Literature on search and some comments

Models of search in **gavazza˙frictions˙2021<empty citation>**

1. Informed vs uninformed: Varian (1980). Also referred as to shoppers and non-shoppers.
2. Simultaneous search: Burdett and Judd (1983). One decides ex-ante the number of searches
3. Sequential search: Rothschild (1973)

Models of search (Ellison)

1. Diamond (1971)
2. Heterogenous search

   Stahl (89) shoppers and non-shoppers as in Varian but the non-shoppers have a search cost hence their search is endogenous, moreover there is heterogeneity among the non shoppers.
3. Clearinghouse: you pay a fixed cost to know all the alternatives and if you do not pay it you are directed to a default option. (Baye and Morgan 2001)

Comments

- Undirected search is the idea that the consumer requests quotes randomly, I think it simplifies the problem.
- why is it that we do not observe the Diamond paradox in Varian(1980)?

  - because buyers do not optimally select how much to search. some individuals search all the options and others none. a way of justifying it is that some have an infinite search cost and others zero search cost, hence neither of the two groups changes their search behavior when prices change.

Varian (1980) — Extension with Initial Offers

Let N sellers. $c = 0$ makes initial offers b_i.

The consumer values the good at v.

In stage 2:

$$\bar{v} = v - (v - \min(b_i)) = \min(b_i)$$

where $v - \min(b_i)$ is the outside option.

Sellers set second stage prices p_i, which are latent prices revealed only if the consumer asks them for an external offer.

With probability $\Pr = 1 - \lambda$: the consumer knows p_i $\forall i$, which can be thought of as having no search costs.

There is no equilibrium in pure strategies. The Bertrand logic applies.

Firms set prices according to $F(p)$, being

1. smooth

2. upper bound $\bar{v}$ (monopoly price)

For any p in the support, $p \cdot Q(p) = \bar{v} \cdot Q(\bar{v})$ applies, where

$$Q(p) = \frac{\lambda}{N} + (1-\lambda)\left(1 - F(p)\right)^{N-1}$$

where the first term is the revenue from uninformed (who choose randomly) and the second term is revenue from informed.

Since $F(\bar{v}) = 1$, then

$$F(p) = 1 - \left(\frac{\lambda}{N(1-\lambda)} \cdot \left(\frac{\bar{v}}{p} - 1\right)\right)^{\frac{1}{N-1}}$$

Then the expected profits are:

$$\pi = \bar{v} \cdot Q(\bar{v}) = \bar{v} \cdot \frac{\lambda}{N}$$

Note that 1) nobody buys in the first stage and 2) since profits are increasing on first stage prices then when doing backwards induction we get that $b_i \geq v, \forall i$.

Essentially the problem is that all the buyers in the second stage can observe the first stage bids and make better bids because otherwise the price p_i is greater than the valuation increase $\bar{v}$.

Comments

- The problem of this model is that everyone buys in the second stage. We need the possibility that $\max(c_i) > \bar{V}$ so that some people take the outside option.

- Search costs are not explicitly modeled, buyers have either 0 or ∞ search costs.

Varian (1980) — extension with assymetric costs.

Let N sellers with marginal cost c_i and $c_i < c_{i+1}$ makes initial offers b_i.

The consumer values the good at v.

In stage 2:

$$\bar{v} = v - (v - \min(b_i)) = \min(b_i)$$

where $v - \min(b_i)$ is the outside option. Assume $\bar{v} > c_N$

Sellers set second stage prices p_i, which are latent prices revealed only if the consumer asks them for an external offer.

With probability $\Pr = 1 - \lambda$: the consumer knows p_i $\forall i$, which can be thought of as having no search costs.

Assume there is an equilibrium where all firms play mixed strategies.

Firms set prices according to $F_i(p)$, being

1. smooth

2. upper bound $\bar{v}$ (monopoly price)

then

$$Q_i(p) = \frac{\lambda}{N} + (1-\lambda)\prod_{j\neq i}(1-F_j(p))$$

where the first term is the revenue from uninformed (who choose randomly) and the second term is revenue from informed.

For any p in the support, $(p-c_i)\cdot Q_i(p) = (\bar{v}-c_i)\cdot Q_i(\bar{v})$ applies.

Moreover since $F_i(\bar{v}) = 1, \forall i$ we have:

$$(p-c_i)\cdot Q_i(p) = (\bar{v}-c_i)\cdot Q_i(\bar{v})$$

$$(p-c_i)\cdot\left[\frac{\lambda}{N} + (1-\lambda)\prod_{j\neq i}(1-F_j(p))\right] = (\bar{v}-c_i)\cdot\frac{\lambda}{N}$$

$$(p-c_i)\cdot\left[(1-\lambda)\prod_{j\neq i}(1-F_j(p))\right] = \frac{\lambda}{N}(\bar{v}-p)$$

$$\prod_{j\neq i}(1-F_j(p)) = \frac{\lambda}{(1-\lambda)N}\frac{\bar{v}-p}{p-c_i}$$

Take the previous equation for firms m and n and divide them, then:

$$\frac{1-F_m(p)}{1-F_n(p)} = \frac{p-c_m}{p-c_n}$$

Note that if

There is no equilibrium in pure strategies. The Bertrand logic applies.

Since $F(\bar{v}) = 1$, then

$$F(p) = 1 - \left(\frac{\lambda}{N(1-\lambda)}\cdot\left(\frac{\bar{v}}{p}-1\right)\right)^{\frac{1}{N-1}}$$

Then the expected profits are:

$$\pi = \bar{v}\cdot Q(\bar{v}) = \bar{v}\cdot\frac{\lambda}{N}$$

Note that 1) nobody buys in the first stage and 2) since profits are increasing on first stage prices then when doing backwards induction we get that $b_i \geq v, \forall i$.

Essentially the problem is that all the buyers in the second stage can observe the first stage bids and make better bids because otherwise the price p_i is greater than the valuation increase $\bar{v}$.

First let's solve Varian with assymetric costs for a duopoly with $c_1 < c_2$.
Claim 1: $F_i(p)$ has support $[c_2, \bar{v}]$ and is smooth
Since $F_i(\bar{v}) = 1$ we have:

$$\begin{aligned}
(p - c_i) \cdot Q_i(p) &= (\bar{v} - c_i) \cdot Q_i(\bar{v}) \\
(p - c_i) \cdot \left[\frac{\lambda}{2} + (1-\lambda)(1 - F_j(p))\right] &= (\bar{v} - c_i) \cdot \frac{\lambda}{2} \\
(p - c_i)\left[(1-\lambda)(1 - F_j(p))\right] &= \frac{\lambda}{2}(\bar{v} - p) \\
1 - F_j(p) &= \frac{\lambda}{(1-\lambda)2}\frac{\bar{v} - p}{p - c_i}
\end{aligned}$$

Claim: $\bar{p}_1 = \bar{p}_2 = \bar{v}$
Assume $\bar{p}_1 < \bar{p}_2$
I THINK THAT SOLVING THE MODEL FOR ASSYMETRIC COSTS IS EQUIVALENT TO SOLVING THE MODEL FOR FIRM SPECIFIC v_i, meaning that putting the heterogeneity in the firm cost or in the the buyer preferences is isomorphic.