Here we explore what determines whether prices are higher/lower in a model with price uncertainty. The simplest model is that firms draw cost c_j from a distribution F_j and then make an offer p_j. The consumer then decides whether to accept one of the offers, their demand is given by $D_j(p)$.

Then firm prices satisfy:

$$p_j(c_j) = \arg\max_{p_j} \int_{\mathbb{R}^{N-1}} (p_j - c_j) D_j(p_j, p_{-j}(c_{-j})) dF(c_{-j}) \tag{1}$$

where $p_{-j}(c_{-j}) = (p_n(c_n))_{n \neq j}$ and $F(c_{-j})$ is the distribution of costs for the rival firms.

The FOC is:

$$\int \left[D_j(\cdot) - (p_j - c_j) \frac{\partial D_j(\cdot)}{\partial p_j} \right] dF(c_{-j}) = 0 \tag{2}$$

Define $\pi(p_j, p_{-j}, c_j) = (p_j - c_j) D_j(p_j, p_{-j})$ the profits of firm j given prices and costs. Then the FOC is:

$$\int \frac{\partial \pi(p_j, p_{-j}(c_{-j}), c_j)}{\partial p_j} dF(c_{-j}) = 0 \tag{3}$$

Assume that $\pi(p_j, p_{-j}, c_j)$ is concave in p_j[1]

there has to be a result that says that increasing the uncertainty over other firm costs increases or decreses the price.

0.1 Example: logit demand

With logit demand we have that profits are given by:

$$\pi(p_j, p_{-j}, c_j) = (p_j - c_j) \frac{e^{v_j - p_j}}{1 + \sum_k e^{v_k - p_k}} \tag{4}$$

and the FOC is:

$$D_j(p_j, p_{-j}) + (p_j - c_j) \frac{\partial D_j(p_j, p_{-j})}{\partial p_j} = 0 \tag{5}$$

since $\frac{\partial D_j(p_j, p_{-j})}{\partial p_j} = -D_j(p_j, p_{-j})(1 - D_j(p_j, p_{-j}))$ we have that:

$$D_j(p_j, p_{-j}) - (p_j - c_j) D_j(p_j, p_{-j})(1 - D_j(p_j, p_{-j})) = 0 \tag{6}$$

[1] Standard assumption, required for the FOC to provide the optimal price.

Without uncertainty without uncertainty, using equation 6 we have that:

$$1-(p_j-c_j)(1-D_j(p_j,p_{-j}))=0 \implies p_j(c_j)=c_j+\frac{1}{1-D_j(p_j,p_{-j})} \tag{7}$$

With uncertainty With uncertainty, using equation 6 we have that:

$$\int \left[1-(p_j-c_j)(1-D_j(p_j,p_{-j}(c_{-j})))\right] D_j(p_j,p_{-j}(c_{-j}))dF(c_{-j})=0 \tag{8}$$

$$\mathbb{E}\int \left[1-(p_j-c_j)(1-D_j(p_j,p_{-j}(c_{-j})))\right] D_j(p_j,p_{-j}(c_{-j}))dF(c_{-j})=0 \tag{9}$$

Symmetric case

To simplify the proof let's consider the symmetric duopoly.

Assume that a mean preserving spread of the cost distribution F leads to a higher expected price. Then, given that the

The previous cases where too complicated because we want to determine the effect of a mean preserving speread of the distribution of costs on the prices, but is not clear what is the effect of the mean preserving spread on the distribution of rival prices which is what enters directly the profits of the firm.