$ and $\lambda > \frac{r(v^h - c^h)}{\int_0^{\bar{y}^T}(v^h - y)\,\mathrm{d}F(y) + \int_{\bar{y}^T}^{c^h}\delta M_G^{\Delta T+t}(y)\,\mathrm{d}F(y) - (v^h - c^h)F(c^h)} \equiv \lambda_{rtb}^T$ where $M_G^{\Delta T+t}(y)$ is given by

$$M_G^{\Delta T+t}(y) = \pi_S^T\left(v^h - v^l\right) + \left(1 - \pi_S^T\right)\left(v^h - y\right)$$

and $\bar{y}^T$ is as given in Lemma 2.

If none of these conditions hold, B accepts the high price from S if $\pi_S^T \leqslant \bar{\pi}_S$. A flexible seller offers the low price if $\pi_B^T \leqslant \bar{\pi}_B$, or if condition (i) *or* (ii) *holds, in which case beliefs are updated such that $\pi_S^T = 1$ if B is offered the low price and $\pi_S^T = 0$ if B is offered the high price. These strategies and beliefs form an equilibrium in a subgame starting in the bargaining phase.*

Proof. Conditions (i) and (ii) of Lemma 3 identify an outside option for B that is a credible threat when bargaining with S. Suppose both types of seller have demanded c^h and both types of buyer have offered v^l so far up to round T. Then $\pi_S^T = \pi_S^0$ and $\pi_B^T = \pi_B^0$. By Lemma 1, a flexible buyer returns at most once to the (unknown type of) seller.

Suppose $\pi_S^T = \pi_S^0 \leqslant \bar{\pi}_S$. From the proof of Lemma 2 we know that then RTB cannot be optimal (since $\bar{\pi}_S(y) > \bar{\pi}_S > \pi_S^T$ and $\bar{\pi}_S(p^*) > \bar{\pi}_S > \pi_S^T$). Then the expected return from search is just $v^h - p^*$, and opting out requires $p^* < c^h$, or in terms of the arrival rate, $\lambda > \lambda_{c^h}$.[8] But since the seller knows the search parameters, he would prefer to offer the low price in that case. Otherwise, if $p^* > c^h$ then B does not take the outside option, and the game proceeds as if there were no such option, i.e. by Proposition 1, we have an equilibrium with $n_S = 2$ and $n_B = 1$.

Now suppose $\pi_S^T = \pi_S^0 > \bar{\pi}_S$. Then a flexible buyer would opt out in round T if $v^h - c^h < M_N^{\Delta T+t}$, where $M_N^{\Delta T+t}$ is given by (12). The buyer would opt out either because search *per se* is profitable or because RTB is profitable. Suppose, first, that given B opts out, $\delta M_G^{\Delta T+t} < M_N^{\Delta T+t}$, i.e. RTB is not profitable. Then the expected return from taking the outside option is $v^h - p^*$, and the condition to opt out is again given by $p^* < c^h$. The latter is therefore a sufficient condition for revealing S's type independent of B's beliefs. In the case where $p^* > c^h$, we have $\delta M_G^{\Delta T+t} > M_N^{\Delta T+t}$, i.e. continuing search is not optimal at time $\Delta T + t$. Then from (12) we have

$$M_N^{\Delta T+t} = \frac{\lambda}{r+\lambda}\int_0^\infty \max\left\{v^h - y, \delta M_G^{\Delta T+t}\right\}\mathrm{d}F(y). \tag{18}$$

The value of $\delta M_G^{\Delta T+t}$ (and optimal behavior after facing the high price again upon RTB) depends on the offer y found at time $\Delta T + t$ compared to p^* and c^h. For $p^* < c^h$ ($\lambda > \lambda_{c^h}$), it

[8] One can easily check that if $p^* < c^h$, then the arrival rate that identifies p^* is greater than the required arrival rate that would make search for any $y < c^h$ profitable:

$$\lambda_{p^*} = \frac{(v^h - p^*)r}{\int_0^{p^*}(v^h - y)\,\mathrm{d}F(y) - F(p^*)(v^h - p^*)} > \frac{(v^h - c^h)r}{\int_0^{c^h}(v^h - y)\,\mathrm{d}F(y) - F(c^h)(v^h - c^h)} = \lambda_{c^h}.$$

Inspection of the denominator of λ_{c^h} shows that it can be rewritten as $\int_0^{p^*}(v^h - y)\,\mathrm{d}F(y) + \int_{p^*}^{c^h}(v^h - y)\,\mathrm{d}F(y) - [F(p^*)(v^h - c^h) + (F(c^h) - F(p^*))(v^h - c^h)]$, and this is greater than the denominator of the arrival rate for p^* if $\int_{p^*}^{c^h}(v^h - y)\,\mathrm{d}F(y) - [F(p^*) - F(c^h)](v^h - c^h) \geqslant 0$, which is always true since the integrand $v^h - y$ is never less than $v^h - c^h$.

was already shown that this is a sufficient condition for a flexible seller to demand the low price in round 1 of bargaining.

If $p^* \geqslant c^h$ then B would prefer to accept an offer $y < c^h$ from search rather than continue search, since $p^* \geqslant c^h > y$. Then the expected payoff from RTB is

$$\delta M_G^{\Delta T+t}(y) = \delta\left[\pi_S^T\left(v^h - v^l\right) + \left(1 - \pi_S^T\right)\left(v^h - y\right)\right] \tag{19}$$

and the expected value from opting out as given by (18) is

$$M_N^{\Delta T+t} = \frac{\lambda}{r + \lambda F(c^h)}\left[\int_0^{\bar{y}^T}\left(v^h - y\right)\mathrm{d}F(y) + \int_{\bar{y}^T}^{c^h}\delta M_G^{\Delta+t}(y)\,\mathrm{d}F(y)\right] \tag{20}$$

where $\bar{y}^T$ is as defined in Lemma 2. When the price a seller demands in the first round of bargaining is c^h, the buyer would then start search if the expected return from search $M_N^{\Delta T+t}$ as defined in (20) is greater than $v^h - c^h$. This gives a condition for the arrival rate:

$$\lambda > \frac{r(v^h - c^h)}{\int_0^{\bar{y}}(v^h - y)\,\mathrm{d}F(y) + \int_{\bar{y}}^{c^h}\delta M_G^{\Delta T+t}(y)\,\mathrm{d}F(y) - (v^h - c^h)F(c^h)} \equiv \lambda_{rtb}^{\Delta T}. \tag{21}$$

Given $\lambda > \lambda_{rtb}^{\Delta T}$, the threat to search for an outside offer $y < c^h$ is credible, even when $p^* \geqslant c^h$, as long as it is sufficiently likely to return to a flexible seller, i.e. $\pi_S^T > \bar{\pi}_S$. But since S knows the search parameter λ, a flexible seller would not wait for B to return, but he would offer the low price immediately.

By Proposition 1, $\pi_B^0 \leqslant \bar{\pi}_B$ is a sufficient condition for a flexible seller to offer the low price immediately. Otherwise, if it is optimal for a flexible seller to demand the low price at some T (given by condition (i) and (ii) in the lemma), B will update his prior such that $\pi_S^T = 0$ when he is offered a high price in round T. Then $\pi_S^T < \bar{\pi}_S$ and neither RTB nor continuing bargaining can be optimal. B opts out if $p^* < c^h$ and accepts the high price if $p^* \geqslant c^h$, thus a flexible seller's strategy of revealing his type is consistent with B's beliefs in round T. Note that assumption (A2) is used to establish that if $c^h = p^*$, B will choose the agreement with S on the high price rather than continuing search. □

Lemma 3 thus defines what a "good" outside option is, i.e. which parameters of the search process represent credible threats so that a flexible seller offers the low price. Then we just have to put these results together to get the equilibrium of the bargaining-search game.

Proposition 3. *The bargaining-search game with two-sided incomplete information and symmetric information about the outside option has a Nash equilibrium, in which two flexible types immediately agree on the low price v^l if at least one of the following conditions hold*:

(i) $p^* < c^h$,
(ii) $\pi_S^0 > \bar{\pi}_S$ *and* $\lambda > \lambda_{rtb}^0$,
(iii) $\pi_B^0 \leqslant \bar{\pi}_B$

where λ_{rtb}^0 is as given by Lemma 3 for $T = 0$. If none of the conditions (i)–(iii) *hold, then two flexible types immediately agree on the high price c^h if $\pi_S^0 \leqslant \bar{\pi}_S$. Otherwise, there is no equilibrium in pure strategies.*

Proof. Lemmas 1–3 give the equilibrium strategies for all the possible subgames. Since delay is costly, Lemma 3 holds in particular for the first bargaining round, i.e. for $T = 0$, and Proposition 3 follows. □

In Regime I, where both agents know the parameters of the distribution of outside offers, the description of the equilibrium also implies the following:

Corollary 2. *On the equilibrium path of the bargaining-search game with symmetric information about the outside option, the buyer never returns to bargaining.*

This follows directly from Lemma 1 and Proposition 3. This result coincides with Muthoo's (1995) result for a bargaining and search game with complete information. The threat to start search in order to change the seller's demand is never tested in equilibrium. There is, however, a difference between this result and Muthoo's. With complete information, the game with an option to move between the bargaining and search process is, in equilibrium, identical to one without such option. With incomplete information, this is not true anymore. The option to return to bargaining may help the flexible buyer to get the high surplus from bargaining even if search *per se* is not profitable. The condition $\pi_S^0 > \bar{\pi}_S$ together with $\lambda > \lambda_{rtb}^0$, which is sufficient for a good outside option in the sense of having a credible threat, would not be applicable.

4.2. Regime II: asymmetric information about the outside option

The previous section showed that it is never optimal to return to the bargaining table when both agents have symmetric information about the outside option. *In equilibrium*, agents in games with both complete and incomplete information should not be searching with the intention to return to bargaining. The key of this result lies in the stationary structure of the bargaining-search game. A flexible seller will never offer the low price *after* the buyer has opted out, but rather *before* he does so. When the expected value of search, including the chance to find an offer lower than c^h soon enough, provides an incentive for the buyer to start the search process, it is also a credible threat for the seller.

Now suppose that there is asymmetric information about the outside option. Only the buyer knows which distribution of outside offers he is facing. The seller believes that with some probability q the buyer has a good outside option, where search parameters satisfy condition (i) or (ii) of Lemma 3. In this case the seller would prefer to demand the low price immediately, irrespective of his prior about B's type. Intuitively, one might think that it does not matter whether the seller knows the exact parameters of the distribution when the outside option is good, i.e. it is not relevant whether $p^* < c^h$, or $\lambda > \lambda_{rtb}^0$, since in either case the flexible seller should reveal his type immediately (and thus the threat to opt out is never carried out). However, with asymmetric information where the seller is uncertain about the quality of the outside option, it turns out that his equilibrium strategy depends on the exact knowledge of the possible parameters of the outside option. That is, the additional uncertainty in the model is about whether the outside option is good or bad, while given that it is good, the seller is certain about its parameters.

Consider the flexible buyer's behavior in the search process. Using Lemma 2 we know that during the search phase, B would, in expectation, only return to the seller at time T if $p^* > \bar{y}^T$. This can be directly verified from the search parameters, and B thus knows if he opts out at time 0 with the intention to return to bargaining (if $p^* > \bar{y}^0$), or if he opts out with the intention to remain in the search process (if $p^* \leqslant \bar{y}^0$). If the latter is the case, the seller would receive zero by demanding the high price to such a buyer.

Then the seller will demand the low price immediately, if the surplus $v^l - c^l$ is higher than the expected payoff from demanding the high price to an unknown type of buyer with uncertain quality of outside offers. A flexible buyer has a bad outside option with probability $1 - q$. In this case, he would immediately accept the seller's high price demand if the seller is sufficiently likely to be inflexible ($\pi_S^0 < \bar{\pi}_S$), as shown in Proposition 1. With probability q, the flexible buyer has a good outside option and would opt out, but possibly return to the seller. The buyer returns in expectation if $p^* > \bar{y}^0$ and he found a sufficiently low offer. If he returns, the flexible seller would then demand the low price and get the low surplus $v^l - c^l$ with delay. The delay depends on how long the buyer has to search for a valid outside offer that is lower than c^h.

Suppose that the seller knows that B faces one of two possible distributions.[9] That is, S thinks that with probability q, B will face a good outside option, in which case he also knows the correct parameters of the distribution.

Proposition 4. *With asymmetric information about the outside option, let q be the seller's probability that B faces a good outside option as characterized by Lemma* 3. *Then*

(i) *for $p^* > \bar{y}^0$, S reveals his type if $q > \bar{q}^{rtb}$. If $q \leqslant \bar{q}^{rtb}$ then delay of an agreement between two flexible agents is expected in equilibrium.*
(ii) *for $p^* \leqslant \bar{y}^0$, S reveals his type if $q > \bar{q}^{\text{search}}$.*

Proof. Since the expected discounting to find an offer lower than c^h is given by $\frac{\lambda F(c^h)}{r+\lambda F(c^h)}$, the seller's expected return from demanding the high price to a flexible buyer with a good outside option and $p^* > \bar{y}^0$ is:

$$R_S^{rtb} = \frac{\lambda F(c^h)}{r + \lambda F(c^h)}\big[F\big(c^h\big) - F\big(\bar{y}^0\big)\big]\delta\big(v^l - c^h\big) \tag{22}$$

while if $p^* \leqslant \bar{y}^0$, the buyer opts out with the intention to remain in the search process. The seller's expected return from demanding the high price to a flexible buyer who opts out when $p^* < \bar{y}^0$ is zero.

When B would opt out expecting to renegotiate, the flexible seller will reveal his type immediately if

$$v^l - c^l \geqslant \pi_B\big[(1-q)\big(c^h - c^l\big) + qR_S^{rtb}\big] + (1-\pi_B)\delta\big(v^l - c^l\big) \tag{23}$$

where R_S^{rtb} denotes the seller's expected return when the buyer returns to bargaining as defined in (22). Note that (A2) is used to establish that S prefers an immediate agreement over a possible renegotiation which yields the same expected payoff. This condition gives a boundary value for the seller's prior q about the quality of B's outside option, where he is just indifferent between the revealing and concealing demand:

$$q \geqslant \frac{\pi_B(c^h - c^l) - (v^l - c^l)(1 - \delta(1-\pi_B))}{\pi_B(c^h - c^l - R_S^{rtb})} \equiv \bar{q}^{rtb}. \tag{24}$$

[9] One can think of this as a situation in which S knows his competitors, but he is not sure whether B can locate sufficiently many of those who have low posted prices.

In other words, if the seller thinks that the probability q with which the buyer faces a good outside option is at least $\bar{q}^{rtb}$, he should immediately demand the low price, given that with a good outside option the buyer would opt out with the intention to RTB ($p^* > \bar{y}^0$). Notice that as R_S^{rtb} increases, the required value of q^{rtb} increases, i.e. for given parameters, S is less likely to reveal his type. Similarly, for increasing π_B^0, $\bar{q}^{rtb}$ increases, and so does the seller's incentive to conceal his type.

Now suppose $p^* < \bar{y}^0$, i.e. with a good outside option the buyer opts out and will not return. Then the flexible seller should demand the low price in round 1 of the bargaining process if $\pi_B^0 < \bar{\pi}_B$ or if

$$v^l - c^l \geqslant \pi_B^0(1-q)\big(c^h - c^l\big) + \big(1-\pi_B^0\big)\delta\big(v^l - c^l\big) \tag{25}$$

which gives a boundary value for q:

$$q \geqslant \frac{\pi_B^0(c^h - c^l) - (v^l - c^l)(1 - \delta(1 - \pi_B^0))}{\pi_B^0(c^h - c^l)} \equiv \bar{q}^{\text{search}}. \tag{26}$$

The seller should immediately reveal his type if he thinks that the probability q with which the buyer faces a good outside option is at least $\bar{q}^{\text{search}}$, given that the buyer would opt out with the intention to search ($p^* \leqslant \bar{y}^0$). $\square$

Notice that $\bar{q}^{\text{search}} < \bar{q}^{rtb}$. This has an important implication for the seller in the model with asymmetric information. His equilibrium strategy depends not only on whether he thinks the buyer can find an outside offer less than c^h soon enough (as in the symmetric information case), but also on his knowledge about the parameters. If $\bar{q}^{rtb} > q > \bar{q}^{\text{search}}$, S would want to reveal his type only if he knew that $p^* < c^h$.

5. Conclusion

This paper studies a noncooperative bargaining model between a buyer and a seller, where both agents have incomplete information about the opponent's valuation for the good to be traded, and where the buyer's outside option is to buy via search. The model interlaces a standard sequential search process and a version of Rubinstein's alternating-offers bargaining process. Two different regimes for the search process are distinguished: *Regime I* with symmetric information about the outside option, where the expected return from search can be used as a credible outside option, and *Regime II* with asymmetric information about the outside option, where the seller believes that the buyer has a good outside option with a given probability q.

In *Regime I*, the threat to opt out is never carried out, if both agents are flexible. Contrary to Muthoo's (1995) result for a game with complete information, the option to return to the bargaining table is not redundant. It ensures that the buyer is offered the high surplus from trade even if search *per se* is not optimal ($p^* > c^h$). However, when two flexible agents bargain, search is never started to induce the bargaining partner to offer a lower price, but only with the intention to follow the optimal stopping rule as in a pure search process. Returning to bargaining is thus not optimal and hence there is no delay of trade.

In *Regime II*, where the seller has only a probability distribution for the buyer's ability to locate a good outside option, delay is possible in equilibrium. The seller's strategy depends on the intention with which the buyer would opt out, i.e. on the knowledge of the distribution parameters of a "good" outside option. The solution determines for which parameters the buyer opts out in order to locate an outside offer and to use it for renegotiation with the seller, or whether the buyer opts out with the intention to continue search according to the optimal search policy without ever returning to the seller.

This model provides a basic framework for a larger variety of bargaining-search problems. It is easy to think about other regimes in a similar setting that would be worth considering, for example if the player without outside option is available only temporarily. Another extension would be a situation where both players are incompletely informed about the distribution function of outside offers. In this case, search would involve learning and the equilibrium strategies would be non-stationary. An interesting application for this type of model might be in automated negotiation and e-commerce, where agents can be programmed to follow rather complex strategies.

Acknowledgments

I would like to thank Ted Bergstrom, Ken Binmore, John McCall, Chris Proulx, Cheng-Zhong Qin, Arthur Robson, Larry Samuelson, as well as two anonymous referees for helpful comments. Special thanks to Morten Bech for discussions and encouragement.

References

Arnold, M., Lippman, S., 1998. Posted prices versus bargaining in markets with asymmetric information. Econ. Inquiry 36, 450–457.

Baucells, M., Lippman, S., 2004. Bargaining with search as an outside option: The impact of the buyer's future availability. Dec. Anal. 1 (4), 235–249.

Bester, H., 1988. Bargaining, search cost and equilibrium price distributions. Rev. Econ. Stud. 55, 201–214.

Binmore, K., 1985. Bargaining and coalitions. In: Roth, A. (Ed.), Game-Theoretic Models of Bargaining. Cambridge Univ. Press, pp. 269–304.

Burdett, K., 1981. A useful restriction on the offer distribution in job search models. In: Eliasson, G., Holmlund, B., Stafford, F. (Eds.), Studies in Labor Market Behavior. IUI Conference Report, Stockholm, Sweden.

Chatterjee, K., Lee, C., 1998. Bargaining and search with incomplete information about outside options. Games Econ. Behav. 22, 203–237.

Chatterjee, K., Samuelson, L., 1987. Bargaining with two-sided incomplete information: An infinite horizon model with alternating offers. Rev. Econ. Stud. 54, 175–192.

Chatterjee, K., Samuelson, L., 1988. Bargaining under two-sided incomplete information: The unrestricted offers case. Operations Res. 36, 605–618.

Chikte, S., Deshmukh, S., 1987. The role of external search in bilateral bargaining. Operations Res. 35, 198–205.

Cramton, P., 1992. Strategic delay in bargaining with two-sided uncertainty. Rev. Econ. Stud. 59, 205–225.

Diamond, P., Maskin, E., 1979. An equilibrium analysis of search and breach of contract, I: Steady states. Bell J. Econ. 10, 282–316.

Fudenberg, D., Tirole, J., 1983. Sequential bargaining under incomplete information. Rev. Econ. Stud. 50, 221–248.

Gantner, A., 2003. Essays in bargaining, search and bounded rationality. Dissertation. University of California, Santa Barbara.

Lippman, S., McCall, J., 1976. The economics of job search: A survey. Econ. Inquiry 14, 155–189.

Lippman, S., McCall, J., 1981. The economics of uncertainty: Selected topics and probabilistic methods. In: Arrow, K., Intriligator, M. (Eds.), Handbook of Mathematical Economics. North-Holland, Amsterdam, pp. 211–284.

Mortensen, D., 1978. Specific capital and labor turnover. Bell J. Econ. 9, 572–586.

Mortensen, D., 1982. The matching process as a non-cooperative/bargaining game. In: McCall, J. (Ed.), The Economics of Information and Uncertainty. The Univ. of Chicago Press.

Muthoo, A., 1995. On the strategic role of outside options in bilateral bargaining. Operations Res. 43 (2), 292–297.

Ordover, J., Rubinstein, A., 1986. A sequential concession game with asymmetric information. Quart. J. Econ. 101, 97–108.

Rubinstein, A., 1982. Perfect equilibrium in a bargaining model. Econometrica 50, 97–108.

Rubinstein, A., 1985. A bargaining model under incomplete information. Econometrica 53, 1151–1172.

Rubinstein, A., Wolinsky, A., 1985. Equilibrium in a market with sequential bargaining. Econometrica 53, 1133–1150.

Schelling, T., 1965. The Strategy of Conflict. Harvard Univ. Press.
Shaked, A., Sutton, J., 1984. Involuntary unemployment as a perfect equilibrium in a bargaining model. Econometrica 52, 1351–1364.
Sobel, J., Takahashi, I., 1983. A multistage model of bargaining. Rev. Econ. Stud. 50, 97–108.